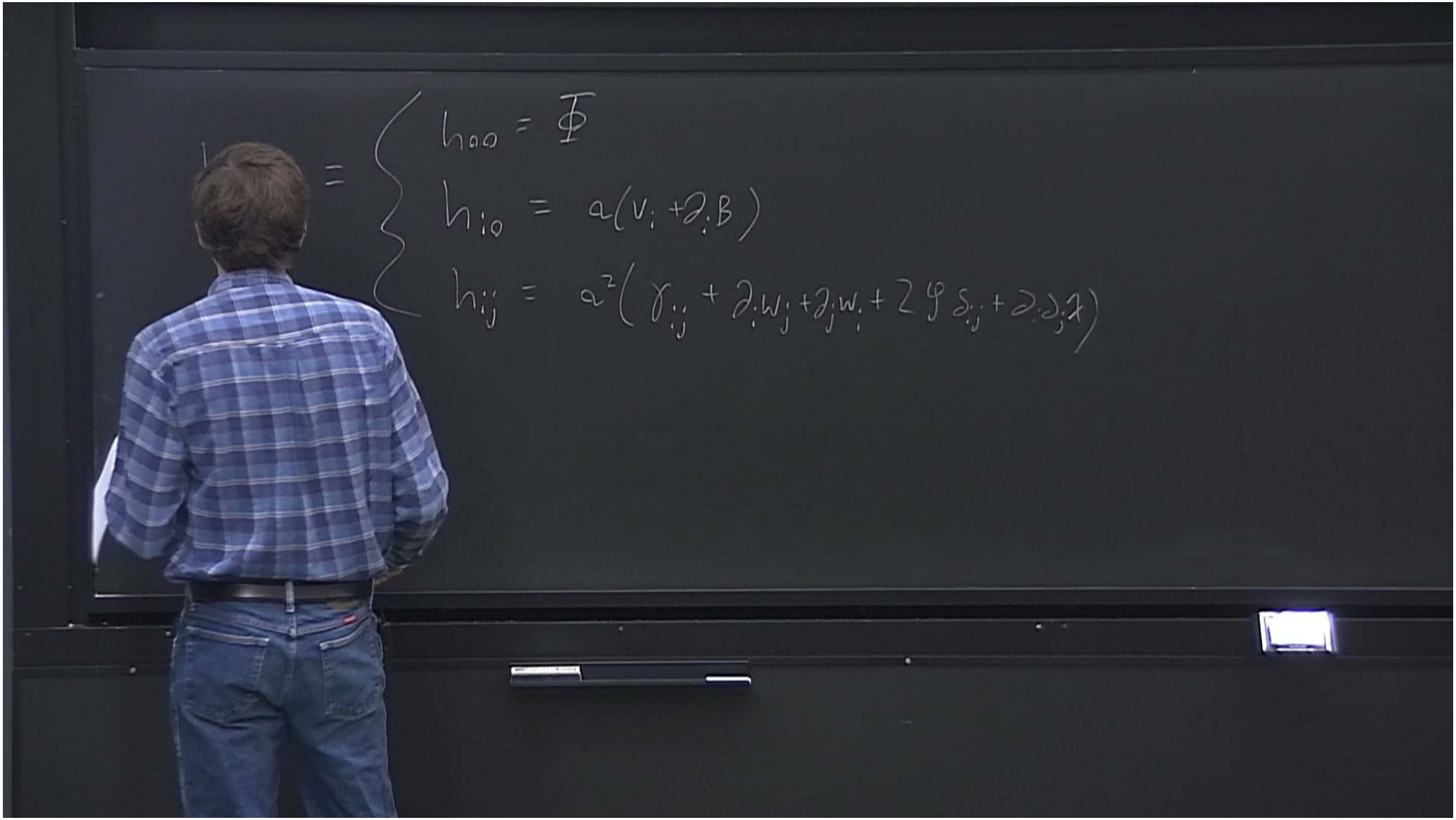
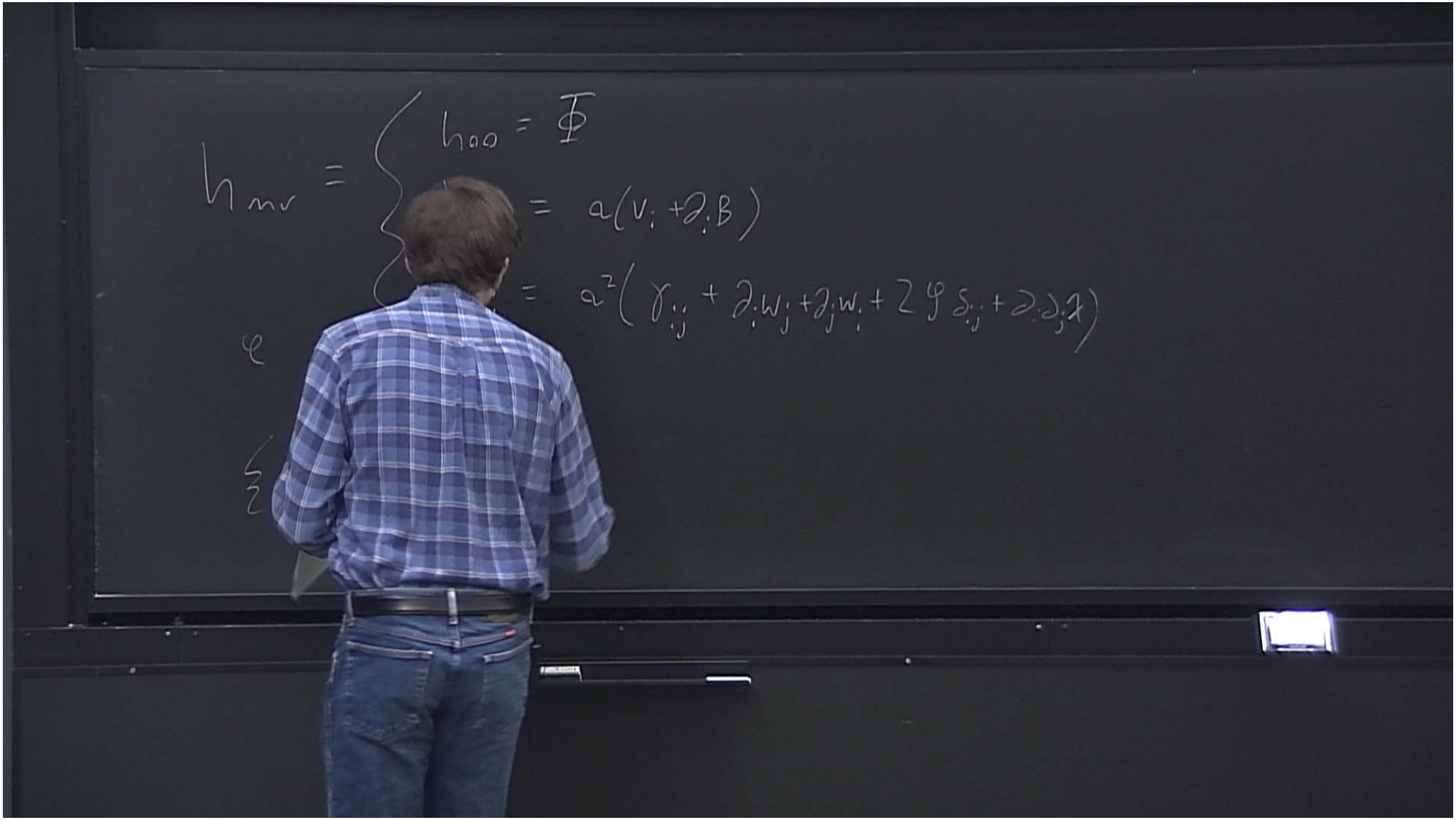
Title: Cosmology Review-14

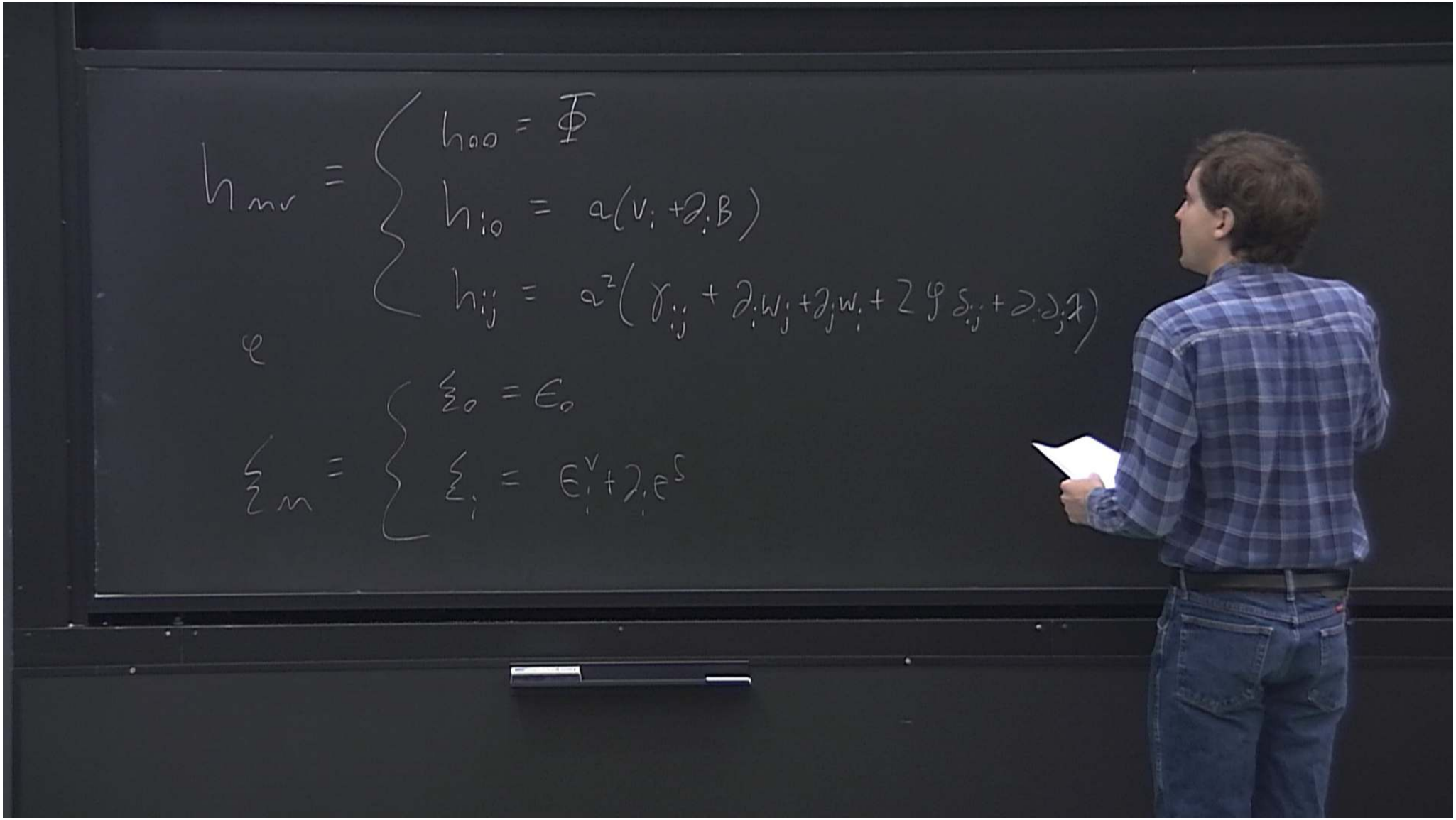
Date: Feb 12, 2015 11:30 AM

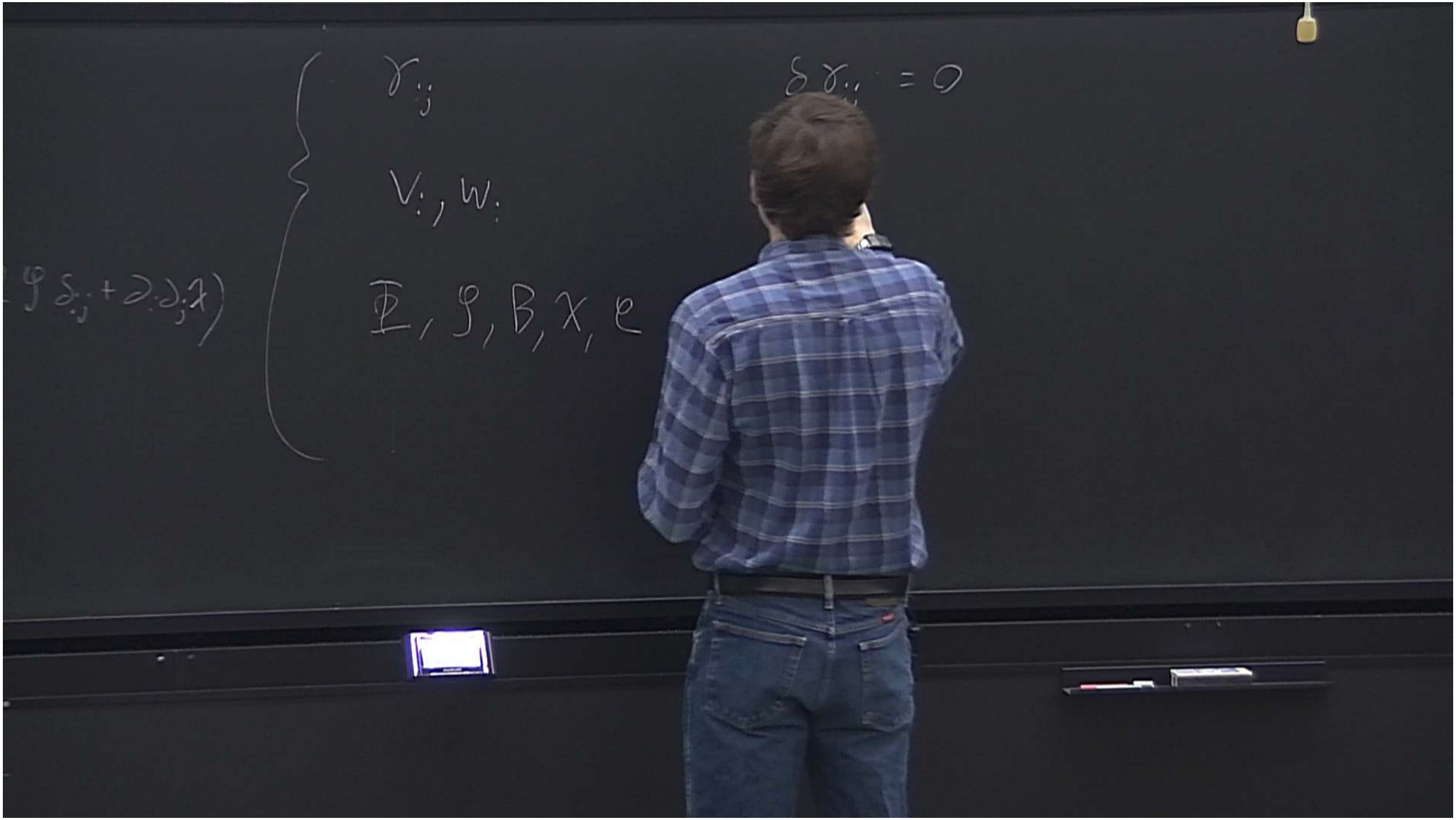
URL: http://pirsa.org/15020037

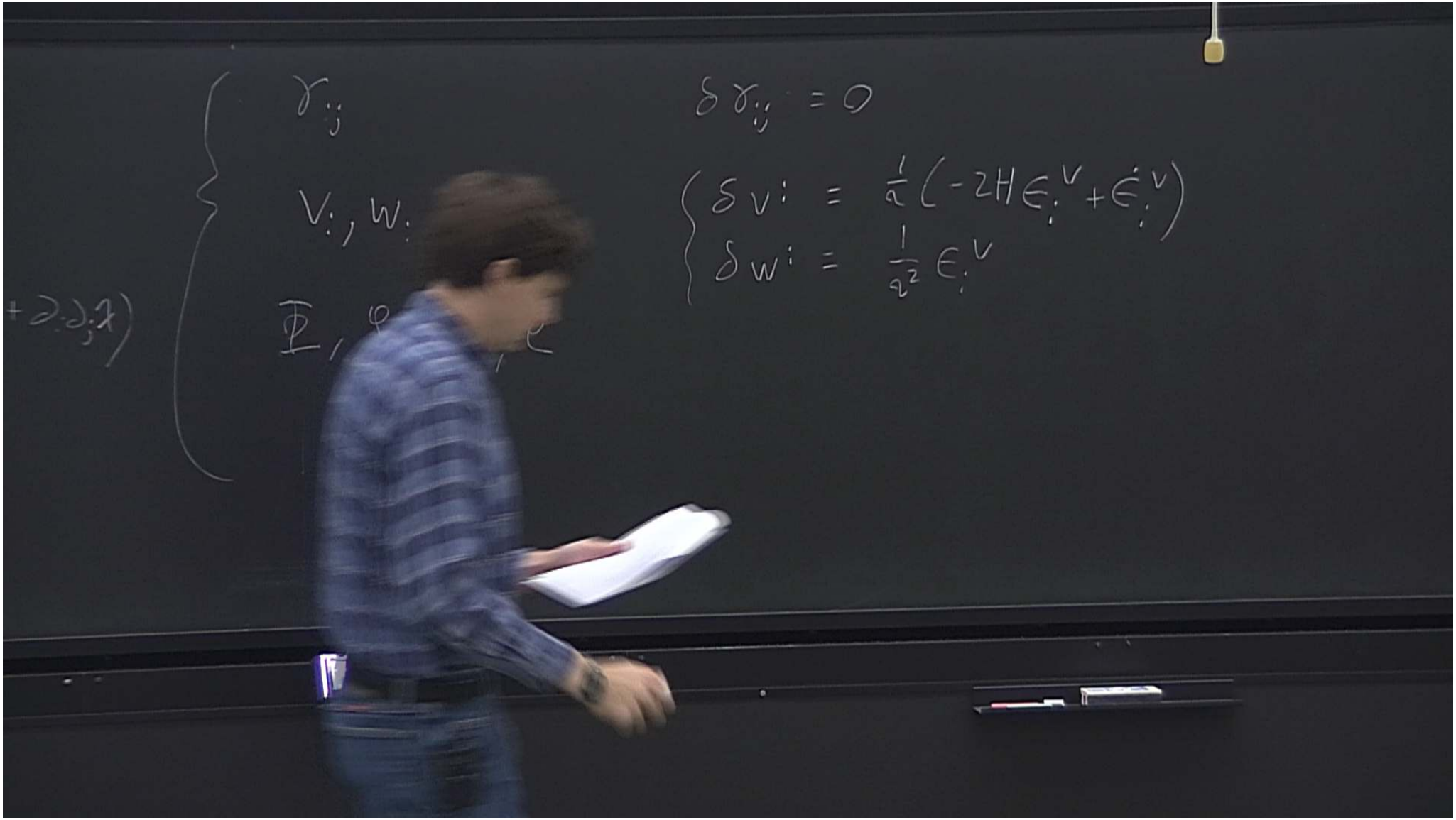
Abstract:

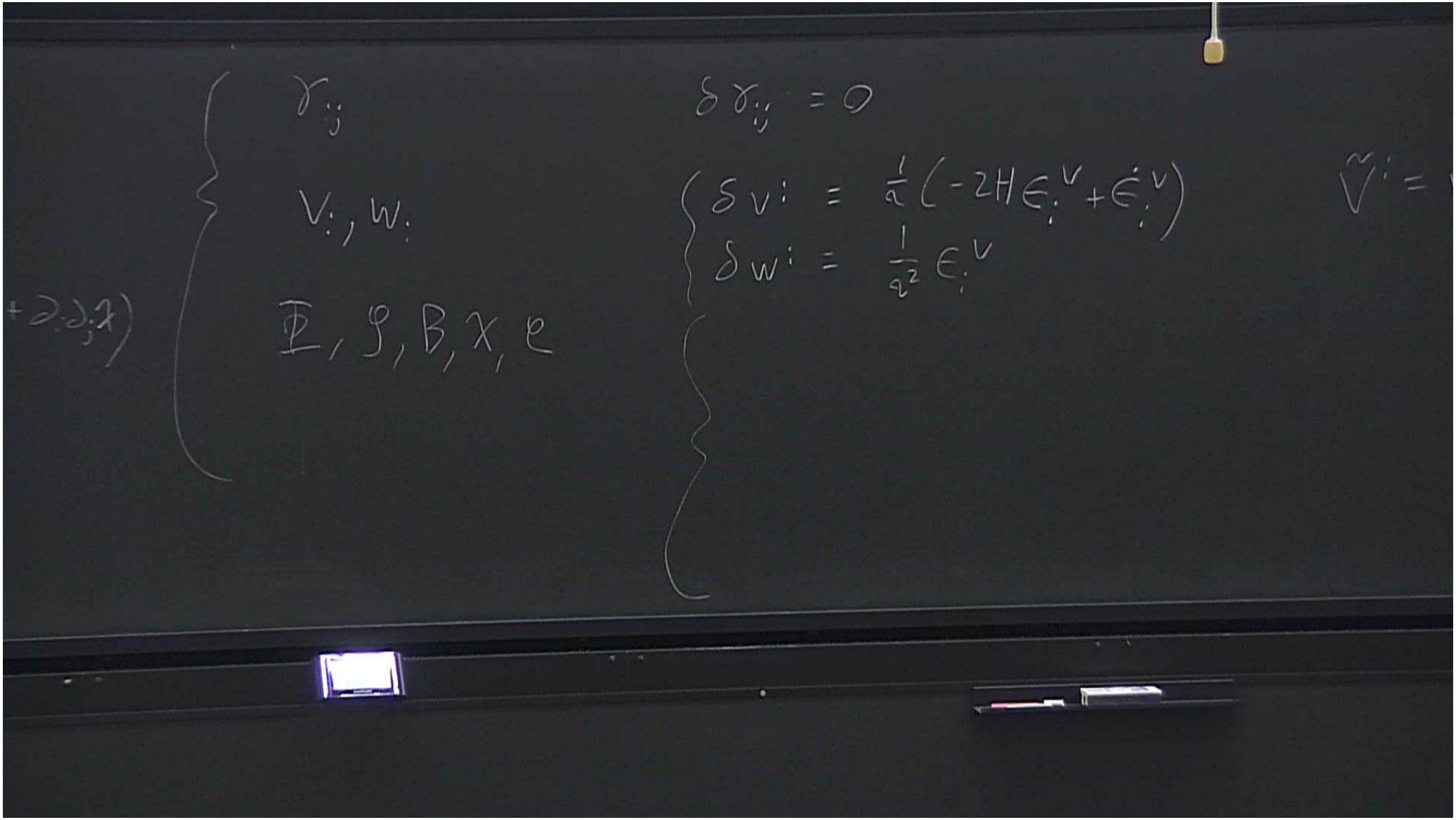

$h_{00} = 2\Phi$
$h_{i0} = a(v_i + \partial_i B)$
$h_{ij} = a^2(\gamma_{ij} + \partial_i w_j + \partial_j w_i + 2\varphi\delta_{ij} + \partial_i\partial_j\chi)$

