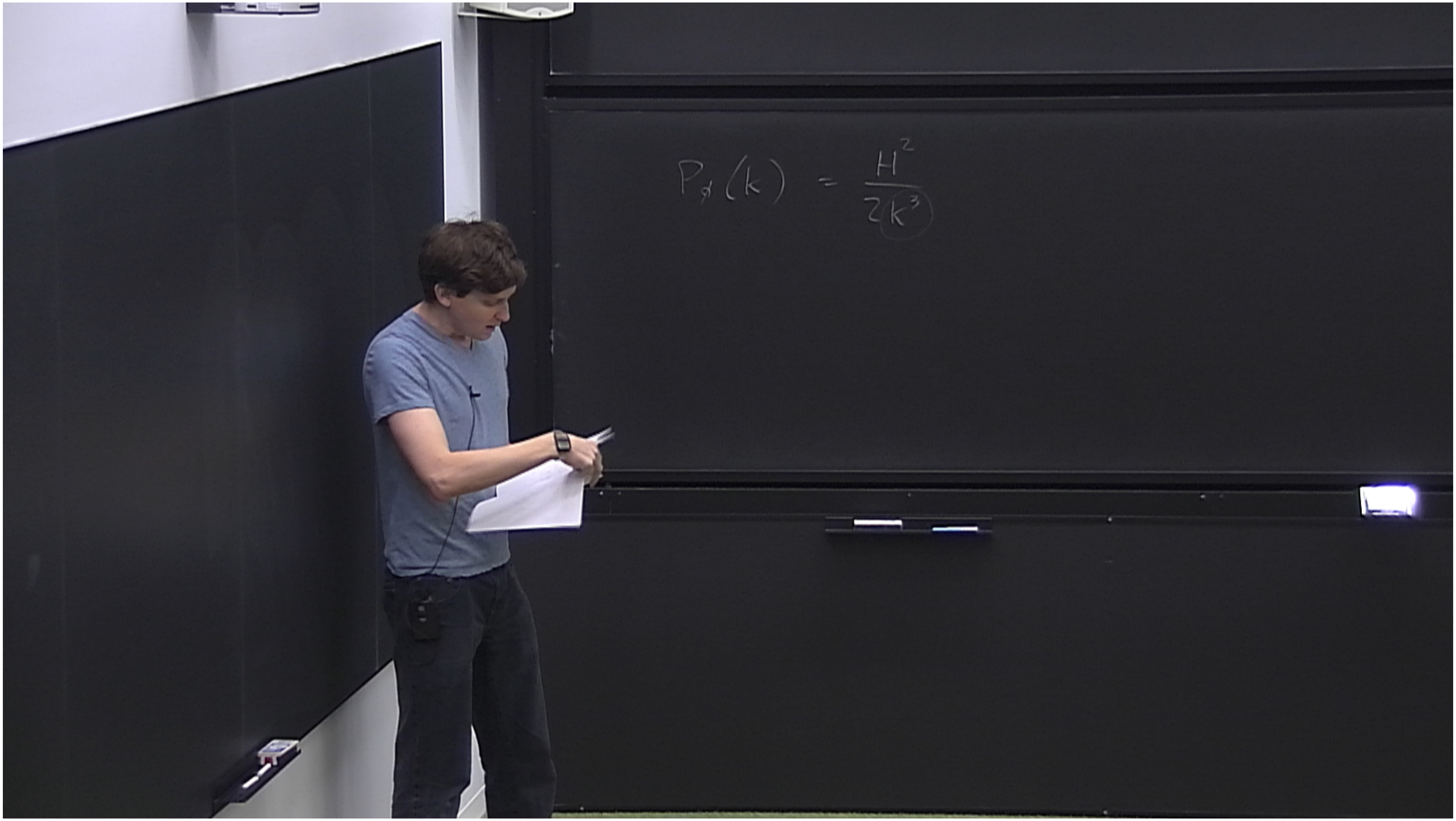
Title: Cosmology Review-13

Date: Feb 11, 2015 11:30 AM

URL: http://pirsa.org/15020036

Abstract:


$P_\phi(k) = \frac{H^2}{2k^3}$

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$$n_s - 1 \equiv k\frac{d}{dk}\ln\left[k^3 P(k)\right]$$

$$\begin{cases} n_s = 1 & \text{scale invariant} \\ n_s < 1 & \text{red tilted} \\ n_s > 1 & \text{blue tilted} \end{cases}$$

