

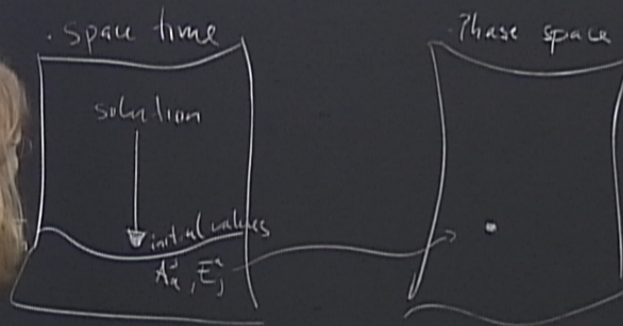
Title: Quantum Gravity Review-5

Date: Feb 23, 2015 10:15 AM

URL: <http://www.pirsa.org/15020056>

Abstract:

Constraint algebra



$$\begin{aligned} \dot{E} &= \{E, H\} \\ \dot{A} &= \{A, H\} \\ \text{constraint } \dot{H} &= \{H', H\} \stackrel{\text{possibly different Lagrangian}}{=} \begin{matrix} 0 \\ \{H' = 0\} \end{matrix} \end{aligned}$$

$$\{H[N], H[N']\} = H[N''[A, E]]$$

$$\begin{aligned} \dot{E} &= \{E, H\} \\ \dot{A} &= \{A, H\} \\ \text{constraint } \dot{H} &= \{H', H\} \end{aligned}$$

possibly
different
Lagrangian mult.

$$\stackrel{\text{for all}}{=} \begin{matrix} 0 \\ \text{or} \\ 0 \end{matrix}$$

$$\{H[N], H[N']\} = H[N''[A, E]]$$

constraints

$$= \int \left(\partial_b (\lambda')^b \epsilon_{k l m} E^{b m} \lambda^l - \epsilon_{k l m} (\lambda')^k (\partial_b \lambda^l + \epsilon^{l p q} A_{b p} \lambda^q) E^{b m} \right. \\ \left. - \epsilon_{k l m} (\lambda')^k A_b^l \epsilon^{m p q} E^{b q} \lambda^p \right) d^2 x$$

$$\{ \Lambda^a, \Lambda^b \} = f^{ab}_c \Lambda^c$$

$$g[\Lambda^a]$$

$$\partial_b \Lambda^a + \epsilon^{apq} A_{bp} \Lambda^q = E^{bm}$$

$$A_b^a (\epsilon^{apq} E^{bq} \Lambda^q) d^2 x$$

$$= (\delta_{b'k} \delta_{a'c} - \delta_{a'k} \delta_{b'c})$$

$$= \int \epsilon_{jlm} (\Lambda^j)' \Lambda^l g^m d^2 x$$

$$[\Lambda^a, \Lambda^b] = \epsilon^{abc} (\Lambda^c)' T_c$$

$$\Lambda = \Lambda^a T_a \quad \text{SO(3)-Lie Algebra generator}$$

$$\Lambda^k (\partial_b \Lambda^l + \epsilon^{lpq} A_{bp} \Lambda^q) E^{bm}$$

$$\Lambda^k A_b^l (\epsilon^{mpq} E^{bg} \Lambda^q) d^2 x$$

$$\delta^{k'l'm} = (\delta^{k'l} \delta^{l'm})$$

$$\Lambda^l) d^2 x$$

$$\epsilon_{jlm} (\Lambda^j)' \Lambda^l g^{lm} d^2 x$$

$$[\Lambda', \Lambda] = \epsilon_{jk} (\Lambda^j)' (\Lambda^k)' T_c$$

$$\Lambda = \Lambda' T' \quad \text{SO(3)-Lie Algebra generator}$$

$$\{g[\Lambda'], g[\Lambda]\} = g[[\Lambda', \Lambda]]$$

$$\Lambda^R (\partial_b \Lambda^e + \epsilon^{epq} A_{bp} \Lambda^q) E^{bm}$$

$$\Lambda^R A_b^e (\epsilon^{mq} E^{bg} \Lambda^q) d^2 x$$

$$\epsilon^{k'l'm} = (\delta_k^{l'} \delta_l^{m'} - \delta_l^{k'} \delta_k^{m'})$$