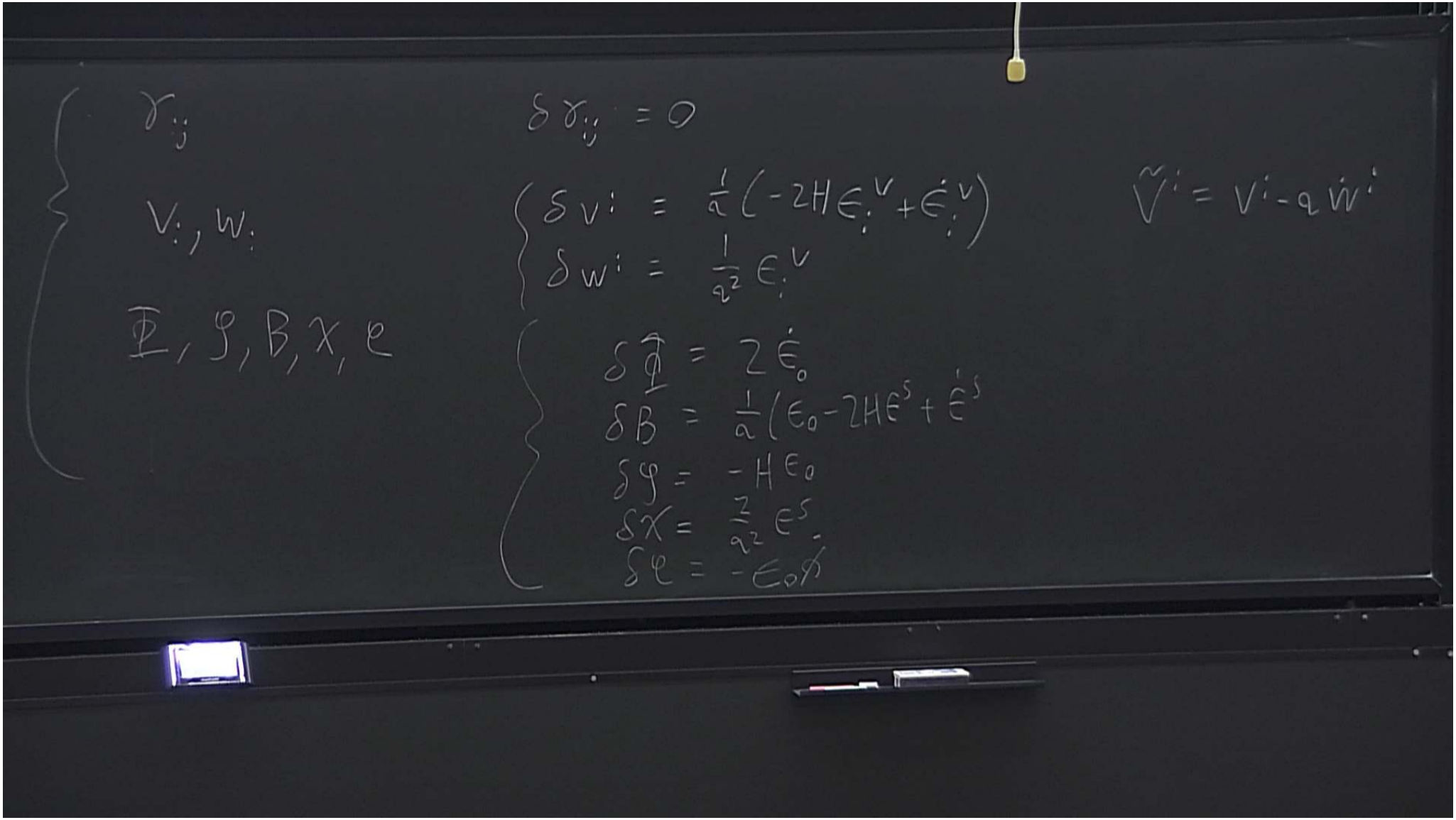

$h_{\mu\nu} = \begin{cases} h_{00} = 2\Phi \\ = a(V_i + \partial_i B) \\ = a^2(\gamma_{ij} + \partial_i w_j + \partial_j w_i + 2\psi\delta_{ij} + \partial_i\partial_j \chi) \end{cases}$

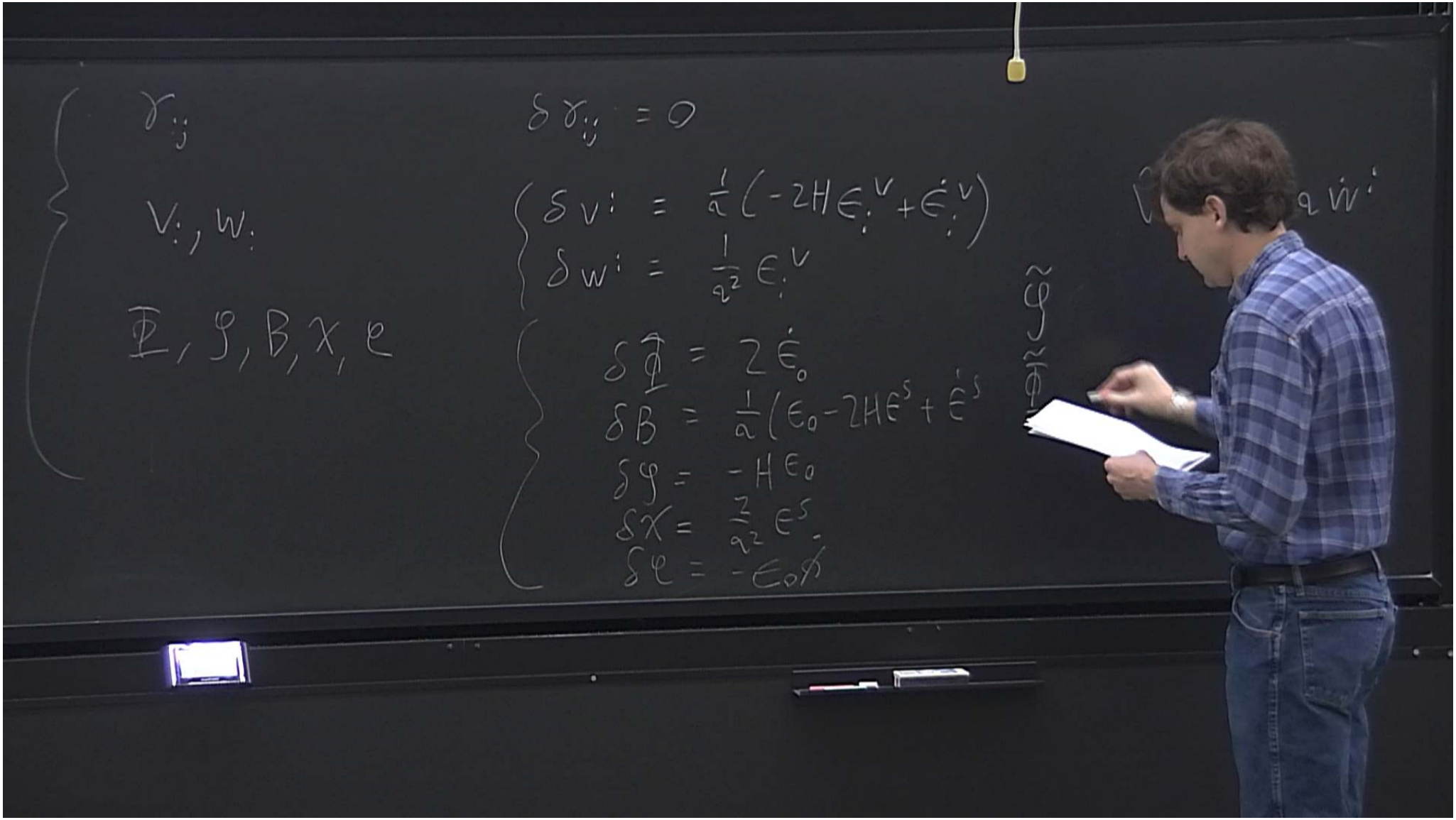

$h_{\mu\nu} = \begin{cases} h_{00} = 2\Phi \\ h_{i0} = a(V_i + \partial_i B) \\ h_{ij} = a^2(\gamma_{ij} + \partial_i W_j + \partial_j W_i + 2\psi\delta_{ij} + \partial_i\partial_j T) \end{cases}$
e
$\xi_\mu = \begin{cases} \xi_0 = \epsilon_0 \\ \xi_i = \epsilon_i^V + \partial_i \epsilon^S \end{cases}$

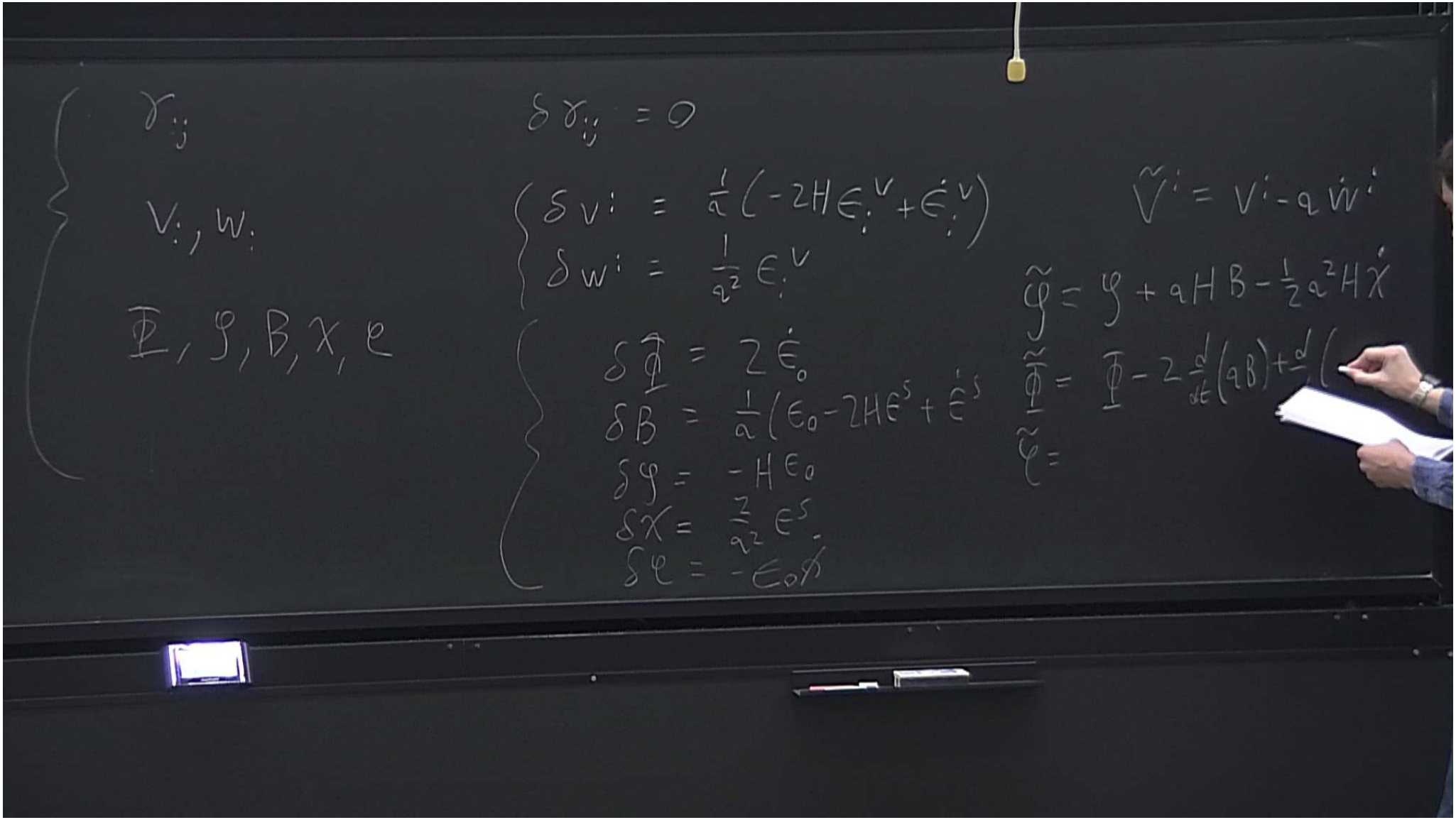

γ_{ij}
$\delta\gamma_{ij} = 0$
V_i, W_i
Σ, s, B, X, e

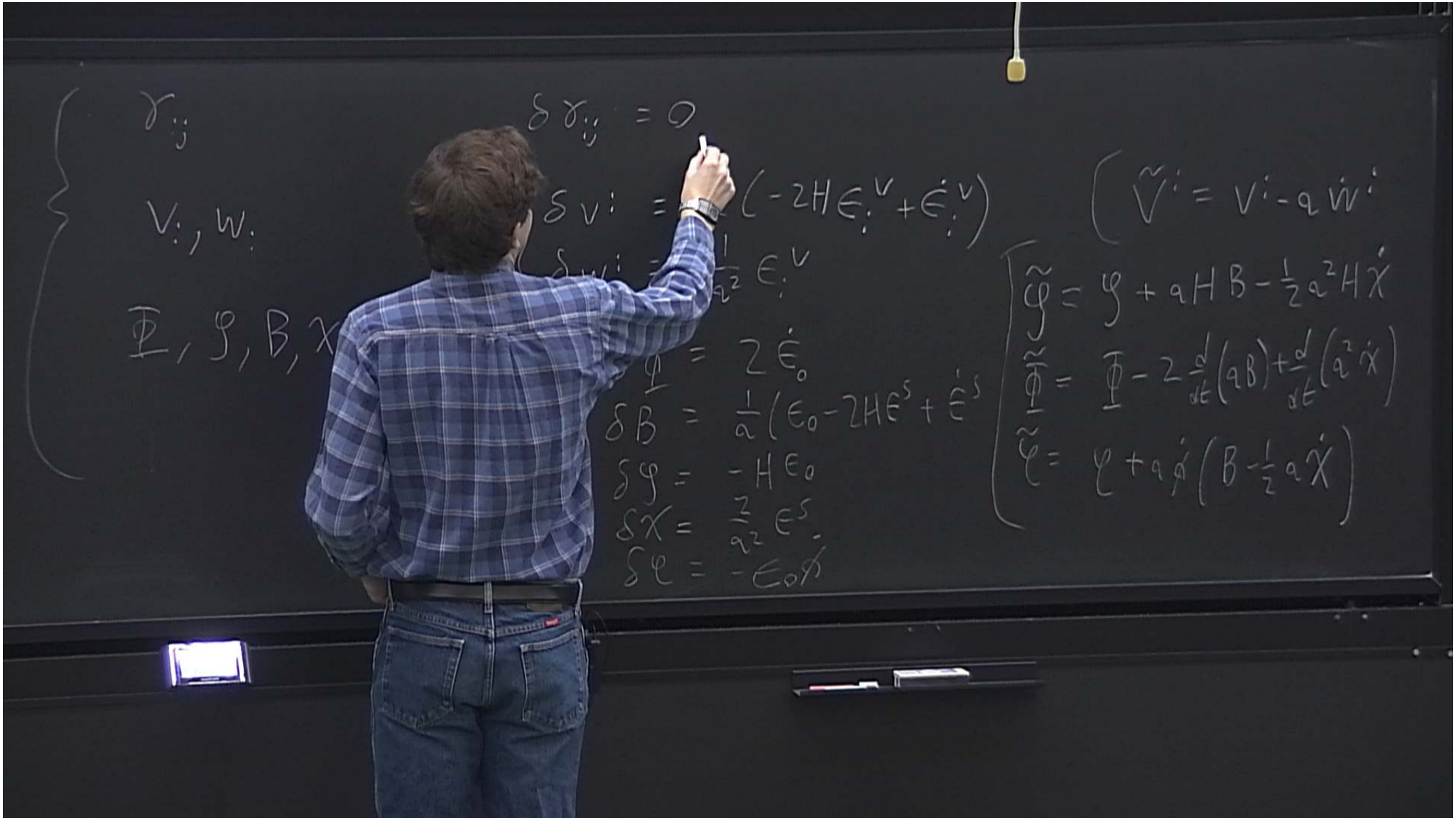

γ_{ij}
$\delta\gamma_{ij} = 0$
V_i, W_i
$\delta V_i = \frac{1}{a}(-2H\epsilon_i^V + \dot{\epsilon}_i^V)$
$\delta W_i = \frac{1}{a^2}\epsilon_i^V$

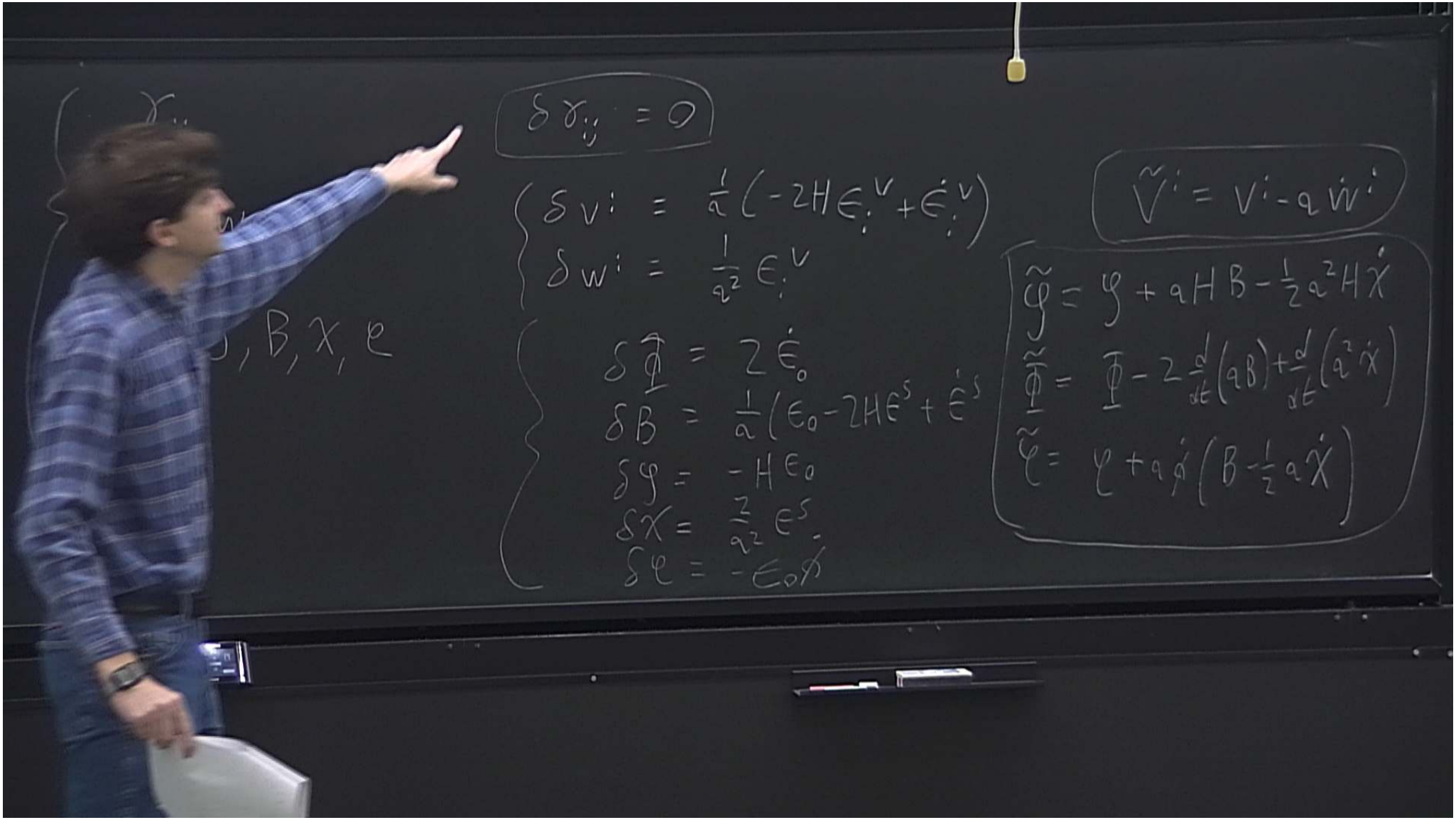

γ_{ij}
V_i, W_i
Σ, φ, B, X, e
$\delta\gamma_{ij} = 0$
$\delta v_i = \frac{1}{a}(-2H\epsilon_i^v + \epsilon_i^v)$
$\delta w_i = \frac{1}{a^2}\epsilon_i^v$
$\tilde{V}_i =$

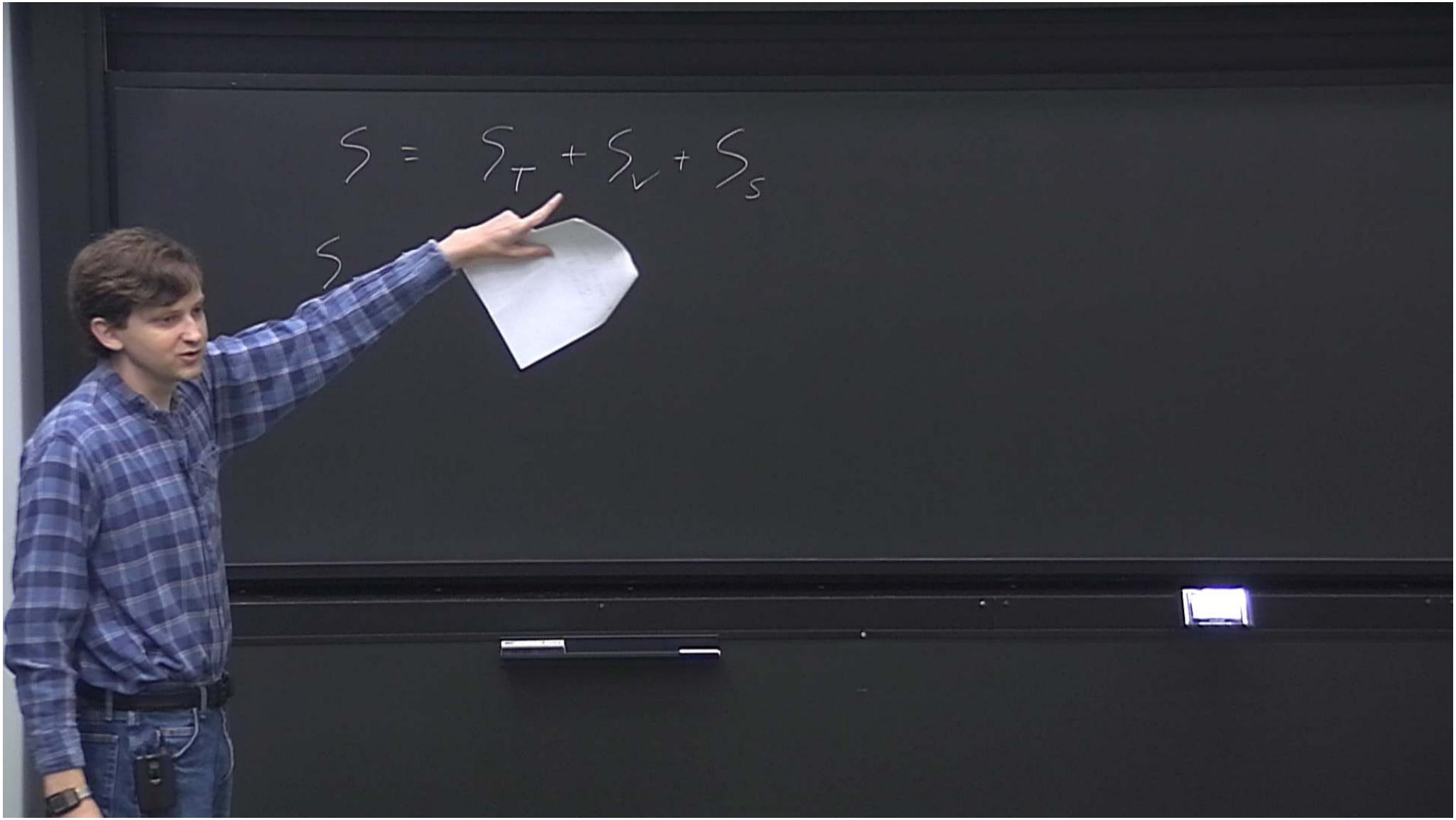

γ_{ij}
$\delta\gamma_{ij} = 0$
V_i, W_i
$\delta v_i = \frac{1}{a}(-2H\epsilon_i^V + \dot{\epsilon}_i^V)$
$\delta w_i = \frac{1}{a^2}\epsilon_i^V$
$\tilde{V}_i = V_i - a\dot{W}_i$
$\Phi, \Psi, B, \chi, \varrho$
$\delta\Phi = 2\dot{\epsilon}_0$
$\delta B = \frac{1}{a}(\epsilon_0 - 2H\epsilon^S + \dot{\epsilon}^S)$
$\delta\psi = -H\epsilon_0$
$\delta\chi = \frac{2}{a^2}\epsilon^S$
$\delta\varrho = -\epsilon_0\bar{\varrho}$

