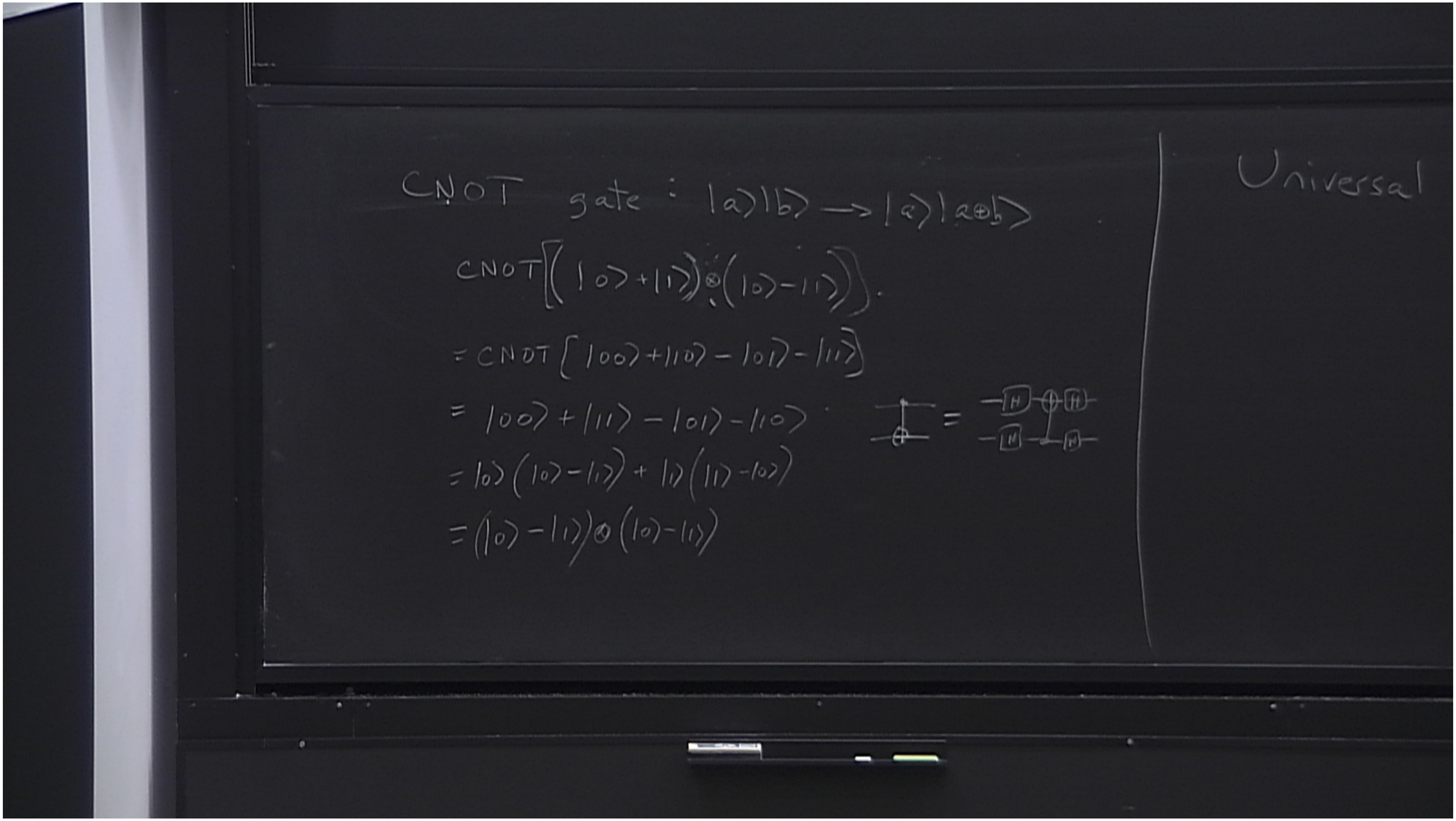
