

Title: Beyond Standard Model-5

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URL: <http://pirsa.org/15020045>

Abstract:

$$P_\mu, J^{\mu\nu}$$

$$Q_\alpha, \bar{Q}_\alpha$$

$$\{Q_\alpha, \bar{Q}_\alpha\} = 2\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$$

$$[P_\mu, Q_\alpha] = 0 \dots$$

Chiral Supermultiplet: $\underline{\Phi} = (\underbrace{\phi(x)}_{\text{complex scalar}}, \underbrace{\psi_\alpha(x)}_{\text{Weyl fermion}}, \underbrace{F}_{\text{auxiliary field}})$

$$S = \int d^4x (|\partial\phi|^2 + i\bar{\psi}\dot{\sigma}\psi + F^\dagger F)$$

$$\left\{ \begin{array}{l} Q\phi \rightarrow \psi \\ Q\psi \rightarrow \sigma^\mu \partial_\mu \phi + F \\ QF \rightarrow \partial^2 \phi \end{array} \right.$$

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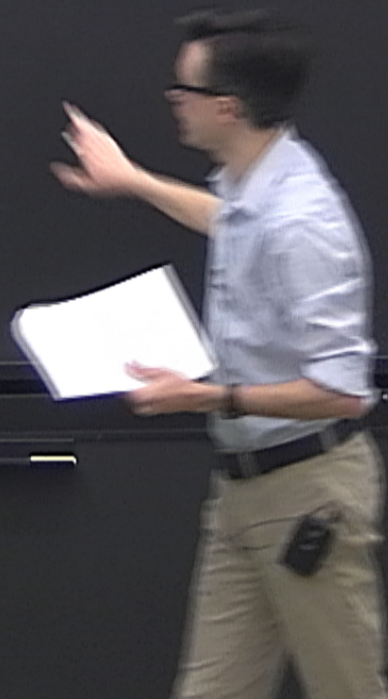
DOFs: Bosonic: 2 + 2
Fermionic: 4

off-shell

$\sigma^\mu_{\alpha\dot{\alpha}} P_\mu$

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$$\mathcal{L}_{\text{int}} = F \cdot \frac{\partial W}{\partial \Phi} \Big|_{\theta} - \frac{1}{2} \frac{\partial^2 W}{\partial \Phi^2} \Big|_{\theta} \Psi \Psi + (\text{h.c.})$$

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$$\frac{\partial \mathcal{L}}{\partial F} = F^\dagger + \frac{\partial W}{\partial \Phi} \Big|_x = 0 \quad \text{plug back in} \quad \mathcal{L}_{\text{int}} = \left(-\frac{1}{2} \frac{\partial^2 W}{\partial \Phi^2} \Big|_x + \text{h.c.} \right) - \left| \frac{\partial W}{\partial \Phi} \Big|_x^2$$

$$\frac{\partial \mathcal{L}}{\partial \Phi} = F^\dagger + \frac{\partial W}{\partial \Phi} = 0 \quad \text{plug back in} \quad \mathcal{L}_{int} = \left(-\frac{1}{2} \frac{\partial^2 W}{\partial \Phi^2} \Big|_{\Phi} + h.c. \right) - \left| \frac{\partial W}{\partial \Phi} \right|^2$$

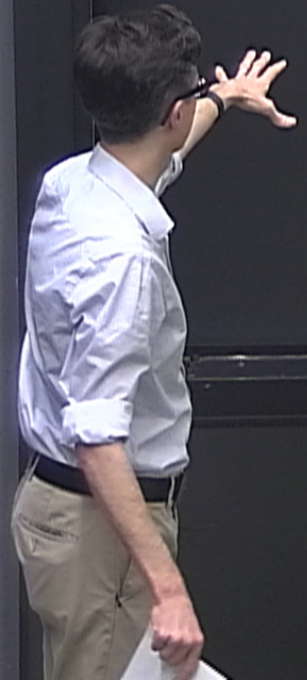
eg 1. Wess-Zumino Model, Φ

$$W = \frac{1}{2} m \Phi^2 + \frac{\lambda}{3!} \Phi^3$$

$$\mathcal{L}_{int} = \left[\frac{1}{2} (m + \lambda \phi) \psi \psi + h.c. \right] + \left| m \phi + \frac{\lambda}{2} \phi^2 \right|^2$$

