Title: Quantum Gravity Review-8

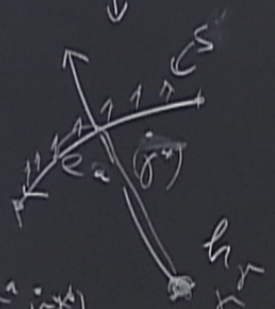
Date: Feb 26, 2015 10:15 AM

URL: http://www.pirsa.org/15020059

Abstract:

Fluxes

$$e_a = e_a^j T^j = \tilde{E}_{ba} \tilde{t}^{bj} T^j$$



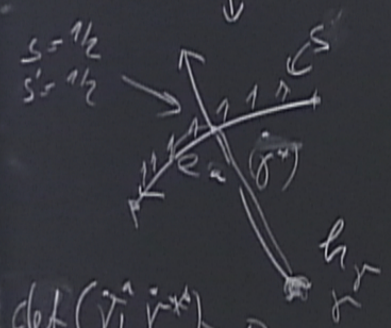
$E_\gamma =$

$\det(\vec{\gamma}_i \vec{\gamma}^*) > 0$

$h_{\gamma, \gamma^*}(s)$

Fluxes

$$e_a = e_a^j T^j = \tilde{E}_{ba} \tilde{E}^{bj} T^j$$



$E_\gamma =$

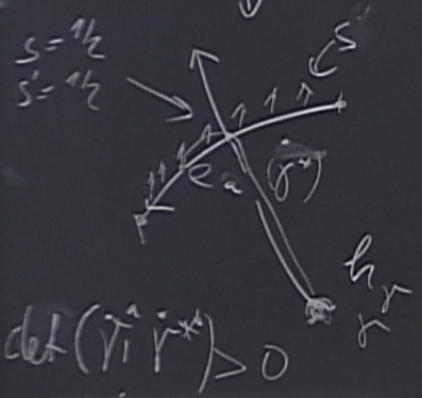
$\det(\vec{x}_i \vec{x}'^a) > 0$

$h_{\gamma, \vec{x}'}(s)$ parallel transport from $\vec{x}'(0)$ to $\vec{x}'(s)$

Fluxes

$$e_a = e_a^j T^j = \tilde{E}_{ba} \tilde{E}^{bj} T^j$$

$$E_\gamma = h_{\gamma, \gamma^*}(s)\, e_a(\gamma^*(s))\, \left(h_{\gamma, \gamma^*}(s)\right)^{-1}$$



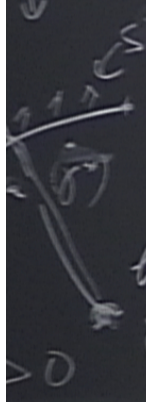
$\det(\dot{\gamma}, \dot{\gamma}^*) > 0$

$h_{\gamma, \gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$

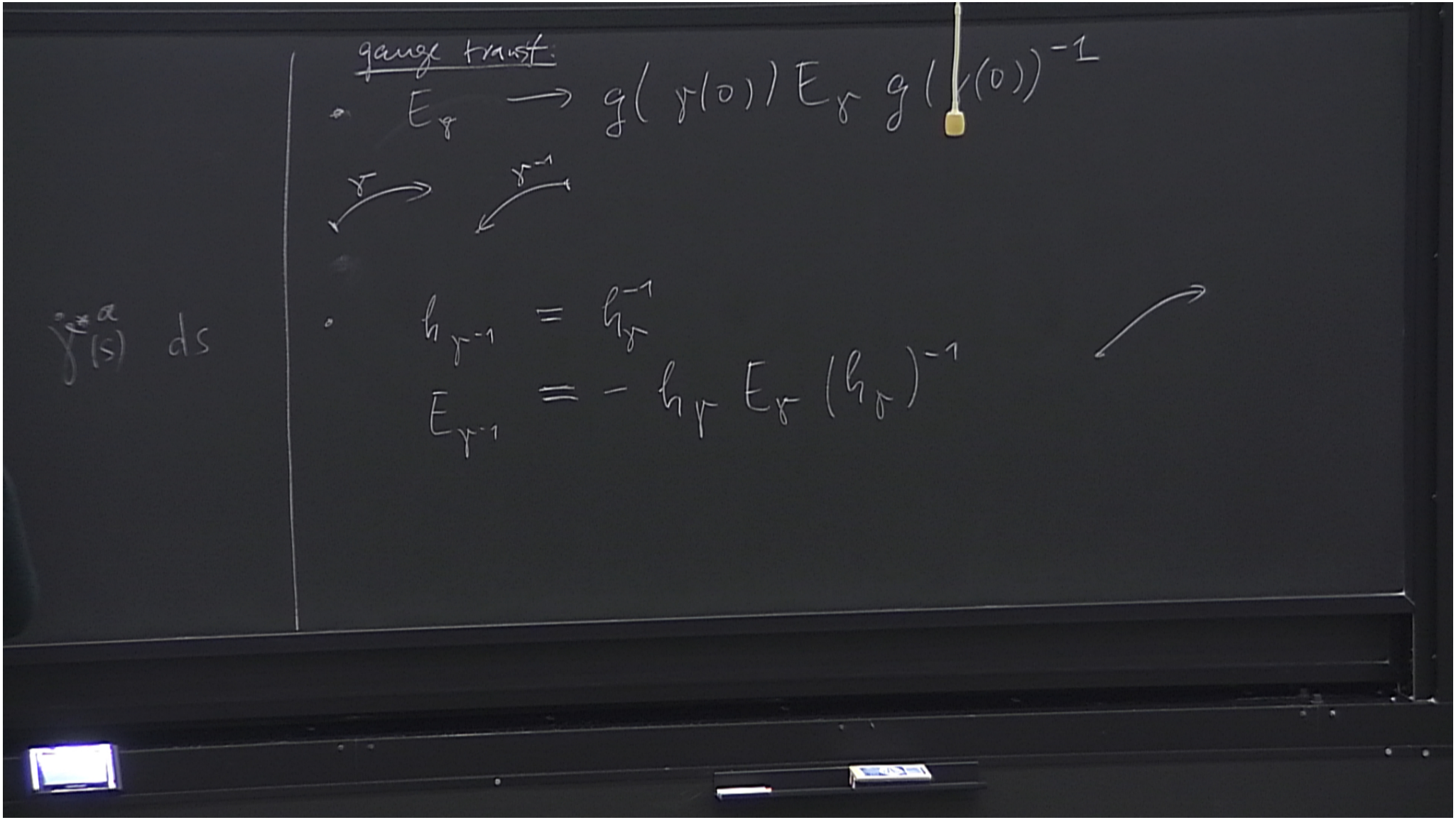
Fluxes

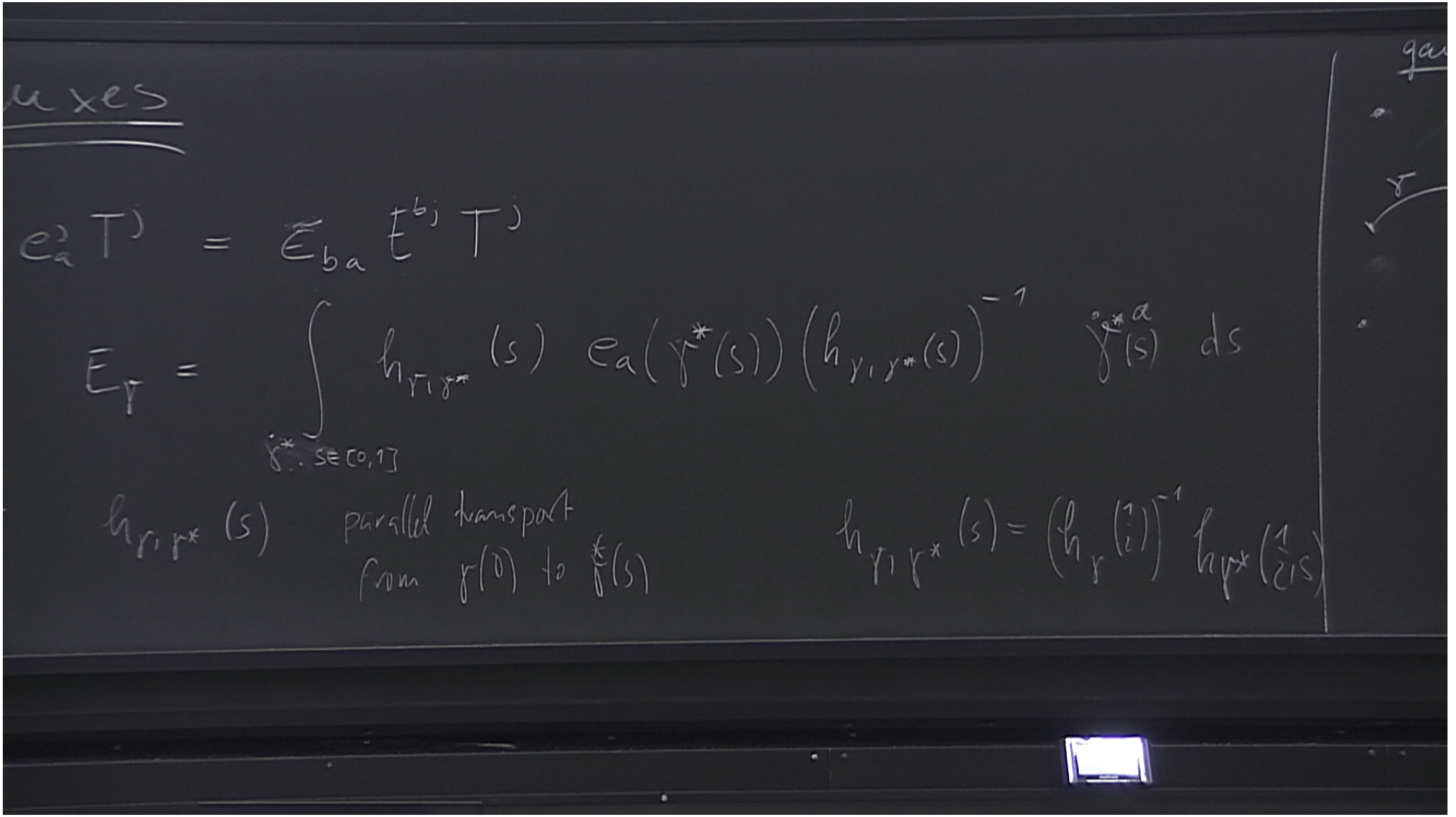
$$e_a = e_a^j T^j = \tilde{E}_{ba} \tilde{E}^{bj} T^j$$

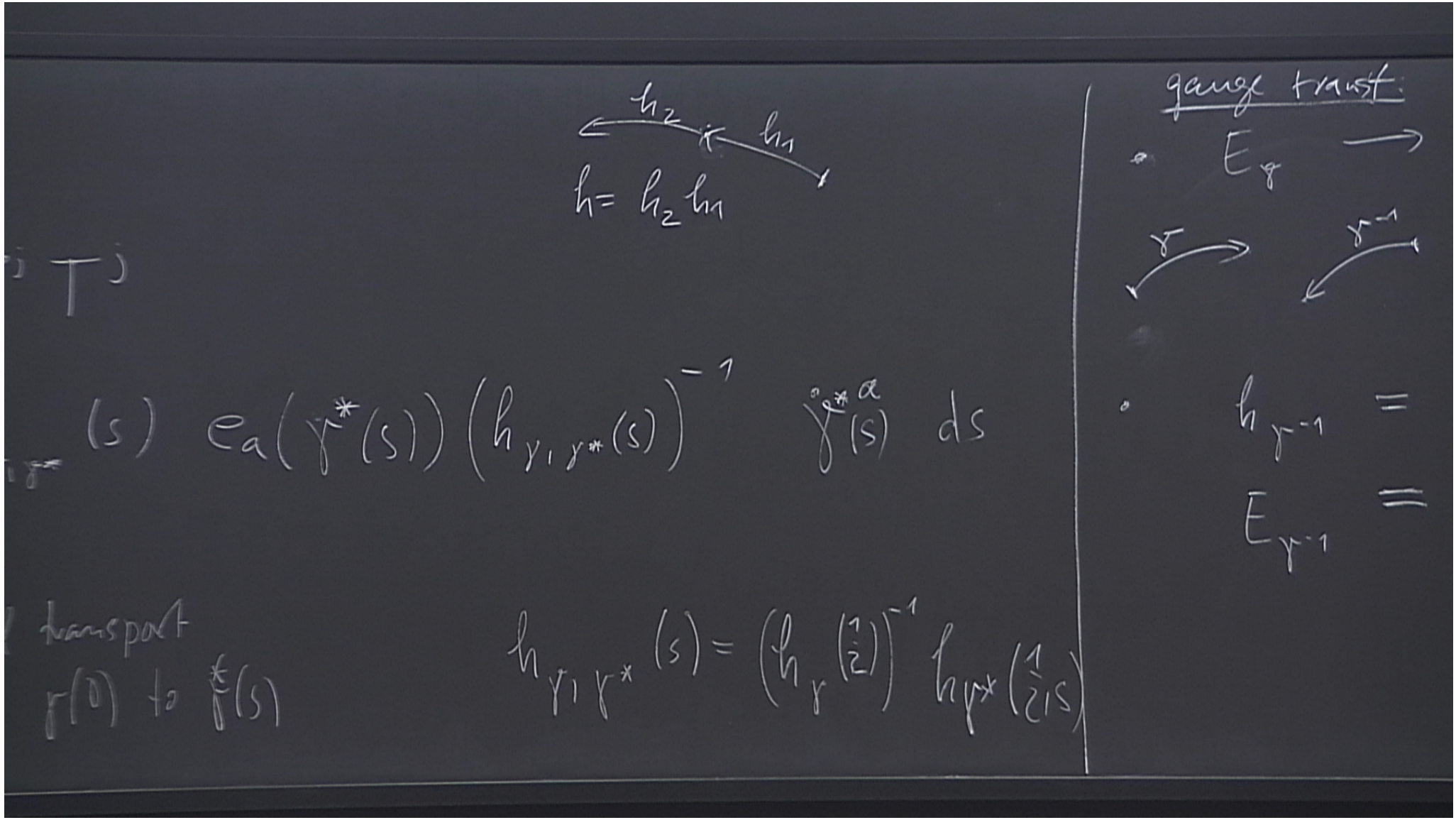
$$E_\gamma = \int_{\gamma^*,\, s\in[0,1]} h_{\gamma,\gamma^*}(s)\, e_a(\gamma^*(s))\, (h_{\gamma,\gamma^*}(s))^{-1}\, \dot{\gamma}^{*a}(s)\, ds$$



$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$


gauge transf.
$E_\gamma \longrightarrow g(\gamma(0)) E_\gamma g(\gamma(0))^{-1}$
γ
γ^{-1}
$\dot{\gamma}^a(s)\, ds$
$h_{\gamma^{-1}} = h_\gamma^{-1}$
$E_{\gamma^{-1}} = -h_\gamma E_\gamma (h_\gamma)^{-1}$


uxes
$e_a^j T^j = \tilde{E}_{ba} E^{bj} T^j$
$E_\gamma = \int_{\gamma^*,\ s\in[0,1]} h_{\gamma,\gamma^*}(s)\, e_a(\gamma^*(s)) \left(h_{\gamma,\gamma^*(s)}\right)^{-1} \dot{\gamma}^{*a}(s)\, ds$
$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$
$h_{\gamma,\gamma^*}(s) = \left(h_\gamma(\tfrac{1}{2})\right)^{-1} h_{\gamma^*}(\tfrac{1}{2}s)$
gau
γ


h_2
h_1
$h = h_2 h_1$
T^j
$(s)\ e_a(\gamma^*(s))\left(h_{\gamma,\gamma^*(s)}\right)^{-1}\ \dot{\gamma}^{*a}(s)\ ds$
transport
$\gamma(0)$ to $\gamma^*(s)$
$h_{\gamma,\gamma^*}(s) = \left(h_\gamma(\frac{1}{2})\right)^{-1} h_{\gamma^*}(\frac{1}{2}s)$
gauge transf.
E_γ
γ
γ^{-1}
$h_{\gamma^{-1}} =$
$E_{\gamma^{-1}} =$

Fluxes

$e_a = e_a^j T_j = E^b_{ja} \tilde{E}^{ja} T_j$

$E_\gamma = \int_{\gamma^* \text{ sec. of } S} h_{\gamma, \gamma^*(s)} \, e_a(\gamma^*(s)) \, (h_{\gamma, \gamma^*(s)})^{-1} \, \dot{\gamma}^a(s) \, ds$

$\det(\dot{\gamma}, \dot{\gamma}^*) > 0$

$h_{\gamma, \gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma(s)$

$h_{\gamma, \gamma^*}(s) = (h_\gamma(\tfrac{1}{2}))^{-1} \, h_{\gamma^*}(\tfrac{1}{2}|s)$

$h = h_2 h_1$

gauge transf.

