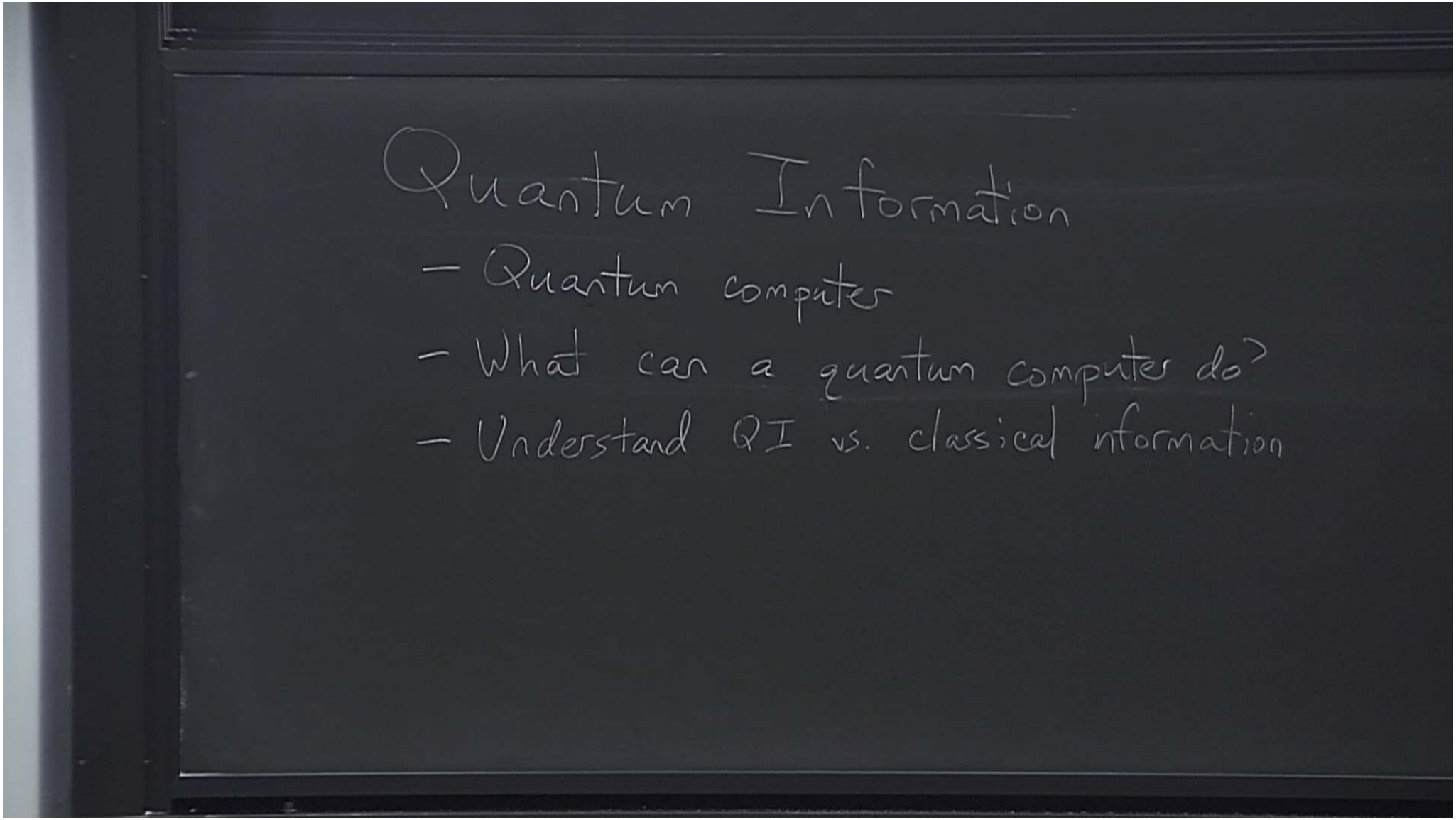
Title: Quantum Information Review-1

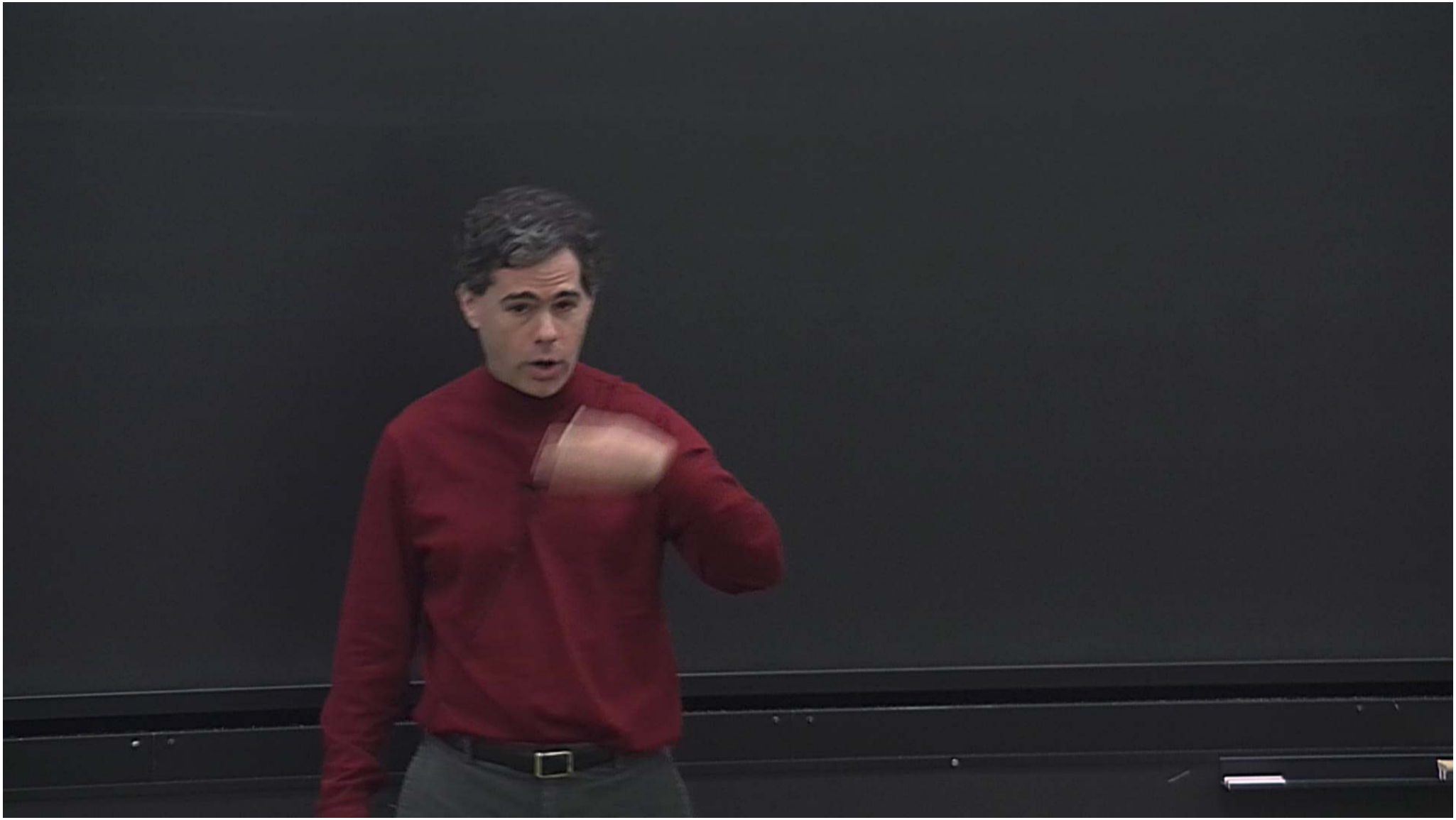
Date: Feb 17, 2015 11:30 AM

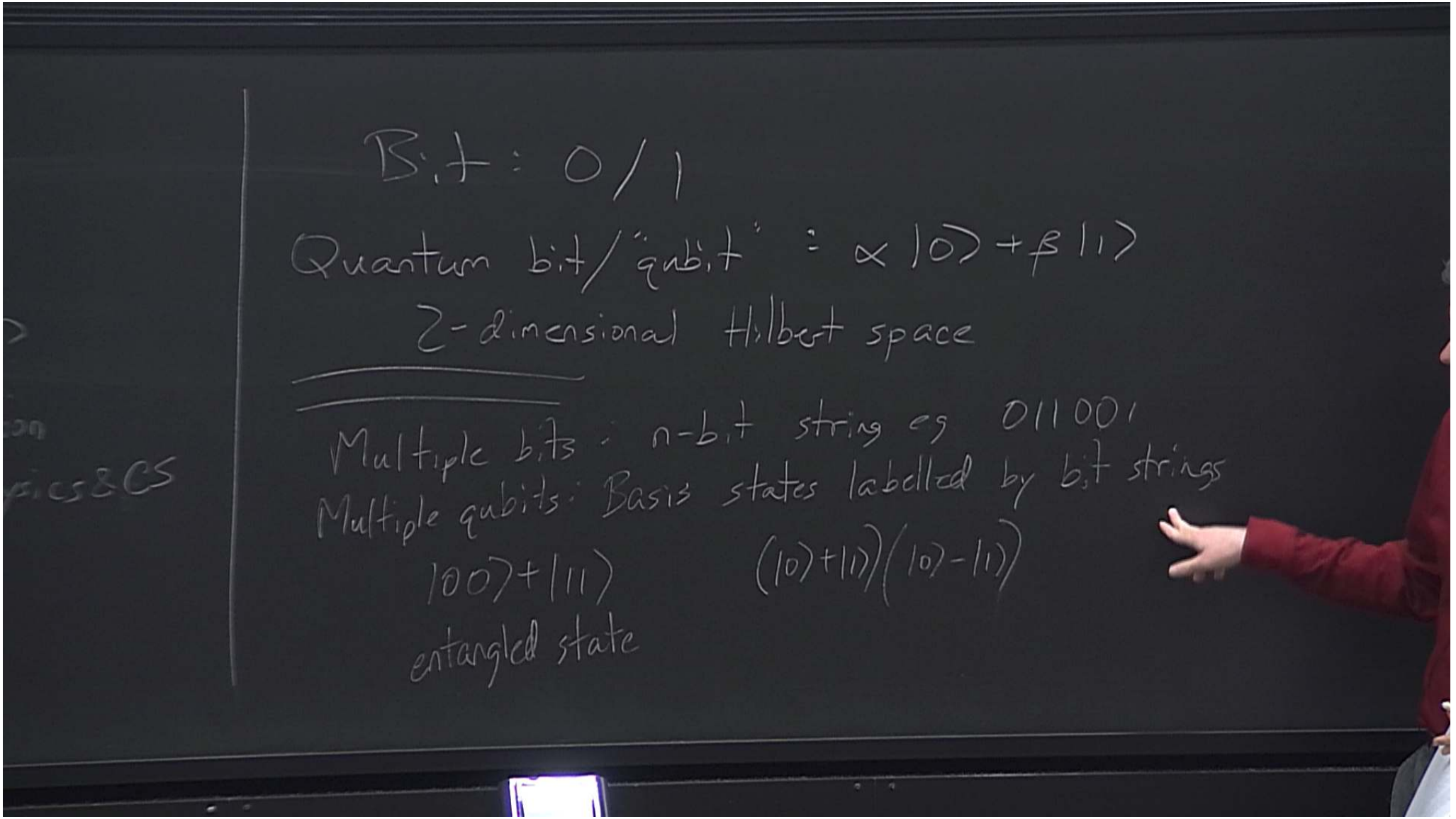
URL: http://pirsa.org/15020061

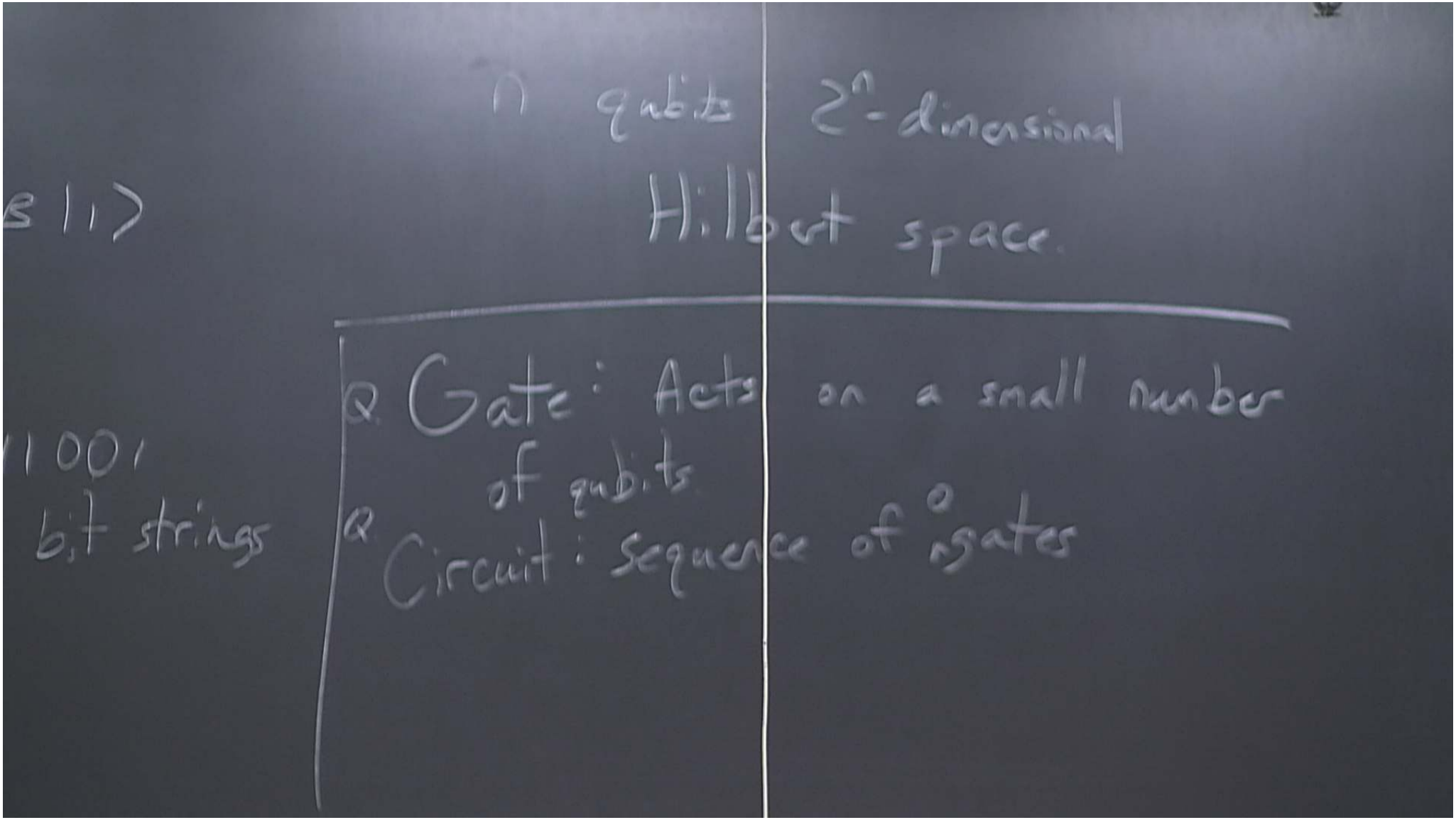
Abstract:


Quantum Information
- Quantum computer
- What can a quantum computer do?
- Understand QI vs. classical information

Quantum Information
– Quantum computer
– What can a quantum computer do?
– Understand QI in classical information




Bit: 0/1
Quantum bit/"qubit": $\alpha|0\rangle + \beta|1\rangle$
2-dimensional Hilbert space
Multiple bits: n-bit string eg 011001
Multiple qubits: Basis states labelled by bit strings
$|00\rangle + |11\rangle$
$(|0\rangle + |1\rangle)(|0\rangle - |1\rangle)$
entangled state



$\beta|1\rangle$

n qubits 2^n-dimensional

Hilbert space.

