Title: Coulomb Branch and the Moduli Space of Instantons

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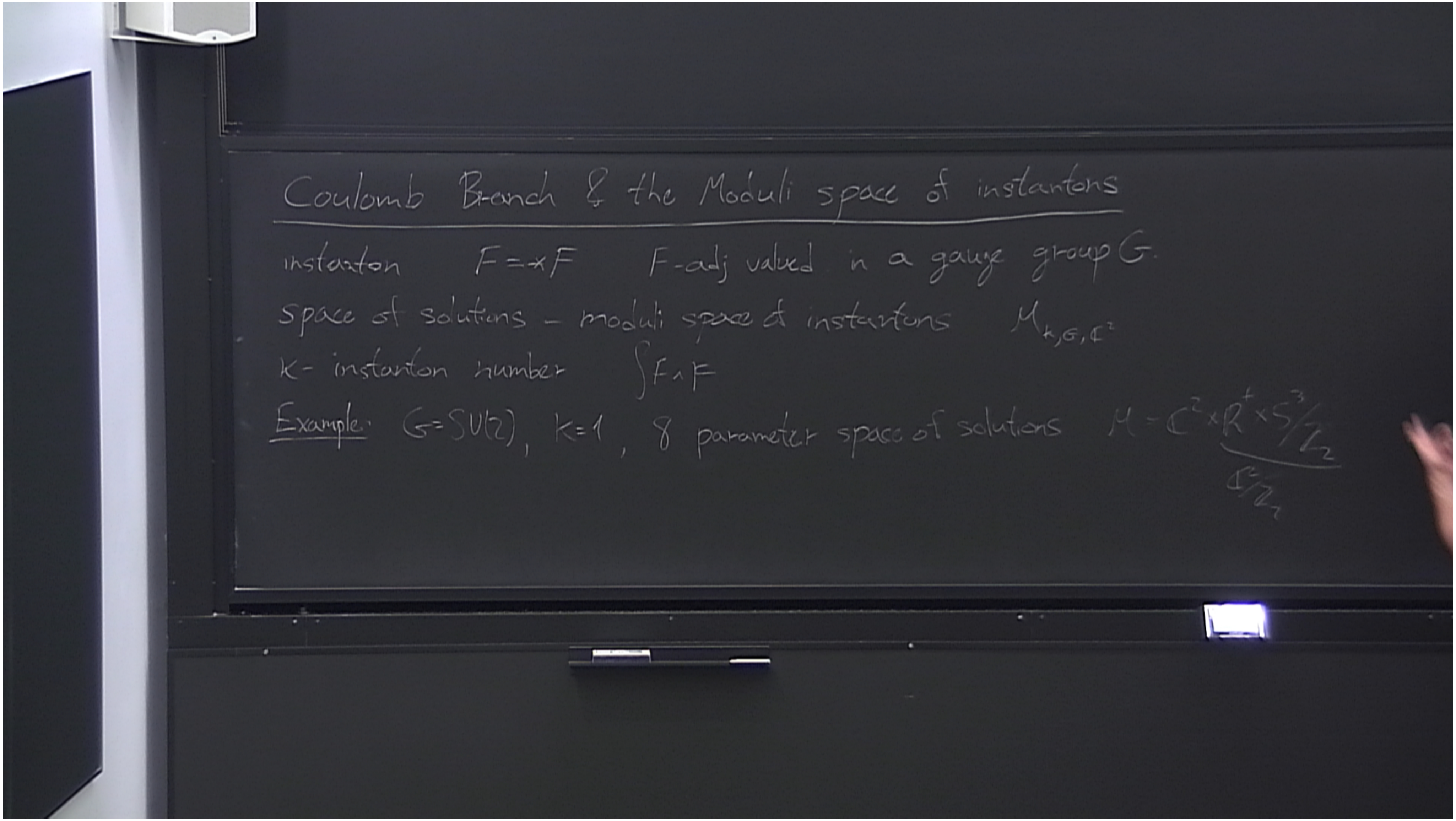
Abstract: <p>The moduli space of k G instantons on C^2, where G is a classical gauge group, has a well known HyperKahler quotient formulation known as the ADHM construction. The extension to exceptional groups is an open problem.
In string theory this is realized using a system of branes, and the moduli space of instantons is identified with the Higgs branch of a particular supersymmetric gauge theory with 8 supercharges.
A less known, and less studied aspect of moduli spaces of instantons is that they can be realized as the Coulomb branch of a supersymmetric gauge theory in 2+1 dimensions.
Recent developments on the understanding of the Coulomb branch gives us a nice solution to the problem where G is an exceptional group, thus allowing a systematic study of these moduli spaces which was unavailable so far.
I will discuss these developments, and present the corresponding quivers, and the Coulomb branch Hilbert Series - the main tool which lead to the recent progress.</p>

Coulomb Branch & the Moduli space of instantons

instanton $\quad F = *F \quad$ F-adj valued in a gauge group G.

space of solutions – moduli space of instantons $\quad \mathcal{M}_{k,G,\mathbb{C}^2}$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k – instanton number $\int F \wedge F$
Example: $G = SU(2)$, $k=1$, 8 parameter space of solutions $\mathcal{M} = \mathbb{C}^2 \times \underbrace{\mathbb{R}^+ \times S^3/\mathbb{Z}_2}_{\mathbb{C}^2/\mathbb{Z}_2}$

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the moduli space is a HK cone of dim $k h^\vee$

↑ dual coxeter number

$$M_{1,G,\mathbb{C}^2} = \mathbb{C}^2 \times \underbrace{\tilde{M}_{1,G,\mathbb{C}^2}}_{\text{reduced instanton moduli space}}$$

H commutant of $SU(2)$ in G

$$\underbrace{\frac{G}{SU(2)\times H} \times \frac{\mathbb{C}^2}{\mathbb{Z}_2}}$$

$$\frac{\mathbb{R}^+ \times S^3/\mathbb{Z}_2}{\mathbb{C}^2/\mathbb{Z}_2}$$

instantons

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$\mathcal{M}_{k,G,\mathbb{C}^2}$

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ADHM: solution to the moduli space of instantons G

$SU(N)$, $Sp(N)$, $SO(N)$ classical groups

Set of algebraic equations HK quotient

$\times \mathbb{R}^+ \times S^3/\mathbb{Z}_2$

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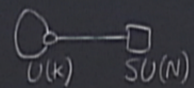
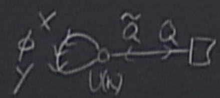
ADHM quiver

$G = SU(N)$



Higgs branch $\longleftrightarrow$ Moduli space of k $SU(N)$ instantons on $\mathbb{C}^2$



$$W = \mathrm{Tr}\,\phi[X,Y] - \mathrm{Tr}\,\phi Q \tilde{Q}$$

F-terms

$$[X,Y] = Q\tilde{Q}$$

$Y \quad X \quad \phi_{k\times k}$

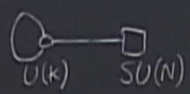
$Q_{k\times N}$

$\tilde{Q}_{N\times k}$

$$\dim(\text{Higgs br}) = kN + k^2 - k^2 = kN$$

ADHM quiver

$G = SU(N)$



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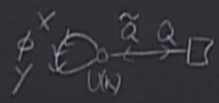

