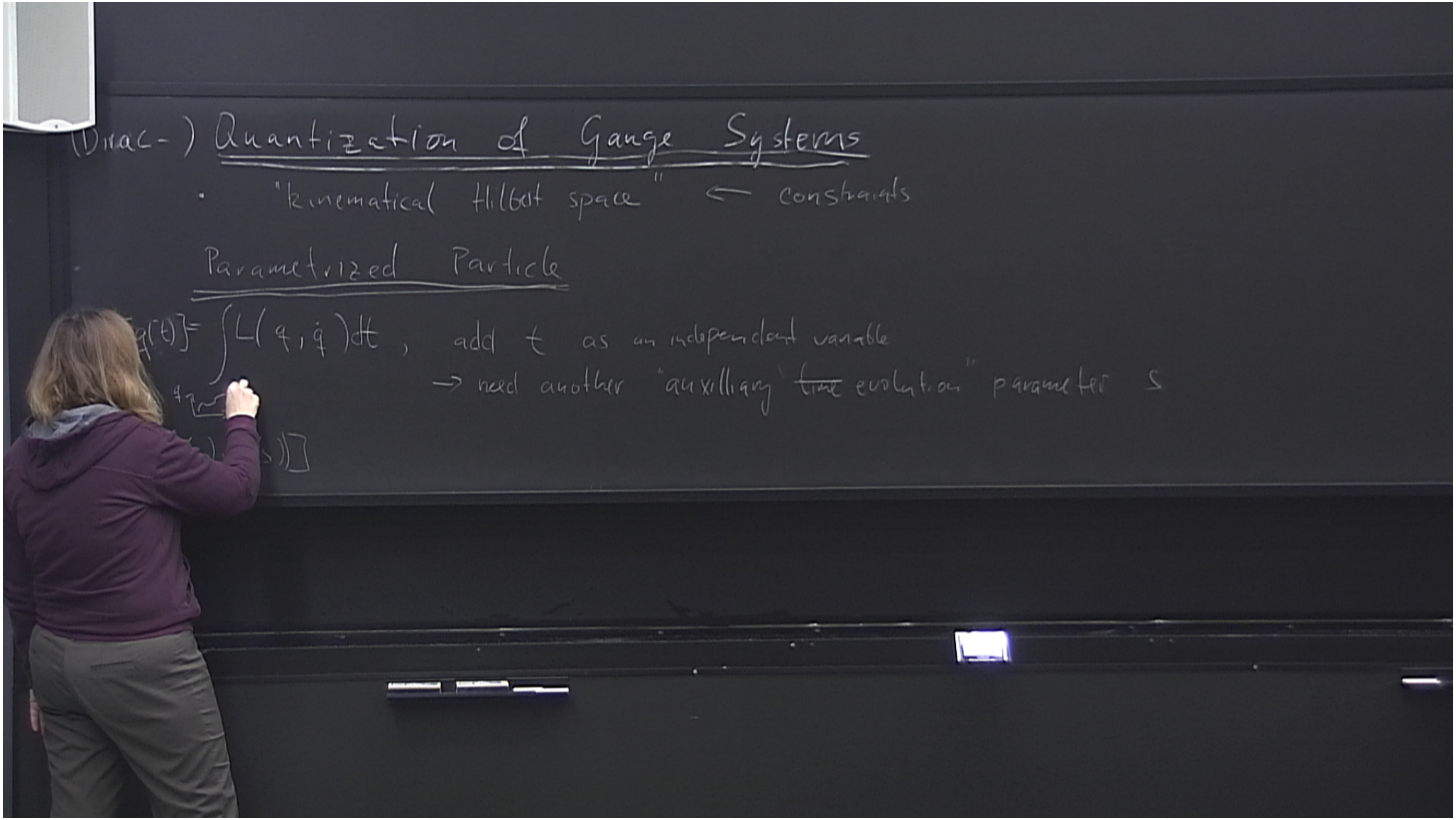
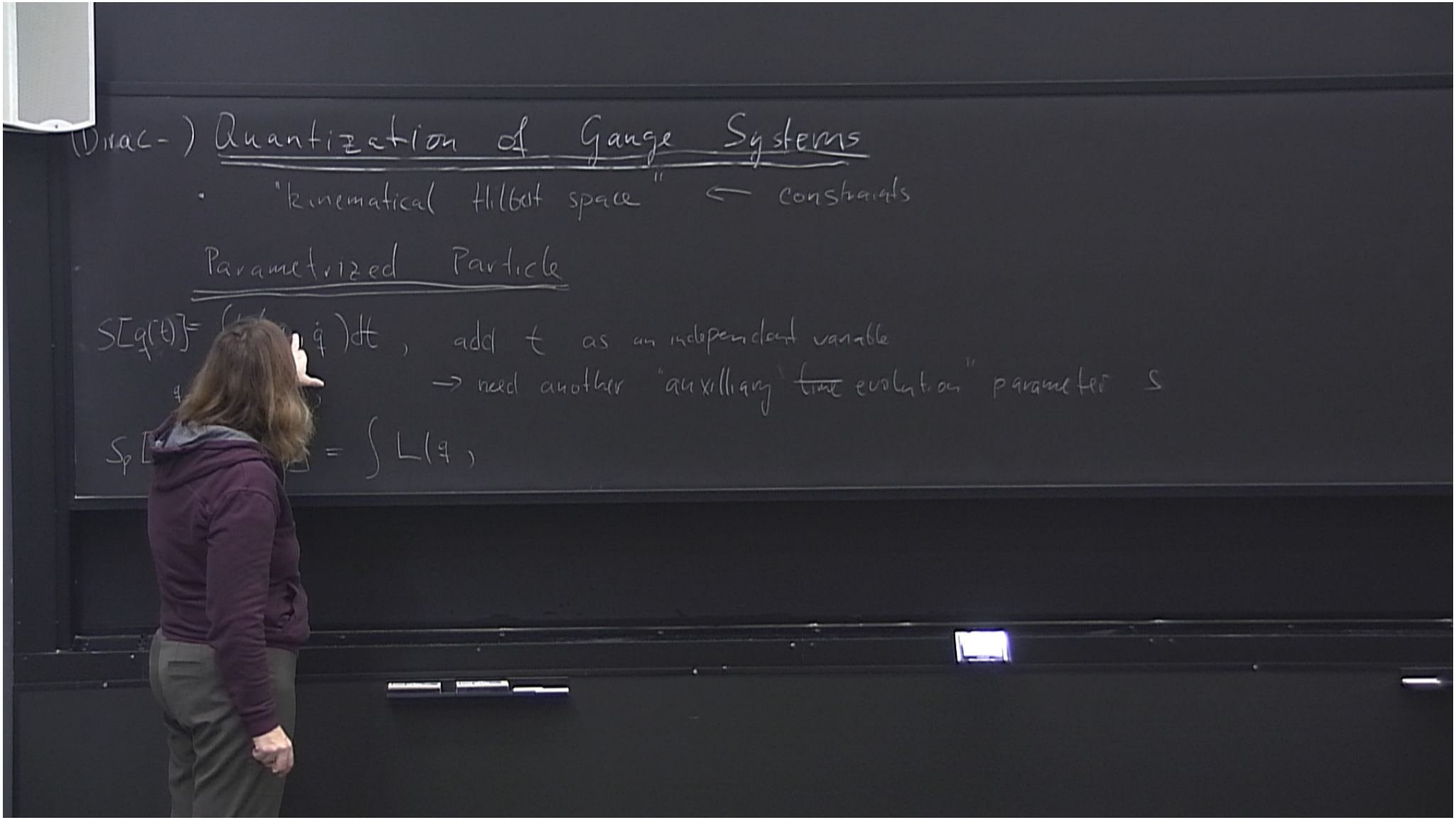
Title: Quantum Gravity Review-6