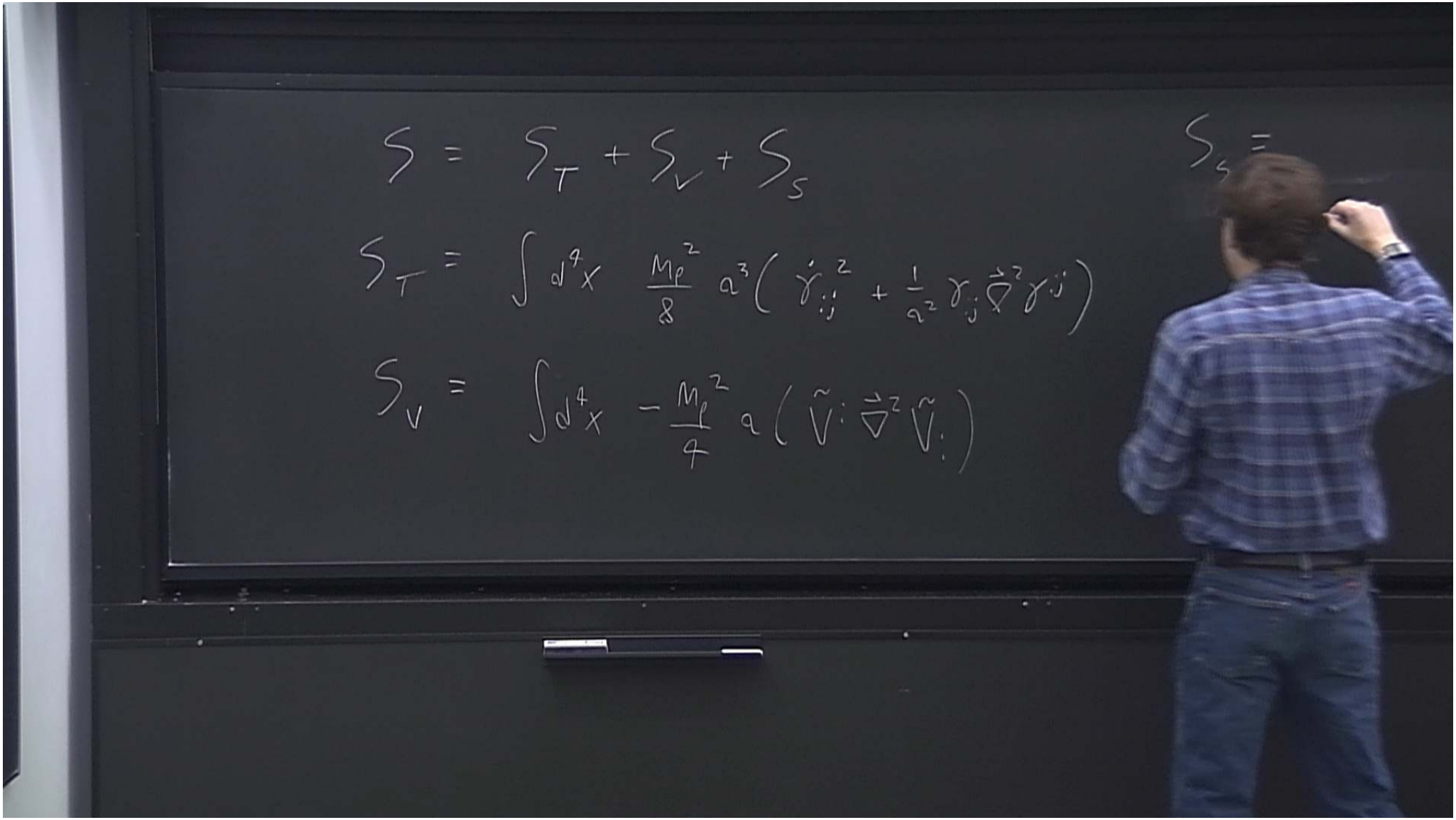

γ_{ij}
V_i, W_i
$\Phi, \varphi, B, \chi, \varrho$
$\delta\gamma_{ij} = 0$
$\delta V_i = \frac{1}{a}(-2H\epsilon_i^V + \dot{\epsilon}_i^V)$
$\delta W_i = \frac{1}{a^2}\epsilon_i^V$
$\delta\Phi = 2\dot{\epsilon}_0$
$\delta B = \frac{1}{a}(\epsilon_0 - 2H\epsilon^S + \dot{\epsilon}^S$
$\delta\varphi = -H\epsilon_0$
$\delta\chi = \frac{2}{a^2}\epsilon^S$
$\delta\varrho = -\epsilon_0\dot{\bar{\varphi}}$
$\tilde{\varphi}$
$\tilde{\Phi}$
$a\dot{W}_i$


γ_{ij}
V_i, W_i
$\Phi, \varphi, B, \chi, \varrho$
$\delta\gamma_{ij} = 0$
$\delta V_i = \frac{1}{a}(-2H\epsilon_i^V + \dot{\epsilon}_i^V)$
$\delta W_i = \frac{1}{a^2}\epsilon_i^V$
$\delta\Phi = 2\epsilon_0$
$\delta B = \frac{1}{a}(\epsilon_0 - 2H\epsilon^S + \dot{\epsilon}^S)$
$\delta\varphi = -H\epsilon_0$
$\delta\chi = \frac{2}{a^2}\epsilon^S$
$\delta\varrho = -\epsilon_0\dot{\rho}$
$\tilde{V}_i = V_i - a\dot{W}_i$
$\tilde{\varphi} = \varphi + aHB - \frac{1}{2}a^2H\dot{\chi}$
$\tilde{\Phi} = \Phi - 2\frac{d}{dt}(aB) + \frac{d}{dt}($
$\tilde{\varrho} =$


γ_{ij}
V_i, W_i
Φ, φ, B, χ
$\delta\gamma_{ij} = 0$
$\delta V_i = \frac{1}{a}(-2H\epsilon_i^V + \dot{\epsilon}_i^V)$
$\delta W_i = \frac{1}{a^2}\epsilon_i^V$
$\delta\Phi = 2\dot{\epsilon}_0$
$\delta B = \frac{1}{a}(\epsilon_0 - 2H\epsilon^S + \dot{\epsilon}^S)$
$\delta\varphi = -H\epsilon_0$
$\delta\chi = \frac{2}{a^2}\epsilon^S$
$\delta\varrho = -\epsilon_0\dot{\bar{\varrho}}$
$\tilde{V}_i = V_i - a\dot{W}_i$
$\tilde{\varphi} = \varphi + aHB - \frac{1}{2}a^2H\dot{\chi}$
$\tilde{\Phi} = \Phi - 2\frac{d}{dt}(aB) + \frac{d}{dt}(a^2\dot{\chi})$
$\tilde{\varrho} = \varrho + a\dot{\bar{\varrho}}(B - \frac{1}{2}a\dot{\chi})$


$\delta\gamma_{ij} = 0$
$\delta v^i = \frac{1}{a}(-2H\epsilon^{iv} + \dot{\epsilon}^{iv})$
$\delta w^i = \frac{1}{a^2}\epsilon^{iv}$
$\tilde{V}^i = V^i - a\dot{w}^i$
$\delta\varphi = \dot{\epsilon}_0$
$\delta B = \frac{1}{a}(\epsilon_0 - 2H\epsilon^s + \dot{\epsilon}^s)$
$\delta\varphi = -H\epsilon_0$
$\delta\chi = \frac{2}{a^2}\epsilon^s$
$\delta\varrho = -\epsilon_0\bar{\rho}$
$\tilde{\varphi} = \varphi + aHB - \frac{1}{2}a^2H\dot{\chi}$
$\tilde{\Phi} = \Phi - 2\frac{d}{dt}(aB) + \frac{d}{dt}(a^2\dot{\chi})$
$\tilde{\varrho} = \varrho + a\dot{\bar{\rho}}(B - \frac{1}{2}a\dot{\chi})$


S = S_T + S_V + S_S
S

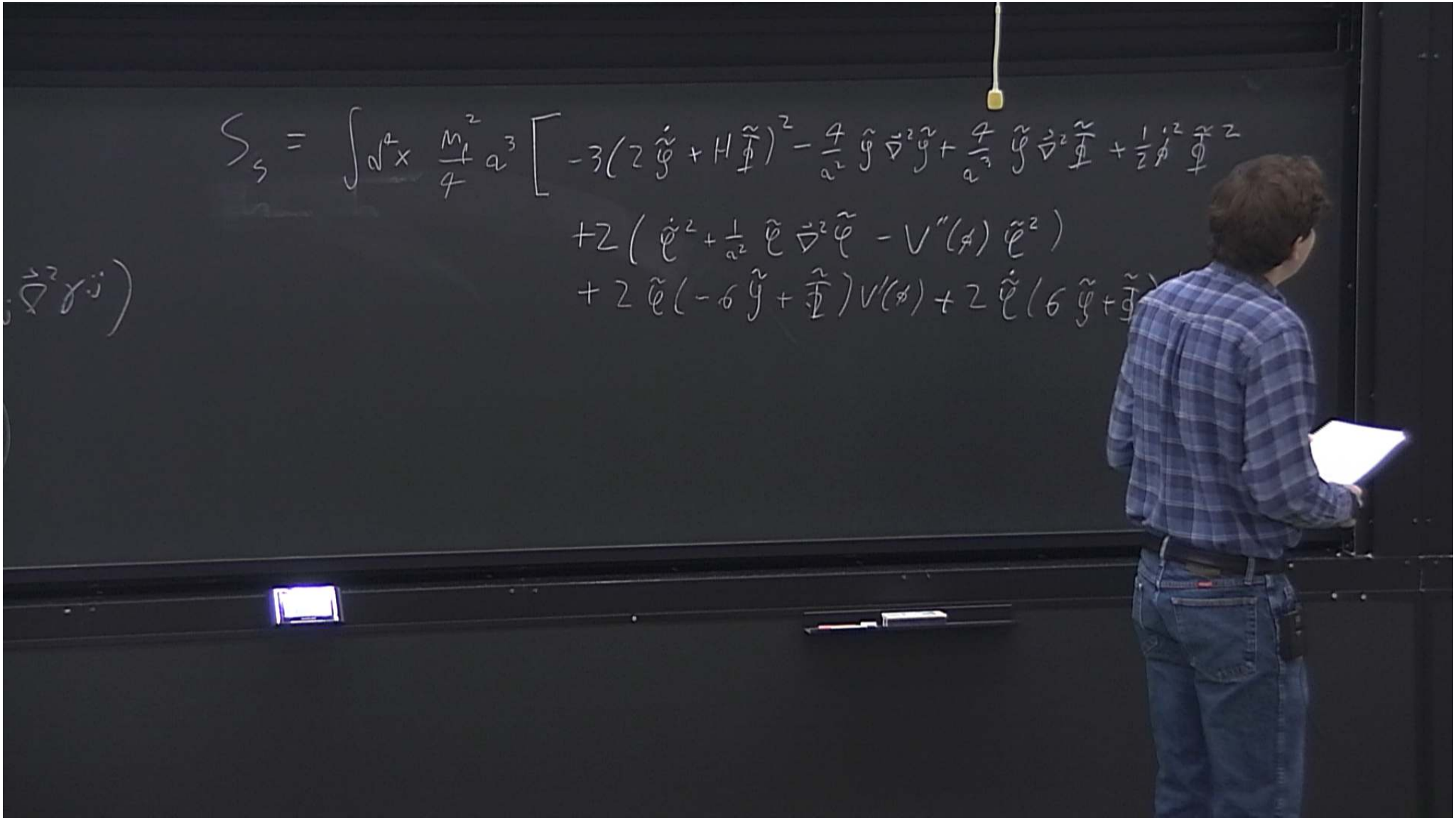

$S = S_T + S_V + S_S$
$S_S =$
$S_T = \int d^4x \ \frac{M_p^2}{8} a^3 \left(\dot{\gamma}_{ij}^2 + \frac{1}{a^2} \gamma_{ij} \vec{\nabla}^2 \gamma_{ij} \right)$
$S_V = \int d^4x \ -\frac{M_p^2}{4} a \left(\tilde{V}_i \vec{\nabla}^2 \tilde{V}_i \right)$

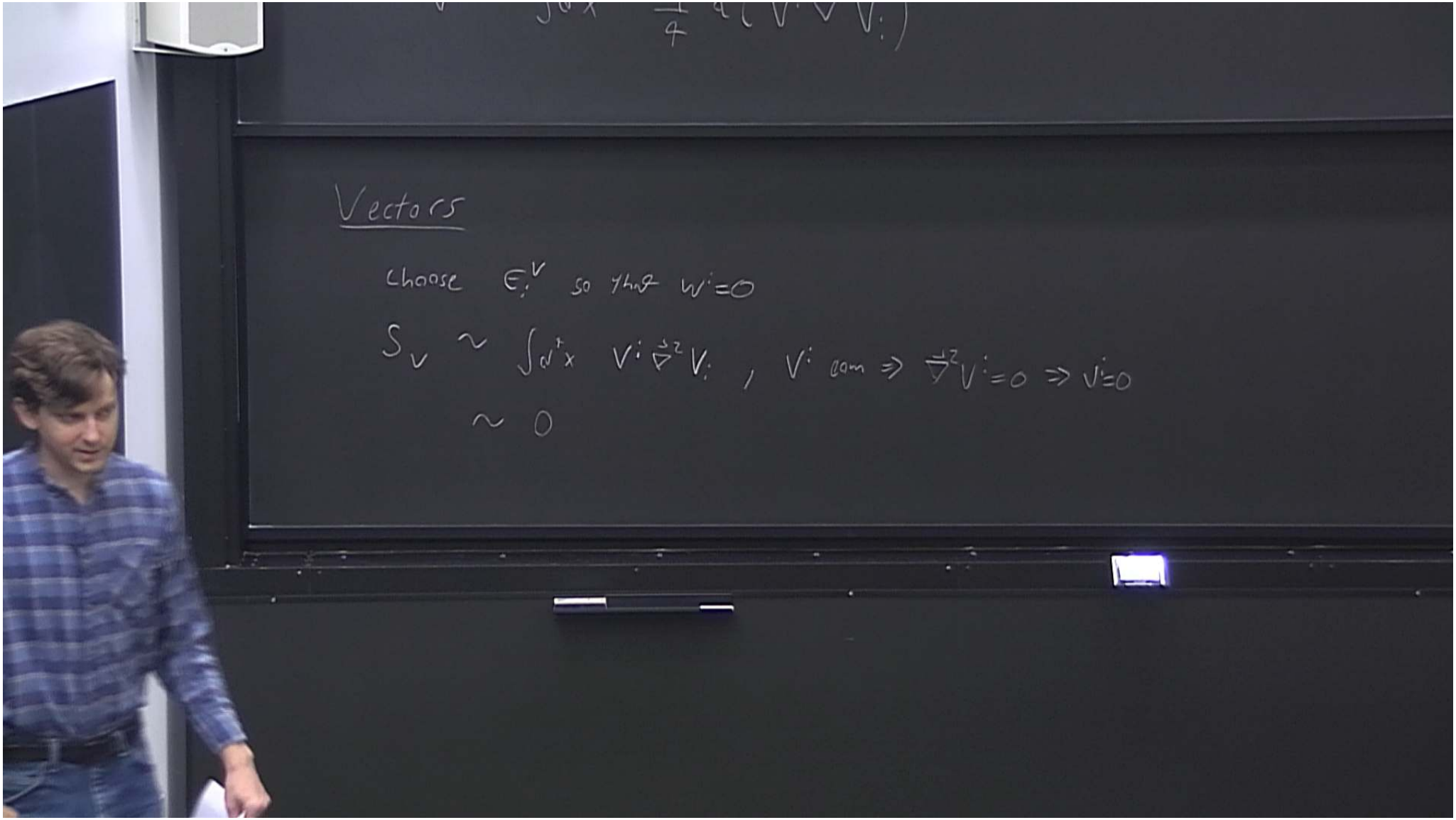
$$S_S = \int d^4x \, \frac{M_P^2}{4} a^3 \Big[-3\big(2\dot{\tilde{\varphi}} + H\tilde{\pi}\big)^2 - \frac{4}{a^2}\tilde{\varphi}\vec{\nabla}^2\tilde{\varphi} + \frac{4}{a^3}\tilde{\varphi}\vec{\nabla}^2\tilde{\pi} + \frac{1}{2}\dot{\pi}^2 - \frac{1}{2}\vec{\nabla}^2\pi^2$$

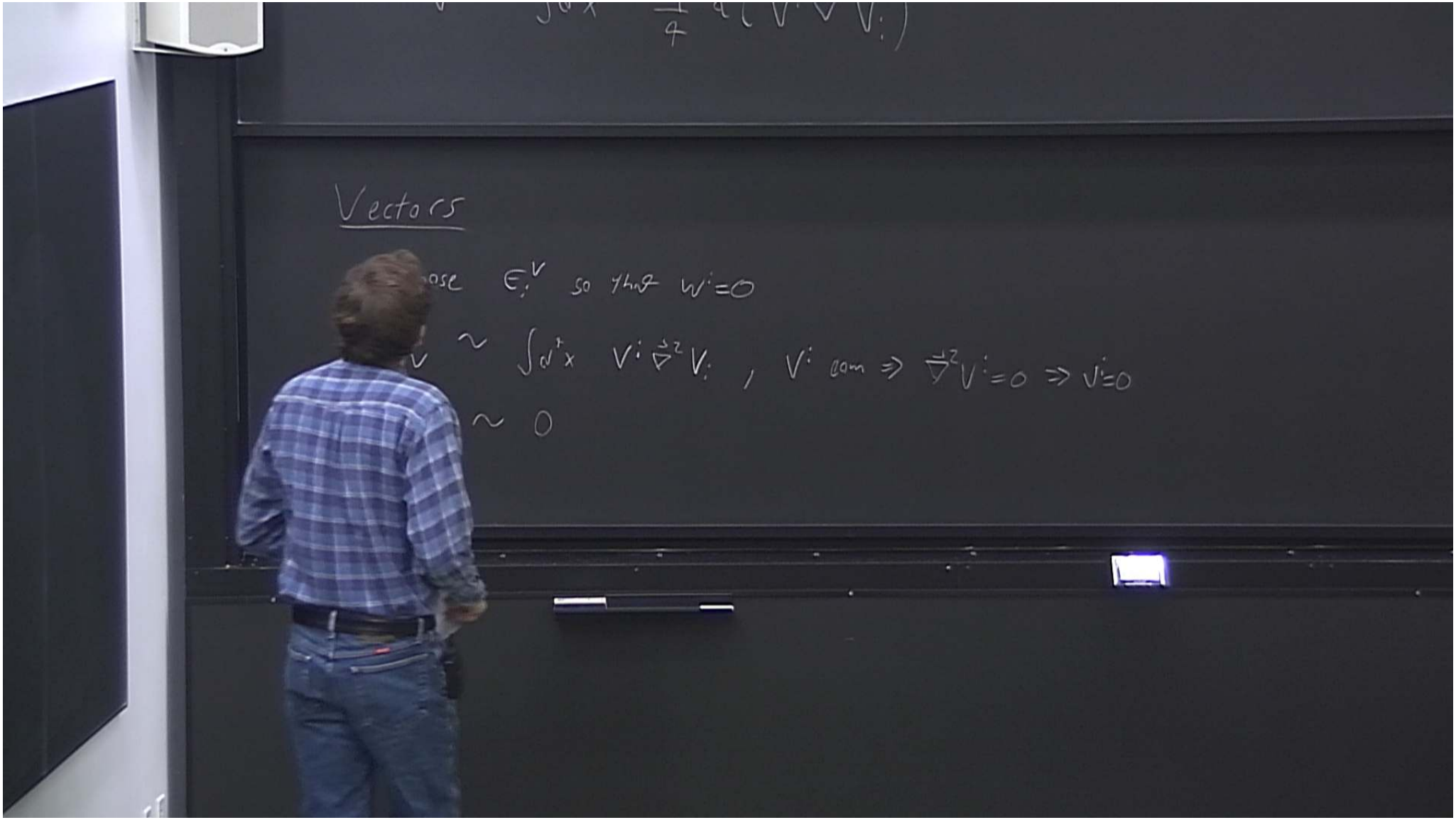
$$+2\Big(\dot{\tilde{\varphi}}^2 + \frac{1}{a^2}\tilde{\varphi}\vec{\nabla}^2\tilde{\varphi} - V''(\phi)\tilde{\varphi}^2\Big)$$

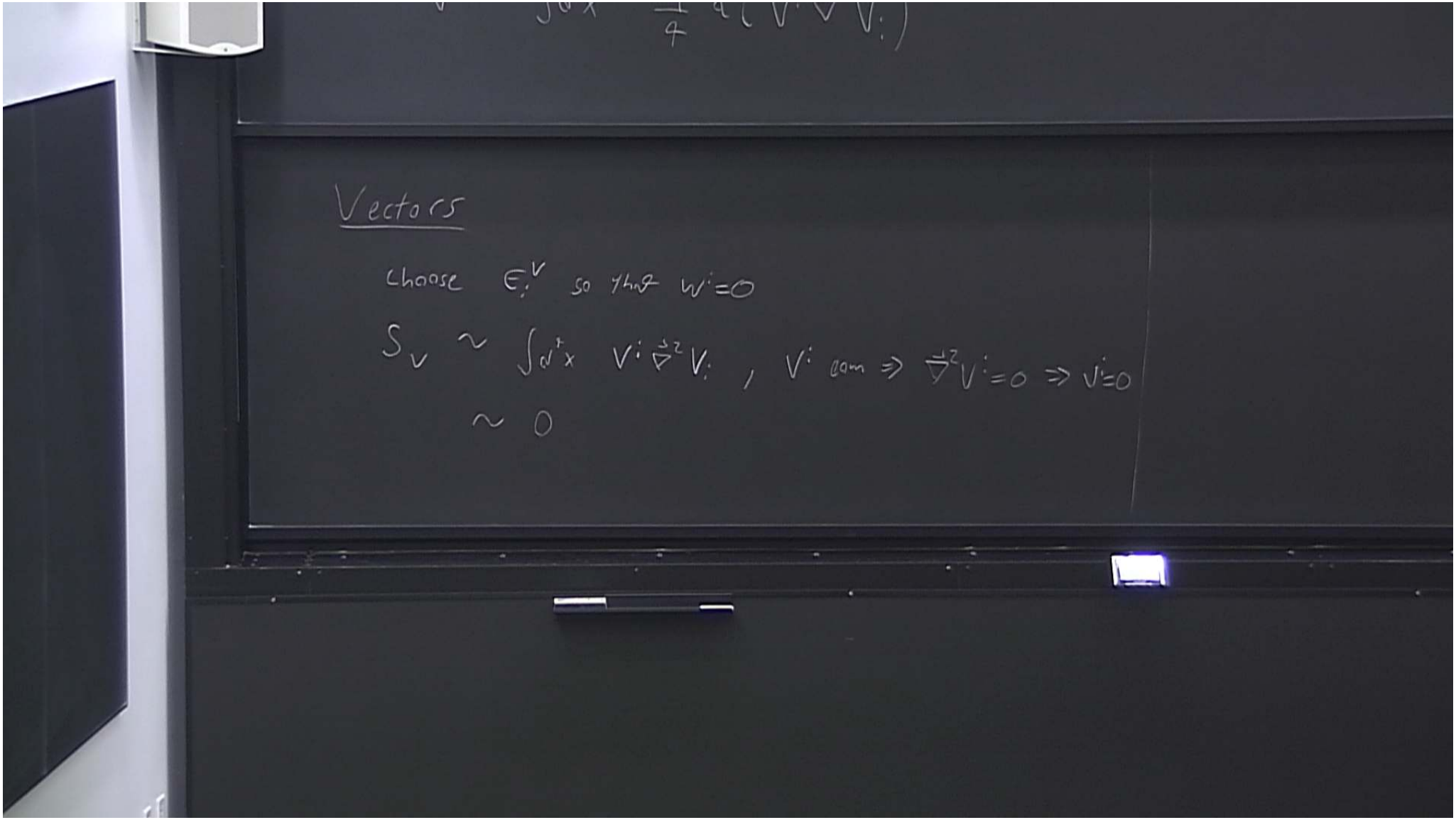
$$+2\tilde{\varphi}\big(-6\dot{\tilde{\varphi}} + \vec{\nabla}^2\tilde{\pi}\big)V'(\phi) + 2\dot{\tilde{\varphi}}\big(6\tilde{\varphi} + \vec{\nabla}^2\tilde{\pi}\big)\dot{\phi}\Big]$$

