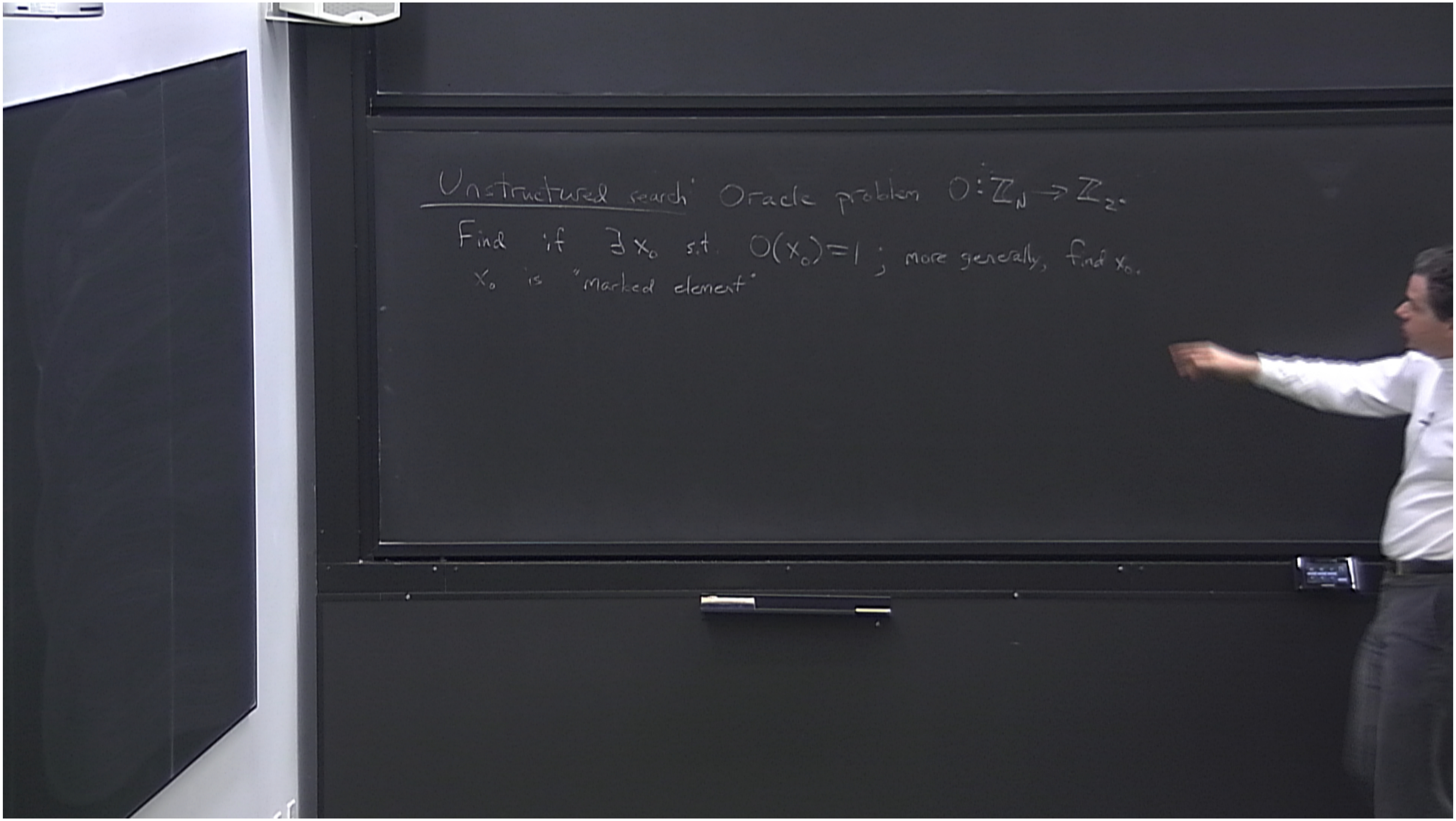
Title: Quantum Information Review-9

Date: Feb 27, 2015 11:30 AM

URL: http://pirsa.org/15020071

Abstract:


Unstructured search: Oracle problem $O: \mathbb{Z}_N \to \mathbb{Z}_2$.
Find if $\exists x_0$ s.t. $O(x_0)=1$; more generally, find x_0.
x_0 is "marked element"

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Abstraction of NP-complete problems