Date: Feb 24, 2015 10:15 AM

URL: http://www.pirsa.org/15020057

Abstract:


(Dirac-) Quantization of Gauge Systems
"kinematical Hilbert space" ← constraints
Parametrized Particle
$S[q(t)] = \int L(q,\dot{q})\,dt$, add t as an independent variable
→ need another "auxiliary time evolution" parameter s


(Dirac-) Quantization of Gauge Systems
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Parametrized Particle
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Parametrized Particle

$S[q(t)] = \int L(q, \dot{q})\, dt$, add t as an independent variable

$q(s), t(s) \to q(t)$

→ need another "auxiliary ~~time~~ evolution" parameter s

$\frac{dt}{ds}\frac{dq}{dt} = \frac{dq}{ds}$

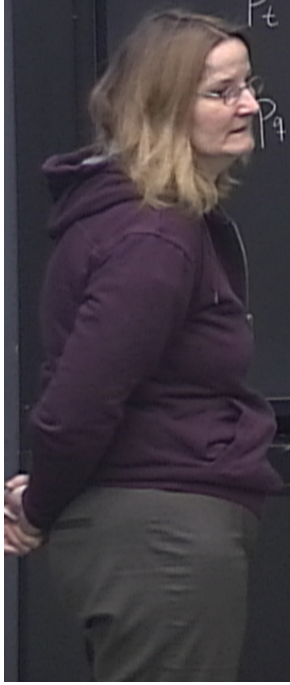
$S_p[q(s), t(s)] = \int L(q, q'/t')\, t'\, ds$

Canonical Analysis S_p

Hamiltonian in the unparametrized system

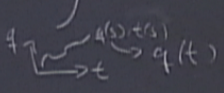
$P_t = \frac{\delta S_p}{\delta t'} = L - \frac{\partial L}{\partial \dot{q}}\frac{q'}{(t')} = -P_q \dot{q} + L = -h(q, P_q)$

$P_q = \frac{\delta S_p}{\delta q'} = t' \frac{\partial L}{\partial q}\frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', q'/t')$



Parametrized Particle

$S[q(t)] = \int L(q, \dot{q})\, dt$, add t as an independent variable



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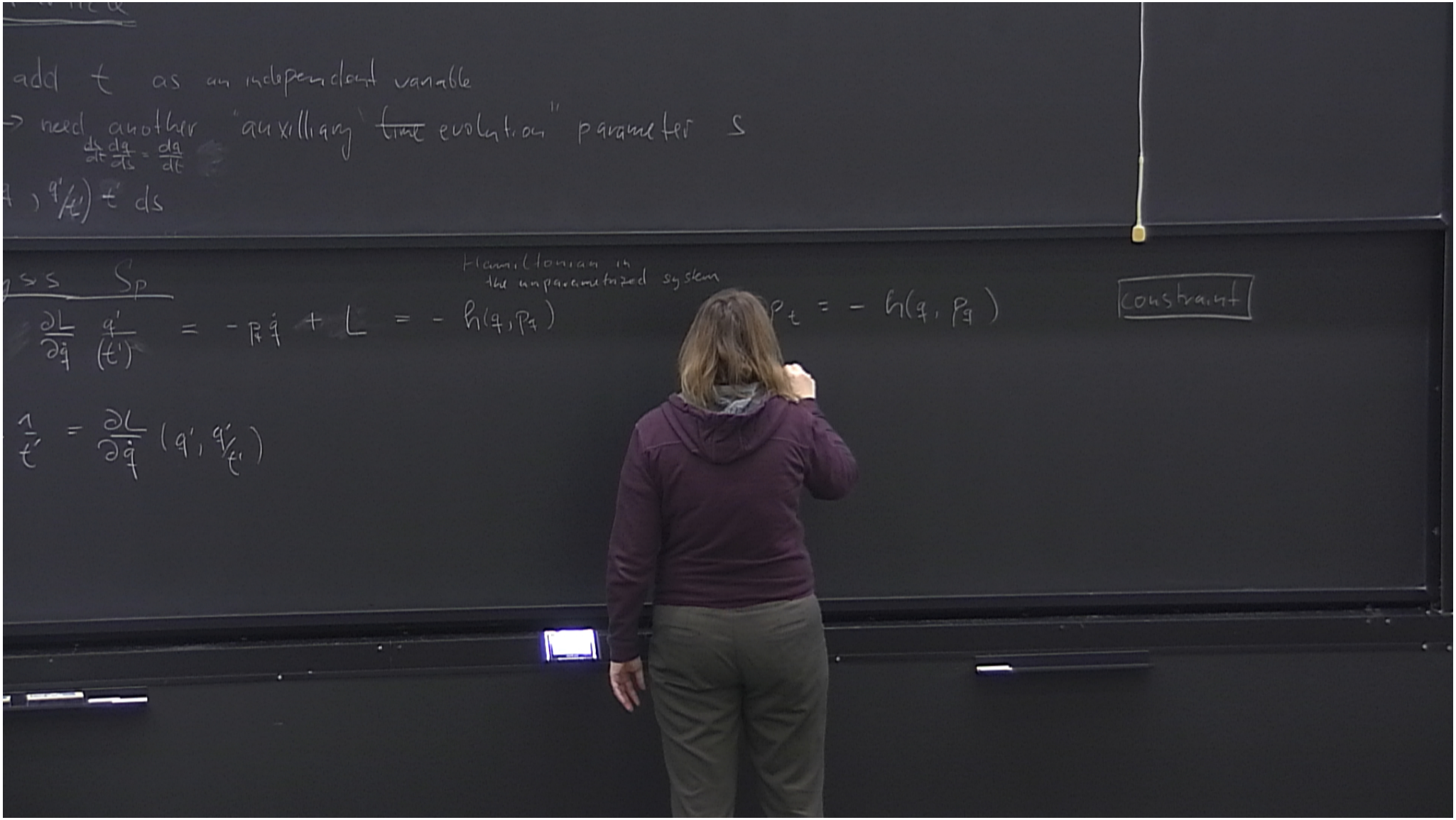
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$$P_q = \frac{\delta S_p}{\delta q'} = t' \frac{\partial L}{\partial q} \frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', q'/t')$$


add t as an independent variable
→ need another "auxiliary time evolution" parameter s
$\frac{dt}{ds}\frac{dq}{dt} = \frac{dq}{ds}$
q'/t'
$t'\,ds$
S_p
Hamiltonian in the unparametrized system
$\frac{\partial L}{\partial \dot{q}}\frac{q'}{(t')} = -p_t\dot{q} + L = -h(q,p_q)$
$p_t = -h(q,p_q)$
constraint
$\frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', q'/t')$

add t as an independent variable

→ need another "auxiliary ~~time~~ evolution" parameter s

$\frac{dt}{ds}\frac{dq}{dt} = \frac{dq}{ds}$

$(\ldots, q', t')\, t'\, ds$

$s \to s_p$

Hamiltonian in the unparametrized system

$$\frac{\partial L}{\partial \dot{q}}\frac{q'}{(t')} = -p_t \dot{q} + L = -h(q, p_q)$$

$$p_t = -h(q, p_q)$$

constraint

$$C = p_t + h(q, p_q) \overset{!}{=} 0$$

$$\{t, p_t\} = +1$$

$$\frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', \frac{q'}{t'})$$

$S[q(t)] = \int L(q, \dot{q})\, dt$, add t as an independent variable

$q(s) \to q(t)$, $t(s) \to t$ → need another "auxiliary ~~time~~ evolution" parameter s

