Title: Quantum Gravity Review-1

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URL: http://www.pirsa.org/15020050

Abstract:

Quantum Gravity & Quantum Geometry

- "A first course on loop QG" Gambini, Pullin
- A. Perez, gr-qc/0409061

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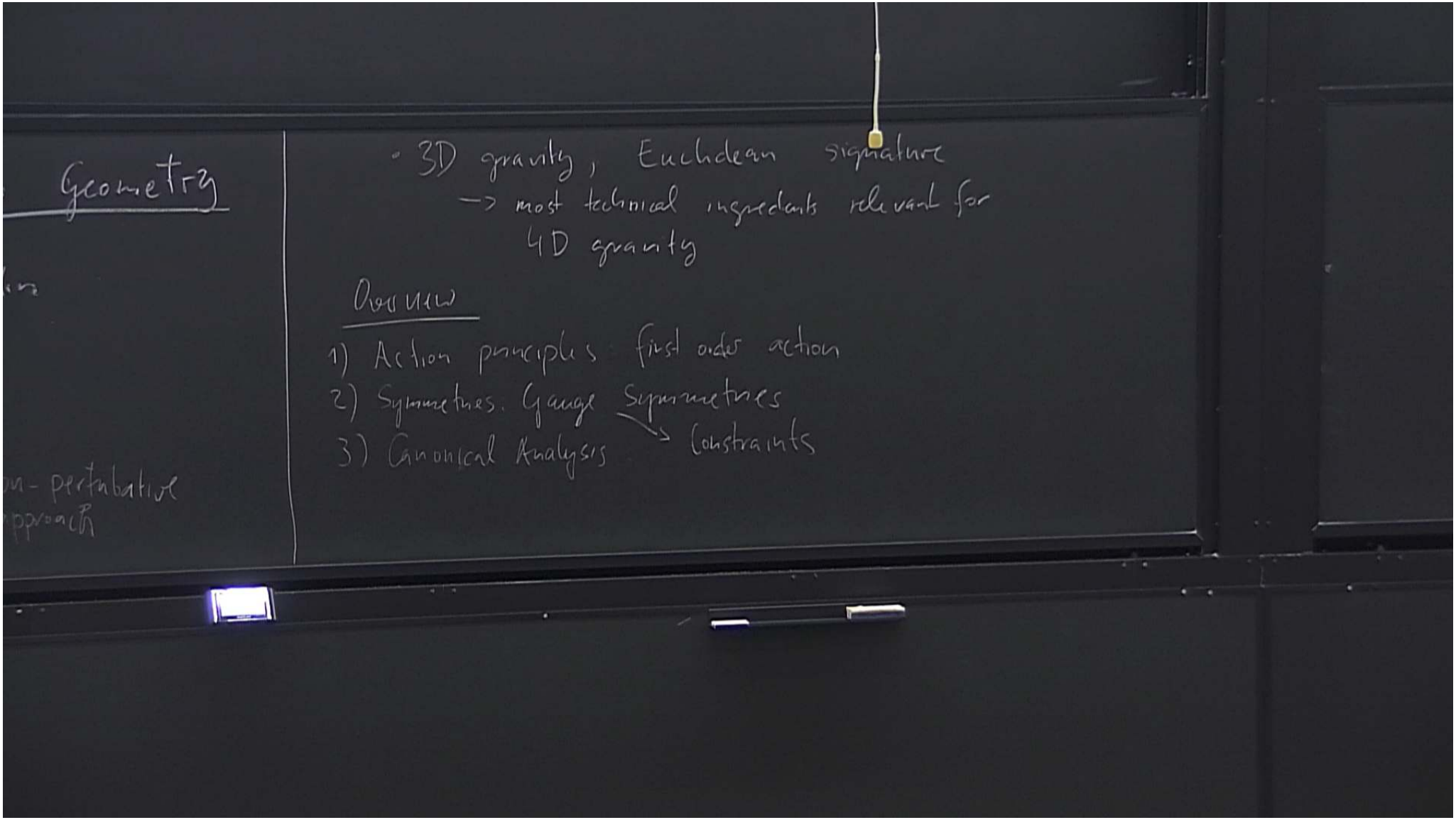
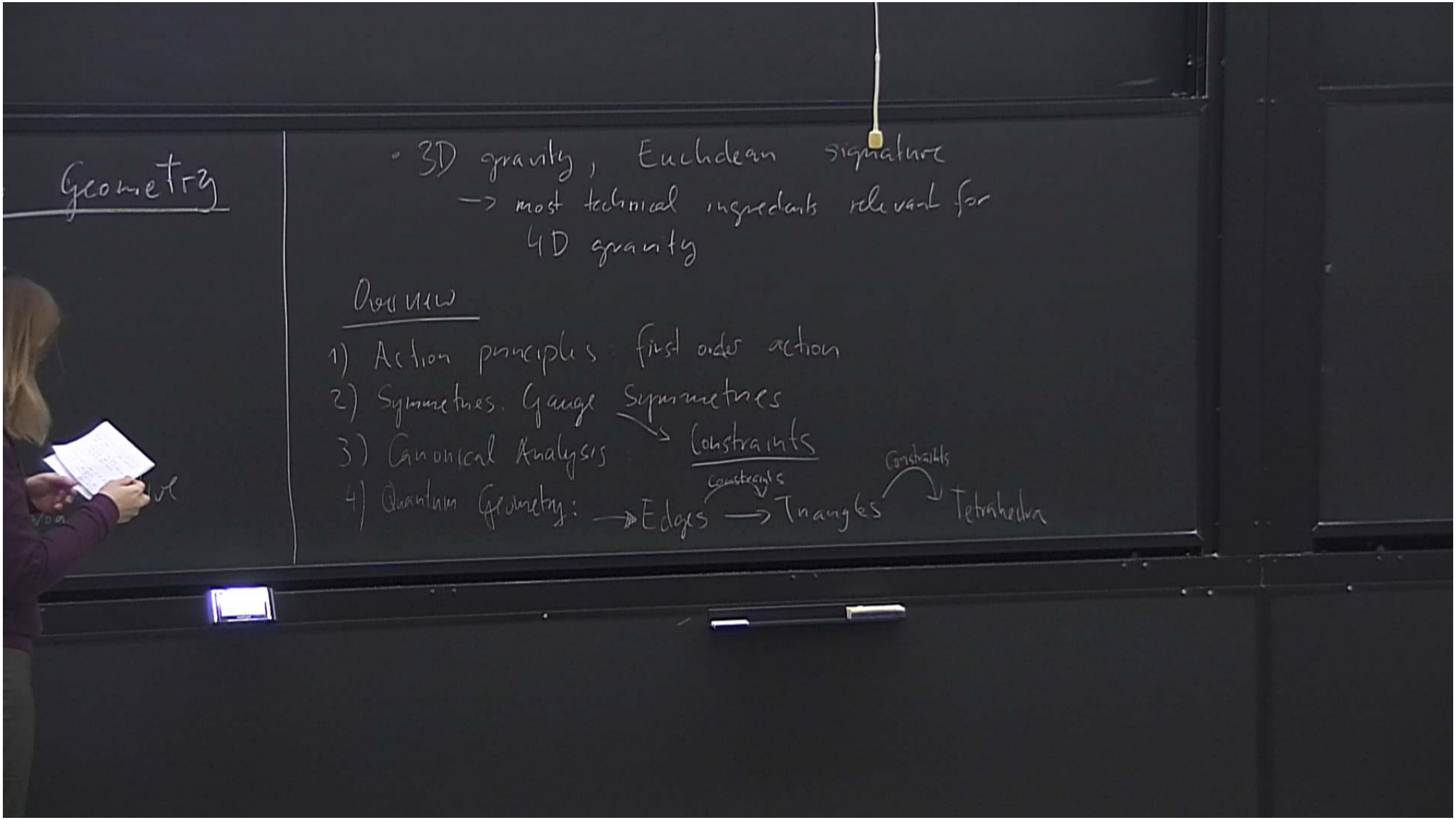
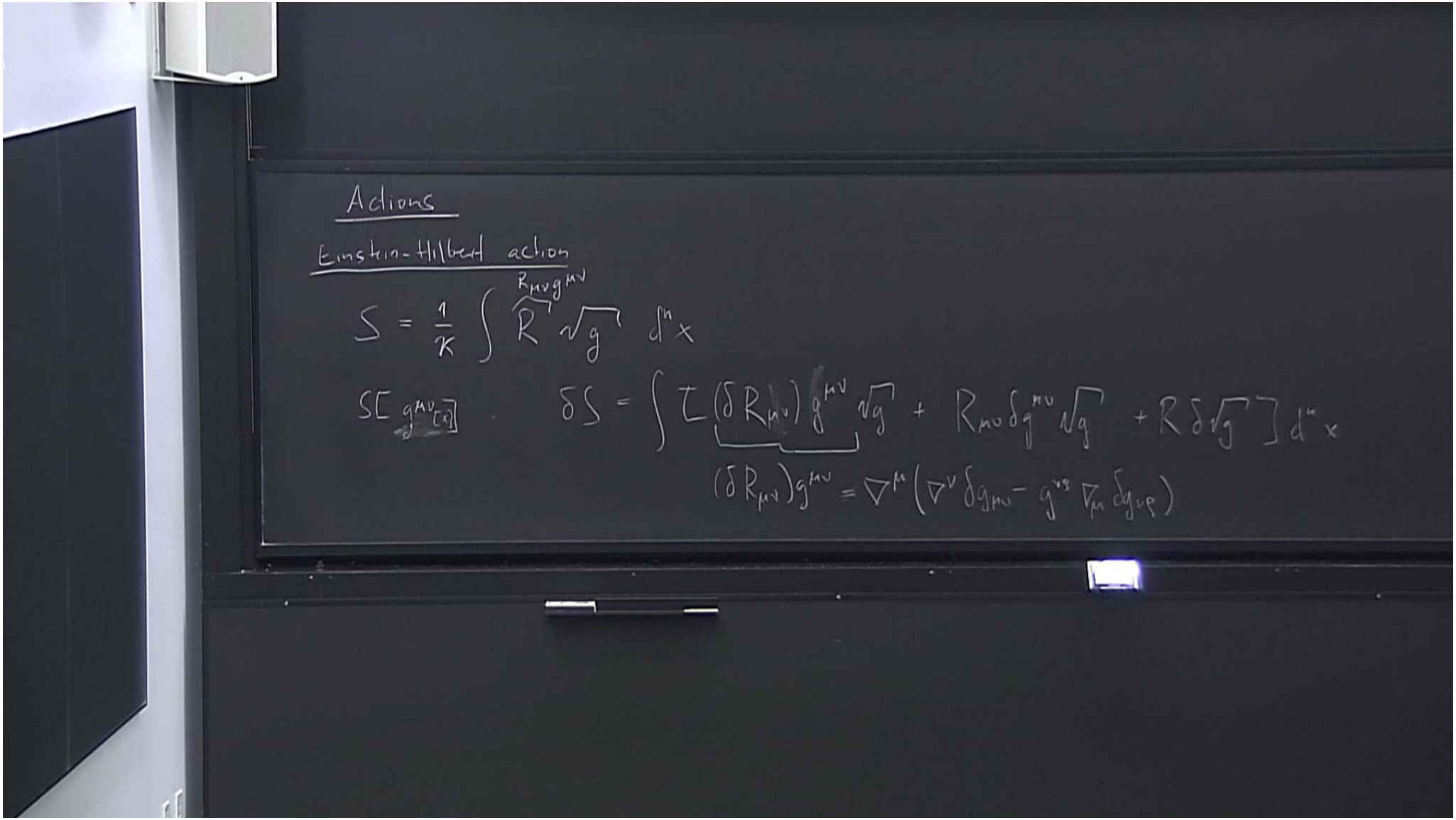
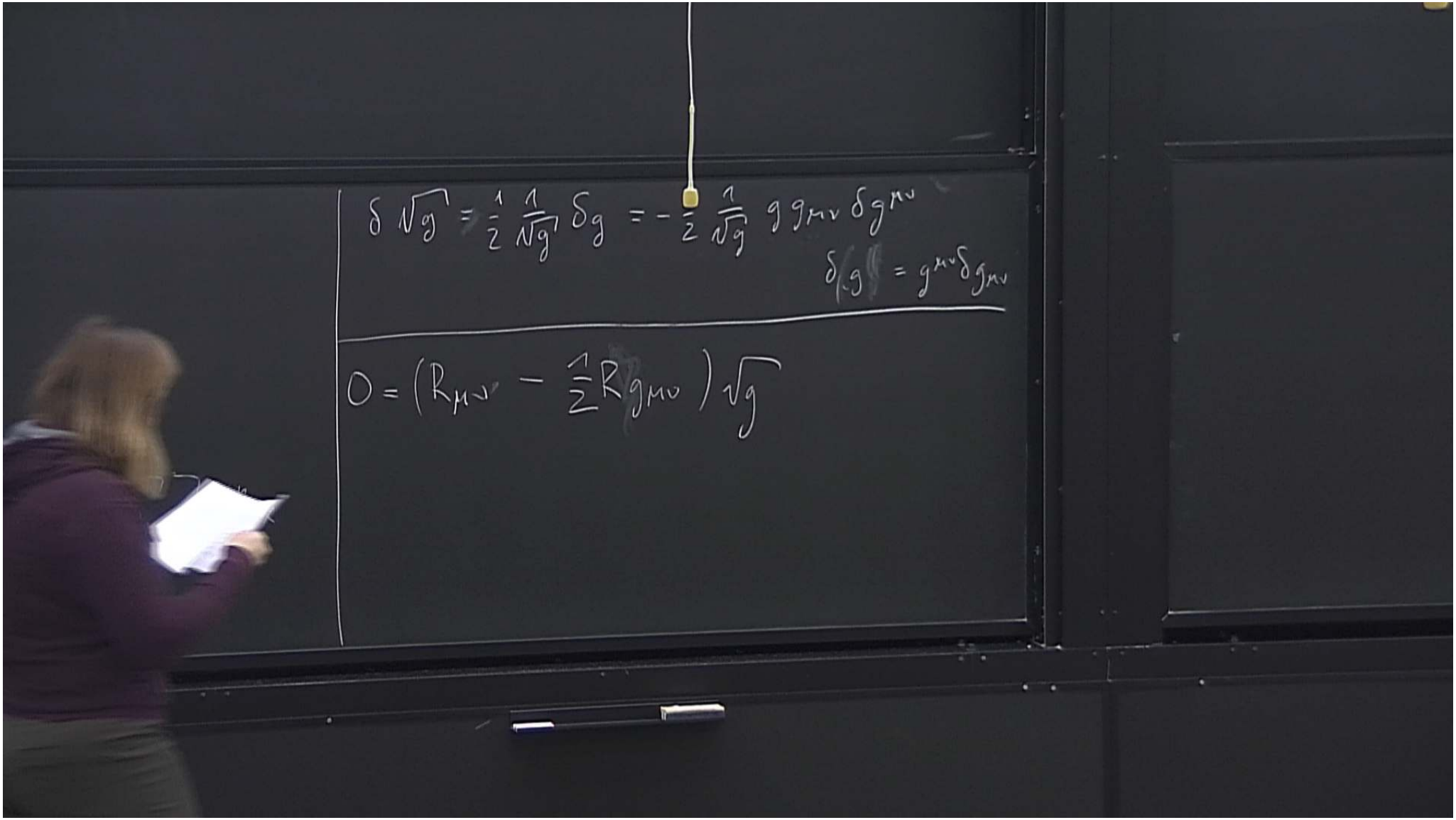
Gravity = Property of space time

