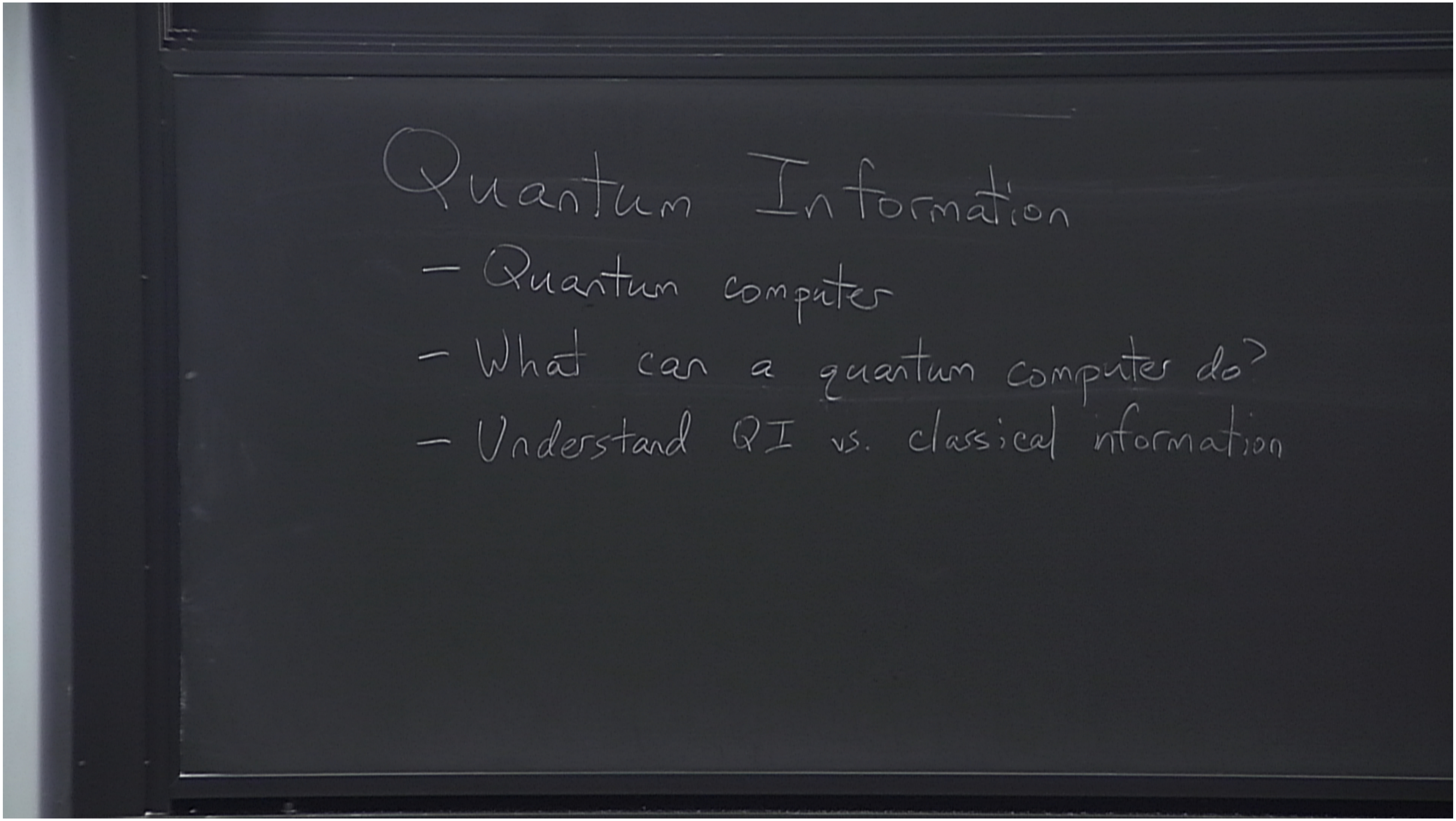
Title: Quantum Information Review-1

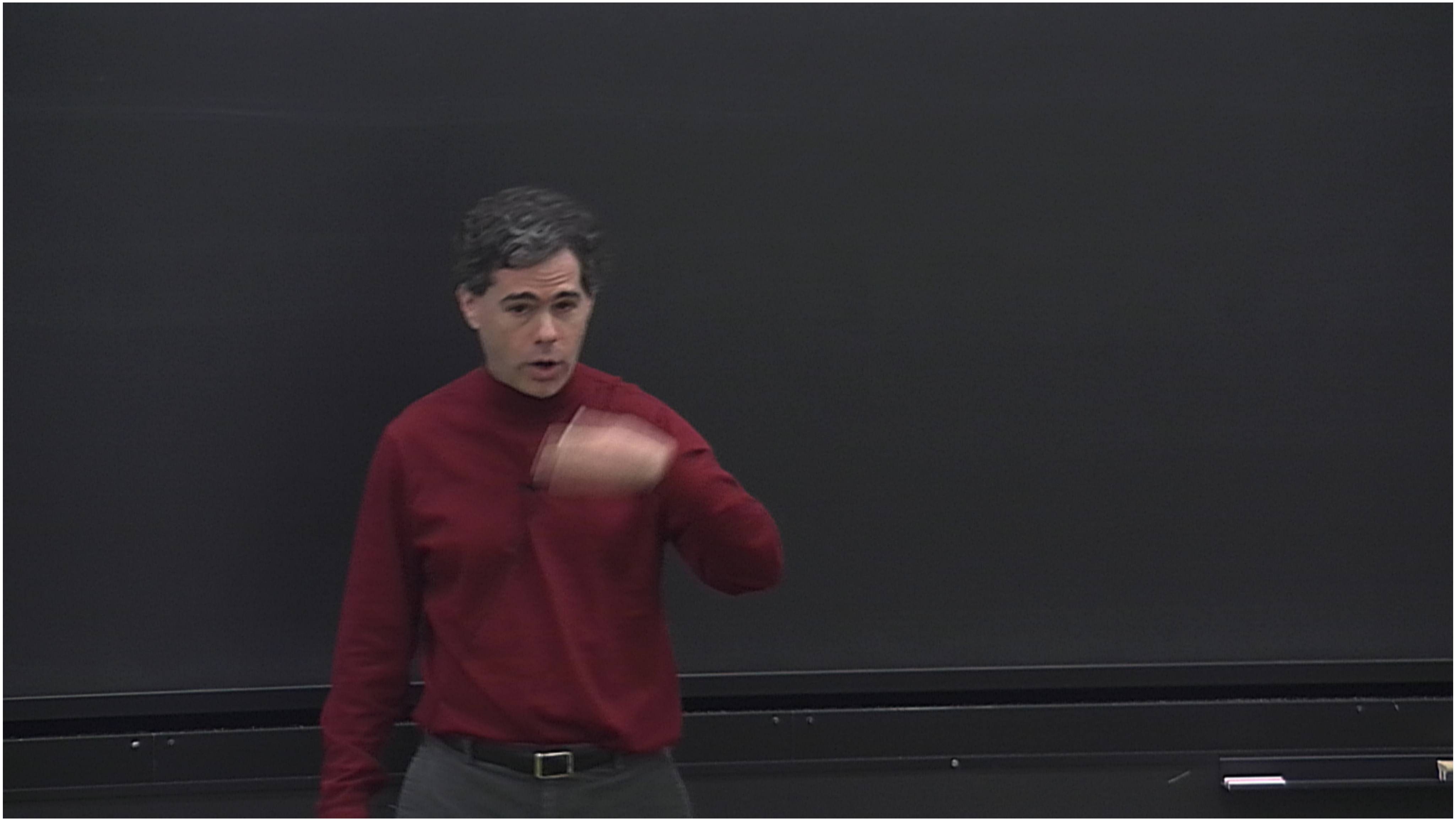
Date: Feb 17, 2015 11:30 AM

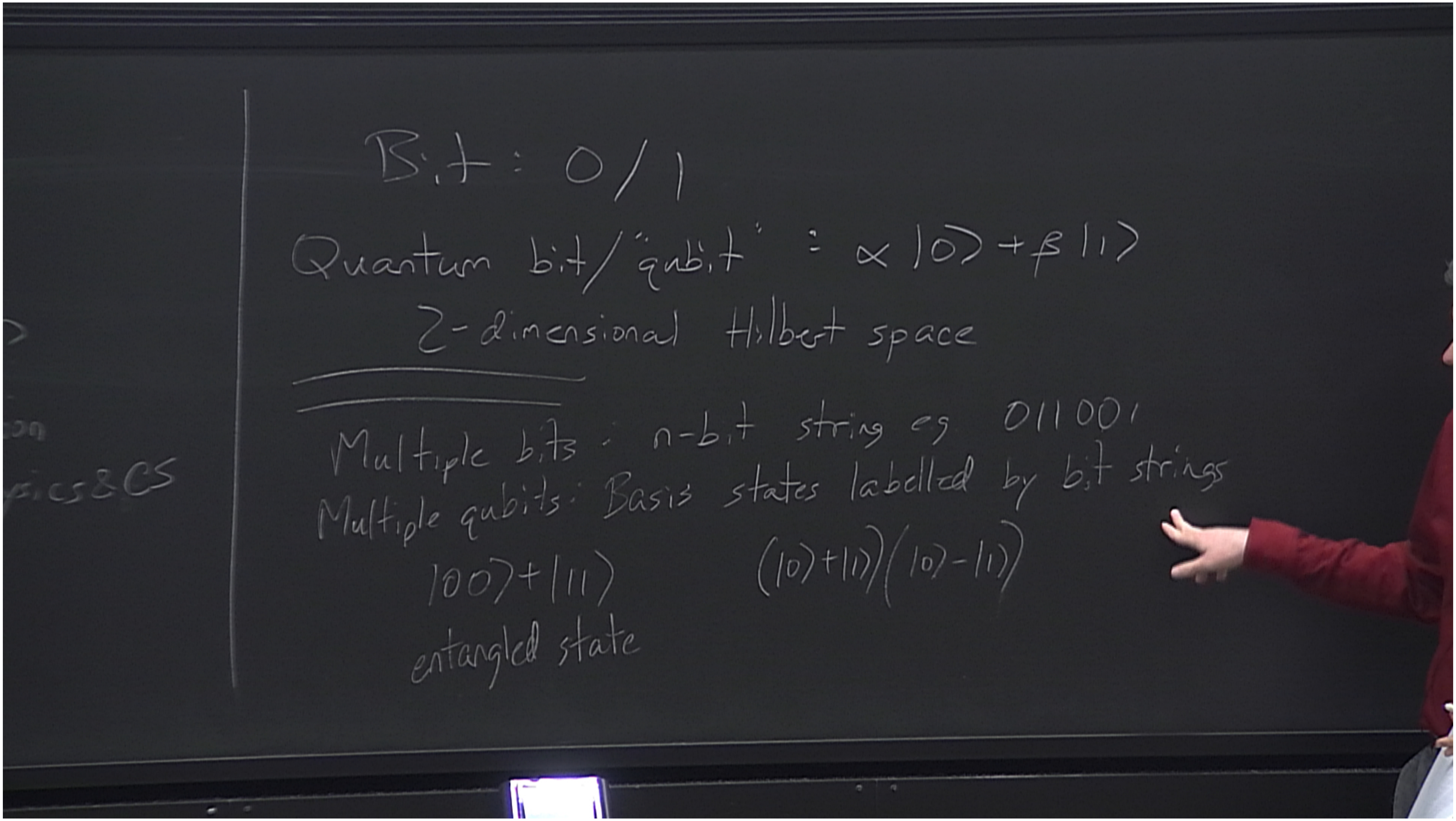
URL: http://pirsa.org/15020061

Abstract:


Quantum Information
- Quantum computer
- What can a quantum computer do?
- Understand QI vs. classical information


Quantum Information
- Quantum computer
- What can a quantum computer do?
- Understand QI vs classical information




Bit : 0/1
Quantum bit/"qubit" : $\alpha|0\rangle+\beta|1\rangle$
2-dimensional Hilbert space
Multiple bits : n-bit string e.g. 011001
Multiple qubits: Basis states labelled by bit strings
$|00\rangle+|11\rangle$
$(|0\rangle+|1\rangle)(|0\rangle-|1\rangle)$
entangled state

n qubits | 2^n-dimensional Hilbert space.

