Title: Quantum Gravity Review-3

Date: Feb 19, 2015 10:15 AM

URL: http://www.pirsa.org/15020052

Abstract:

Global & Local Symmetries. Noether's thms

$$S = \int d^4x \, \mathcal{L}(\varphi^a, \partial_\mu \varphi^a) \quad ; \quad \delta_s \varphi^a$$

(only variation of the fields, no change of space time coord.)

$$0 \doteq \delta_s S = \int d^4x \; \partial_\mu (\epsilon X^\mu)$$

$$= \int d^4x \left(\frac{\partial \mathcal{L}}{\partial \varphi^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \right) \delta_s \varphi^a + \int d^4x \; \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \delta_s \varphi^a \right)$$

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$$= \int d^4x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi^a)} \right)}_{L_a} \delta_s \varphi^a + \int d^4x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi^a)} \delta_s \varphi^a \right)$$

$$= \int d^n x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \right)}_{L_a} \delta_s \varphi^a + \int d^n x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \delta_s \right.$$

$$0 = \int d^n x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \delta_s \varphi^a - \epsilon \lambda^\mu \right) + \int L_a \, \delta_s \varphi^a \, d^n x$$

<u>Two cases</u>: Global symmetry: $\delta_{glob} \varphi^a = \varepsilon B^a$

Local symmetry: $\delta_{gauge} \varphi^a = \varepsilon(x) B^a + C^{a\mu} \partial_\mu \varepsilon(x)$

$$= \int d^n x \underbrace{\left(\frac{\partial \mathcal{L}}{\partial \varphi^a} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi^a)} \right)}_{L_a} \delta_s \varphi^a + \int d^n x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \delta_s \varphi^a \right.$$

$$0 = \int d^n x \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \delta_s \varphi^a - \varepsilon \lambda^\mu \right) + \int L_a \, \delta_s \varphi^a \, d^n x$$

Two cases: Global symmetry: $\delta_{glob} \varphi^a = \varepsilon B^a$

Local symmetry: $\delta_{gauge} \varphi^a = \varepsilon(x) B^a + C^{a\mu} \partial_\mu \varepsilon(x)$;

- Assume: global symm., ε = const, $L_a = 0$: $j^\mu = \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} B^a - \lambda^\mu \right)$ currents

$$\int_R \partial_\mu j^\mu \, dx^n = 0$$

$$R = [t_1, t_2] \times \Sigma \implies Q^{(1)} = Q^{(2)}$$

$$Q = \int_\Sigma j^0 \, d^{n-1}x \,, \qquad \dot{Q} = 0$$

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Noether's 1st thm

EOM satisfied

$$0 = \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} \left(\varepsilon B^a + C^{a\nu} \partial_\nu \varepsilon \right) \right)$$

$$= \varepsilon \, \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} B^a \right)$$

$$+ (\partial_\mu \varepsilon) \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} B^a \right) + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\nu} \right) (\partial_\nu \varepsilon)$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\mu} \partial_\nu \partial_\mu \varepsilon$$

$$+(\partial_\mu \varepsilon)\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} B^a\right) + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\nu}\right)(\partial_\nu \varepsilon)$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\mu} \partial_\nu \partial_\mu \varepsilon$$

$$\partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} B^a\right) = 0$$

$$\partial_0 \partial_0 \varphi = (\ldots\ldots)$$

$$\partial_0 \varphi,\ \partial_n \varphi,\ \ldots \partial_0 \partial_n \varphi$$

$$+ (\partial_\mu \varepsilon)\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi^a)} B^a\right) + \partial_\mu\left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi^a)} C^{a\nu}\right)(\partial_\nu\varepsilon)$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu\varphi^a)} C^{a\nu}\,\partial_\nu\partial_\mu\varepsilon$$

$$\partial_\mu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi^a)}B^a\right)=0$$

$$\partial_0\partial_0\varphi = (\ldots\ldots)$$

$$\partial_0\varphi,\ \partial_n\varphi,\ \ldots\ \partial_0\partial_n\varphi$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi^a)}B^a + \partial_\nu\left(\frac{\partial\mathcal{L}}{\partial(\partial_\nu\varphi^a)}C^{a\mu}\right)=0$$

$$\frac{\partial\mathcal{L}}{\partial(\partial_\mu\varphi^a)}C^{a\nu} + \frac{\partial\mathcal{L}}{\partial(\partial_\nu\varphi^a)}C^{a\mu}=0$$

Gauge Symmetries in 3D first order gravity action

Internal rotations :

$$\delta^R_\Lambda e^j_\mu = \underline{\epsilon^j{}_{ki}}\, e^k_\mu \Lambda^i$$

$$\delta^R_\Lambda \omega^m_\mu = \partial_\mu \Lambda^m + \epsilon^m{}_{li}\, \omega^l_\mu \Lambda^i$$