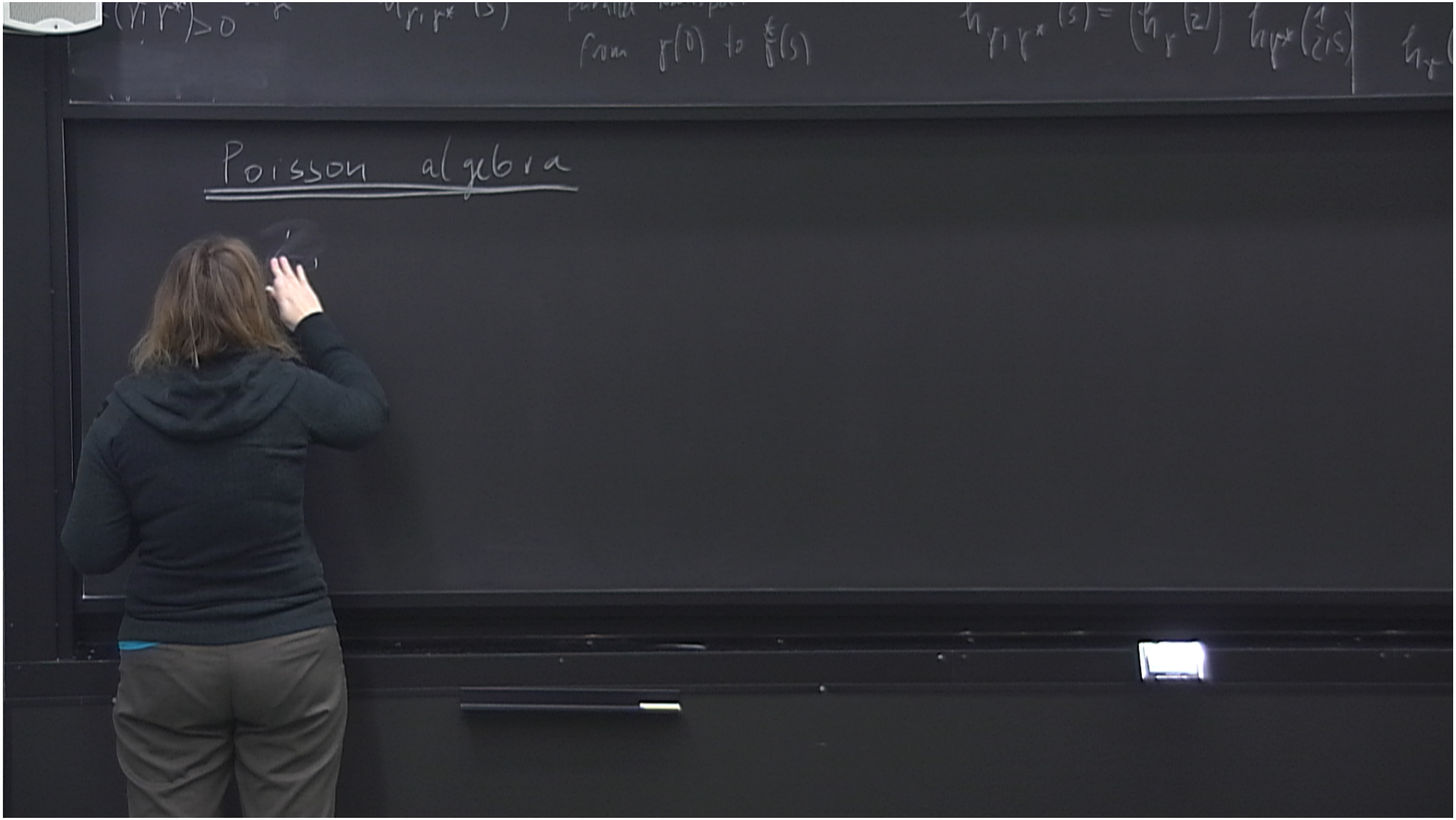
$E_\gamma \to g(\gamma(0)) \, E_\gamma \, g(\gamma(0))^{-1}$

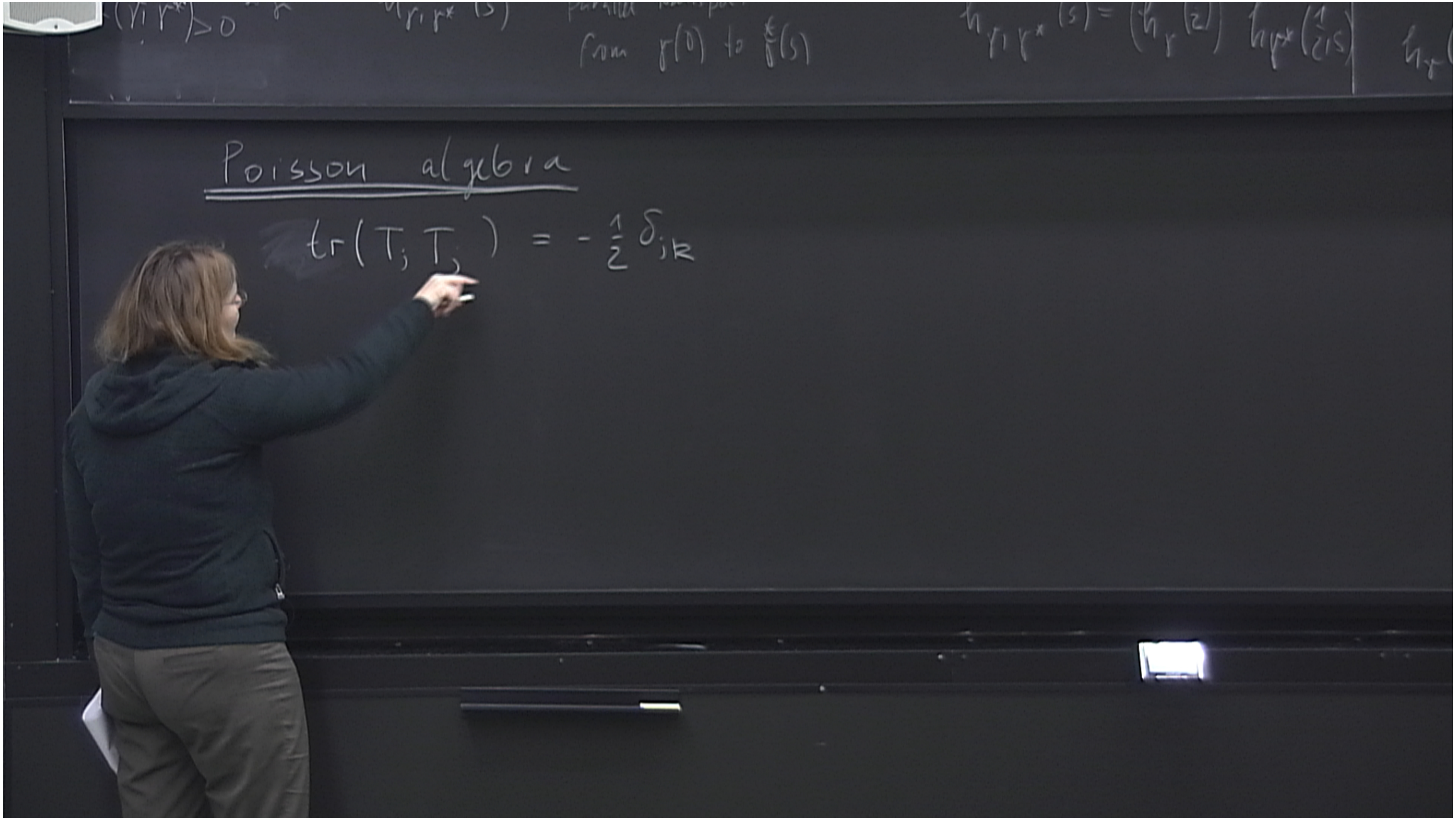
$h_{\gamma^{-1}} = h_\gamma^{-1}$

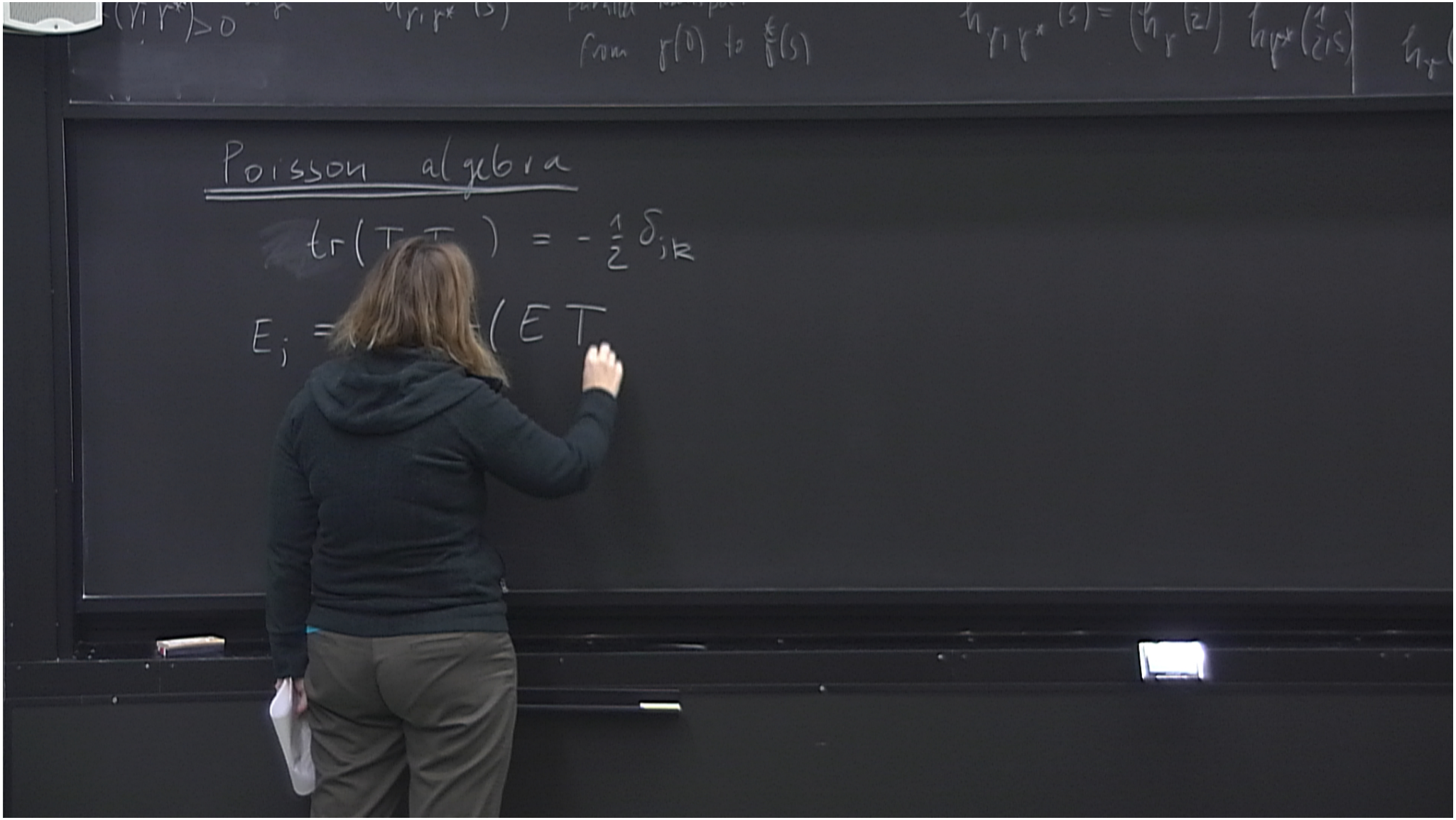
$E_{\gamma^{-1}} = -h_\gamma E_\gamma (h_\gamma)^{-1}$

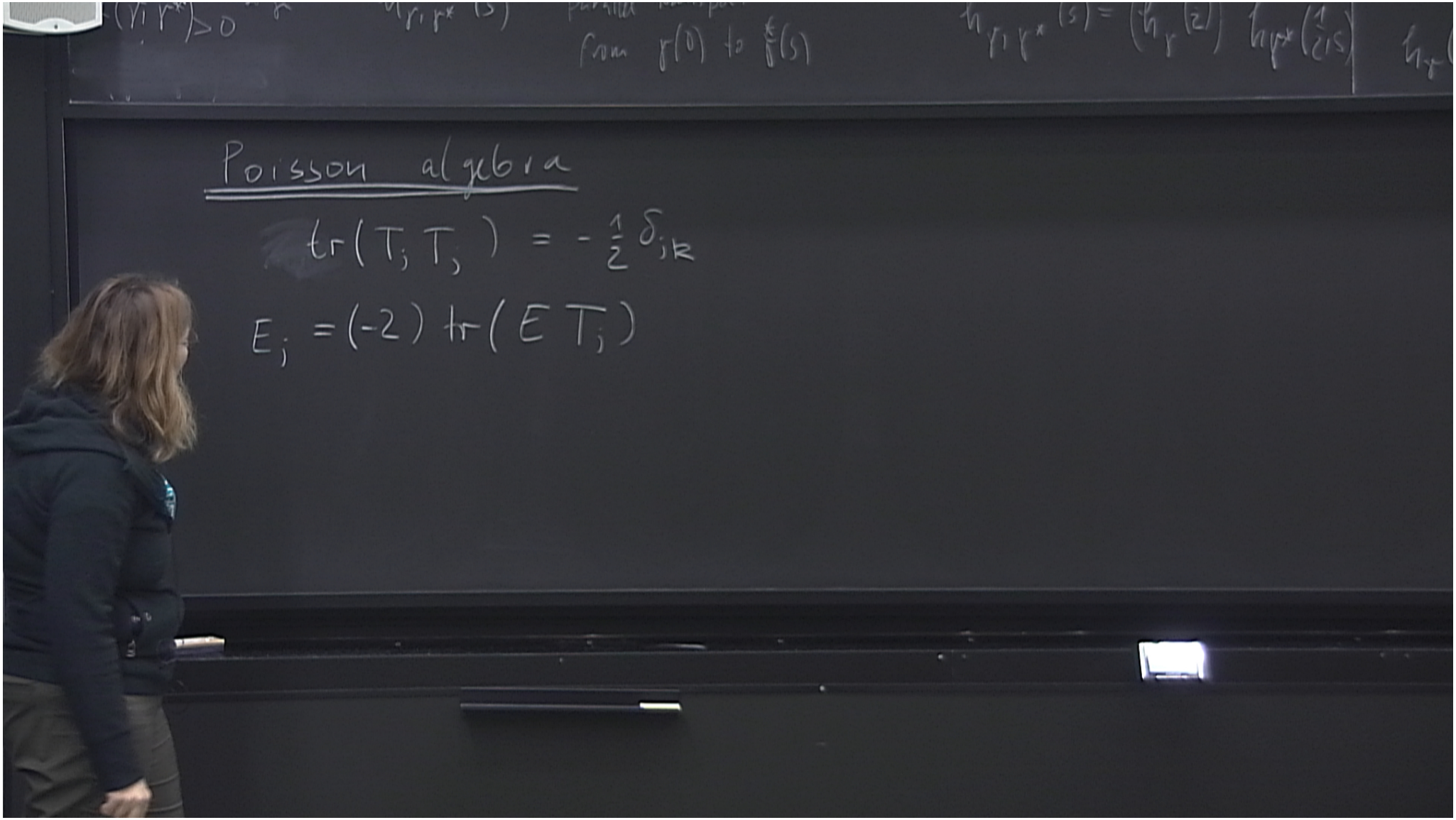
$h_\gamma(s) = \mathrm{P}[-\int_\gamma A] = \sum_{n=0} (-1)^n \int_0^s ds_n \int_0^{s_n} ds_{n-1} \cdots \int_0^{s_2} A(s_1) \cdots A(s_n)$