s $|1\rangle$

1001

bit strings

Q. Gate: Acts on a small number of qubits.

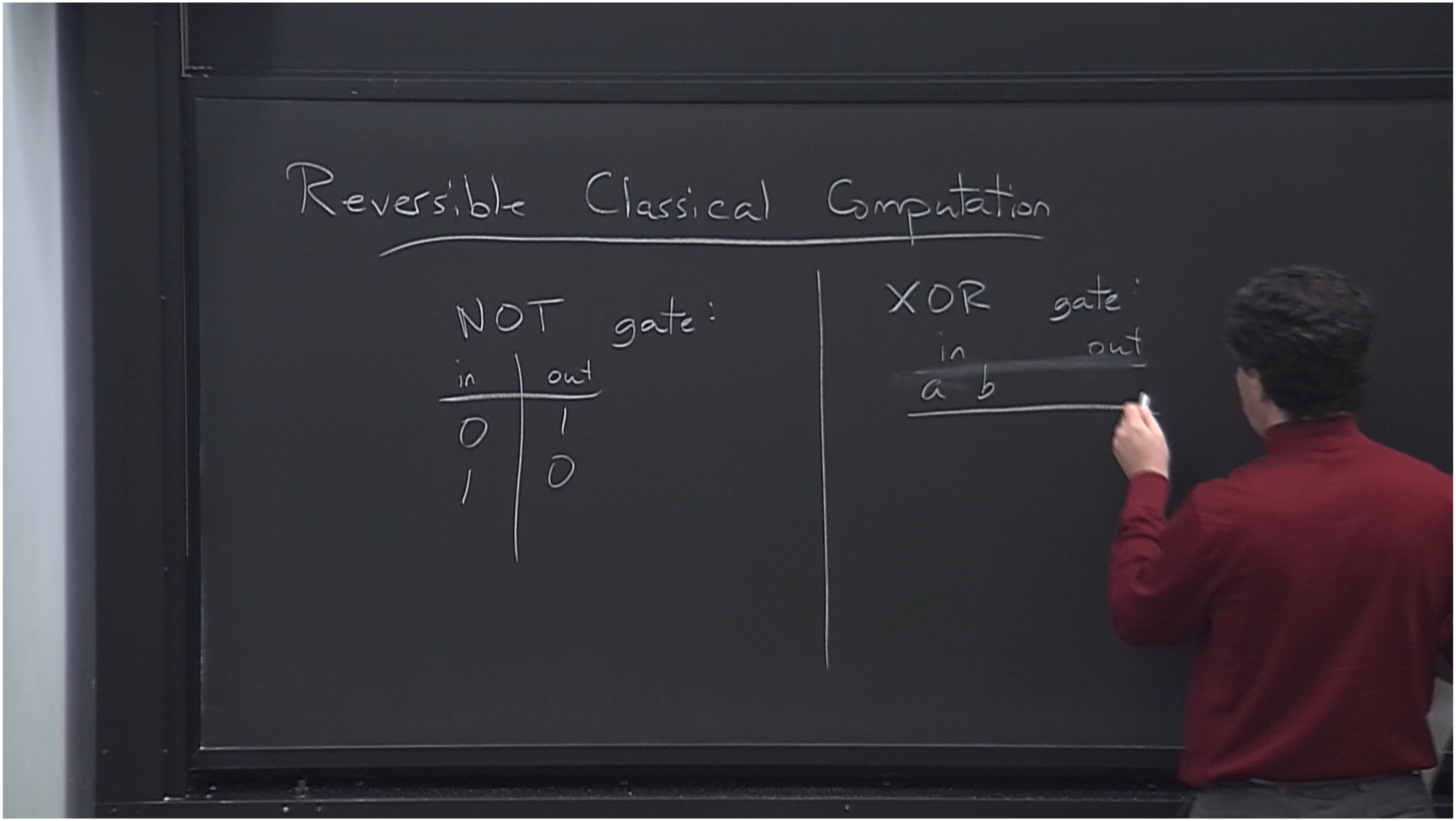
Q. Circuit: sequence of Q gates

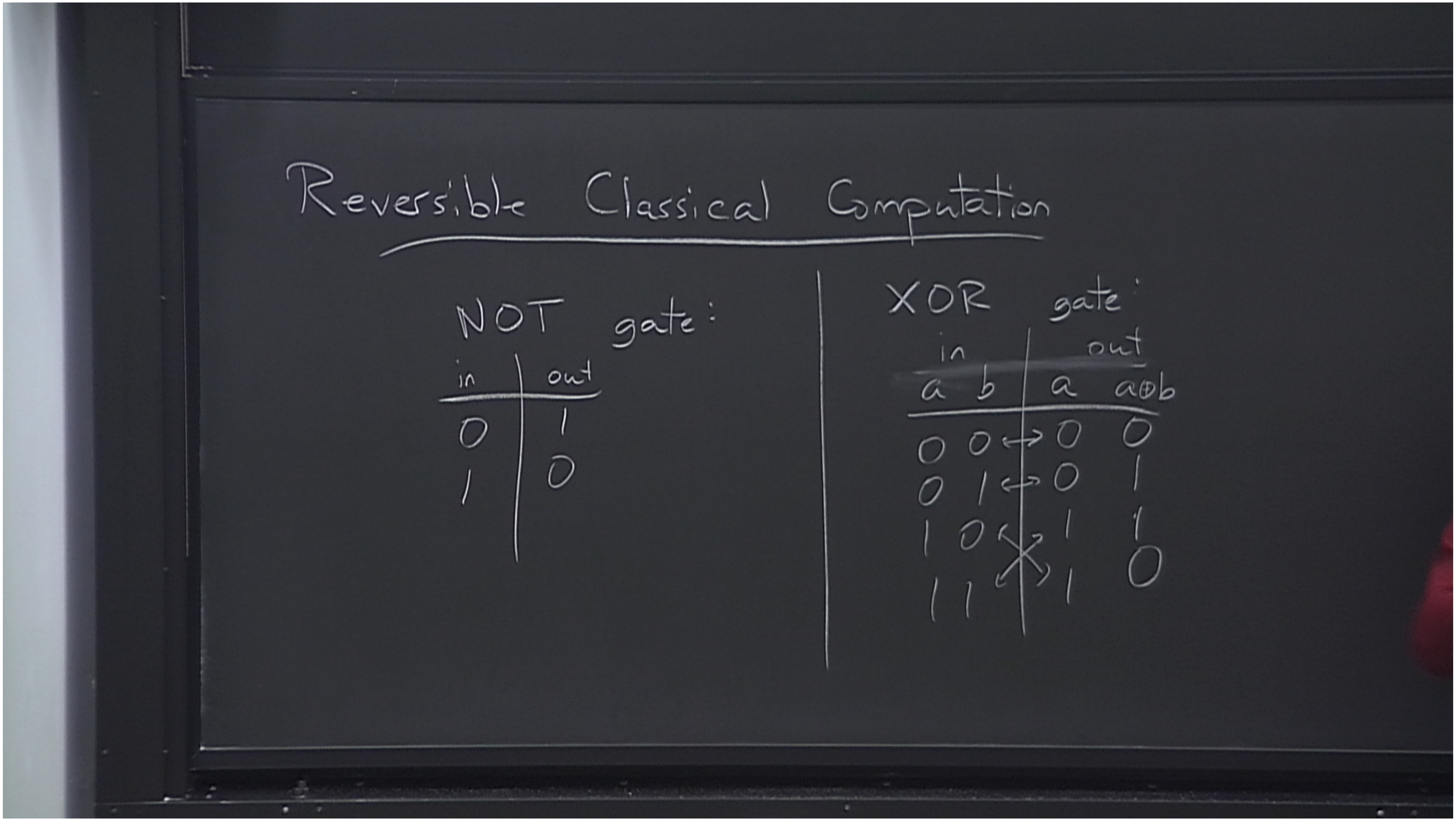
Circuit diagrams:

Each line represents 1 bit

Time goes left to right.

NOT $a -\!\!\oplus\!\!-$ NOT $a = a \oplus 1$ $\quad a -\!\!\oplus\!\!-\!\!\oplus\!\!- a$