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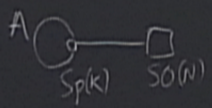
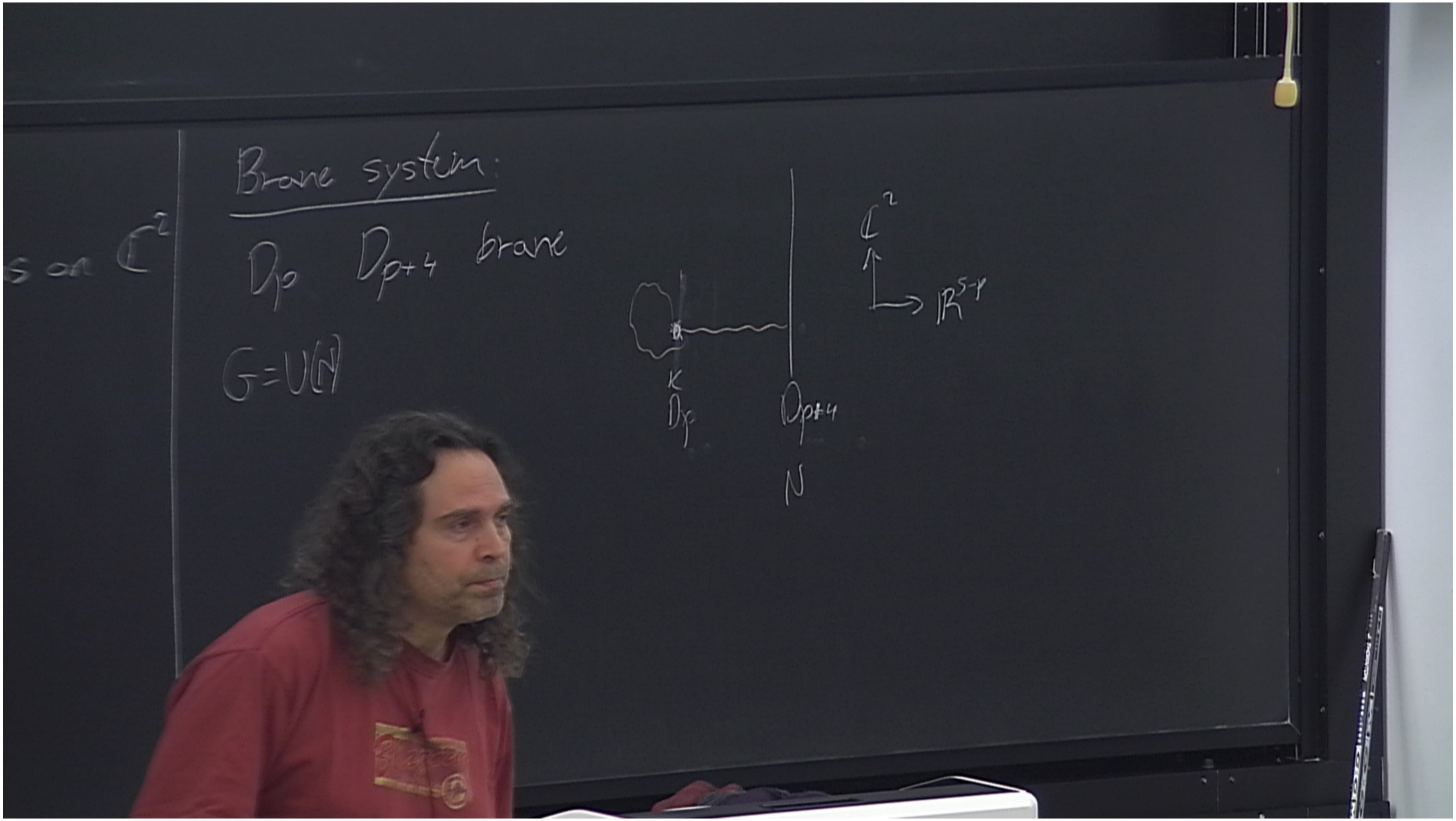
$G = SO(N)$

$Sp(N)$



$Q_{k\times N}$

$\tilde{Q}_{N\times k}$

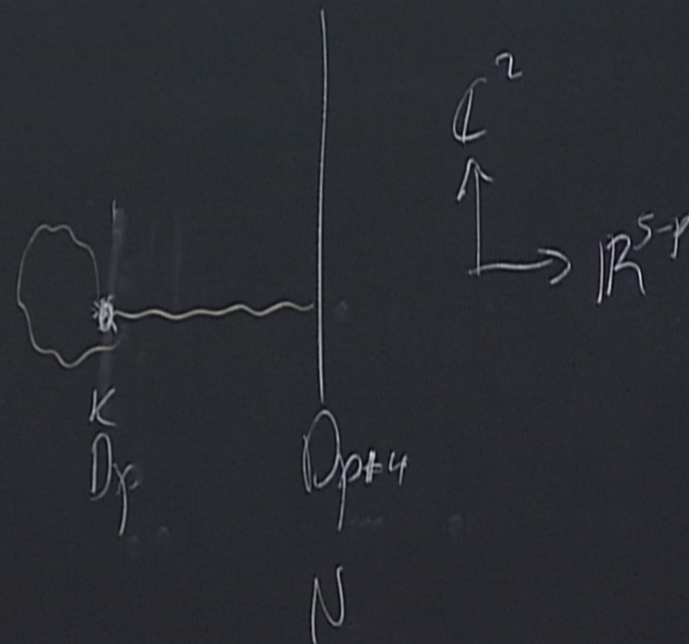

Brane system:
D_p D_{p+4} brane
$G = U(N)$
$\mathbb{C}^2$
$\mathbb{R}^{5-p}$
K
D_p
D_{p+4}
N

$\mathbb{C}^2$

Brane system:

D_p D_{p+4} brane

$G = U(N)$



Example: $G = SU(2)$, $k=1$, 8 parameter space of solutions $\mathcal{M} = \mathbb{C}^2 \times \underbrace{\mathbb{R} \times S^3/\mathbb{Z}_2}_{\mathbb{C}^2/\mathbb{Z}_2}$

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set of algebraic equations HK qu