$\frac{dt}{ds} = \frac{dq}{ds} \cdot \frac{dq}{dt}$

$S_P[(q(s), t(s))] = \int L(q, \frac{q'}{t'})\, t'\, ds$

Canonical Analysis S_P

Hamiltonian in the unparametrized system

$p_t = \frac{\delta S_P}{\delta t'} = L - \frac{\partial L}{\partial \dot{q}} \frac{q'}{(t')} = -p_q \dot{q} + L = -h(q, p_q)$

$p_q = \frac{\delta S_P}{\delta q'} = t' \frac{\partial L}{\partial q'} \frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', \frac{q'}{t'})$

$H = p_t t' + p_q q' - L t'$

$p_t = -h(q, p_q)$

$C = p_t + h(q, p_q) \doteq 0$

$\{t, p_t\} = +1$

$S[q(t)] = \int L(q, \dot{q})\, dt$, add t as an independent variable

$q(t) \to q(t(s))$ → need another "auxiliary ~~time~~ evolution" parameter s

$\frac{dt}{ds}\frac{dq}{dt} = \frac{dq}{ds}$

$S_p[(q(s), t(s))] = \int L(q, \frac{q'}{t'})\, t'\, ds$

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$C = p_t + h(q, p_q) \doteq 0$

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$p_q = \frac{\delta S_p}{\delta q'} = t' \frac{\partial L}{\partial q'} \frac{1}{t'} = \frac{\partial L}{\partial \dot{q}}(q', \frac{q'}{t'})$

$H = p_t t' + \underbrace{p_q q'}_{p_q \dot{q} t'} - L t' =$

$S[q(t)] = \int L(q, \dot{q})\, dt$, add t as an independent variable

$q(s) \to q(t)$ $\quad t(s) \to t$ $\quad \frac{dq}{ds} = \frac{dq}{dt}\frac{dt}{ds}$

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$S_p[(q(s), t(s))] = \int L(q, \frac{q'}{t'})\, t'\, ds$

Canonical Analysis S_p

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$C = p_t + h(q, p_q) \doteq 0$

$\{t, p_t\} = +1$

$H = p_t t' + \underbrace{p_q q'}_{p_q \dot{q} t'} - L t' = t'(p_t + h(p, q)) = t' C$

$$S_p[(q(s), t(s))] = \int L(q, q'/t') \, t' \, ds$$

$$H = NC$$

$$t' = \{t, H\} = N$$

$$q' = \{q, H\} = N\{q, h\}$$

"gauge transformation"