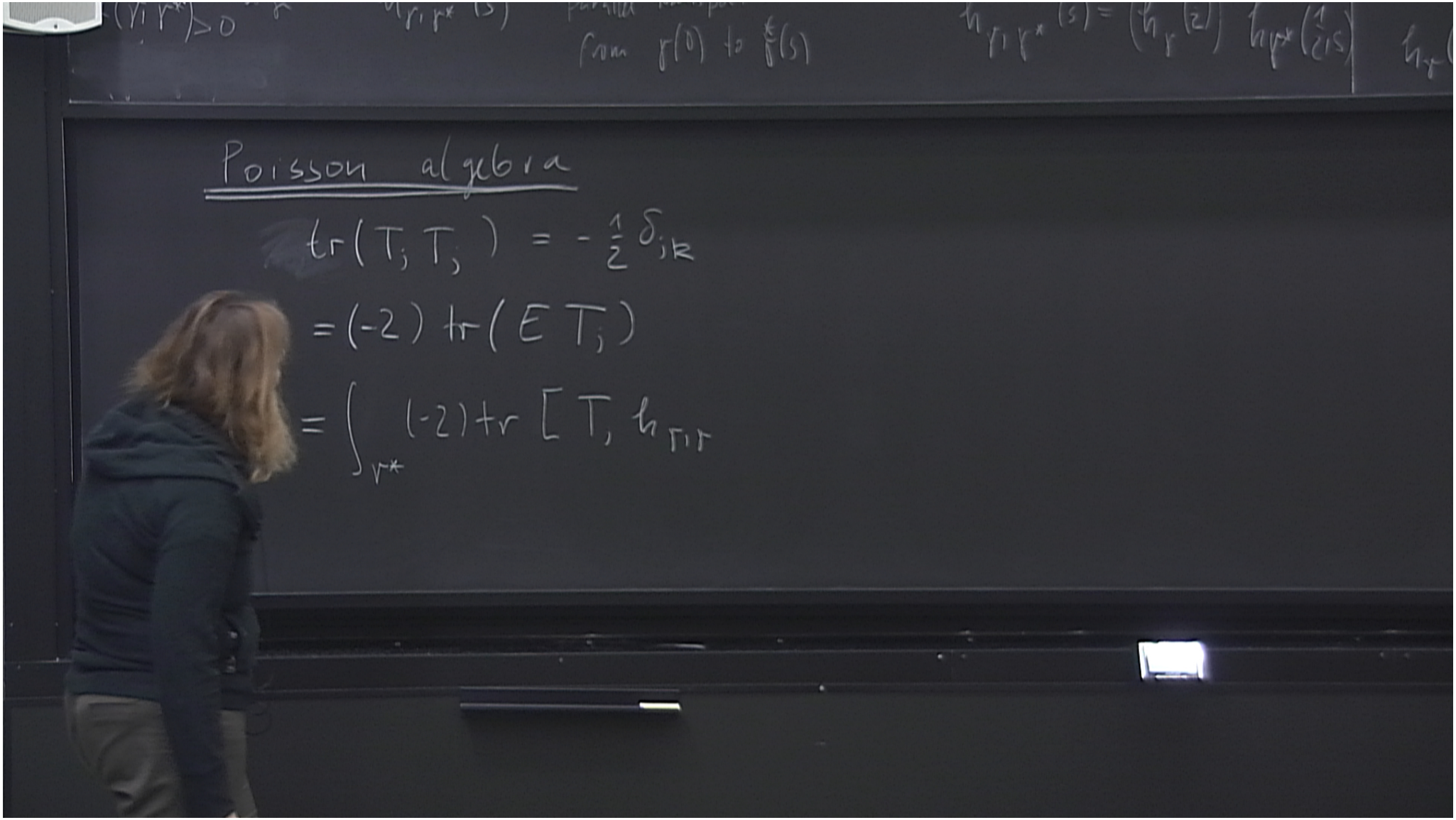
Poisson algebra

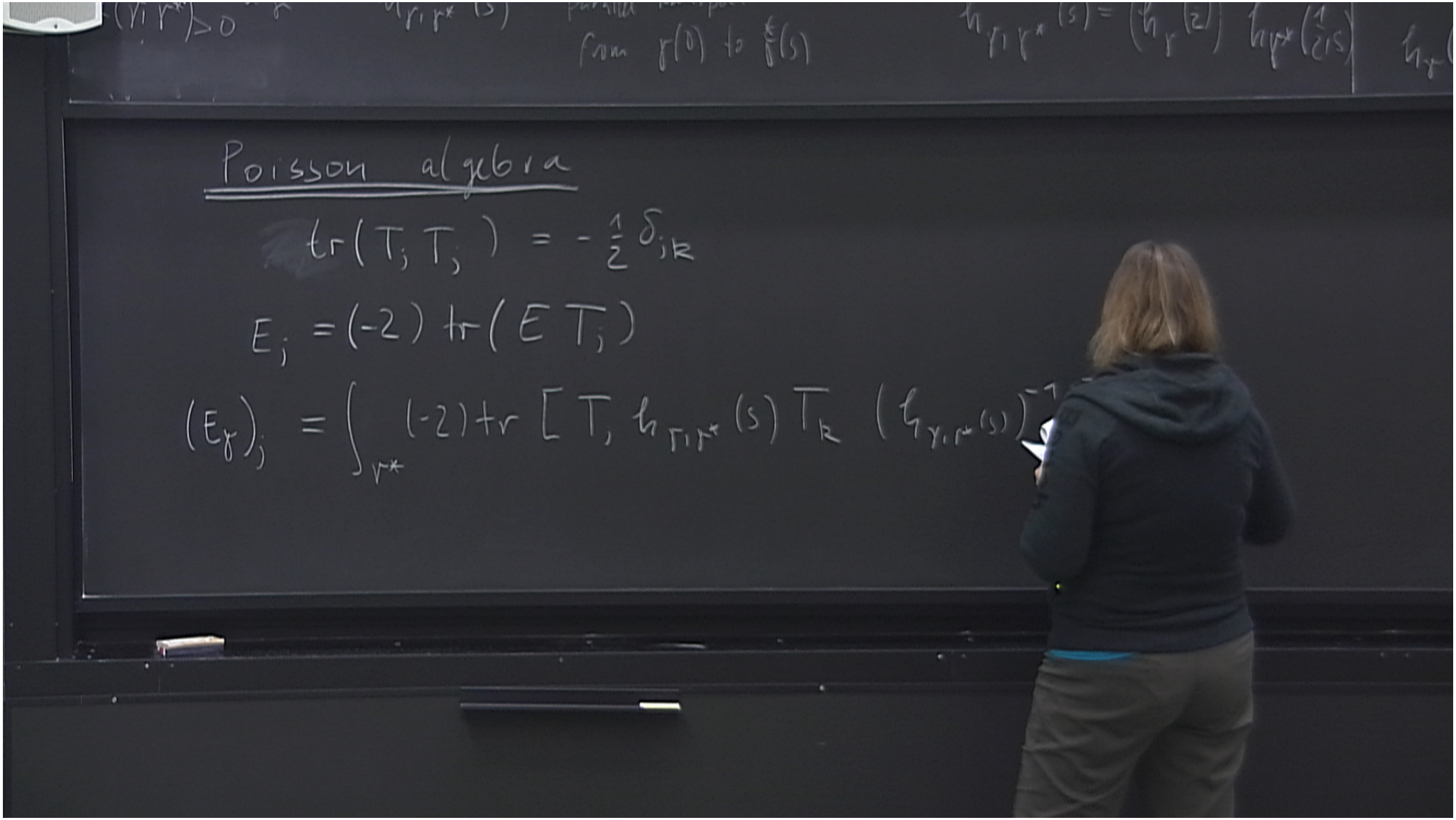

Poisson algebra

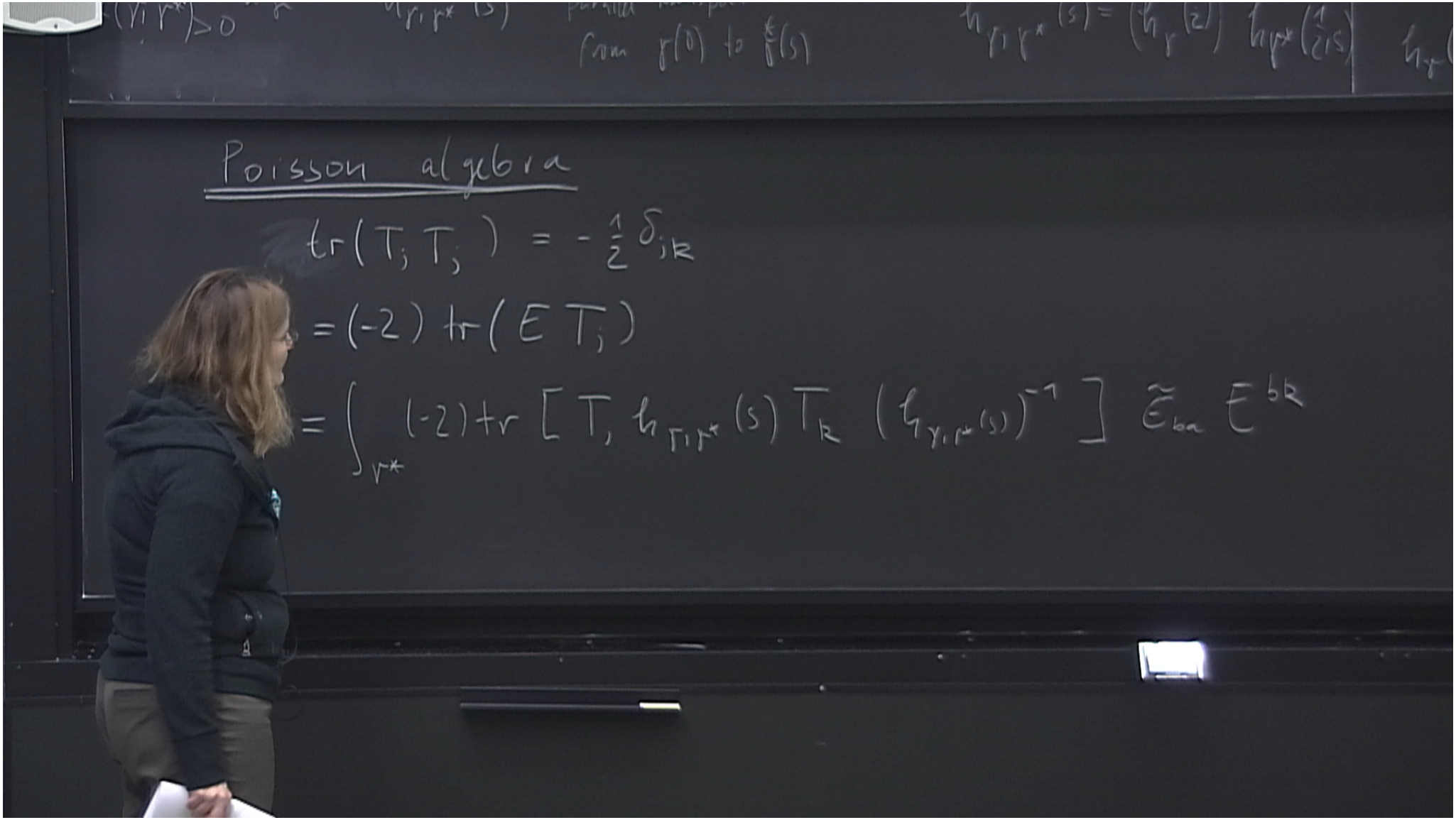

Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$

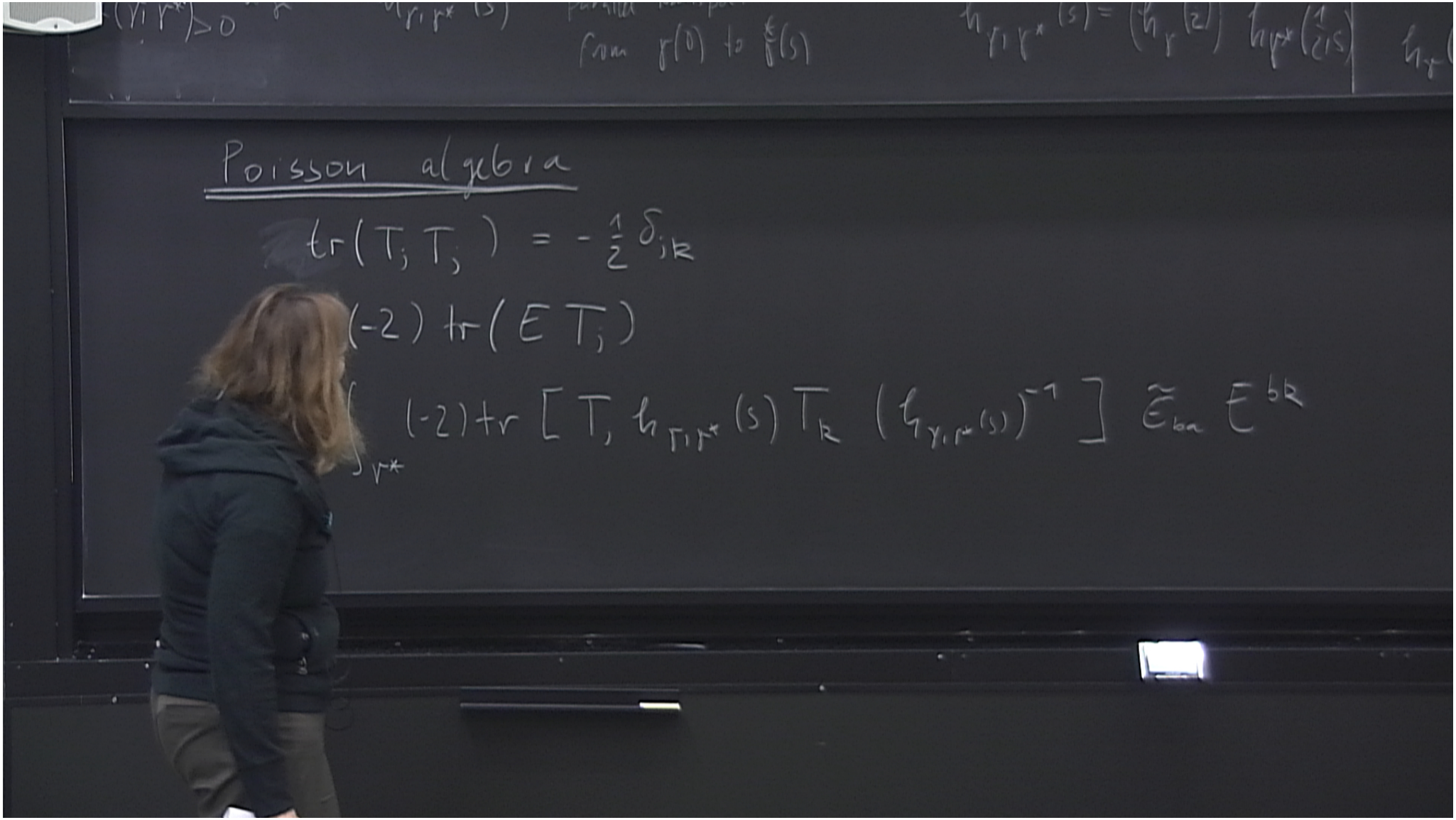

Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ij}$
$E_i = (ET_i$

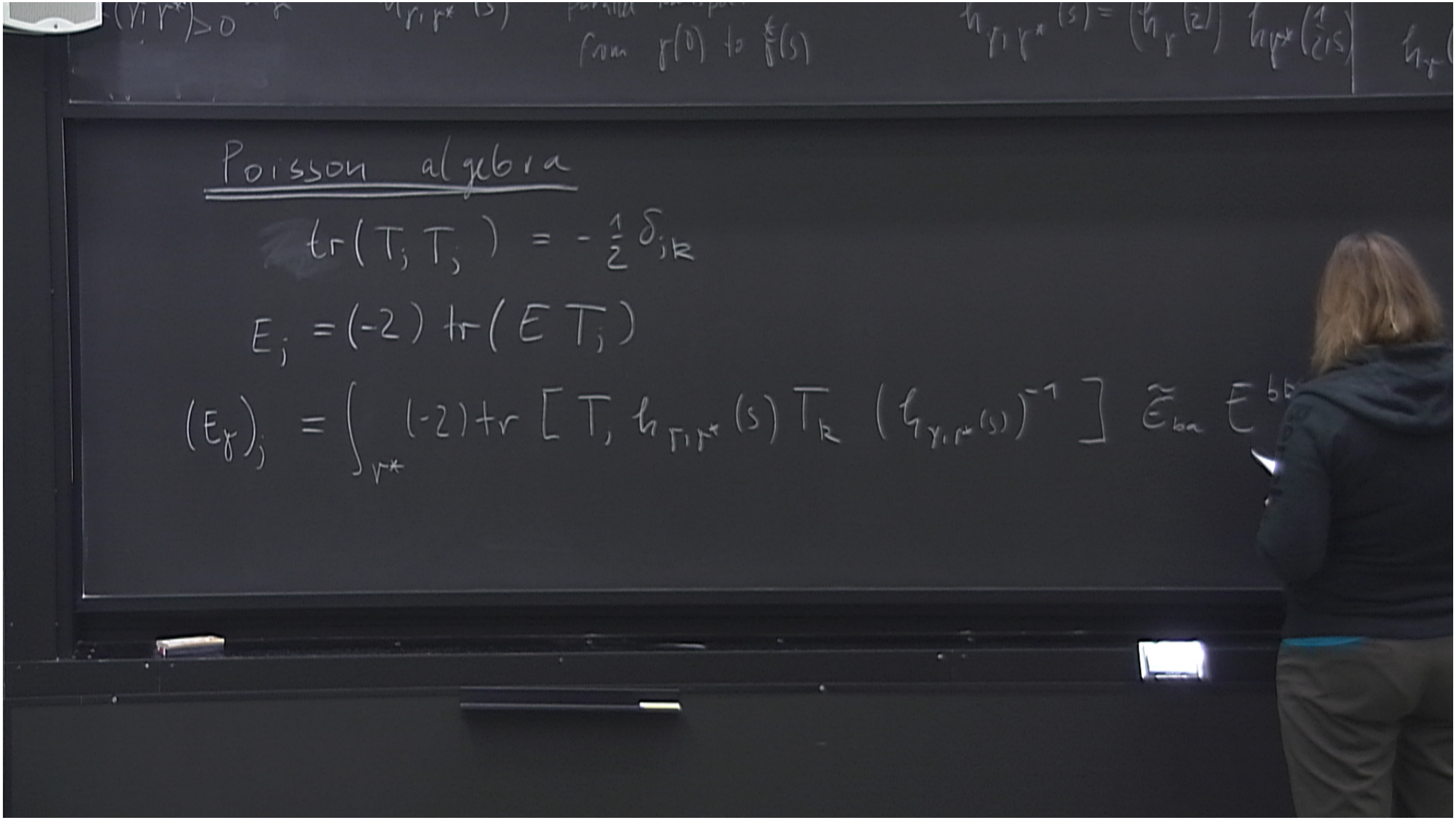

Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$
$E_i = (-2)\,tr(E\,T_i)$


Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$
$= (-2)\, tr(E T_i)$
$= \int_{\gamma^*} (-2)\, tr[T, h_{\gamma,\gamma}$


Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$
$E_i = (-2)\,tr(E\,T_i)$
$(E_\gamma)_i = \int_{\gamma^*} (-2)\,tr\,[T_i\, h_{\gamma,\gamma^*}(s)\, T_k\, (h_{\gamma,\gamma^*}(s)$


Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{jk}$
$= (-2)\, tr(E T_j)$
$= \int_{\gamma^*} (-2)\, tr\left[T_i\, h_{\gamma,\gamma^*}(s)\, T_k\, (h_{\gamma,\gamma^*}(s))^{-1}\right] \tilde{\epsilon}_{ba}\, E^{bk}$


Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$
$(-2)\,tr(E\,T_i)$
$(-2)\,tr\left[T_i\, h_{\gamma;\gamma^*}(s)\, T_k\, (h_{\gamma;\gamma^*}(s))^{-1}\right]\tilde{\epsilon}_{ba}\,E^{bk}$
$\int_{\gamma^*}$


Poisson algebra
$tr(T_i T_j) = -\frac{1}{2}\delta_{ik}$
$E_i = (-2)\, tr(E T_i)$
$(E_\gamma)_i = \int_{\gamma^*} (-2)\, tr\left[T_i\, h_{\gamma,\gamma^*}(s)\, T_k\, (h_{\gamma,\gamma^*}(s))^{-1}\right] \tilde{\varepsilon}_{ba}\, E^{b}$

$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$

$h_{\gamma,\gamma^*}(s) = \left(h_\gamma\left(\tfrac{1}{2}\right)\right)^{-1} h_{\gamma^*}\left(\tfrac{1}{2}, s\right)$

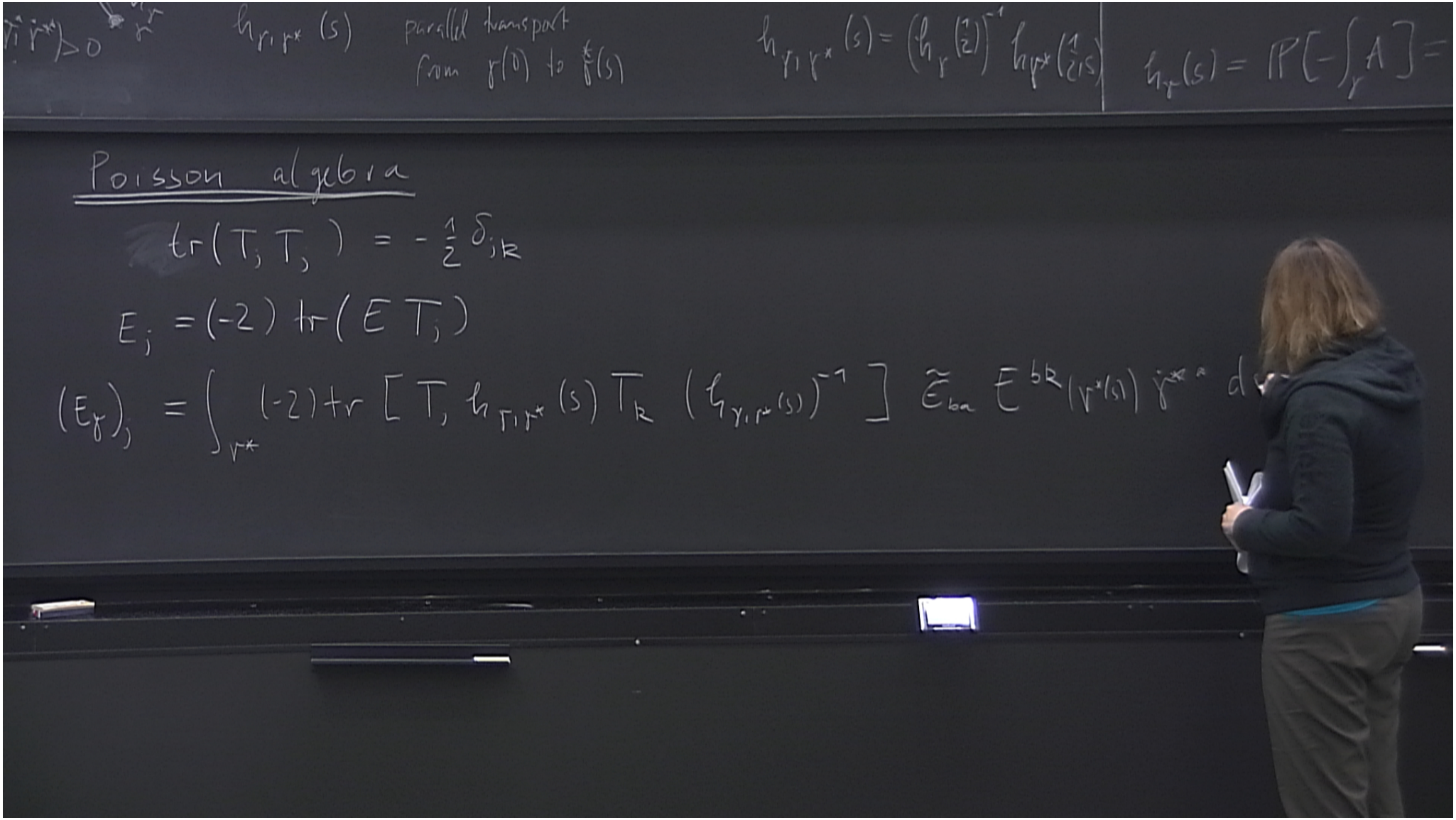
$h_\gamma(s) = \mathrm{P}\left[\exp\int_\gamma A\right] =$

