

Title: Quantum Gravity Review-2

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Abstract:

Triads and Spin connection

- objects w/

$$g_{\mu\nu} = e_{\mu}^i e_{\nu}^j \delta_{ij} \quad \text{co-triads}$$

$$g^{\mu\nu} e_{\nu}^i \delta_{ij} = e_j^{\mu} \quad \text{-frame triads}$$

$$e_j^{\mu} e_{\mu}^i = e_j^i \quad e_i^{\mu} e_{\mu}^j = \delta_i^j \quad g_{\mu\nu} = \delta_{ij}$$

- objects w/ space time indices $\mu, \nu, \dots$
 & internal indices $i, j, \dots$

← space time tensor
 ← transformation rule
 w/ internal rotations

- covariant derivative ∇_μ (wrt $\Gamma_{\mu\nu}^{\rho}$ L-C-connection)
 ↳ w/ internal indices: $D_\mu \omega_{\mu k}^j$ spin-connection

$$D_\mu \phi^i = \partial_\mu \phi^i + \omega_{\mu k}^i \phi^k$$

$$D_\mu \phi_j = \partial_\mu \phi_j - \omega_{\mu}^k{}_j \phi_k$$

$$D_\mu v_\nu^i = \partial_\mu v_\nu^i - \Gamma_{\mu\nu}^\rho v_\rho^i + \omega_{\mu k}^i v_\nu^k$$

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$$D_\mu v_\nu^i = \partial_\mu v_\nu^i - \Gamma_{\mu\nu}^\rho v_\rho^i + \omega_{\mu k}^i v_\nu^k$$

time indices $\mu, \nu, \dots$
 spatial indices $i, j, \dots$

← space time tensor
 ← transformation rule
 w/ internal rotations

∇_μ (wrt $\Gamma_{\mu\nu}^\sigma$ L-C-connection)

$\partial_\mu \psi^i + \omega_{\mu k}^j \psi^k$ spin-connection

$+ \omega_{\mu k}^j \psi^k$

$- \omega_{\mu j}^k \psi^j$

$- \Gamma_{\mu\nu}^\sigma \psi^\nu + \omega_{\mu k}^j \psi^k$

$\omega' \sim$ inhom.

$(\partial_\mu \psi^i) \sim$ inhom.
 $R^\alpha_\mu \partial_\alpha R^\mu_\nu$

Compatib. condition

$$\underline{D_\mu e^j_\nu} = 0$$

$$\partial_\mu e^j_\nu - \Gamma_{\mu\nu}^s e^j_s + \omega_{\mu k}^j e^k_\nu = 0$$

$$\boxed{\omega_{\mu k}^j = -e^{\nu}_k \nabla_\mu e^j_\nu}$$

spin-connection

$$\tilde{\Gamma}_{\mu\nu}^s = e^s_{, \mu} \omega_{\mu R}^k e^k_{, \nu} + e^s_{, \nu} \partial_{\mu} e^k_{, \nu}$$

When is $\tilde{\Gamma} = \Gamma$?

$$0 = \nabla_{\mu} \underbrace{(e^{\nu}_R e^s_{, \nu} g_{\nu s})}_{\delta_{Rj}} = -\omega_{\mu j R} - \omega_{\mu k j}$$

$$\omega_{\mu j R} = \omega_{\mu R}^l \delta_{lj}$$

metric compatible of
 $\Leftrightarrow$ antisymmetry
 of ω

$$\tilde{\Gamma}_{\mu\nu}^s = e_{\mu}^s \omega_{\mu R}^j e_{\nu}^k + e_{\mu}^s \partial_{\mu} e_{\nu}^j$$

then is $\tilde{\Gamma} = \Gamma$?

$$= \nabla_{\mu} \left(\underbrace{e_{\mu}^{\nu} e_{\nu}^s}_{\delta_{\mu}^s} g_{\nu s} \right) = -\omega_{\mu j R} - \omega_{\mu k j}$$

$$\omega_{\mu j R} = \omega_{\mu R}^l \delta_{lj}$$

$$\nabla_{\mu}(\delta_{\mu}^k) = 0$$

metric compatible of Γ
 $(\Rightarrow)$ anti-symmetry
 of ω

$$e^{\mu}_i e^{\nu}_j = e^{\mu}_j e^{\nu}_i \quad \text{triads} \\ g_{\mu\nu} = \delta_{ij}$$