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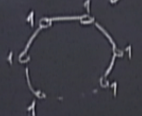
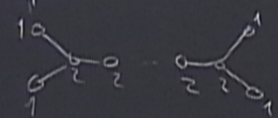
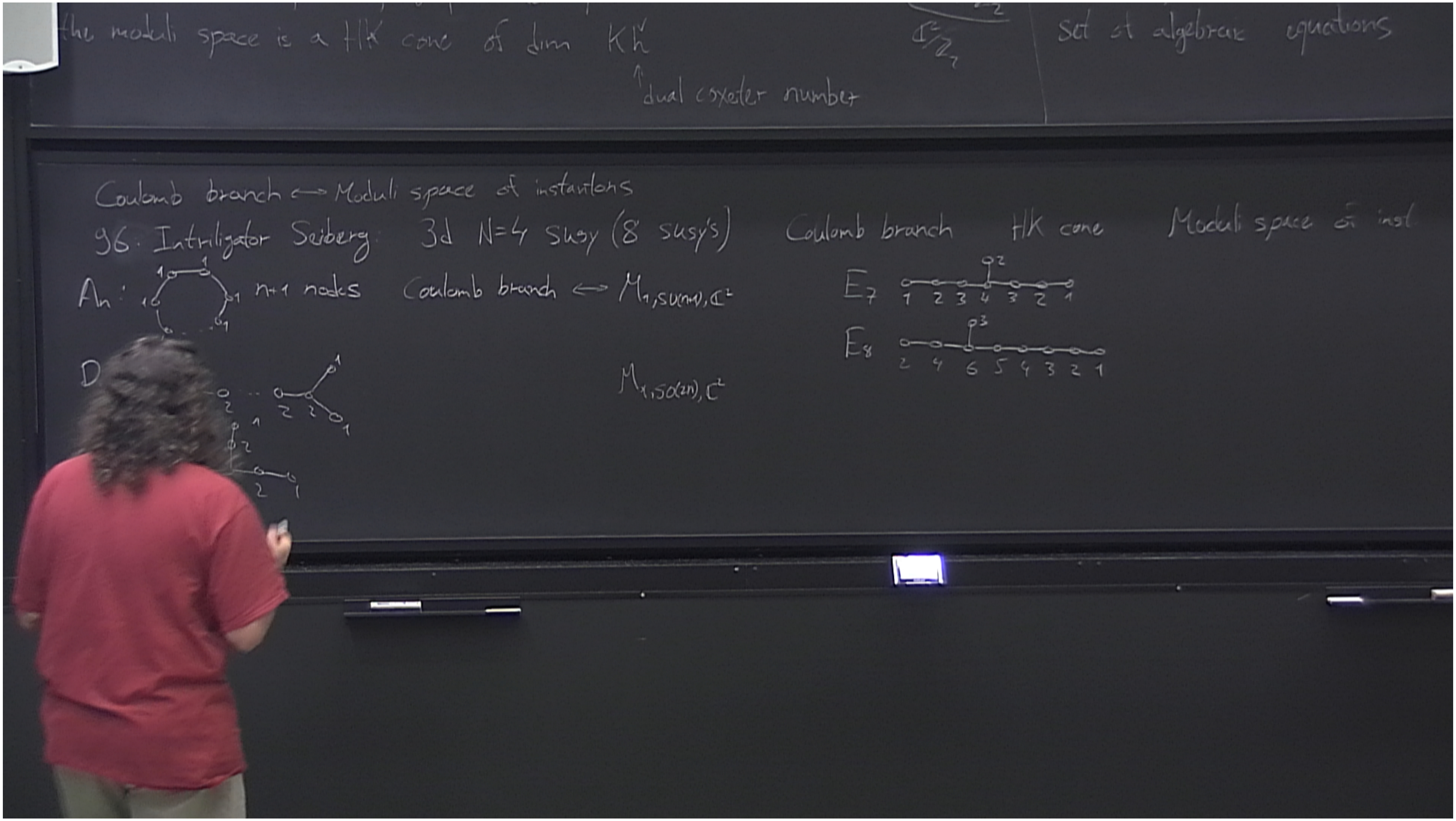
96 Intriligator Seiberg 3d $N=4$ susy (8 susy's) Coulomb branch HK cone Moduli space of inst

A_n: 

$n+1$ nodes Coulomb branch $\longleftrightarrow$ $\mathcal{M}_{1, SU(n+1), \mathbb{C}^2}$

D_n: 