\downarrow $m = \text{fermion mass}$

\downarrow $m^2 |\phi|^2$
 \downarrow complex scalar



$\frac{\partial W}{\partial \Phi} \Big|_{\Phi} = 2 \frac{\partial W}{\partial \Phi^2} \Big|_{\Phi} \Psi \Psi + (h.c.)$
 $\frac{\partial W}{\partial \Phi} \Big|_{\Phi} = 0$ plug back in
 $\mathcal{L}_{int} = \left(-\frac{1}{2} \frac{\partial^2 W}{\partial \Phi^2} \Big|_{\Phi} \Psi \Psi + h.c. \right) - \left| \frac{\partial W}{\partial \Phi} \Big|_{\Phi} \right|^2$

SS - Zmino Model, Φ
 $= \frac{1}{2} m \Phi^2 + \frac{\lambda}{3!} \Phi^3$

$\Phi \rightarrow \begin{matrix} \psi \\ \psi \end{matrix} \sim \lambda$

$\mathcal{L}_{int} = \left[\frac{1}{2} (m + \lambda \alpha) \Psi \Psi + h.c. \right] + \left| m \Phi + \frac{\lambda}{2} \Phi^2 \right|^2$

$m = \text{fermion mass}$
 $m^2 |\Phi|^2$
 $\lambda^2 |\Phi|^4$

$\left(\frac{m}{2} \lambda \alpha^2 \Phi^2 + h.c. \right)$

$$\frac{\partial \mathcal{L}}{\partial \Phi} = F^\dagger + \frac{\partial W}{\partial \Phi} = 0 \quad \text{plug back in} \quad \mathcal{L}_{int} = \left(-\frac{1}{2} \frac{\partial^2 W}{\partial \Phi^2} \psi \psi + h.c. \right) - \left| \frac{\partial W}{\partial \Phi} \right|^2$$

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$$\mathcal{L}_{int} = \left[\frac{1}{2} (m + \lambda \alpha) \psi \psi + h.c. \right]$$

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\downarrow
 $m^2 |\Phi|^2$
 \downarrow
complex scalar

\nearrow $\left(\frac{m \lambda \alpha^2 \Phi^2 + h.c. \right)$

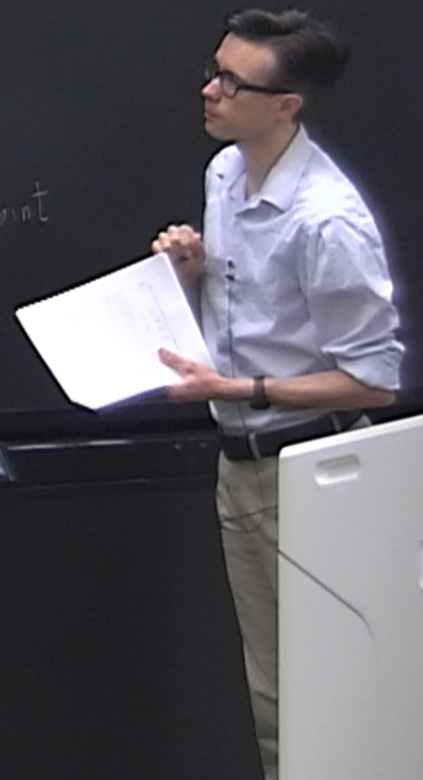
\searrow $\frac{\lambda^2}{4} |\Phi|^4$

Vector (Real) Supermultiplet $V = (\lambda, A^\mu, D)$ real scalar, auxiliary
"Weyl" vector

$$S = \int d^4x \left(\bar{\lambda} i \bar{\sigma} \cdot \partial \lambda - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} D^2 \right)$$