Reversible Classical Computation
NOT gate:
in out
0 1
1 0
XOR gate:
in out
a b


Reversible Classical Computation
NOT gate:
in | out
0 | 1
1 | 0
XOR gate:
in | out
a b | a a⊕b
0 0 ↔ 0 0
0 1 ↔ 0 1
1 0 | 1 1
1 1 | 1 0

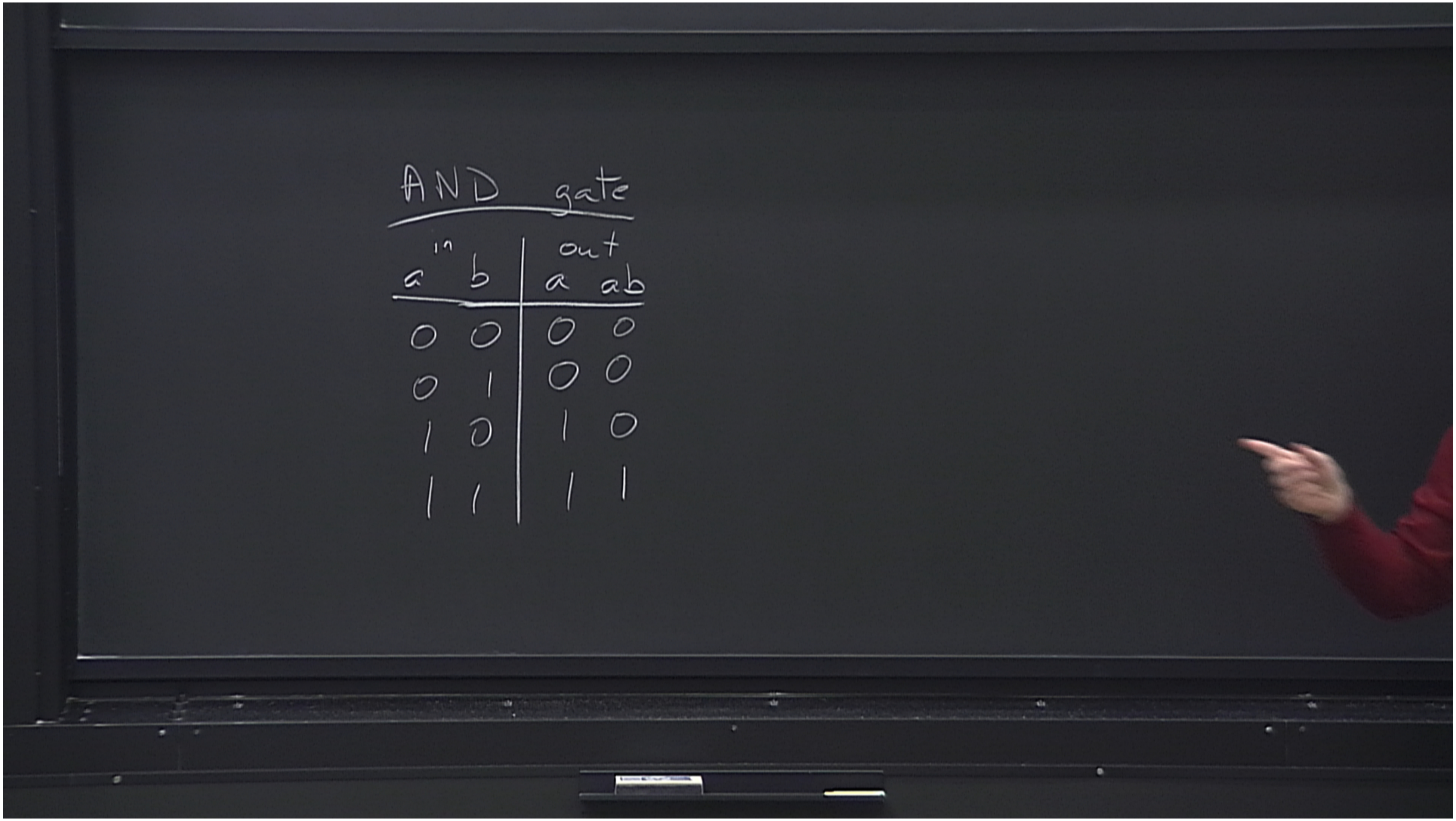
Reversible Classical Computation

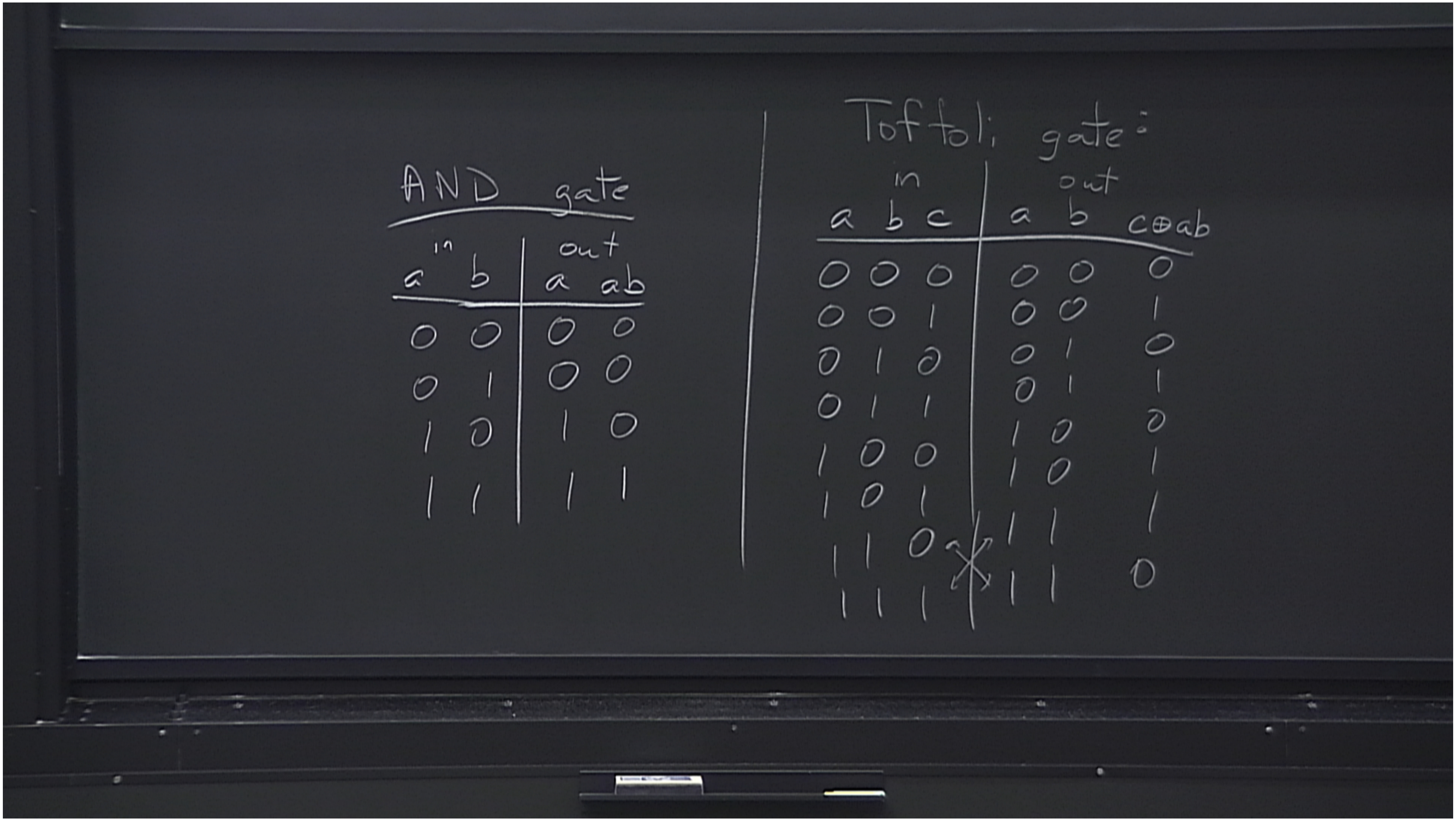
NOT gate:

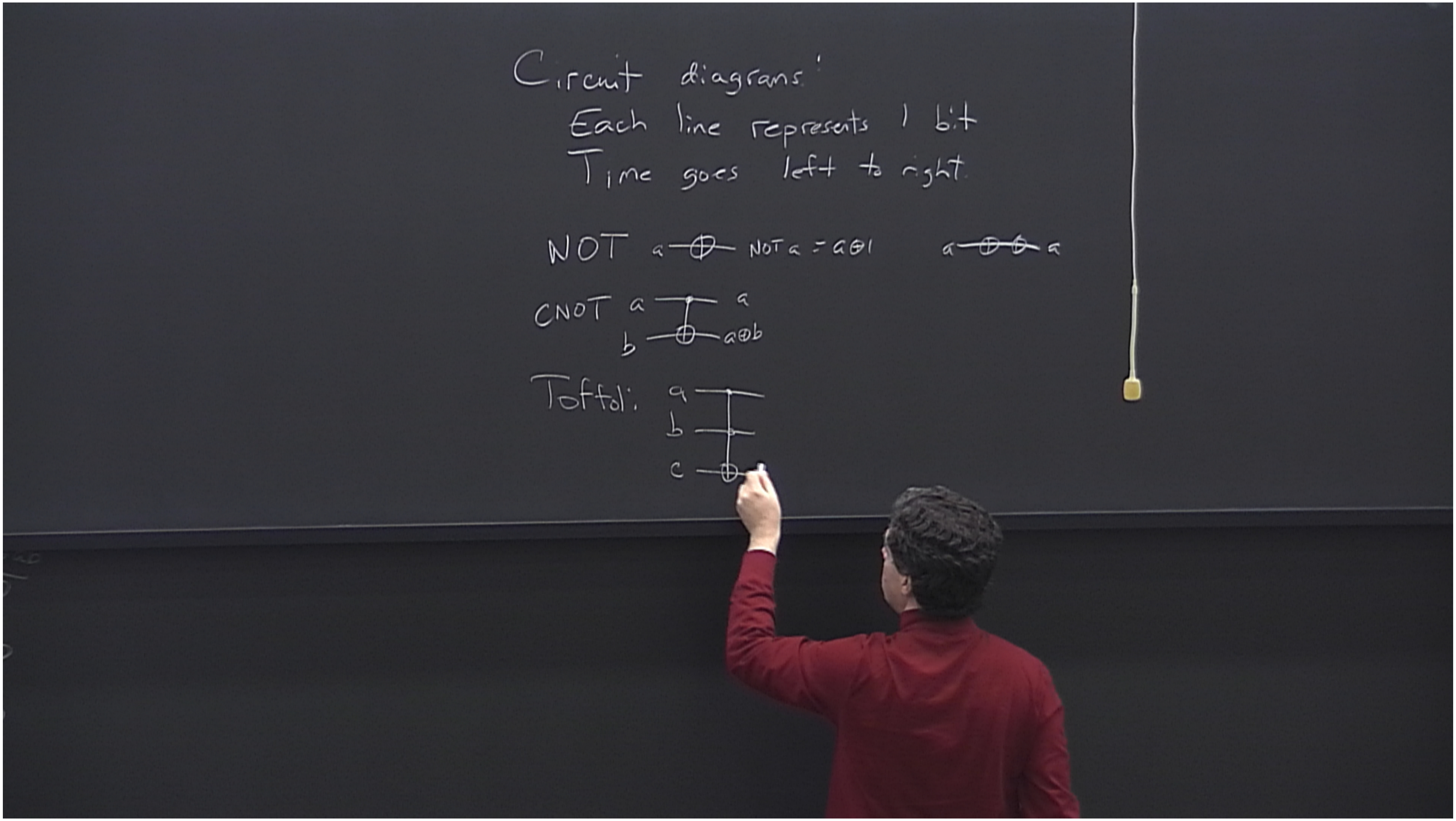
in	out
0	1
1	0

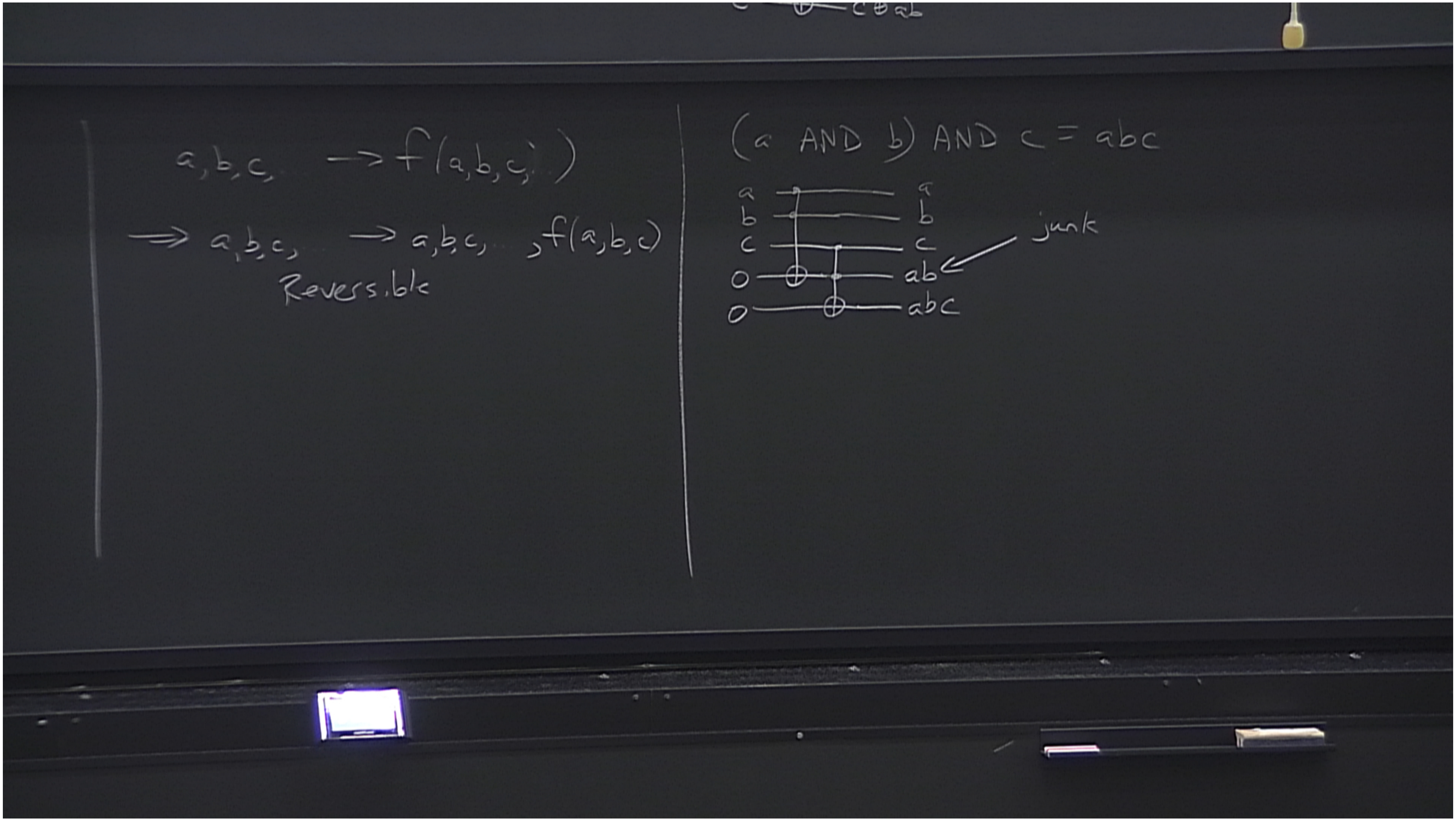
CNOT/ XOR gate:

in			out	
a	b		a	$a \oplus b$
0	0	$\leftrightarrow$	0	0
0	1	$\leftrightarrow$	0	1
1	0		1	1
1	1		1	0