Input potential witness

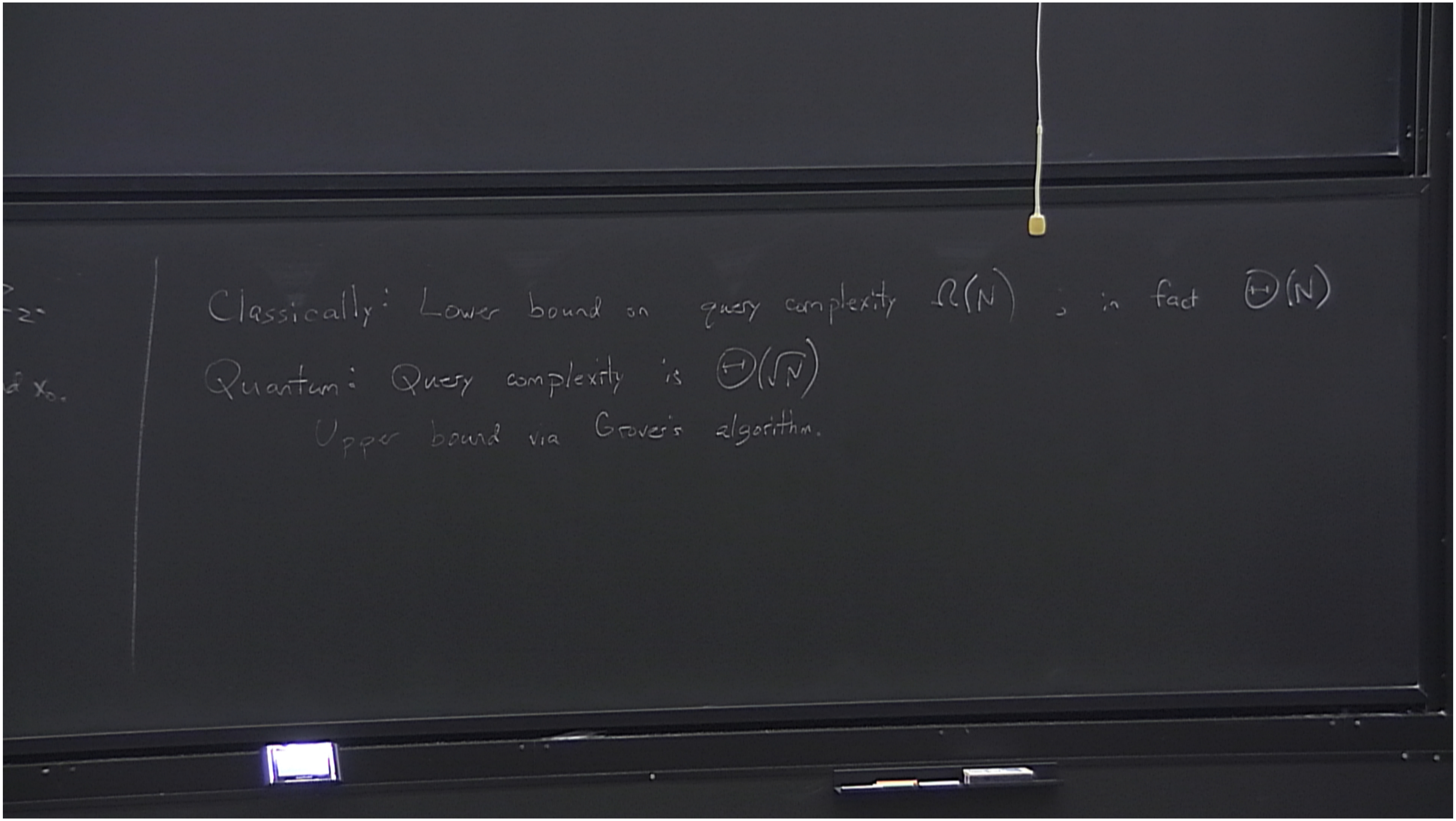
Oracle = verifier function

$z.$

$d\ x_0.$

Classically: Lower bound on query complexity $\Omega(N)$; in fact $\Theta(N)$

Quantum: Query complexity is $\Theta(\sqrt{N})$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Upper bound via Grover's algorithm.

