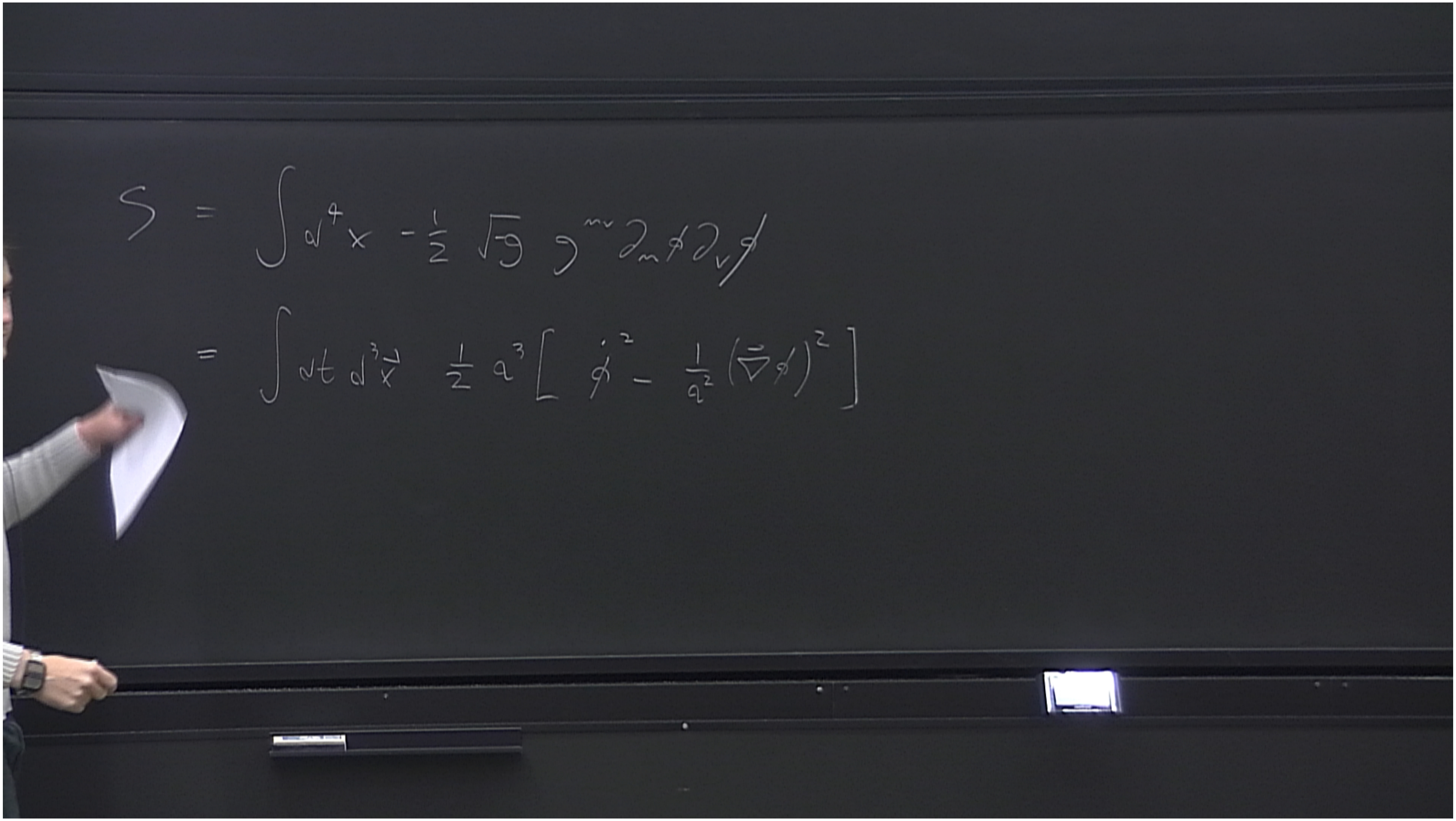
Title: Cosmology Review-11

Date: Feb 09, 2015 11:30 AM

URL: http://pirsa.org/15020034

Abstract:



$S = \int d^4x \, -\frac{1}{2} \sqrt{-g} \, g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi$
$= \int dt \, d^3\vec{x} \; \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2} (\vec{\nabla} \phi)^2 \right]$