$P(k)]$

n_s const. $\quad P(k) \sim k^{-3+(n_s-1)}$

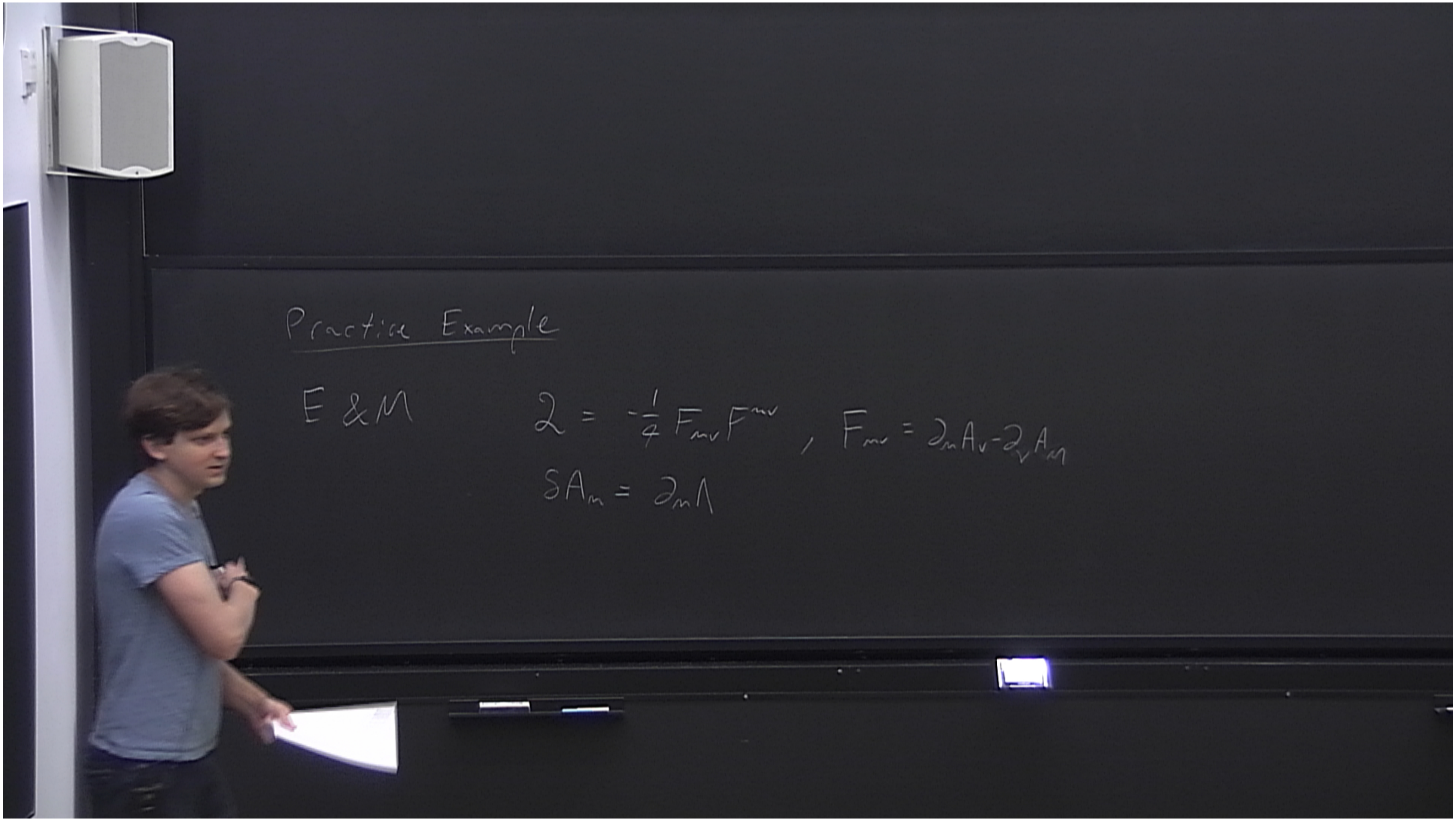
variant

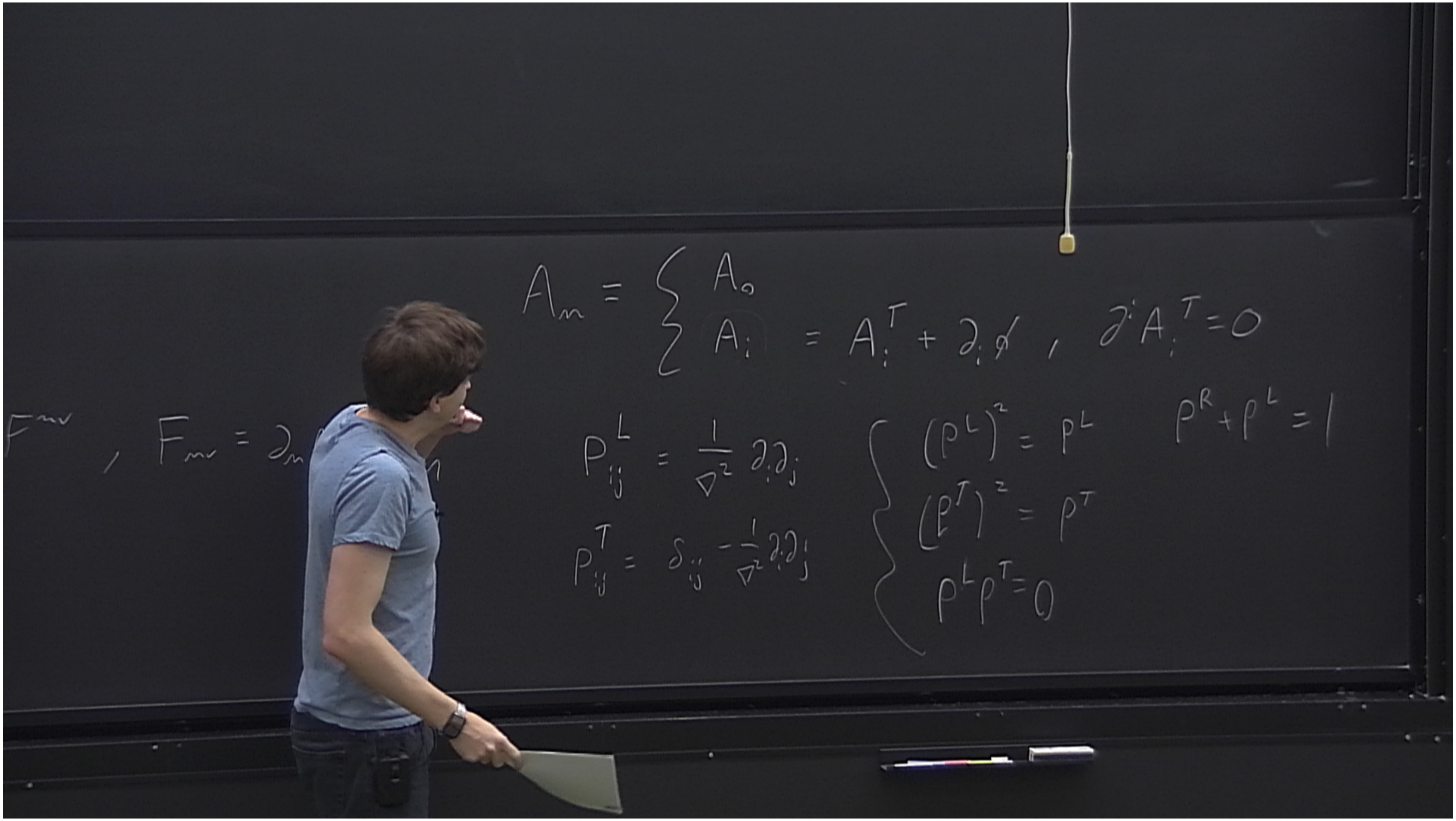
$n_s \neq$ const. $\quad$ running

ilted

Planck: $n_s = 0.96 \pm 0.01$

ted


Practice Example
E & M
$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$
$\delta A_\mu = \partial_\mu \Lambda$



$A_\mu = \begin{cases} A_0 \\ A_i = A_i^T + \partial_i \alpha, \quad \partial^i A_i^T = 0 \end{cases}$
$F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu$
$P_{ij}^L = \frac{1}{\nabla^2}\partial_i\partial_j$
$P_{ij}^T = \delta_{ij} - \frac{1}{\nabla^2}\partial_i\partial_j$
$(P^L)^2 = P^L$
$(P^T)^2 = P^T$
$P^L P^T = 0$
$P^R + P^L = 1$

$$\delta A_m = \partial_m \Lambda$$

$$P^T_{ij} = \delta_{ij}$$

$$\mathcal{L} = \frac{1}{2}\left(\dot{A}_i^{T\,2} + A_i^T \vec{\nabla}^2 A_i^T\right) - \frac{1}{2}\left(A_0 - \dot{\phi}\right)\vec{\nabla}^2\left(A_0 - \dot{\phi}\right)$$

$$\delta A_0 = \dot{\Lambda}$$

$$\delta A_i^T = 0$$

$$\delta A_i = \partial_i \Lambda \Rightarrow \delta\left(A_i^T + \partial_i \phi\right) = \partial_i \Lambda \Rightarrow$$