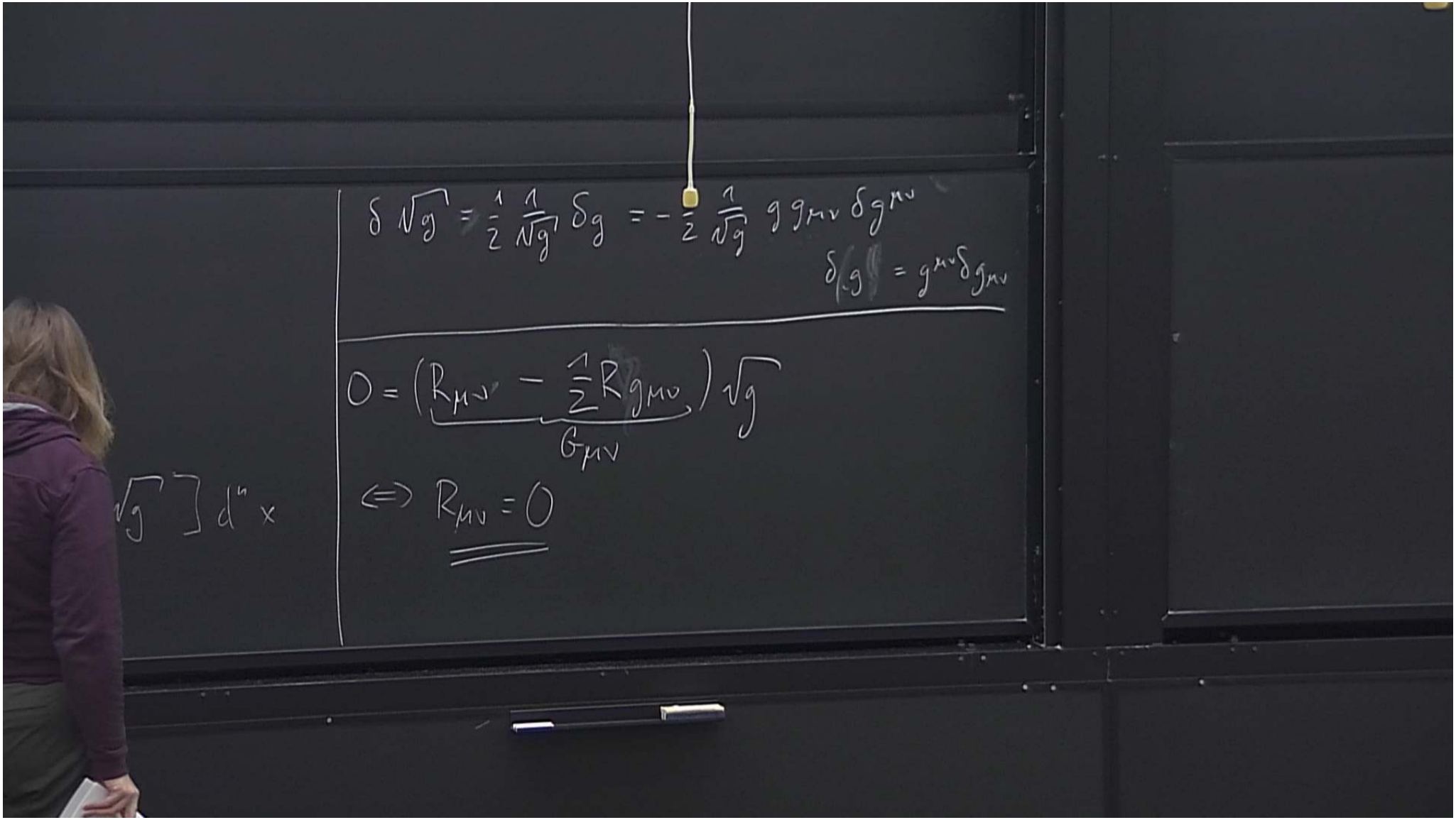
Quantum Gravity = Quantum space time → non-perturbative approach