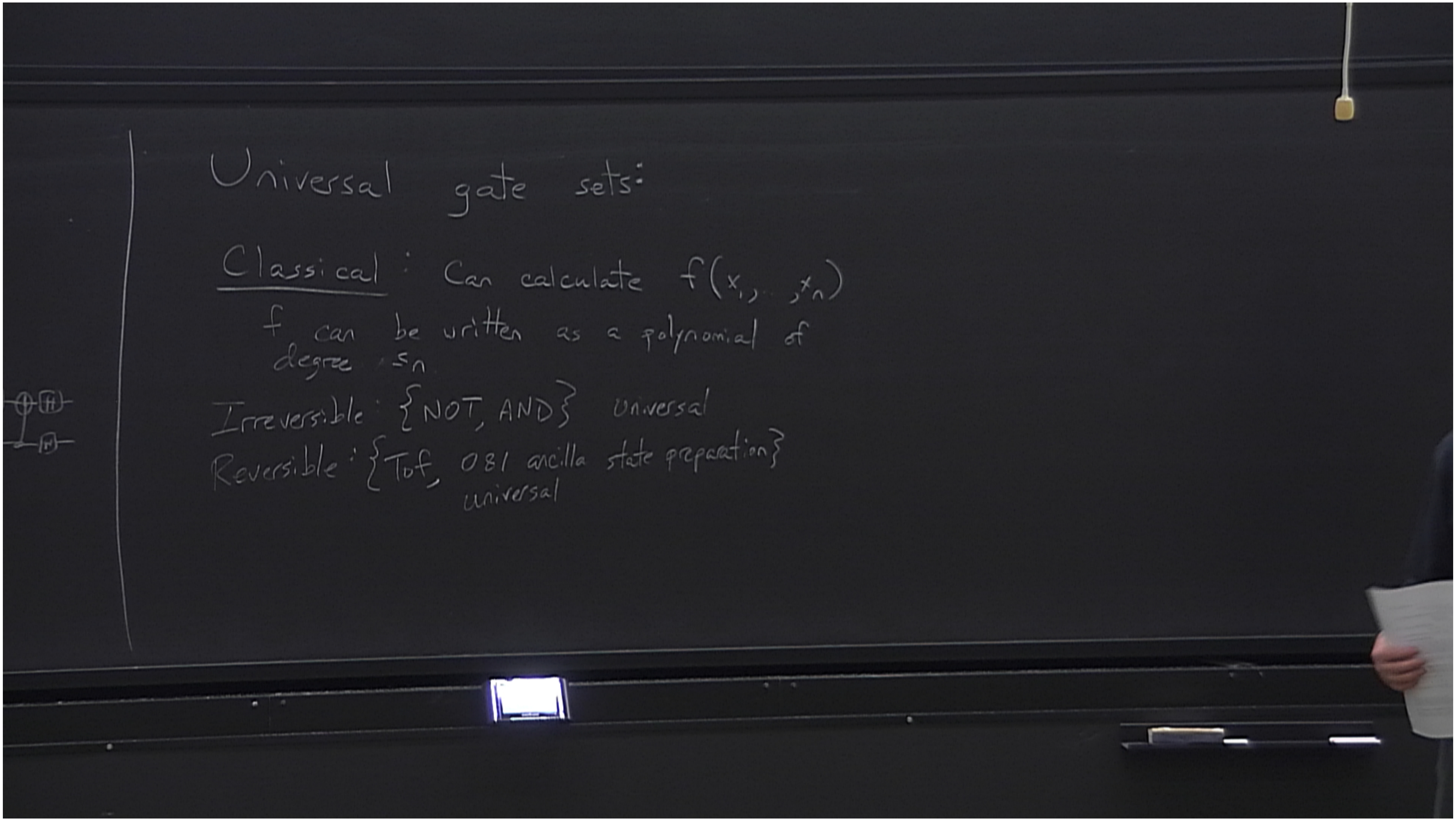
Title: Quantum Information Review-2