AND gate
in
out
a b
a ab
0 0 0 0
0 1 0 0
1 0 1 0
1 1 1 1


AND gate
in | out
a b | a ab
0 0 | 0 0
0 1 | 0 0
1 0 | 1 0
1 1 | 1 1
Toffoli gate:
in | out
a b c | a b c⊕ab
0 0 0 | 0 0 0
0 0 1 | 0 0 1
0 1 0 | 0 1 0
0 1 1 | 0 1 1
1 0 0 | 1 0 0
1 0 1 | 1 0 1
1 1 0 | 1 1 1
1 1 1 | 1 1 0

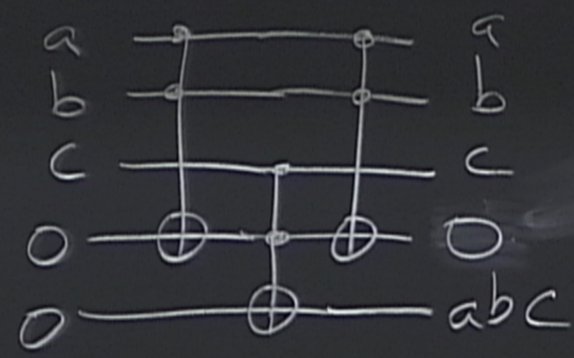

Circuit diagrams:
Each line represents 1 bit
Time goes left to right.
NOT a
NOT a = a⊕1
a
a
CNOT a
b
a
a⊕b
Toffoli: a
b
c


a, b, c, → f(a, b, c,)
⇒ a, b, c, → a, b, c, , f(a, b, c)
Reversible
(a AND b) AND c = abc
a
b
c
0
0
a
b
c
ab
abc
junk

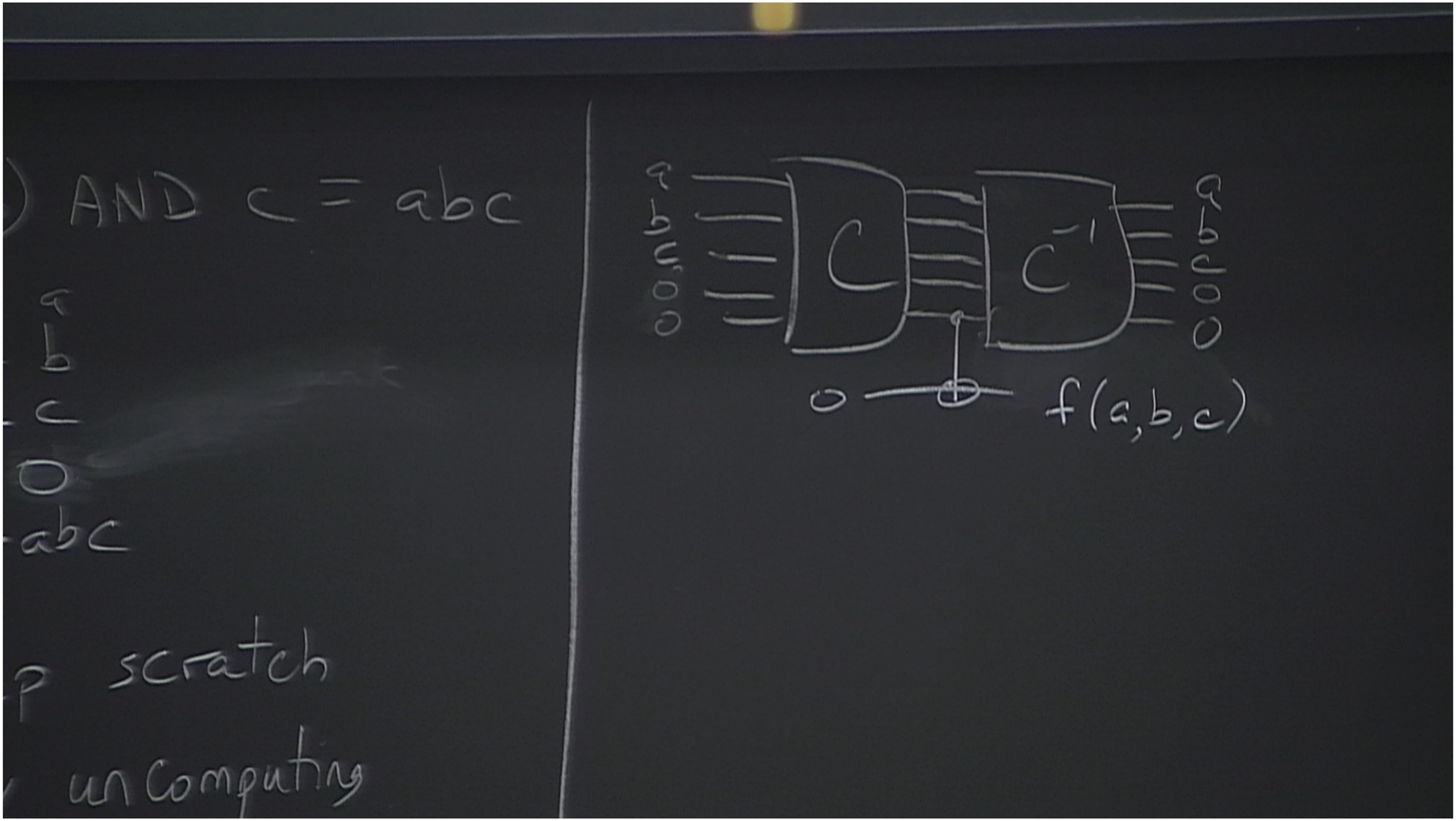
b, c, ...)

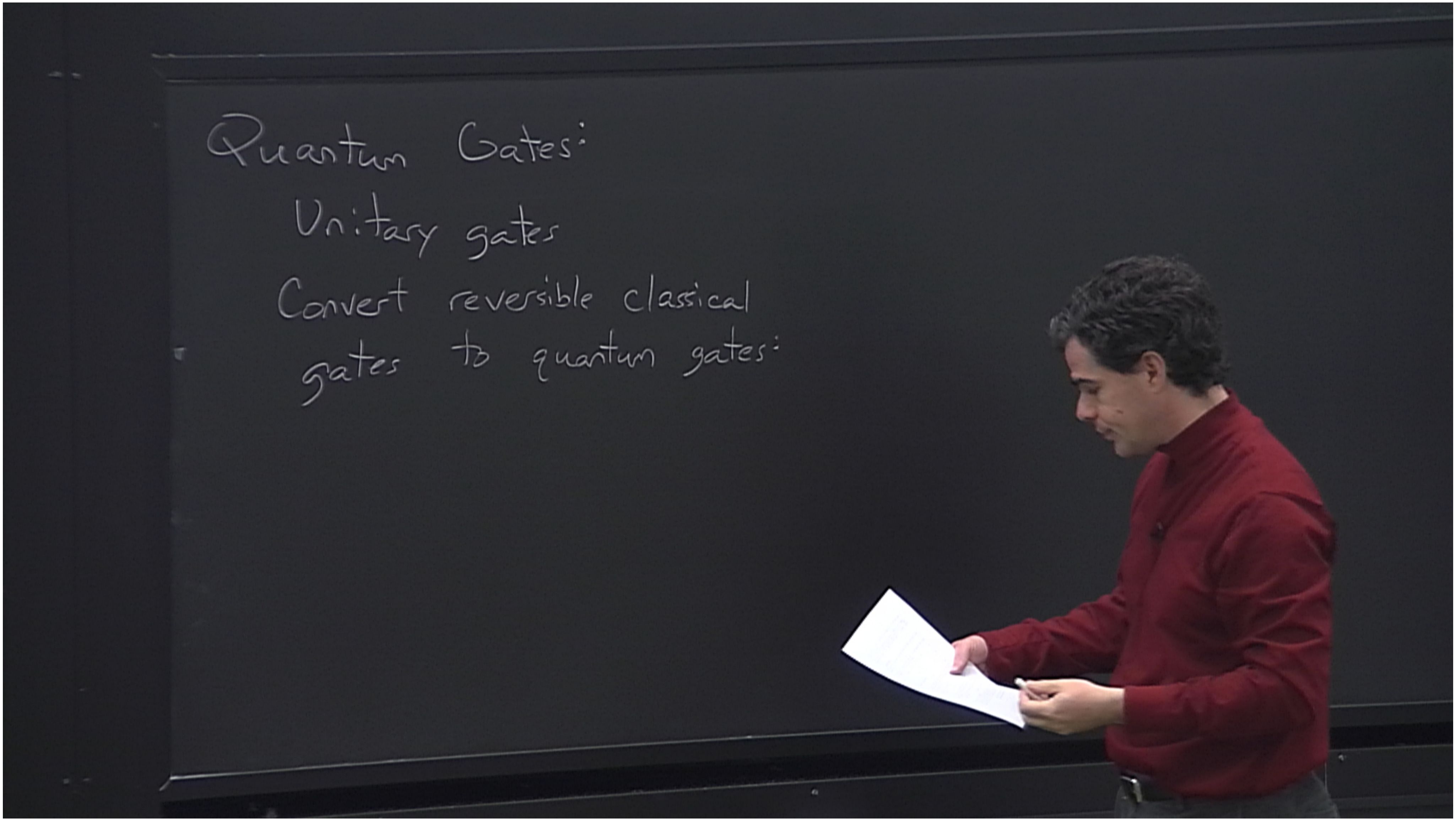
..., f(a, b, c)

$$(a \text{ AND } b) \text{ AND } c = abc$$



Clear up scratch bits by uncomputing them


AND c = abc
scratch
uncomputing
C
C⁻¹
f(a,b,c)