DoF	off-shell	on-shell
boson	3+1	2
fermion	4	2

U(1) gauge invariance
 λ, D transforming as adjoint



boson

Fermion

4

2

λ, D transforming as adjoint

$$d_\mu \rightarrow D_\mu \quad ; \quad \Phi_i \rightarrow \left(e^{-i\alpha^a t^a} \right)_{ij} \Phi_j$$
$$[D_\mu \Phi]_i = d_\mu \Phi_i + ig t_{ij}^a A_\mu^a \Phi_j$$

boson

Fermion

4

2

λ, D transforming as adjoint

$$d_\mu \rightarrow D_\mu, \quad \Phi_i \rightarrow (e^{-i\alpha^a t^a})_{ij} \Phi_j$$

$$[D_\mu \Phi]_i = d_\mu \Phi_i + ig t_{ij}^a A_\mu^a \Phi_j$$

$$-\mathcal{L} = \underbrace{\sqrt{2}g (\Phi_i^\dagger t_{ij}^a \Psi_j)}_{\text{gauge ferm}} \lambda_\mu^a + \sqrt{2}g \bar{\lambda}^a (\bar{\Psi} t^a \Phi) - g (\Phi^\dagger t^a \Phi) D^a$$



boson

Fermion

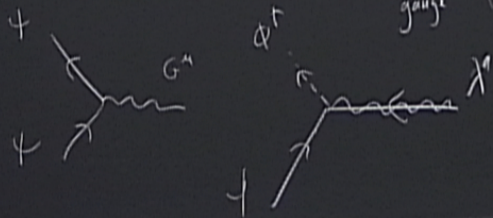
4 2

λ, D transformation of ψ

$$d_\mu \rightarrow D_\mu, \quad \Phi_i \rightarrow (e^{-i\alpha^a t^a})_{ij} \Phi_j$$

$$[D_\mu \Phi]_i = d_\mu \Phi_i + ig t_{ij}^a A_\mu^a \Phi_j$$

$$-L = \sqrt{2} g (\underbrace{\Phi_i^\dagger t_{ij}^a \Psi_j}_{\text{gauge ferm}}) \lambda^a + \sqrt{2} g \bar{\lambda}^a (\bar{\Psi} t^a \Phi) - g (\Phi^\dagger t^a \Phi) D^a$$



λ, D transforming as adjoint

2

$$\rightarrow (e^{-i\alpha^a t^a})_{ij} \Phi_j$$
$$\Phi_i + ig t_{ij}^a A_\mu^a \Phi_j$$
$$\underbrace{(\bar{\psi}_j \gamma^\mu \lambda_\mu^a)}_{\text{Ferm}} + \sqrt{2} g \bar{\lambda}^a (\bar{\psi} t^a \psi) - \underbrace{g (\Phi^\dagger t^a \Phi)}_{\frac{1}{2} D^{a2}} D^a$$

∇V_D

$$D^a - g (\Phi^\dagger t^a \Phi) = 0 = \frac{\partial \mathcal{L}}{\partial D^a}$$
$$\hookrightarrow -\mathcal{L} = \frac{1}{2} D^{a2} = \frac{g^2}{2} |\Phi^\dagger t^a \Phi|^2 = V_D$$
$$V_{\text{scalar}} = V_F + V_D = \left| \frac{\partial W}{\partial \Phi} \right|^2 + \frac{1}{2} (D^a)^2$$