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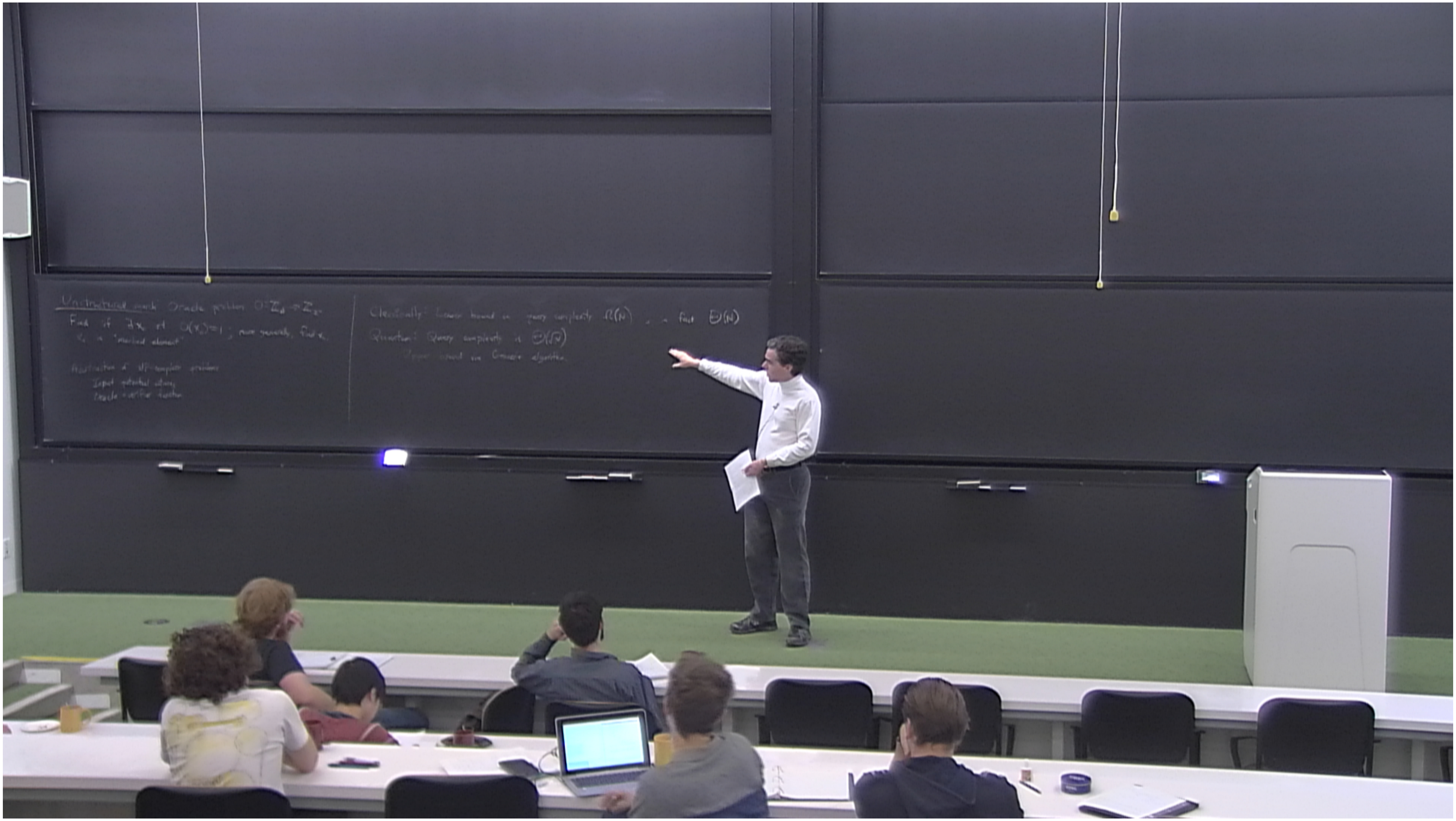
Unstructured search Oracle problem