Geometry
- 3D gravity, Euclidean signature
-> most technical ingredients relevant for 4D gravity
Overview
1) Action principles first order action
2) Symmetries. Gauge Symmetries
3) Canonical Analysis
Constraints


Geometry
• 3D gravity, Euclidean signature
→ most technical ingredients relevant for 4D gravity
Overview
1) Action principles first order action
2) Symmetries. Gauge Symmetries
3) Canonical Analysis
Constraints
4) Quantum Geometry: Edges → Triangles → Tetrahedra
constraints
Constraints


Actions
Einstein-Hilbert action
$S = \frac{1}{\kappa}\int \overbrace{R}^{R_{\mu\nu}g^{\mu\nu}}\sqrt{g}\,d^n x$
$S[g_{\mu\nu}(x)]$
$\delta S = \int [(\delta R_{\mu\nu})g^{\mu\nu}\sqrt{g} + R_{\mu\nu}\delta g^{\mu\nu}\sqrt{g} + R\delta\sqrt{g}]\,d^n x$
$(\delta R_{\mu\nu})g^{\mu\nu} = \nabla^\mu(\nabla^\nu \delta g_{\mu\nu} - g^{\nu\rho}\nabla_\mu \delta g_{\nu\rho})$


$\delta\sqrt{g} = \frac{1}{2}\frac{1}{\sqrt{g}}\delta g = -\frac{1}{2}\frac{1}{\sqrt{g}} g\, g_{\mu\nu}\delta g^{\mu\nu}$
$\delta g = g^{\mu\nu}\delta g_{\mu\nu}$
$0 = \left(R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}\right)\sqrt{g}$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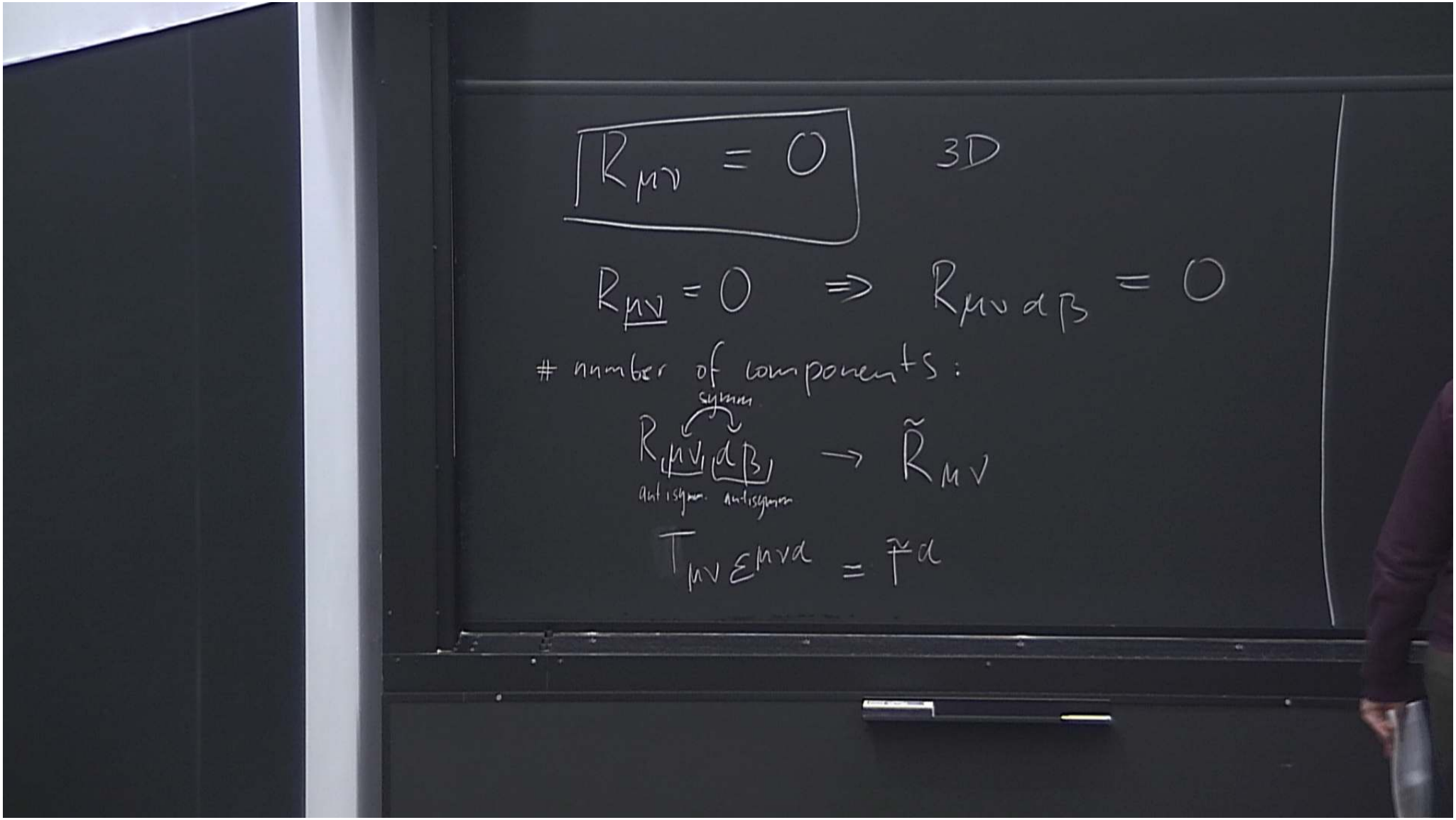
$\delta g = g^{\mu\nu}\delta g_{\mu\nu}$
$0 = (\underbrace{R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu}}_{G_{\mu\nu}})\sqrt{g}$
$\Leftrightarrow R_{\mu\nu} = 0$
$\sqrt{g}\,]\,d^n x$


$R_{\mu\nu} = 0$
3D
$R_{\mu\nu} = 0 \Rightarrow R_{\mu\nu\alpha\beta} = 0$
number of components:
symm.
$R_{[\mu\nu][\alpha\beta]} \rightarrow \tilde{R}_{\mu\nu}$
antisym.
antisym.
$T_{\mu\nu}\varepsilon^{\mu\nu\alpha} = \tilde{T}^{\alpha}$

$R_{\mu\nu} = 0$ 3D

$R_{\mu\nu} = 0 \Rightarrow R = 0$

number of components

$R_{[\mu\nu][\alpha\beta]}$ → symmetric

antisymm. antisymm.