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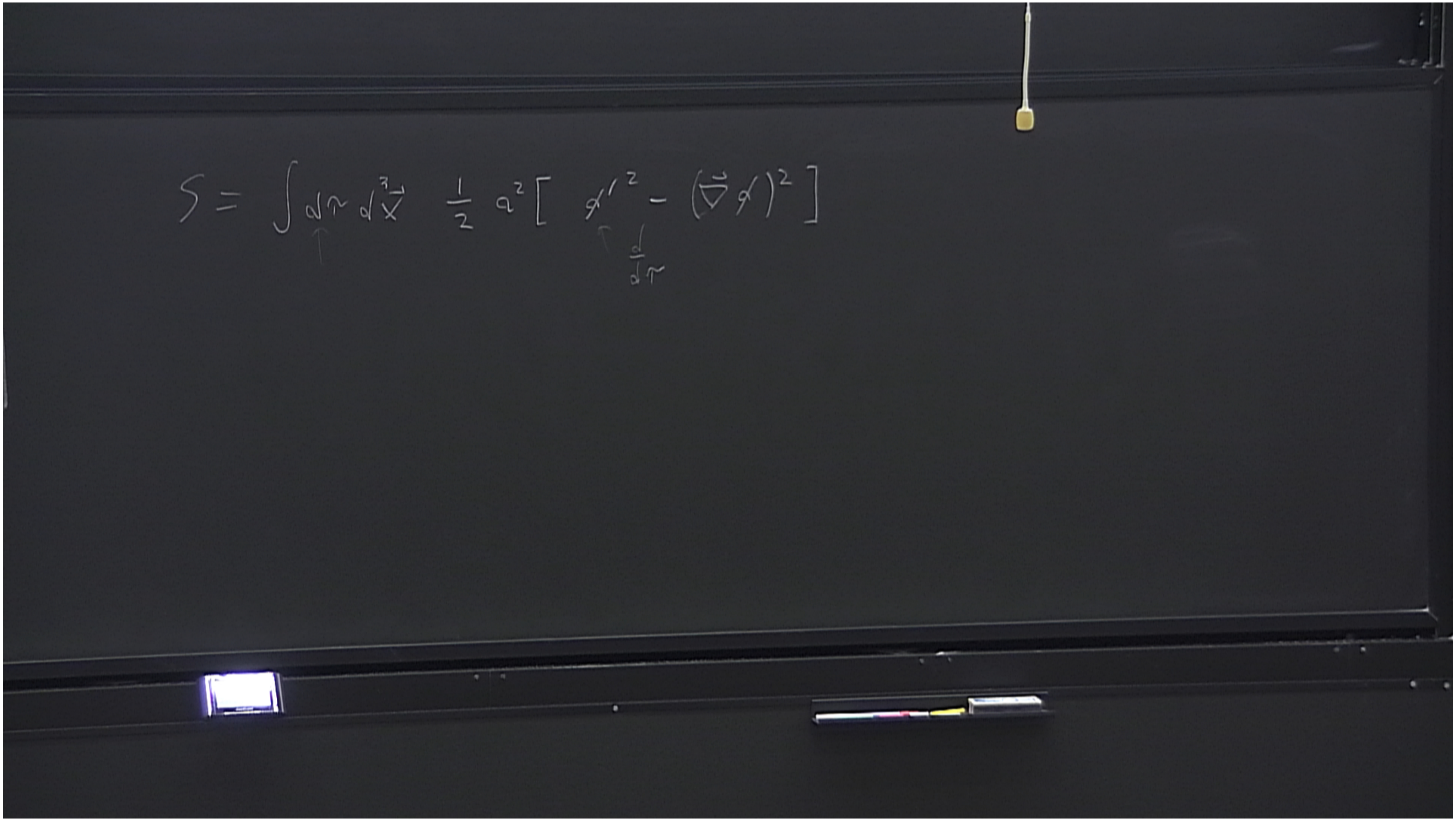
conformal time $\quad d\tau = \dfrac{dt}{a}$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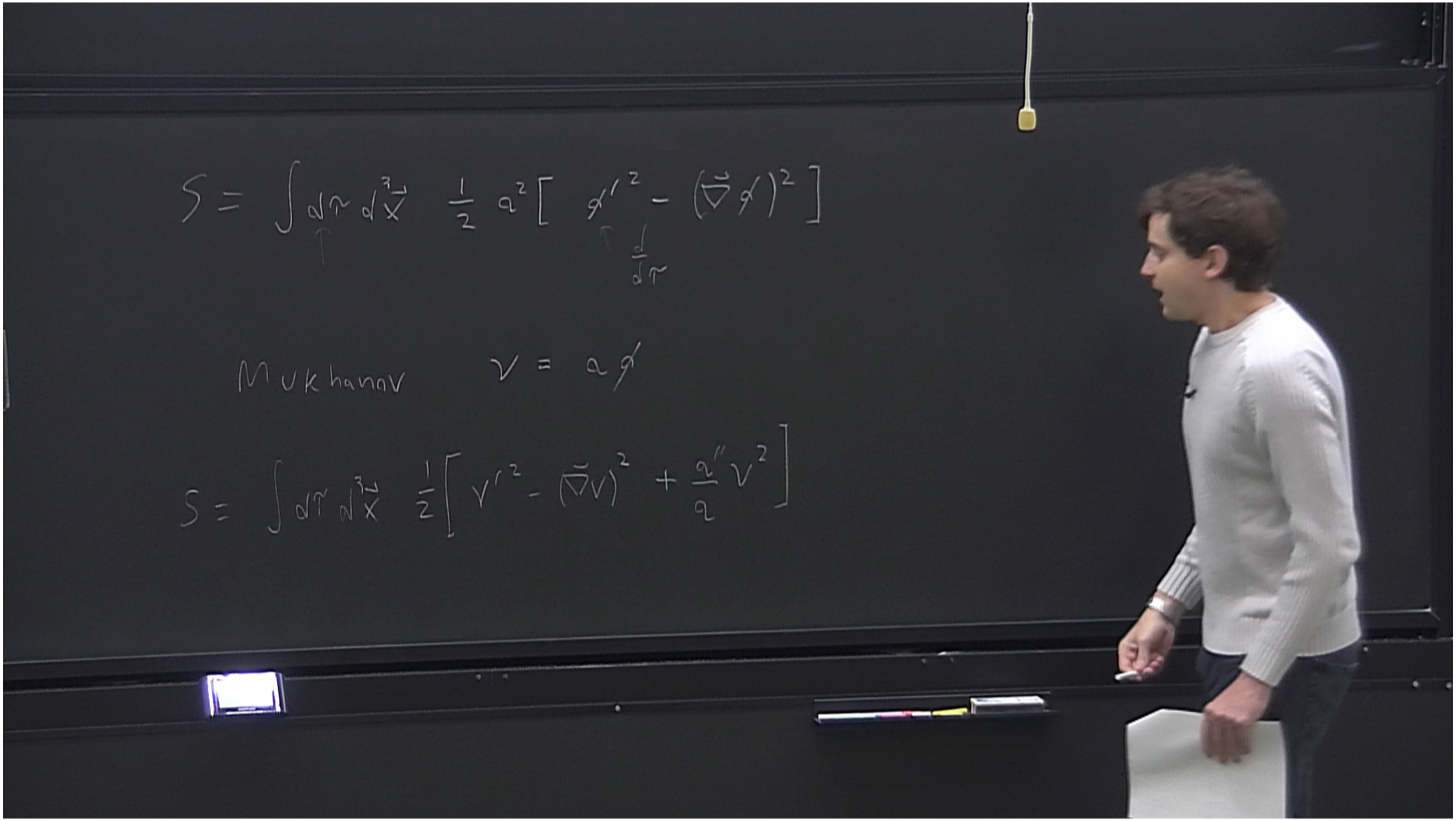
$\frac{d}{d\tau}$