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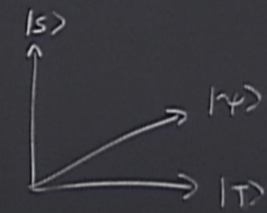
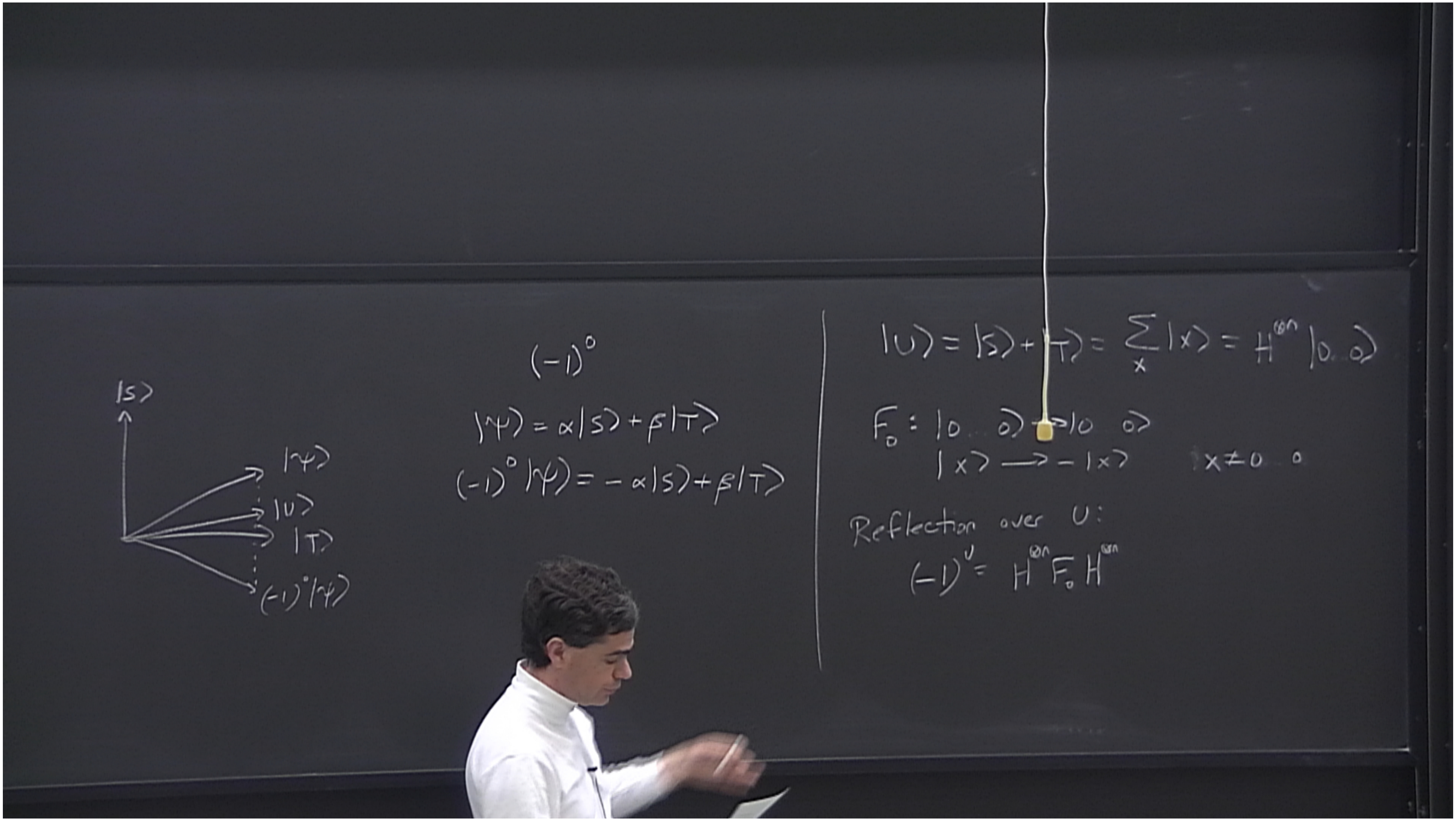
Grover's algorithm w/ 1 marked element x_0:

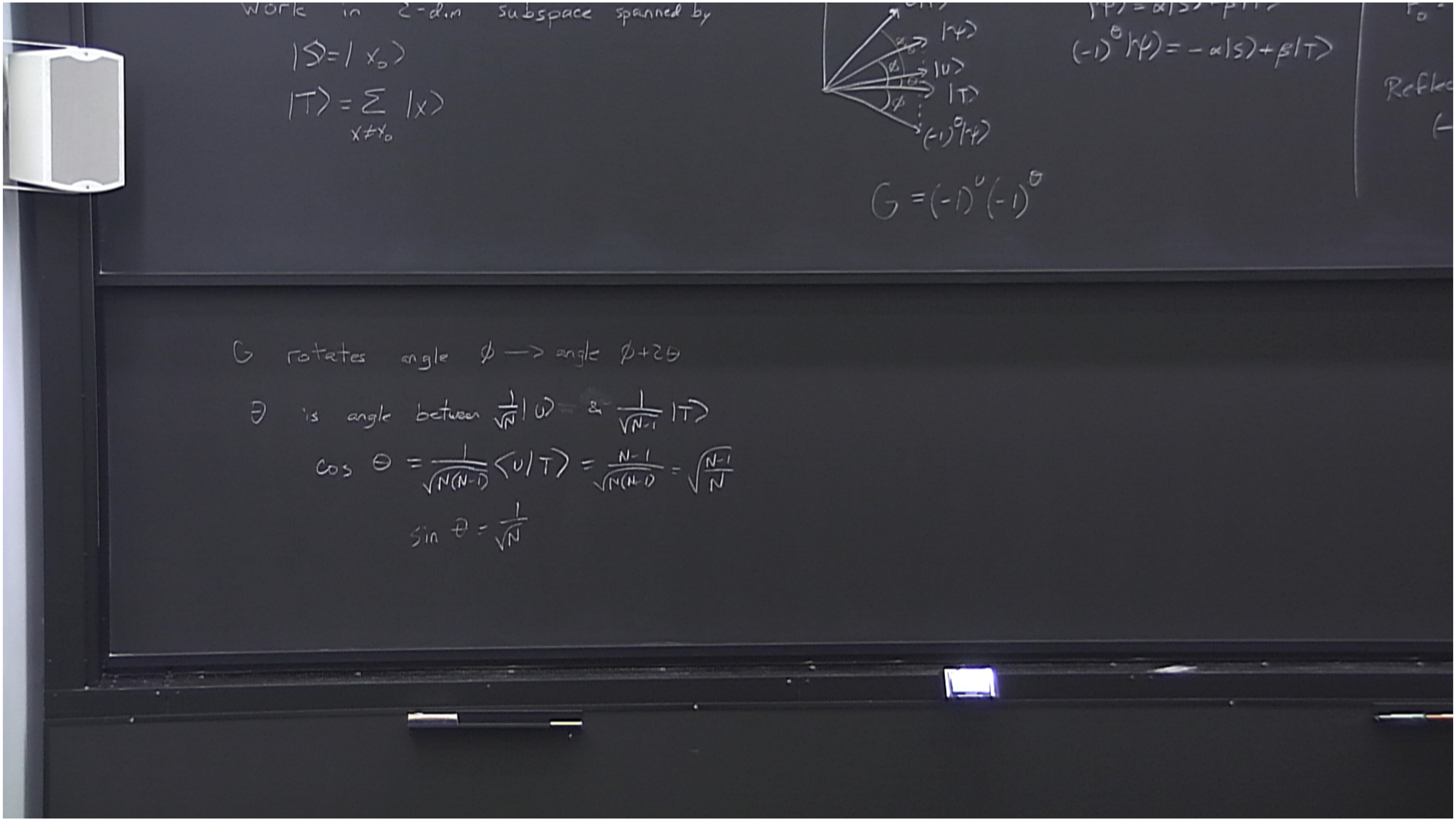
Work in 2-dim subspace spanned by

$$|S\rangle = |x_0\rangle$$

$$|T\rangle = \sum_{x \neq x_0} |x\rangle$$




|S⟩
|ψ⟩
|U⟩
|T⟩
(−1)^0 |ψ⟩
(−1)^0
|ψ⟩ = α|S⟩ + β|T⟩
(−1)^0 |ψ⟩ = −α|S⟩ + β|T⟩
|U⟩ = |S⟩ + |T⟩ = Σ_x |x⟩ = H^⊗n |0…0⟩
F_0 : |0…0⟩ → |0…0⟩
|x⟩ → −|x⟩   x ≠ 0…0
Reflection over U:
(−1)^U = H^⊗n F_0 H^⊗n


Work in 2-dim subspace spanned by
$|S\rangle = |x_0\rangle$
$|T\rangle = \sum_{x \neq x_0} |x\rangle$
$|\psi\rangle$
$|U\rangle$
$|T\rangle$
$(-1)^{\theta}|\psi\rangle$
$(-1)^{\theta}|\psi\rangle = -\alpha|S\rangle + \beta|T\rangle$
$G = (-1)^{U}(-1)^{\theta}$
G rotates angle $\phi \longrightarrow$ angle $\phi + 2\theta$
θ is angle between $\frac{1}{\sqrt{N}}|U\rangle$ & $\frac{1}{\sqrt{N-1}}|T\rangle$
$\cos\theta = \frac{1}{\sqrt{N(N-1)}}\langle U|T\rangle = \frac{N-1}{\sqrt{N(N-1)}} = \sqrt{\frac{N-1}{N}}$
$\sin\theta = \frac{1}{\sqrt{N}}$

Grover's algorithm:

1) Initialize to $|\psi_0\rangle = |u\rangle = H^{\otimes n}|0\ldots 0\rangle$ angle $\phi_0 = \theta$

2) Apply $G = (-1)^u(-1)^\theta$ M times. Each application of G uses 1 oracle call. After kth application of G

$$|\psi_k\rangle = G|\psi_{k-1}\rangle, \text{ angle } \phi_k = \phi_{k-1} + 2\theta = (2k+1)\theta$$