$T_{\mu\nu}\varepsilon^{\mu\nu\alpha} =$

· Locally flat manifolds.

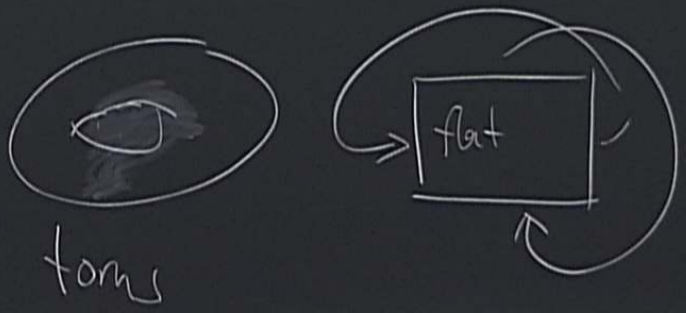
· Global dof: depend on the topology of the manifold

time $\mathbb{R} \times \Sigma^{2d}$

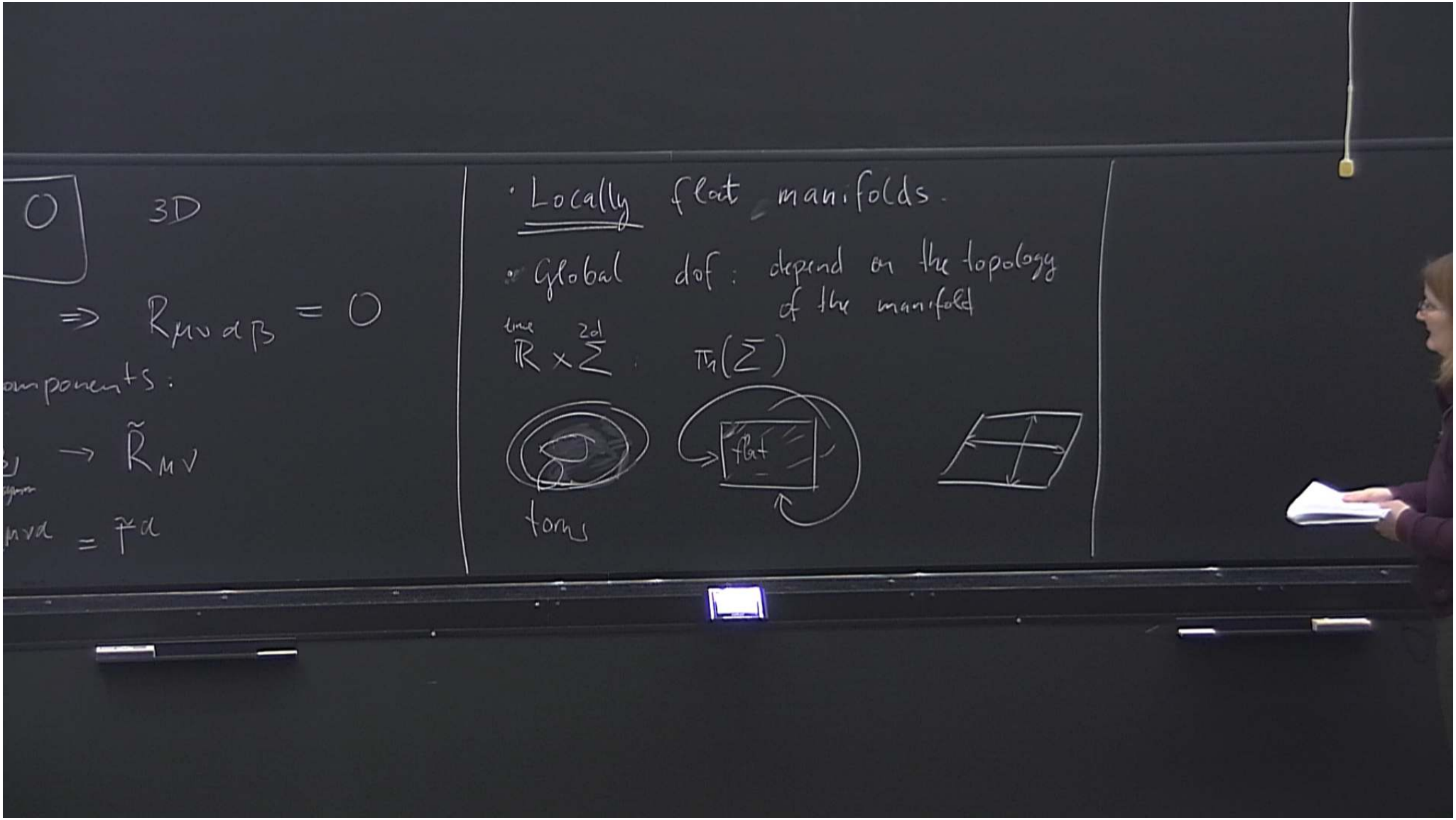
$\nu \, d\beta = 0$

- Locally flat manifolds.
- Global dof: depend on the topology of the manifold

time 2d
$\mathbb{R} \times \Sigma$:



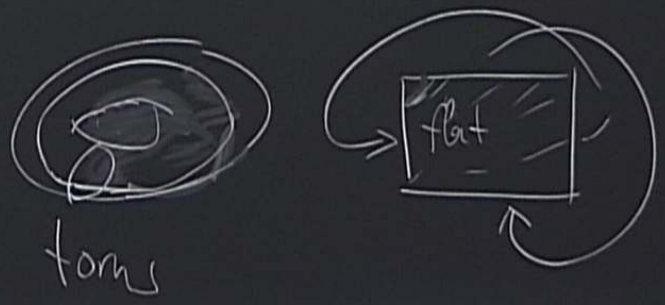
tori


3D
$\Rightarrow R_{\mu\nu\alpha\beta} = 0$
components:
$\rightarrow \tilde{R}_{MV}$
time
$\mathbb{R} \times \Sigma^{2d}$
$\pi_1(\Sigma)$
flat
tori