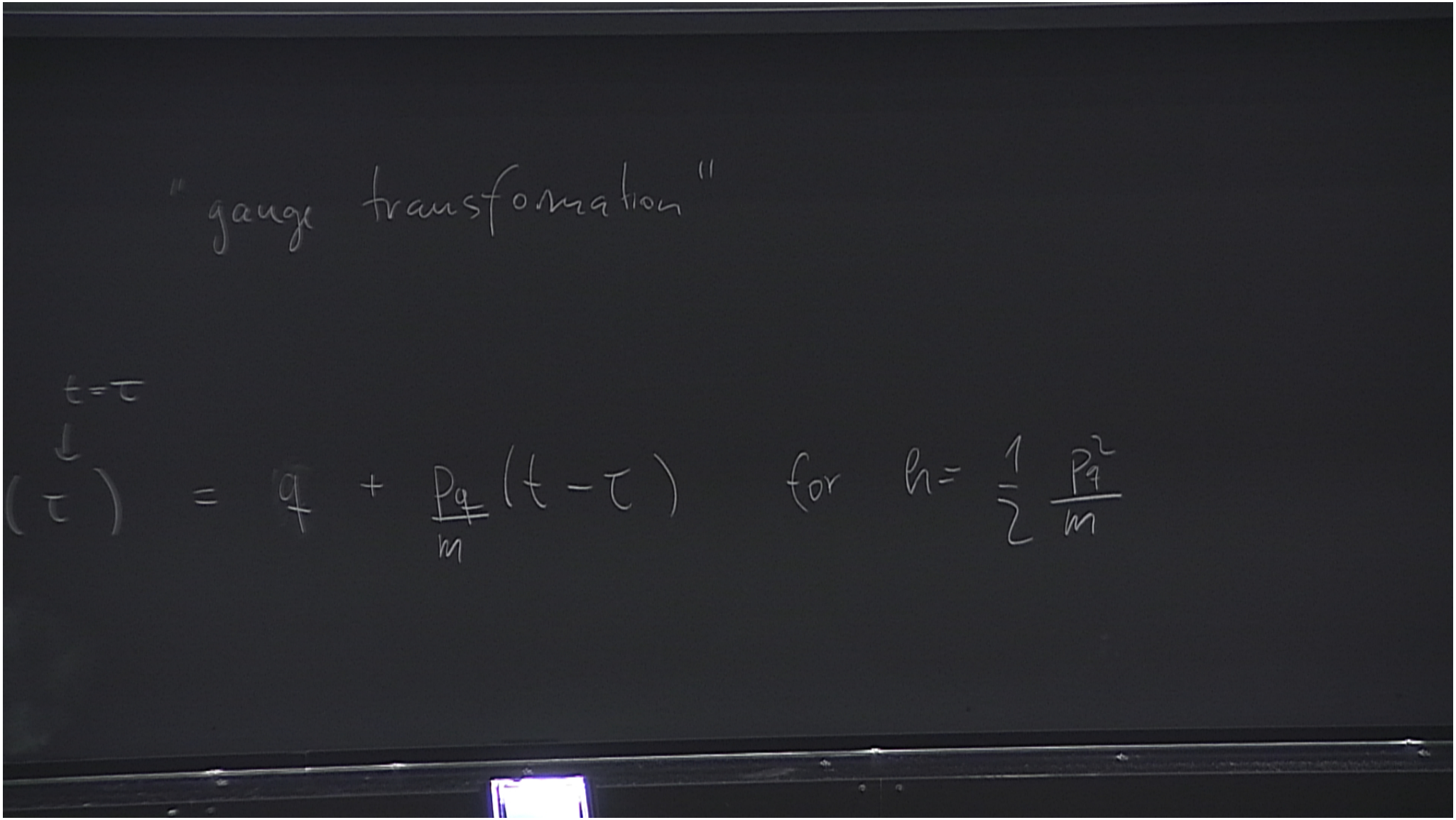
Dirac observables

gauge fixing (family) $T_\tau = t - \tau$

$$F_q(\tau) = q + P_q(t - \tau)$$

$t = \tau$




"gauge transfomation"
t=τ
(τ) = q + Pq/m (t − τ)
for h = 1/2 Pq²/m

$q'(t')\,t'\,ds$

$= N$

$= N\{q, h\}$

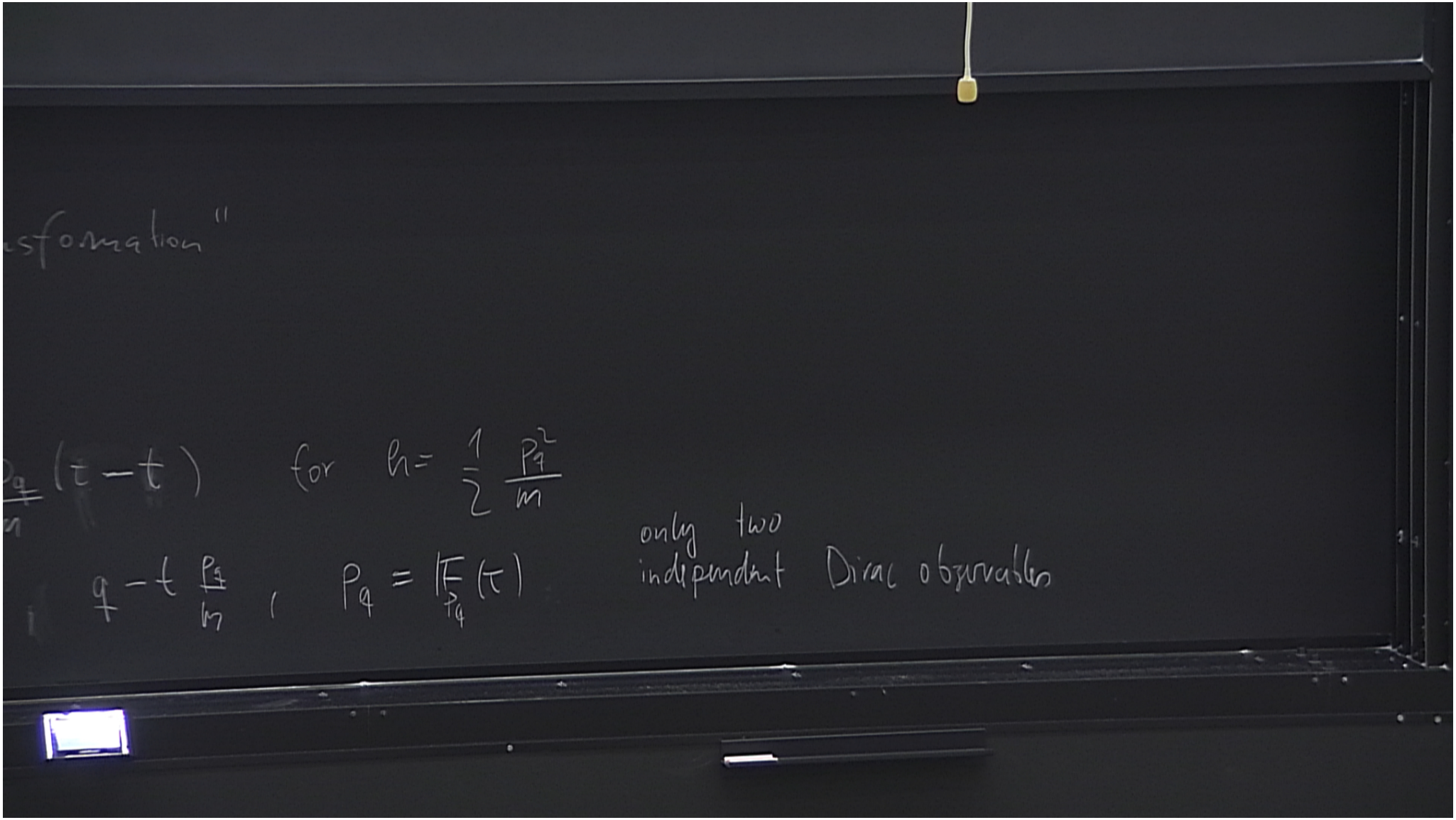
"gauge transformation"

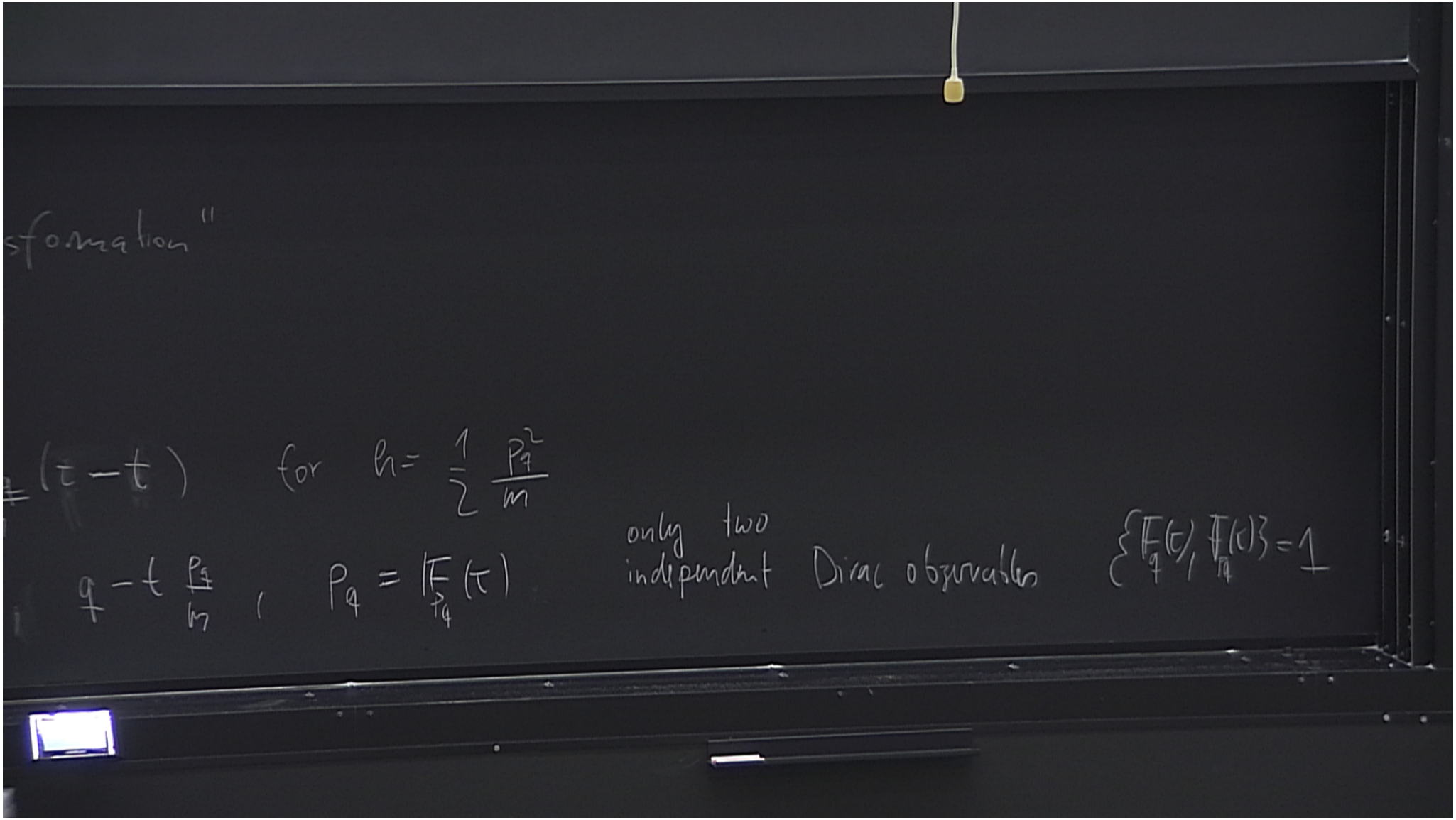
$\tau = t - \tau$

$t = \tau$

$F_q(\tau) = q + \frac{p_q}{m}(t - t)$ for $h = \frac{1}{2}\frac{p_q^2}{m}$

$\{F_q(t), C\} = 0$


sformation"
$(t-t)$ for $h=\frac{1}{2}\frac{p_q^2}{m}$
$q-t\frac{p_q}{m}$, $p_q=F_q(t)$
only two
independent Dirac observables


sformation"
(t−t)
for
$h = \frac{1}{2}\frac{p_q^2}{m}$
$q - t\frac{p_q}{m}$,
$P_q = F_q(t)$
only two independent Dirac observables
$\{F_q(t), F_q(t)\} = 1$