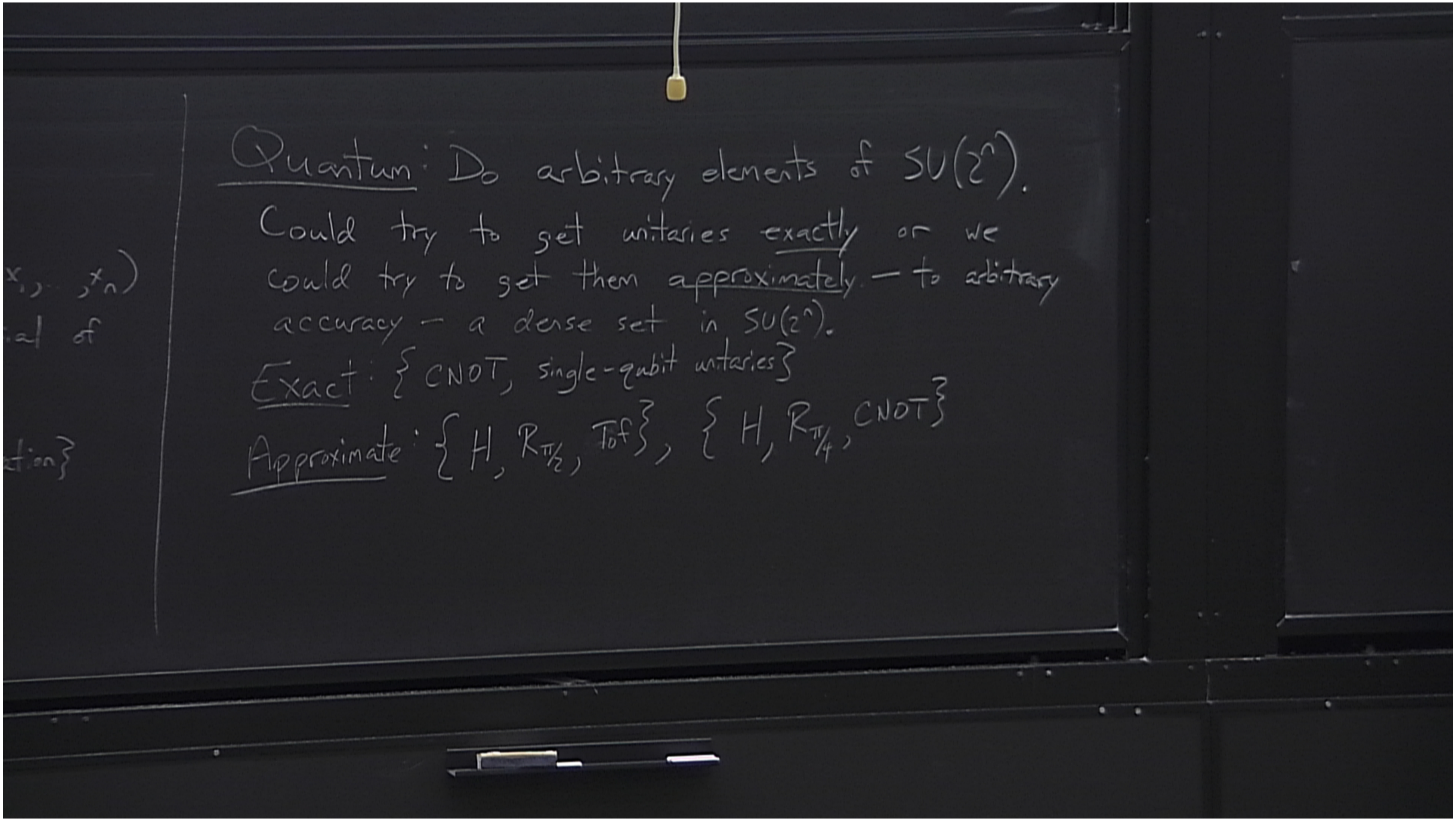
Date: Feb 18, 2015 11:30 AM

URL: http://pirsa.org/15020062

Abstract:


CNOT gate : |a⟩|b⟩ ⟶ |a⟩|a⊕b⟩
CNOT[(|0⟩+|1⟩)⊗(|0⟩−|1⟩)].
= CNOT[|00⟩+|10⟩−|01⟩−|11⟩]
= |00⟩+|11⟩−|01⟩−|10⟩
= |0⟩(|0⟩−|1⟩)+|1⟩(|1⟩−|0⟩)
= (|0⟩−|1⟩)⊗(|0⟩−|1⟩)
H
Universal


Universal gate sets:
Classical: Can calculate $f(x_1, \ldots, x_n)$
f can be written as a polynomial of degree $\leq n$
Irreversible: {NOT, AND} universal
Reversible: {Tof, 0&1 ancilla state preparation}
universal