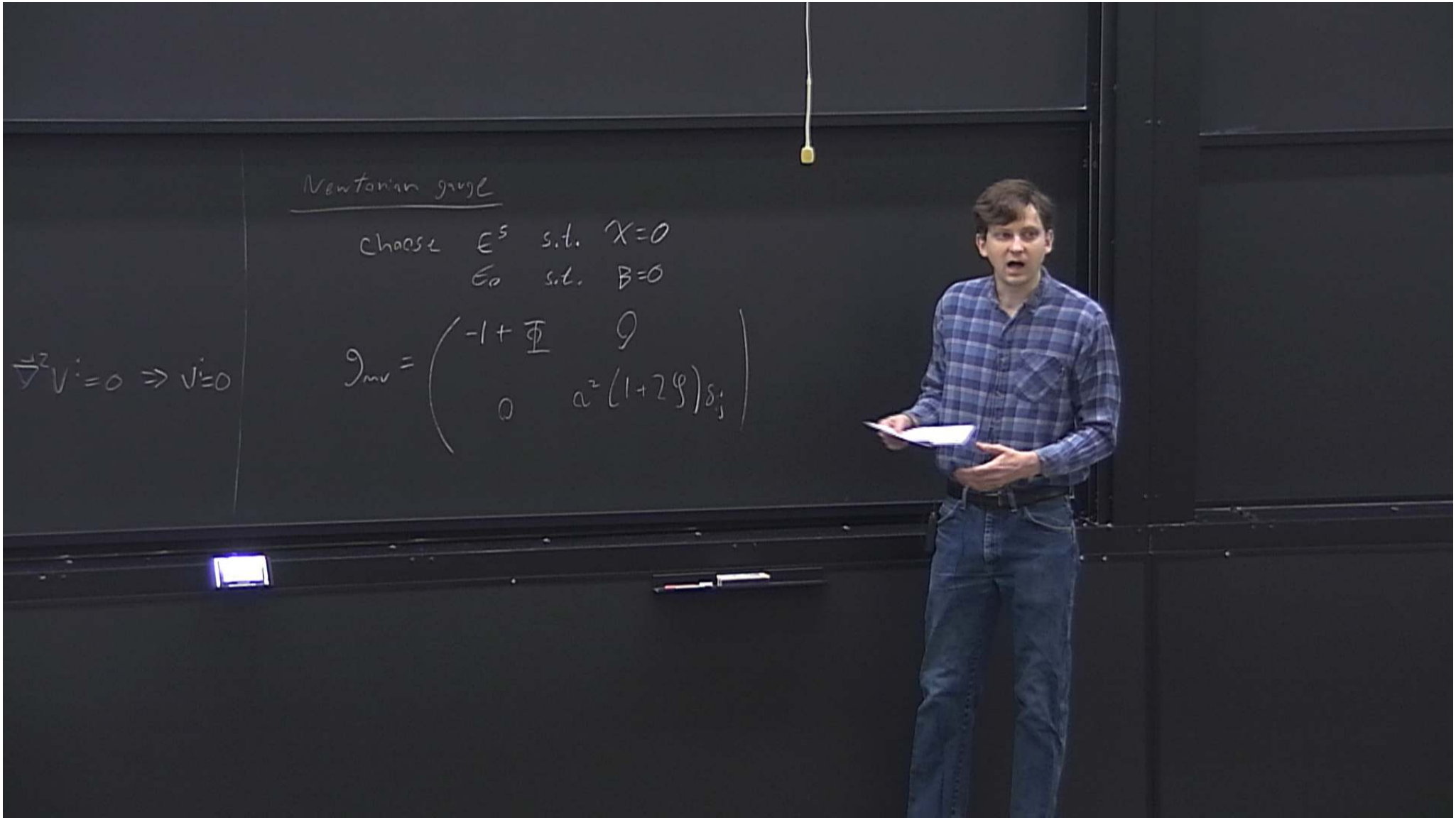
$$\vec{\nabla}^2\gamma_{ij}\Big)$$

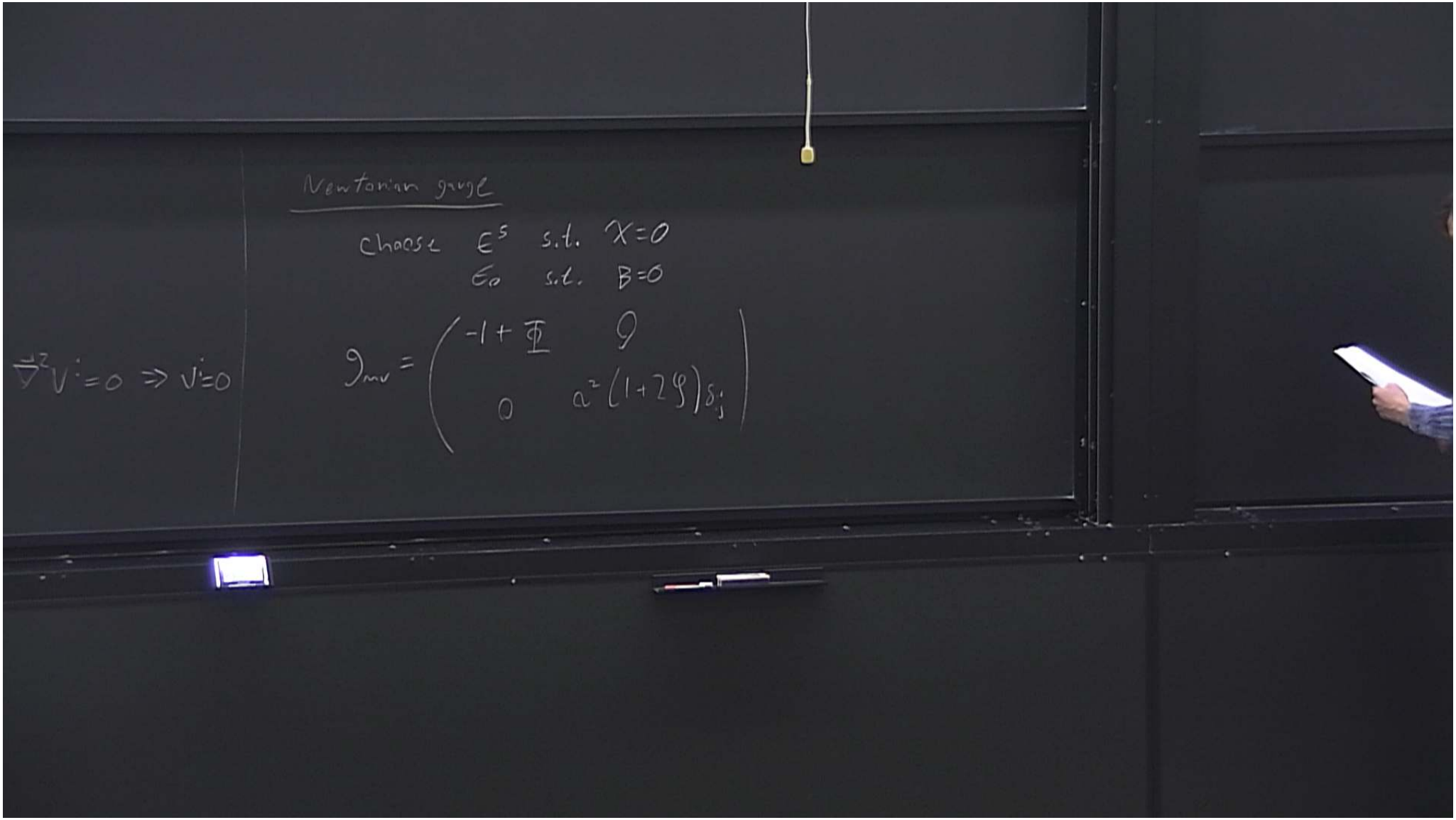

$S_s = \int d^4x \, \frac{m_p^2}{4} a^3 \Big[-3(2\dot{\tilde{\varphi}} + H\tilde{\Phi})^2 - \frac{4}{a^2}\tilde{\varphi}\vec{\nabla}^2\tilde{\varphi} + \frac{4}{a^3}\tilde{\varphi}\vec{\nabla}^2\tilde{\Phi} + \frac{1}{2}\dot{\phi}^2\tilde{\Phi}^2$
$+2\left(\dot{\tilde{\varphi}}^2 + \frac{1}{a^2}\tilde{\varphi}\vec{\nabla}^2\tilde{\varphi} - V''(\phi)\tilde{\varphi}^2\right)$
$+2\tilde{\varphi}(-6\tilde{\varphi} + \tilde{\Phi})V'(\phi) + 2\dot{\tilde{\varphi}}(6\tilde{\varphi} + \tilde{\Phi}$
$\vec{\nabla}^2\gamma^{ij})$

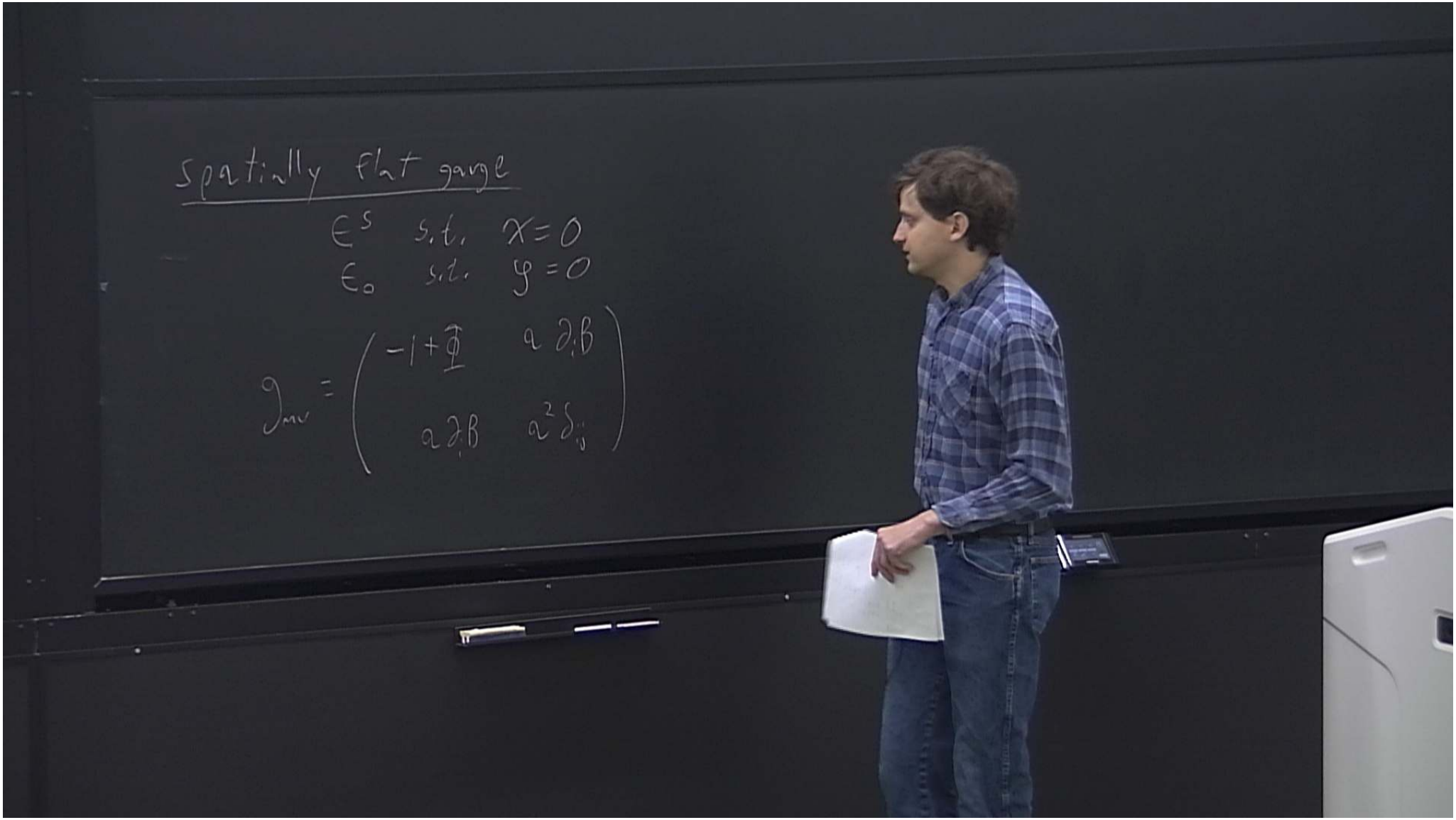

Vectors
choose ϵ_i^V so that $W^i = 0$
$S_V \sim \int d^4x\, V_i \vec{\nabla}^2 V_i$, V^i com $\Rightarrow \vec{\nabla}^2 V^i = 0 \Rightarrow V^i = 0$
~ 0

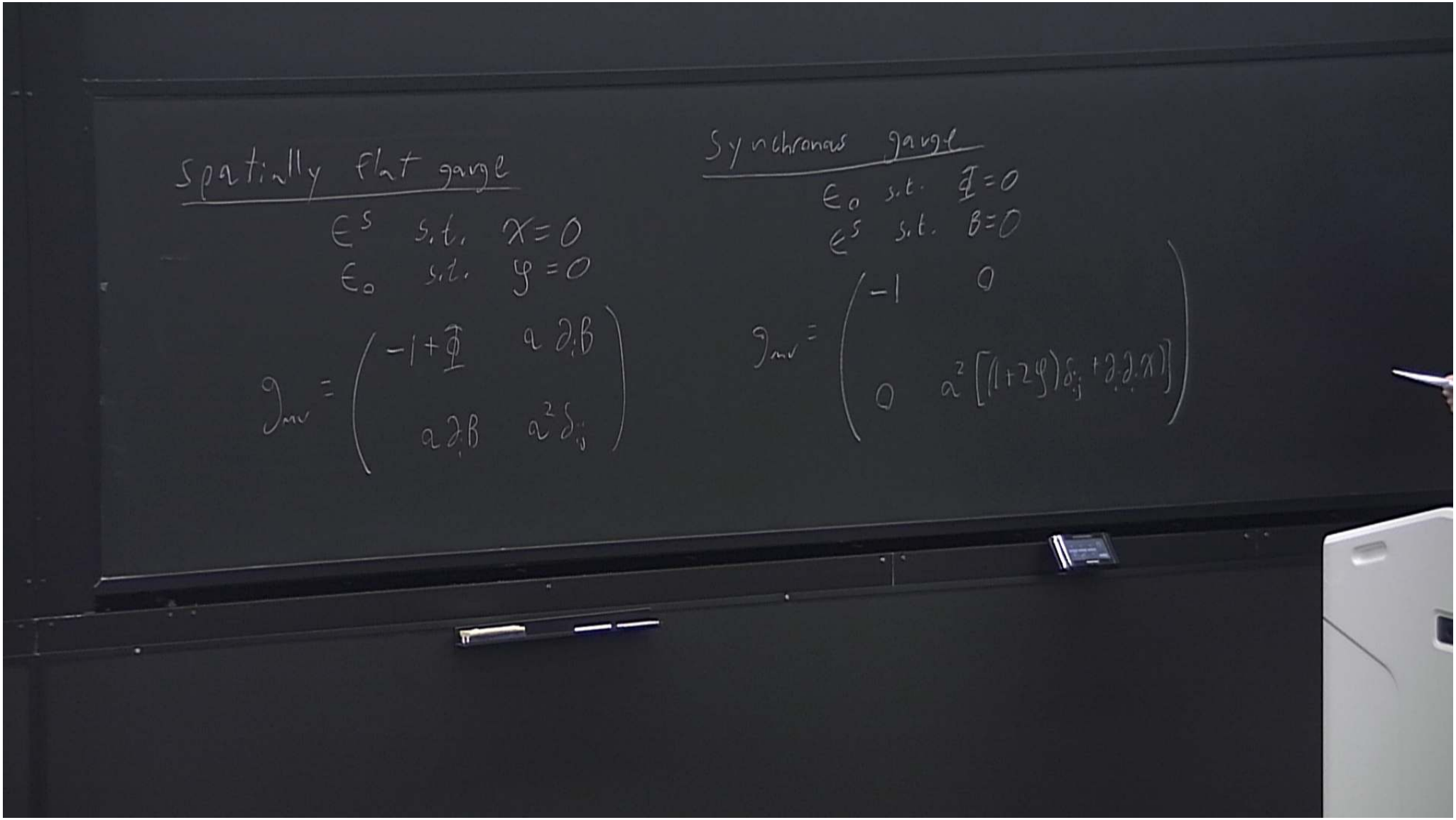

Vectors
$\sim \int d^3x \; V_i \vec{\nabla}^2 V_i$, V_i com $\Rightarrow \vec{\nabla}^2 V_i = 0 \Rightarrow V_i = 0$
~ 0

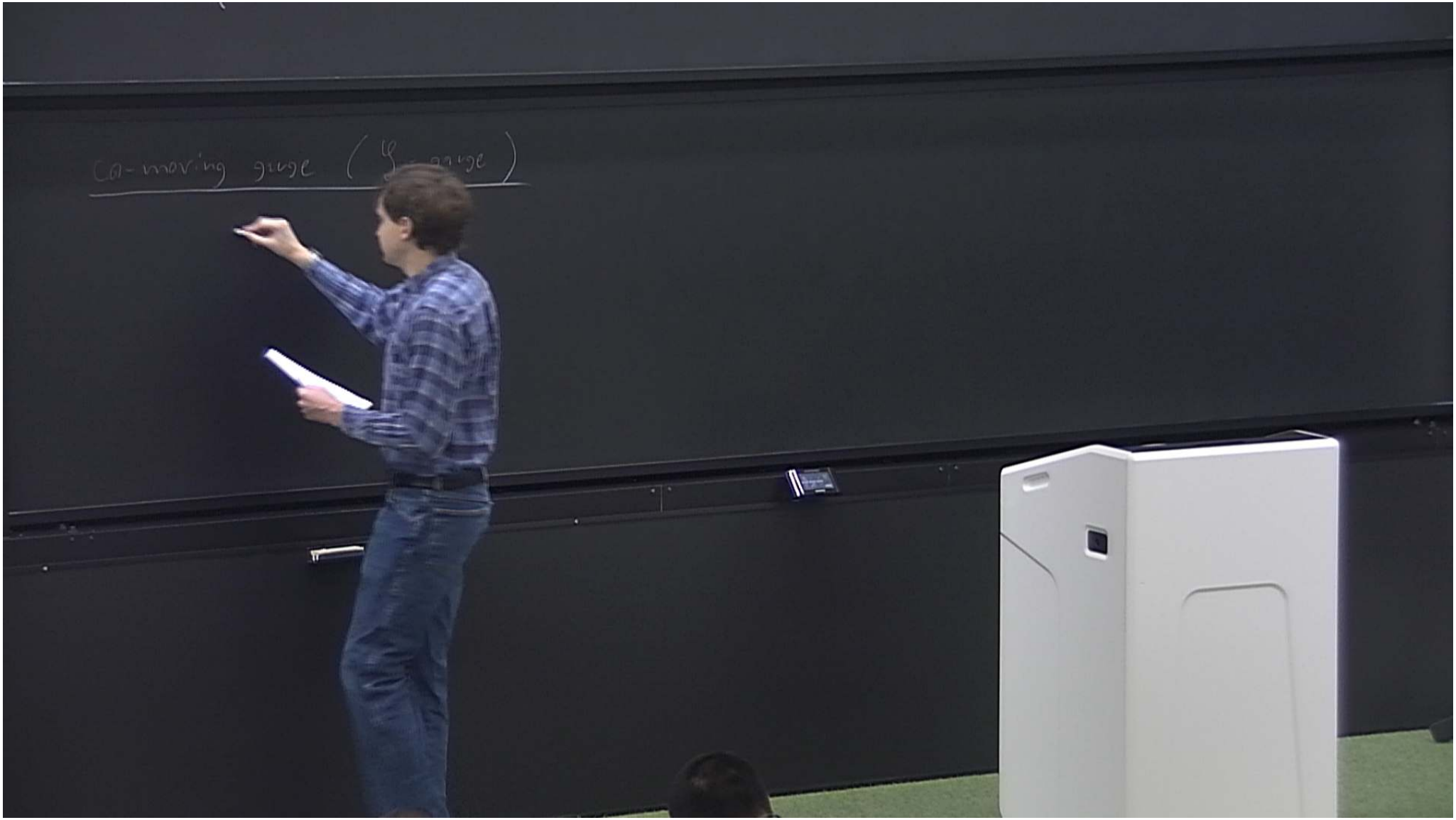

Vectors
Choose ϵ_i^V so that $W^i = 0$
$S_V \sim \int d^4x \; V^i \vec{\nabla}^2 V_i$, V^i com $\Rightarrow \vec{\nabla}^2 V^i = 0 \Rightarrow V^i = 0$
~ 0

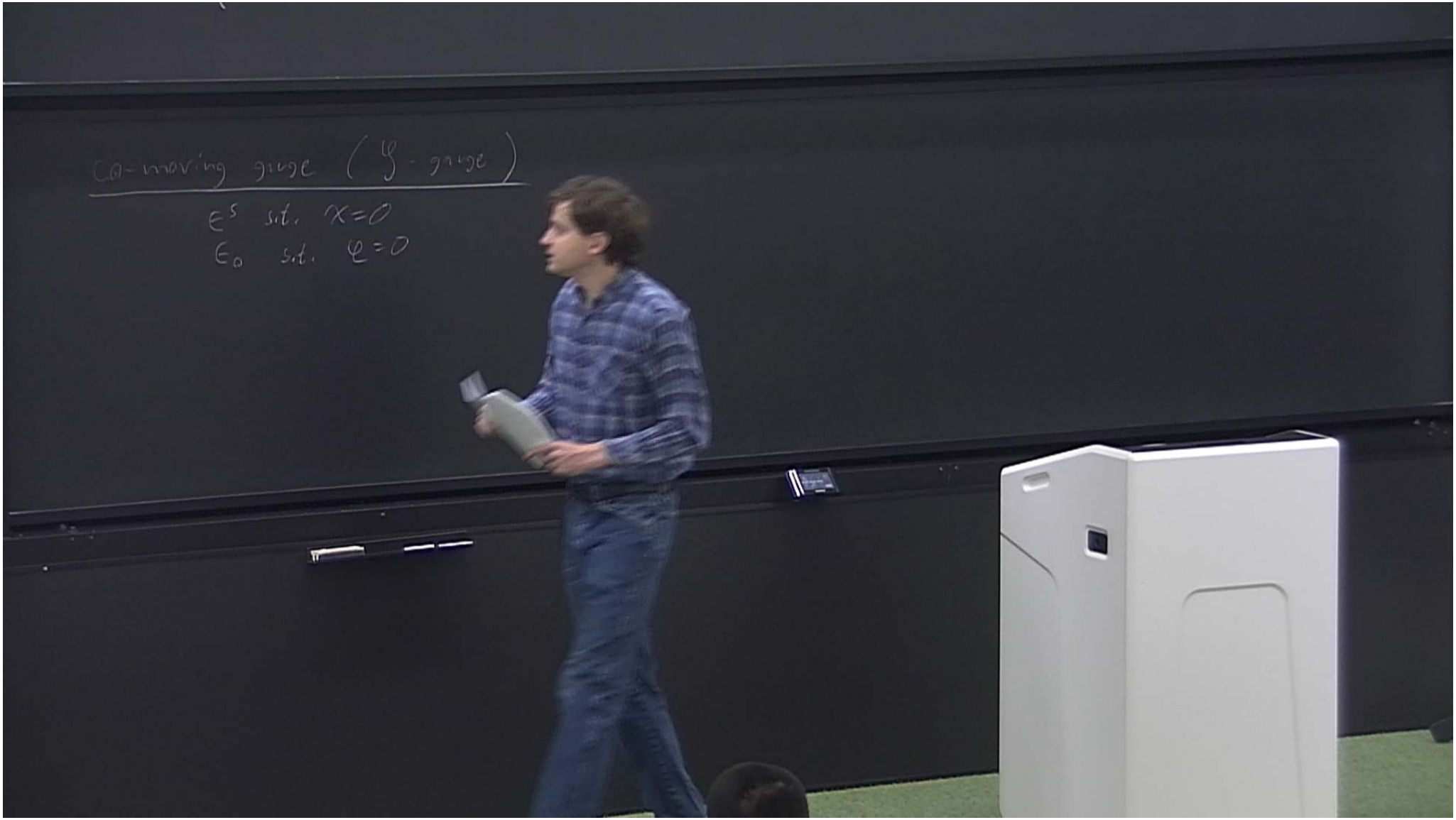

Newtonian gauge
Choose ϵ^s s.t. $\chi = 0$
ϵ_0 s.t. $B = 0$
$\vec{\nabla}^2 V = 0 \Rightarrow V = 0$
$g_{\mu\nu} = \begin{pmatrix} -1+\Phi & 0 \\ 0 & a^2(1+2\Psi)\delta_{ij} \end{pmatrix}$

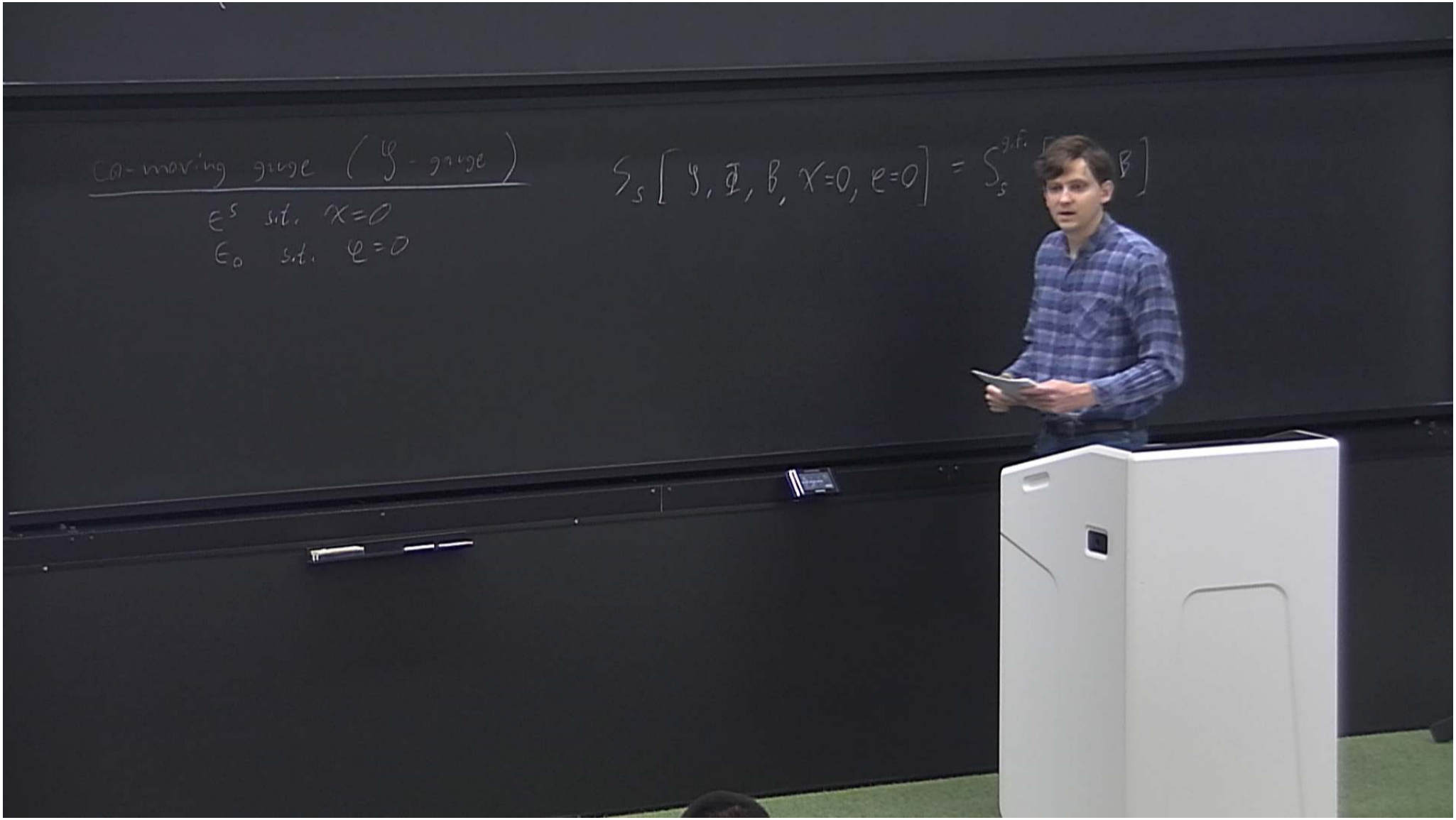

Newtonian gauge
Choose ϵ^s s.t. $\chi = 0$
ϵ_0 s.t. $B = 0$
$\vec{\nabla}^2 V^i = 0 \Rightarrow V^i = 0$
$g_{\mu\nu} = \begin{pmatrix} -1+\Phi & g \\ 0 & a^2(1+2\Psi)\delta_{ij} \end{pmatrix}$