$\mathcal{M}_{1, SO(2n), \mathbb{C}^2}$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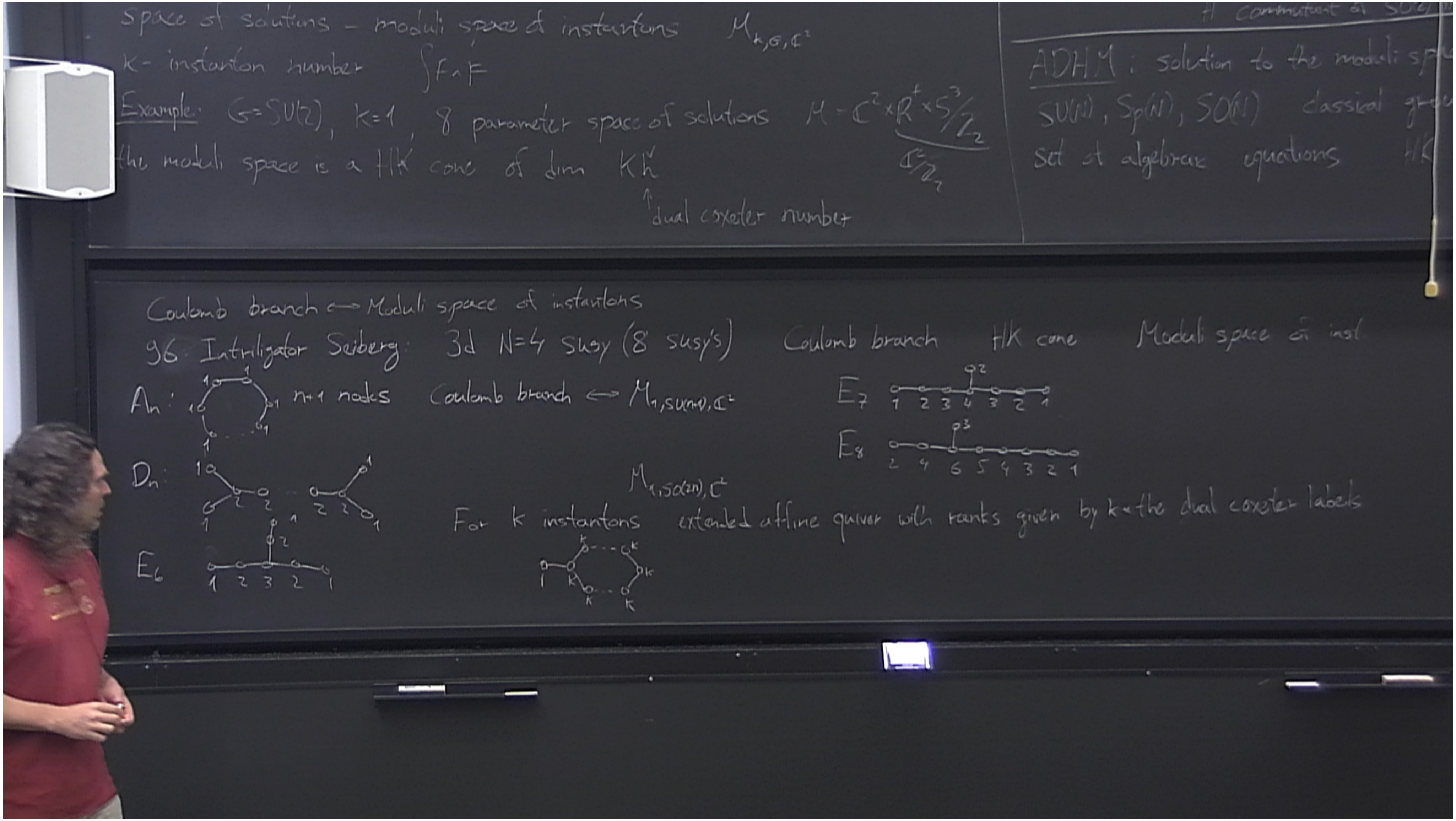
↑ dual coxeter number
C²/Z₂
Set of algebraic equations
Coulomb branch ⟷ Moduli space of instantons
96. Intriligator Seiberg: 3d N=4 susy (8 susy's) Coulomb branch HK cone Moduli space of inst
Aₙ: n+1 nodes Coulomb branch ⟷ M₁,SU(n+1),C²
M₁,SO(2n),C²
E₇
1 2 3 4 3 2 1
E₈
2 4 6 5 4 3 2 1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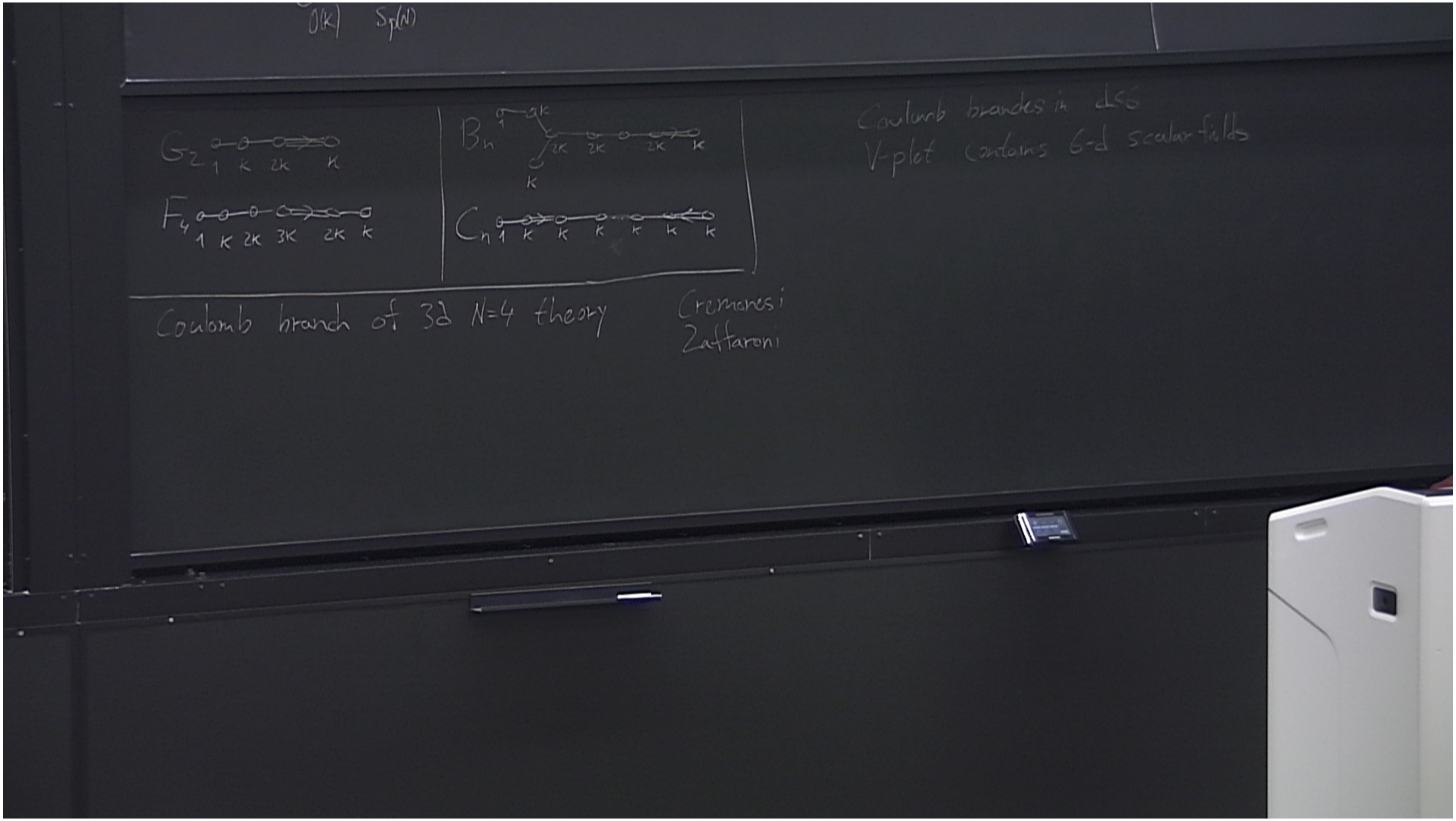
A_n: n+1 nodes Coulomb branch ⟷ $\mathcal{M}_{1,SU(n+1),\mathbb{C}^2}$
D_n:
$\mathcal{M}_{1,SO(2n),\mathbb{C}^2}$
E_6
1 2 3 2 1
E_7
1 2 3 4 3 2 1
E_8
2 4 6 5 4 3 2 1
For k instantons extended affine quiver with ranks given by k × the dual coxeter labels