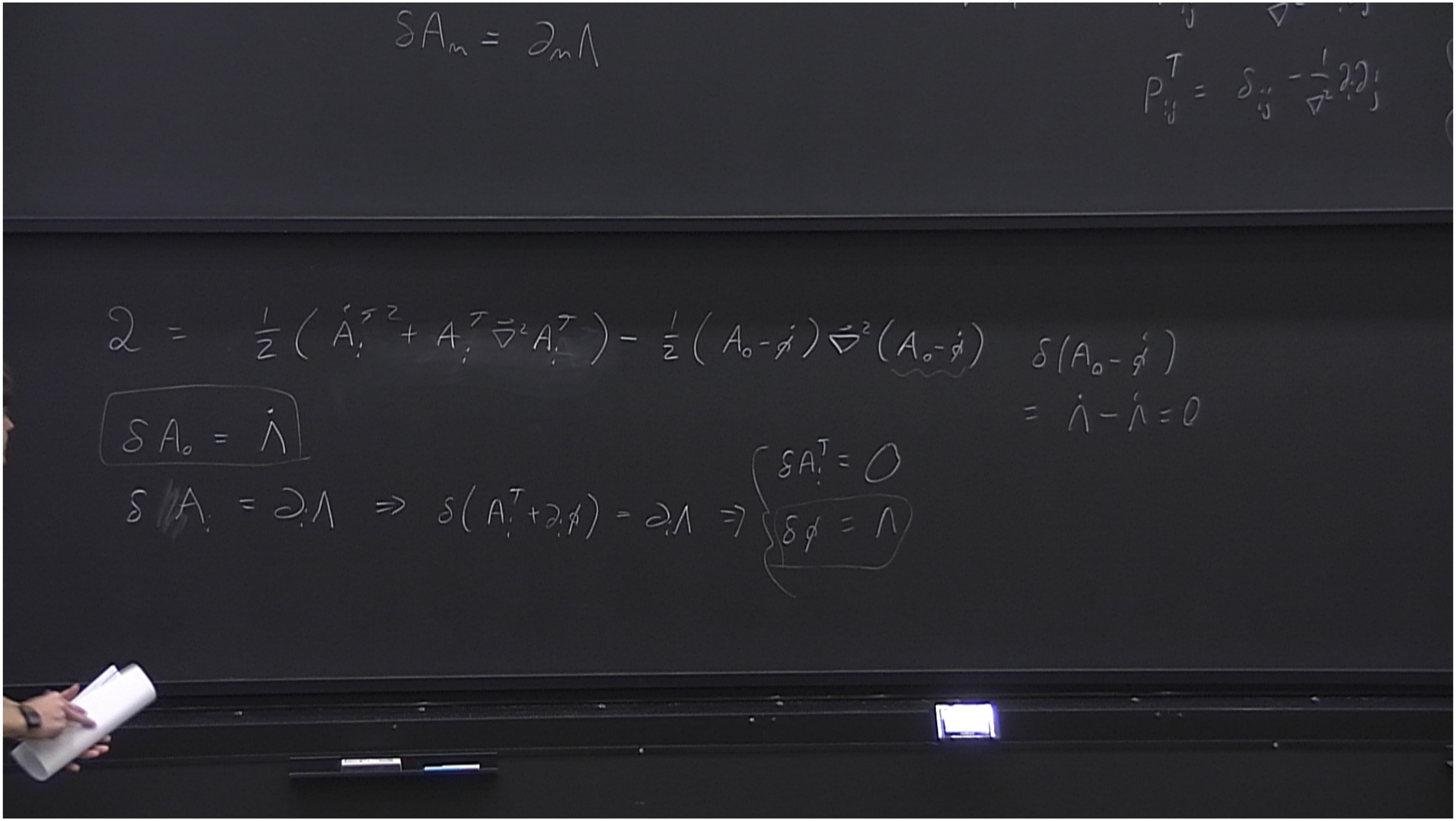
$$\delta A_\mu = \partial_\mu \Lambda$$

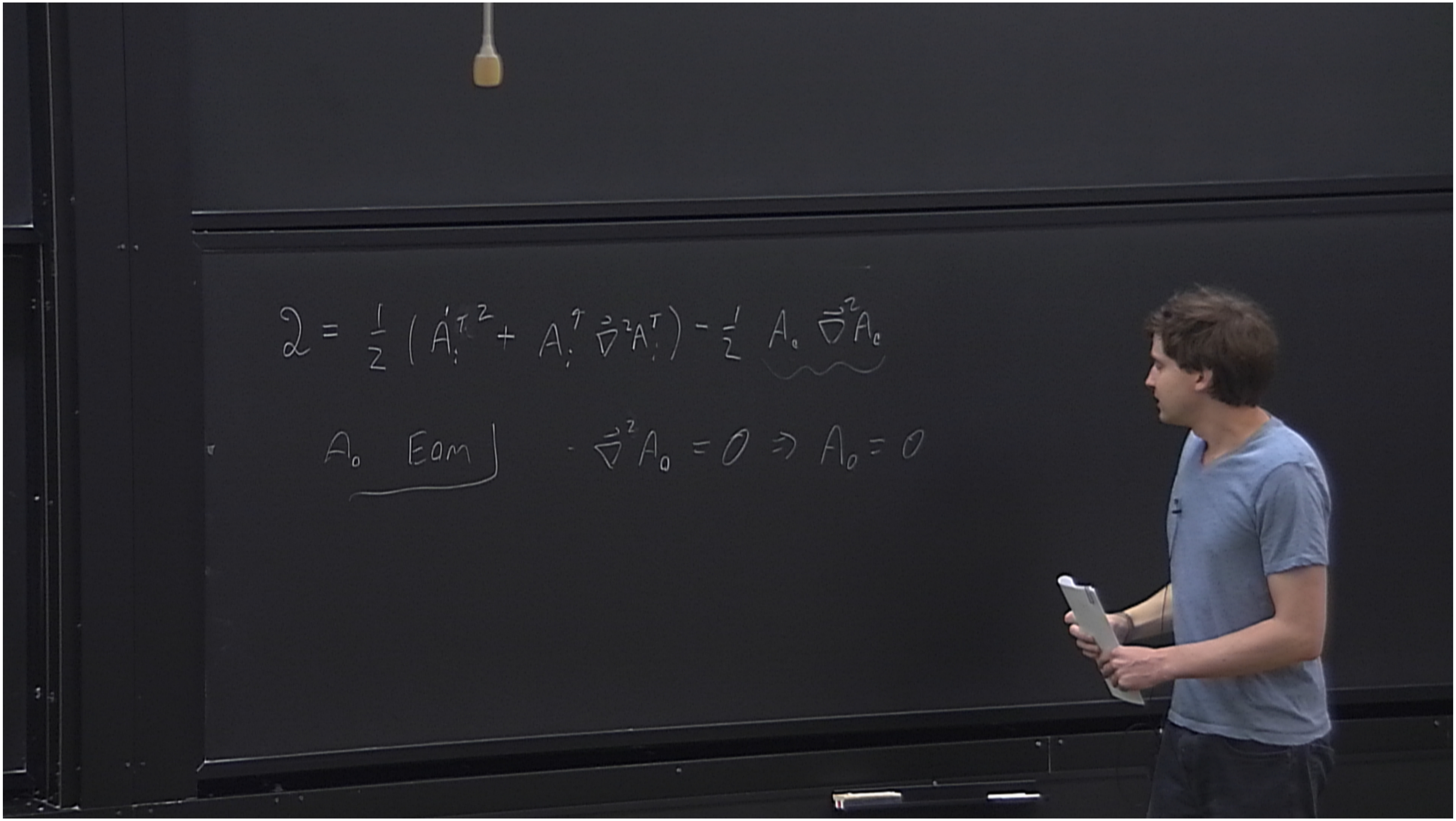
$$P^T_{ij} = \delta_{ij} - \frac{1}{\nabla^2}\partial_i\partial_j$$