$\omega \wedge d\beta = 0$

- Locally flat manifolds.
- Global dof: depend on the topology of the manifold

time 2d
$\mathbb{R} \times \Sigma$: $\pi_1(\Sigma)$

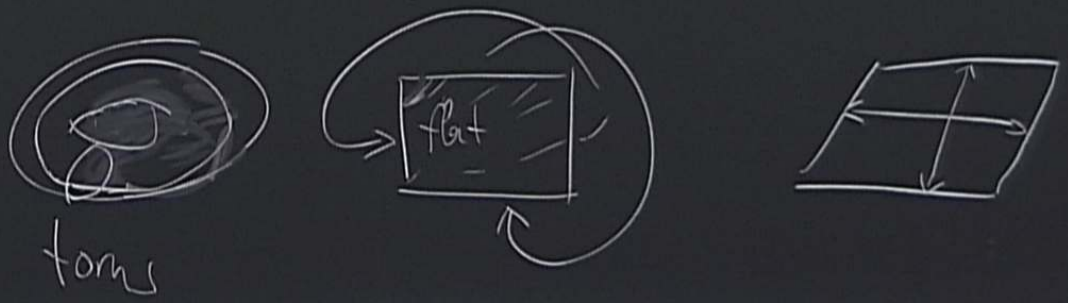


torus

$d\beta = 0$

- Locally flat manifolds.
- Global dof: depend on the topology of the manifold

time 2d
$\mathbb{R} \times \Sigma$: $\pi_1(\Sigma)$

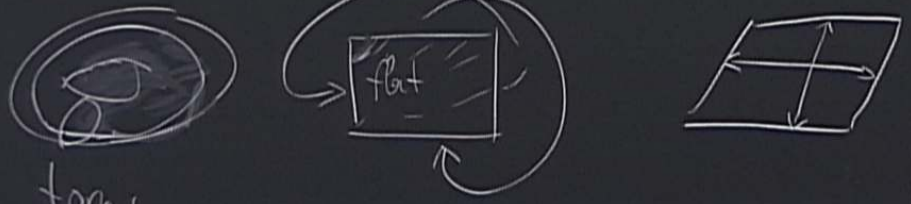


torus

- Locally flat manifolds

- Global dof: depend on the topology of the manifold

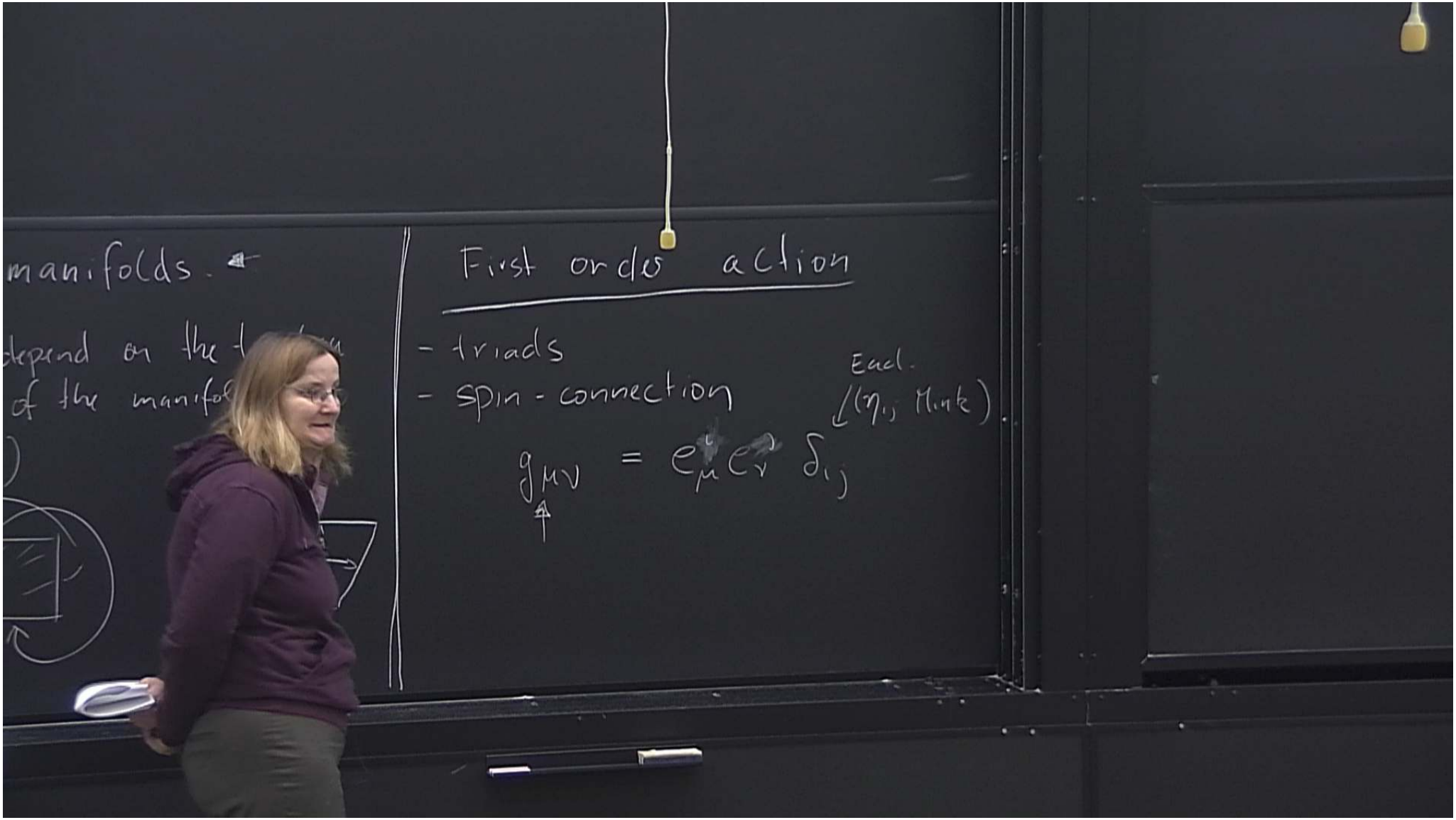
time
$\mathbb{R} \times \Sigma^{2d}$: $\pi_1(\Sigma)$