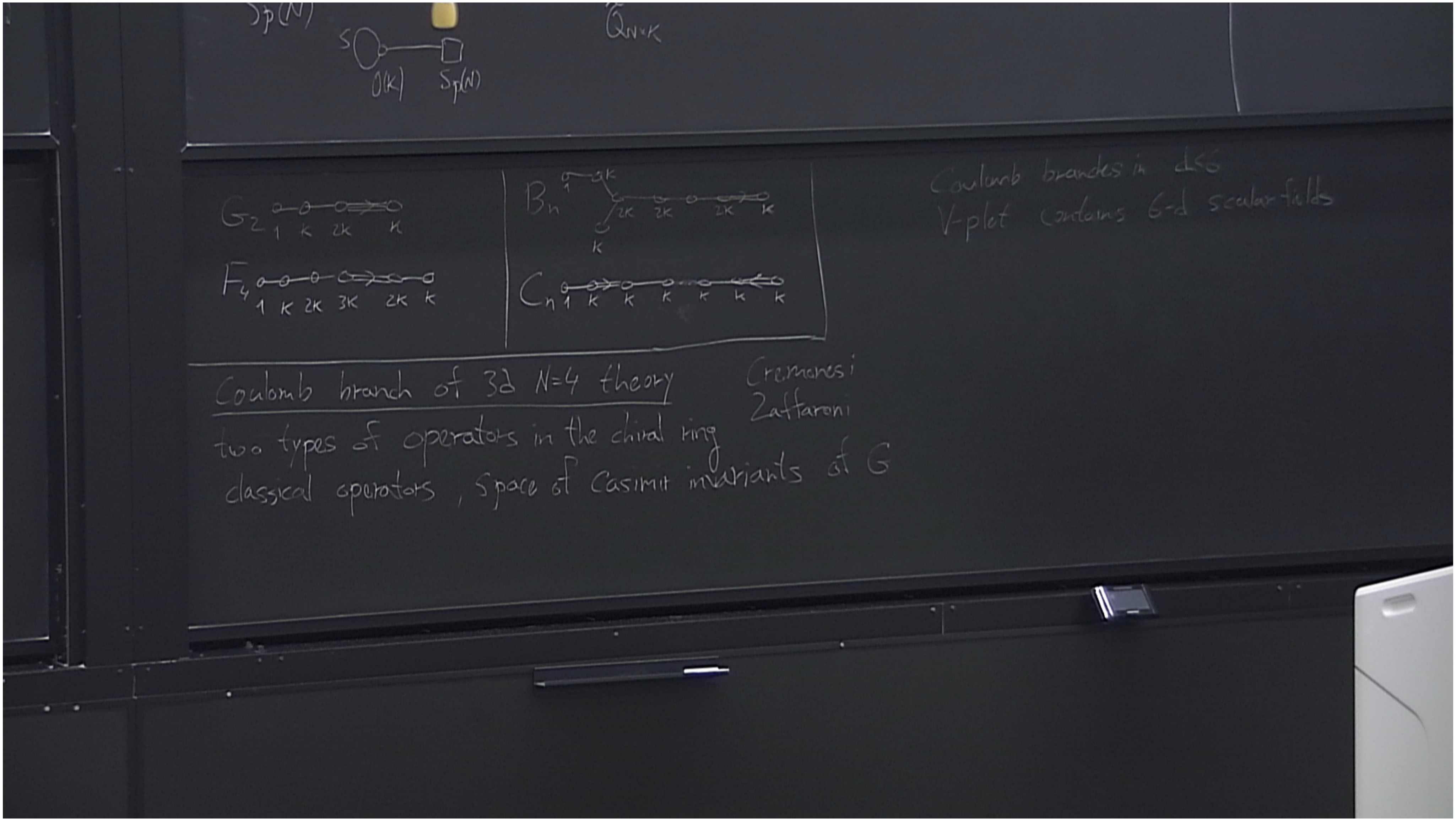
Cremonesi
Ferlito
Mekareeya

Moduli space of inst.

$k \times$ the dual coxeter labels



O(k) Sp(N)
Coulomb branches in d<6
V-plet contains 6-d scalar fields
G2 1 k 2k k
Bn 1 k 2k 2k 2k k k
F4 1 k 2k 3k 2k k
Cn 1 k k k k k k
Coulomb branch of 3d N=4 theory
Cremonesi
Zaffaroni


Sp(N)
O(k) Sp(N)
Q_{N×K}
G2 1 k 2k k
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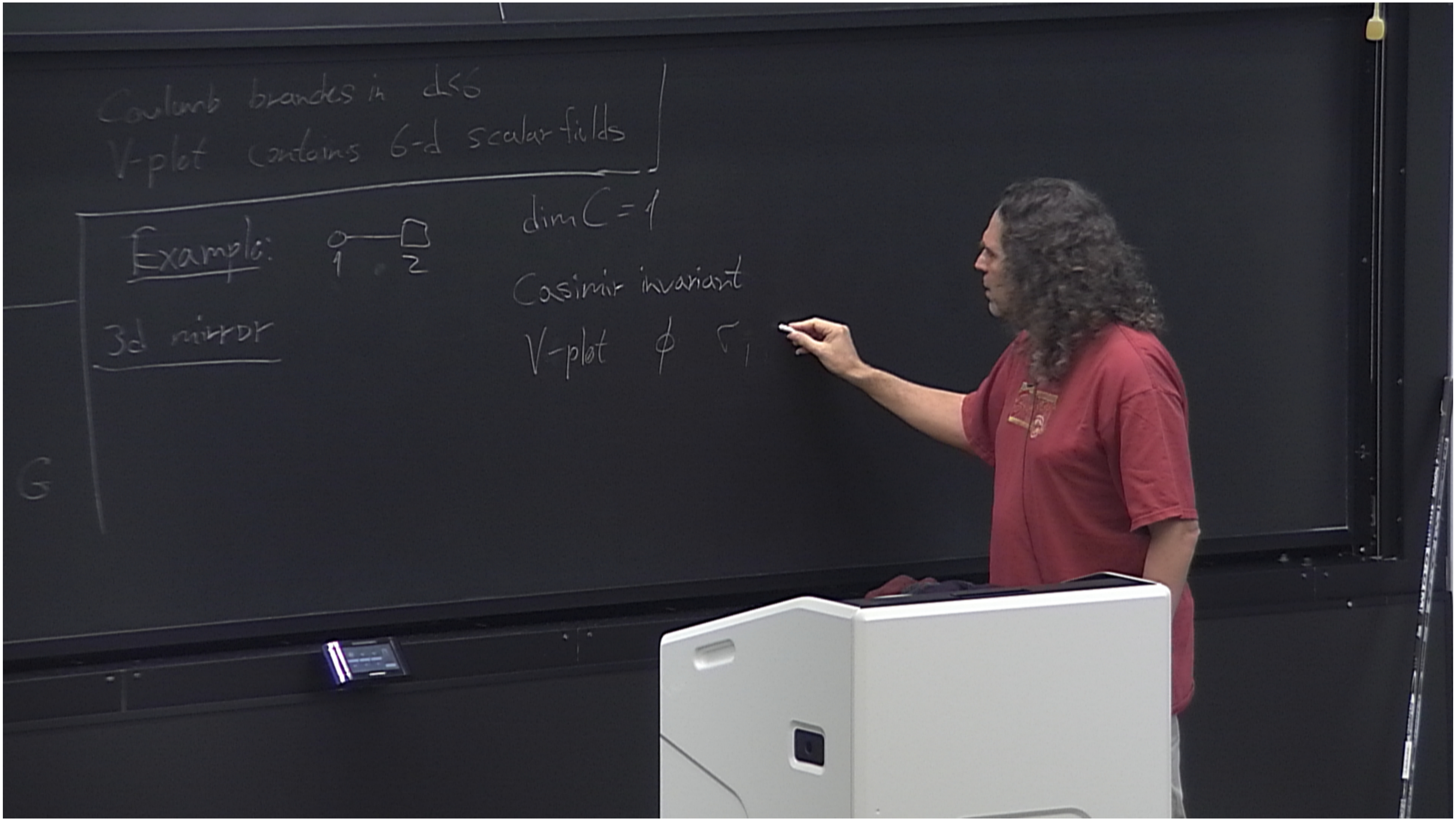
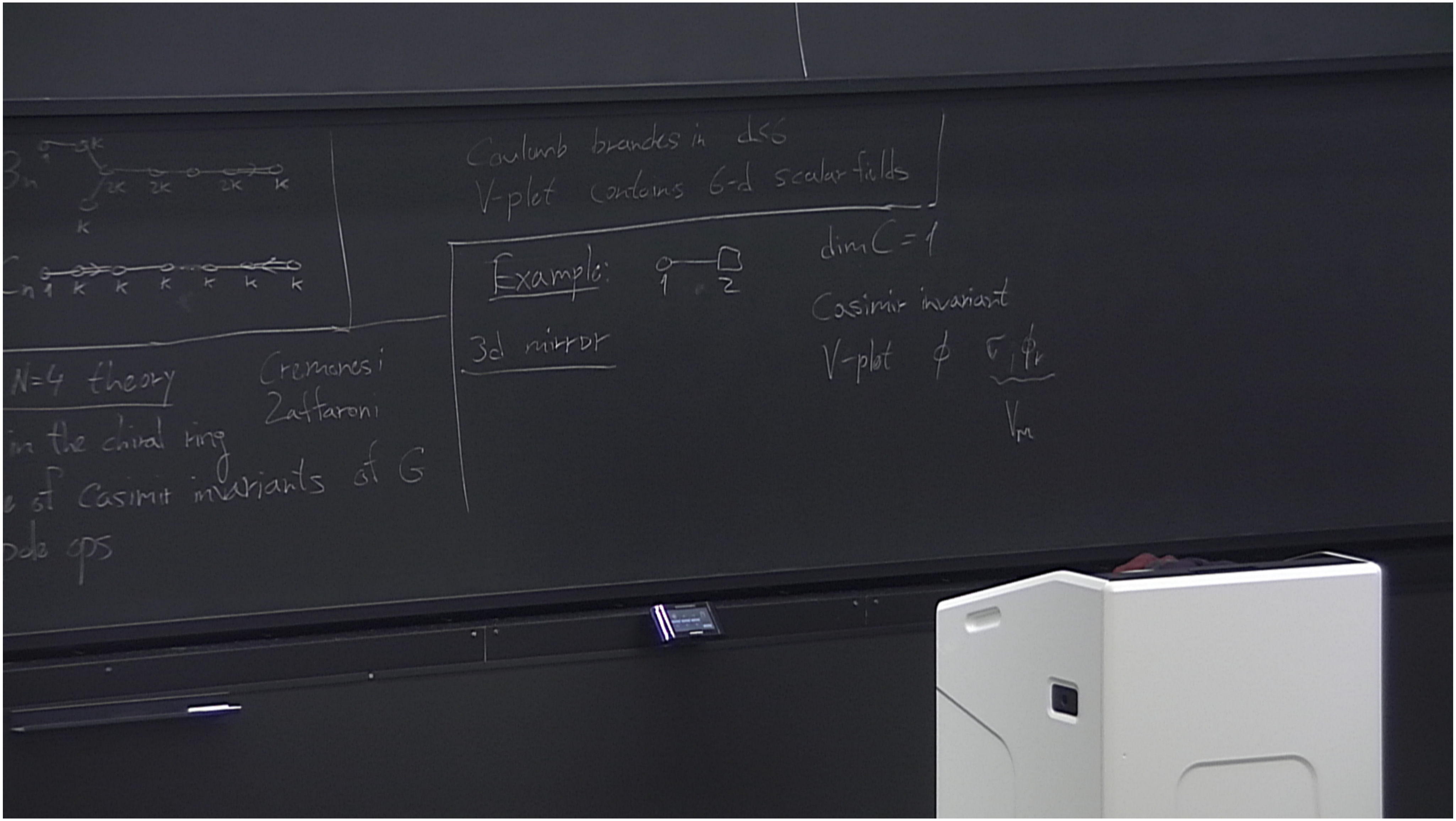
Coulomb branches in d=6
V-plet contains 6-d scalar fields
Coulomb branch of 3d N=4 theory
Cremonesi
Zaffaroni
two types of operators in the chiral ring
classical operators, space of Casimir invariants of G