$p_t \psi = -\frac{\partial \psi}{\partial t}$

$C = 0$, $\{F, C\} = 0$

Observables should be invariant under constraint flow

$\hat{C}\psi = 0$

wave function is invariant under gauge trafos

$C_{cr} = P_q$, $\Rightarrow C_{cr}$ generates translations in q

$\hat{P}_q \psi(q, \dots) = 0$ · constant in q

$p_t \psi = -\frac{\partial}{\partial t}$

$C = 0 \quad , \quad \{F, C\} = 0$

Observables should be invariant under constraint flow

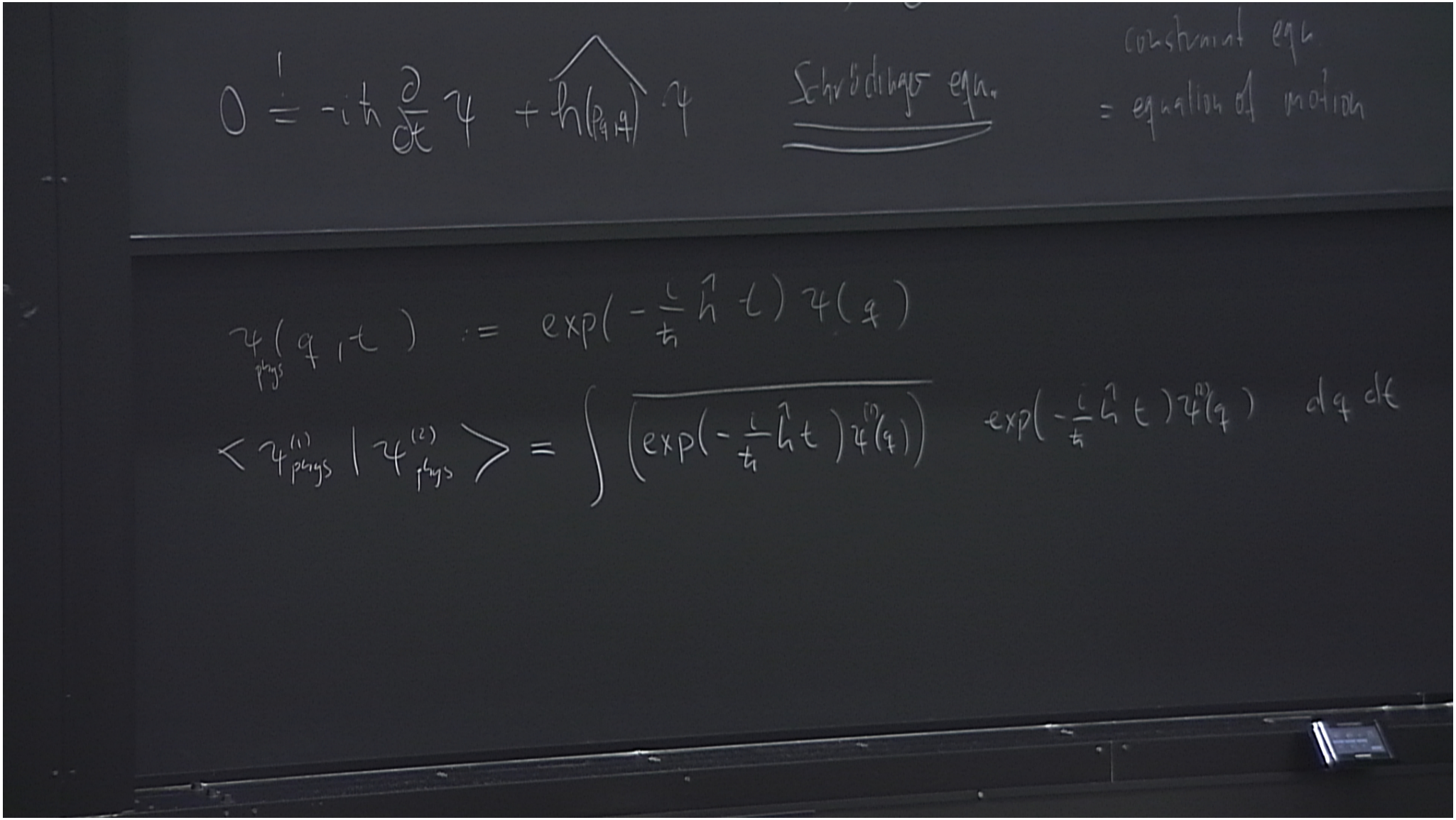
$\hat{C}\psi = 0$

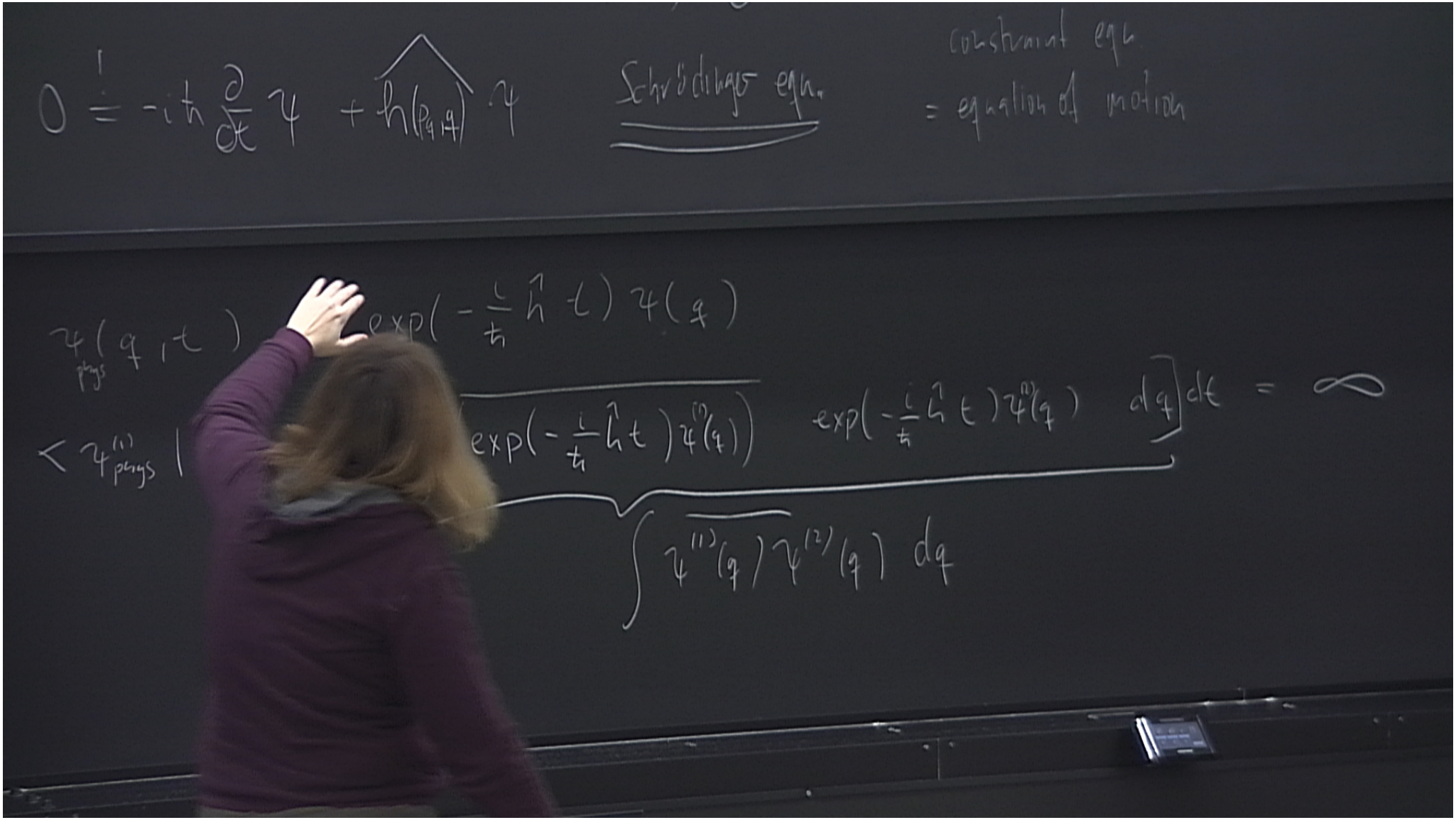
wave function is invariant under gauge trafos

$C_x = P_x \quad , \Rightarrow C_x$ generates translations in q

$\hat{P}_q \psi(q, \dots) = 0$ - constant in q

$0 \stackrel{!}{=} -i\hbar \frac{\partial}{\partial t}\psi + h(p_q, q)\cdot\psi$ Schrödinger eqn.


$0 \stackrel{!}{=} -i\hbar \frac{\partial}{\partial t} \psi + \hat{h}(p_q, q)\,\psi$
Schrödinger eqn.
constraint eqn
= equation of motion
$\psi_{phys}(q,t) := \exp\left(-\frac{i}{\hbar}\hat{h}\,t\right)\psi(q)$