Poisson algebra

$$\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$$

$$E_i = (-2)\,\mathrm{tr}(E\,T_i)$$

$$(E_\gamma)_i = \int_{\gamma^*} (-2)\,\mathrm{tr}\left[T_i\, h_{\gamma,\gamma^*}(s)\, T_k\, \left(h_{\gamma,\gamma^*}(s)\right)^{-1}\right] \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds$$



$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$

$h_{\gamma,\gamma^*}(s) = \left(h_\gamma\left(\frac{1}{2}\right)\right)^{-1} h_{\gamma^*}\left(\frac{1}{2}, s\right)$

$h_\gamma(s) = \mathrm{P}\left[-\int_\gamma A\right] =$

Poisson algebra

$$\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$$

$$(-2)\,\mathrm{tr}(E T_i)$$

$$(-2)\,\mathrm{tr}\left[T_i\, h_{\gamma,\gamma^*}(s)\, T_k\, \left(h_{\gamma,\gamma^*}(s)\right)^{-1}\right] \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds$$

$\dot{\gamma}^a \dot{\gamma}^{*a}) > 0$

$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$

$h_{\gamma,\gamma^*}(s) = \left(h_\gamma\left(\frac{1}{2}\right)\right)^{-1} h_{\gamma^*}\left(\frac{1}{2}\right) h_\gamma(s)$

$h_\gamma(s) = \mathrm{P}\left[-\int_\gamma A\right] =$

Poisson algebra

$$\mathrm{tr}(T_j T_k) = -\frac{1}{2}\delta_{jk}$$

$$= (-2)\,\mathrm{tr}(E T_j)$$

$$= \int_{\gamma^*} \underbrace{(-2)\,\mathrm{tr}\left[T_j\, h_{\gamma,\gamma^*}(s)\, T_k\, \left(h_{\gamma,\gamma^*}(s)\right)^{-1}\right]}\, \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds$$

$h_{\gamma,\gamma^*}(s)$ parallel transport from $\gamma(0)$ to $\gamma^*(s)$

$h_{\gamma,\gamma^*}(s) = \left(h_\gamma\left(\frac{1}{2}\right)\right)^{-1} h_{\gamma^*}\left(\frac{1}{2}\leq s\right)$

$h_\gamma(s) = \mathcal{P}\left[-\int_\gamma A\right] =$

Poisson algebra

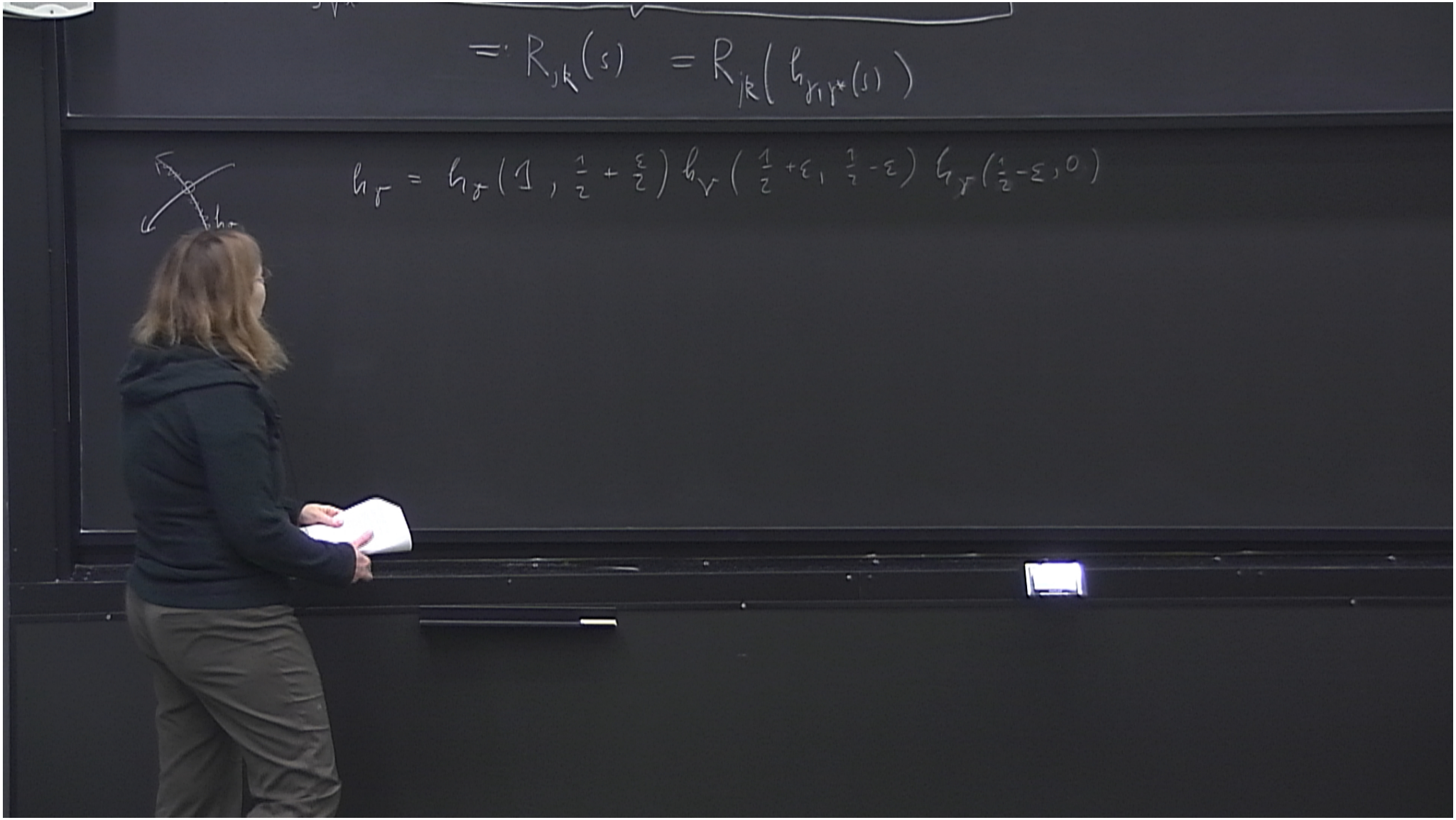
$R(g)\,T \sim g\,T\,g^{-1}$

$$\mathrm{tr}(T_i T_j) = -\frac{1}{2}\delta_{ij}$$

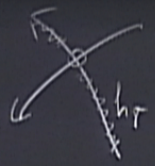
$E_i = (\quad)(E T_i)$

$$(E_\gamma)_i \quad \left[\quad (s)\, T_k \left(h_{\gamma,\gamma^*(s)}\right)^{-1}\right] \tilde{\varepsilon}_{ba}\, E^{bk}(\gamma^*(s))\, \dot{\gamma}^{*a}\, ds$$

$$=: R_{ik}(s) = R_{ik}\left(h_{\gamma,\gamma^*}(s)\right)$$


$=: R_{jk}(s) = R_{jk}(h_{\gamma_1\gamma^*}(s))$
$h_\gamma = h_\gamma(1, \frac{1}{2}+\frac{\varepsilon}{2})\, h_\gamma(\frac{1}{2}+\varepsilon, \frac{1}{2}-\varepsilon)\, h_\gamma(\frac{1}{2}-\varepsilon, 0)$

$= R_{jk}(s) = R_{jk}(h_{\gamma_1\gamma^*}(s))$



$h_\gamma = h_\gamma(1, \frac{1}{2} + \frac{\varepsilon}{2})\, h_\gamma(\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon)\, h_\gamma(\frac{1}{2} - \varepsilon, 0)$

$= 1$

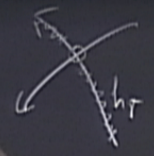
$$=: R_{jk}(s) \quad = R_{jk}\left(h_{\gamma_1\gamma^*}(s)\right)$$



$$h_\gamma = h_\gamma\left(1, \tfrac{1}{2}+\tfrac{\varepsilon}{2}\right) \underbrace{h_\gamma\left(\tfrac{1}{2}+\varepsilon, \tfrac{1}{2}-\varepsilon\right)}_{\gamma_{1/2}+\varepsilon/2} h_\gamma\left(\tfrac{1}{2}-\varepsilon, 0\right)$$

$$= \mathbb{1} - \int_{\gamma_{1/2}-\varepsilon/2} T_\ell \, \dot\gamma^c(s') \, A_c^\ell(\gamma(s')) \, ds' + \mathcal{O}(\varepsilon^2)$$

$$= R_{jk}(s) \quad = R_{jk}\left(h_{\gamma_1\gamma^*}(s)\right)$$

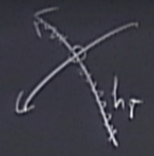


$$h_\gamma = h_\gamma\left(1, \tfrac{1}{2} + \tfrac{\varepsilon}{2}\right) \underbrace{h_\nu\left(\tfrac{1}{2}+\varepsilon, \tfrac{1}{2}-\varepsilon\right)}_{\gamma_0+\varepsilon/2} h_\gamma\left(\tfrac{1}{2}-\varepsilon, 0\right)$$

$$= \mathbb{1} - \int_{\gamma_0-\varepsilon/2} T_e \, j^c(s') \, A_c^e(\gamma(s')) \, ds' + O(\varepsilon^2)$$

$$\{(E_\gamma)_j, h_\gamma\}$$

$$= R_{jk}(s) \quad = R_{jk}\left(h_{\gamma,\gamma^*}(s)\right)$$

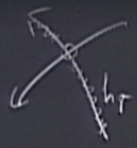


$$h_\gamma = h_\gamma\left(1, \tfrac{1}{2} + \tfrac{\varepsilon}{2}\right) \underbrace{h_\nu\left(\tfrac{1}{2}+\varepsilon, \tfrac{1}{2}-\varepsilon\right)}_{\gamma_2+\varepsilon/2} h_\gamma\left(\tfrac{1}{2}-\varepsilon, 0\right)$$

$$= \mathbb{1} - \int_{\gamma_2-\varepsilon/2} T_\ell \, j^c(s') \, A_c^\ell(\gamma(s')) \, ds' + \mathcal{O}(\varepsilon^2)$$

$$\xi\{(E_\gamma)_j, h_\gamma\} = -(\quad) \, T_\ell \, h_\gamma\left(\tfrac{1}{2} - \tfrac{\varepsilon}{2}, 0\right)$$

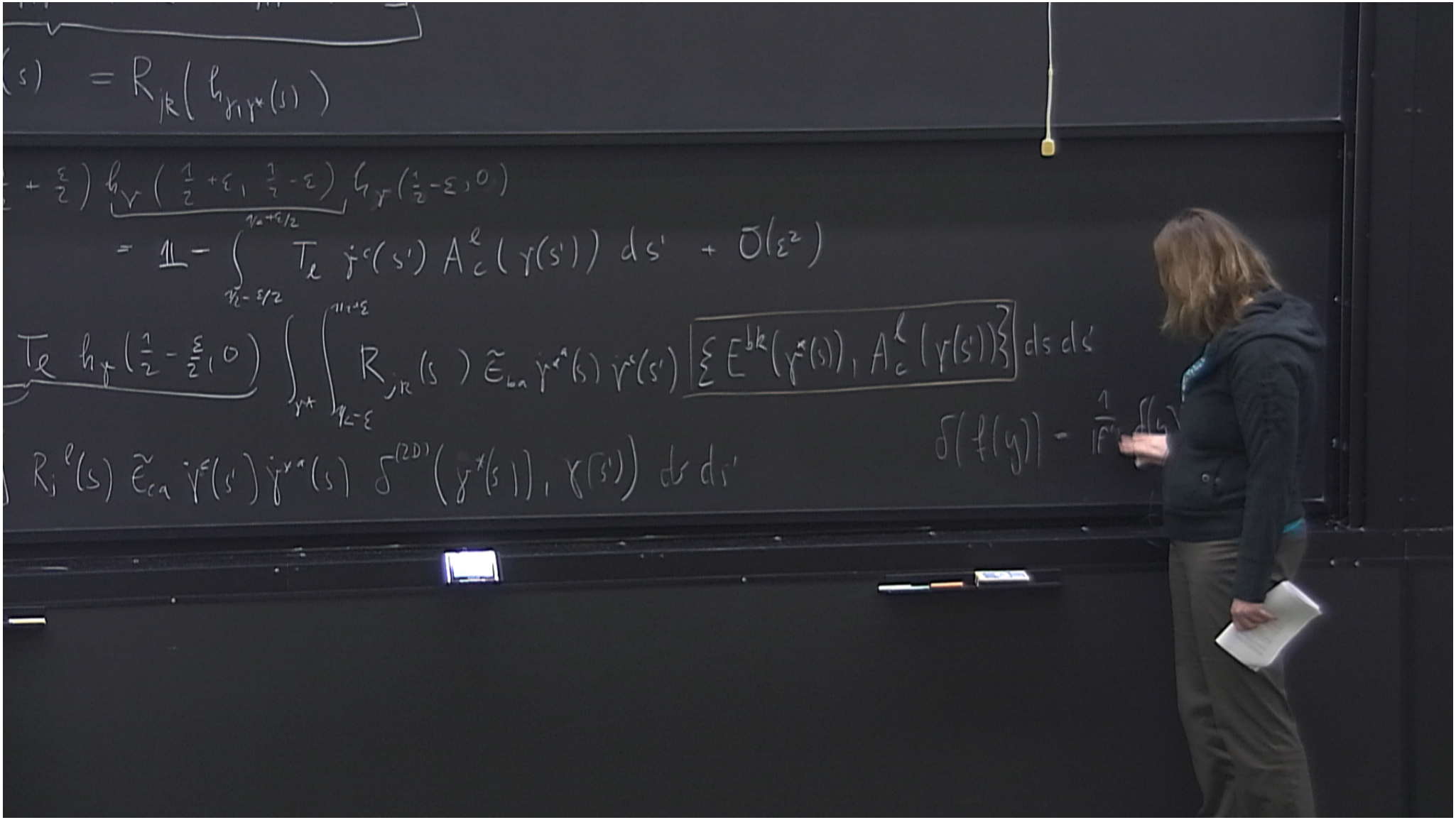
$$(E_\gamma)_j = \int_{\gamma^*} \underbrace{(-2)\,\mathrm{tr}\left[\tau_j\, h_{\gamma_1\gamma^*}(s)\, \tau_k\, h_{\gamma_1\gamma^*}(s)\right]}_{=: R_{jk}(s) = R_{jk}(h_{\gamma_1\gamma^*}(s))} \dot\gamma^*(s)\, ds$$