Coulomb branches in d<6
V-plet contains 6-d scalar fields
Example:
1
2
dim C = 1
Casimir invariant
3d mirror
V-plet ϕ
G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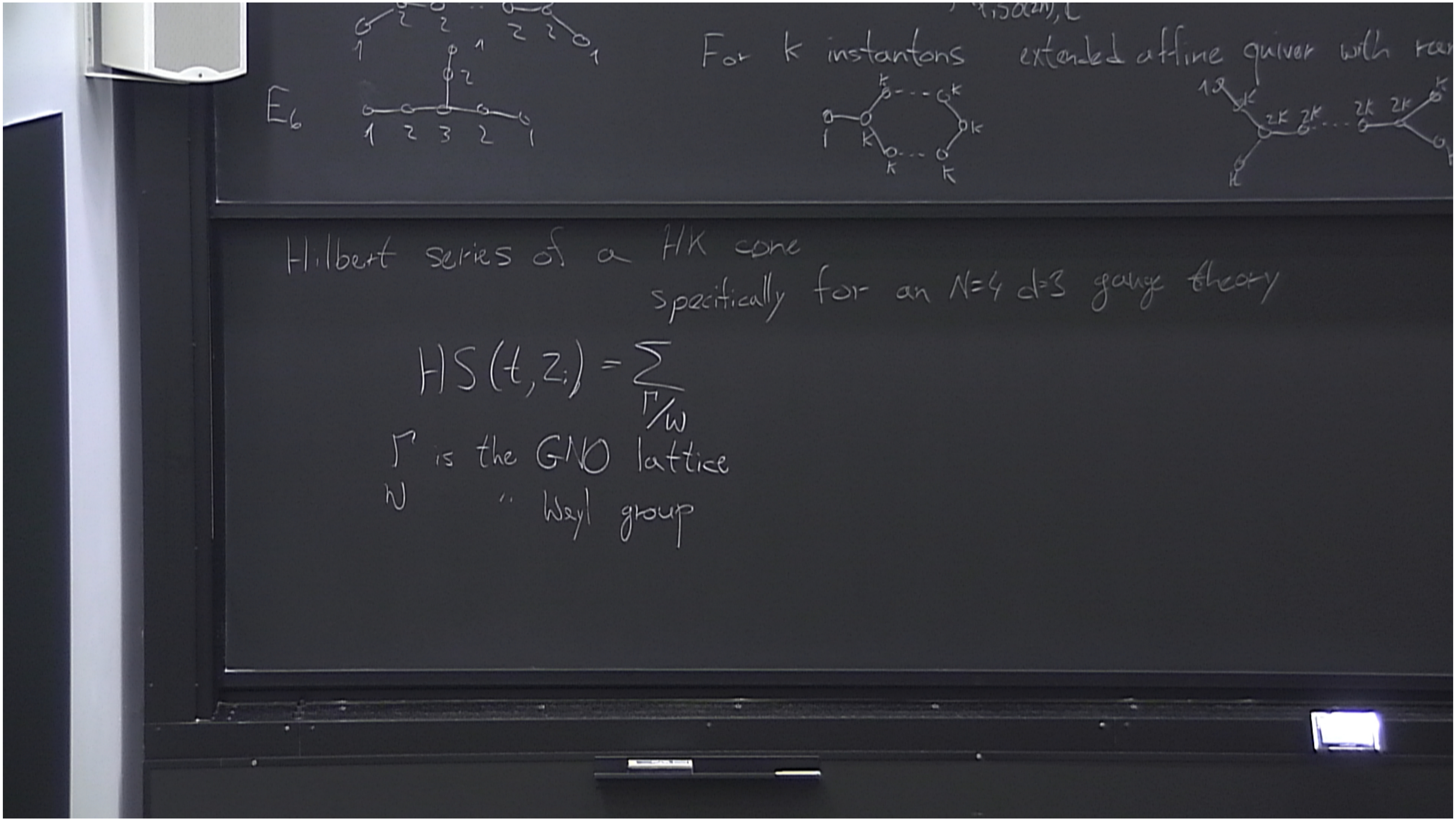
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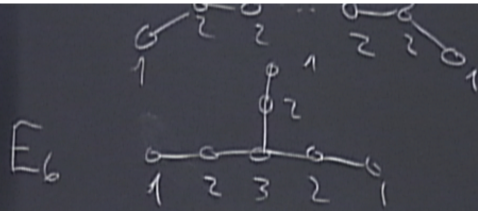
E6
Hilbert series of a HK cone
specifically for an N=4 d=3 gauge theory
HS(t, z_i) = Σ_{Γ/W}
Γ is the GNO lattice
W " Weyl group