Quantum Gates:
Unitary gates
Convert reversible classical
gates to quantum gates:

Quantum Gates:

Unitary gates

Convert reversible classical gates to quantum gates:

$$\text{CNOT}\,|a,b\rangle = |a, a\oplus b\rangle$$

$$\text{CNOT}\left(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle\right)$$

$$=\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\delta|10\rangle$$

$$\text{Tof}\,|a,b,c\rangle = |a,b,c\oplus ab\rangle$$

Quantum Gates:

Unitary gates

Convert reversible classical gates to quantum gates:

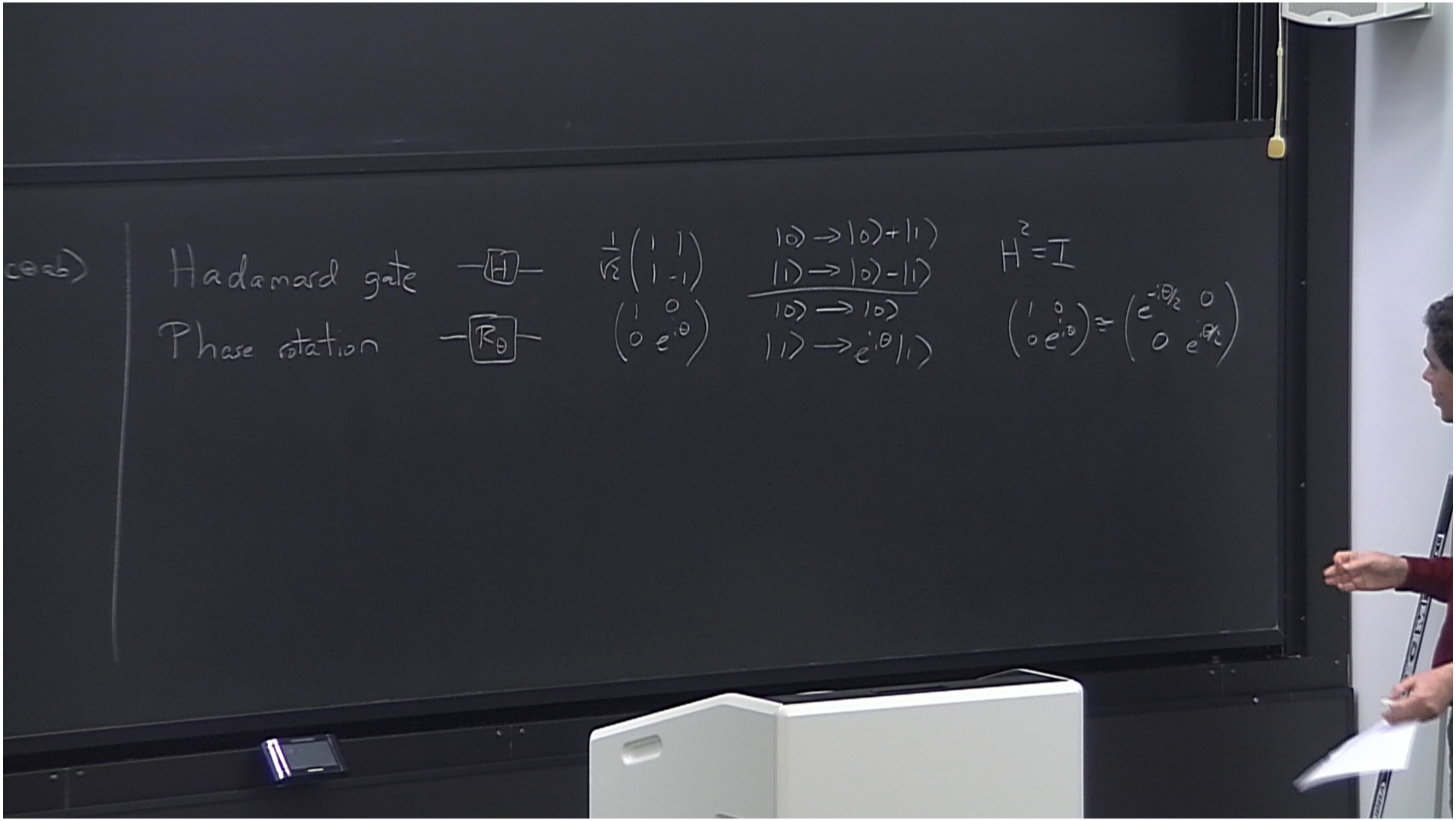
$$\mathrm{CNOT}\,|a, b\rangle = |a, a \oplus b\rangle$$

$$\mathrm{CNOT}\left(\alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle\right)$$

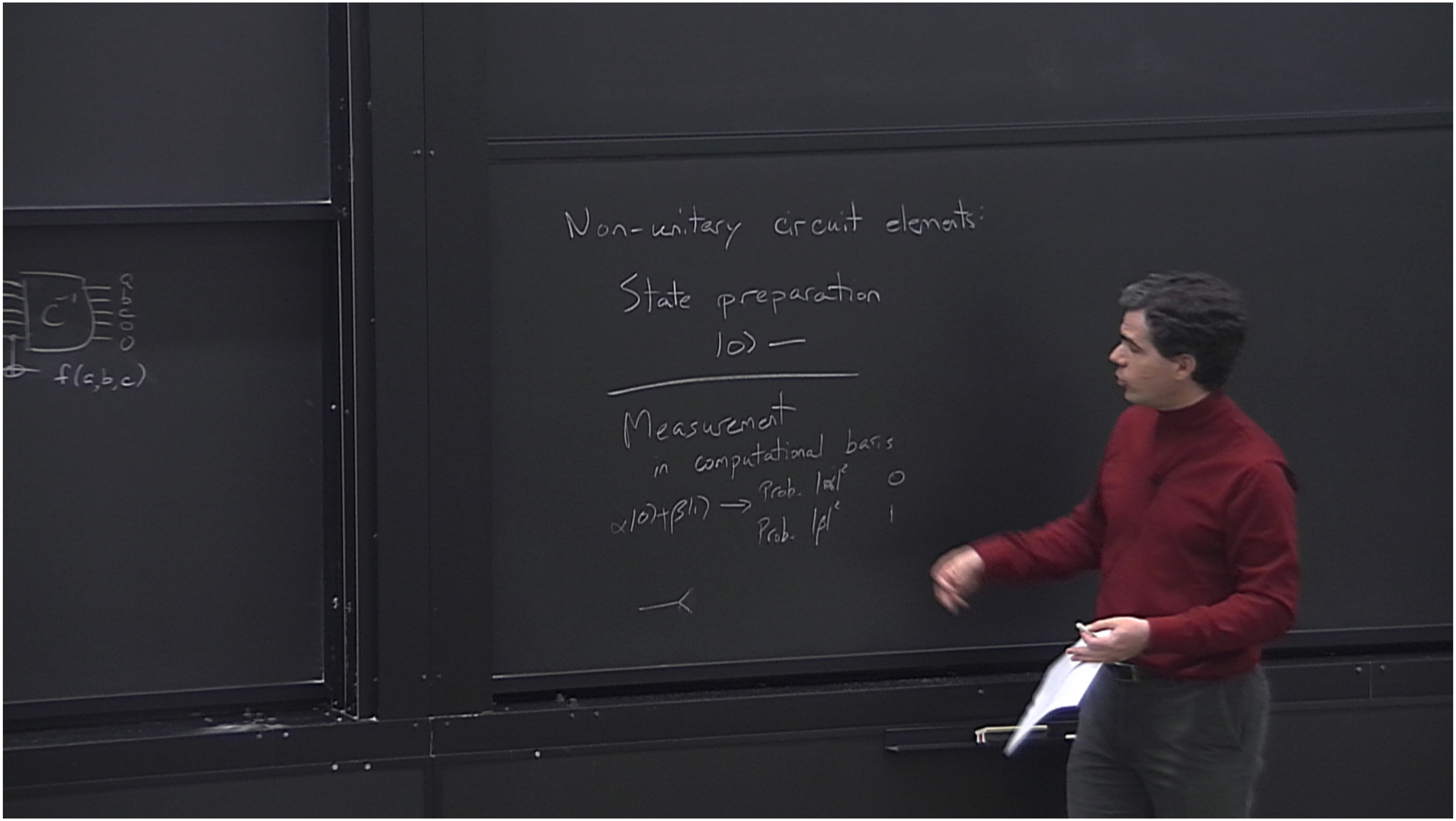
$$= \alpha|00\rangle + \beta|01\rangle + \gamma|11\rangle + \delta|10\rangle$$