$$\Lambda^e) d^2 x = \int \left[\epsilon^{epq} \Lambda^q \right] g^m d^2 x$$

$$[\Lambda', \Lambda] = \epsilon_{,r}^{l} (\Lambda')^i (\Lambda^k) T_c$$

$$\Lambda = \Lambda' T' \quad \text{SO(3)-Lie Algebra generator}$$

$$\{g[\Lambda'], g[\Lambda]\} = g[[\Lambda', \Lambda]]$$

$$\{F[N], F[N']\} = 0$$

$$\partial_b \Lambda^e + \epsilon^{epq} A_{bp} \Lambda^q E^{bm}$$

$$\Lambda^e \left(A_b^e \left(\epsilon^{mq} E^{bg} \Lambda^q \right) d^2 x \right)$$

$$E^{b'c'm} = \left(\delta^{b'c} \delta^{m'} - \delta^{b'c'} \delta^{m} \right)$$

$$\Lambda^e) d = \int \left[\epsilon_{jlm} (\Lambda')^j \Lambda^l \right] G^m d^2 x$$

$$[\Lambda', \Lambda] = \epsilon_{jkl} (\Lambda')^j (\Lambda^k) T_l$$

$$\Lambda = \Lambda' T' \quad \text{SO(3)-Lie Algebra generator}$$

$$\{g[\Lambda'], g[\Lambda]\} = g[[\Lambda', \Lambda]]$$

$$\{F[N], F[N']\} = 0$$

$$\{F[N], g[\Lambda]\}$$

$$\Lambda^{\ell} (\partial_b \Lambda^{\ell} + \epsilon^{\ell pq} A_{bp} \Lambda^q) E^{bm}$$

$$\Lambda^{\ell} A_b^{\ell} (\epsilon^m_{pq} E^{bq} \Lambda^q) d^2 x$$

$$\epsilon^{k'l'm} = (\delta_k^{k'} \delta_l^{l'} - \delta_l^{k'} \delta_k^{l'})$$

$$\Lambda^{\ell} d^2 x = \int \boxed{\epsilon_{jlm} (\Lambda')^j \Lambda^l} g^m d^2 x$$

$$[\Lambda', \Lambda] = \epsilon_{j\ell} (\Lambda')^j (\Lambda^{\ell}) T_{\ell}$$

$$\Lambda = \Lambda' T' \quad \text{SO(3)-Lie Algebra generator}$$

$$\boxed{\{g[\Lambda'], g[\Lambda]\} = g[[\Lambda', \Lambda]]}$$

$$\{F[N], F[N']\} = 0$$

$$\{F[N], g[\Lambda]\} = F[[N, \Lambda]]$$

Phase space for gauge systems

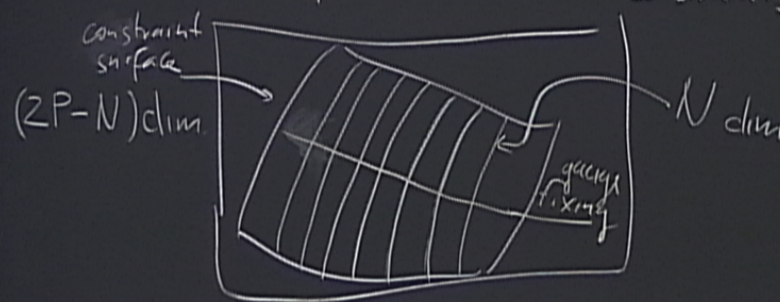
$$\vec{C} = \{C_i, C_j\} = f_{ij}^k C_k$$