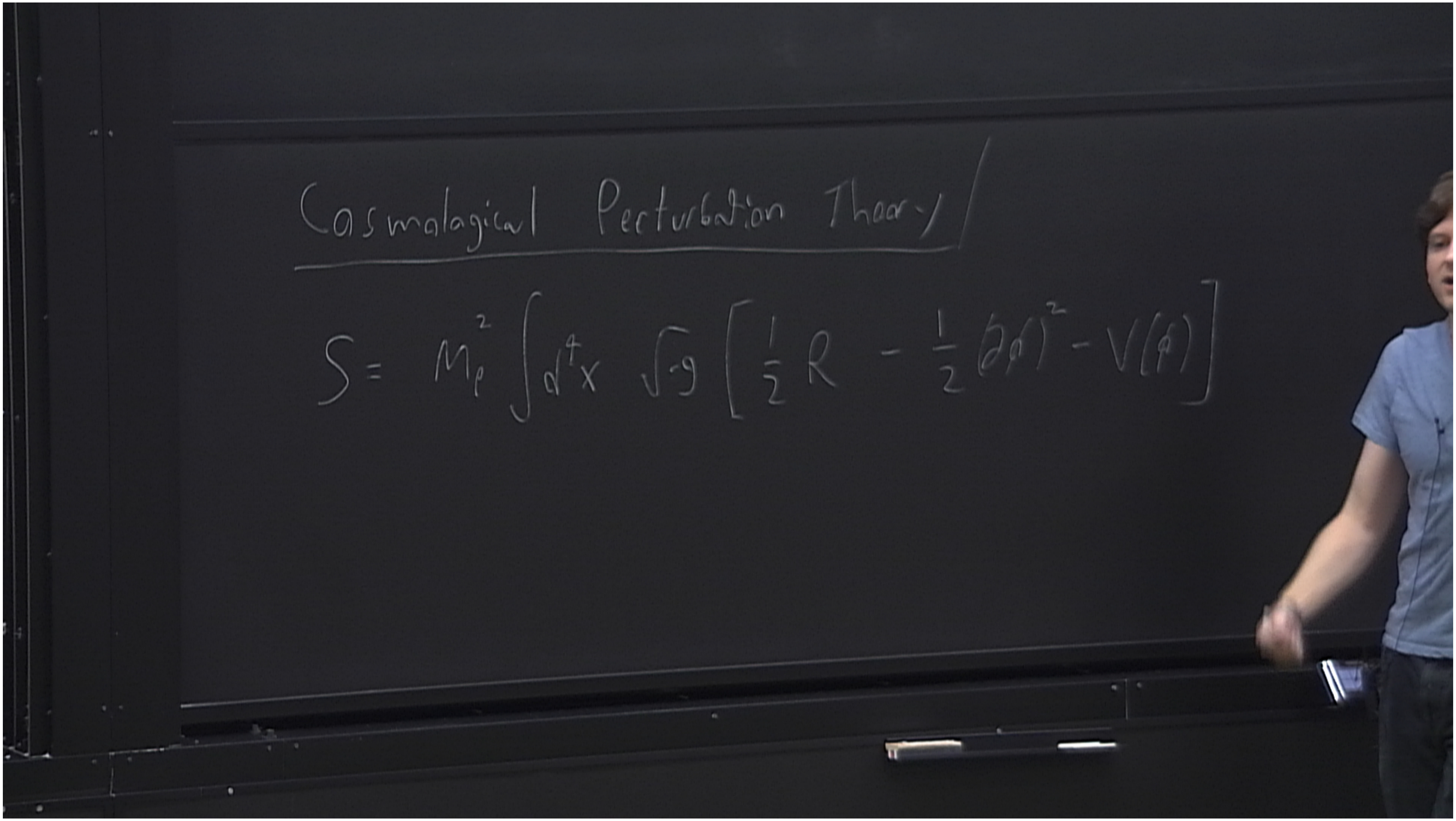
$$\mathcal{L} = \frac{1}{2}\left(\dot{A}_i^{T\,2} + A_i^T \vec{\nabla}^2 A_i^T\right) - \frac{1}{2}\left(A_0 - \dot{\phi}\right)\vec{\nabla}^2\left(A_0 - \dot{\phi}\right) \qquad \delta$$

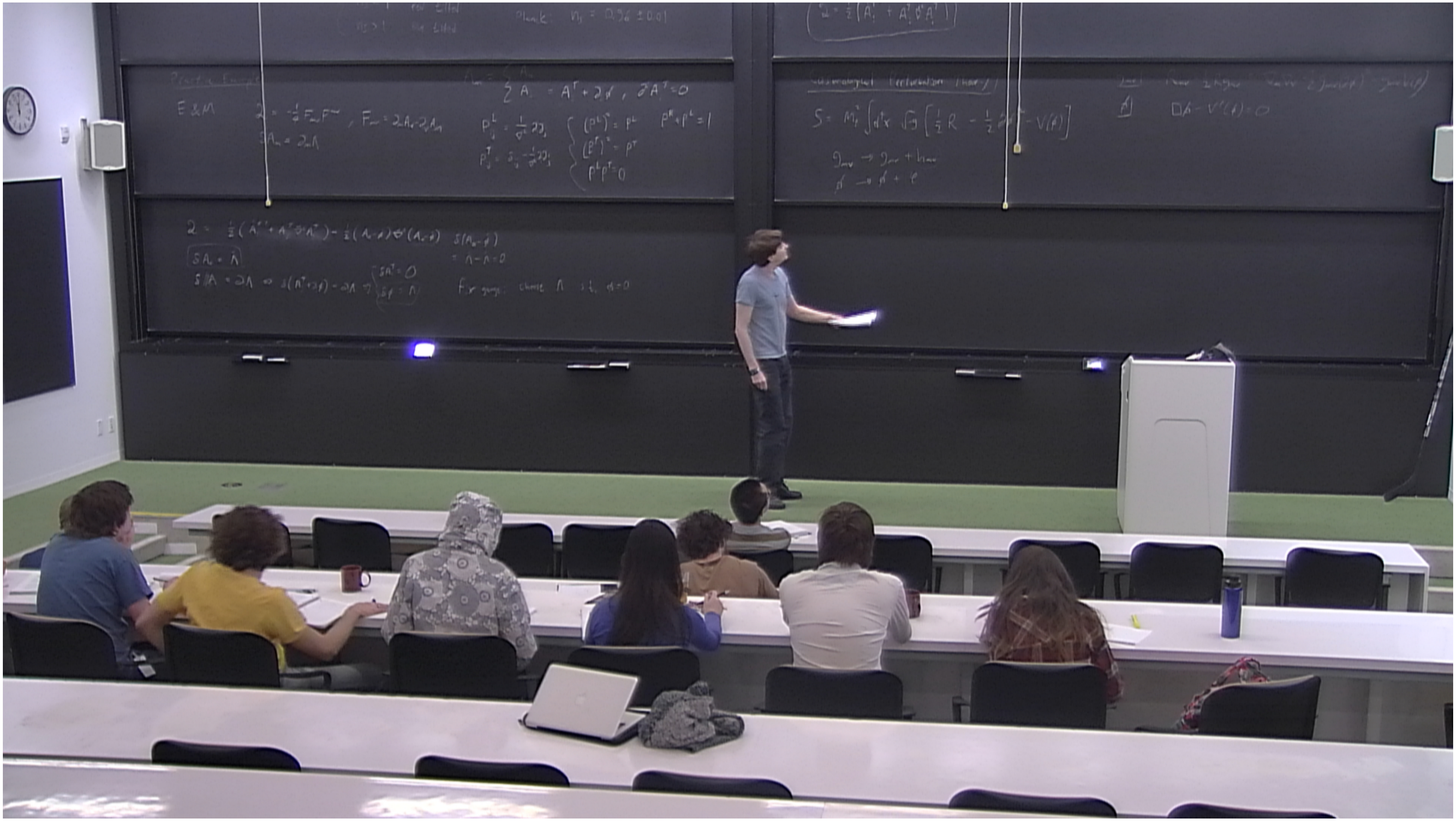
$$\boxed{\delta A_0 = \dot{\Lambda}}$$

$$\delta A_i = \partial_i \Lambda \Rightarrow \delta\left(A_i^T + \partial_i \phi\right) = \partial_i \Lambda \Rightarrow \begin{cases} \delta A_i^T = 0 \\ \boxed{\delta \phi = \Lambda} \end{cases}$$


$\delta A_m = \partial_m \Lambda$
$P^T_{ij} = \delta_{ij} - \frac{1}{\nabla^2}\partial_i\partial_j$
$\mathcal{L} = \frac{1}{2}\left(\dot{A}_i^{T\,2} + A_i^T \vec{\nabla}^2 A_i^T\right) - \frac{1}{2}\left(A_0 - \dot{\phi}\right)\vec{\nabla}^2\left(A_0 - \dot{\phi}\right)$
$\delta(A_0 - \dot{\phi}) = \dot{\Lambda} - \dot{\Lambda} = 0$
$\boxed{\delta A_0 = \dot{\Lambda}}$
$\delta A_i = \partial_i \Lambda \Rightarrow \delta(A_i^T + \partial_i \phi) = \partial_i \Lambda \Rightarrow \delta A_i^T = 0,\ \boxed{\delta\phi = \Lambda}$