$|1001$

bit strings

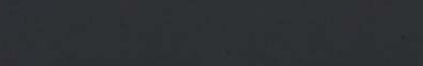
Q. Gate: Acts on a small number of qubits

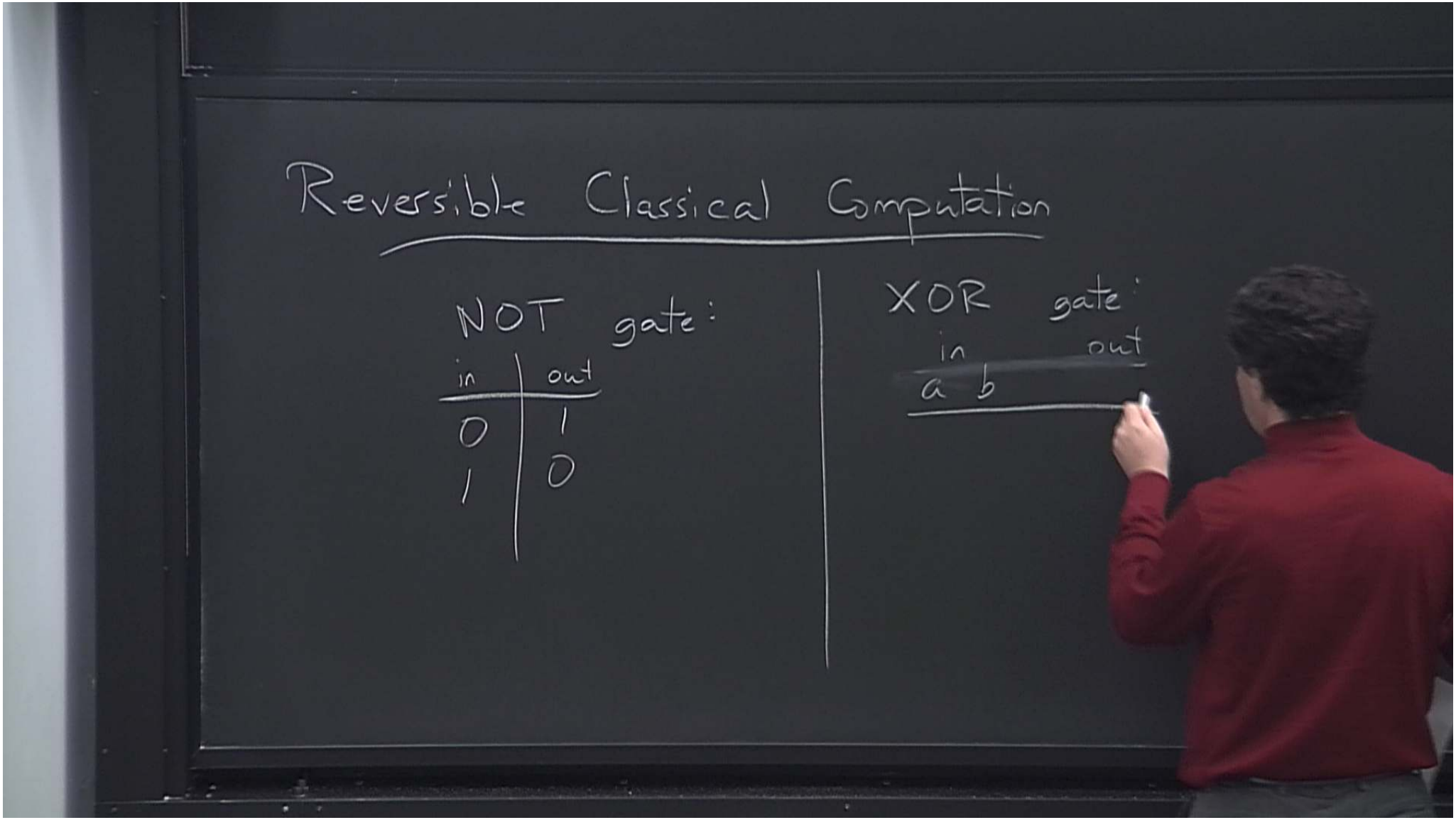
Q. Circuit: sequence of gates

Circuit diagrams:

Each line represents 1 bit

Time goes left to right

NOT  NOT a = a⊕1


Reversible Classical Computation
NOT gate:
in | out
0 | 1
1 | 0
XOR gate:
in out
a b

Reversible Classical Computation

NOT gate:

in	out
0	1
1	0

XOR gate:

in		out	
a	b	a	a⊕b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

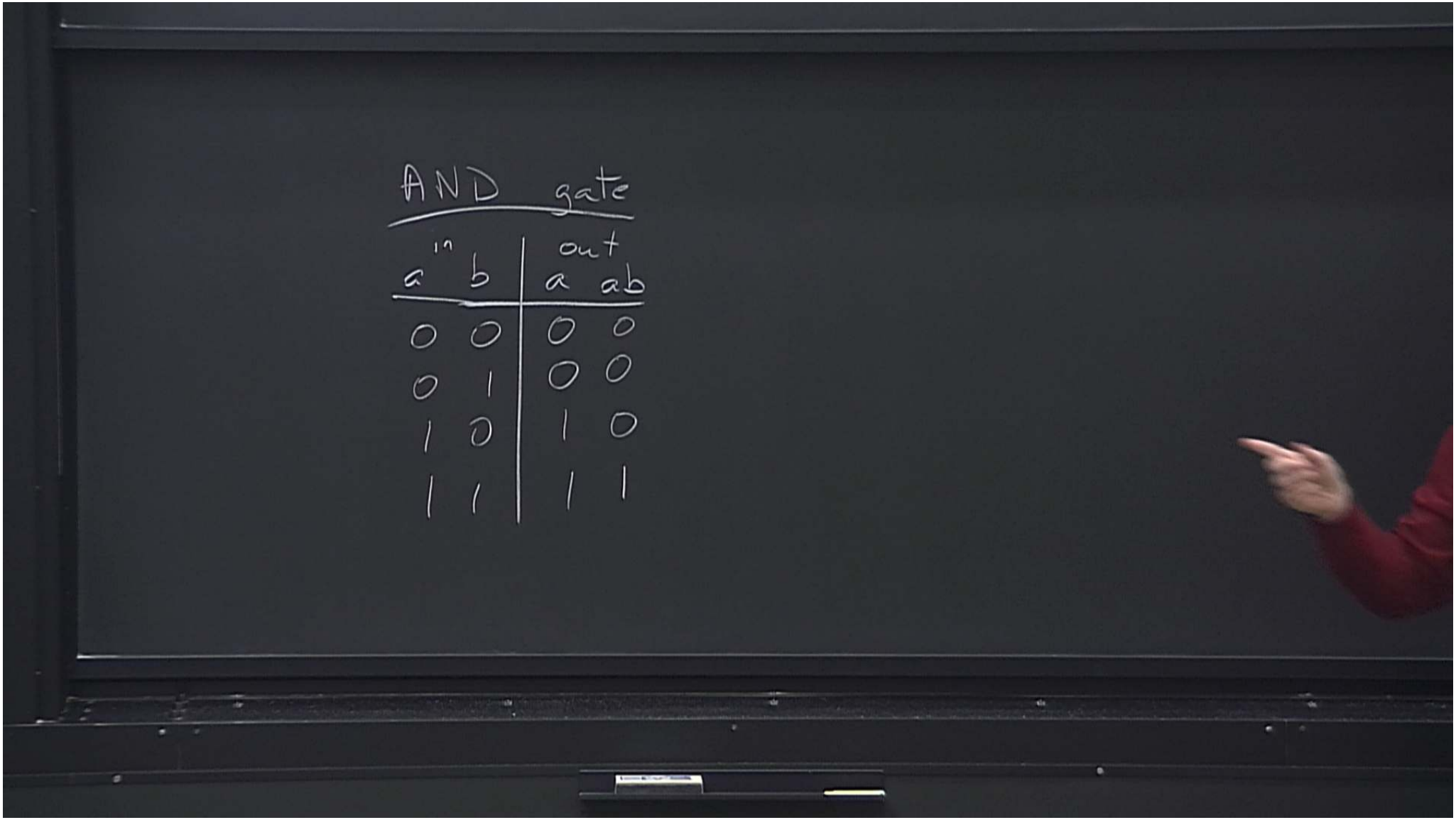
Reversible Classical Computation

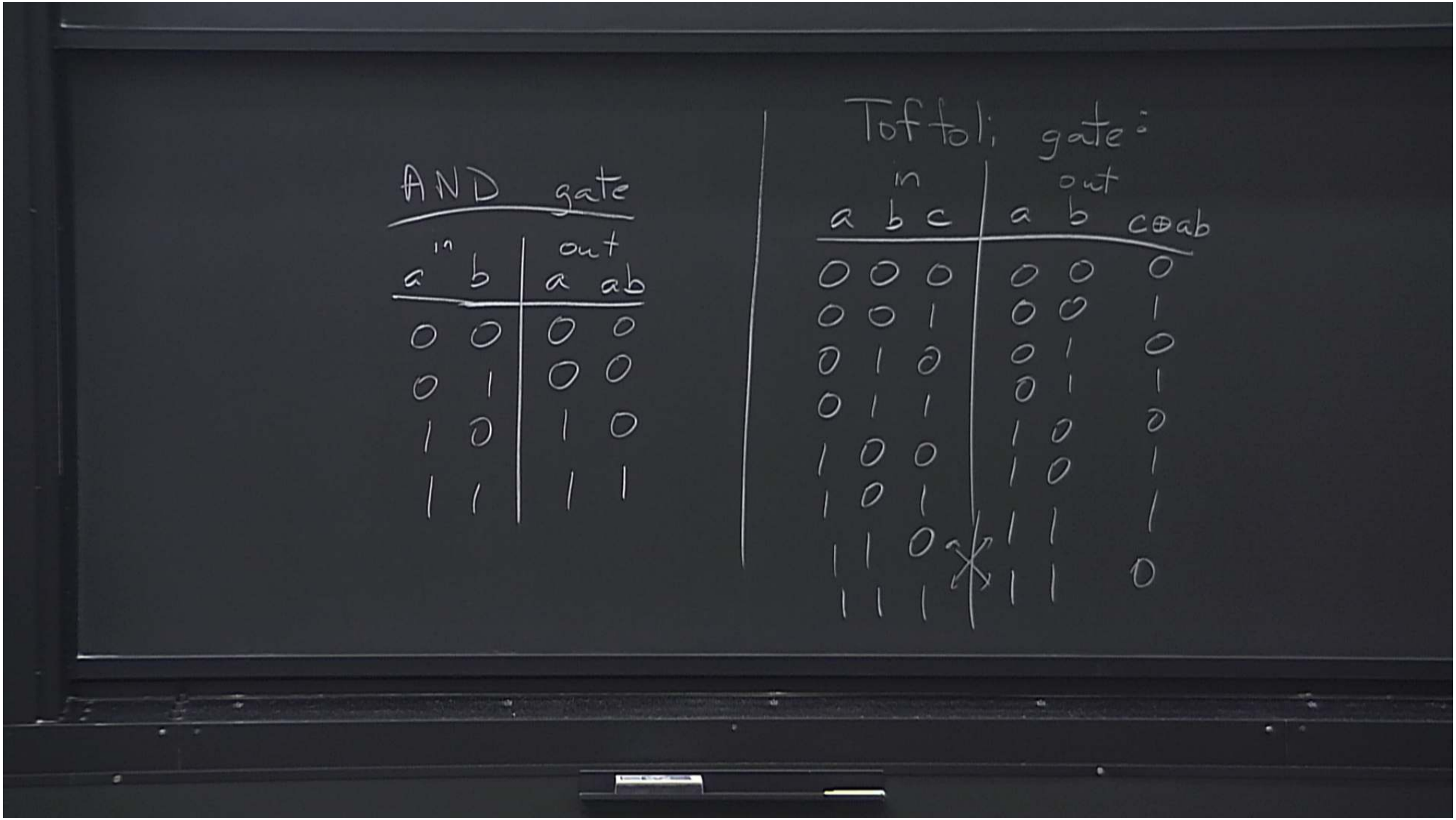
NOT gate:

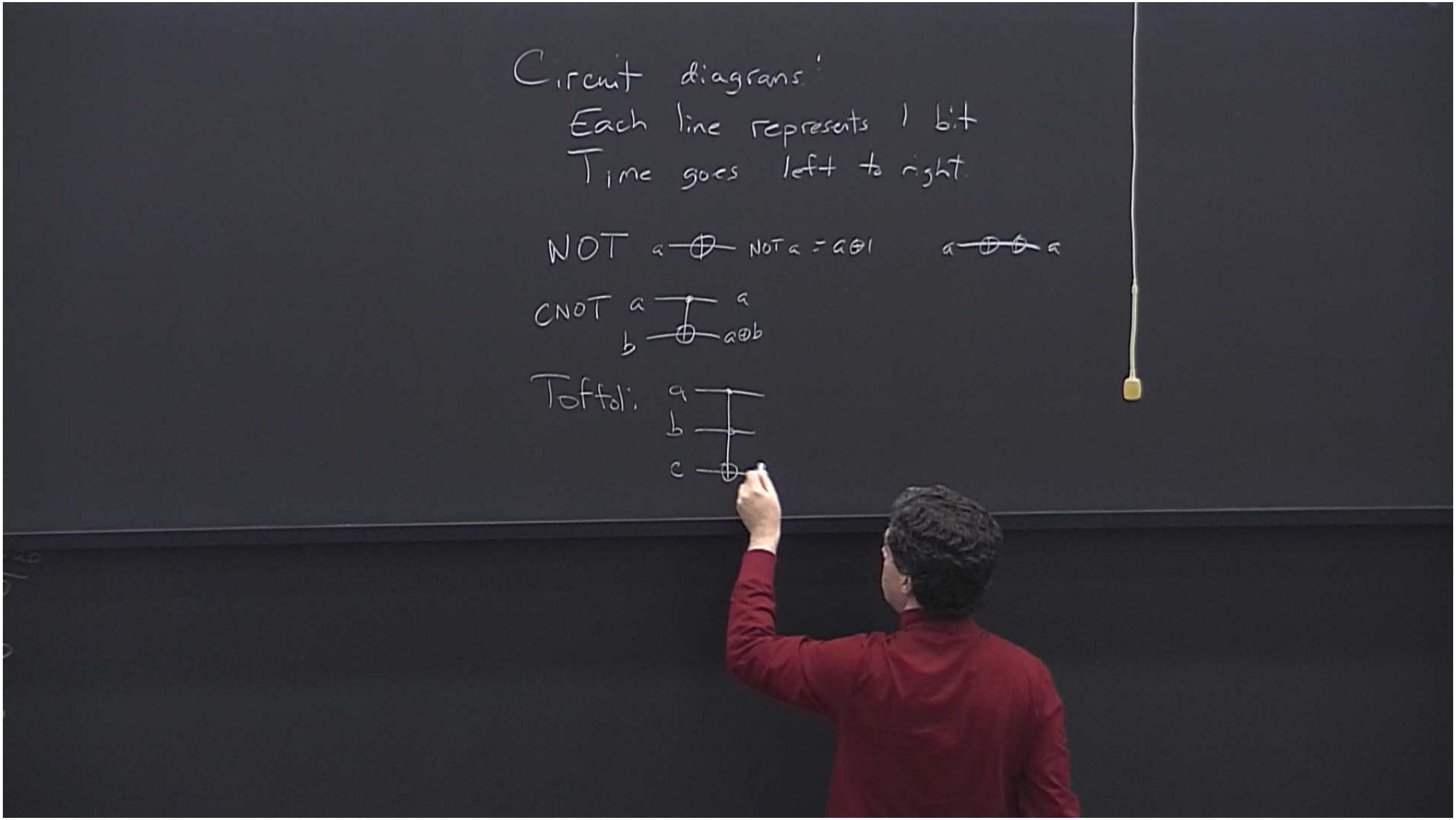
in	out
0	1
1	0

CNOT / XOR gate:

in		out	
a	b	a	a⊕b
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0


AND gate
in
a b
out
a ab
0 0 0 0
0 1 0 0
1 0 1 0
1 1 1 1


AND gate
in
a b
out
a ab
Toffoli gate:
in
a b c
out
a b c⊕ab

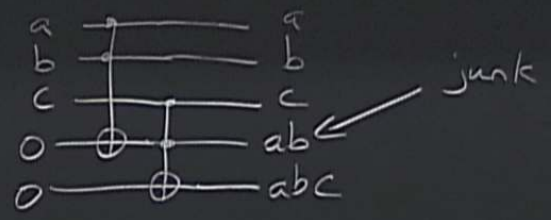

Circuit diagrams:
Each line represents 1 bit
Time goes left to right
NOT
a
NOT a = a⊕1
a
a
CNOT
a
b
a
a⊕b
Toffoli:
a
b
c