$\langle \psi^{(1)}_{phys} \mid \psi^{(2)}_{phys} \rangle = \int \overline{\left(\exp\left(-\frac{i}{\hbar}\hat{h}\,t\right)\psi^{(1)}(q)\right)}\ \exp\left(-\frac{i}{\hbar}\hat{h}\,t\right)\psi^{(2)}(q)\ dq\,dt$


$0 \overset{!}{=} -i\hbar \frac{\partial}{\partial t} \psi + \hat{h}(p_q, q)\, \psi$
Schrödinger eqn.
constraint eqn.
= equation of motion
$\psi(q,t) = \exp(-\frac{i}{\hbar} \hat{h} t)\, \psi(q)$
$\int \overline{\exp(-\frac{i}{\hbar} \hat{h} t)\, \psi^{(1)}(q)}\ \exp(-\frac{i}{\hbar} \hat{h} t)\, \psi^{(2)}(q)\ dq\, dt = \infty$
$\int \overline{\psi^{(1)}(q)}\, \psi^{(2)}(q)\, dq$

$$0 \overset{!}{=} -i\hbar \frac{\partial}{\partial t}\psi + \hat{h}(p_q, q)\,\psi$$

Schrödinger eqn.

constraint eqn.
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$$\psi_{phys}(q,t) := \exp\left(-\frac{i}{\hbar}\hat{H}\,t\right)\psi(q)$$

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$$\int \overline{\psi^{(1)}(q)}\,\psi^{(2)}(q)\,dq$$

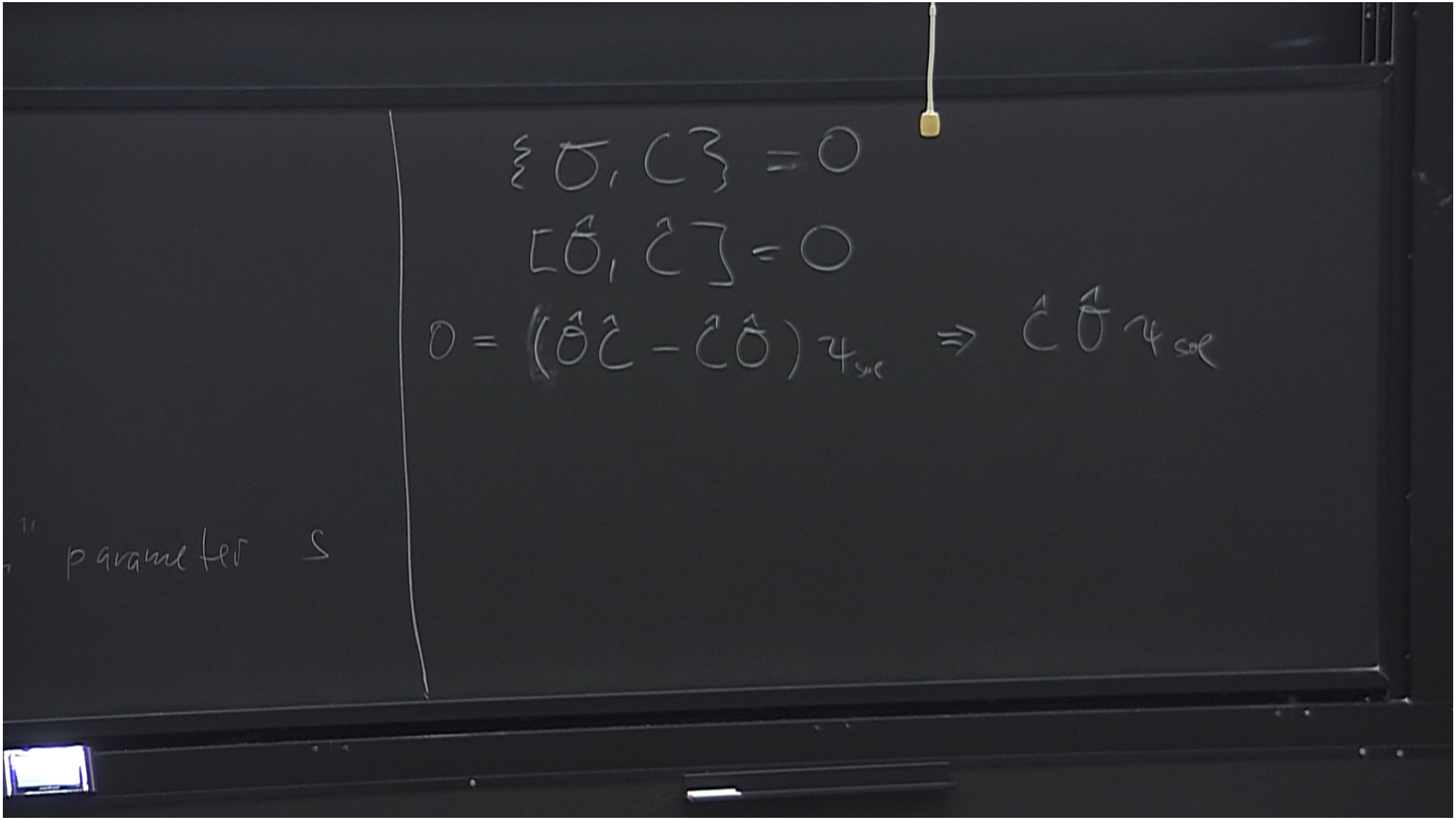
$0 = -i\hbar \frac{\partial}{\partial t} \psi + h(p_q, q)\, \psi$ Schrödinger eqn. = equation of motion