$$D_{\mu} \phi^i = \partial_{\mu} \phi^i + \omega^i_{\mu k} \phi^k$$

$$D_{\mu} \phi_j = \partial_{\mu} \phi_j - \omega^k_{\mu j} \phi_k$$

$$D_{\mu} v^i_{\nu} = \partial_{\mu} v^i_{\nu} - \Gamma^s_{\mu\nu} v^i_s + \omega^i_{\mu k} v^k_{\nu}$$

$$\begin{aligned} D_{\mu} e^i_{\nu} - D_{\nu} e^i_{\mu} &= \partial_{\mu} e^i_{\nu} - \partial_{\nu} e^i_{\mu} - \Gamma^s_{\mu\nu} e^i_s + \Gamma^s_{\nu\mu} e^i_s + \omega^i_{\mu k} e^k_{\nu} - \omega^i_{\nu k} e^k_{\mu} = 0 \\ &= \partial_{\mu} e^i_{\nu} - \partial_{\nu} e^i_{\mu} + \omega^i_{\mu k} e^k_{\nu} - \omega^i_{\nu k} e^k_{\mu} \\ &= (\Gamma^s_{\mu\nu} - \Gamma^s_{\nu\mu}) e^i_s \end{aligned}$$

$$e^{\mu}_{i} e^{\nu}_{j} = e^{\mu\nu}_{ij} \quad \text{triads} \\ g_{\mu\nu} = \delta_{ij}$$

$$D_{\mu} \phi^i = \partial_{\mu} \phi^i + \omega_{\mu k}^i \phi^k$$

$$D_{\mu} \phi_j = \partial_{\mu} \phi_j - \omega_{\mu}^k{}_j \phi_k$$

$$D_{\mu} v^i_{\nu} = \partial_{\mu} v^i_{\nu} - \Gamma_{\mu\nu}^{\rho} v^i_{\rho} + \omega_{\mu k}^i v^k_{\nu}$$

$$D_{\mu} e^j_{\nu} - D_{\nu} e^j_{\mu} = \partial_{\mu} e^j_{\nu} - \partial_{\nu} e^j_{\mu} - \Gamma_{\mu\nu}^{\rho} e^j_{\rho} + \Gamma_{\nu\mu}^{\rho} e^j_{\rho} + \omega_{\mu k}^j e^k_{\nu} - \omega_{\nu k}^j e^k_{\mu} \stackrel{!}{=} 0$$

$$T_{\mu\nu}^j = \partial_{\mu} e^j_{\nu} - \partial_{\nu} e^j_{\mu} + \omega_{\mu k}^j e^k_{\nu} - \omega_{\nu k}^j e^k_{\mu}$$

$$= (\Gamma_{\mu\nu}^{\rho} - \Gamma_{\nu\mu}^{\rho}) e^j_{\rho}$$

$$a_{\nu}^j = \partial_{\mu} r_{\nu}^j - \Gamma_{\mu\nu}^{\rho} r_{\rho}^j + \omega_{\mu k}^j r_{\nu}^k$$

$$+ \Gamma_{\nu\mu}^{\rho} e_{\rho}^j + \omega_{\mu k}^j e_{\nu}^k - \omega_{\mu k}^j e_{\mu}^k = 0$$

$$e_{\nu}^k - \omega_{\nu k}^j e_{\mu}^k = (d_{\omega} e^j)_{\mu\nu}$$

$$\begin{aligned}
 R_{\mu\nu\sigma}^{\quad\sigma} v_{\sigma} &= (\nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu}) (\overbrace{e_{\sigma}^{\quad\sigma}}^{v_{\sigma}}) \\
 &= e_{\sigma}^{\quad\sigma} (\nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu}) v_{\sigma} \\
 &= e_{\sigma}^{\quad\sigma} [\partial_{\mu} \omega_{\nu\sigma k} - \partial_{\nu} \omega_{\mu\sigma k} + \omega_{\mu\sigma l} \omega_{\nu k}^{\quad l} - \omega_{\nu\sigma l} \omega_{\mu k}^{\quad l}] e^{\sigma k} v_{\sigma}
 \end{aligned}$$

$$\begin{aligned}
 R_{\mu\nu\sigma}^{\quad\sigma} v_{\sigma} &= (\nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu}) (\overbrace{e_{\sigma}^{\quad\sigma}}^{v_{\sigma}}) \\
 &= e_{\sigma}^{\quad i} (\nabla_{\mu} \nabla_{\nu} - \nabla_{\nu} \nabla_{\mu}) v_i \\
 &= e_{\sigma}^{\quad i} \underbrace{[\partial_{\mu} \omega_{\nu jk} - \partial_{\nu} \omega_{\mu jk} + \omega_{\mu jk}^{\quad l} \omega_{\nu l}^{\quad k} - \omega_{\nu jk}^{\quad l} \omega_{\mu l}^{\quad k}]}_{F_{\mu\nu jk}} e^{\sigma k} v_{\sigma}
 \end{aligned}$$