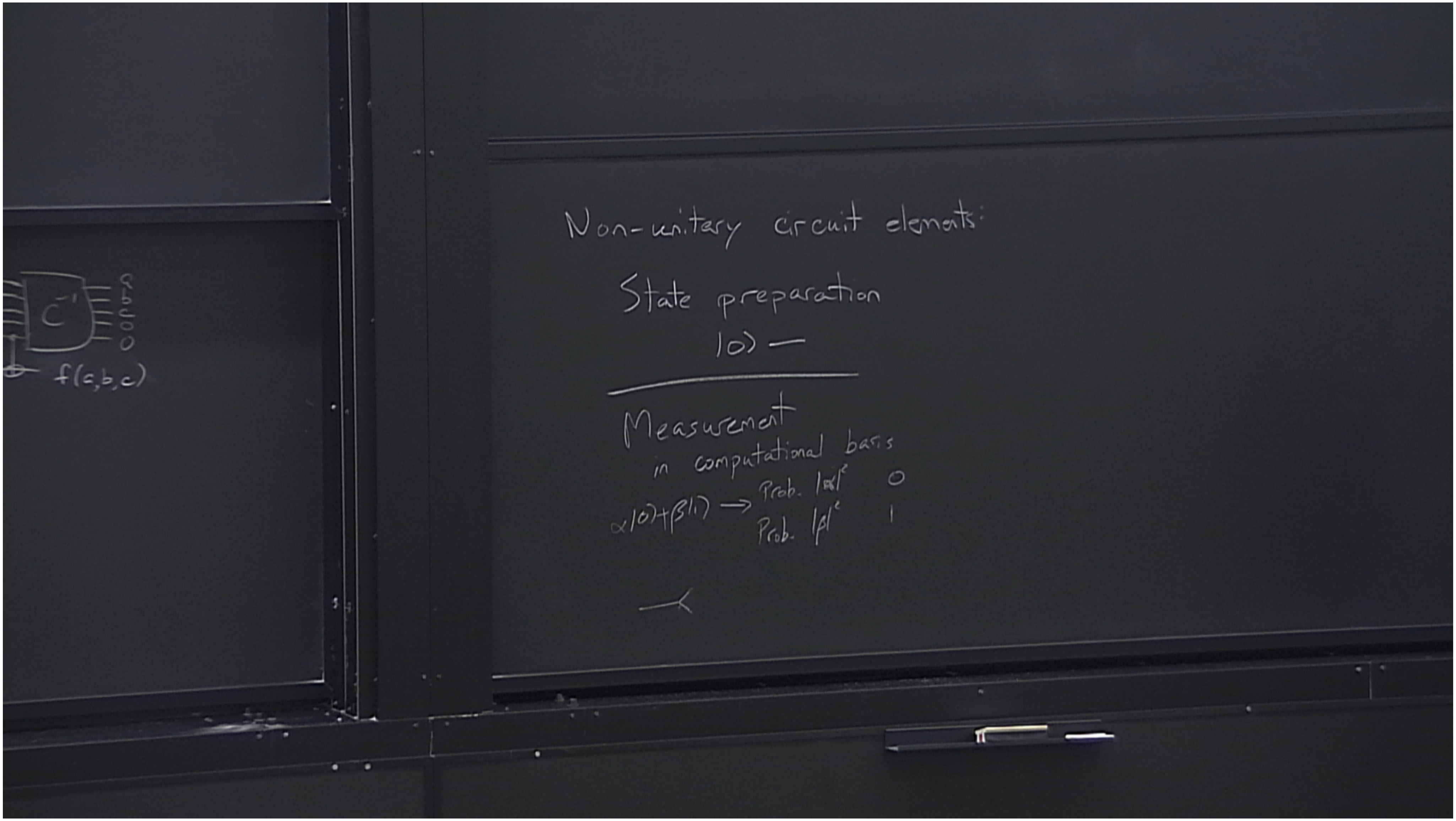
$$\mathrm{Tof}\,|a, b, c\rangle = |a, b, c \oplus ab\rangle$$

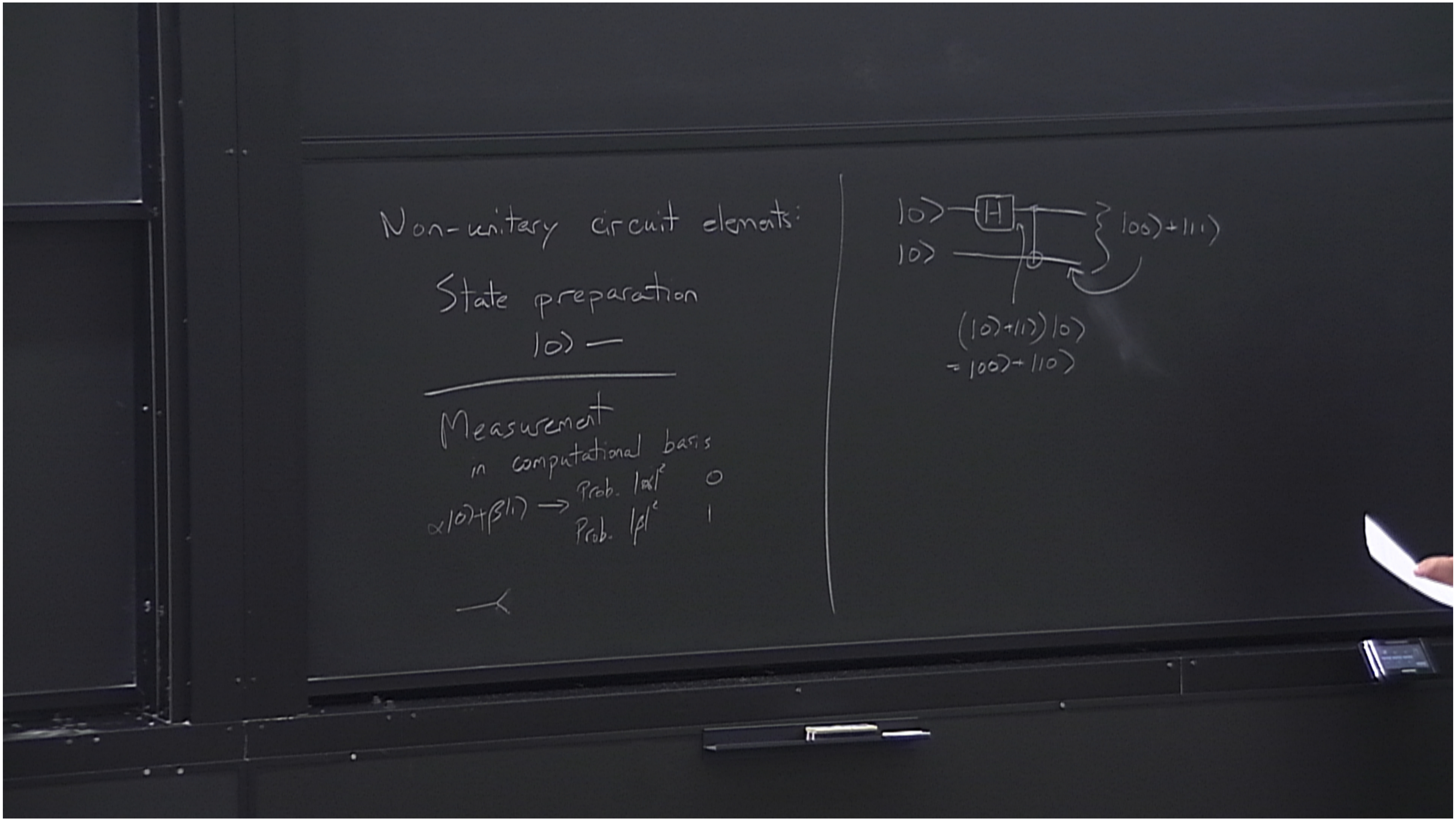
$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & & 0 \\ 0 & 1 & & \\ & & 0 & 1 \\ 0 & & 1 & 0 \end{pmatrix}$$