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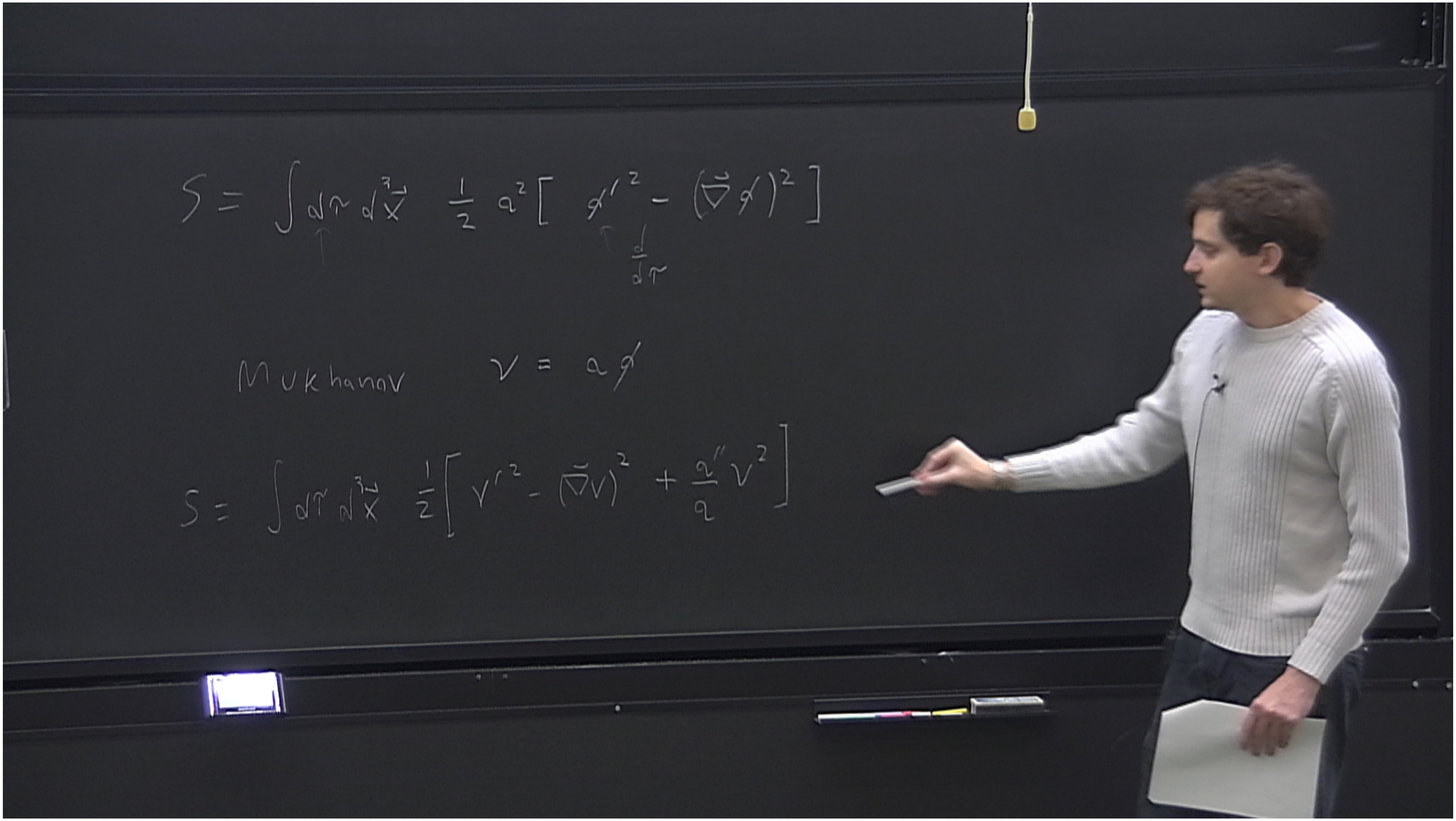
Mukhanov $v = a\phi$