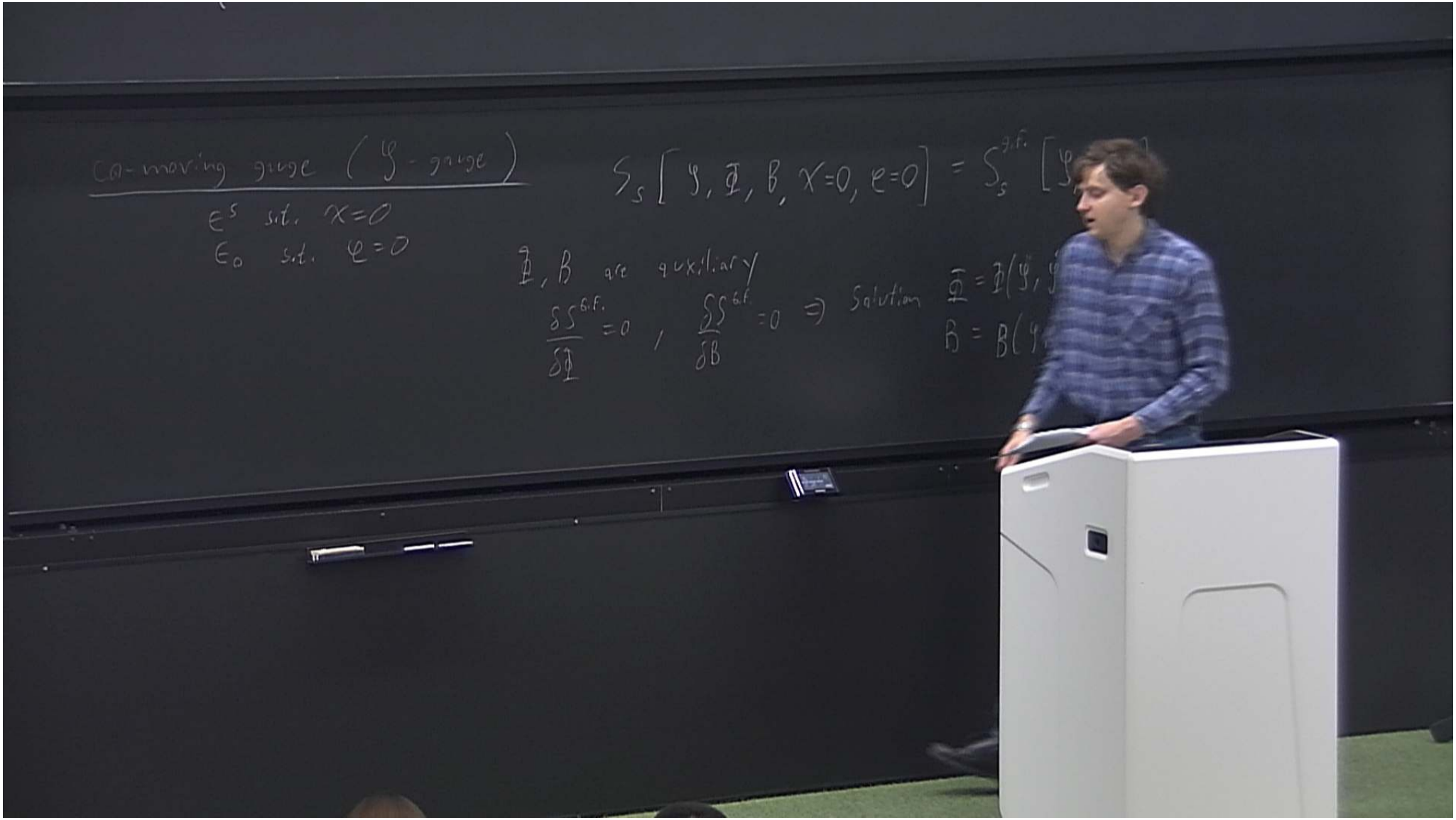

Spatially flat gauge
ϵ^S s.t. $\chi = 0$
ϵ_0 s.t. $\gamma = 0$
$g_{\mu\nu} = \begin{pmatrix} -1+\Phi & a\partial_i B \\ a\partial_i B & a^2\delta_{ij} \end{pmatrix}$

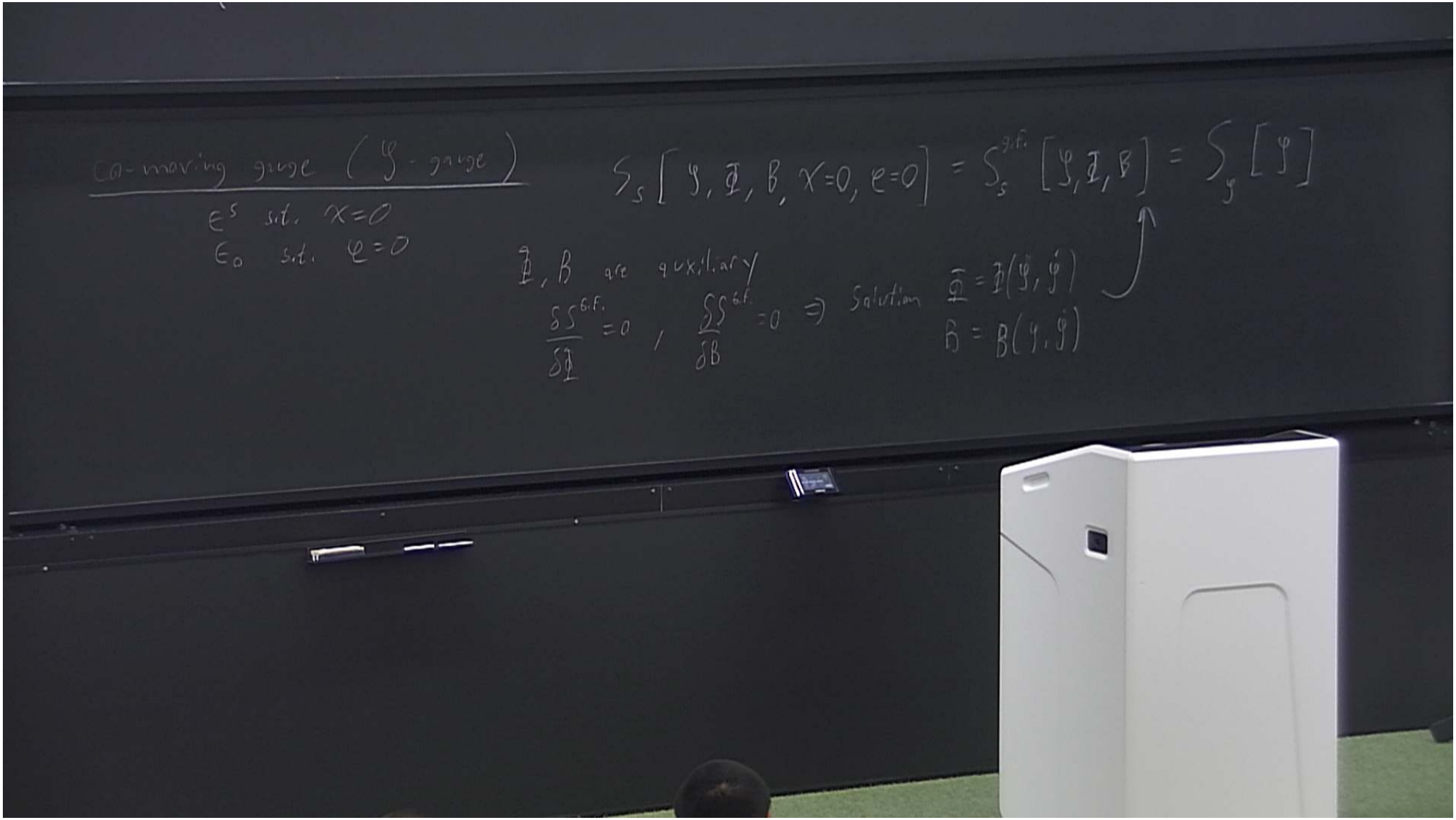

Spatially flat gauge
ϵ^S s.t. $\chi = 0$
ϵ_0 s.t. $\varphi = 0$
$g_{\mu\nu} = \begin{pmatrix} -1+\Phi & a\,\partial_i B \\ a\,\partial_i B & a^2\delta_{ij} \end{pmatrix}$
Synchronous gauge
ϵ_0 s.t. $\Phi = 0$
ϵ^S s.t. $B = 0$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2\left[(1+2\varphi)\delta_{ij} + \partial_i\partial_j\chi\right] \end{pmatrix}$

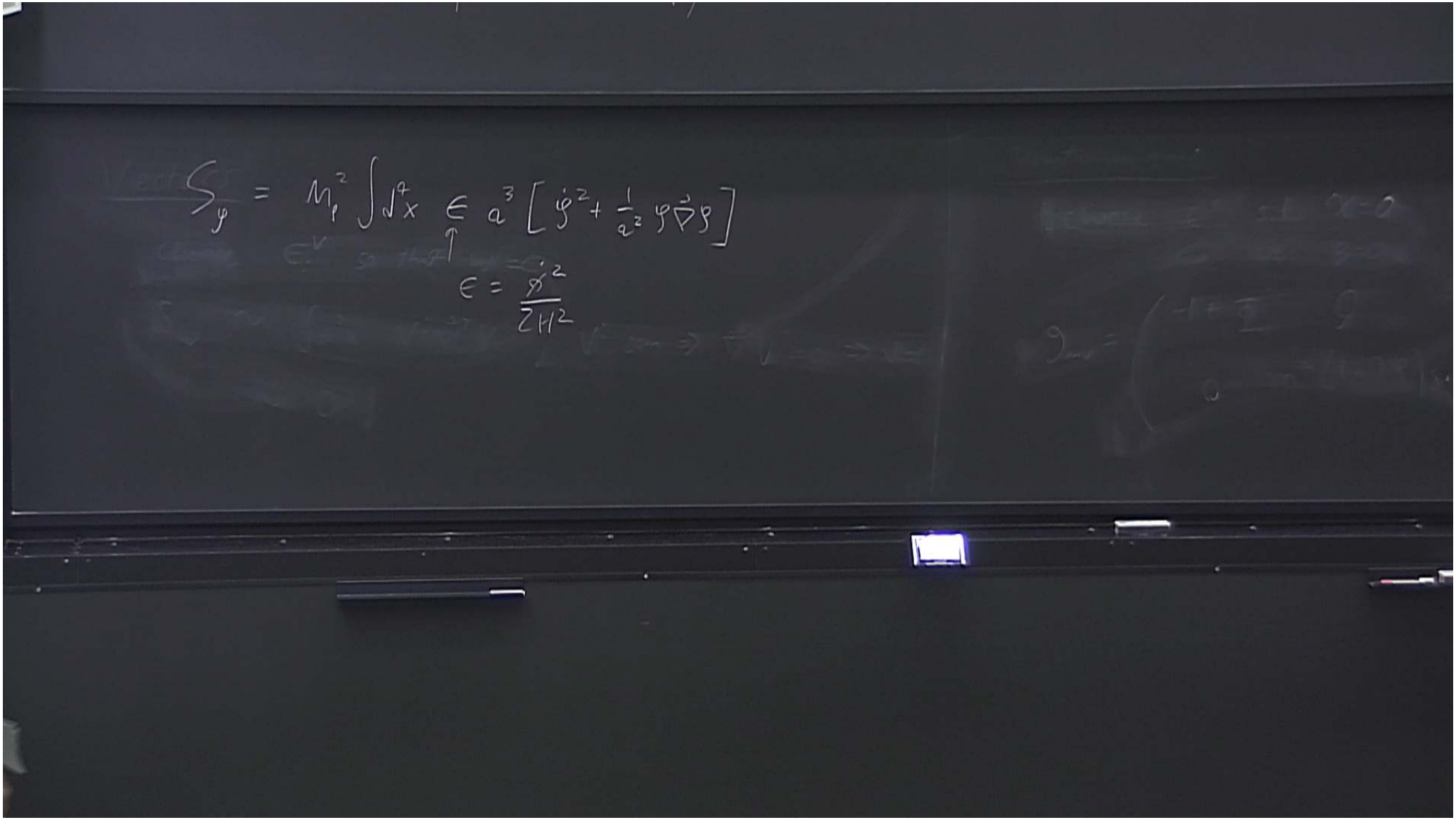

Co-moving gauge

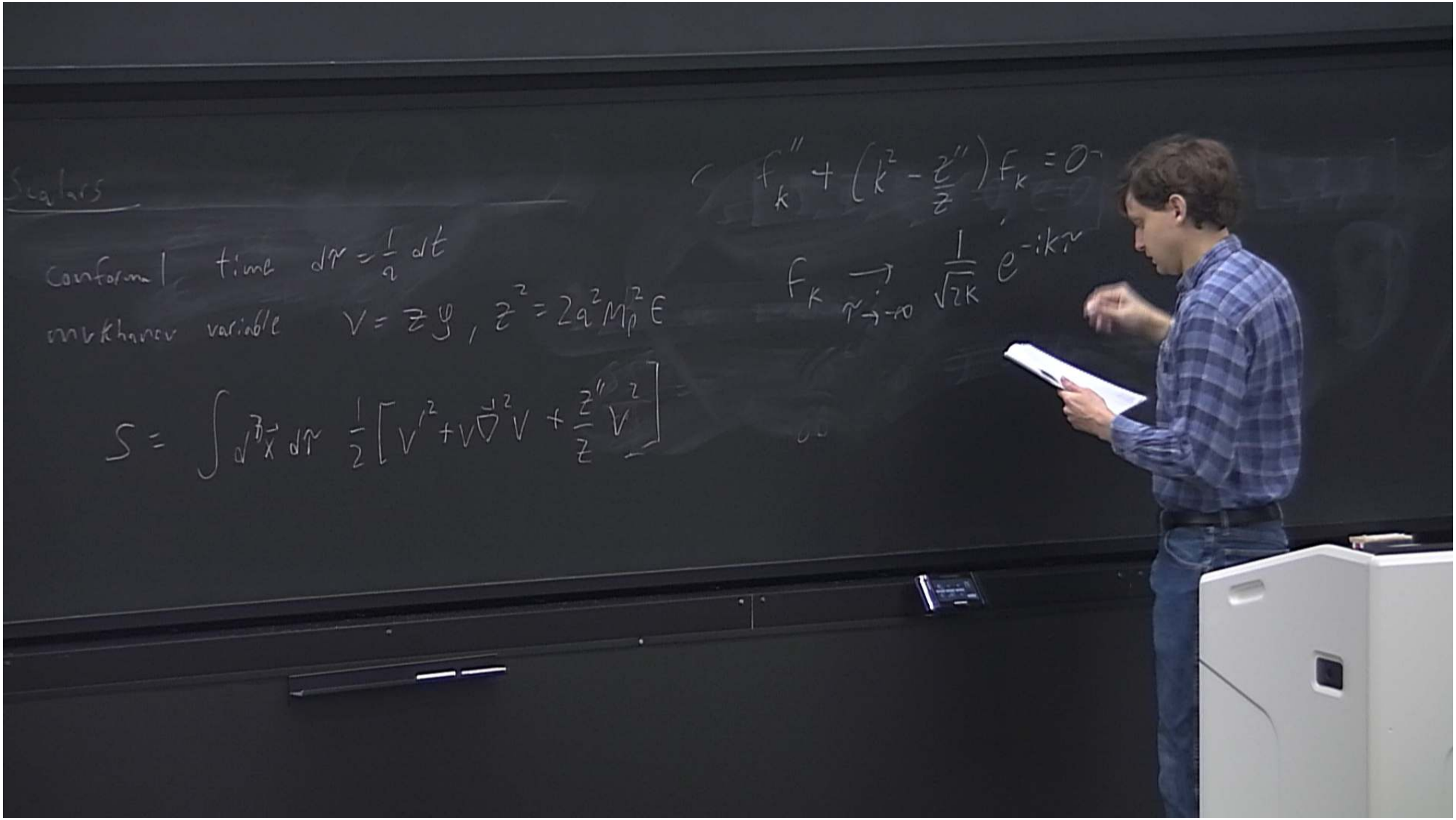

Co-moving gauge (S-gauge)
ϵ^S s.t. $\chi=0$
ϵ_0 s.t. $\varrho=0$

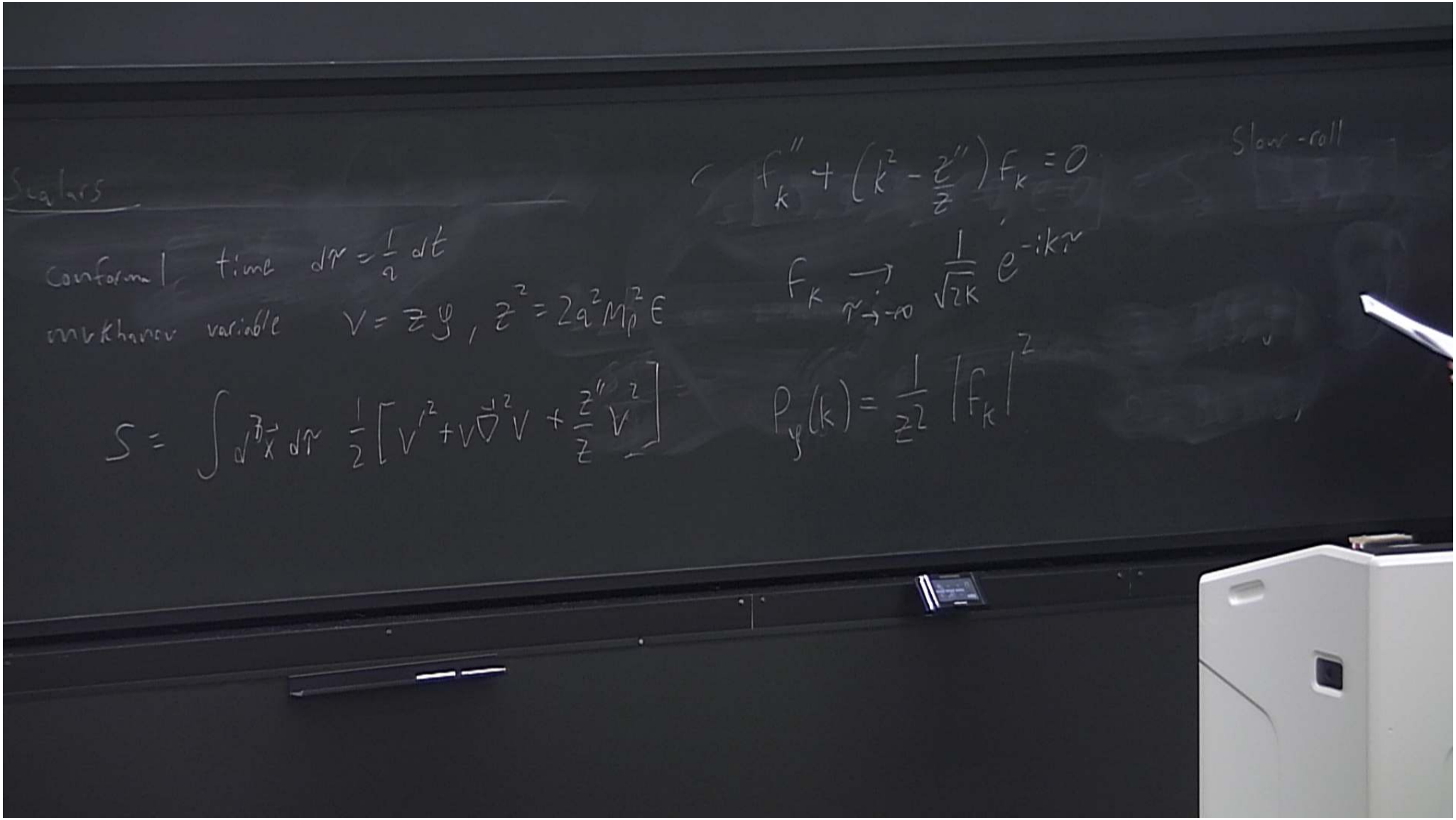

Co-moving gauge (ζ - gauge)
ε^s s.t. χ=0
ε_0 s.t. φ=0
S_s [ζ, Φ, B, χ=0, φ=0] = S_s^{g.f.}

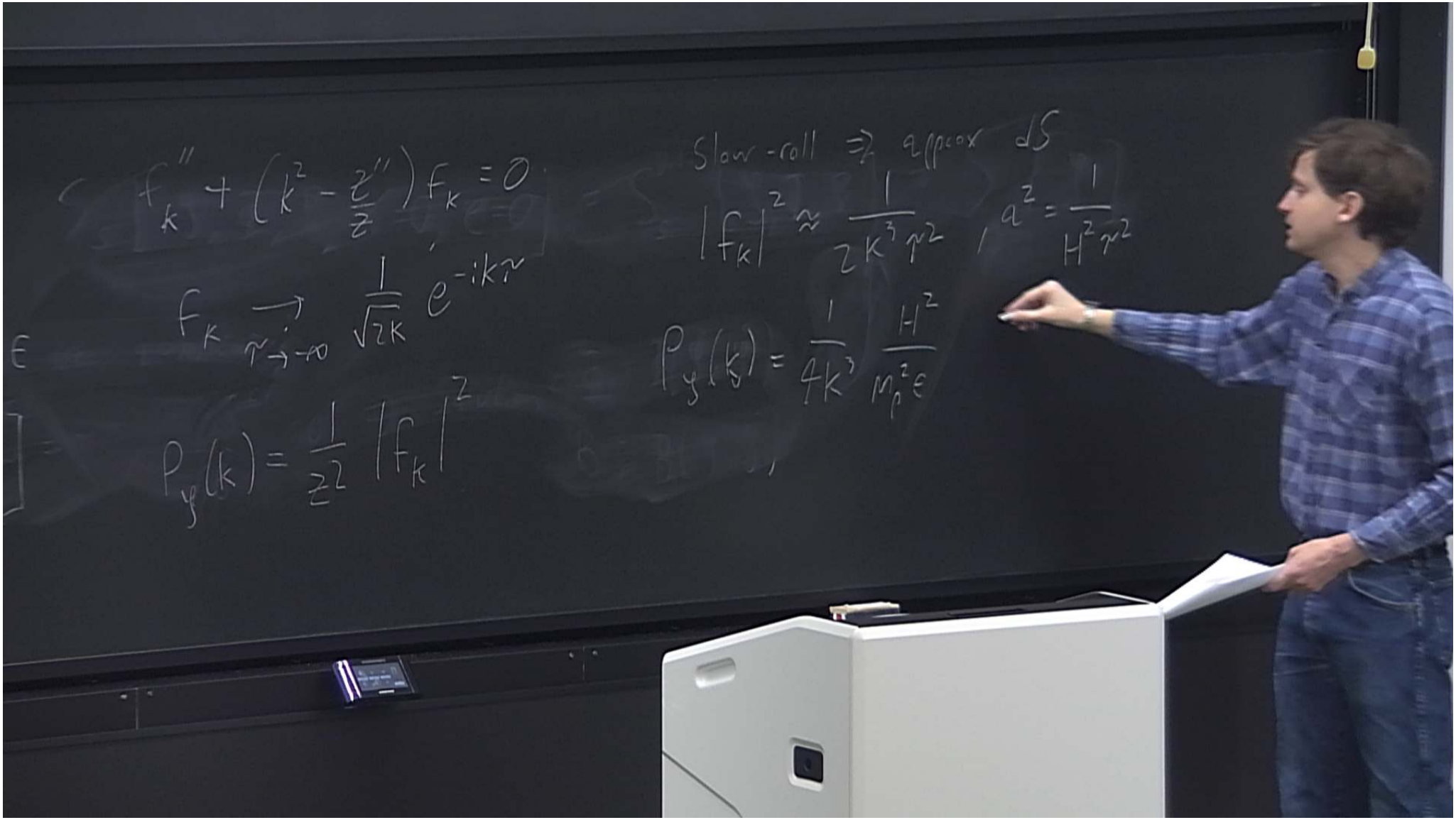

Co-moving gauge (φ - gauge)
ε^s s.t. χ=0
ε_0 s.t. φ=0
S_s [φ, Φ, B, χ=0, φ=0] = S_s^g.f. [φ
Φ, B are auxiliary
δS^g.f./δΦ = 0 , δS^g.f./δB = 0 ⇒ Solution Φ = Φ(φ, φ
B = B(φ