Quantum: Do arbitrary elements of $SU(2^n)$.
Could try to get unitaries exactly or we
could try to get them approximately — to arbitrary
accuracy — a dense set in $SU(2^n)$.
Exact: {CNOT, single-qubit unitaries}
Approximate: $\{H, R_{\pi/2}, \text{Tof}\}$, $\{H, R_{\pi/4}, \text{CNOT}\}$

No-Cloning Theorem:

$\nexists$ quantum operation

$$|\psi\rangle \longrightarrow |\psi\rangle|\psi\rangle$$

Proof:

$$|\phi\rangle \longrightarrow |\phi\rangle|\phi\rangle$$

$$|\psi\rangle \longrightarrow |\psi\rangle|\psi\rangle$$

By linearity, $|\phi\rangle + |\psi\rangle \longrightarrow |\phi\rangle|\phi\rangle + |\psi\rangle|\psi\rangle$

$$(|\phi\rangle+|\psi\rangle)\otimes(|\phi\rangle+|\psi\rangle)$$

ration

$\rangle|\psi\rangle$

$|\phi\rangle|\phi\rangle+|\psi\rangle|\psi\rangle$

$(|\phi\rangle+|\psi\rangle)\otimes(|\phi\rangle+|\psi\rangle)$

But

$|0\rangle/|1\rangle$

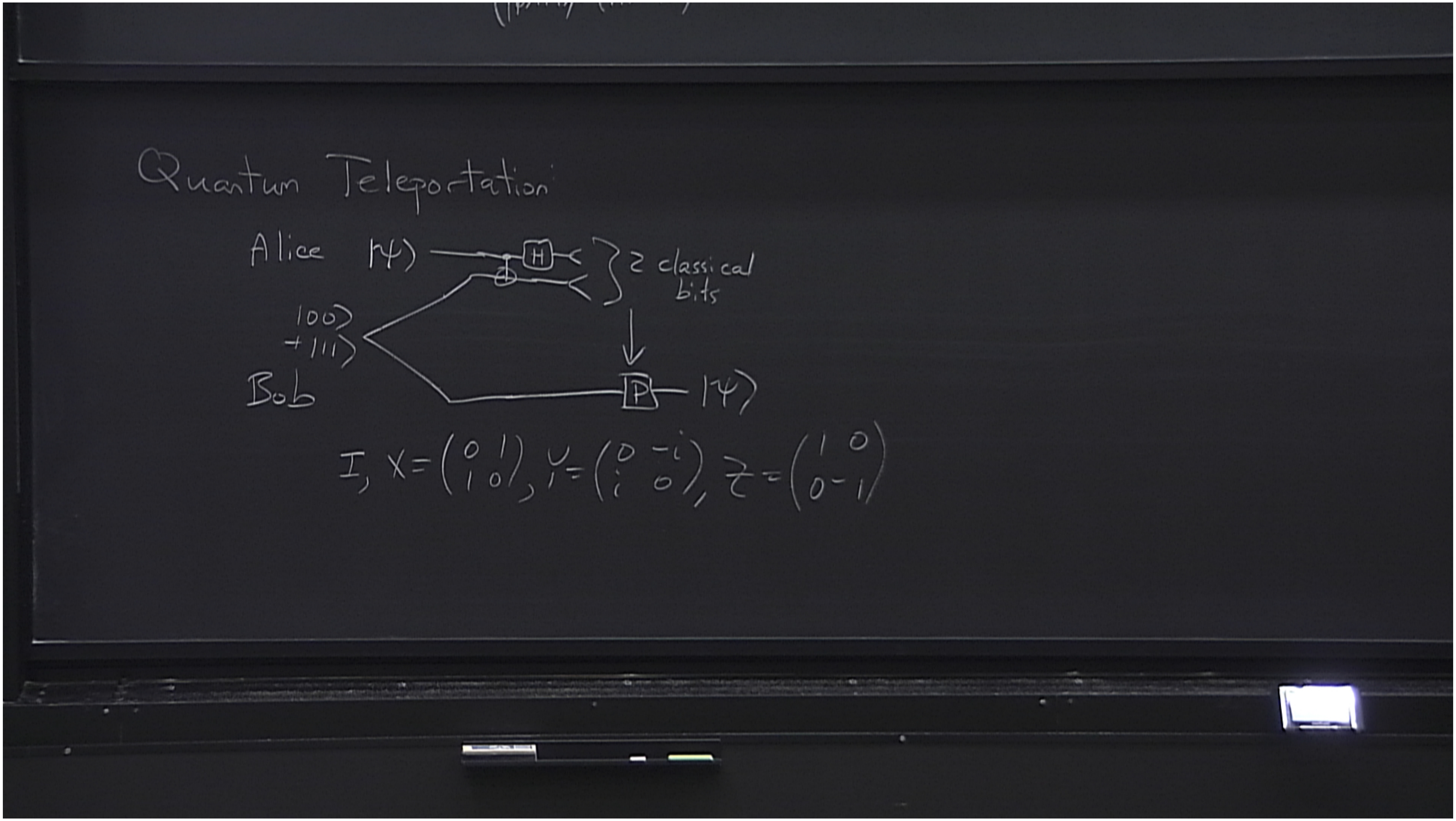
$|0\rangle$

$|00\rangle/|11\rangle$

instead

$\alpha|0\rangle+\beta|1\rangle \rightarrow \alpha|00\rangle+\beta|11\rangle$

Entangled



Quantum Teleportation:
Alice
$|\psi\rangle$
H
2 classical bits
$|00\rangle + |11\rangle$
Bob
P
$|\psi\rangle$
$I, X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Distance between quantum states:

Fidelity:

$$F(|\psi\rangle, |\phi\rangle) = |\langle\psi|\phi\rangle|$$

Operational interpretation of fidelity

Make measurement $\{|\psi\rangle\langle\psi|, I - |\psi\rangle\langle\psi|\}$

If state is $|\phi\rangle$, prob. of 1st outcome is $F(|\psi\rangle, |\phi\rangle)^2$.

$$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}(|\psi\rangle\langle\psi|\sigma)$$