$\mathcal{L} = \frac{1}{2}\left(A_i'^{T\,2} + A_i^T \vec{\nabla}^2 A_i^T\right) - \frac{1}{2} A_0 \vec{\nabla}^2 A_0$
A_0 EOM
$-\vec{\nabla}^2 A_0 = 0 \Rightarrow A_0 = 0$


Cosmological Perturbation Theory
$S = M_p^2 \int d^4x \sqrt{-g} \left[\frac{1}{2} R - \frac{1}{2} (\partial \phi)^2 - V(\phi) \right]$



$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\phi \to \phi + \varphi$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - h^{\mu\nu} + \cdots$$

$$\sqrt{-\tilde{g}} = \sqrt{-g}\left(1 + \frac{1}{2} h + \cdots\right)$$

$$\mathcal{L}^{(2)} = \sqrt{-g}\, M_p^2 \Big[-\frac{1}{8} \nabla_\lambda h_{\mu\nu} \nabla^\lambda h^{\mu\nu} + \frac{1}{4} \nabla_\lambda h^{\mu\nu} \nabla_\nu h_\mu{}^\lambda - \frac{1}{4} \nabla_\mu h^{\mu\nu}$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\phi \to \phi + \varphi$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - h^{\mu\nu} + \cdots$$

$$\sqrt{-\tilde{g}} = \sqrt{-g}\left(1 + \frac{1}{2}h + \cdots\right)$$

$$\mathcal{L}^{(2)} = \sqrt{-g}\, M_p^2 \Big[-\frac{1}{8} \nabla_\lambda h_{\mu\nu} \nabla^\lambda h^{\mu\nu} + \frac{1}{4} \nabla_\lambda h^{\mu\nu} \nabla_\nu h_\mu{}^\lambda - \frac{1}{4} \nabla_\mu h^{\mu\nu} \nabla_\nu h + \frac{1}{8} (\nabla h)^2$$

$$S = M_p^2 \int d^4x \sqrt{-g}\left[\frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$\Box\phi - V'(\phi) = 0$$

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

$$\phi \to \phi + \varphi$$

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} - h^{\mu\nu} + \dots$$

$$\sqrt{-\tilde{g}} = \sqrt{-g}\left(1 + \frac{1}{2}h + \dots\right)$$

$$\mathcal{L}^{(2)} = \sqrt{-g}\, M_p^2 \Big[-\frac{1}{8}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \frac{1}{4}\nabla_\lambda h^{\mu\nu}\nabla_\nu h_\mu{}^\lambda - \frac{1}{4}\nabla_\mu h^{\mu\nu}\nabla_\nu h + \frac{1}{8}(\nabla h)^2 + \frac{1}{4}V(\phi)\left[h_{\mu\nu}h^{\mu\nu} - \frac{1}{2}h^2\right]$$

$$+ h_{\mu\nu}\nabla^\mu\phi\nabla^\nu\varphi - \frac{1}{2}h\left[\nabla_\mu\phi\nabla^\mu\varphi + \varphi V'(\phi)\right]$$

$$-\frac{1}{2}\left[(\partial\varphi)^2 + V''(\phi)\varphi^2\right]\Big]$$

diff symmetry

$$\delta g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

$$\delta \phi = \xi^\mu \partial_\mu \phi$$

$$P^T_{ij} = \delta_{ij} - \frac{1}{\nabla^2}\partial_i\partial_j \qquad \begin{cases} (P^T)^2 = P^T \\ P^L P^T = 0 \end{cases}$$

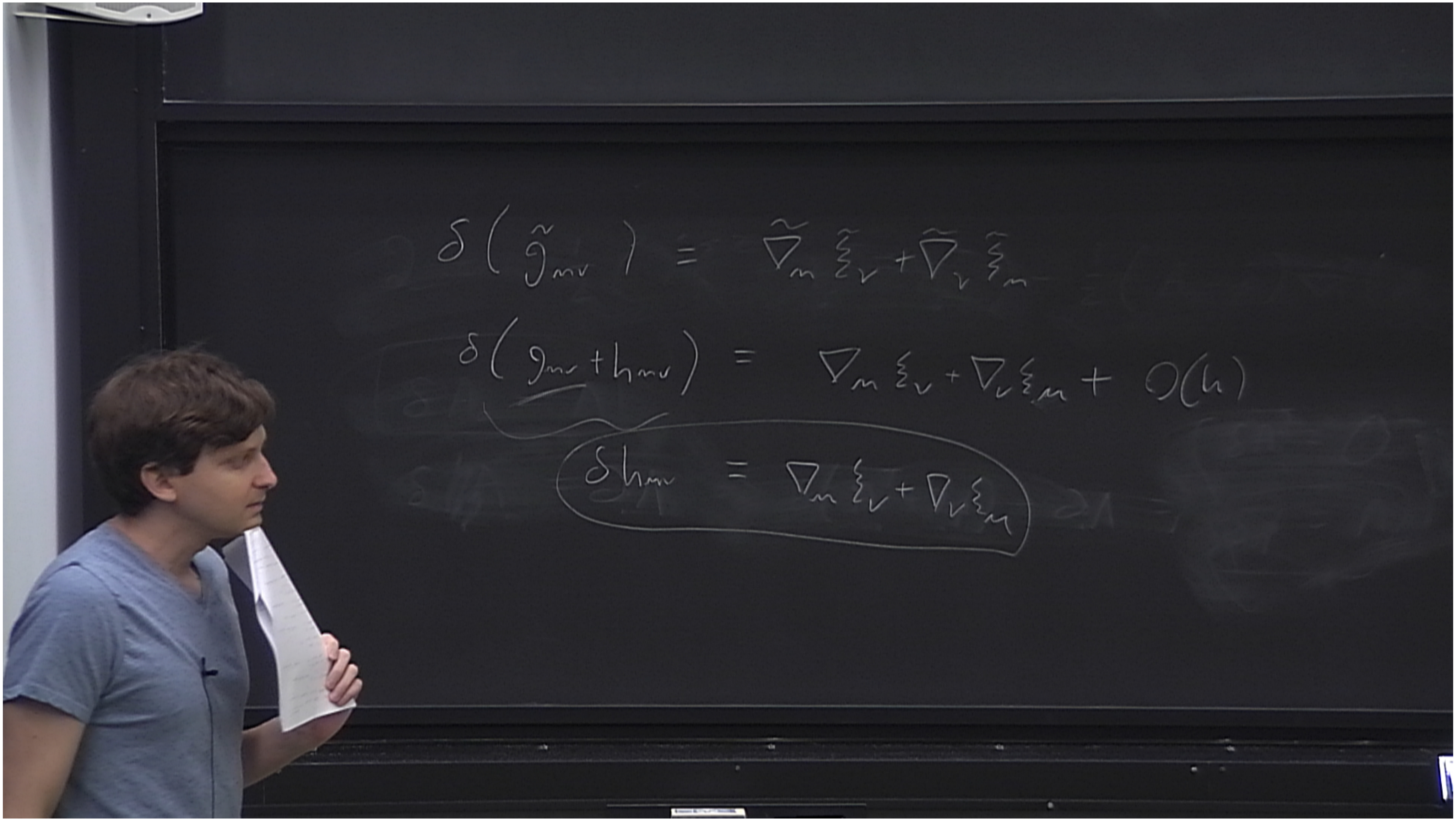
$$x^\mu \to f^\mu(x) = x^\mu + \xi^\mu(x)$$

$$\phi(x) \longrightarrow \phi(x') = \phi(f(x)) = \phi(x+\xi) = \phi(x) + \xi^\mu \partial_\mu \phi + \cdots$$

$$\delta\phi = \xi^\mu \partial_\mu \phi$$

$$g_{\mu\nu}(x) \longrightarrow \frac{\partial f^{-1\alpha}}{\partial x^\mu}\frac{\partial f^{-1\beta}}{\partial x^\nu} g_{\mu\nu}(f(x)) \Rightarrow \delta g_{\mu\nu} = \xi^\lambda \partial_\lambda g_{\mu\nu} + \partial_\mu \xi^\lambda g_{\nu\lambda} + \partial_\nu \xi^\lambda g_{\mu\lambda}$$

$$= \xi^\lambda \nabla_\lambda g_{\mu\nu} + \nabla_\mu \xi^\lambda g_{\nu\lambda} + \nabla_\nu \xi^\lambda g_{\mu\lambda}$$