first class system

phase space

N indep. constraints

phase space dependent

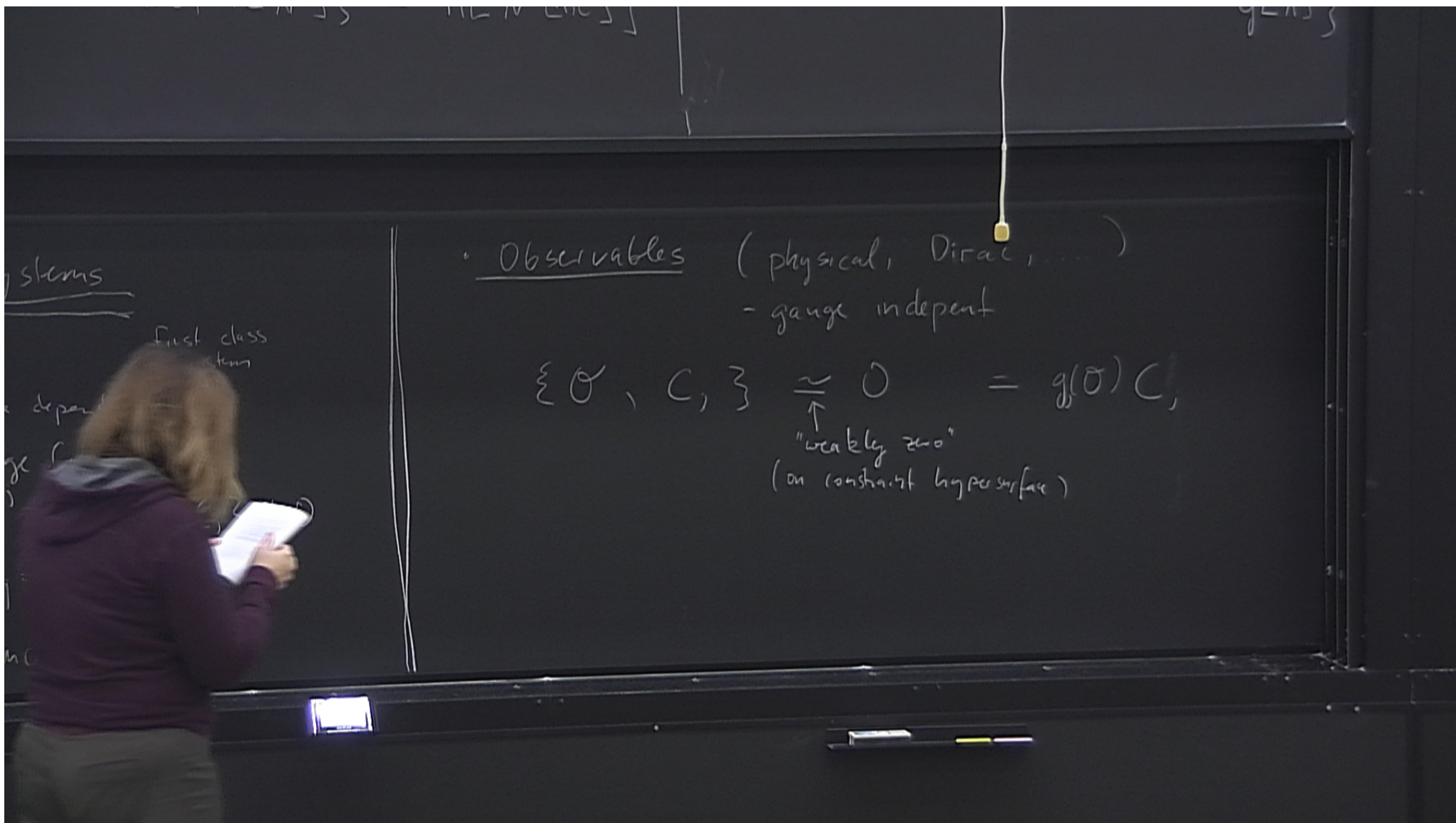


gauge fixing

$$N = T^1 \quad \det \{T^1, C_i\} \neq 0$$

$$C_i = 0, T^1 = 0$$

reduced phase space



systems

first class
system

dependent

fixing

$$\det \{T^i, C_j\} \neq 0$$

$$T^i = 0, T^j = 0$$

reduced phase space

• Observables (physical, Dirac, ...)
- gauge independent

$$\{O, C_j\} \stackrel{\sim}{=} 0 = g(O) C_j$$

↑
"weakly zero"
(on constraint hypersurface)

4D Gravity - None. Explicitly, for compact spatial mfd's
- 10 for asymp. flat

systems

first class
system

dependent

fixing

$$\det \{T^i, C, \} \neq 0$$

$$= 0, T^i = 0$$

reduced phase space

• Observables (physical, Dirac, ...)
- gauge independent

$$\{ \mathcal{O}, C, \} \stackrel{\sim}{=} 0 = g(\mathcal{O}) C,$$

↑
"weakly zero"
(on constraint hypersurface)

4D Gravity - None. Explicitly, for compact spatial mfd's
- 10 for asymp. flat
- Thm (Tone): "non-local"