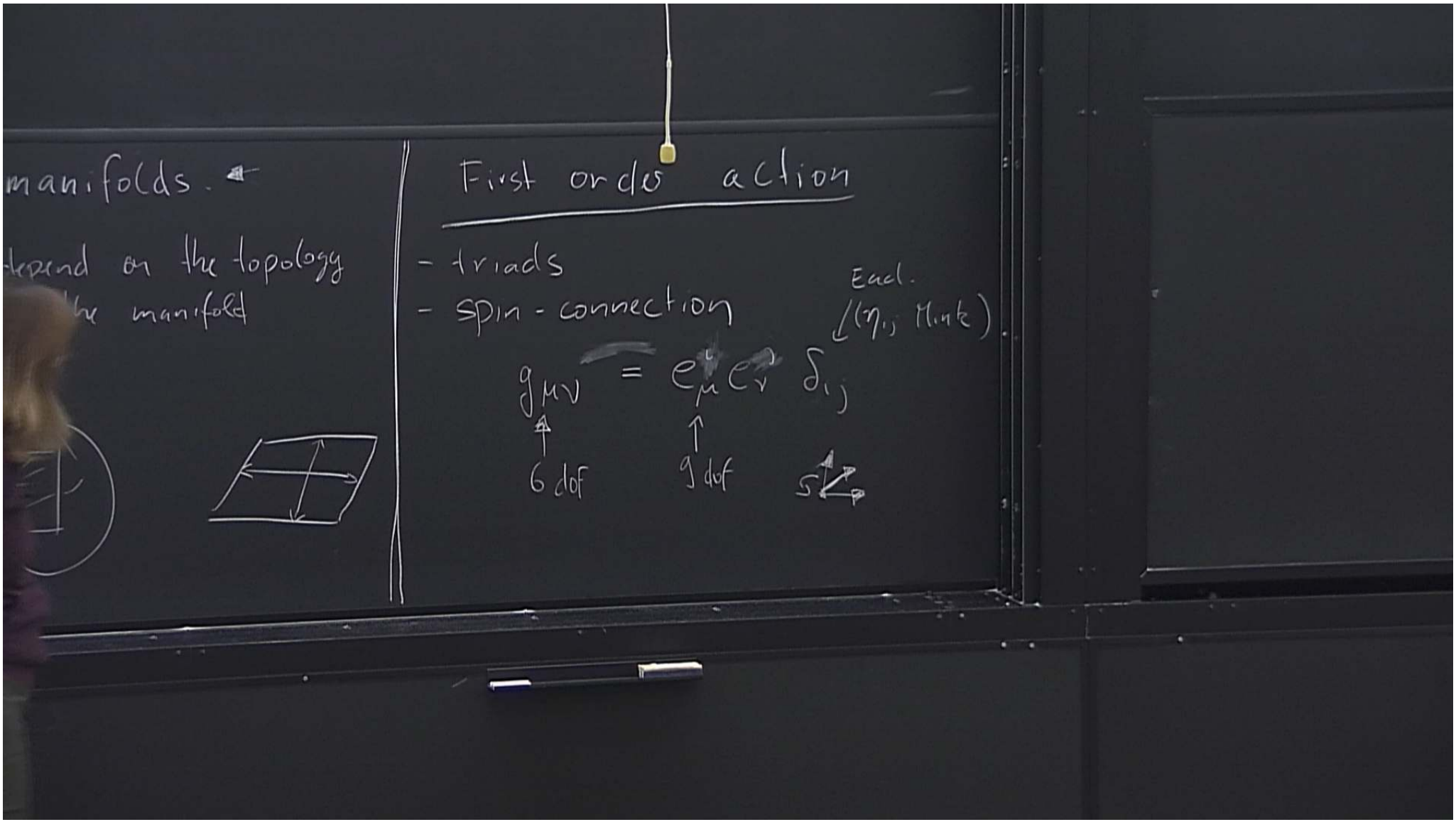


tori

First order action

- triads
- spin-connection


manifolds.
depend on the
of the manifo
First order action
- triads
- spin-connection
Eucl.
(η_ij, Mink)
g_μν = e^i_μ e^j_ν δ_ij


manifolds.
depend on the topology
the manifold
First order action
- triads
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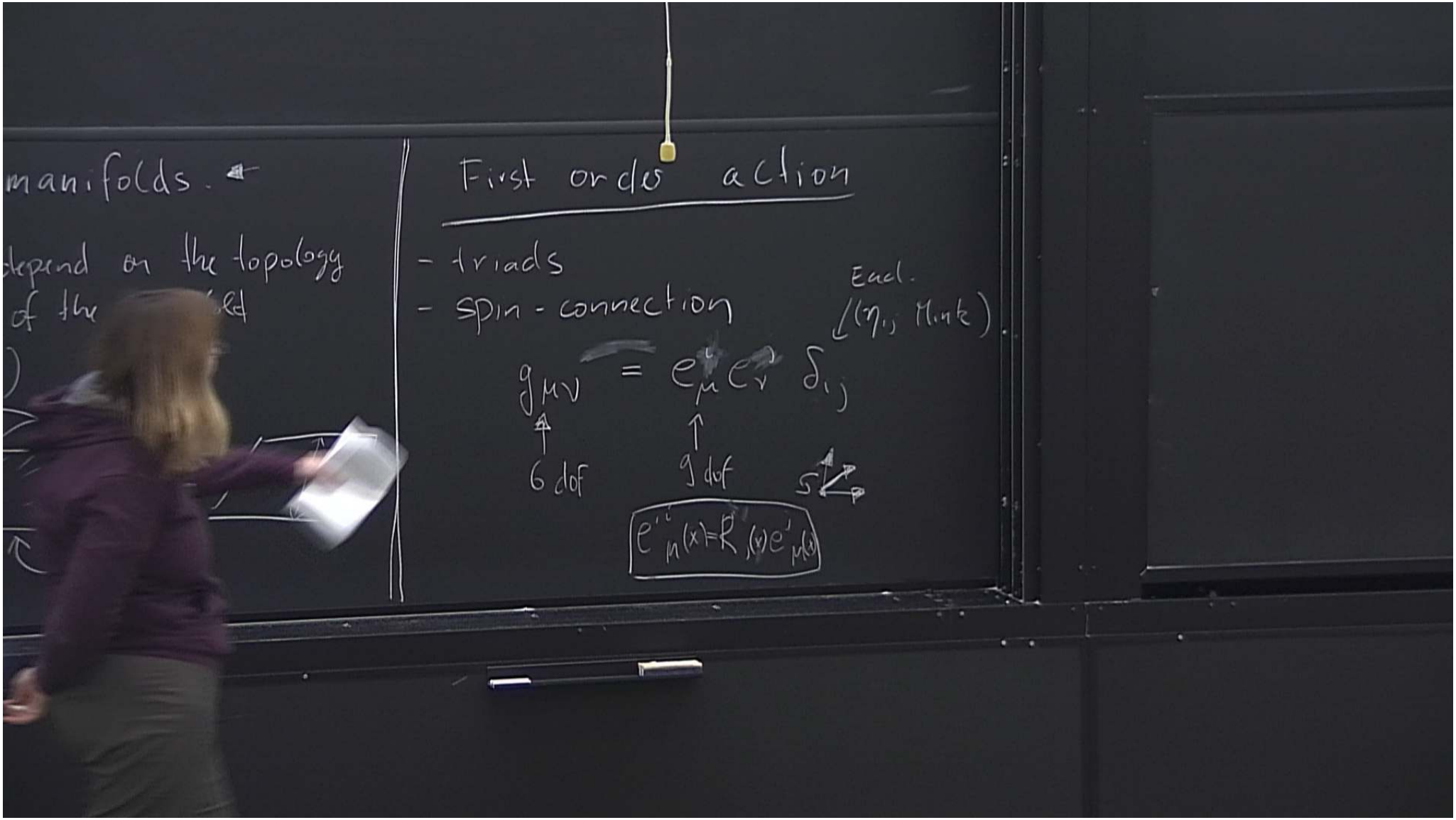
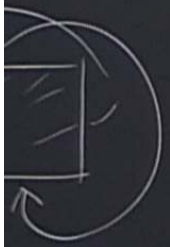
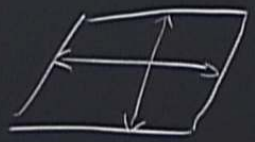
Eucl.
(η_{ij} Mink)
$g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$
6 dof
9 dof