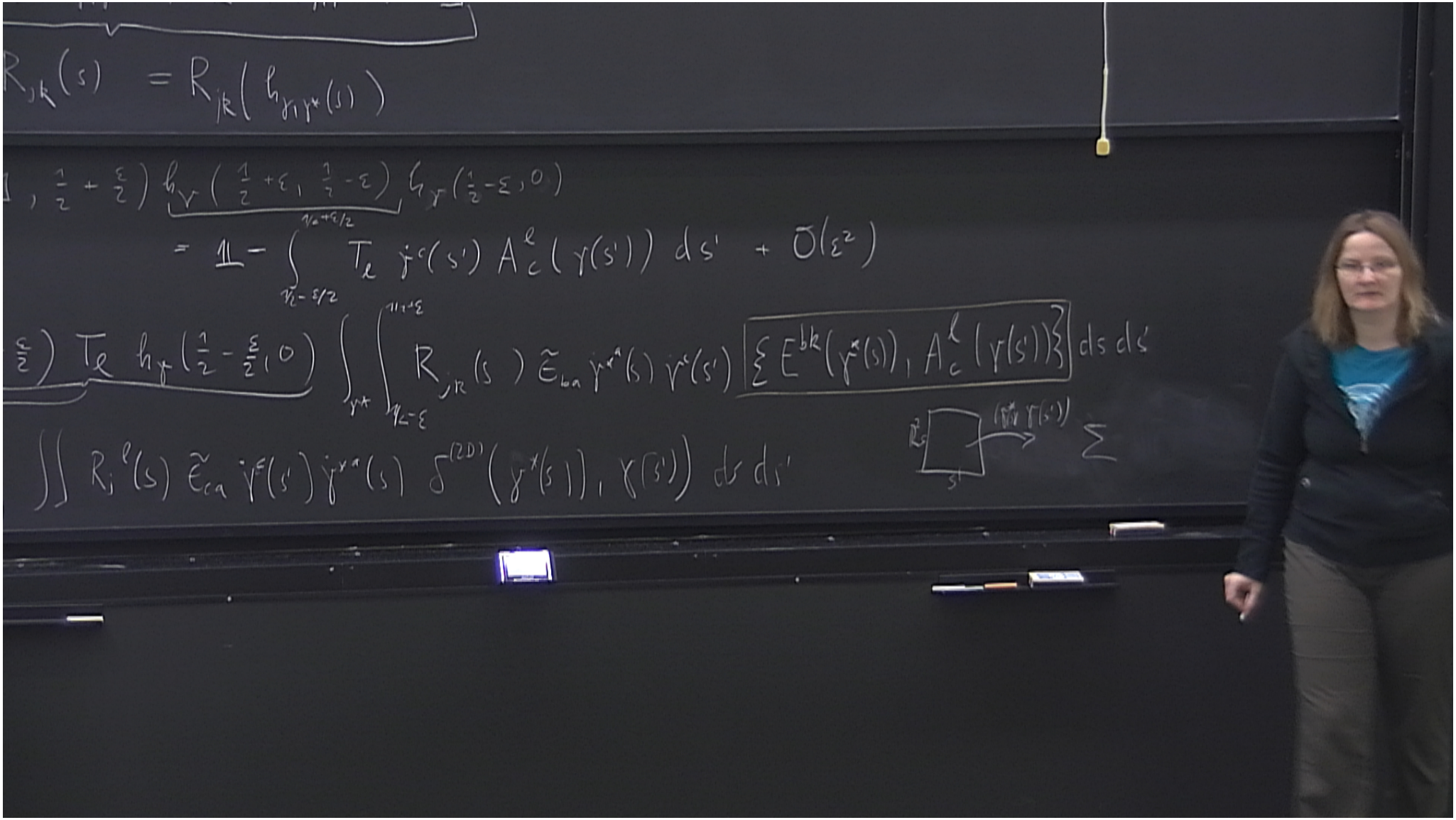


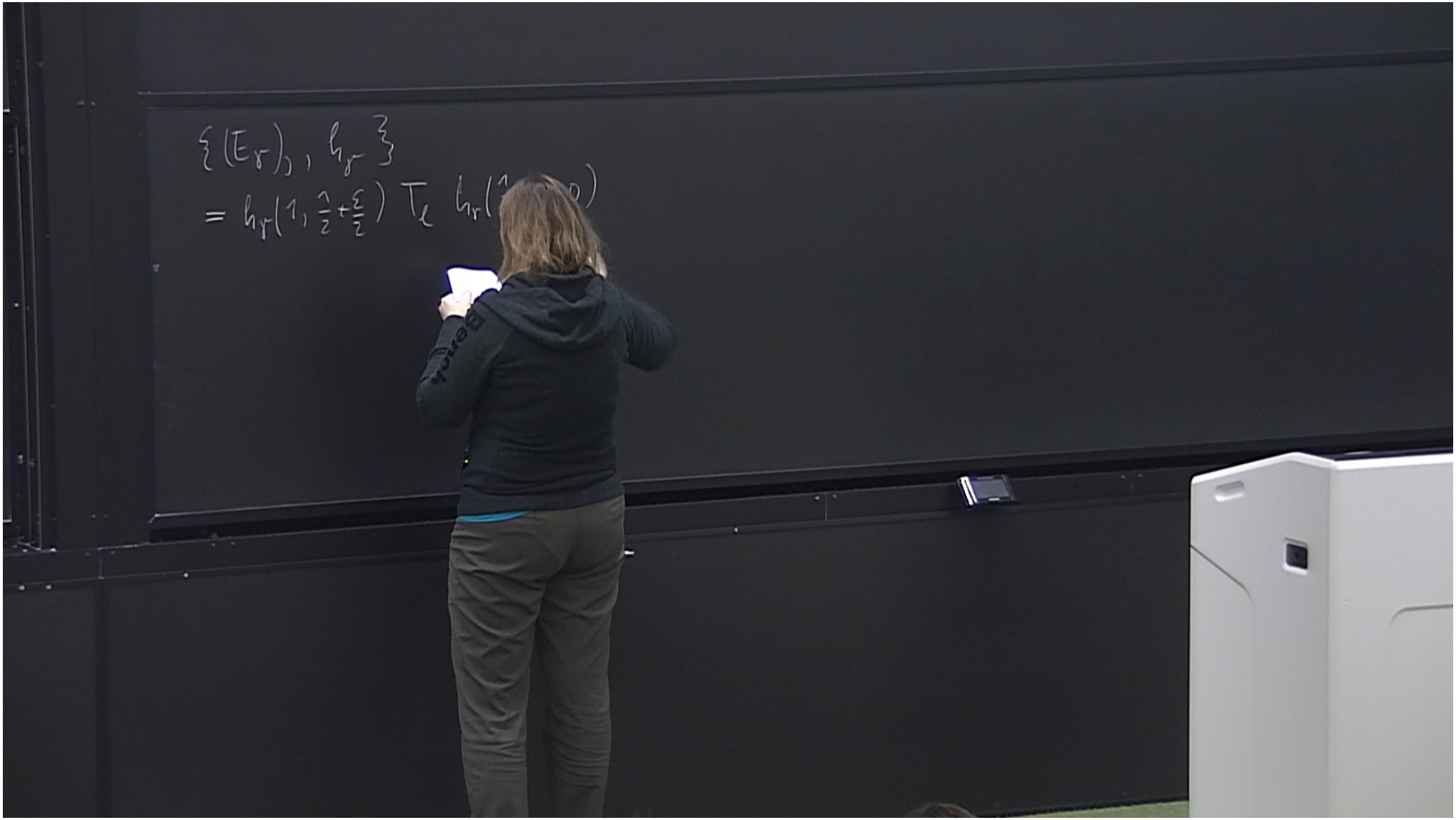
$$h_\gamma = h_\gamma\left(1, \tfrac{1}{2}+\tfrac{\varepsilon}{2}\right) \underbrace{h_\gamma\left(\tfrac{1}{2}+\varepsilon, \tfrac{1}{2}-\varepsilon\right)}_{\gamma_2+\varepsilon/2} h_\gamma\left(\tfrac{1}{2}-\varepsilon, 0\right)$$

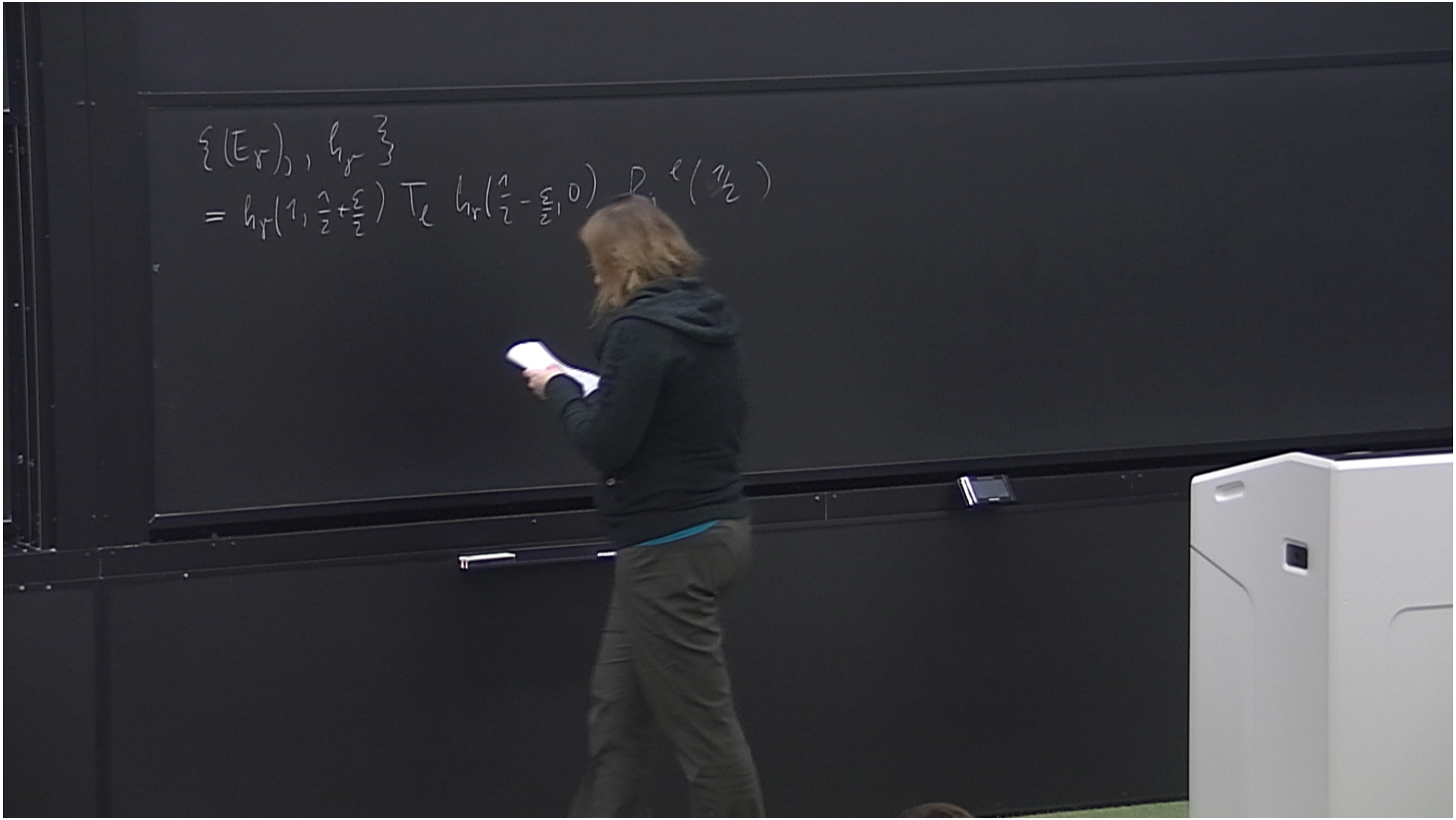
$$= \mathbb{1} - \int_{\gamma_2-\varepsilon/2} T_e\, \dot\gamma^c(s')\, A^e_c(\gamma(s'))\, ds' + \mathcal{O}(\varepsilon^2)$$

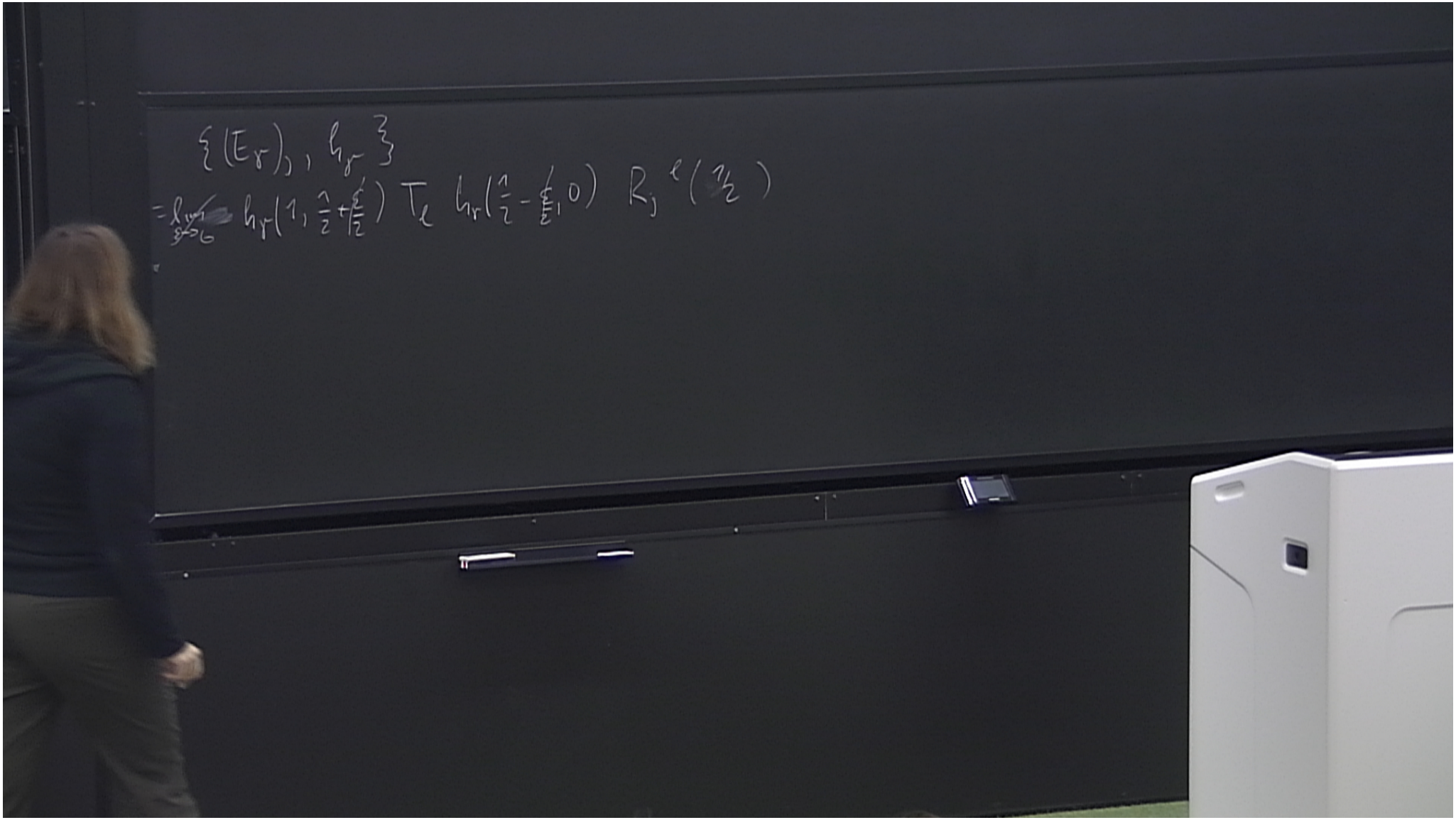
$$\xi\{(E_\gamma)_j, h_\gamma\}\xi = -h_\gamma\left(1, \tfrac{1}{2}+\tfrac{\varepsilon}{2}\right) T_e\, h_\gamma\left(\tfrac{1}{2}-\tfrac{\varepsilon}{2}, 0\right) \int_{\gamma^*} \int_{\gamma_2-\varepsilon}^{\gamma_2+\varepsilon} R_{jk}(s)\, \tilde\varepsilon_{bca}\, \dot\gamma^{*a}(s)\, \dot\gamma^c(s')\, \xi E^{bk}(\gamma^*(s)), A^e_c$$

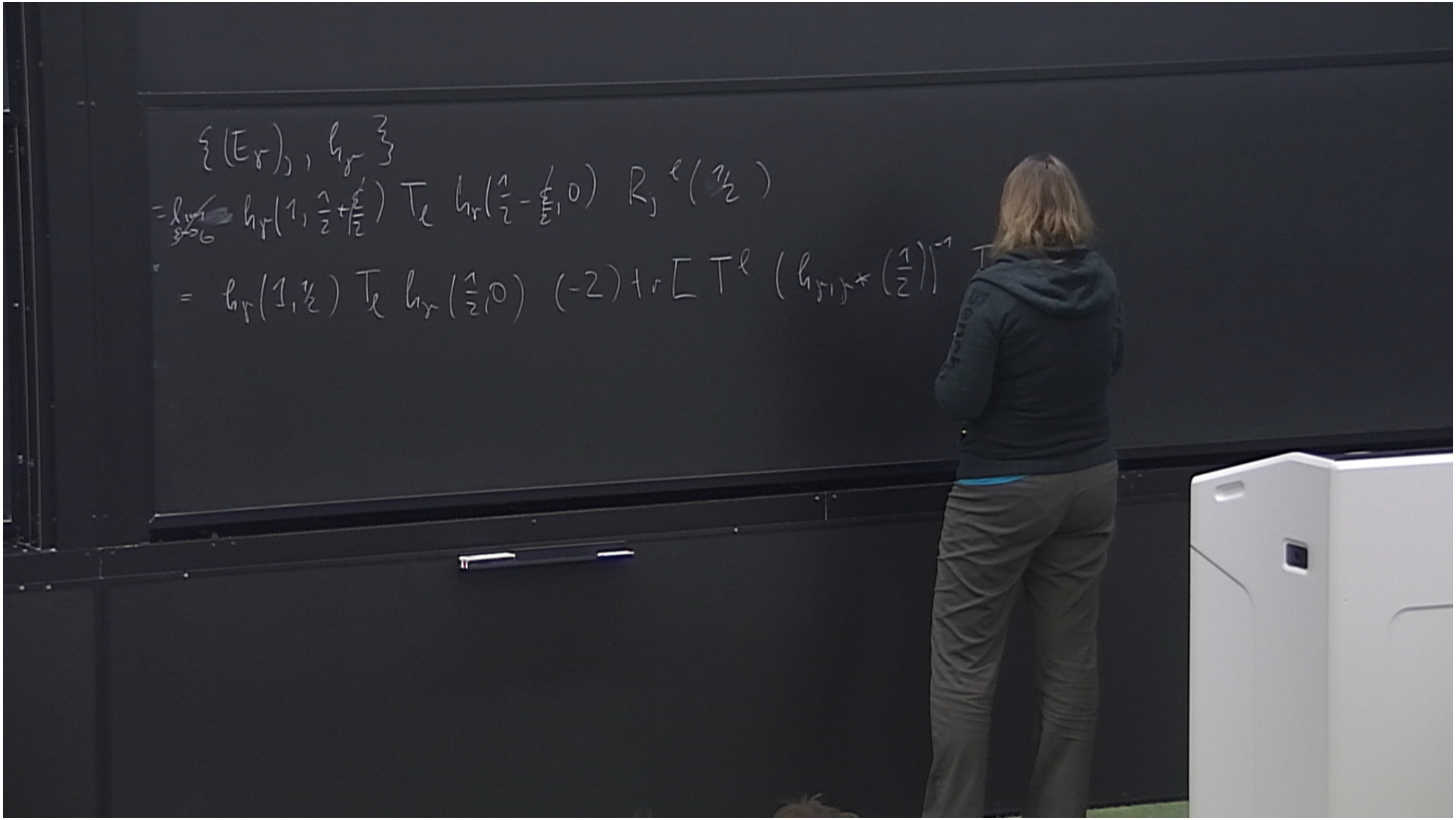

$= R_{jk}(h_{\gamma_1\gamma^*}(s))$
$= \mathbb{1} - \int_{\gamma_2-\varepsilon/2}^{\gamma_2+\varepsilon/2} T_\ell \dot{\gamma}^c(s') A_c^\ell(\gamma(s'))\, ds' + \mathcal{O}(\varepsilon^2)$
$T_\ell\, h_\gamma(\frac{1}{2}-\frac{\varepsilon}{2},0) \int_{\gamma^*}^{\gamma_1+\varepsilon} \int_{\gamma_2-\varepsilon}^{\gamma_2+\varepsilon} R_{jk}(s)\, \tilde{\epsilon}_{ba}\, \dot{\gamma}^{*a}(s)\, \dot{\gamma}^c(s') \{\xi E^{bk}(\gamma^*(s)), A_c^\ell(\gamma(s'))\}\, ds\, ds'$
$R_i^\ell(s)\, \tilde{\epsilon}_{ca}\, \dot{\gamma}^c(s')\, \dot{\gamma}^{*a}(s)\, \delta^{(2D)}(\gamma^*(s)), \gamma(s'))\, ds\, ds'$
$\delta(f(y)) = \frac{1}{|f'|}$