$\delta(\tilde{g}_{\mu\nu}) = \tilde{\nabla}_\mu \xi_\nu + \tilde{\nabla}_\nu \xi_\mu$
$\delta(g_{\mu\nu} + h_{\mu\nu}) = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu + O(h)$
$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$

$$\mathcal{L} = \mathcal{L}_n + \mathcal{L}_{n+1} + \mathcal{L}_{n+2} + \cdots$$

$$\delta\phi = F(\phi) = F_m + F_{m+1} + \cdots$$

$m+n$

$$\frac{\delta\mathcal{L}}{\delta\phi}\delta\phi = 0$$

$$\underbrace{\frac{\delta}{\delta\phi}}\left(\mathcal{L}_n + \mathcal{L}_{n+1} + \cdots\right)\left(F_m + F_{m+1} + \cdots\right) = 0$$

$$\frac{\delta}{\delta\phi}\mathcal{L}_n F_m = 0$$

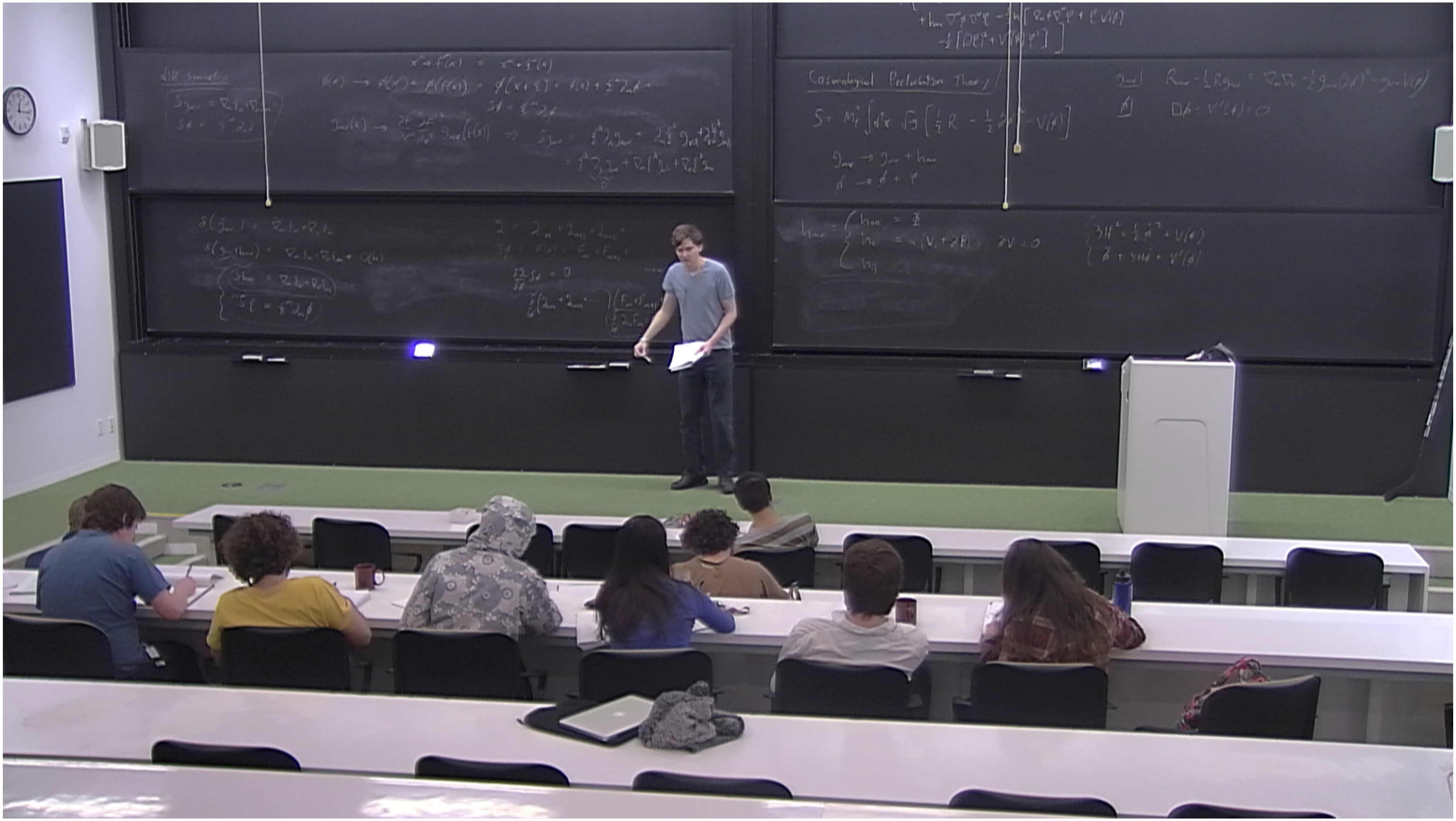
$$\delta(\tilde{g}_{\mu\nu}) = \tilde{\nabla}_\mu \tilde{\xi}_\nu + \tilde{\nabla}_\nu \tilde{\xi}_\mu$$