3) Measure to get a candidate x_0

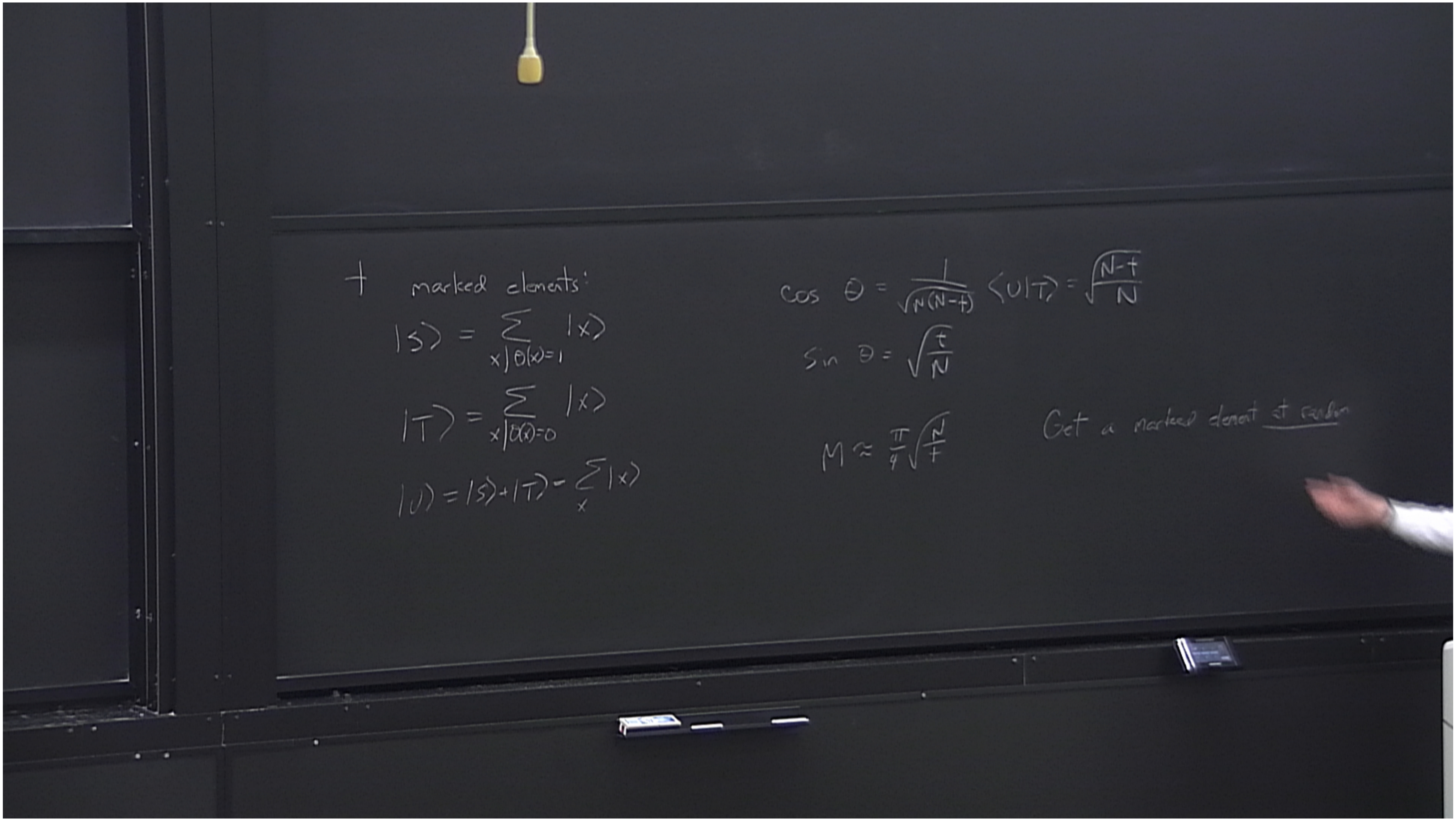
4) Verify x_0 with 1 oracle call.