manifolds.
depend on the topology
of the
First order action
- triads
- spin-connection
Eucl.
$g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$
6 dof
9 dof
$e^i_\mu(x) = R^i_j(x) e^j_\mu(x)$

manifolds.

depend on the topology of the manifold





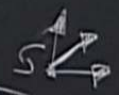
First order action

- triads
- spin-connection

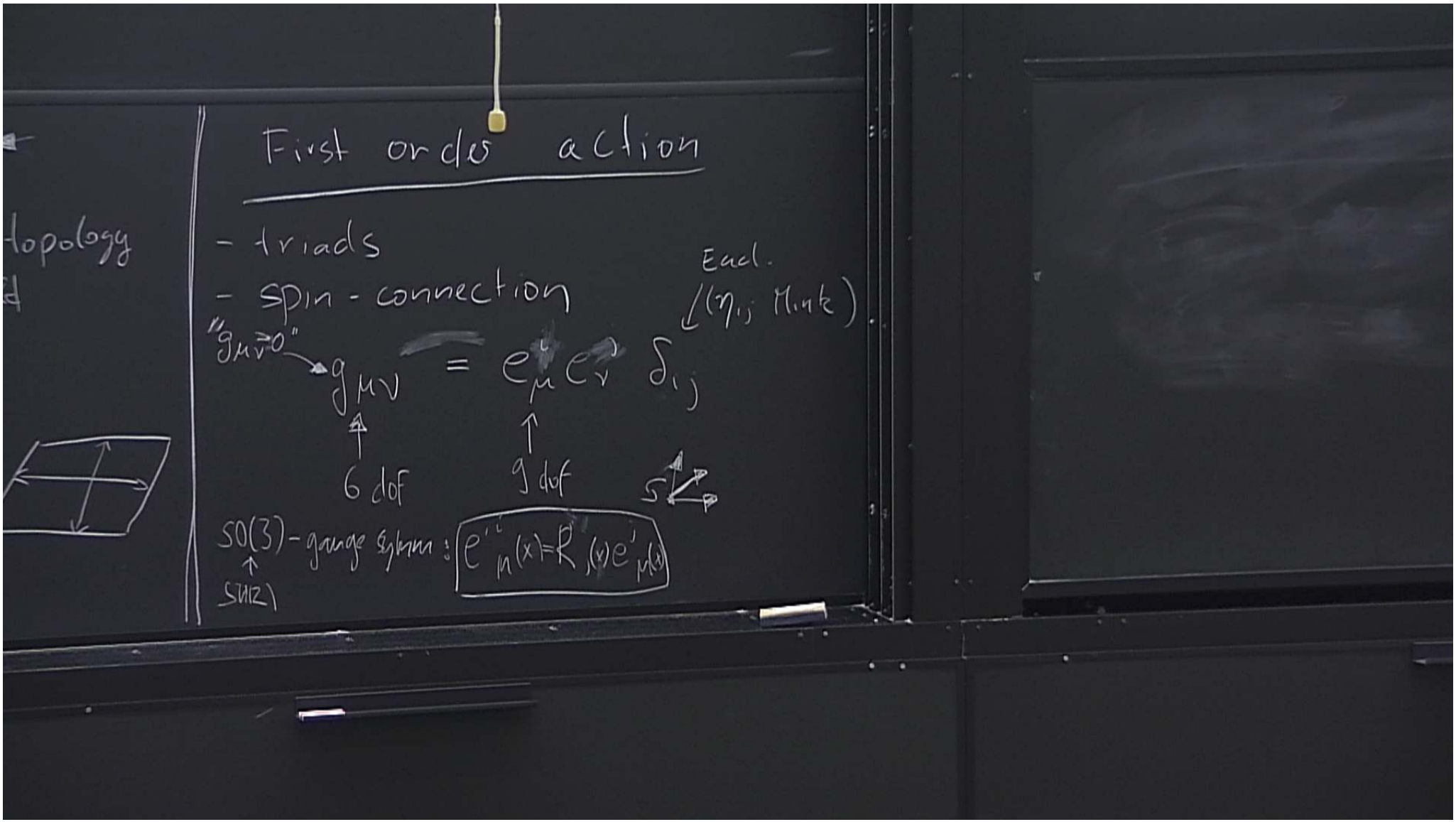
Eucl. $(\eta_{ij} = \text{Mink})$

$$g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$$

6 dof — 9 dof

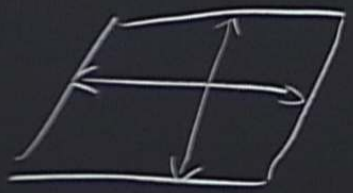


SO(3) (SU(2)) - gauge symm: $e'^i_\mu(x) = R^i_j(x) e^j_\mu(x)$


topology
First order action
- triads
- spin - connection
Eucl.
(η_{ij} Mink)
"$g_{\mu\nu} \to 0$"
$g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$
6 dof
9 dof
SO(3) - gauge symm : $e'^i_\mu(x) = R^i_j(x) e^j_\mu(x)$
SU(2)

olds.

on the topology
manifold



First order action

$\dot{\phi} = \pi$

- triads (e)
- spin-connection (ω)

Eucl. / (η_ij Mink)

"$g_{\mu\nu} \geq 0$" → $g_{\mu\nu} = e^i_\mu e^j_\nu \delta_{ij}$

$g_{\mu\nu}$: 6 dof

$e^i_\mu e^j_\nu \delta_{ij}$: 9 dof

SO(3) (↑ SU(2)) - gauge symm: $e'^i_\mu(x) = R^i_j(x) e^j_\mu(x)$