$$R_{\mu\nu\sigma}^{\quad\sigma} = e_{\sigma}^{\quad i} e^{\sigma k} F_{\mu\nu jk}$$

$$R_{\mu\nu\sigma} = e_j^\sigma e_k^\sigma F_{\mu\nu}^{jk}$$

$$F_{\mu\nu}^{jk}$$

$$R = R_{\mu\nu} g^{\mu\nu} = e_j^\mu e_k^\sigma F_{\mu\nu}^{jk}(\omega)$$

$$\sqrt{g} = \dots |\det(e)|$$

$$e^{\mu}_{j} e^{\nu}_{k} = e^{\mu}_{j} e^{\nu}_{k} \quad g_{\mu\nu} = \delta_{ij}$$

$$D_{\mu} \phi_j = \partial_{\mu} \phi_j - \omega_{\mu}^k \phi_k$$

$$D_{\mu} v^j_{\mu} = \partial_{\mu} v^j_{\mu} - \Gamma^j_{\mu\nu} v^{\nu}$$

3D First order action

$$\omega_{\mu jk} = \epsilon_{jlk} \omega_{\mu}^l$$

$$F_{\mu\nu jk} = \epsilon_{jlk} F_{\mu\nu}^l$$

$$D_{\mu} \phi_j = \partial_{\mu} \phi_j + \epsilon_{jkl} \omega_{\mu}^k \phi^l$$

$$F_{\mu\nu}^l = \partial_{\mu} \omega_{\nu}^l - \partial_{\nu} \omega_{\mu}^l + \epsilon^{ljk} \omega_{\mu j} \omega_{\nu k}$$

$$T_{\mu\nu}^l = \partial_{\mu} e_{\nu}^l - \partial_{\nu} e_{\mu}^l$$

$$g_{\mu\nu} = e_{\mu}^i e_{\nu}^j \delta_{ij}$$

$$g^{\mu\nu} e_{\nu}^i \delta_{ij} = e_{\mu}^i$$

$$e_{\mu}^i e_{\nu}^j = e_{\mu}^i e_{\nu}^j$$

$$g_{\mu\nu} = \delta_{ij}$$

ω -triads

frame
triads

- covariant derivative

∇_{μ} (with $\Gamma_{\mu\nu}^{\rho}$ L-C-connection)
w/ internal indices $D_{\mu} \omega'_{\mu k}$ spin-connection

$$D_{\mu} \phi^i = \partial_{\mu} \phi^i + \omega'_{\mu k} \phi^k$$

$$D_{\mu} \phi_j = \partial_{\mu} \phi_j - \omega_{\mu}^k \phi_k$$

$$D_{\mu} v_{\nu}^i = \partial_{\mu} v_{\nu}^i - \Gamma_{\mu\nu}^{\rho} v_{\rho}^i + \omega_{\mu k} v_{\nu}^k$$

← transformation rule
w/ internal rotation

$\omega' \sim$ inhom

$(\partial_{\mu} \omega'_{\nu}^i) \sim$ inhom
 $R^i_{\ j} \partial_{\mu} R^j_{\ k}$

3D First order action

$$= \epsilon_{ijk} \omega_{\mu}^l$$

$$F_{\mu\nu}{}^k = \epsilon_{ijk} F_{\mu\nu}^i$$

$$\partial_{\mu} \phi_j + \epsilon_{ijk} \omega_{\mu}^k \phi^i$$

$$\partial_{\mu} \omega_{\nu}^l - \partial_{\nu} \omega_{\mu}^l + \epsilon^{ijk} \omega_{\mu}^j \omega_{\nu}^k$$

$$T_{\mu\nu}^l = \partial_{\mu} e_{\nu}^l - \partial_{\nu} e_{\mu}^l + \epsilon^{ijk} \omega_{\mu}^j e_{\nu}^k - \epsilon^{ijk} \omega_{\nu}^j e_{\mu}^k$$

$$R_{\mu\nu\sigma}{}^\rho = e_\mu{}^\alpha e_\nu{}^\beta e_\sigma{}^\gamma e^\rho{}_\delta F_{\alpha\beta\gamma}{}^\delta$$

$$R = R_{\mu\nu} g^{\mu\nu} = e_\mu{}^\alpha e_\nu{}^\beta F_{\alpha\beta}{}^{\mu\nu}(\omega)$$