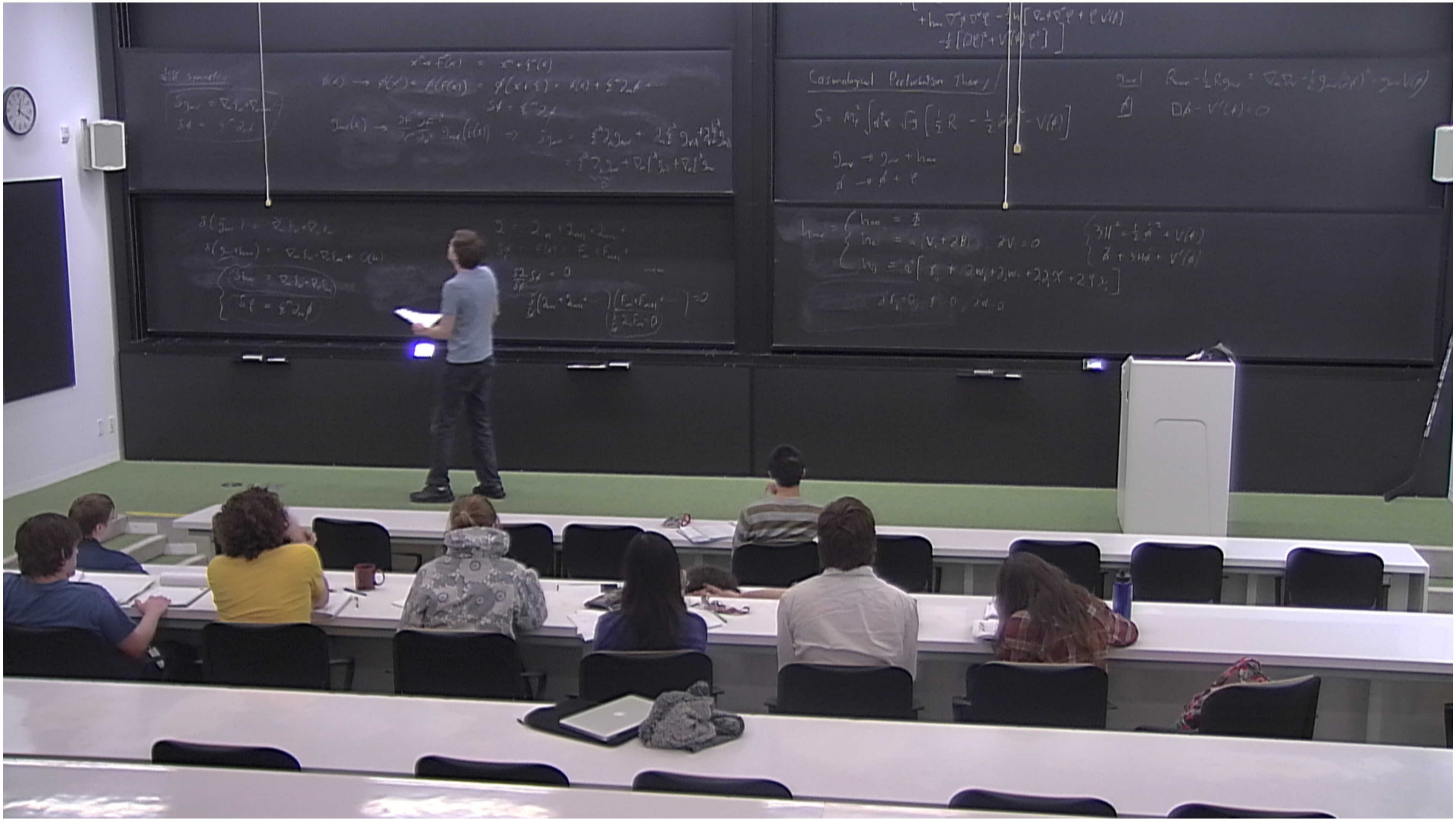
$$\delta(g_{\mu\nu} + h_{\mu\nu}) = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu + O(h)$$

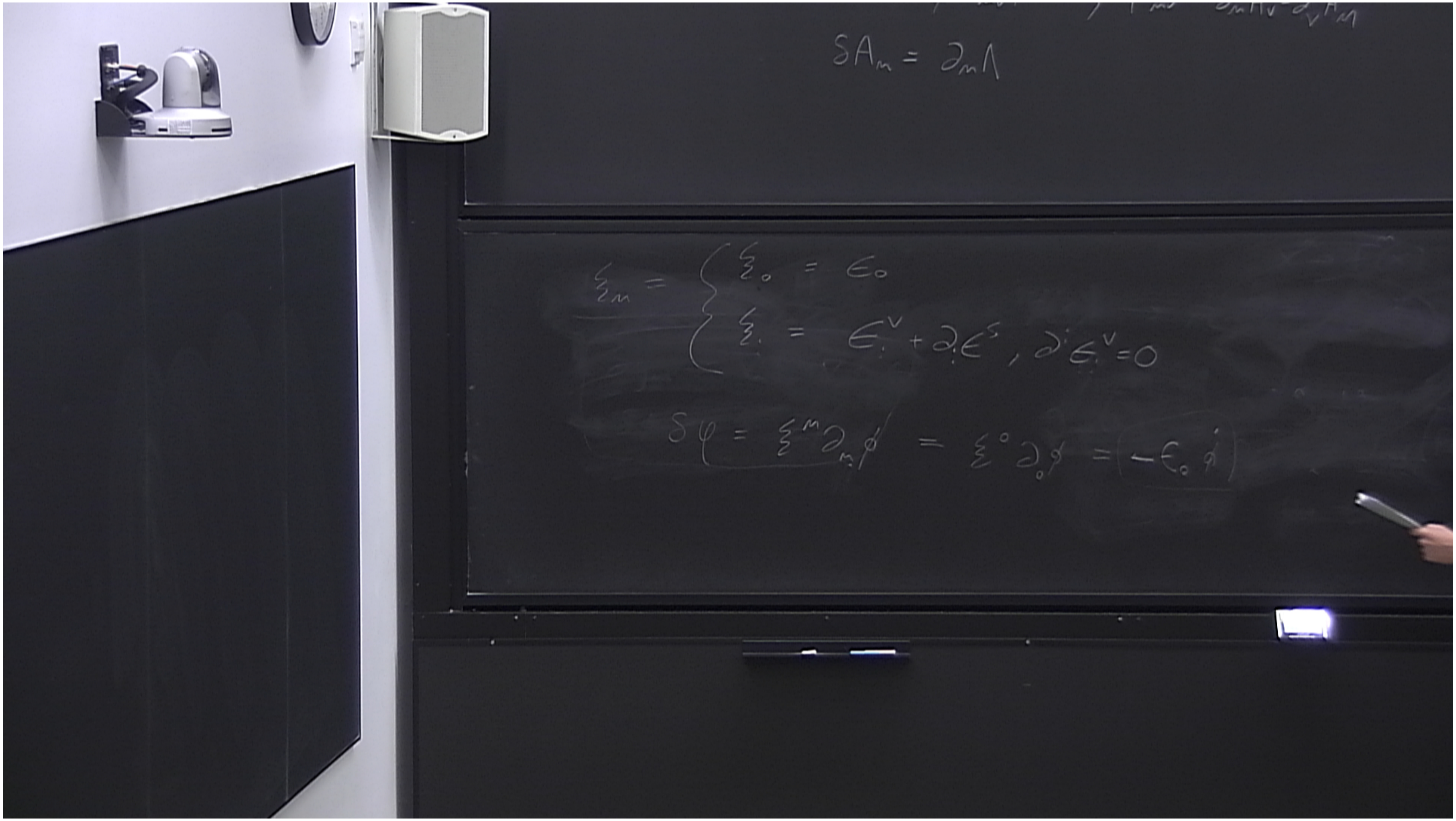
$$\delta h_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$