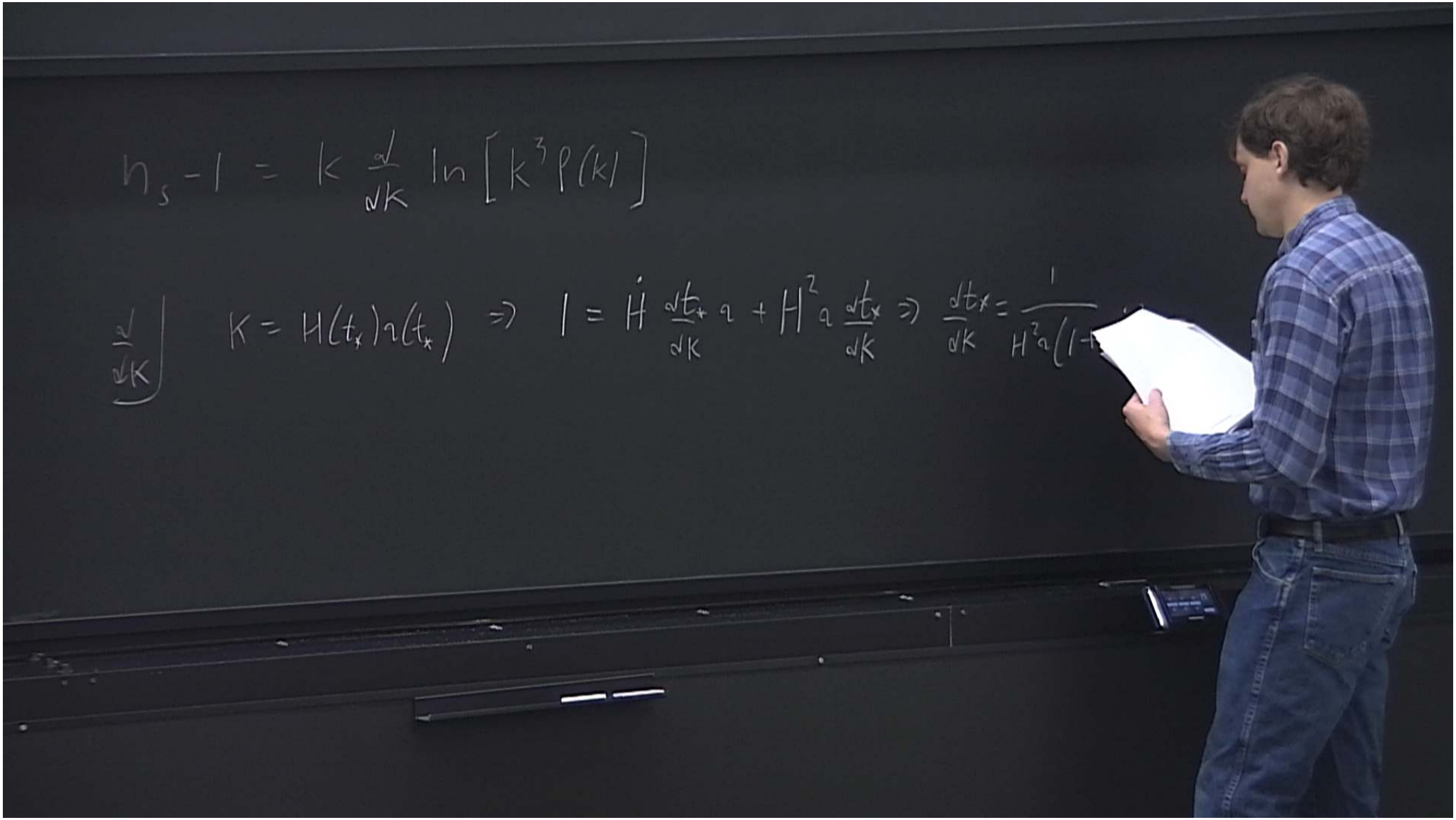

Co-moving gauge (ϑ-gauge)
ϵ^s s.t. $\chi=0$
ϵ_0 s.t. $\varrho=0$
$S_s[\vartheta, \Phi, B, \chi=0, \varrho=0] = S_s^{g.f.}[\vartheta, \Phi, B] = S_\vartheta[\vartheta]$
Φ, B are auxiliary
$\frac{\delta S^{g.f.}}{\delta \Phi} = 0$, $\frac{\delta S^{g.f.}}{\delta B} = 0 \Rightarrow$ Solution $\bar{\Phi} = \Phi(\vartheta, \dot{\vartheta})$
$\bar{B} = B(\vartheta, \dot{\vartheta})$

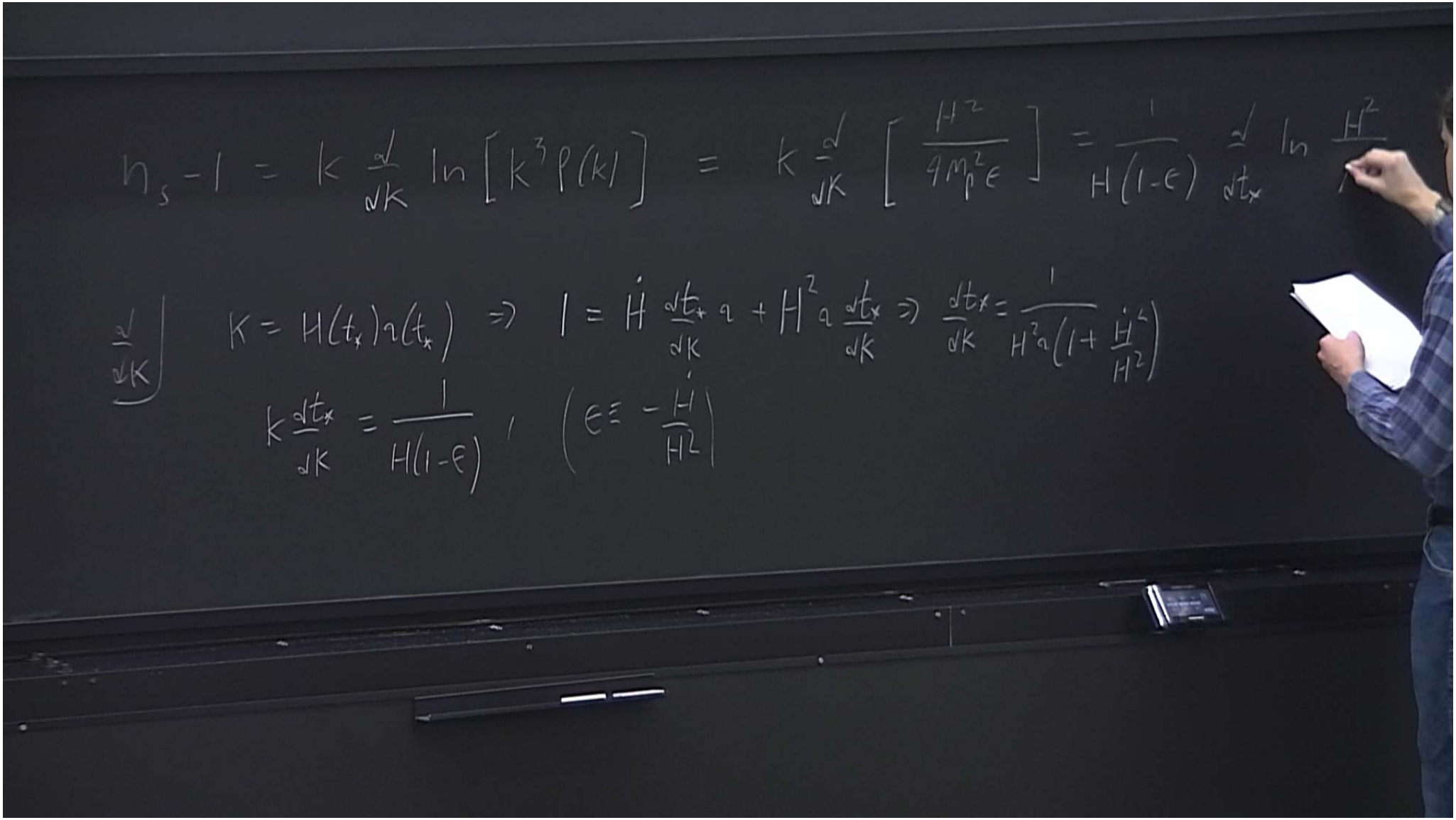

$S_\varphi = M_P^2 \int d^4x \; \epsilon\, a^3 \left[\dot\varphi^2 + \frac{1}{a^2}\varphi\vec\nabla\varphi\right]$
$\epsilon = \frac{\dot\chi^2}{2H^2}$


Scalars
conformal time $d\tau = \frac{1}{a}\,dt$
mukhanov variable $v = z\varphi$, $z^2 = 2a^2 M_p^2 \epsilon$
$S = \int d^3\vec{x}\, d\tau\, \frac{1}{2}\left[v'^2 + v\vec{\nabla}^2 v + \frac{z''}{z} v^2\right]$
$f_k'' + \left(k^2 - \frac{z''}{z}\right) f_k = 0$
$f_k \xrightarrow[\tau \to -\infty]{} \frac{1}{\sqrt{2k}} e^{-ik\tau}$


Scalars
Conformal time $d\tau = \frac{1}{a}\,dt$
Mukhanov variable $v = z\,\varphi$, $z^2 = 2a^2 M_p^2 \epsilon$
$S = \int d^3\vec{x}\,d\tau\ \frac{1}{2}\left[v'^2 + v\vec{\nabla}^2 v + \frac{z''}{z} v^2\right]$
$f_k'' + \left(k^2 - \frac{z''}{z}\right) f_k = 0$
$f_k \xrightarrow[\tau \to -\infty]{} \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$P_\varphi(k) = \frac{1}{z^2} |f_k|^2$
Slow-roll


$f_k'' + \left(k^2 - \frac{z''}{z}\right) f_k = 0$
$f_k \xrightarrow[\tau \to -\infty]{} \frac{1}{\sqrt{2k}} e^{-ik\tau}$
$P_\gamma(k) = \frac{1}{z^2} |f_k|^2$
Slow-roll $\Rightarrow$ approx dS
$|f_k|^2 \approx \frac{1}{2k^3\tau^2}$, $a^2 = \frac{1}{H^2\tau^2}$
$P_\gamma(k) = \frac{1}{4k^3} \frac{H^2}{M_p^2 \epsilon}$


$n_s - 1 = k \frac{d}{dk} \ln\left[k^3 P(k)\right]$
$\frac{d}{dk}$
$k = H(t_*) a(t_*) \Rightarrow 1 = \dot{H} \frac{dt_*}{dk} a + H^2 a \frac{dt_*}{dk} \Rightarrow \frac{dt_*}{dk} = \frac{1}{H^2 a (1-$



$$n_s - 1 = k \frac{d}{dk} \ln\left[k^3 P(k)\right] = k \frac{d}{dk}\left[\frac{H^2}{4 m_p^2 \epsilon}\right] = \frac{1}{H(1-\epsilon)} \frac{d}{dt_*} \ln \frac{H^2}{\ldots}$$

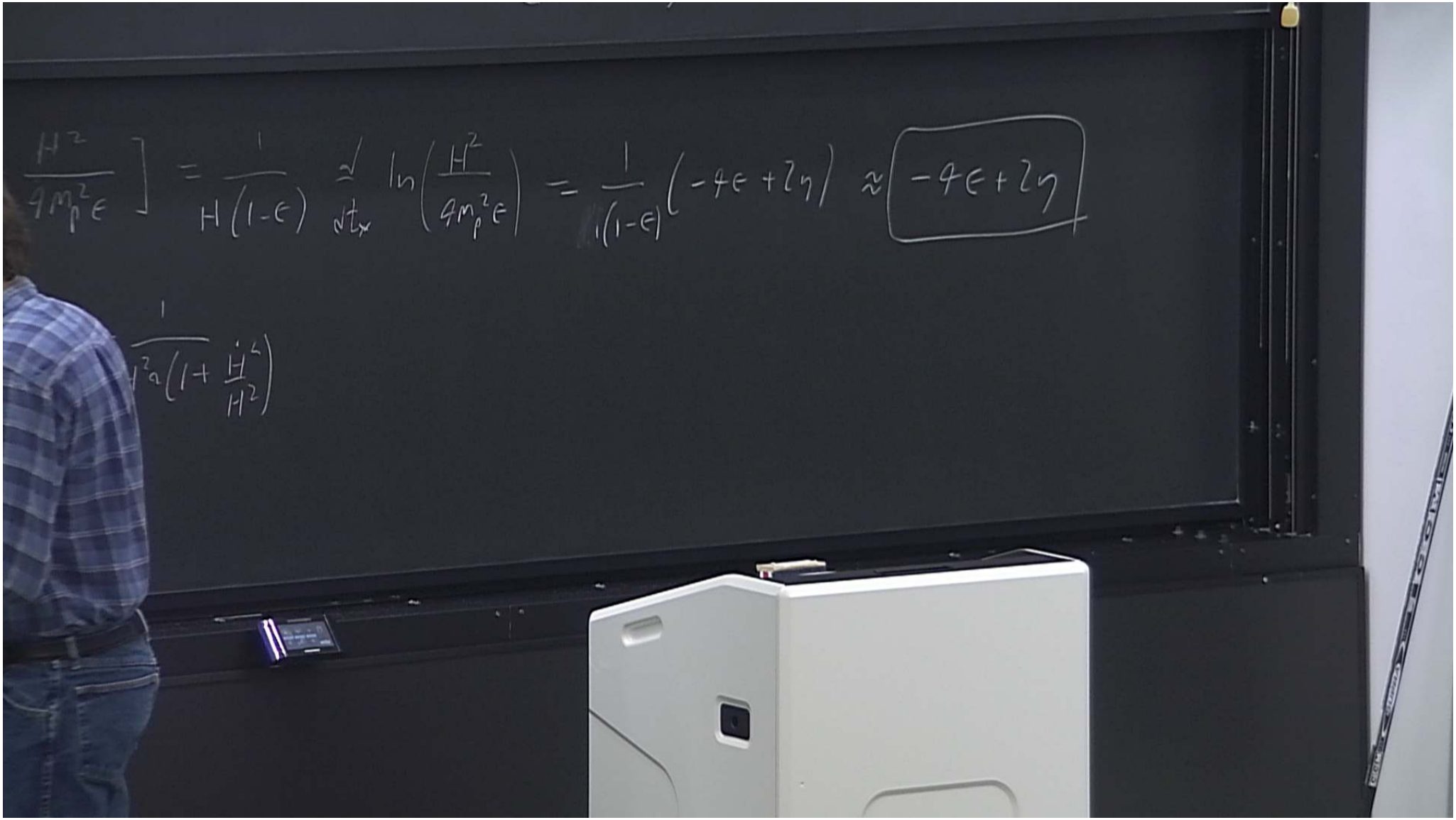
$$\frac{d}{dk}\Big) \quad k = H(t_*)\, a(t_*) \Rightarrow 1 = \dot{H} \frac{dt_*}{dk} a + H^2 a \frac{dt_*}{dk} \Rightarrow \frac{dt_*}{dk} = \frac{1}{H^2 a\left(1 + \frac{\dot{H}^2}{H^2}\right)}$$

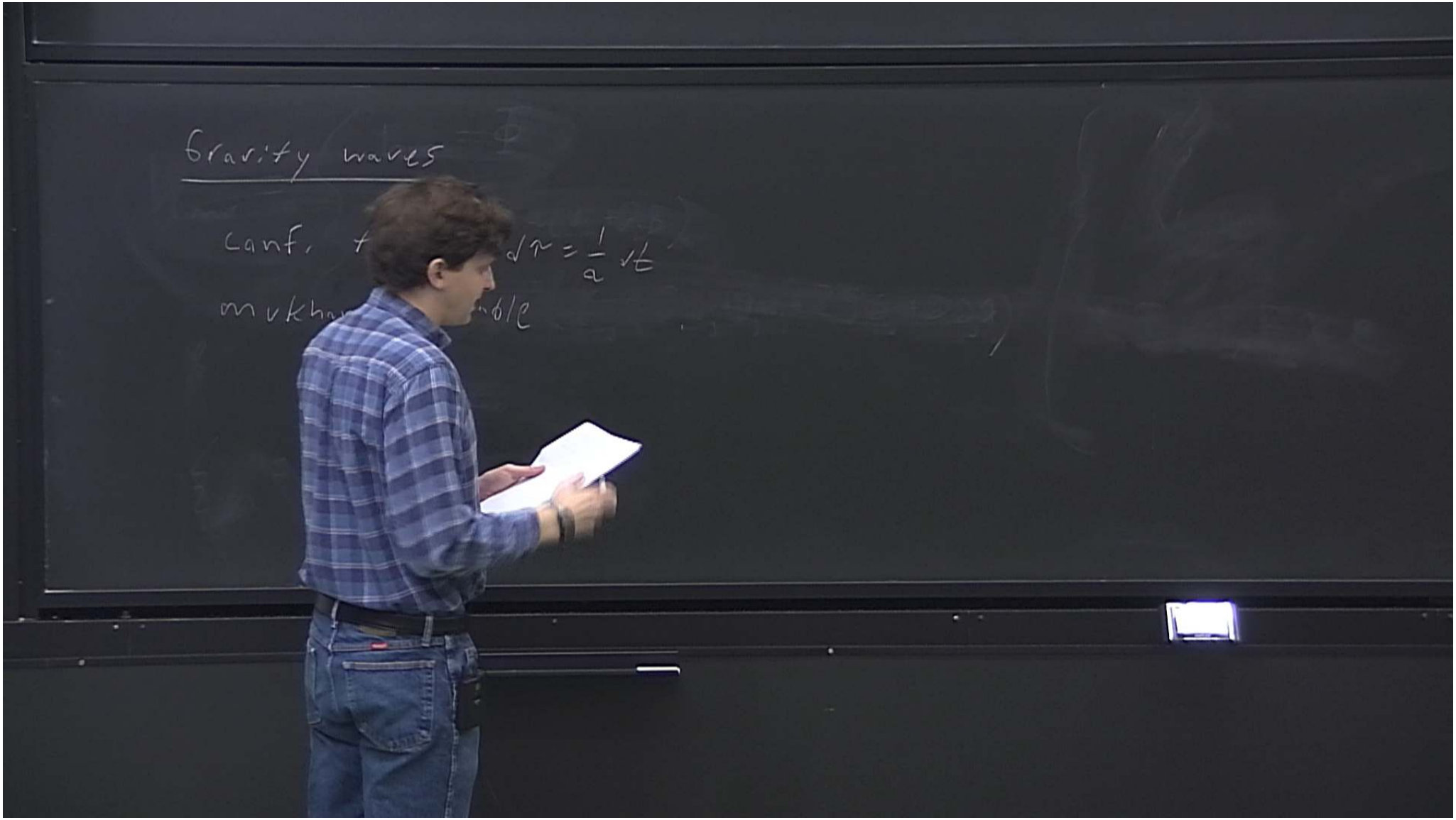
$$k \frac{dt_*}{dk} = \frac{1}{H(1-\epsilon)}, \quad \left(\epsilon \equiv -\frac{\dot{H}}{H^2}\right)$$

$$n_s - 1 = k \frac{d}{dk} \ln\left[k^3 P(k)\right] = k \frac{d}{dk}\left[\frac{H^2}{4m_p^2 \epsilon}\right] = \frac{1}{H(1-\epsilon)} \frac{d}{dt_*} \ln\left(\frac{H^2}{4m_p^2 \epsilon}\right)$$

$$\frac{d}{dk}\Big| \quad k = H(t_*)\, a(t_*) \Rightarrow 1 = \dot{H} \frac{dt_*}{dk} a + H^2 a \frac{dt_*}{dk} \Rightarrow \frac{dt_*}{dk} = \frac{1}{H^2 a\left(1 + \frac{\dot{H}}{H^2}\right)}$$

$$k \frac{dt_*}{dk} = \frac{1}{H(1-\epsilon)}, \quad \left(\epsilon \equiv -\frac{\dot{H}}{H^2}\right)$$


$\frac{H^2}{4m_p^2\epsilon}\Big] = \frac{1}{H(1-\epsilon)} \simeq \ln\left(\frac{H^2}{4m_p^2\epsilon}\right) = \frac{1}{(1-\epsilon)}(-4\epsilon+2\eta) \approx -4\epsilon+2\eta$
$\frac{1}{H^2a\left(1+\frac{\dot{H}}{H^2}\right)}$