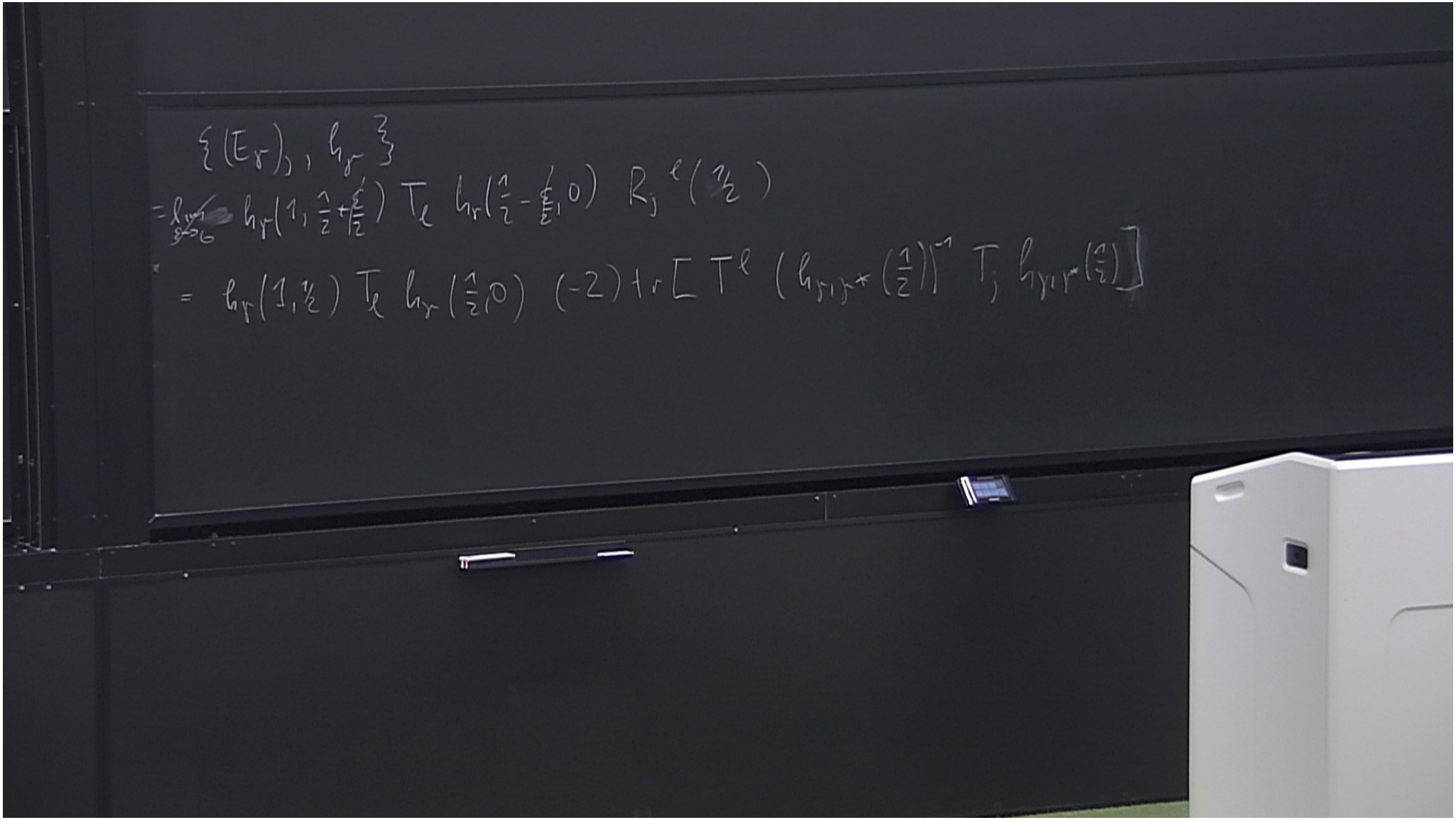

$R_{jk}(s) = R_{jk}(h_{\gamma,\gamma^*}(s))$
$= \mathbb{1} - \int_{\eta_2 - \varepsilon/2}^{\eta_2 + \varepsilon/2} T_\ell \, \dot{\gamma}^c(s') \, A_c^\ell(\gamma(s')) \, ds' + O(\varepsilon^2)$
$\int_{\gamma^*} \int_{\eta_2 - \varepsilon}^{\eta_2 + \varepsilon} R_{jk}(s) \, \tilde{\epsilon}_{ba} \, \dot{\gamma}^{*a}(s) \, \dot{\gamma}^c(s') \, \{\Sigma E^{bk}(\gamma^*(s)), A_c^\ell(\gamma(s'))\} \, ds \, ds'$
$\iint R_i^\ell(s) \, \tilde{\epsilon}_{ca} \, \dot{\gamma}^c(s') \, \dot{\gamma}^{*a}(s) \, \delta^{(2D)}(\gamma^*(s)), \gamma(s')) \, ds \, ds'$
$\mathbb{R}^2$
S^1
Σ

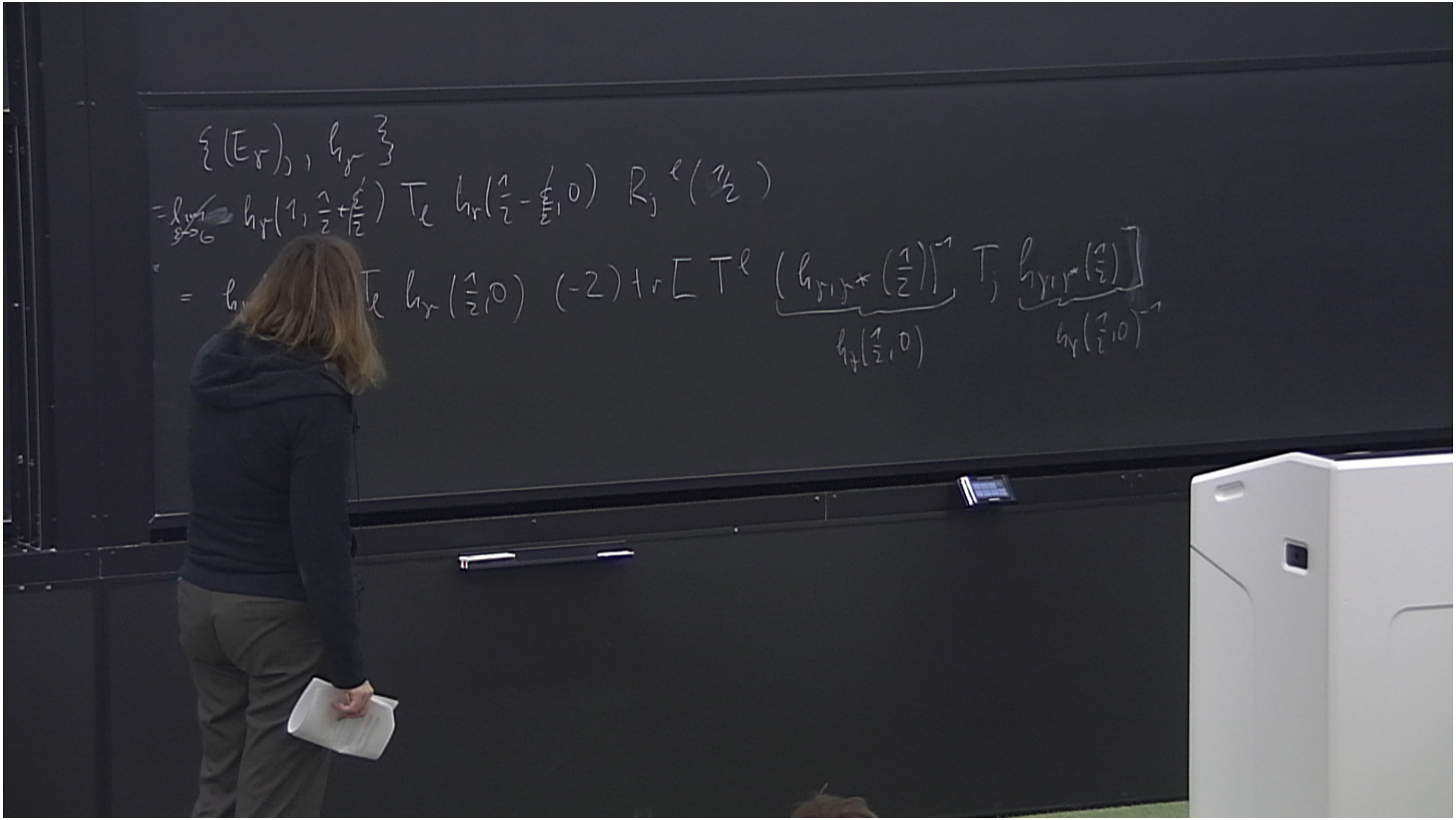

$\{(E_\gamma)_j, h_\gamma\}$
$= h_\gamma(1, \frac{\hat{}}{2} + \frac{\xi}{2})\ T_e\ h_\gamma($

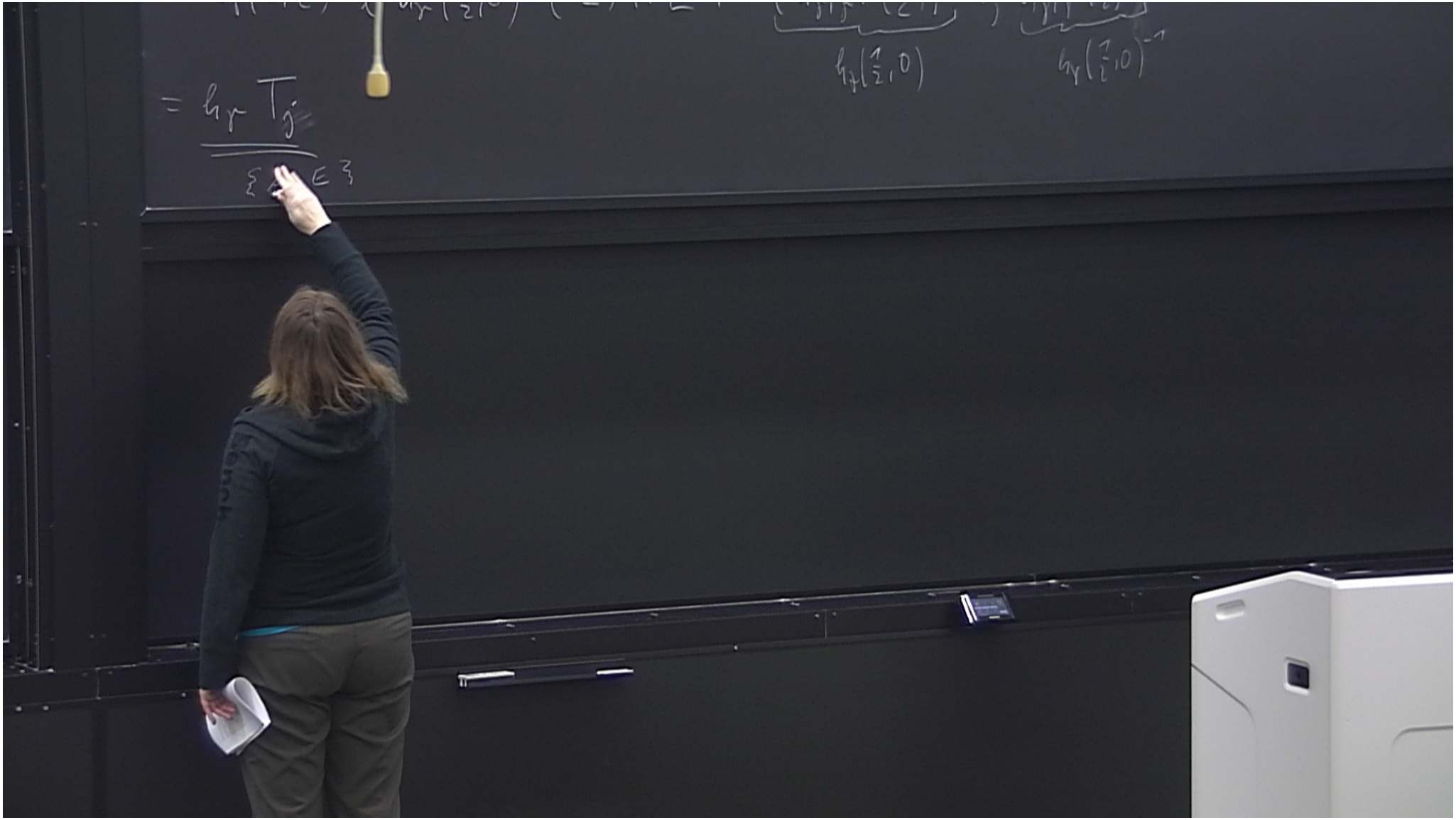

$\{(E_\gamma)_\gamma, h_\gamma\}$
$= h_\gamma(1, \frac{\gamma}{2} + \frac{\xi}{2})\, T_\ell\, h_\gamma(\frac{\gamma}{2} - \frac{\xi}{2}, 0)$

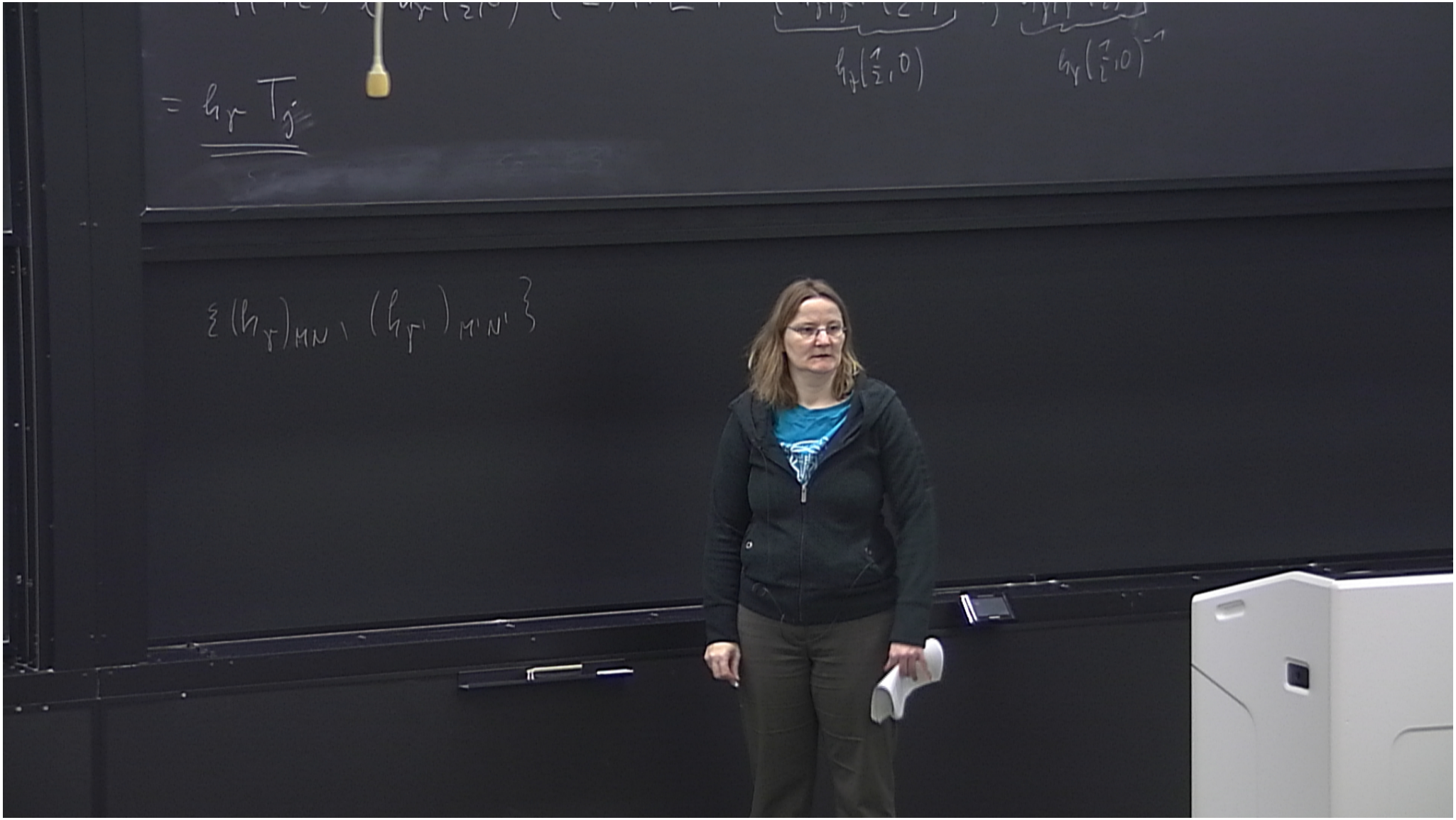

$\{(E_\gamma)_j, h_\gamma\}$
$= h_\gamma(1, \frac{\hat{}}{2} + \frac{\xi}{2})\ T_\ell\ h_r(\frac{\hat{}}{2} - \frac{\xi}{2}, 0)\ R_j{}^\ell(\frac{\pi}{2})$