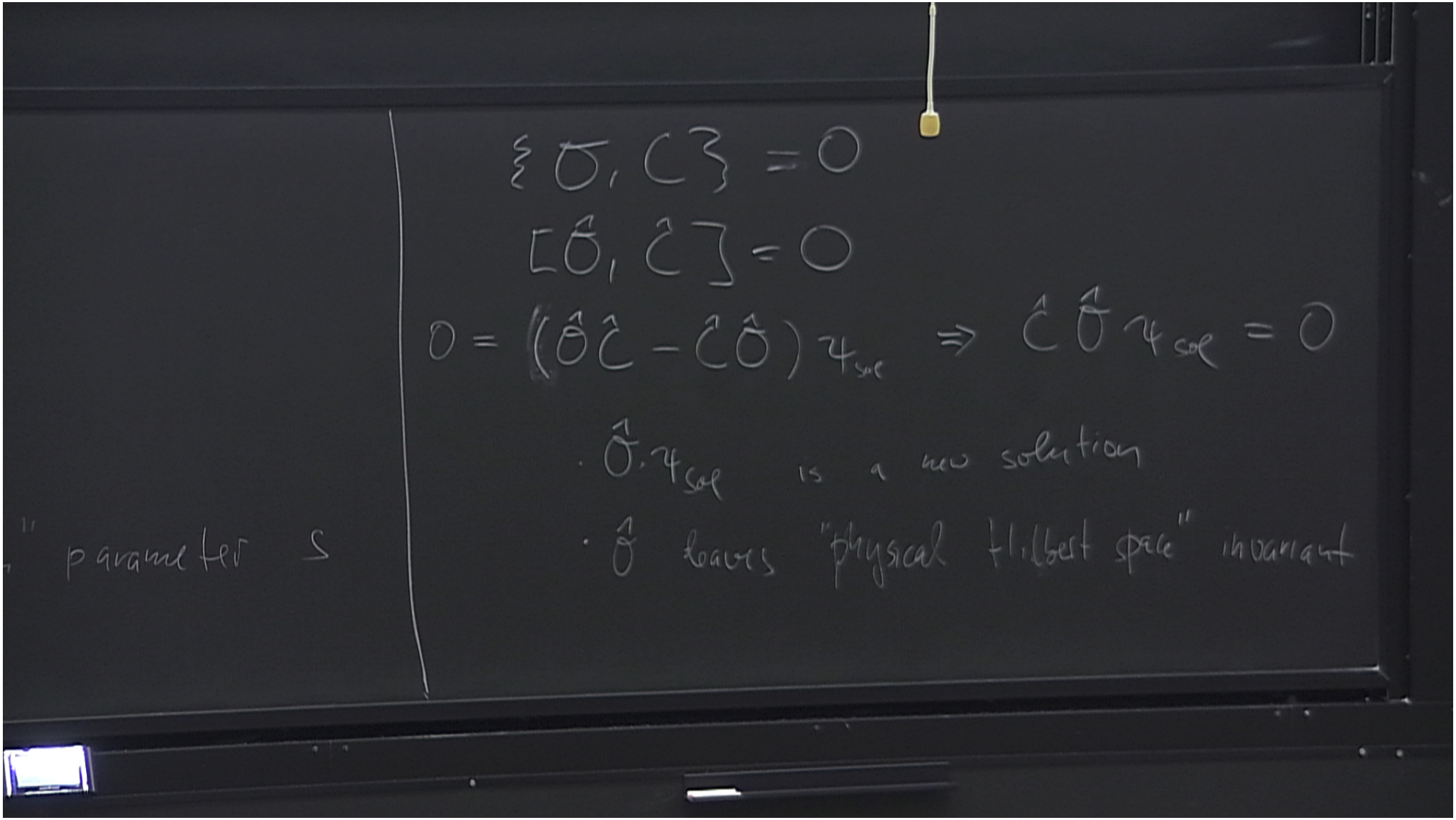
$\psi_{phys}(q, t) := \exp\left(-\frac{i}{\hbar} \hat{h}\, t\right) \psi(q)$

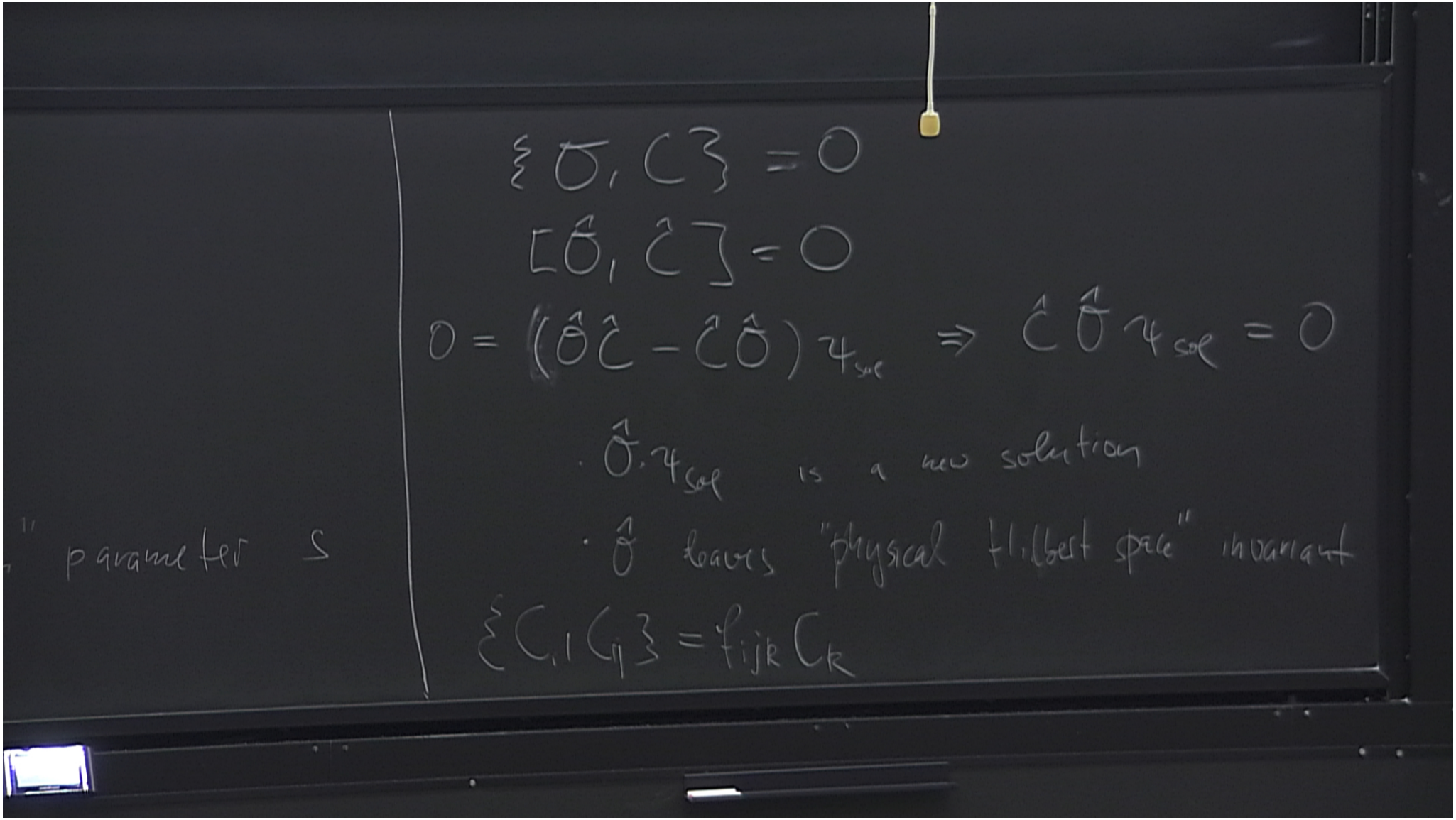
$\langle \psi^{(1)}_{phys} \mid \psi^{(2)}_{phys} \rangle = \int \left(\overline{\exp\left(-\frac{i}{\hbar} \hat{h} t\right) \psi^{(1)}(q)} \right) \exp\left(-\frac{i}{\hbar} \hat{h} t\right) \psi^{(2)}(q)\, dq\, dt =$