$$R = e_\mu{}^\alpha e_\nu{}^\beta e^\gamma{}_\delta F_{\alpha\beta}{}^{\gamma\delta}$$

$$R_{\mu\nu\sigma}{}^\rho = e_\mu{}^\alpha e_\nu{}^\beta e_\sigma{}^\gamma e^\rho{}_\delta F_{\alpha\beta\gamma}{}^\delta$$

$$R = R_{\mu\nu} g^{\mu\nu} = e_\mu{}^\alpha e_\nu{}^\beta F_{\alpha\beta}{}^{\mu\nu}$$

$$R = \left[e_\mu{}^\alpha e_\nu{}^\beta e^\gamma{}_\delta F_{\alpha\beta\gamma}{}^\delta \right]_{\mu\nu}{}^{\mu\nu}$$

$\mu\nu jk$

jk

$$e^{\mu}_{\sigma} e^{\sigma}_{\nu} F^{\mu\nu}_{jk}(\omega)$$

$$F_{\mu\nu l}$$

$$\begin{aligned} & \epsilon^{a_1 \dots a_{n-1} a} A_{a_1 b_1} \dots A_{a_{n-1} b_{n-1}} \\ &= \det(A) (A^{-1})^{ba} \epsilon_{b_1 \dots b_{n-1} b} \end{aligned}$$

$$R_{\mu\nu} = e_{\mu}^{\sigma} e_{\nu}^{\rho} F_{\sigma\rho}$$

$$R = R_{\mu\nu} g^{\mu\nu} = e_{\mu}^{\sigma} e_{\nu}^{\rho} F_{\sigma\rho} g^{\mu\nu}$$

$$R = \left[e_{\mu}^{\sigma} e_{\nu}^{\rho} \right] F_{\sigma\rho}$$

$$= -\epsilon^{\mu\nu\sigma} e_{\sigma}^{\rho} \det(e_{\mu}^{\alpha}) F_{\mu\nu\rho}$$

$$\rightarrow \sqrt{g} R$$

$$\begin{aligned} & \epsilon^{a_1 \dots a_{n-1} a} A_{a_1 b_1} \dots A_{a_{n-1} b_{n-1}} \\ &= \det(A) (A^{-1})^{ba} \epsilon_{b_1 \dots b_{n-1} b} \end{aligned}$$

$$R_{\mu\nu\sigma}{}^\rho = e_\mu{}^\alpha e_\nu{}^\beta e_\sigma{}^\gamma e^\rho{}_\delta F_{\alpha\beta\gamma\delta}$$

$$R = R_{\mu\nu} g^{\mu\nu} = e_\mu{}^\alpha e_\nu{}^\beta F_{\alpha\beta}{}^{\mu\nu}(\omega)$$

$$R = \left[e_\mu{}^\alpha e_\nu{}^\beta \epsilon^{\gamma\delta\epsilon} \right] F_{\alpha\beta\gamma\delta}$$

$$= -\epsilon^{\mu\nu\sigma} e_\sigma{}^\delta \det(e_\mu{}^\alpha) F_{\mu\nu\delta}$$

$$\Rightarrow \sqrt{|g|} R = -\epsilon^{\sigma\mu\nu} e_{\sigma\delta} F_{\mu\nu}{}^\delta$$

$$\begin{aligned} & \epsilon^{a_1 \dots a_{n-1} a} A_{a_1 b_1} \dots A_{a_{n-1} b_{n-1}} \\ &= \det(A) (A^{-1})^b{}_a \epsilon^{a_1 \dots a_{n-1} a} \end{aligned}$$

$$R_{\mu\nu\sigma}{}^{\rho} = e_{\rho}^{\sigma} e^{\sigma k} F_{\mu\nu jk}$$

$$\int R \sqrt{g} d^3x = - \int e e^{\sigma\mu\nu} F_{\mu\nu}^2 d^3x$$

EDM:

δe_{σ}^k

$$F = 0$$

$$R_{\mu\nu\sigma}{}^\rho = e_{\sigma}^{\lambda} e^{\rho}{}_{\lambda} F_{\mu\nu}{}^{\sigma}{}_{\rho}$$

$$\int R \sqrt{g} d^3x = - \int e_{\sigma}^{\lambda} F_{\mu\nu}{}^{\sigma}{}_{\rho} e^{\rho}{}_{\lambda} d^3x$$

EDM:

$\delta e_{\sigma}^{\lambda}$

$$F = 0$$

$\delta \omega$

$$T_{\mu\nu}^i = 0$$

$\Rightarrow \omega_{\mu\nu}^i$ leads to L-C connection