$$\delta \varphi = \xi^\mu \partial_\mu \phi$$

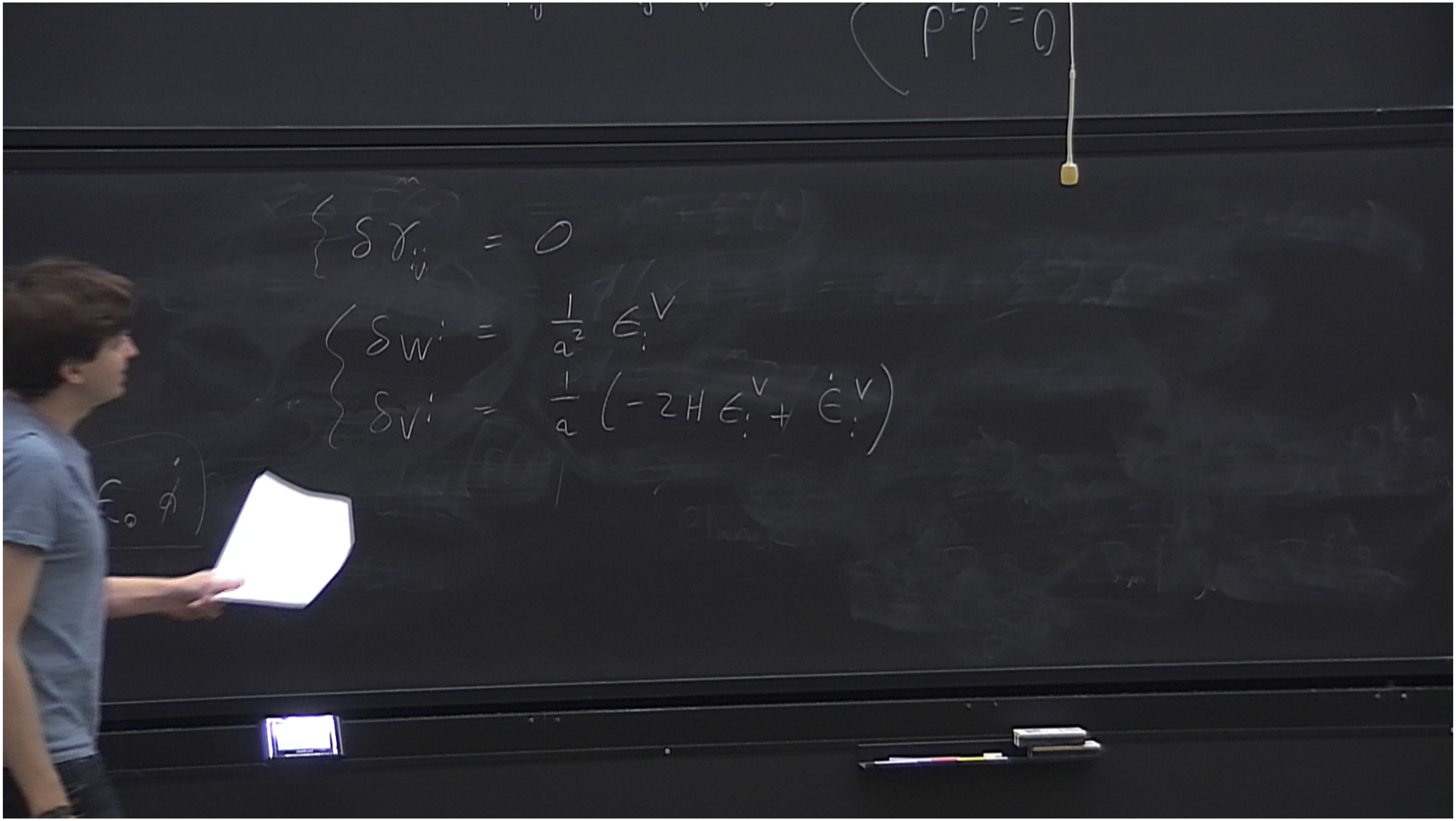
$$
h_{\mu\nu} = \begin{cases} h_{00} = \Phi \\ h_{0i} = a\left(V_i + \partial_i B\right), \quad \partial^i V_i = 0 \\ h_{ij} \end{cases}
$$

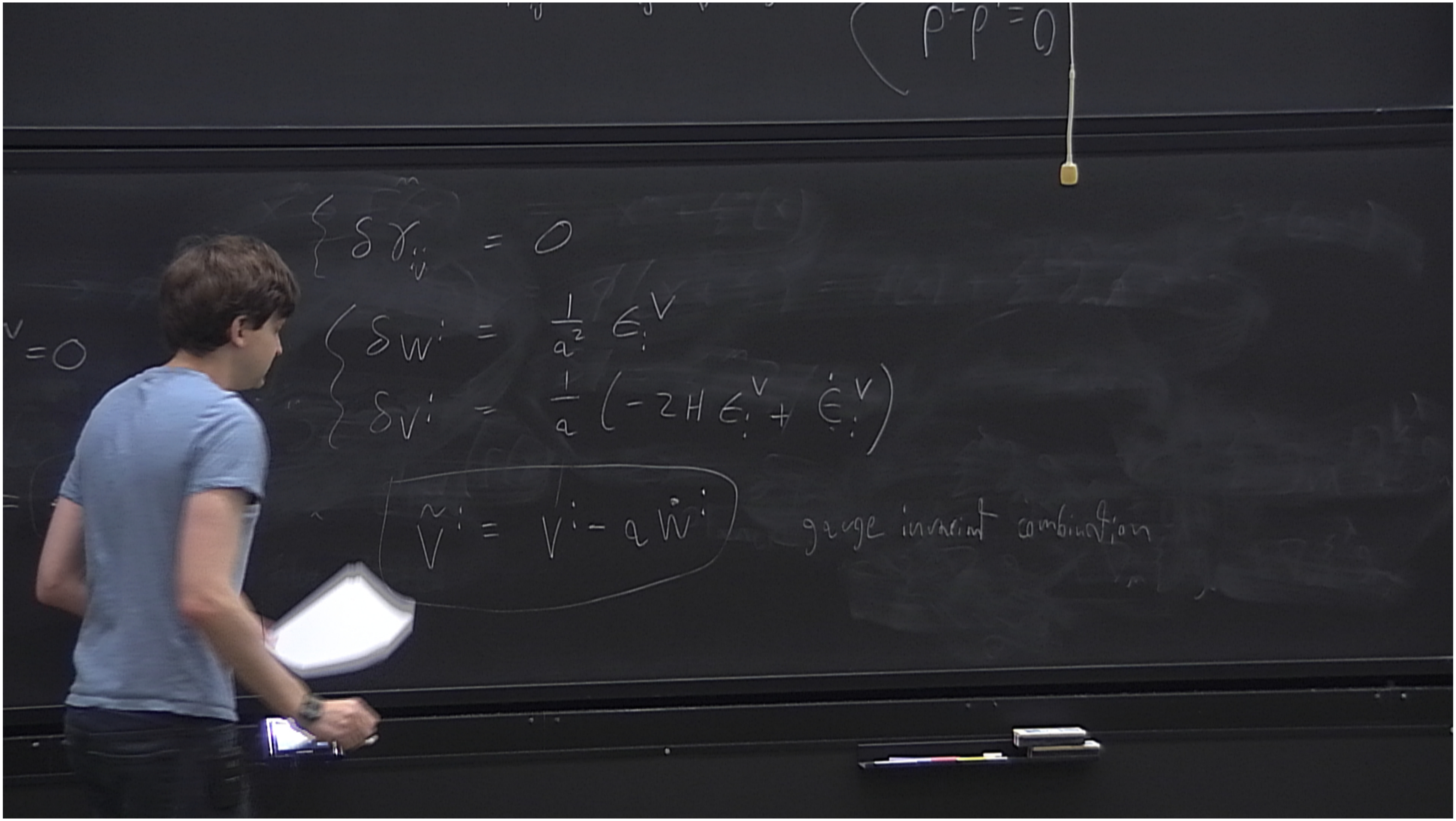






$\delta A_m = \partial_m \Lambda$
$\xi_m = \begin{cases} \xi_0 = \epsilon_0 \\ \xi_i = \epsilon_i^v + \partial_i \epsilon^s , \ \partial^i \epsilon_i^v = 0 \end{cases}$
$\delta \varphi = \xi^m \partial_m \phi = \xi^0 \partial_0 \phi = -\epsilon_0 \dot{\phi}$


$\delta \gamma_{ij} = 0$
$\delta W^i = \frac{1}{a^2} \epsilon_i^V$
$\delta V^i = \frac{1}{a} \left(-2H \epsilon_i^V + \dot{\epsilon}_i^V \right)$


δγ_ij = 0
δw^i = 1/a² ε_i^V
δv^i = 1/a (−2H ε_i^V + ε̇_i^V)
ṽ^i = v^i − a ẇ^i
gauge invariant combination