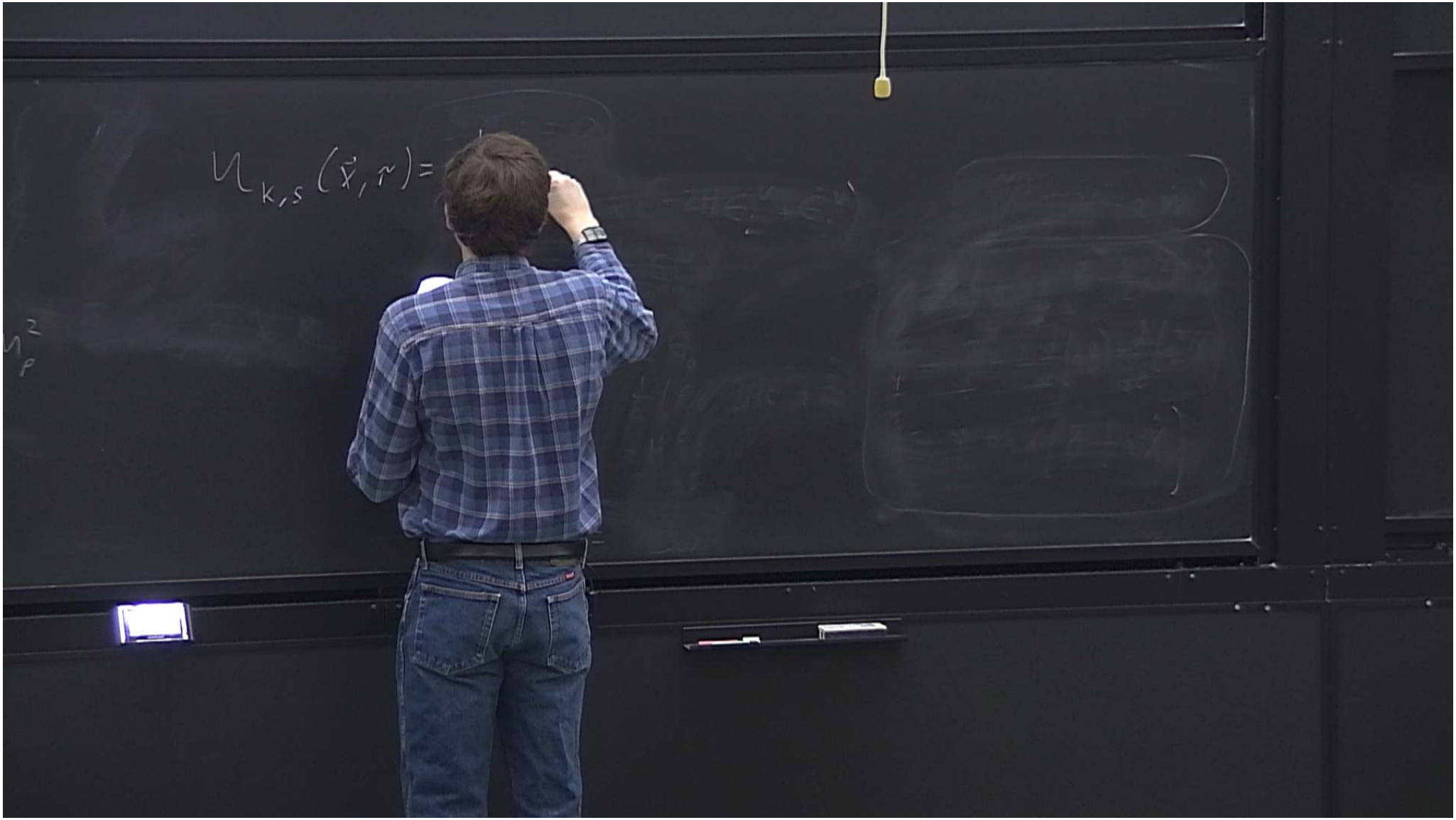

gravity waves

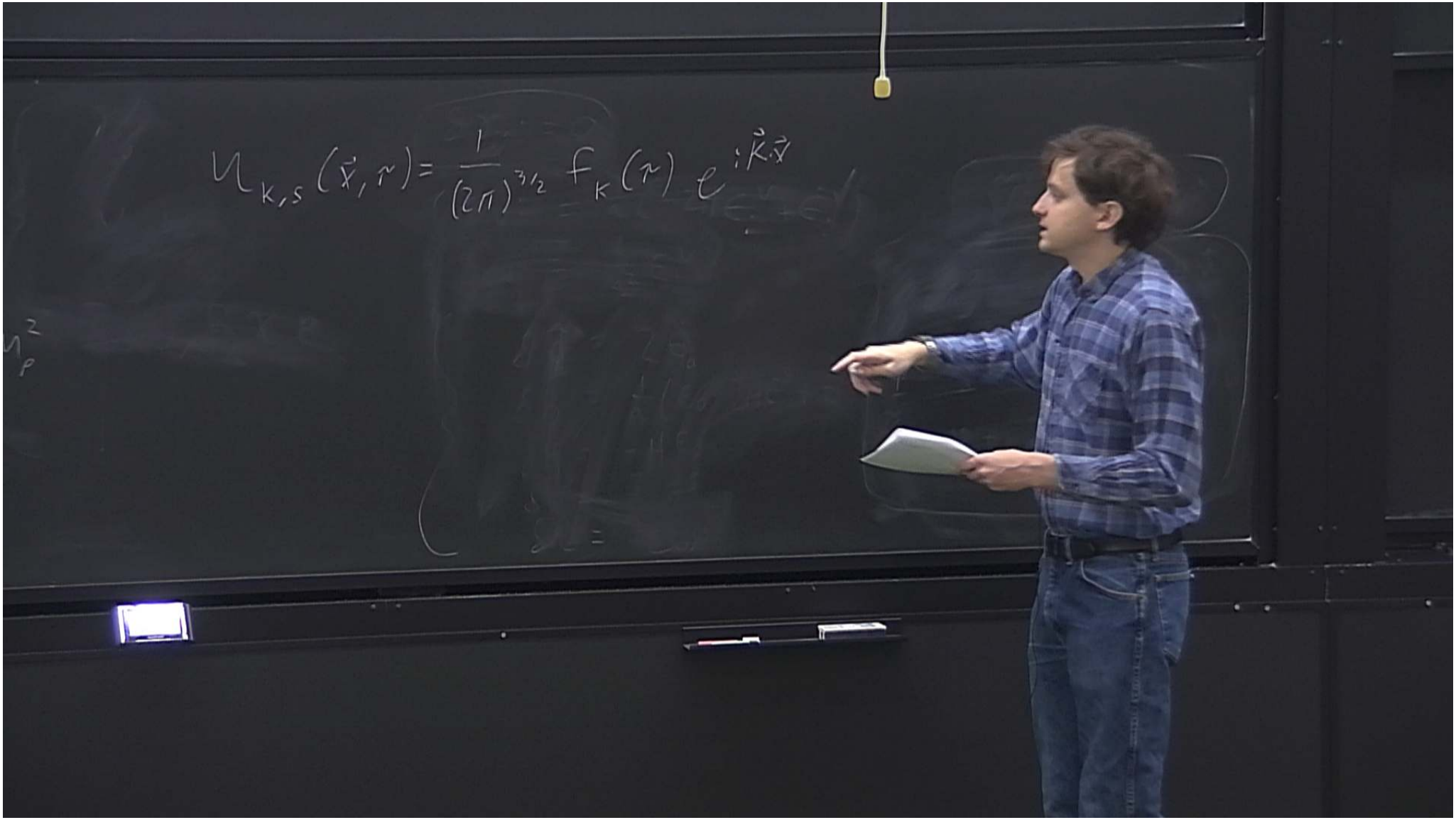
Gravity waves

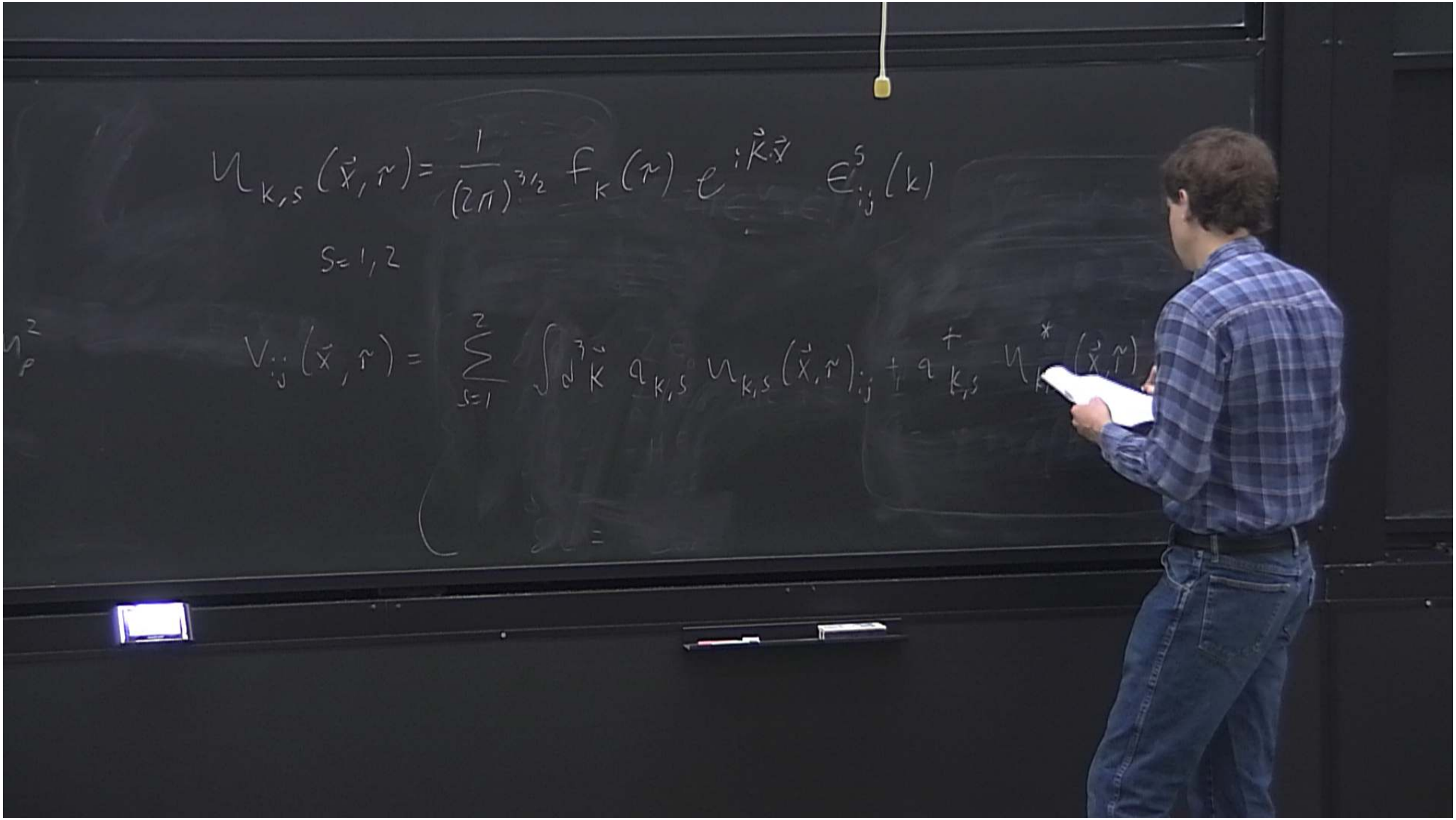
conf. time $d\tau = \frac{1}{a} dt$

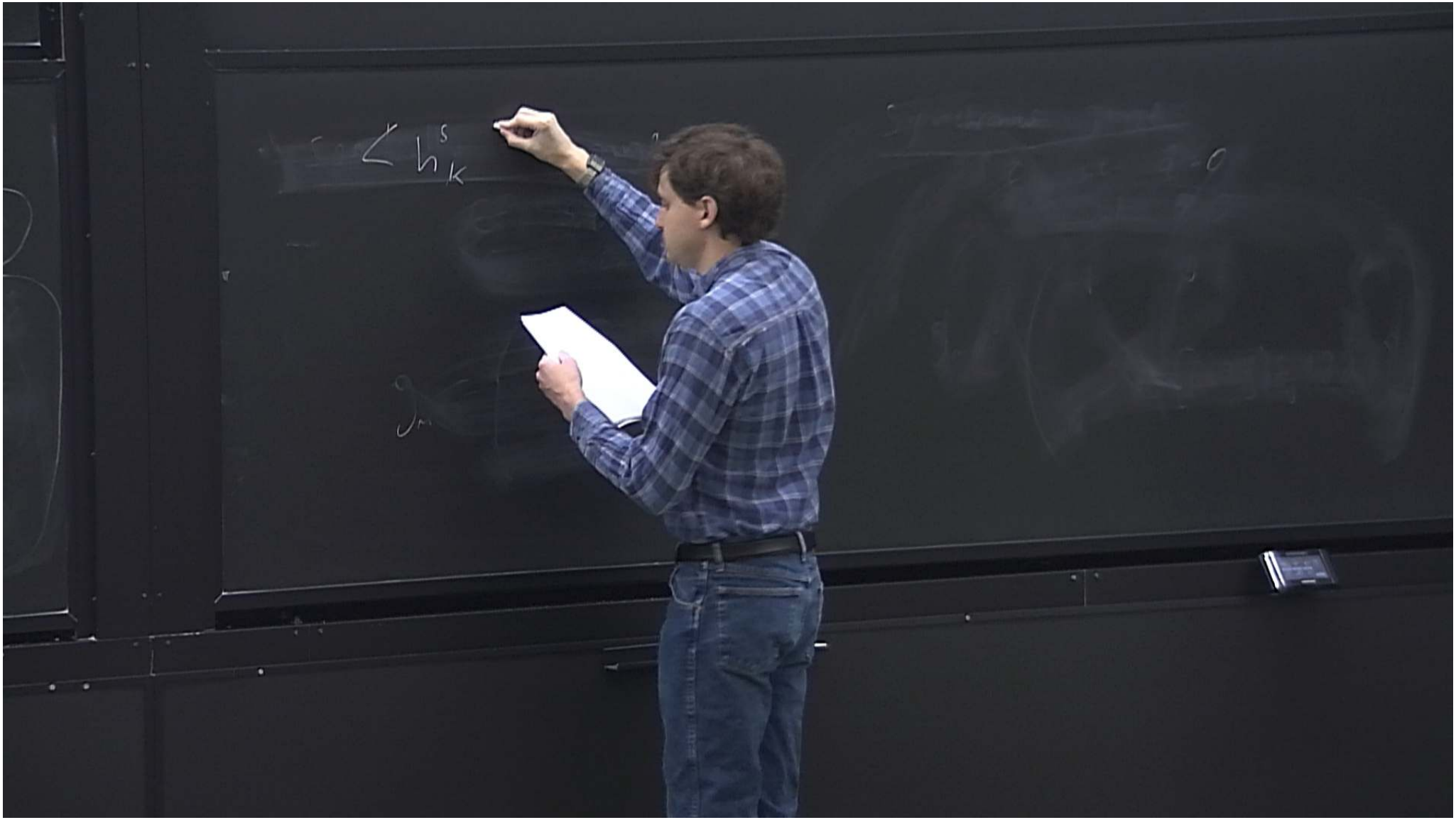
mukhanov variable $V_{ij} = z h_{ij}$, $z^2 = \frac{1}{2} a^2 M_p^2$

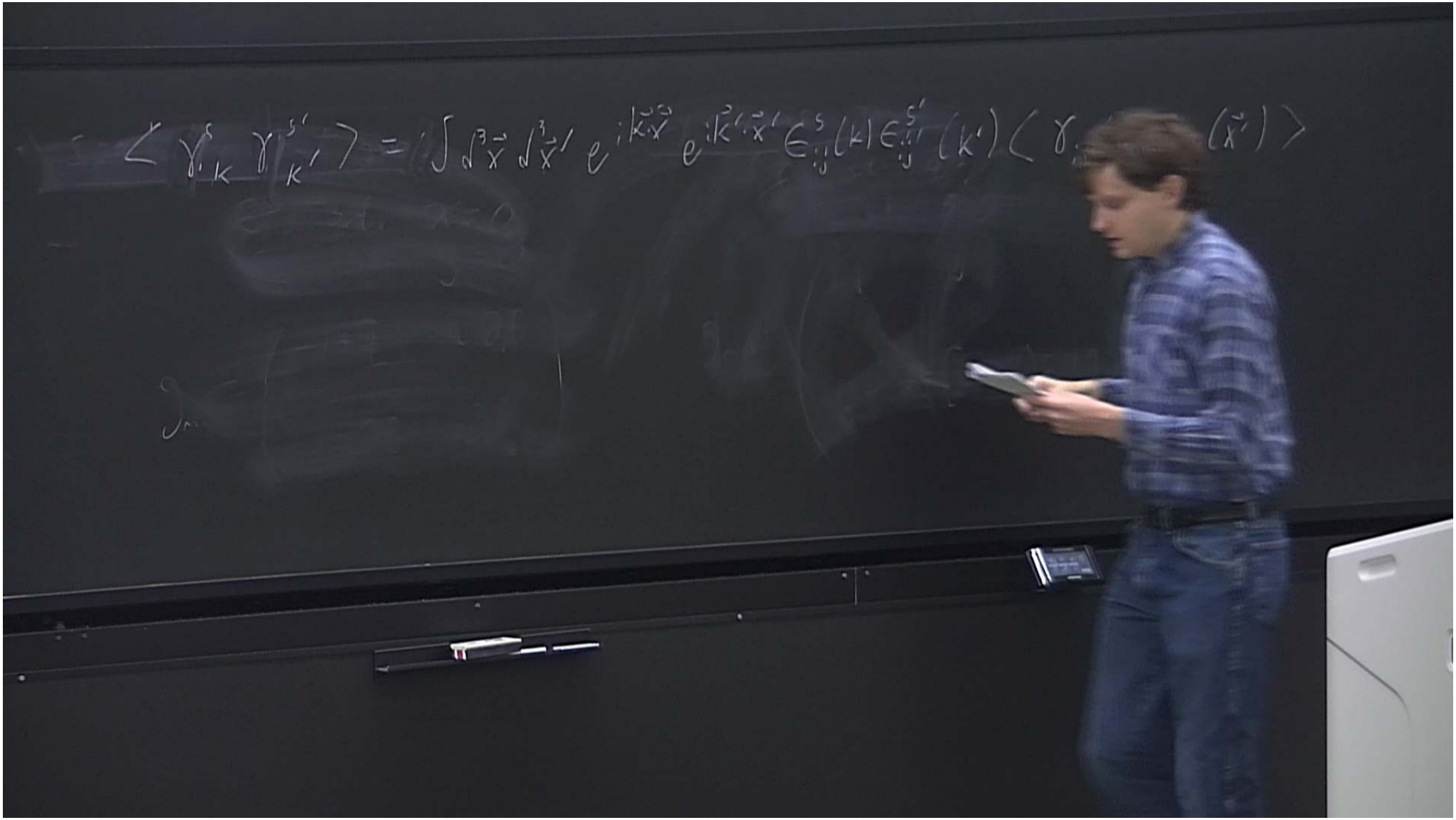
$$S_\gamma = \int d^3x \, d\tau \, \frac{1}{2} \left[V_{ij}'^2 + V_{ij} \vec{\nabla}^2 V^{ij} + \frac{z''}{z} V_{ij}^2 \right]$$

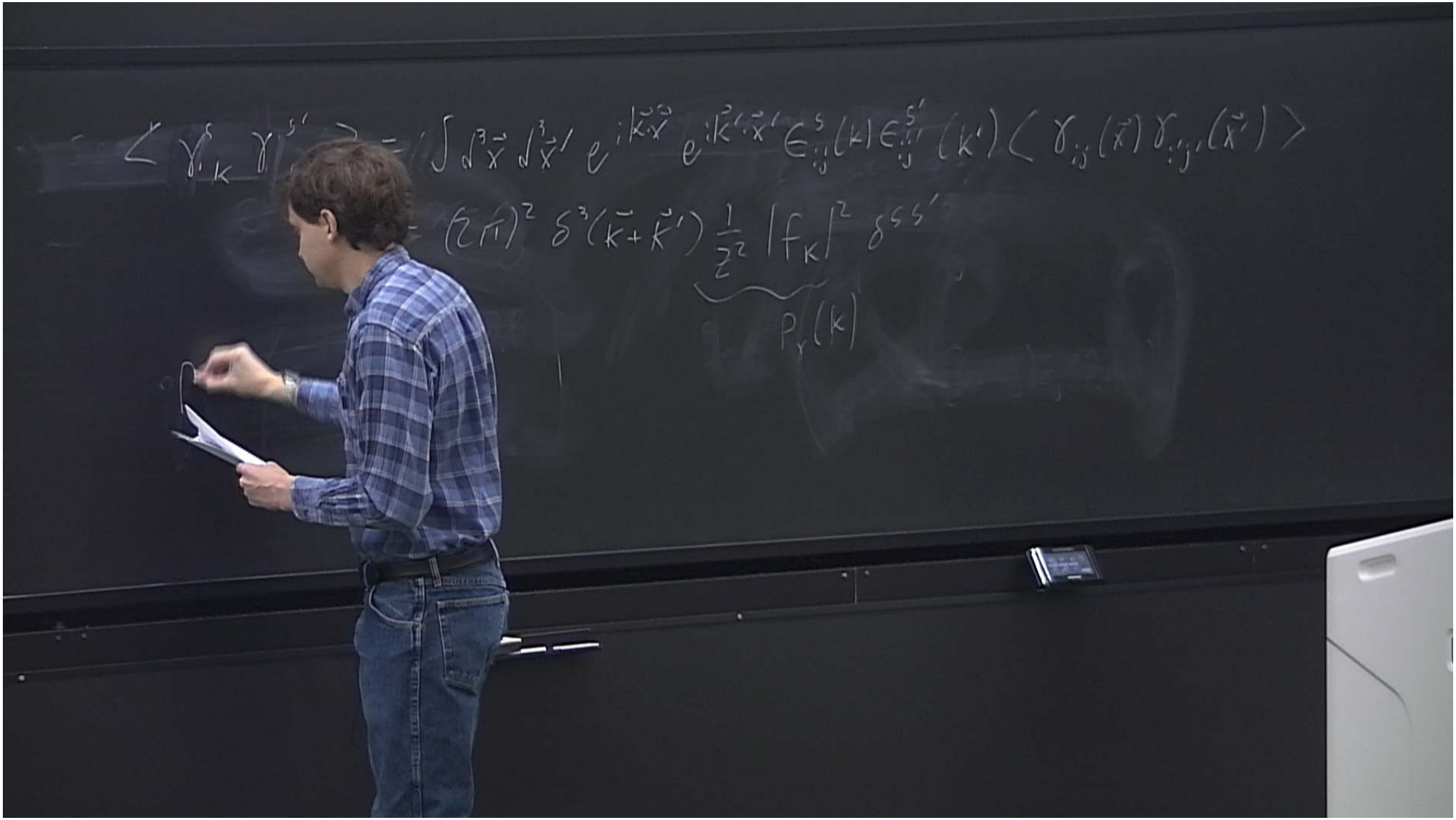

$u_{k,s}(\vec{x},\tau)=$

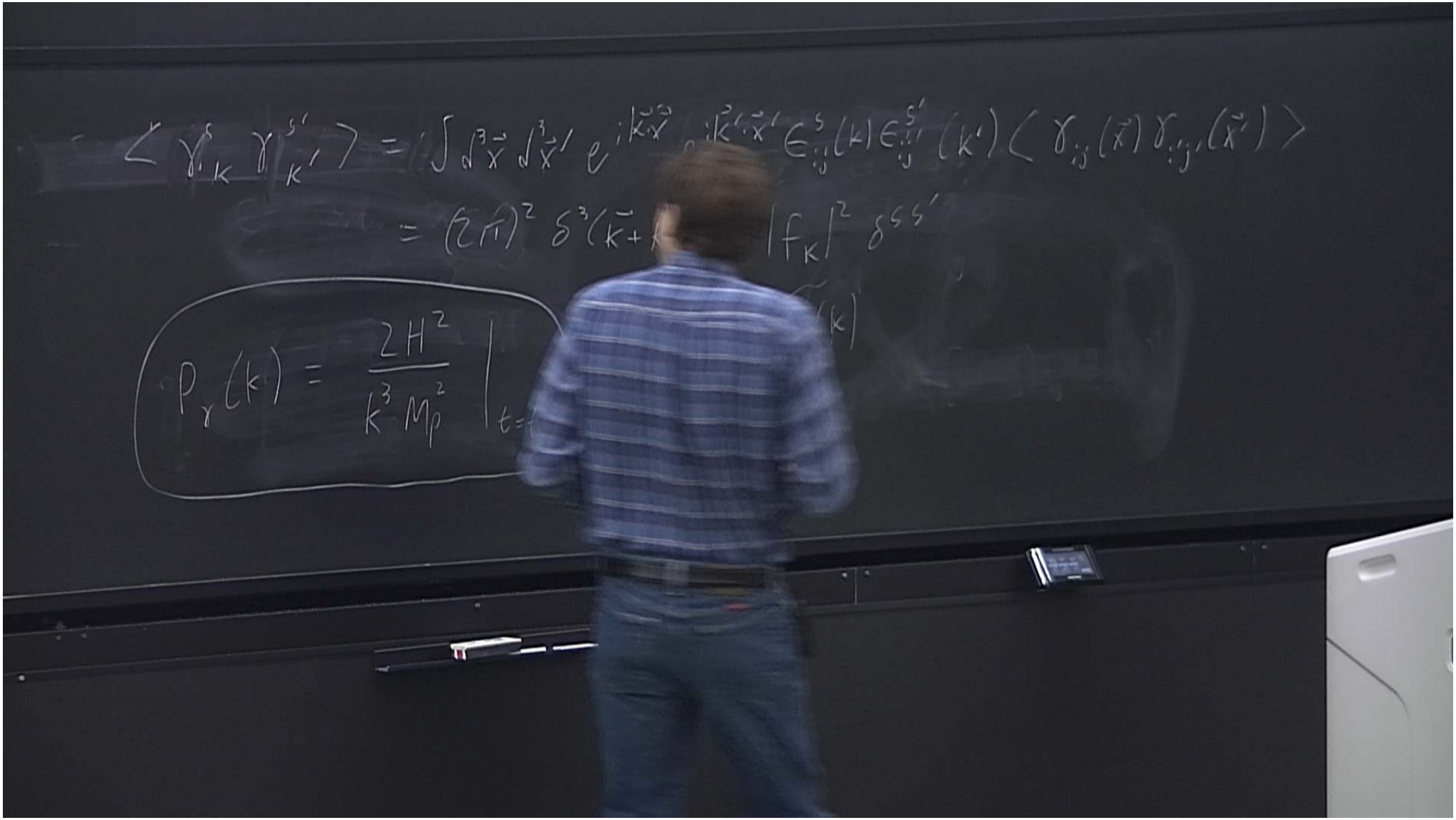

$u_{k,s}(\vec{x},\tau) = \frac{1}{(2\pi)^{3/2}} f_k(\tau)\, e^{i\vec{k}\cdot\vec{x}}$