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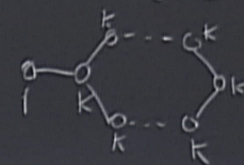
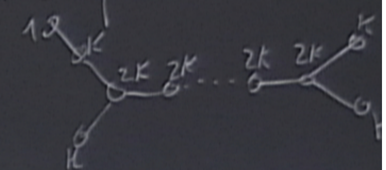




Hilbert series of a HK cone

specifically for an $N=4$ $d=3$ gauge theory

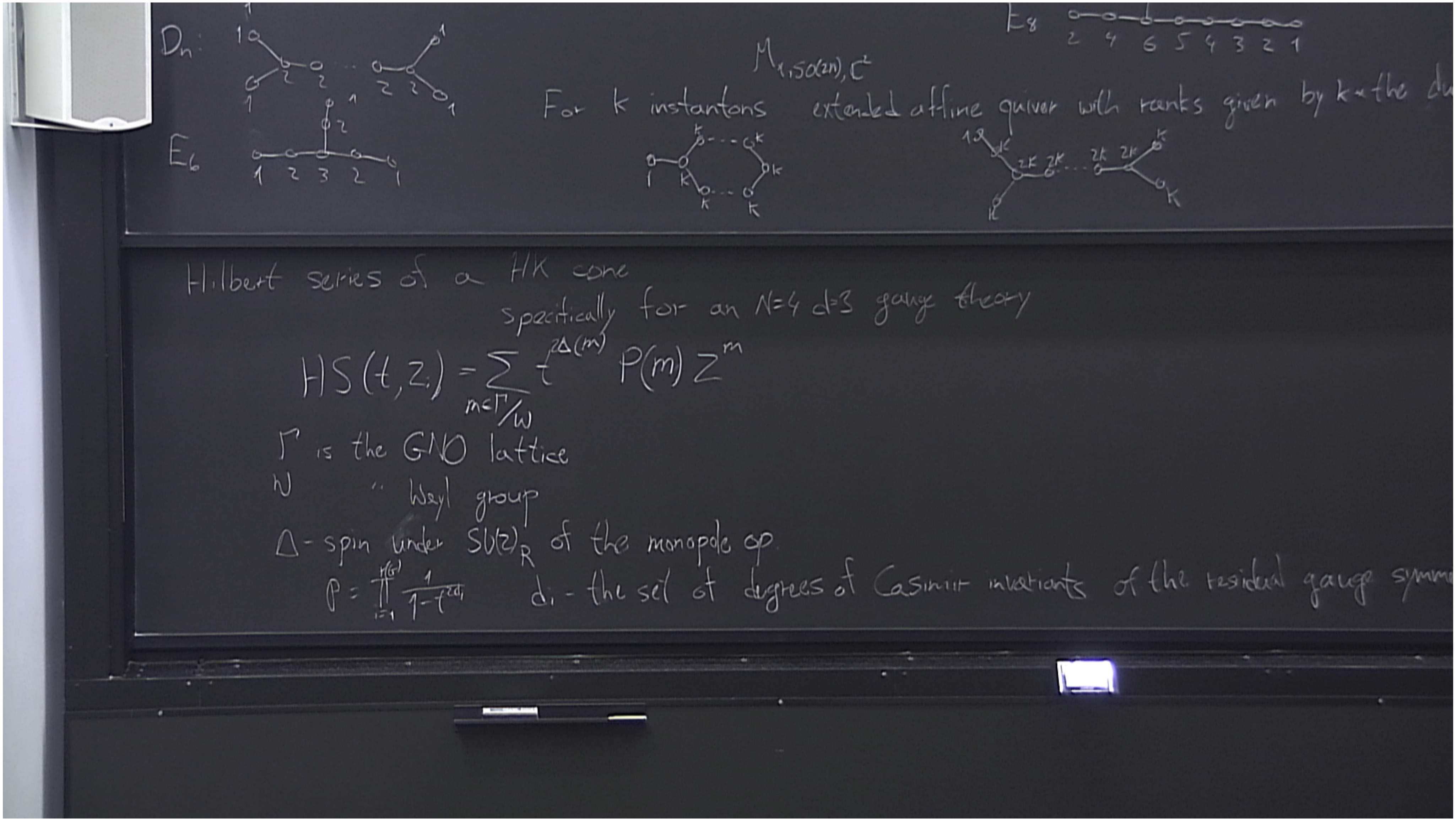
$$HS(t,z_i) = \sum_{m\in\Gamma/W} t^{2\Delta(m)} P(m)$$

Γ is the GNO lattice

W " Weyl group

Δ - spin under $SU(2)_R$ of the monopole op.

$$P = \prod_{i=1}^{r(G)} \frac{1}{1-t^{2d_i}}$$ d_i - the set of degrees of Casimir invariants