"F"²

λ, D transforming as adjoint

2

$$\rightarrow (e^{-i\alpha^a t^a})_{ij} \bar{\Psi}_j$$

$$\bar{\Psi}_i + ig t_{ij}^a A_\mu^a \Psi_j$$

$$\underbrace{\bar{\Psi}_j \lambda^a}_{\text{ferm}} + \sqrt{2} g \bar{\lambda}^a (\bar{\Psi} t^a \Psi) - \underbrace{g (\bar{\Phi} t^a \Phi)}_{\frac{1}{2} D^a{}^2} D^a$$

∇V_D

$$D^a - g (\bar{\Phi} t^a \Phi) = 0 = \frac{\partial \mathcal{L}}{\partial D^a}$$

$$\hookrightarrow -\mathcal{L} \supset \frac{1}{2} D^a{}^2 = \frac{g^2}{2} |\bar{\Phi} t^a \Phi|^2 = V_D$$

$$V_{\text{scalar}} = V_F + V_D = \left| \frac{\partial W}{\partial \Phi} \right|^2 + \frac{1}{2} (D^a)^2$$

"F"²

eg. SUSY QED

U(1) gauge invariance $\rightarrow V = (\lambda, A^\mu, D)$

$$\text{Matter} : \begin{cases} E = (\tilde{E}, E, F_E), & Q = -1 \\ E^c = (\tilde{E}^c, E^c, F_{E^c}), & Q = +1 \end{cases}$$

$$W = m E E^c$$

$$-\mathcal{L} = m^2 |E|^2$$

$$-\mathcal{L} = m^2 (|\tilde{E}|^2 + |\tilde{E}^c|^2) + \underbrace{m E E^c + m \bar{E} \bar{E}^c}_{\text{fermion masses}}$$

fermion masses

$$+ \frac{e^2}{2} (-|\tilde{E}|^2 + |\tilde{E}^c|^2)$$

= D

$$\Psi = \begin{pmatrix} E \\ \bar{E}^c \end{pmatrix} \quad m \bar{\Psi} \Psi$$

$$\begin{cases} F_E = m \tilde{E}^c \\ F_{E^c} = m \tilde{E} \end{cases}$$

Q = -1
Q = +1

$$-\mathcal{L} = m^2 (|\tilde{E}|^2 + |\tilde{E}^c|^2) + \underbrace{m E E^c + m \bar{E} \bar{E}^c}_{\text{fermion masses}}$$

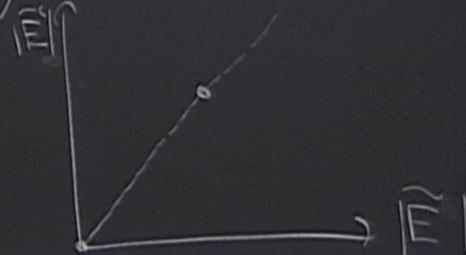
fermion masses

$$+ \frac{e^2}{2} (-|\tilde{E}|^2 + |\tilde{E}^c|^2)^2$$

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$$D = |\tilde{E}| = |\tilde{E}^c|$$

~~$$\begin{cases} F_E = m E^c \\ F_{E^c} = m E \end{cases}$$~~



$$-\mathcal{L} = m^2 (|\tilde{E}|^2 + |\tilde{E}^c|^2) + \underbrace{M\tilde{E}\tilde{E}^c + M\tilde{E}\tilde{E}}_{\text{fermion masses}}$$

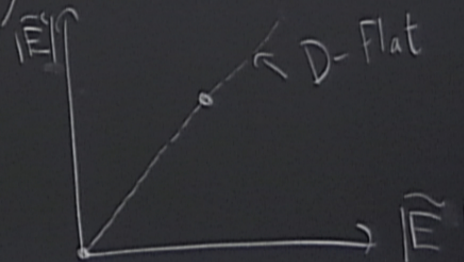
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fermion masses

$$+ \frac{e^2}{2} (-|\hat{E}|^2 + |\tilde{E}^c|^2)^2$$

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~~$$\begin{cases} F_E = m \bar{E}^c \\ F_{E^c} = m \bar{E} \end{cases}$$~~