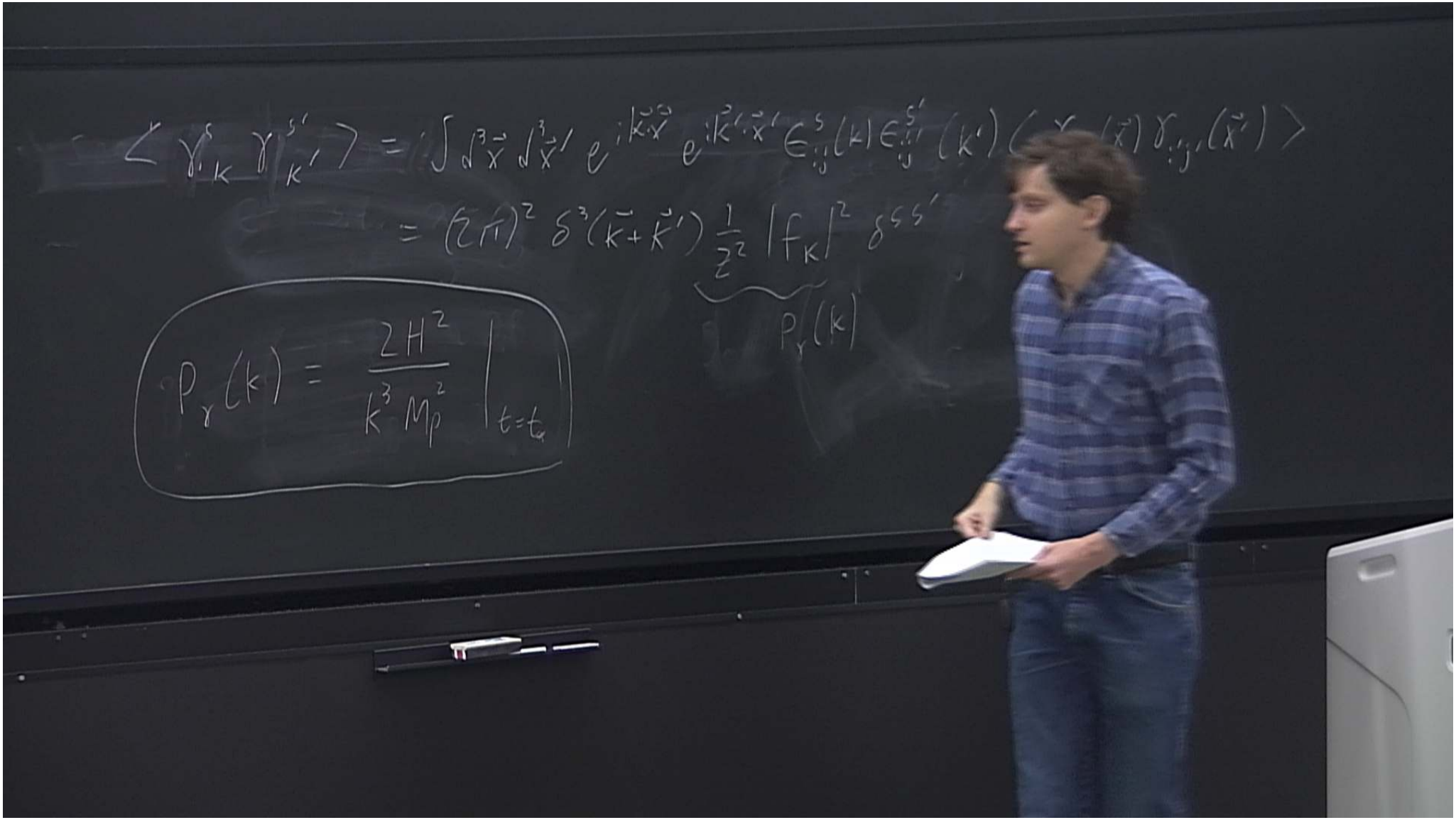

$u_{k,s}(\vec{x},\tau) = \frac{1}{(2\pi)^{3/2}} f_k(\tau)\, e^{i\vec{k}\cdot\vec{x}}\, \epsilon^{s}_{ij}(k)$
$s = 1, 2$
$V_{ij}(\vec{x},\tau) = \sum_{s=1}^{2} \int d^3\vec{k}\; q_{k,s}\, u_{k,s}(\vec{x},\tau)_{ij} + q^{\dagger}_{k,s}\, u^{*}_{k,s}(\vec{x},\tau)$

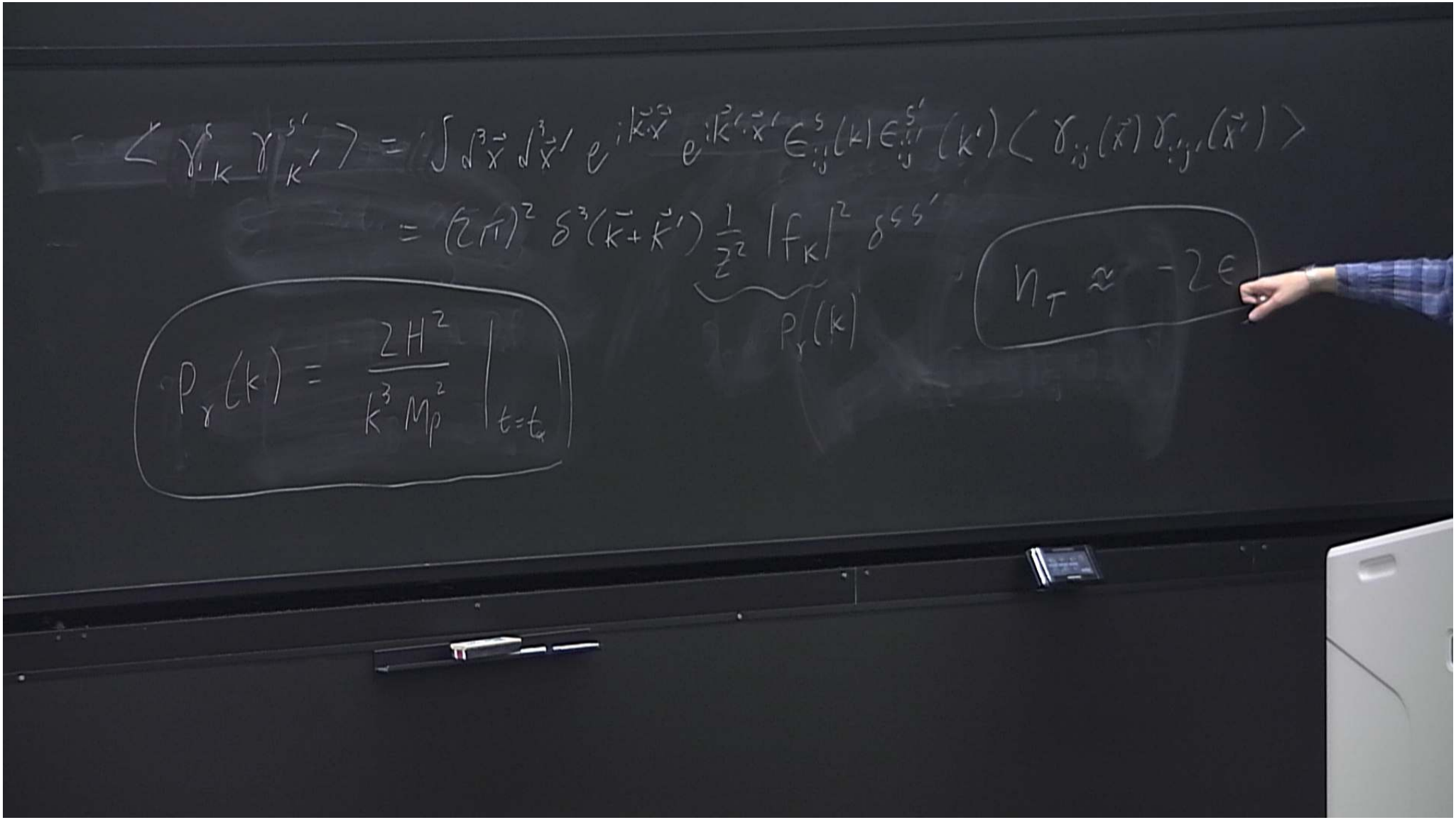


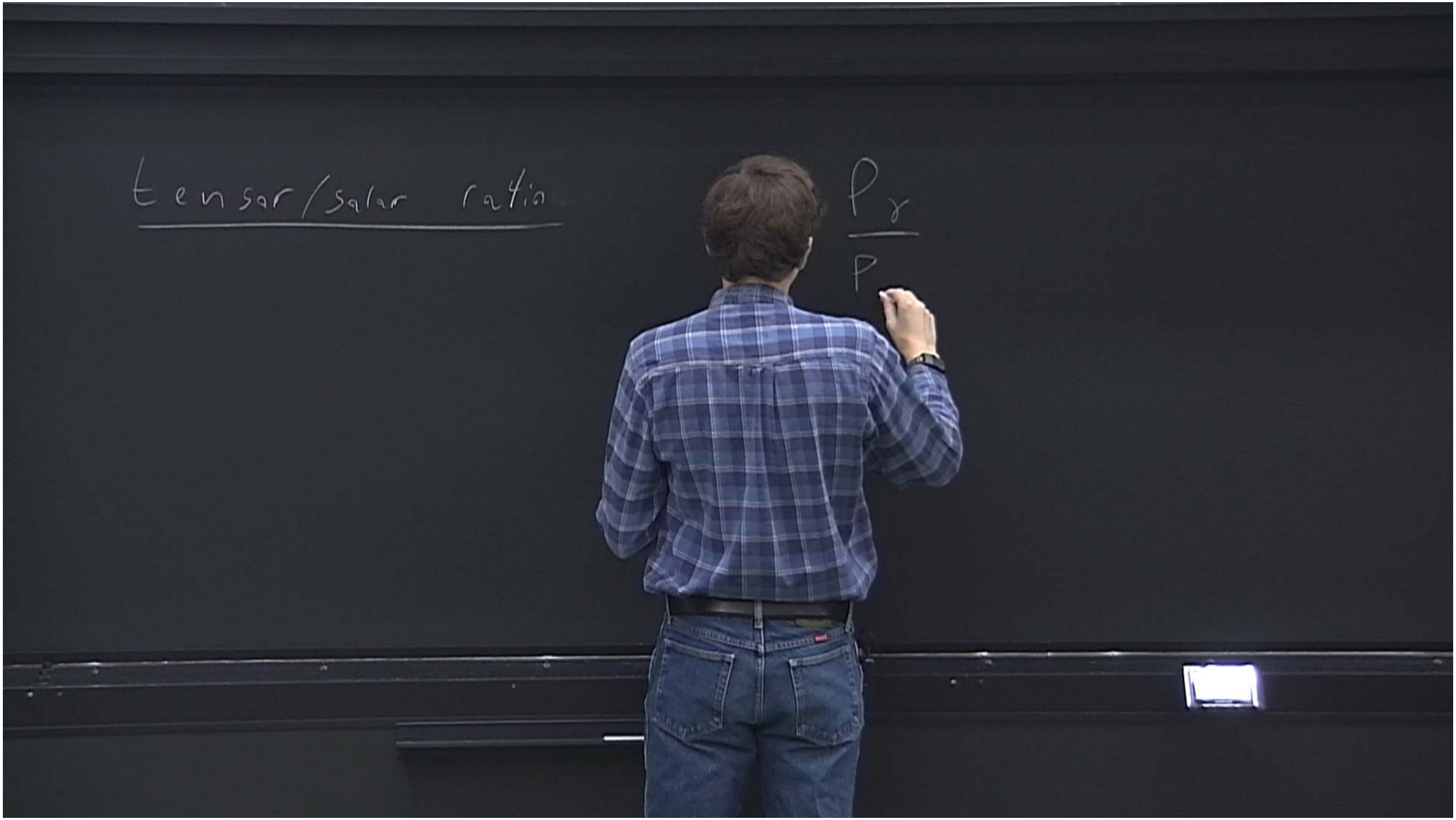


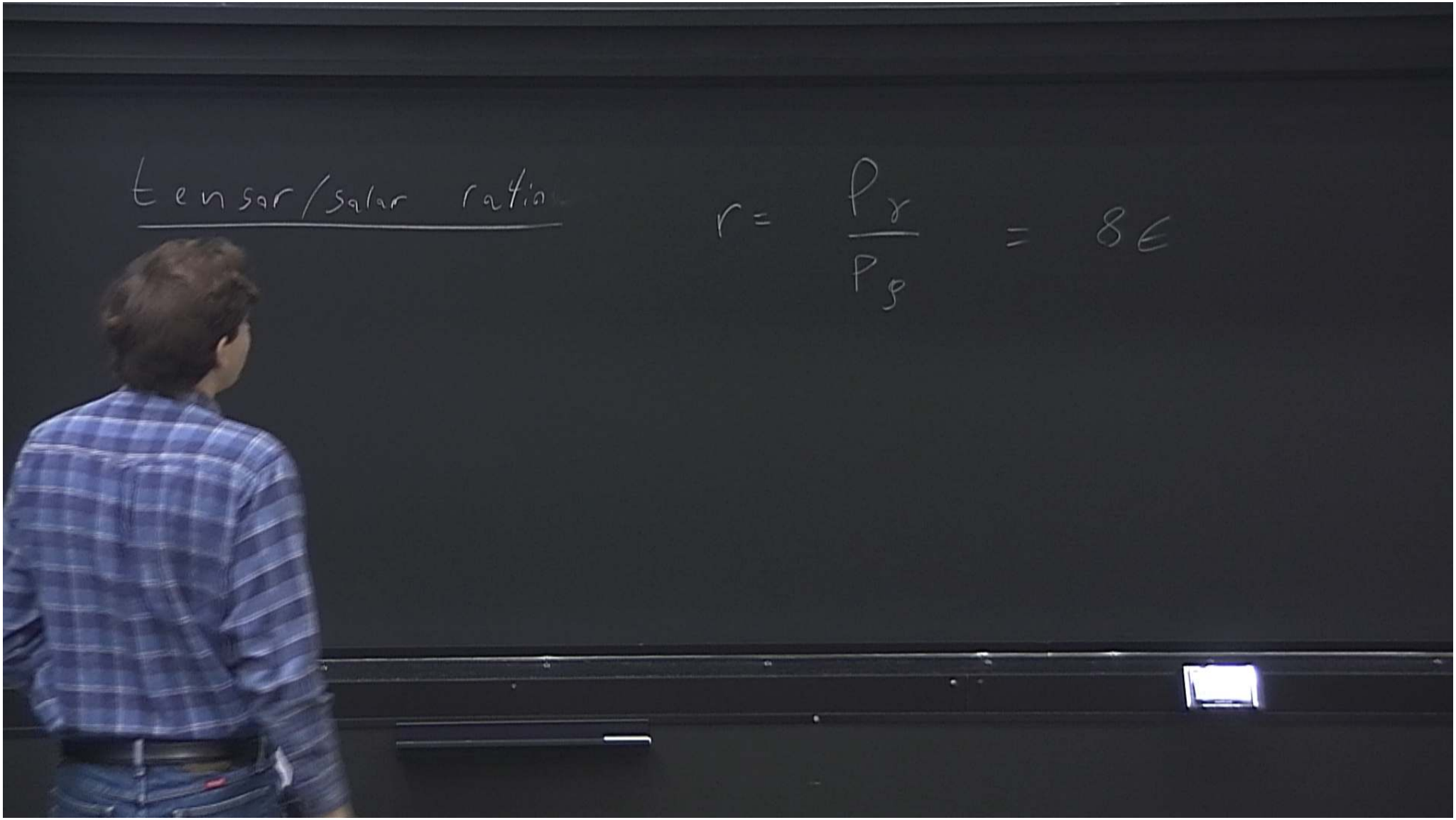

$\langle \gamma^{s}_{ij}(k)\, \gamma^{s'}_{ij}(k') \rangle = \int d^3\vec{x}\, d^3\vec{x}'\, e^{i\vec{k}\cdot\vec{x}}\, e^{i\vec{k}'\cdot\vec{x}'}\, \epsilon^{s}_{ij}(k)\, \epsilon^{s'}_{ij}(k')\, \langle \gamma_{ij}(\vec{x})\, \gamma_{ij}(\vec{x}') \rangle$
$= (2\pi)^2\, \delta^3(\vec{k}+\vec{k}')\, \frac{1}{2^2}\, |f_k|^2\, \delta^{ss'}$
$P_\gamma(k)$

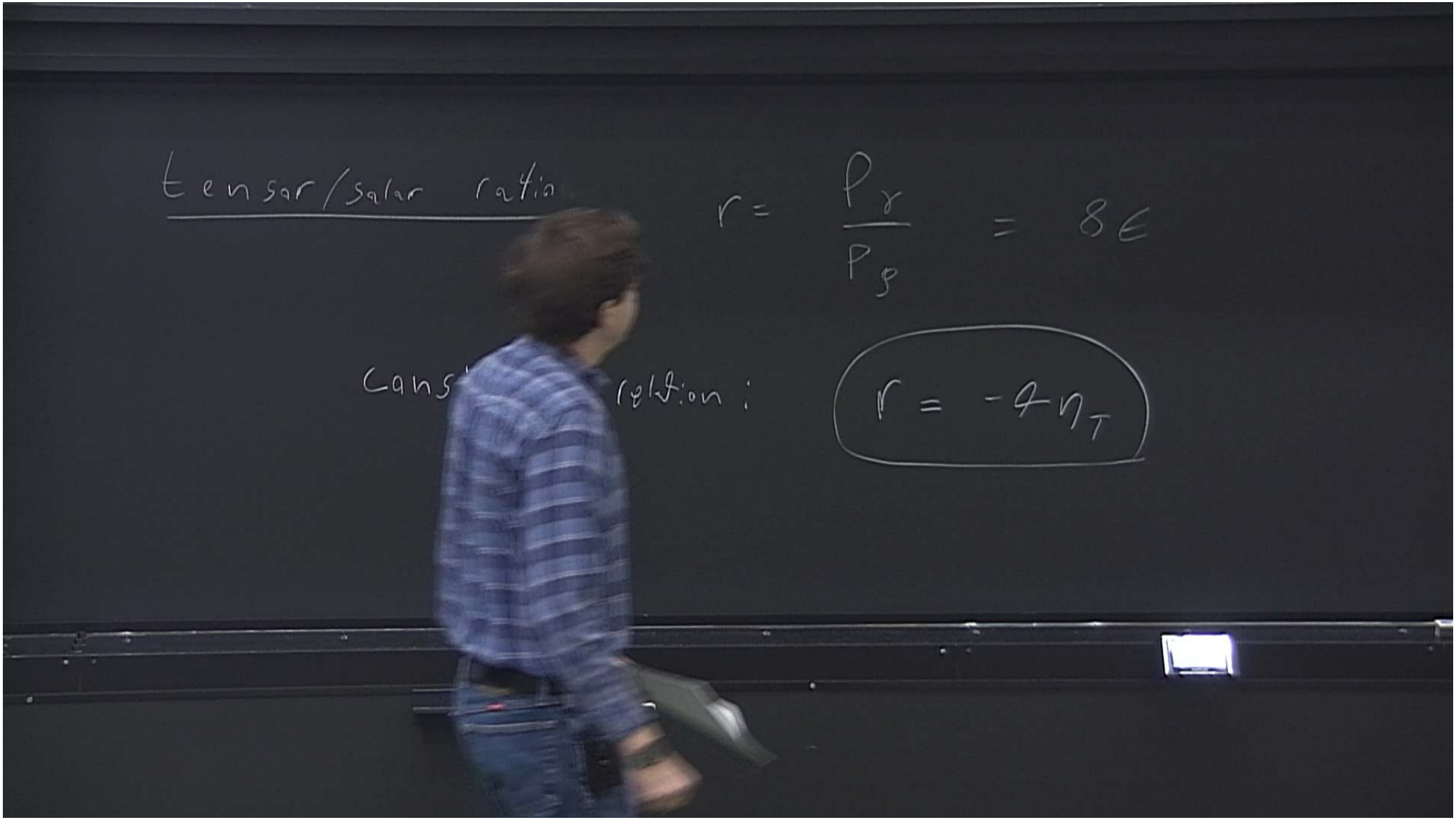

$P_\gamma(k) = \frac{2H^2}{k^3 M_p^2}\Big|_{t=}$

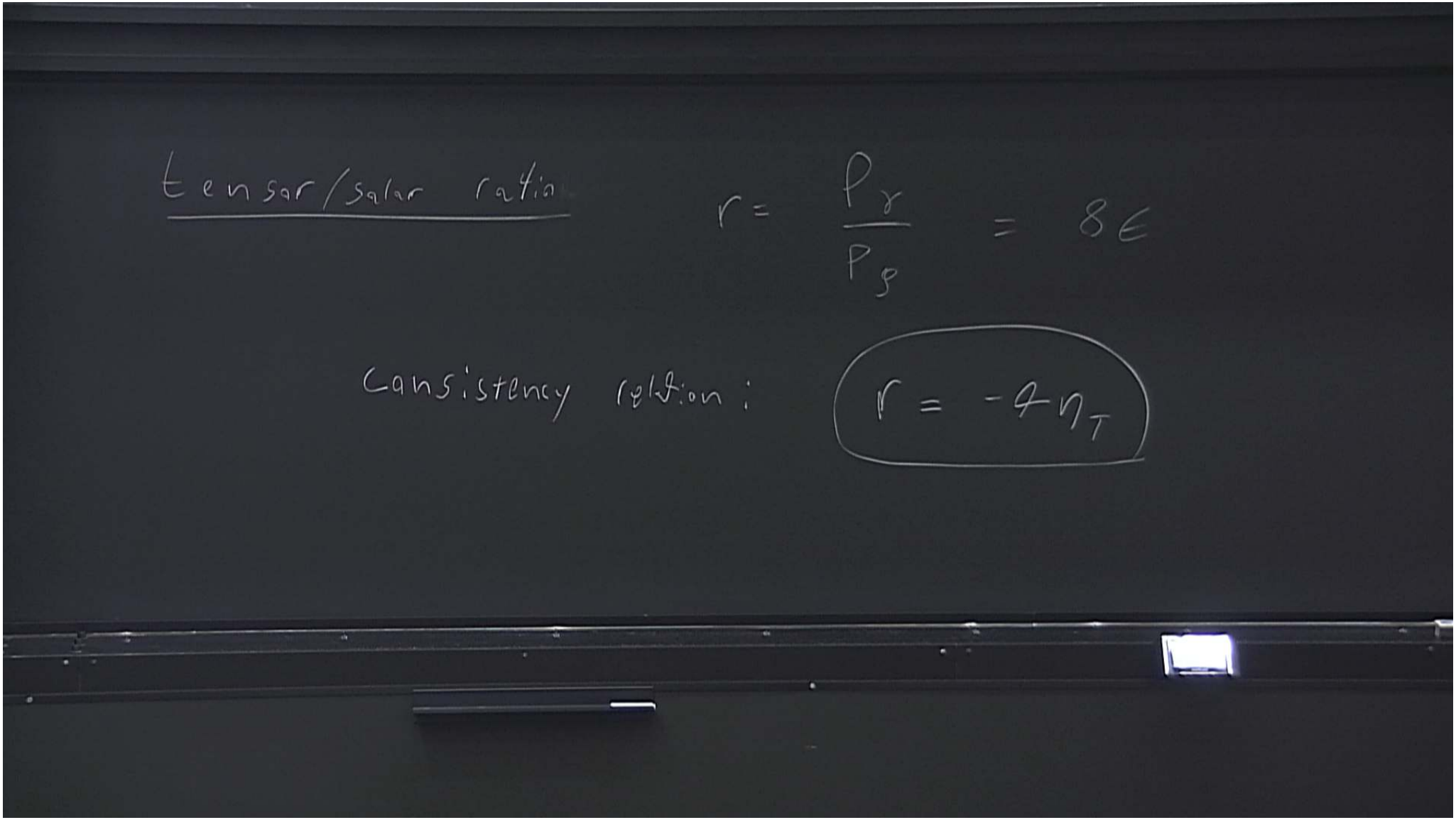

$\langle \gamma^{s}_{\vec{k}} \gamma^{s'}_{\vec{k}'} \rangle = \int d^3\vec{x}\, d^3\vec{x}'\, e^{i\vec{k}\cdot\vec{x}}\, e^{i\vec{k}'\cdot\vec{x}'}\, \epsilon^{s}_{ij}(k)\, \epsilon^{s'}_{ij}(k')\, \langle \gamma_{ij}(\vec{x})\, \gamma_{ij}(\vec{x}') \rangle$
$= (2\pi)^3\, \delta^3(\vec{k}+\vec{k}')\, \frac{1}{2^2}\, |f_k|^2\, \delta^{ss'}$
$P_\gamma(k)$
$P_\gamma(k) = \frac{2H^2}{k^3 M_p^2}\Big|_{t=t_*}$


$\langle \gamma^s_k \gamma^{s'}_{k'} \rangle = \int d^3\vec{x}\, d^3\vec{x}'\, e^{i\vec{k}\cdot\vec{x}}\, e^{i\vec{k}'\cdot\vec{x}'}\, \epsilon^s_{ij}(k)\, \epsilon^{s'}_{ij}(k')\, \langle \gamma_{ij}(\vec{x})\, \gamma_{ij}(\vec{x}') \rangle$
$= (2\pi)^3\, \delta^3(\vec{k}+\vec{k}')\, \frac{1}{2^2}\, |f_k|^2\, \delta^{ss'}$
$P_\gamma(k)$
$P_\gamma(k) = \frac{2H^2}{k^3 M_p^2}\Big|_{t=t_*}$
$n_T \approx -2\epsilon$


tensor/scalar ratio
$\frac{P_\gamma}{P}$


tensor/scalar ratio
$r = \frac{P_T}{P_S} = 8\epsilon$


tensor/scalar ratio
$r = \frac{P_T}{P_S} = 8\epsilon$
Cons relation:
$r = -8 n_T$


tensor/scalar ratio
$r = \frac{P_T}{P_S} = 8\epsilon$
consistency relation:
$r = -8 n_T$