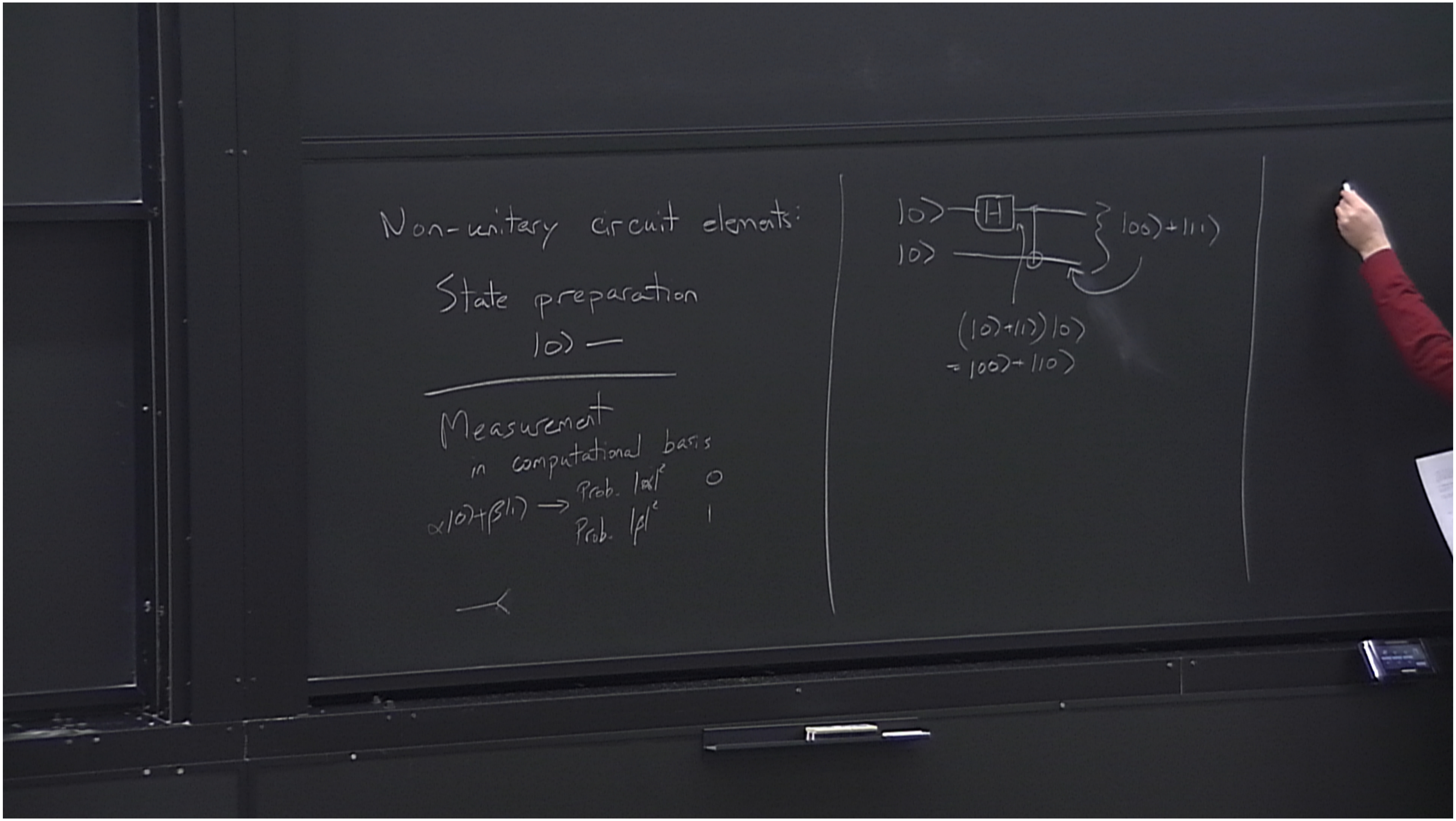


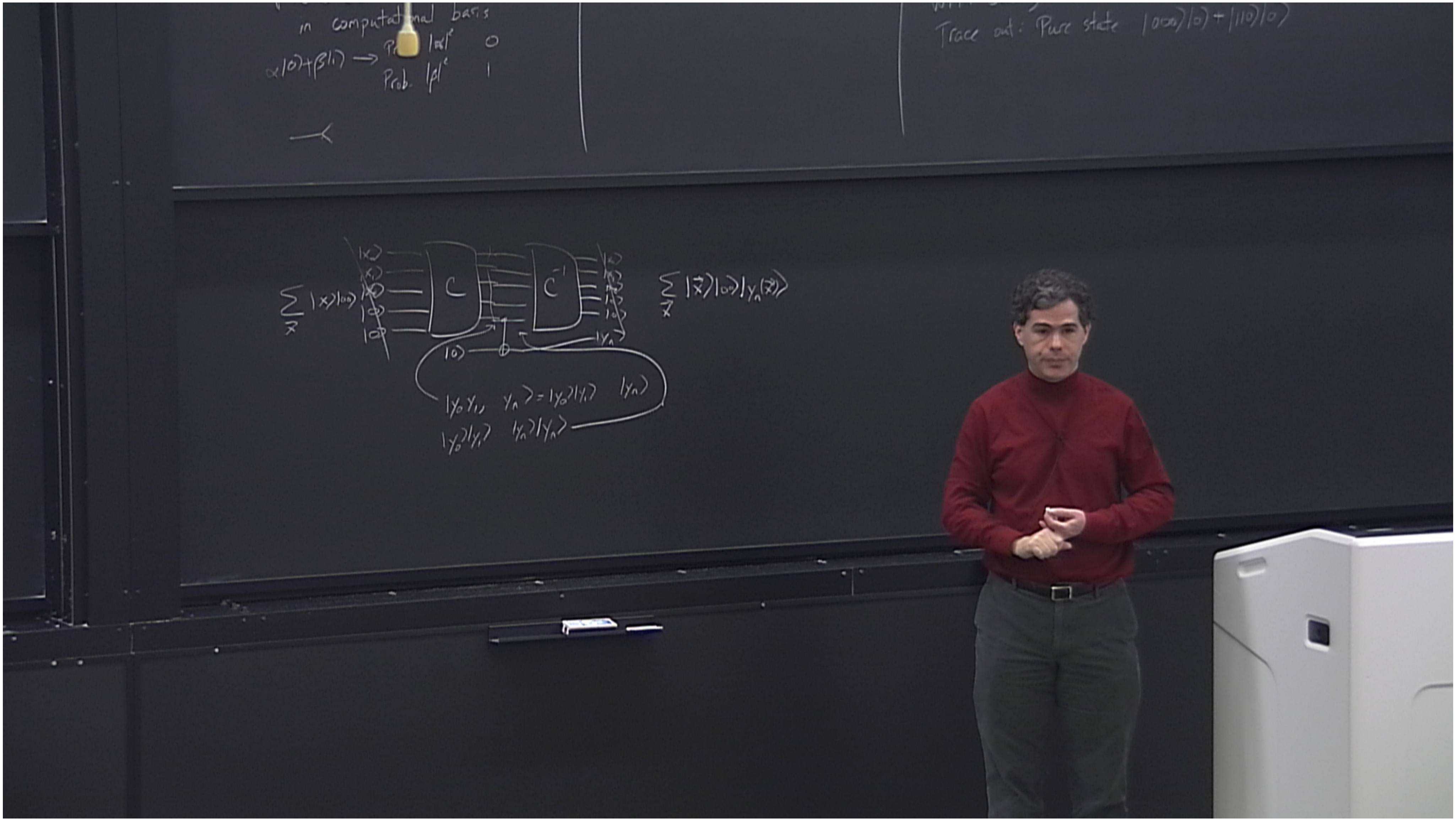
Hadamard gate
$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$
$|0\rangle \to |0\rangle + |1\rangle$
$|1\rangle \to |0\rangle - |1\rangle$
$H^2 = I$
Phase rotation
R_θ
$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$
$|0\rangle \to |0\rangle$
$|1\rangle \to e^{i\theta}|1\rangle$
$\begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix} \simeq \begin{pmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{pmatrix}$


Non-unitary circuit elements:
State preparation
|0⟩ —
Measurement
in computational basis
α|0⟩+β|1⟩ →
Prob. |α|² 0
Prob. |β|² 1
f(a,b,c)


Non-unitary circuit elements:
State preparation
$|0\rangle$ —
Measurement
in computational basis
$\alpha|0\rangle+\beta|1\rangle \rightarrow$ Prob. $|\alpha|^2$ 0
Prob. $|\beta|^2$ 1
C^{-1}
a
b
c
0
0
$f(a,b,c)$


Non-unitary circuit elements:
State preparation
|0⟩ —
Measurement
in computational basis
α|0⟩+β|1⟩ → Prob. |α|² 0
Prob. |β|² 1
|0⟩ H
|0⟩
|00⟩+|11⟩
(|0⟩+|1⟩)|0⟩
= |00⟩+|10⟩


Non-unitary circuit elements:
State preparation
|0⟩ —
Measurement
in computational basis
α|0⟩+β|1⟩ → Prob. |α|² 0
Prob. |β|² 1
|0⟩ H
|0⟩
|00⟩+|11⟩
(|0⟩+|1⟩)|0⟩
= |00⟩+|10⟩


in computational basis
Trace out: Pure state