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$$F(|\psi\rangle, \sigma)^2 = \mathrm{tr}\left(|\psi\rangle\langle\psi|\sigma\right)$$

$$\boxed{F(\rho,\sigma) = \mathrm{tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}}$$

Properties of fidelity:

1) $0 \le F(\rho,\sigma) \le 1$
2) $\rho = \sigma \Leftrightarrow F(\rho,\sigma) = 1$
3) $F(\rho,\sigma) = 0$ iff ρ & σ have support on orthogonal subspaces
4) $F(\rho,\sigma) = F(\sigma,\rho)$
5) $F(U\rho U^\dagger, U\sigma U^\dagger) = F(\rho,\sigma)$ $\forall$ unitaries U
6) $F(\mathcal{E}(\rho), \mathcal{E}(\sigma)) \geq F(\rho,\sigma)$ $\forall$ CPTP map $\mathcal{E}$

7) Uhlmann's thm: $F(\rho,\sigma) = \max_{|\psi\rangle, |\phi\rangle} F(|\psi\rangle, |\phi\rangle)$

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ρ & σ are on Hilbert space Q, Purify using R

$\mathrm{tr}_R |\psi\rangle\langle\psi| = \rho$, $\mathrm{tr}_R |\phi\rangle\langle\phi| = \sigma$

Trace distance: $D(\rho,\sigma) = \frac{1}{2} \operatorname{tr} |\rho - \sigma|$

$$D(\{p_i\}, \{q_i\}) = \frac{1}{2} \sum_i |p_i - q_i|$$

<u>Thm</u>: $D(\rho,\sigma)$ is max over POVMs $\{E_m\}$ of classical statistical distance of $\{\operatorname{tr}(\rho E_m)\}$ & $\{\operatorname{tr}(\sigma E_m)\}$

Prob. (guessing correctly) $= \frac{1}{2}\left(1 + D(\rho,\sigma)\right)$

Pr

1)

2)

3)

4)

of

$\{\mathrm{tr}(\sigma E_m)\}$

Properties:

1) $0 \leq D(\rho,\sigma) \leq 1$
2) $D(\rho,\sigma) = 0 \Leftrightarrow \rho = \sigma$
3) $D(\rho,\sigma) = D(\sigma,\rho)$
4) $D(\rho,\sigma) \leq D(\rho,\eta) + D(\eta,\sigma)$
5) $D(U\rho U^\dagger, U\sigma U^\dagger) = D(\rho,\sigma)$ $\forall$ unitary U
6) $D(\mathcal{E}(\rho),\mathcal{E}(\sigma)) \leq D(\rho,\sigma)$ $\forall$ CPTP $\mathcal{E}$