D_n:
E_8
$M_{k,SO(2n),\mathbb{C}^2}$
For k instantons extended affine quiver with ranks given by k × the d
E_6
Hilbert series of a HK cone
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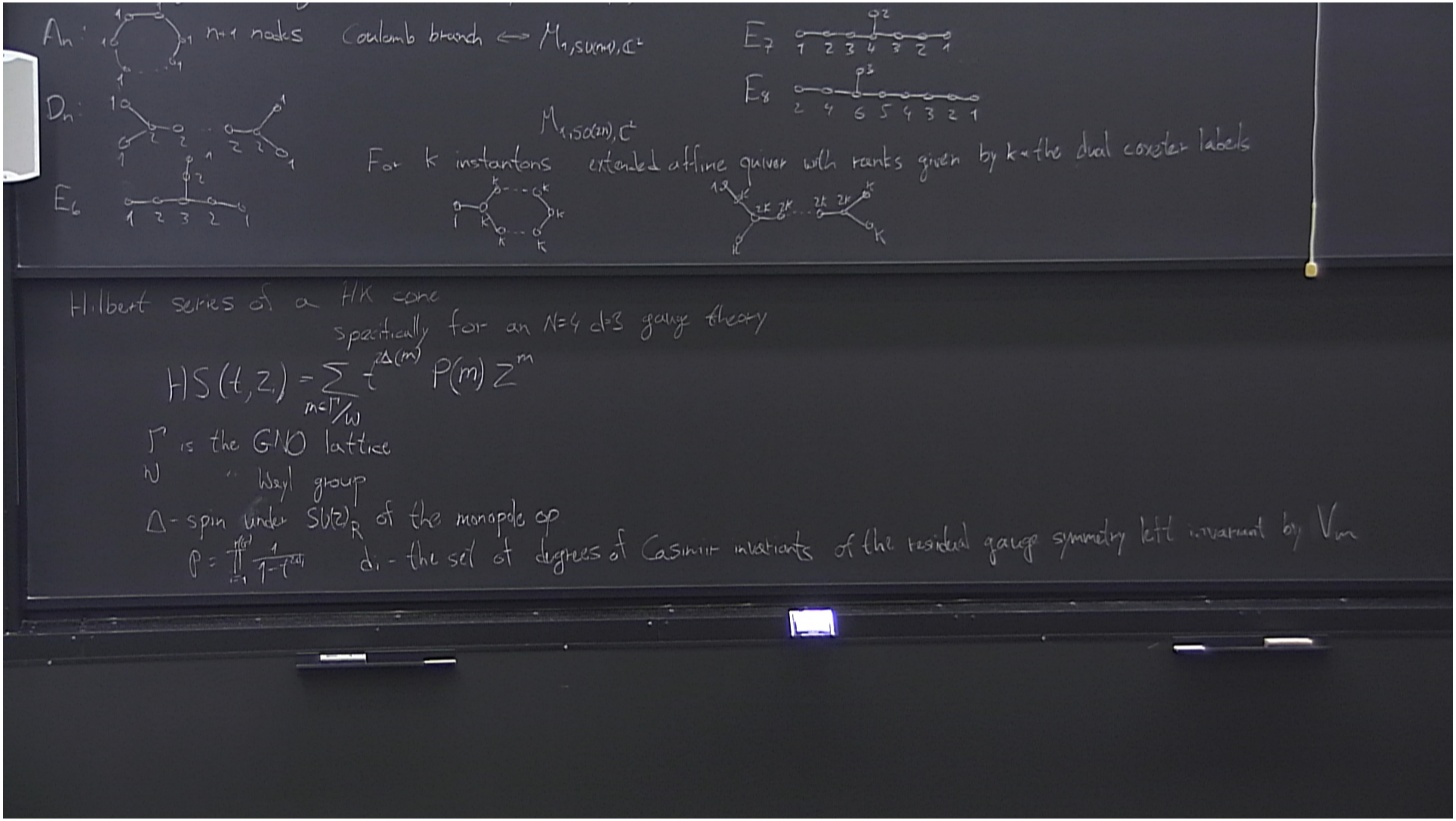

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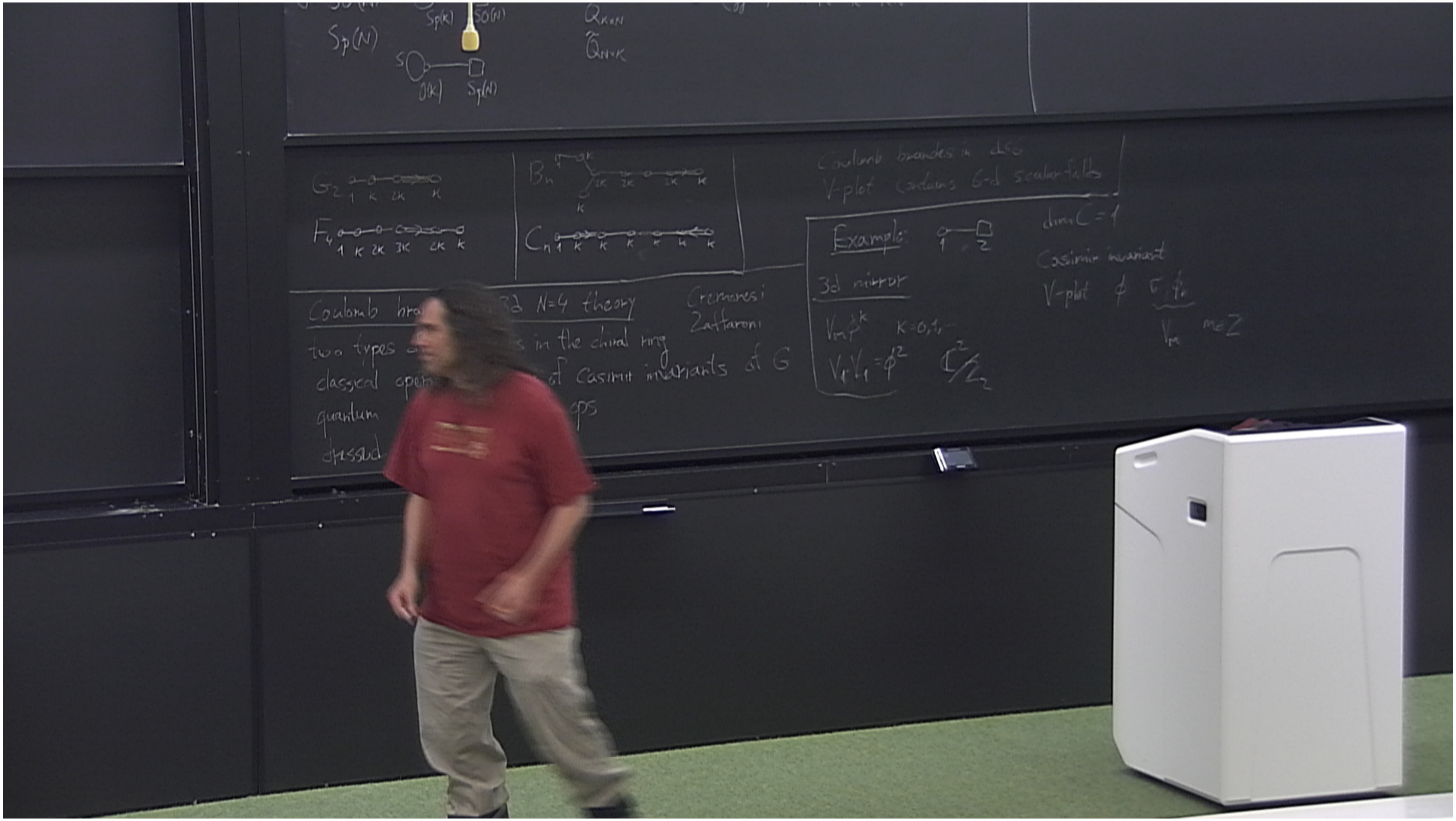
Example
3d mirror
Casimir invariant
3d N=4 theory
Cremonesi
Zaffaroni
in the chiral ring
Casimir invariants of G
two types
classical
quantum
dressed