$$|\psi_k\rangle = \cos((2k+1)\theta)\,|T\rangle + \sin((2k+1)\theta)\,|S\rangle$$

$$M \approx \left\lfloor \frac{\pi}{4}\sqrt{N} \right\rfloor$$

$$(2M+1)\theta = \frac{\pi}{2} \Rightarrow M = \frac{\frac{\pi}{2}\frac{1}{\theta} - 1}{2}$$

$$\sin^{-1}\left(\frac{1}{\sqrt{N}}\right)$$


t marked elements:
$|S\rangle = \sum_{x|\theta(x)=1} |x\rangle$
$|T\rangle = \sum_{x|\theta(x)=0} |x\rangle$
$|U\rangle = |S\rangle + |T\rangle = \sum_x |x\rangle$
$\cos\theta = \frac{1}{\sqrt{N(N-t)}} \langle U|T\rangle = \sqrt{\frac{N-t}{N}}$
$\sin\theta = \sqrt{\frac{t}{N}}$
$M \approx \frac{\pi}{4}\sqrt{\frac{N}{t}}$
Get a marked element at random

t marked elements:

$$|S\rangle = \sum_{x \mid \theta(x)=1} |x\rangle$$

$$|T\rangle = \sum_{x \mid \theta(x)=0} |x\rangle$$

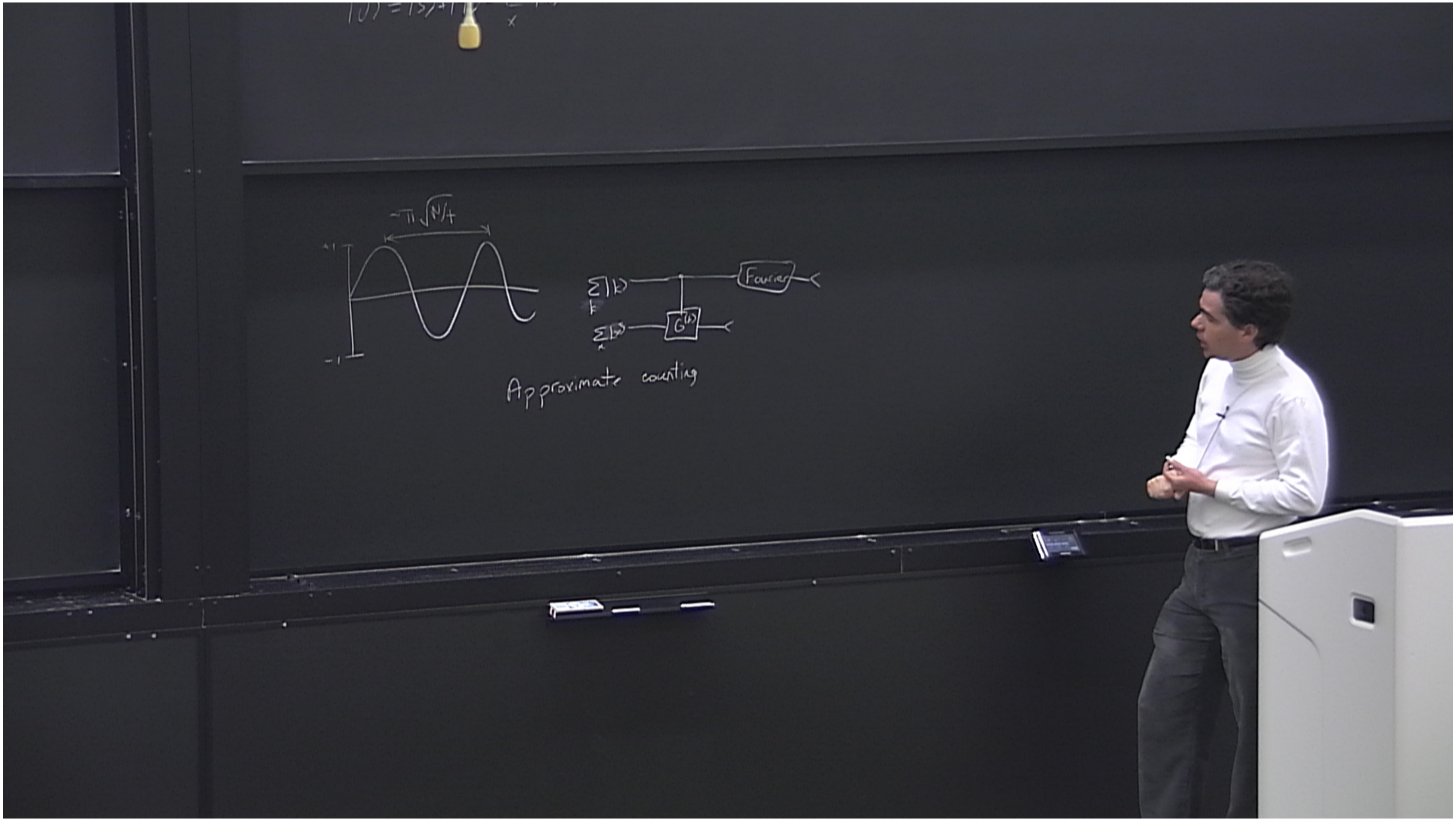
$$|U\rangle = |S\rangle + |T\rangle = \sum_{x} |x\rangle$$