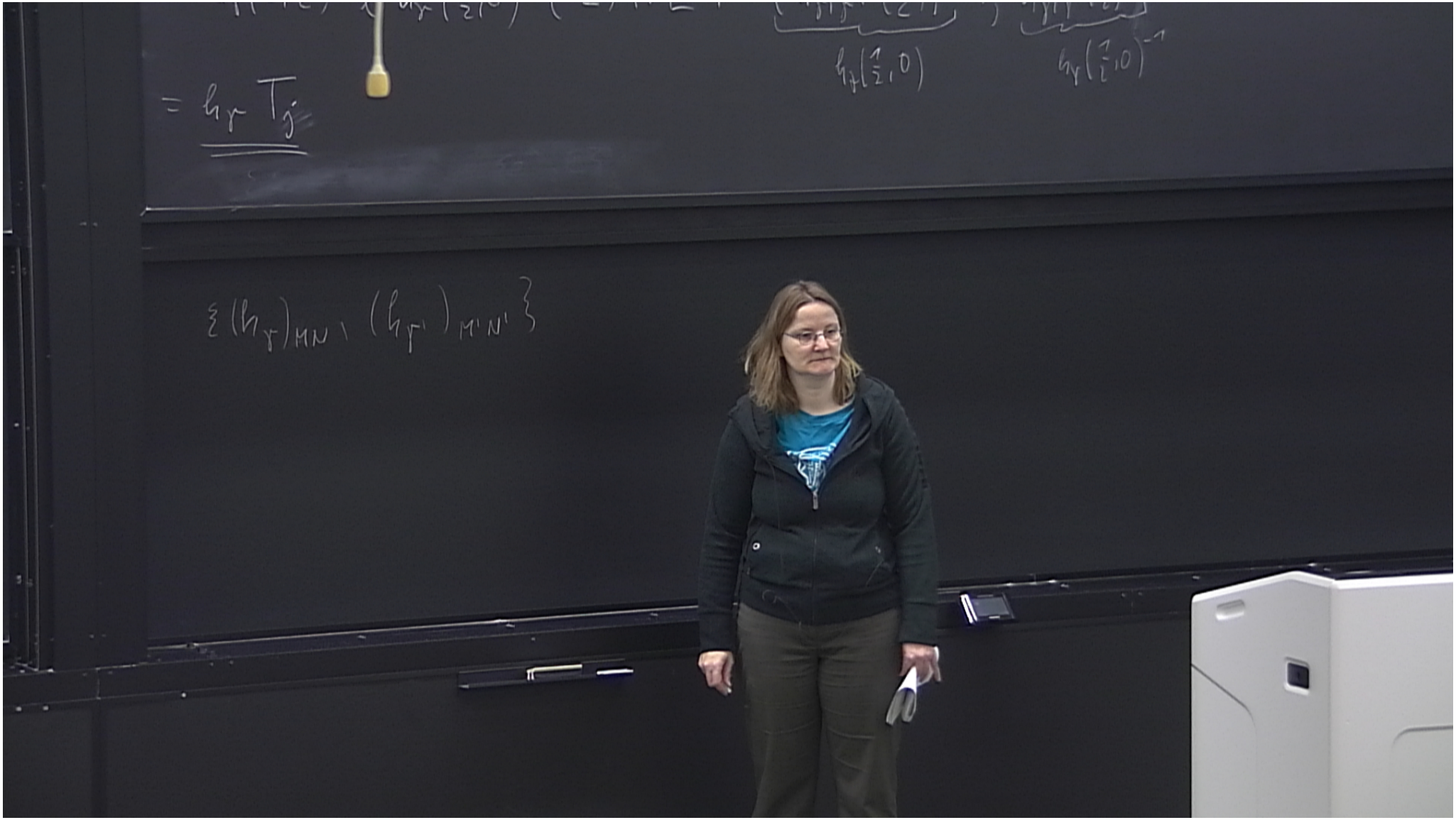

$\{(E_\gamma)_j, h_\gamma\}$
$= h_\gamma(1, \frac{1}{2} + \frac{\varepsilon}{2}) T_\ell h_r(\frac{1}{2} - \frac{\varepsilon}{2}, 0) R_j{}^\ell(\frac{1}{2})$
$= h_\gamma(1, \frac{1}{2}) T_\ell h_r(\frac{1}{2}, 0) (-2) + r[T^\ell (h_{\gamma,\gamma*}(\frac{1}{2}))^{-1}$

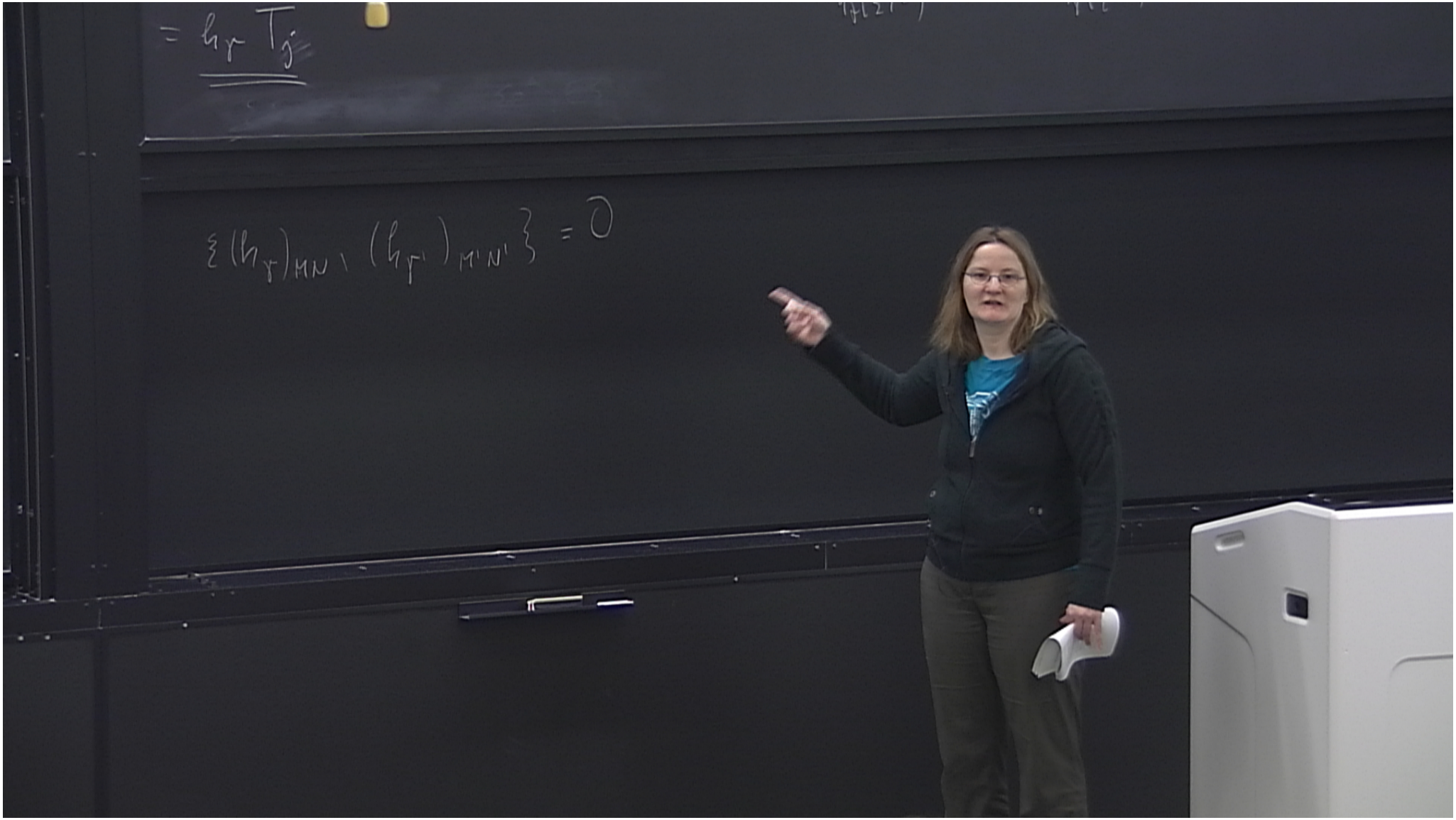

$\{(E_\gamma)_j, h_\gamma\}$
$= h_\gamma(1, \frac{\hat{}}{2} + \frac{\xi}{2}) T_\ell h_r(\frac{\hat{}}{2} - \frac{\xi}{2}, 0) R_j{}^\ell(\frac{\hat{}}{2})$
$= h_\gamma(1, \frac{\hat{}}{2}) T_\ell h_r(\frac{\hat{}}{2}, 0) (-2) \mathrm{tr}[T^\ell (h_{\gamma_1\gamma^*}(\frac{\hat{}}{2}))^{-1} T_j h_{\gamma_1\gamma^*}(\frac{\hat{}}{2})]$

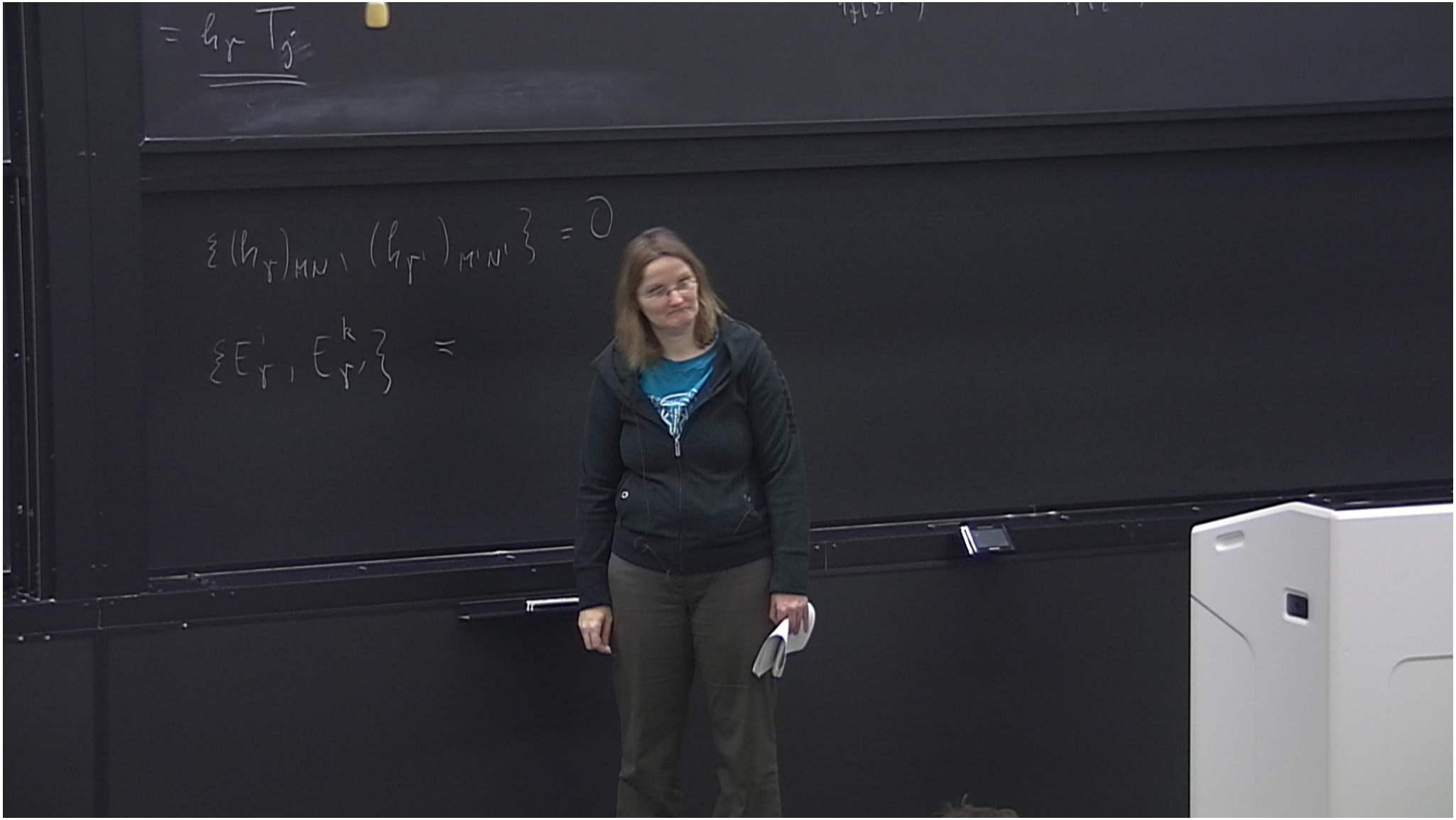

$\{(E_\gamma)_j, h_\gamma\}$
$= h_\gamma(1, \frac{\xi}{2} + \frac{\xi}{2})\, T_\ell\, h_r(\frac{1}{2} - \frac{\xi}{2}, 0)\, R_j{}^\ell(\frac{1}{2})$
$= h_\gamma\, T_\ell\, h_r(\frac{1}{2}, 0)\, (-2) + r[\, T^\ell\, \underbrace{(h_{\gamma_1\gamma^*}(\frac{1}{2}))^{-1}}_{h_\gamma(\frac{1}{2},0)}\, T_j\, \underbrace{h_{\gamma_1\gamma^*}(\frac{1}{2})}_{h_\gamma(\frac{1}{2},0)^{-1}}$

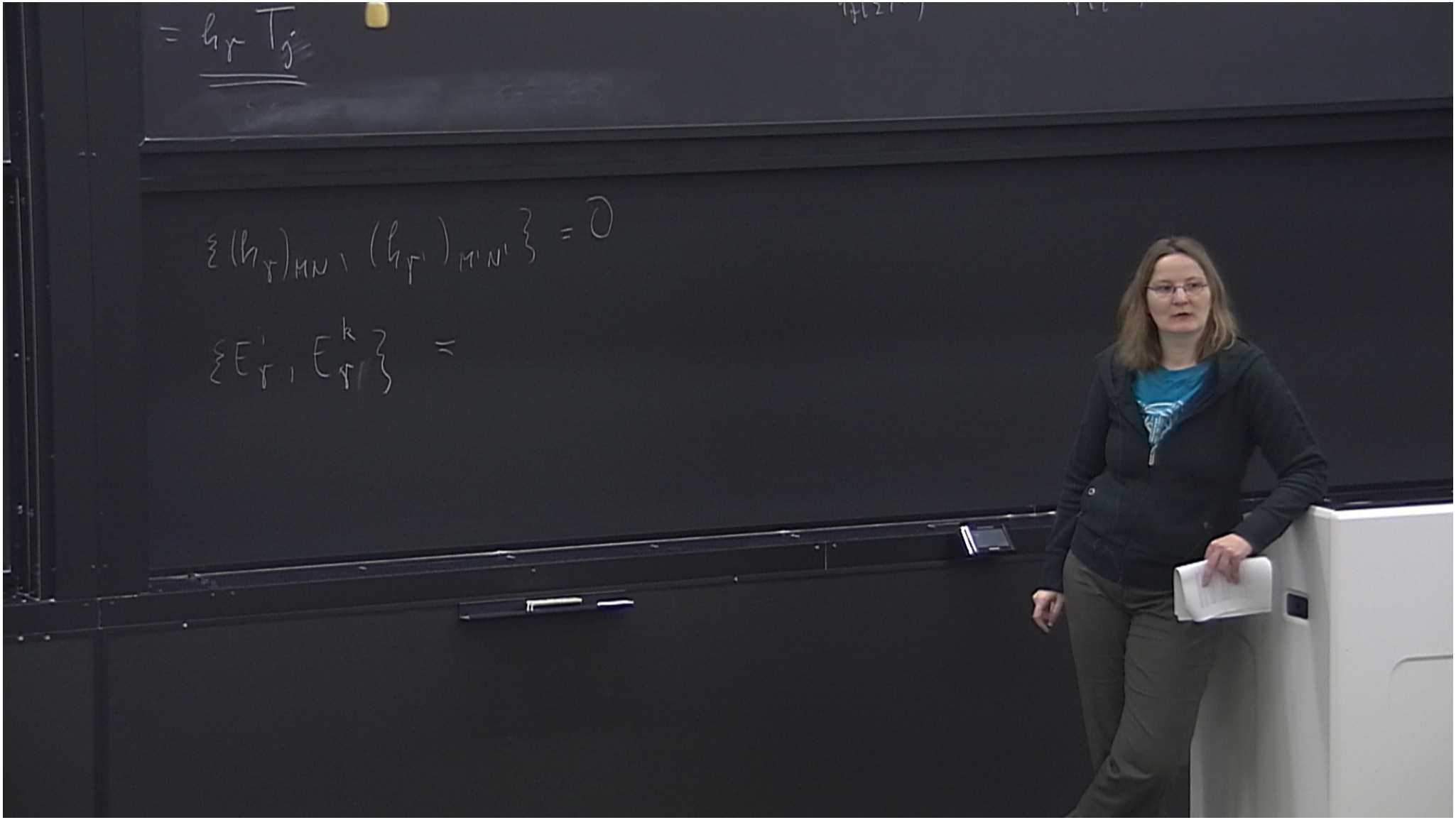


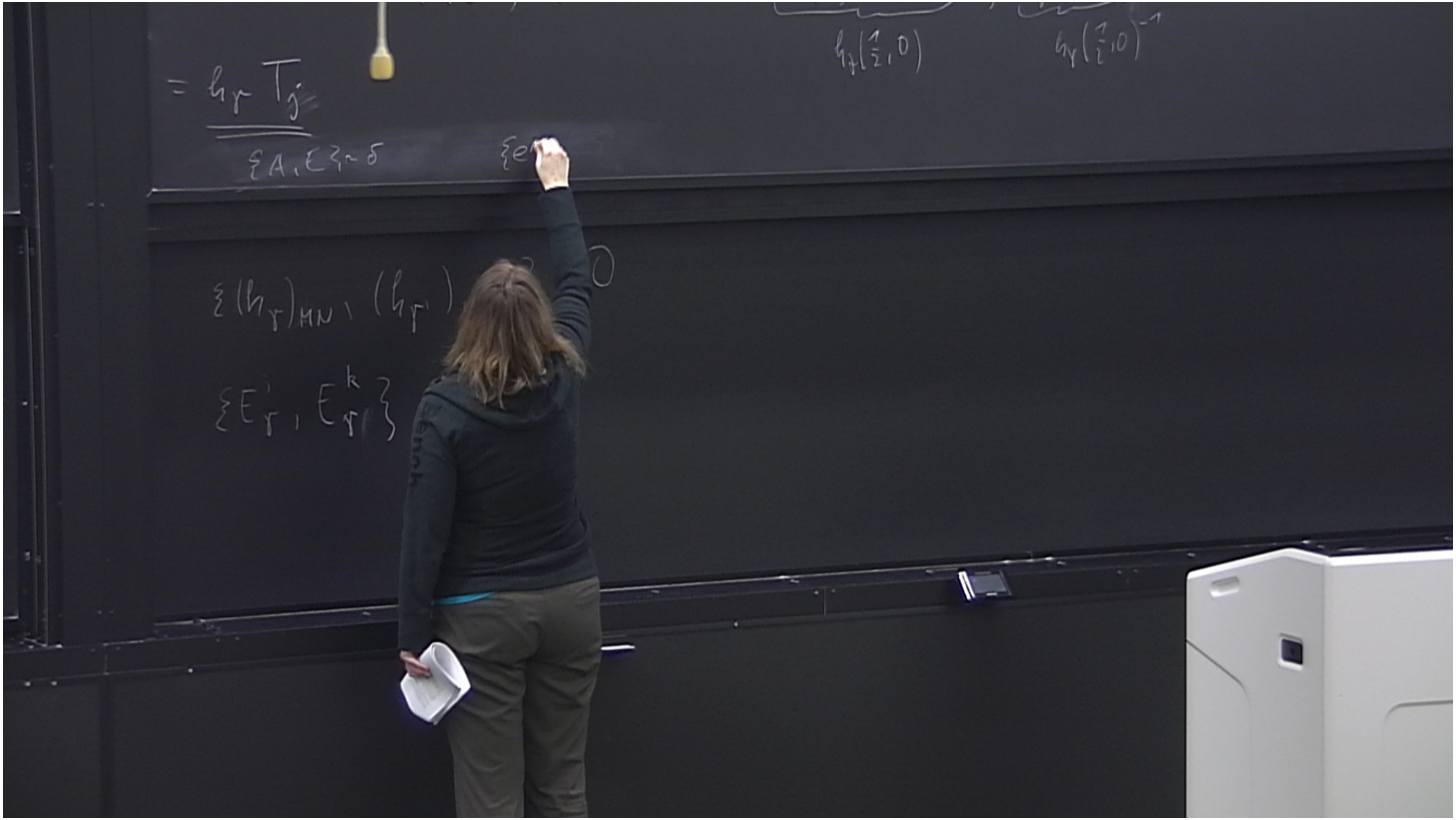

$= h_\gamma T_j$
$\{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\}$
$h_\gamma(\vec{\Sigma}, 0)$
$h_\gamma(\vec{\Sigma}, 0)^{-1}$