$a, b, c, \rightarrow f(a, b, c,)$

$\Rightarrow a, b, c, \rightarrow a, b, c, , f(a, b, c)$

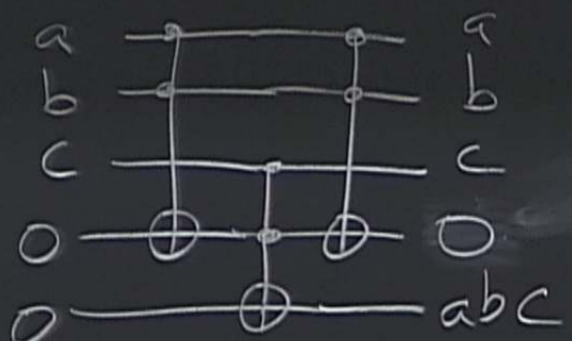
Reversible

(a AND b) AND c = abc

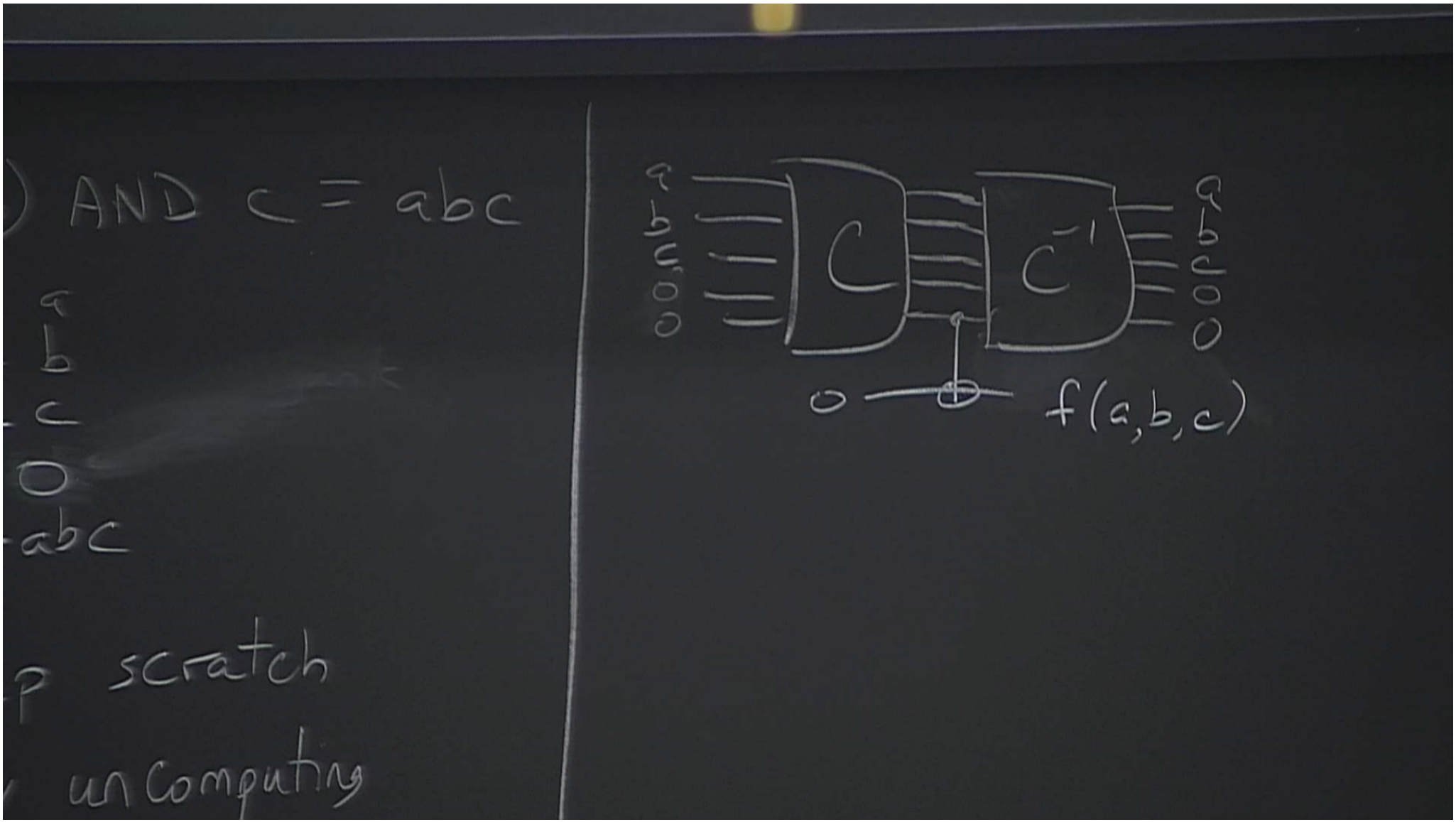


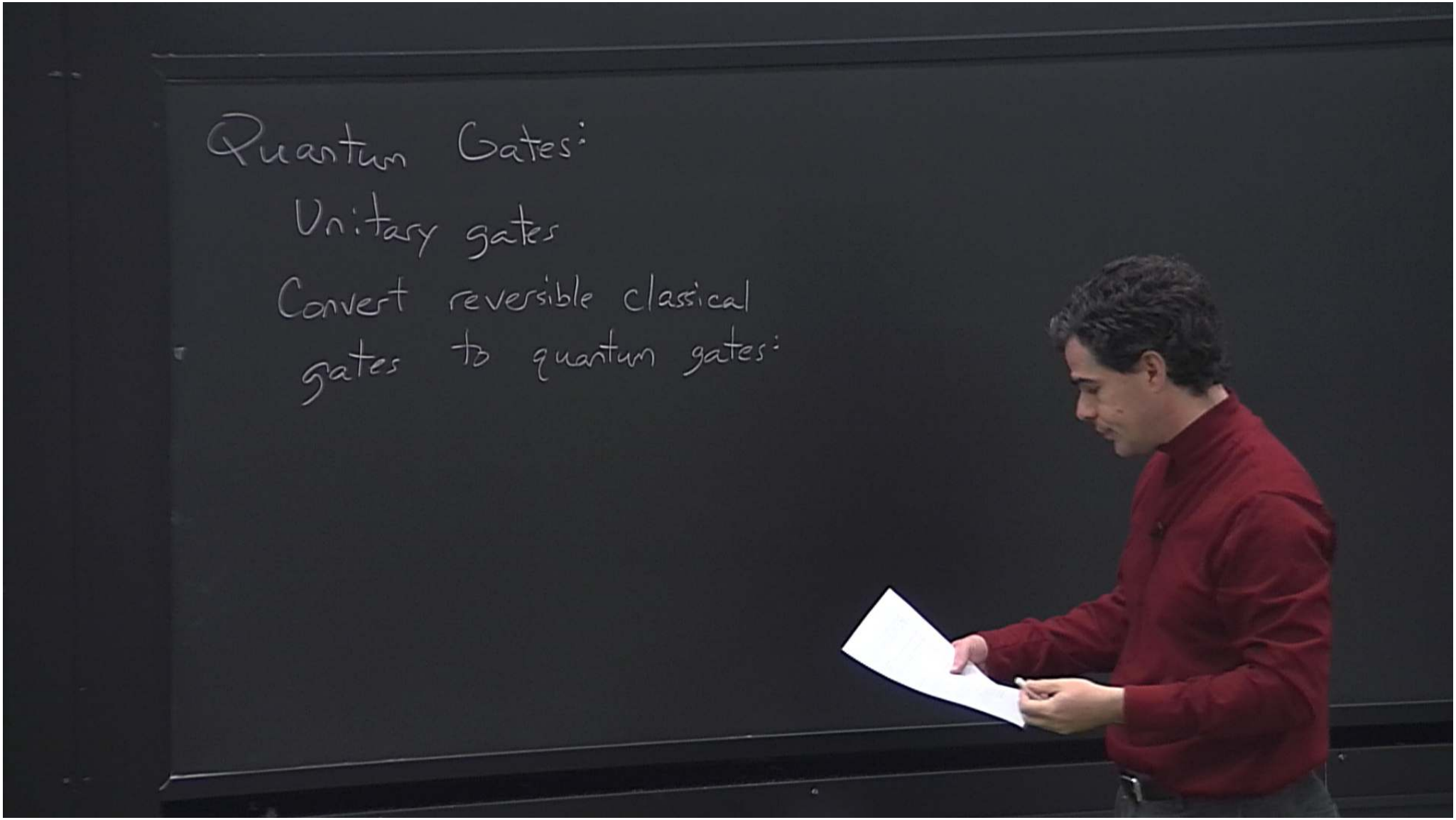
$f(a,b,c)$

(a AND b) AND c = abc



Clear up scratch bits by uncomputing them


) AND c = abc
a
b
c
0
abc
p scratch
uncomputing
a
b
c
0
0
C
C⁻¹
a
b
c
0
0
0
f(a,b,c)


Quantum Gates:
Unitary gates
Convert reversible classical
gates to quantum gates:

Quantum Gates:

Unitary gates

Convert reversible classical gates to quantum gates:

$$\text{CNOT}\,|a,b\rangle = |a, a\oplus b\rangle$$

$$\text{CNOT}(\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle)$$

$$=\alpha|00\rangle+\beta|01\rangle+\gamma|11\rangle+\delta|10\rangle$$

$$\text{Tof}\,|a,b,c\rangle = |a,b,c\oplus ab\rangle$$

Quantum Gates:

Unitary gates

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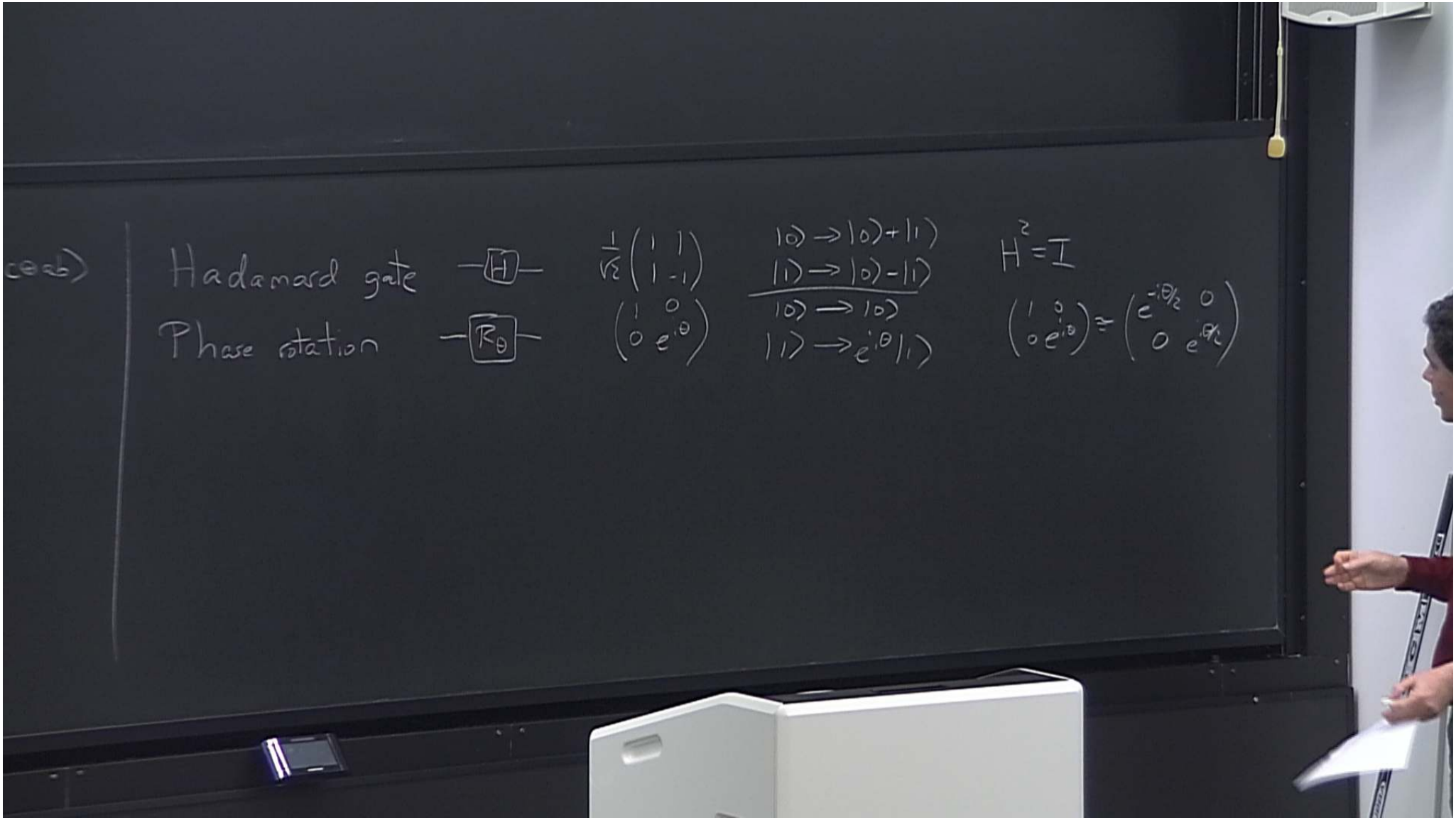
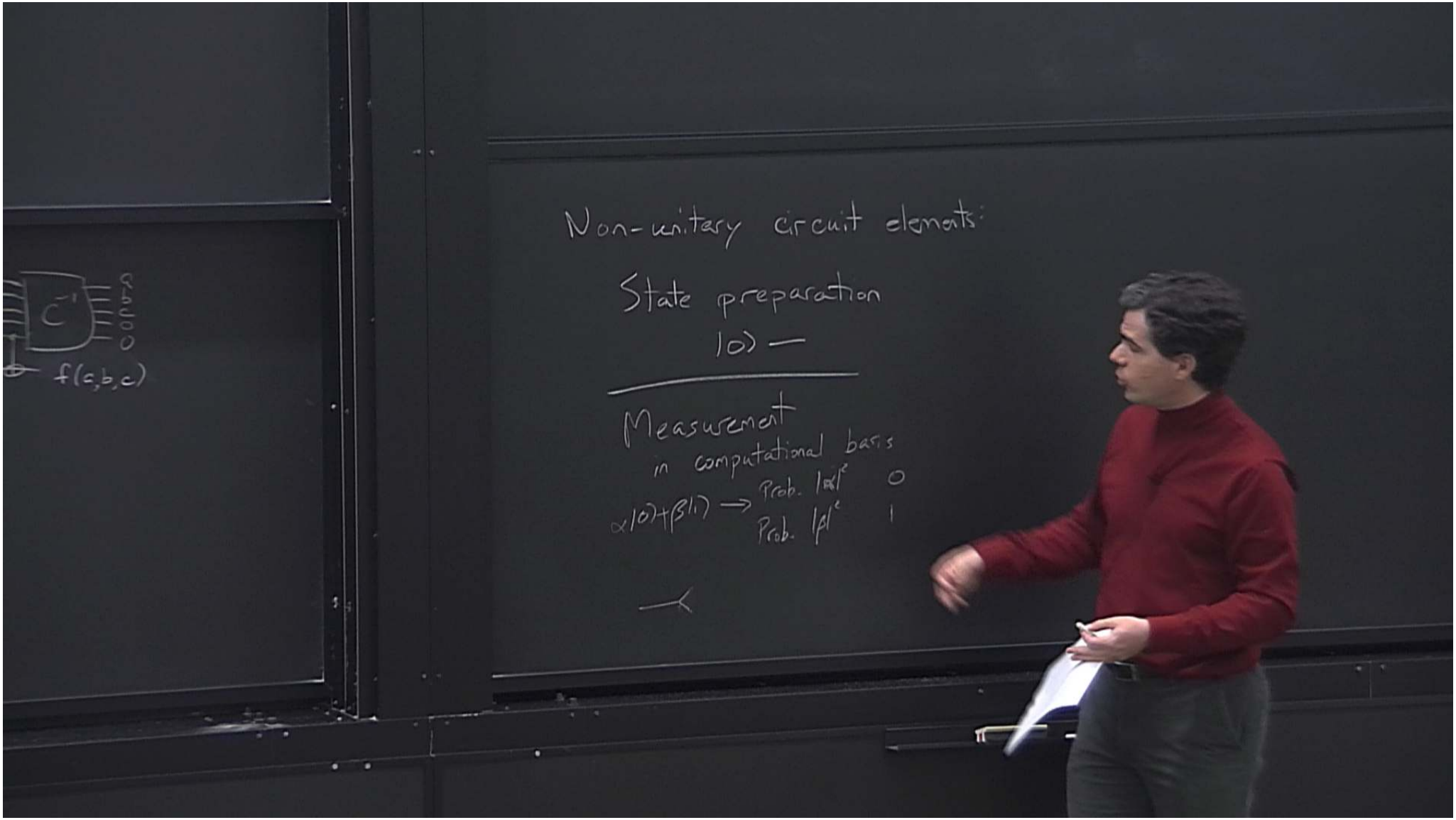
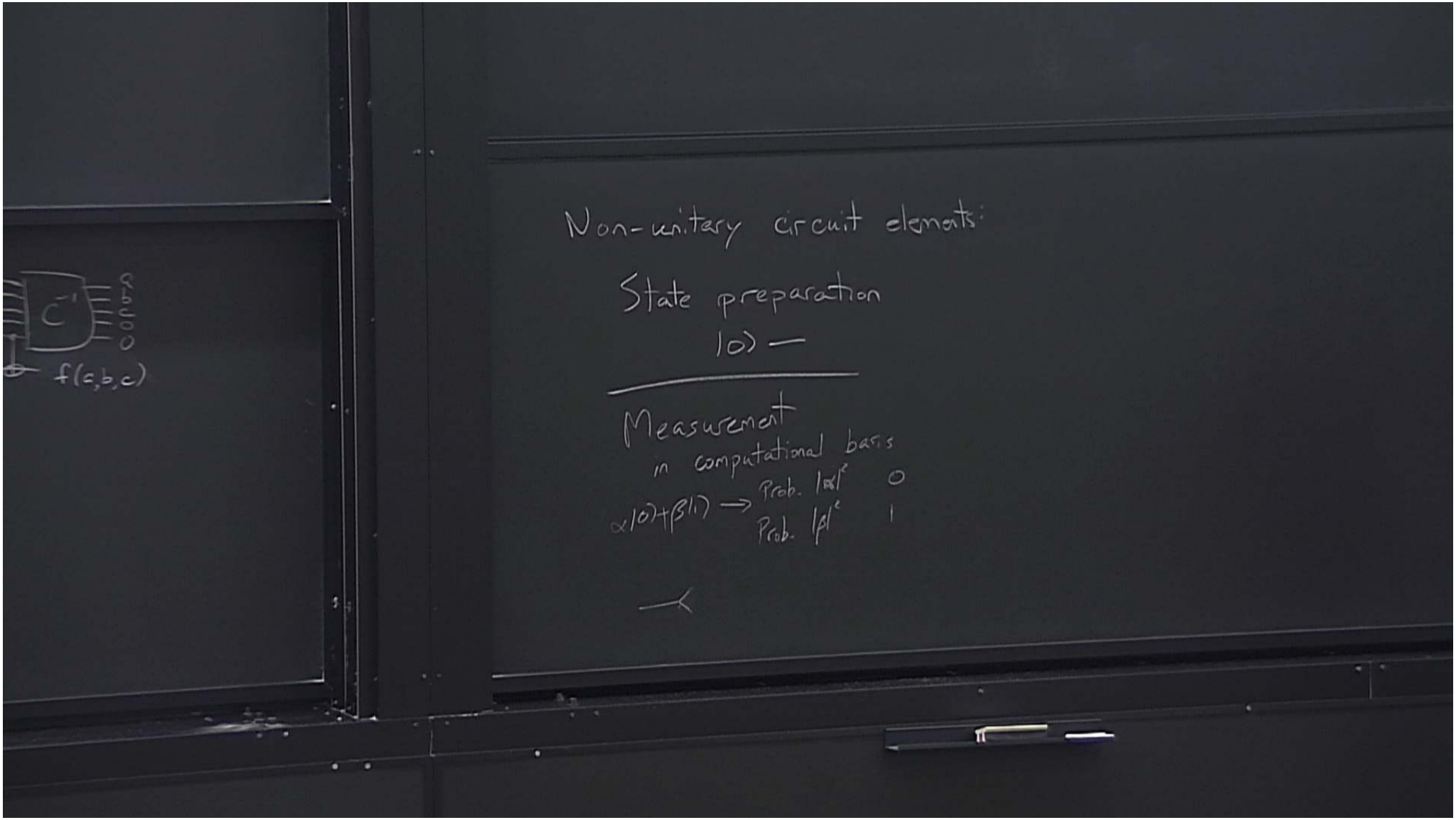
Tof $|a,b,c\rangle = |a,b,c\oplus ab\rangle$