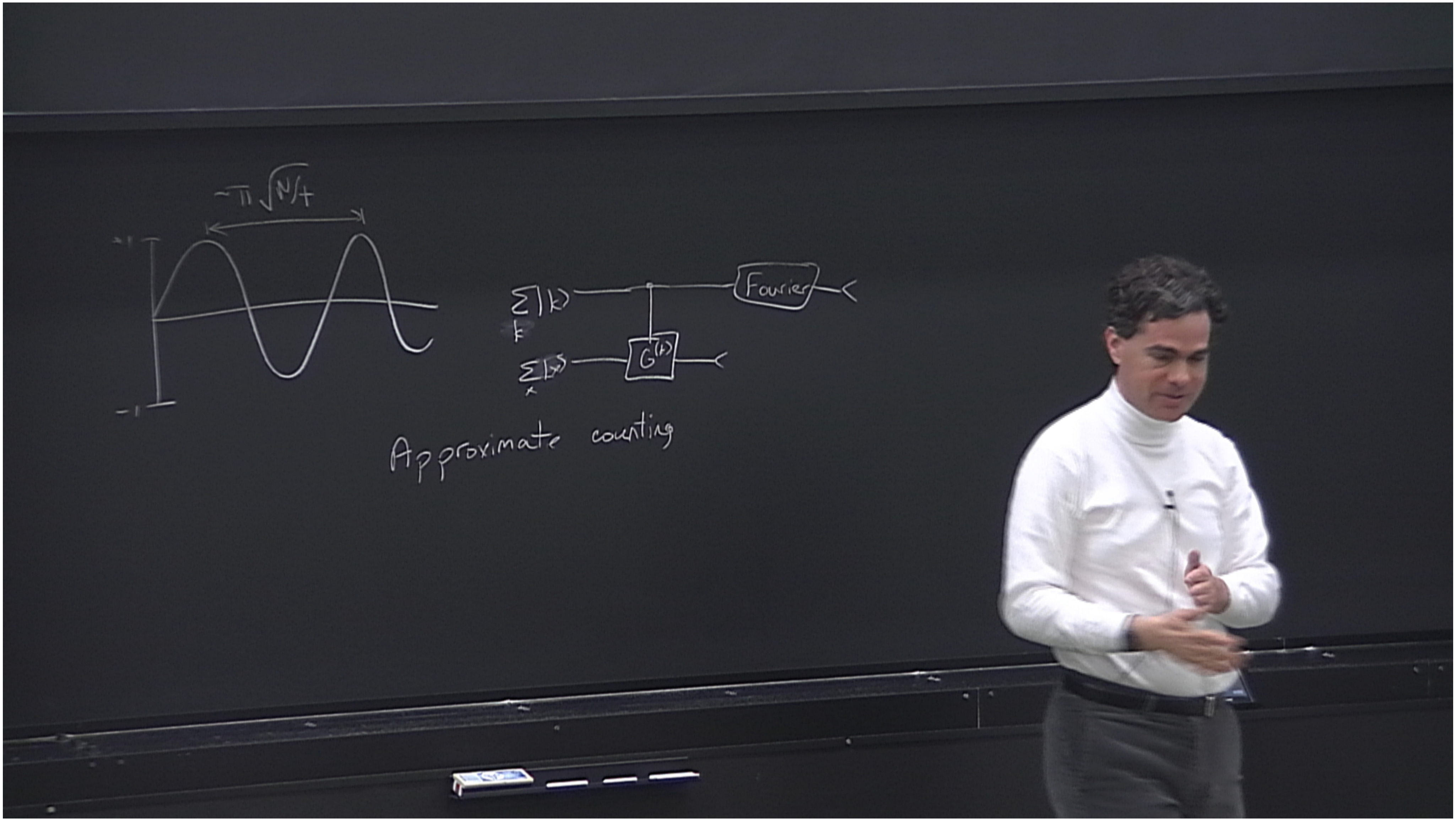
$$\cos\theta = \frac{1}{\sqrt{N(N-t)}} \langle U|T\rangle = \sqrt{\frac{N-t}{N}}$$

$$\sin\theta = \sqrt{\frac{t}{N}}$$

$$M \approx \frac{\pi}{4}\sqrt{\frac{N}{t}}$$

Get a marked element <u>at random</u>


~π √(N/t)
+1
−1
Σ|k⟩
k
Σ|x⟩
x
G^(k)
Fourier
Approximate counting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