$S = \int d\tau\, d^3\vec{x}\ \frac{1}{2} \left[v'^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2 \right]$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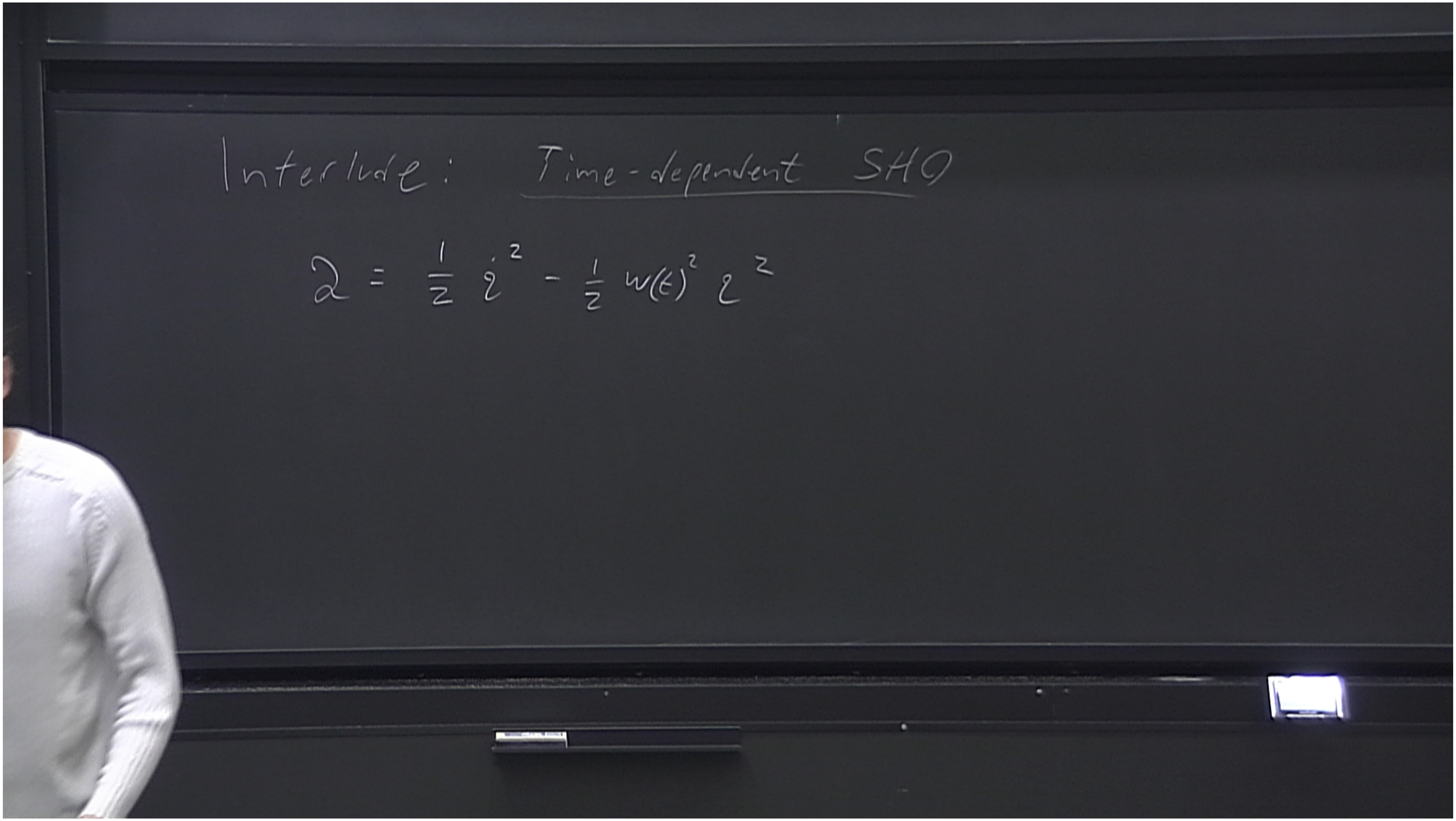
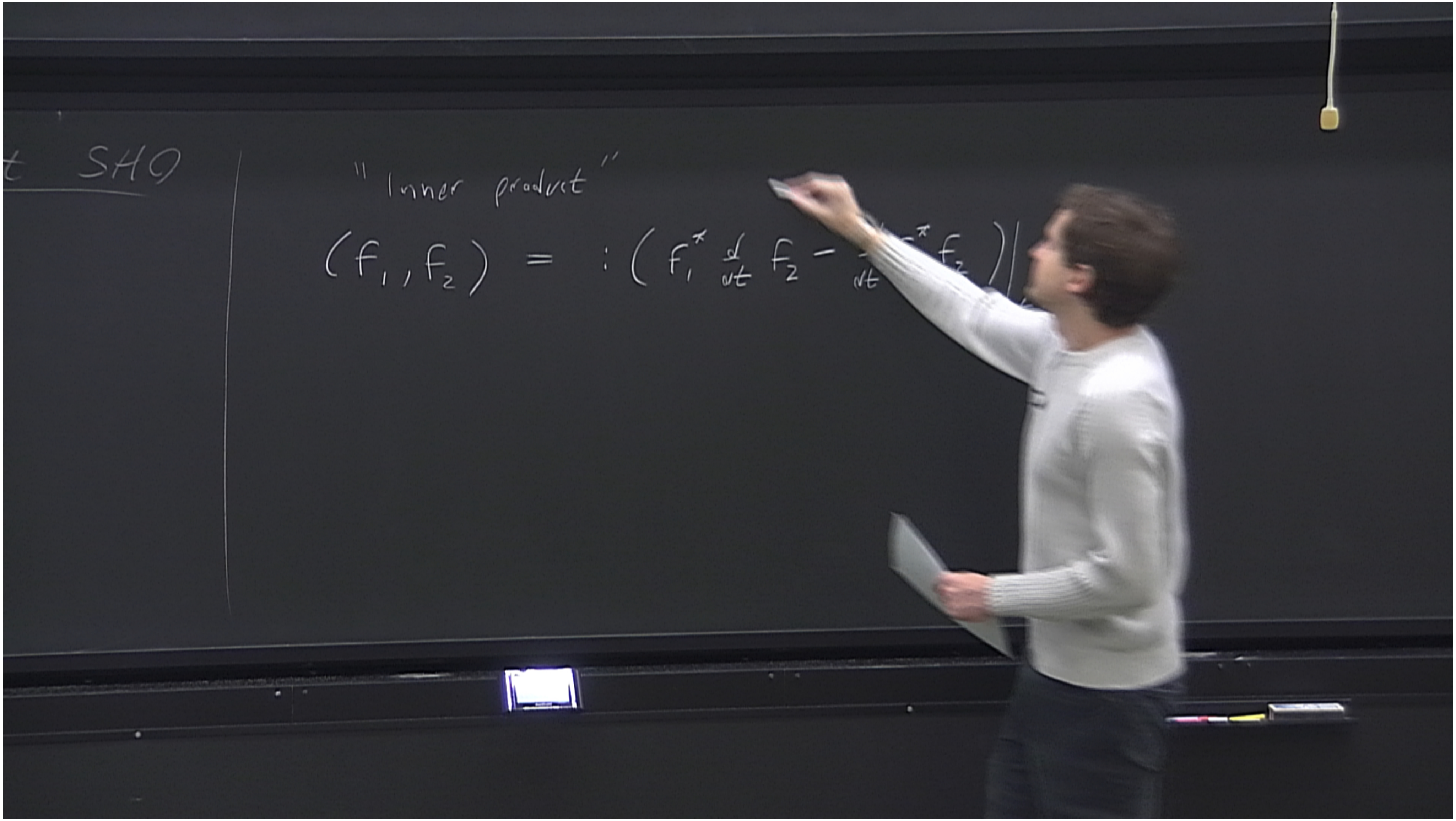
$$' = \frac{d}{d\tau}$$