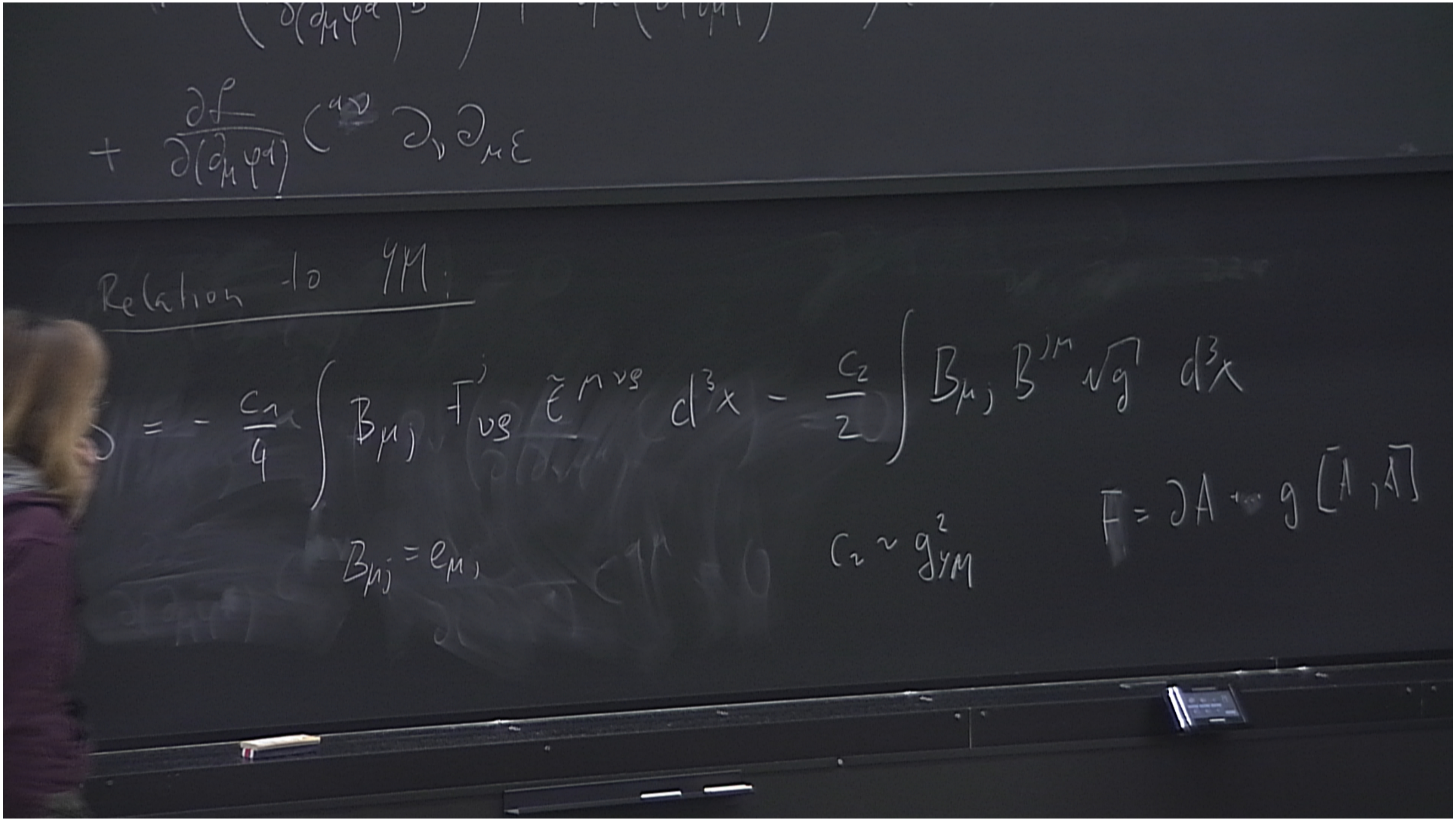
$$R_{ij} = \delta_{ij} + \varepsilon \, \epsilon_{ijk} \Lambda^{k}$$

$$\Lambda^{i} = D_{\mu}^{(\omega)} \lambda_{m}$$

$$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\nu} \partial_\nu \partial_\mu \varepsilon$$

Relation to YM:

$$S = -\frac{c_1}{4}\int B_{\mu j} F^{j}_{\nu\rho}\, \tilde{\varepsilon}^{\mu\nu\rho}\, d^3x - \frac{c_2}{2}\int B_{\mu j} B^{j\mu} \sqrt{g}\, d^3x$$


$+ \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi^a)} C^{a\nu} \partial_\nu \partial_\mu \varepsilon$
Relation to YM:
$S = -\frac{c_1}{4} \int B_{\mu j} F^j_{\nu\rho} \tilde{\varepsilon}^{\mu\nu\rho} d^3x - \frac{c_2}{2} \int B_{\mu j} B^{j\mu} \sqrt{g}\, d^3x$
$F = \partial A + g[A,A]$
$B_{\mu j} = e_{\mu j}$
$c_2 \sim g^2_{YM}$