$$\text{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$


Hadamard gate
H
$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}$
$|0\rangle \to |0\rangle + |1\rangle$
$|1\rangle \to |0\rangle - |1\rangle$
$H^2 = I$
Phase rotation
R_θ
$\begin{pmatrix}1 & 0\\ 0 & e^{i\theta}\end{pmatrix}$
$|0\rangle \to |0\rangle$
$|1\rangle \to e^{i\theta}|1\rangle$
$\begin{pmatrix}1 & 0\\ 0 & e^{i\theta}\end{pmatrix} \approx \begin{pmatrix}e^{-i\theta/2} & 0\\ 0 & e^{i\theta/2}\end{pmatrix}$


Non-unitary circuit elements:
State preparation
$|0\rangle$ —
Measurement
in computational basis
$\alpha|0\rangle+\beta|1\rangle \rightarrow$ Prob. $|\alpha|^2$ 0
Prob. $|\beta|^2$ 1
C^{-1}
a b c 0 0 0
f(a,b,c)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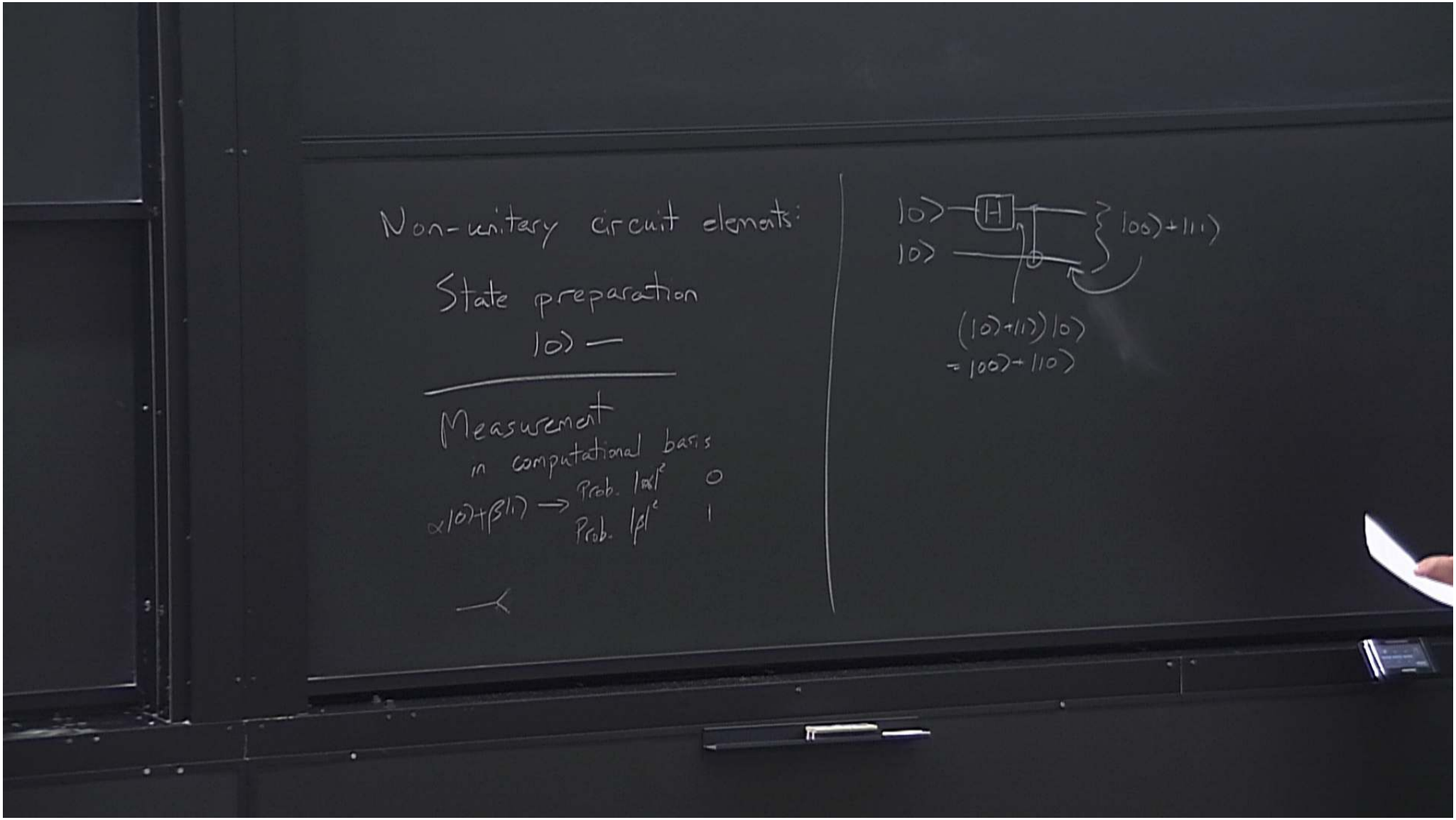
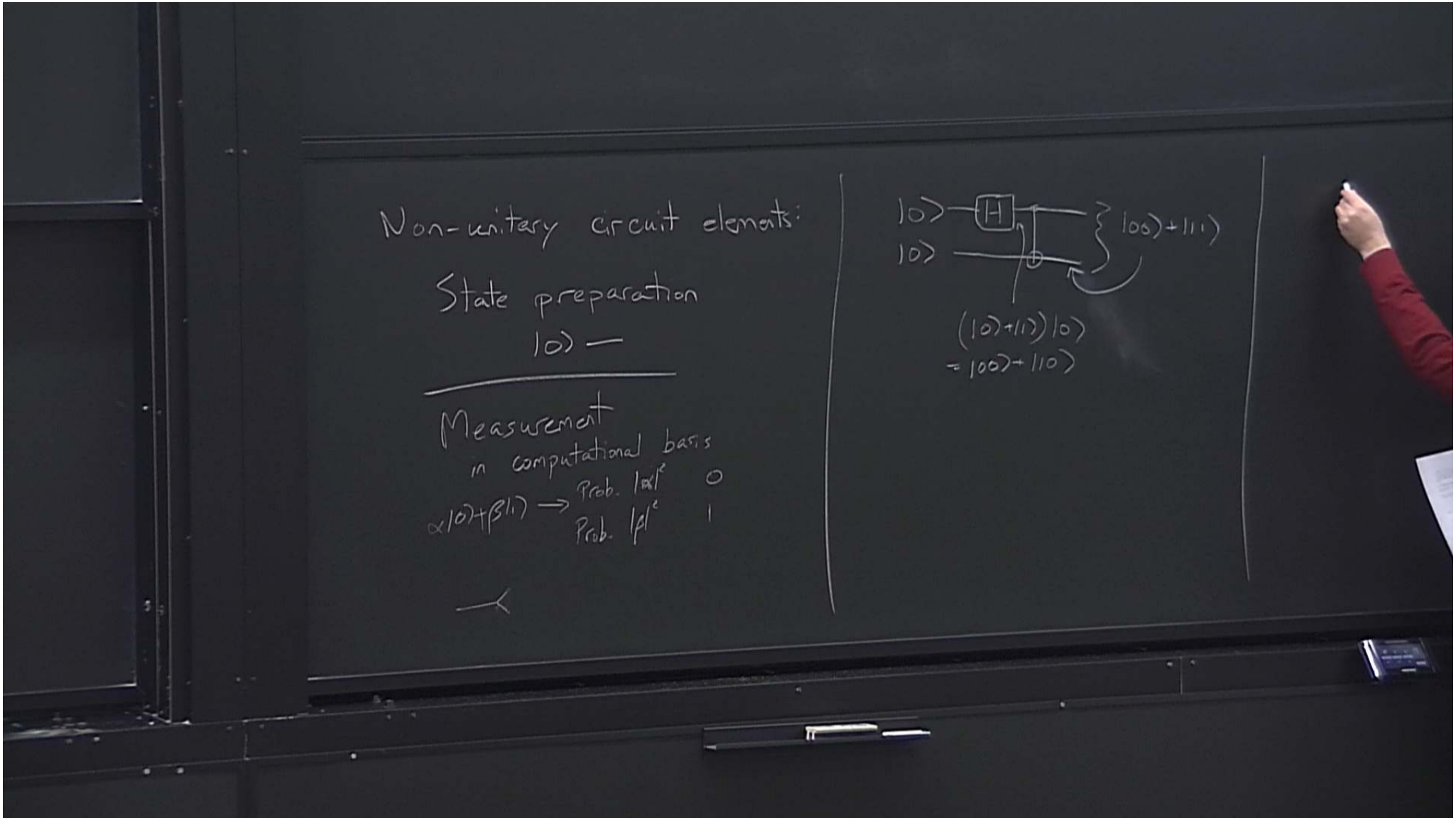