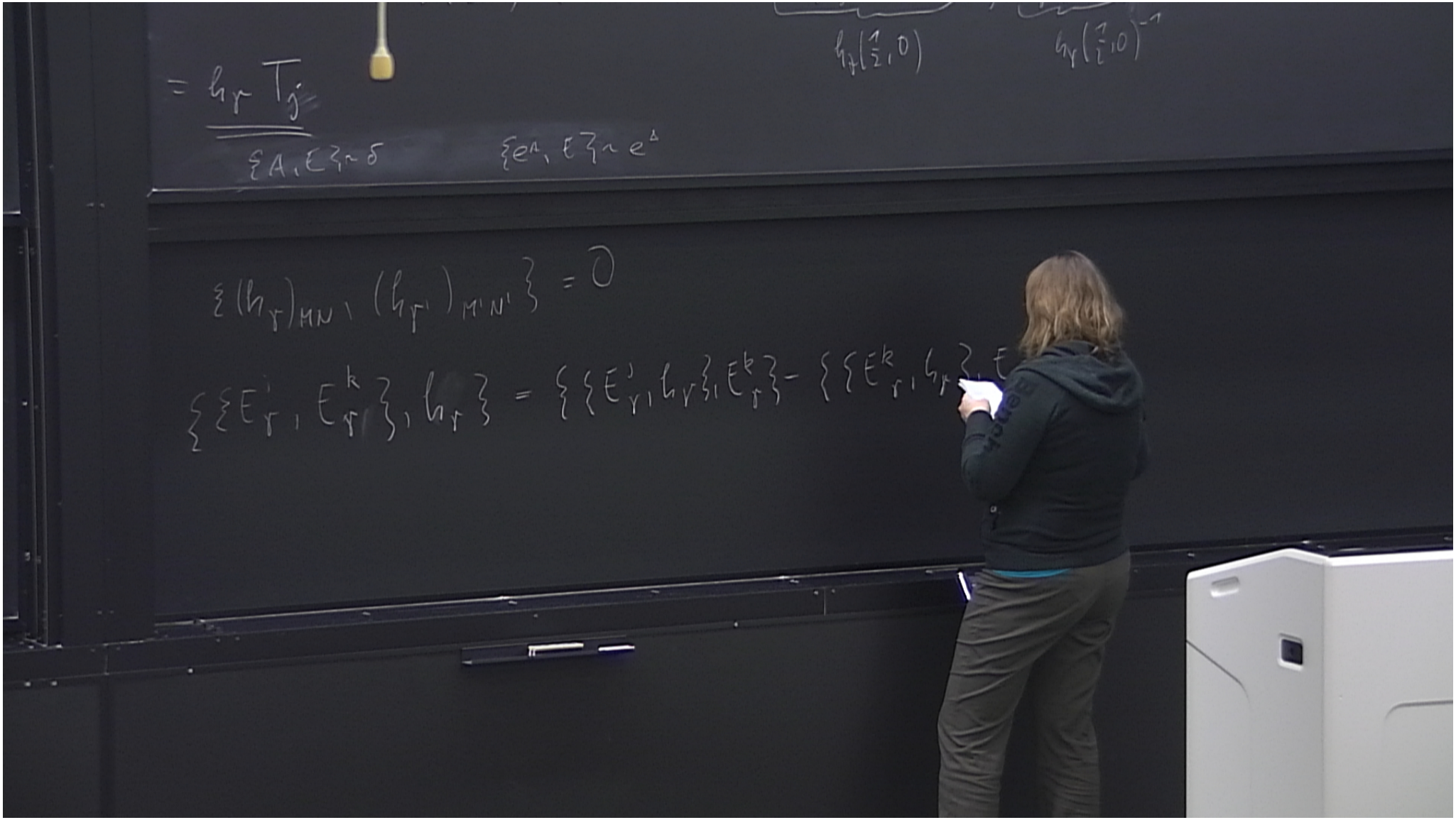

$= h_\gamma T_j$
$\{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\}$
$h_\gamma(\vec{\Sigma}, 0)^{-1}$

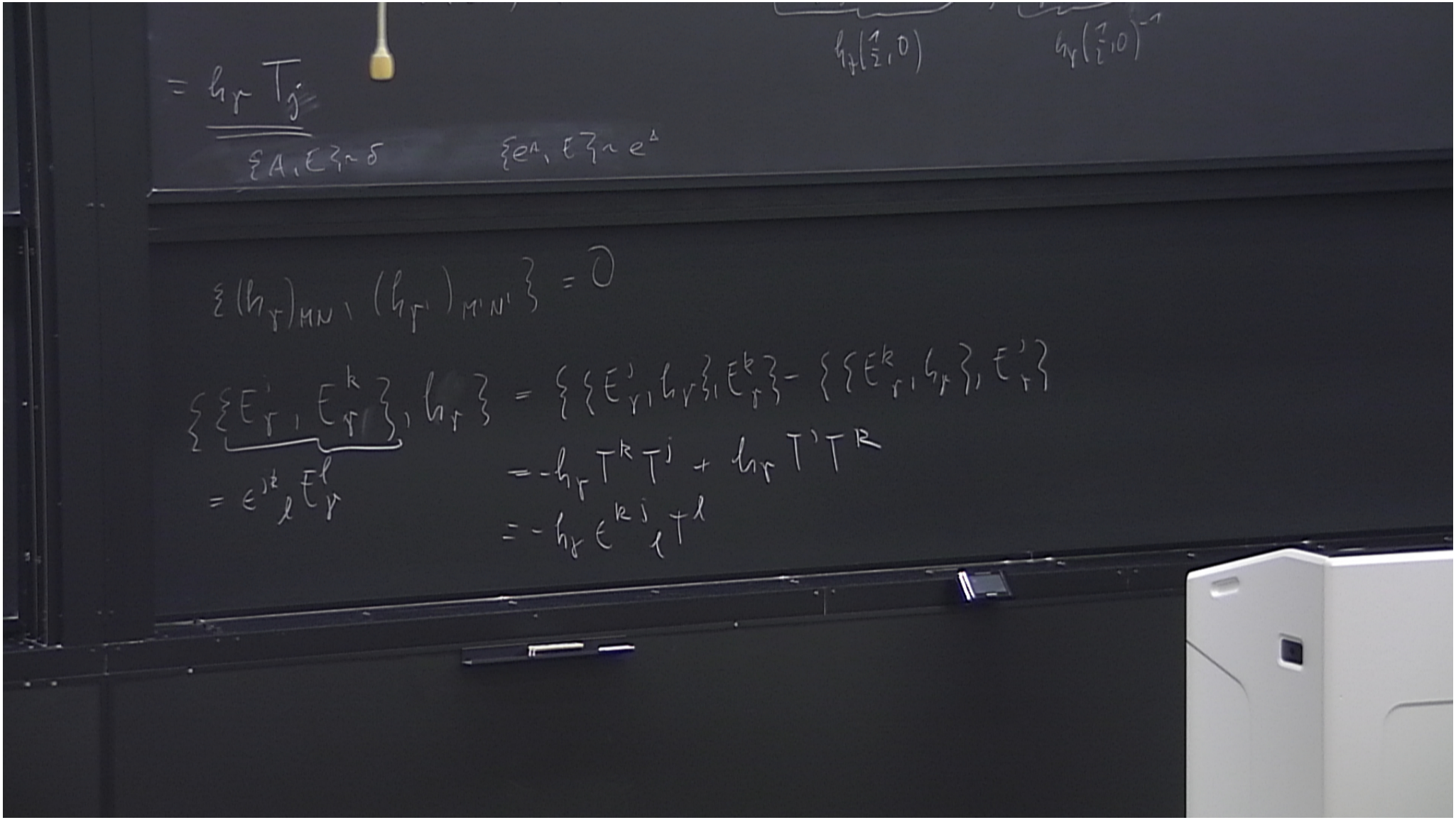

$= h_r T_j$
$\{(h_r)_{MN}, (h_{r'})_{M'N'}\} = 0$

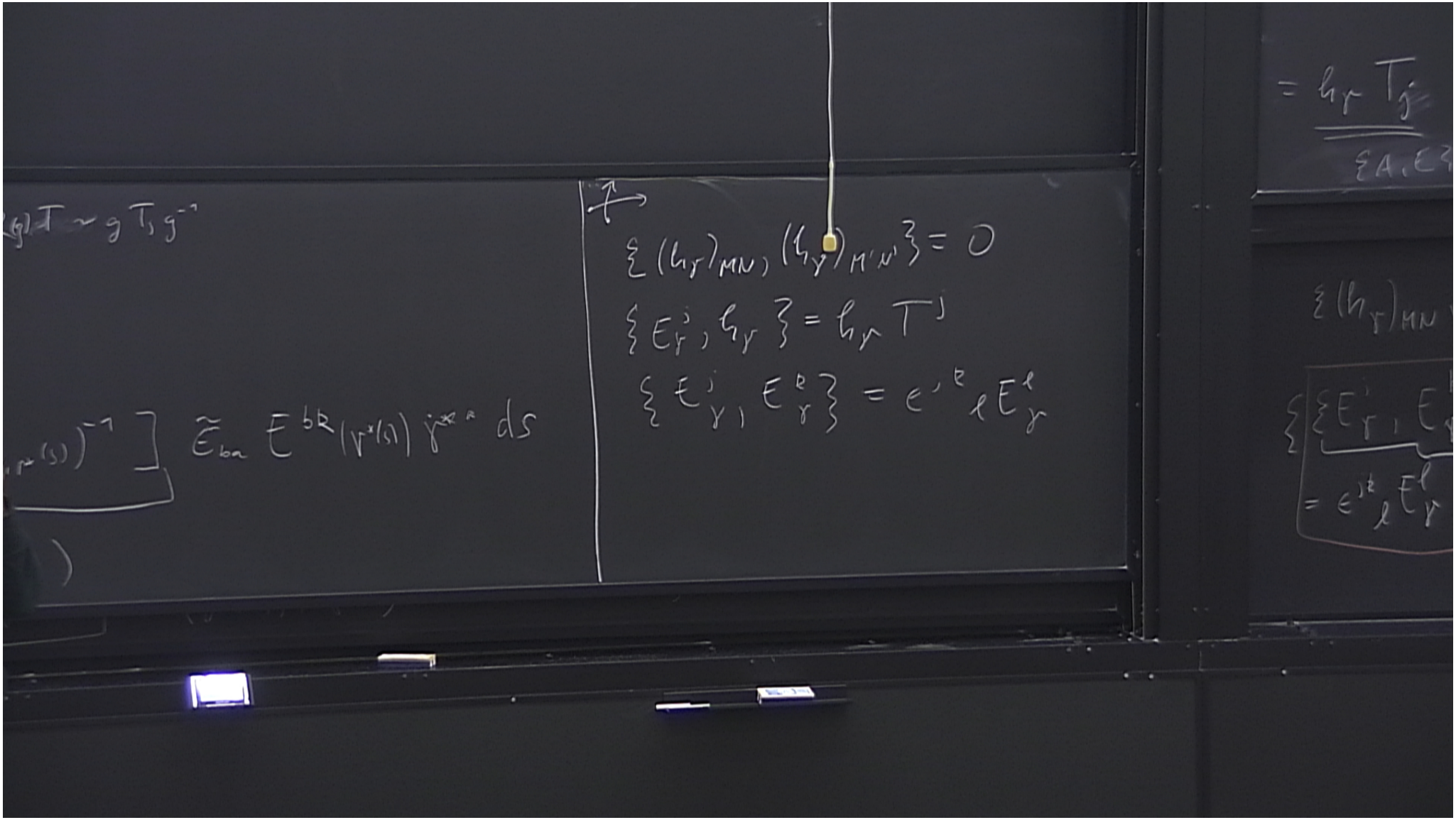

$= h_r T_j$
$\{(h_r)_{MN}, (h_{r'})_{M'N'}\} = 0$
$\{E^i_r, E^k_{r'}\} =$

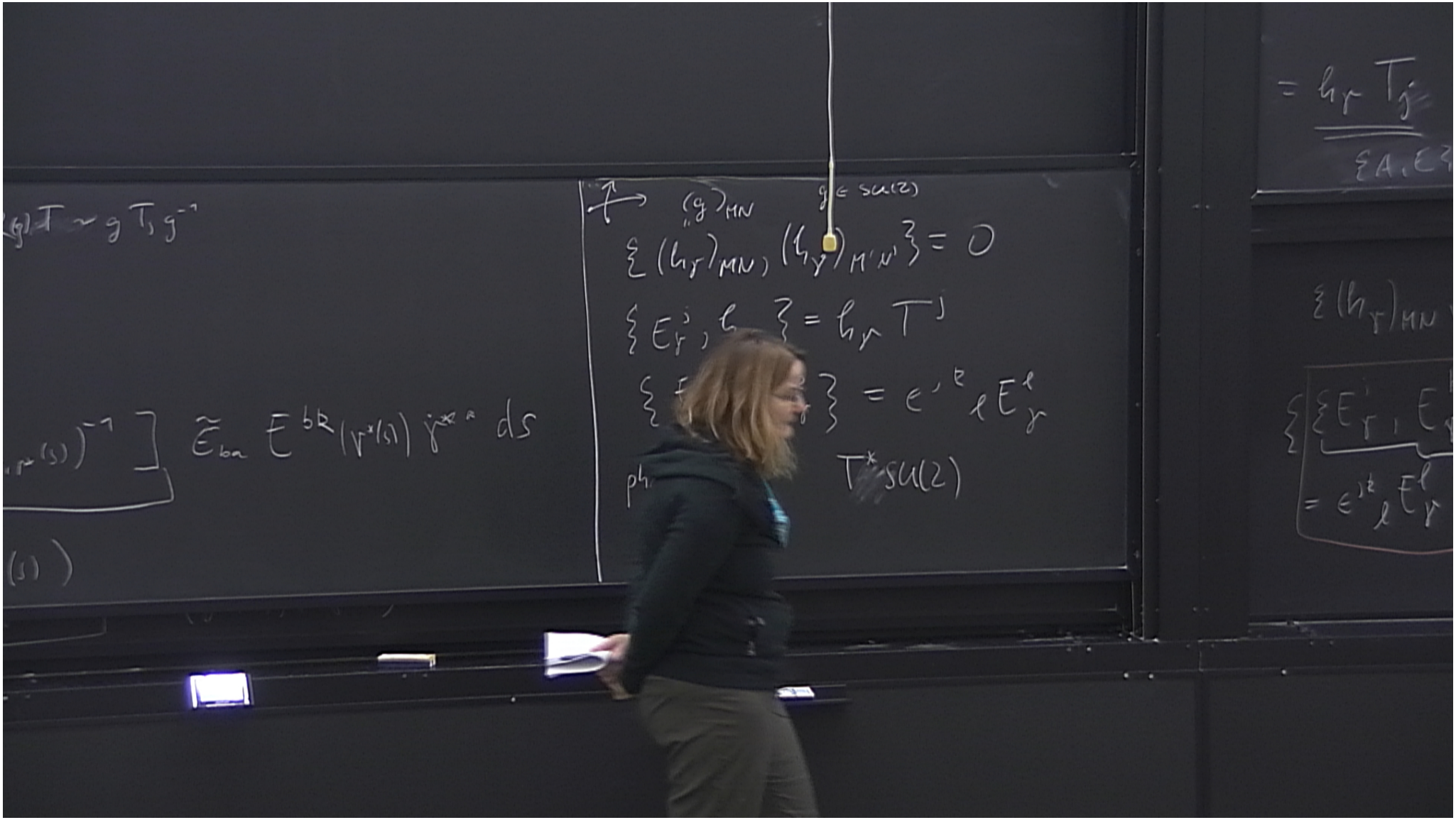

$= h_r T_j$
$\{(h_r)_{MN}, (h_{r'})_{M'N'}\} = 0$
$\{E^i_r, E^k_{r'}\} =$




$= h_r T_j =$
$\{A, E\} \sim \delta$
$\{e^A, E\} \sim e^A$
$\{(h_r)_{MN}, (h_{r'})_{M'N'}\} = 0$
$\{\{E^i_r, E^k_r\}, h_r\} = \{\{E^i_r, h_r\}, E^k_r\} - \{\{E^k_r, h_r\}, E$


$= h_r T_j$
$\{A, E\} \sim \delta$
$\{e^a, E\} \sim e^a$
$h_r(\vec{x}, 0)$
$h_r(\vec{x}, 0)^{-1}$
$\{(h_r)_{MN}, (h_{r'})_{M'N'}\} = 0$
$\{\{E^i_r, E^k_r\}, h_r\} = \{\{E^i_r, h_r\}, E^k_r\} - \{\{E^k_r, h_r\}, E^i_r\}$
$= \epsilon^{ik}{}_l E^l_r$
$= -h_r T^k T^j + h_r T^i T^k$
$= -h_r \epsilon^{kij}{}_l T^l$


$\{(h_\gamma)_{MN}, (h_\gamma)_{M'N'}\} = 0$
$\{E^j_\gamma, h_\gamma\} = h_\gamma T^j$
$\{E^j_\gamma, E^k_\gamma\} = \epsilon^{jk}{}_\ell E^\ell_\gamma$
$= h_\gamma T_j$


$(g)_{MN}$
$g \in SU(2)$
$\{(h_\gamma)_{MN}, (h_{\gamma'})_{M'N'}\} = 0$
$\{E^i_\gamma, \quad\} = h_\gamma T^j$
$= \epsilon^{jk}{}_\ell E^\ell_\gamma$
$T^* SU(2)$
$= h_\gamma T_j$
$\{A, E$
$\{(h_\gamma)_{MN}$
$= \epsilon^{jk}{}_\ell E^\ell_\gamma$