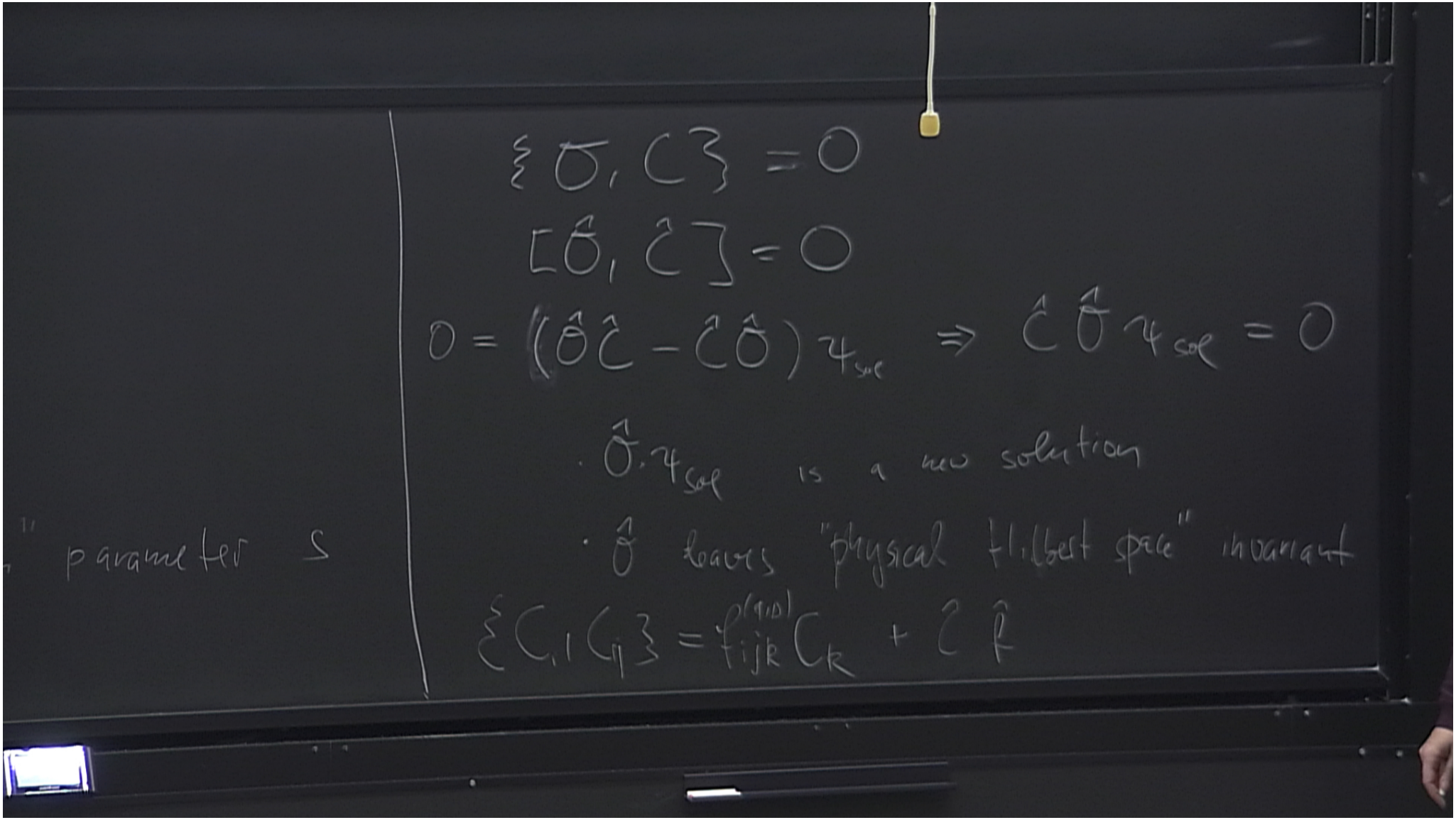
$\int \overline{\psi^{(1)}(q)}\, \psi^{(2)}(q)\, dq$

"Need physical inner product"


$\{\hat{O}, \hat{C}\} = 0$
$[\hat{O}, \hat{C}] = 0$
$0 = (\hat{O}\hat{C} - \hat{C}\hat{O})\,\psi_{sol} \Rightarrow \hat{C}\hat{O}\,\psi_{sol}$
parameter s


{O, C} = 0
[Ô, Ĉ] = 0
0 = (ÔĈ − ĈÔ) ψ_sol ⇒ Ĉ Ô ψ_sol = 0
· Ô · ψ_sol is a new solution
· Ô leaves "physical Hilbert space" invariant
parameter s


$\{O, C\} = 0$
$[\hat{O}, \hat{C}] = 0$
$0 = (\hat{O}\hat{C} - \hat{C}\hat{O})\,\psi_{sol} \Rightarrow \hat{C}\hat{O}\,\psi_{sol} = 0$
$\hat{O}\cdot\psi_{sol}$ is a new solution
$\hat{O}$ leaves "physical Hilbert space" invariant
parameter s
$\{C_i, C_j\} = f_{ijk} C_k$


$\{O, C\} = 0$
$[\hat{O}, \hat{C}] = 0$
$0 = (\hat{O}\hat{C} - \hat{C}\hat{O})\,\psi_{sol} \Rightarrow \hat{C}\hat{O}\,\psi_{sol} = 0$
$\hat{O}\cdot\psi_{sol}$ is a new solution
$\hat{O}$ leaves "physical Hilbert space" invariant
$\{C_i, C_j\} = f_{ijk}^{(q,p)} C_k + C\,\hat{R}$
parameter s