Non-unitary circuit elements:
State preparation
|0⟩ —
Measurement
in computational basis
α|0⟩+β|1⟩ → Prob. |α|² 0
Prob. |β|² 1
|0⟩
|0⟩
H
|00⟩+|11⟩
(|0⟩+|1⟩)|0⟩
= |00⟩+|10⟩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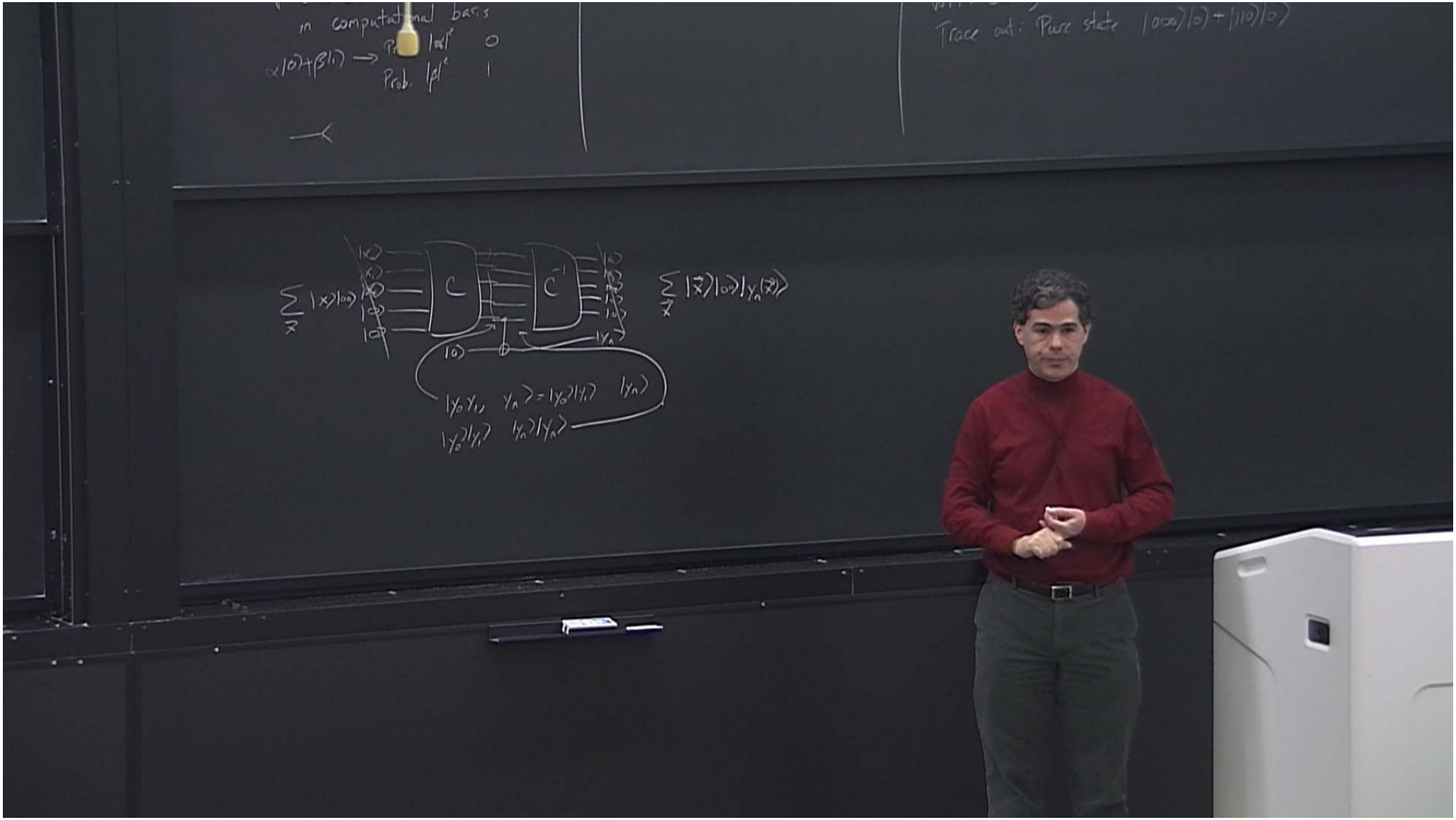
$|0\rangle$
$|0\rangle$
H
$|00\rangle+|11\rangle$
$(|0\rangle+|1\rangle)|0\rangle$
$=|00\rangle+|10\rangle$


in computational basis
Prob. $|\alpha|^2$ 0
Prob. $|\beta|^2$ 1
Trace out: Pure state $|000\rangle|0\rangle + |110\rangle|0\rangle$
$\sum_x |x\rangle|00\rangle$
C
C^{-1}
$\sum_x |\vec{x}\rangle|00\rangle|y_n(\vec{x})\rangle$
$|0\rangle$
$|y_n\rangle$
$|y_0 y_1 \ldots y_n\rangle = |y_0\rangle|y_1\rangle \ldots |y_n\rangle$
$|y_0\rangle|y_1\rangle \ldots |y_n\rangle|y_n\rangle$