$$\frac{a'}{a} v' v = \frac{1}{2} \frac{a'}{a} \frac{d}{d\tau} (v^2)$$

Mukhanov $\quad v = a\phi$

$$S = \int d\tau \, d^3\vec{x} \; \frac{1}{2} \left[v'^2 - (\vec{\nabla} v)^2 + \frac{a''}{a} v^2 \right]$$


Interlude: Time-dependent SHO
$\mathcal{L} = \frac{1}{2}\dot{q}^2 - \frac{1}{2}\omega(t)^2 q^2$


t SHO
"Inner product"
(f1, f2) = i(f1* d/dt f2 −

"Inner product" (wronskian)

$$(f_1, f_2) = i\left(f_1^* \frac{d}{dt} f_2 - \frac{d}{dt} f_1^* f_2\right)\Big|_{t=0} \equiv i\, f_1^* \overleftrightarrow{\partial_t} f_2\Big|_{t=0}$$

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given a solution

SHO

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given a solution f, $(f, f) = 1$

then f^* is linearly indedendent of f, $(f^*, f^*) = -1$,

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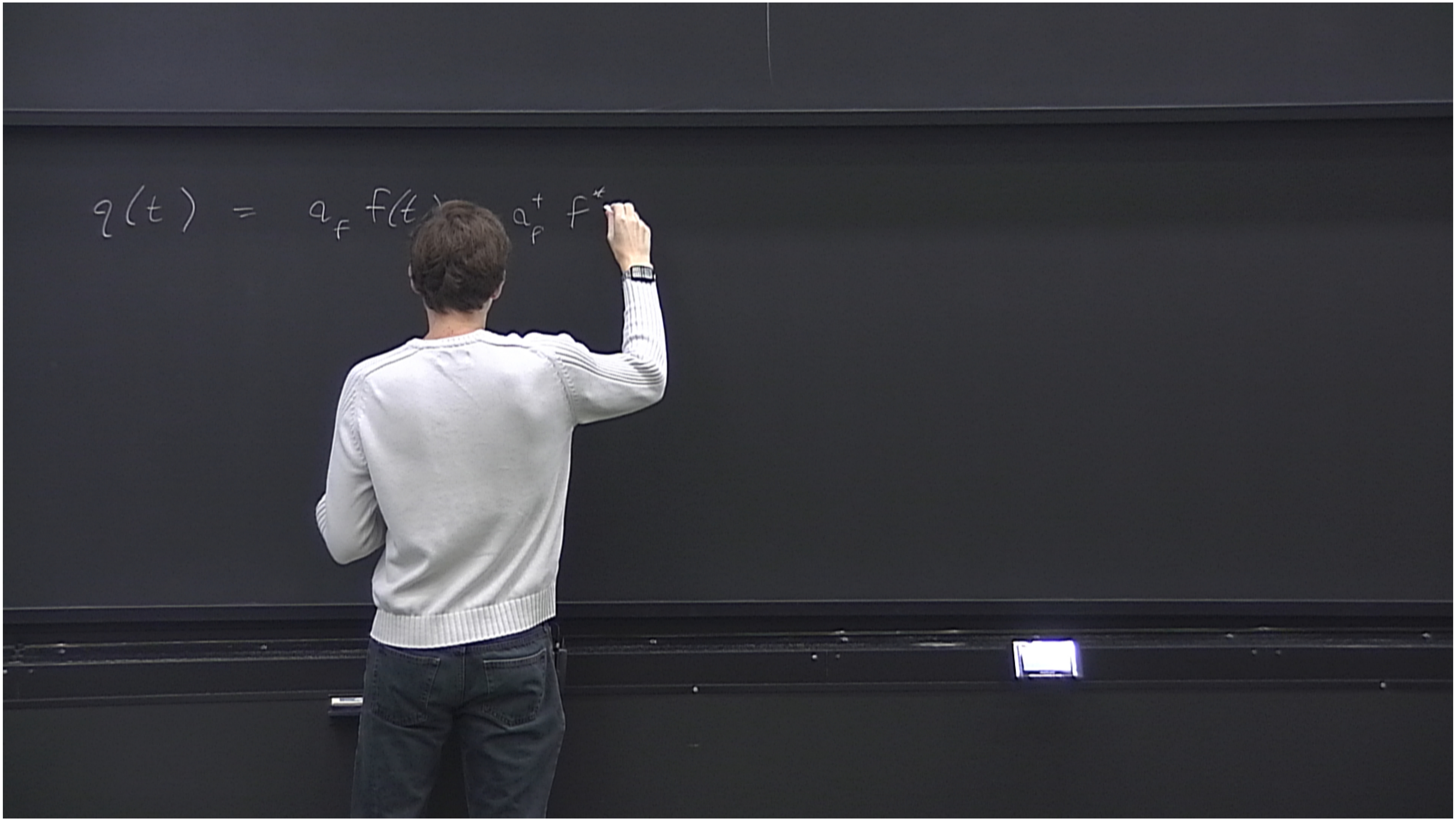
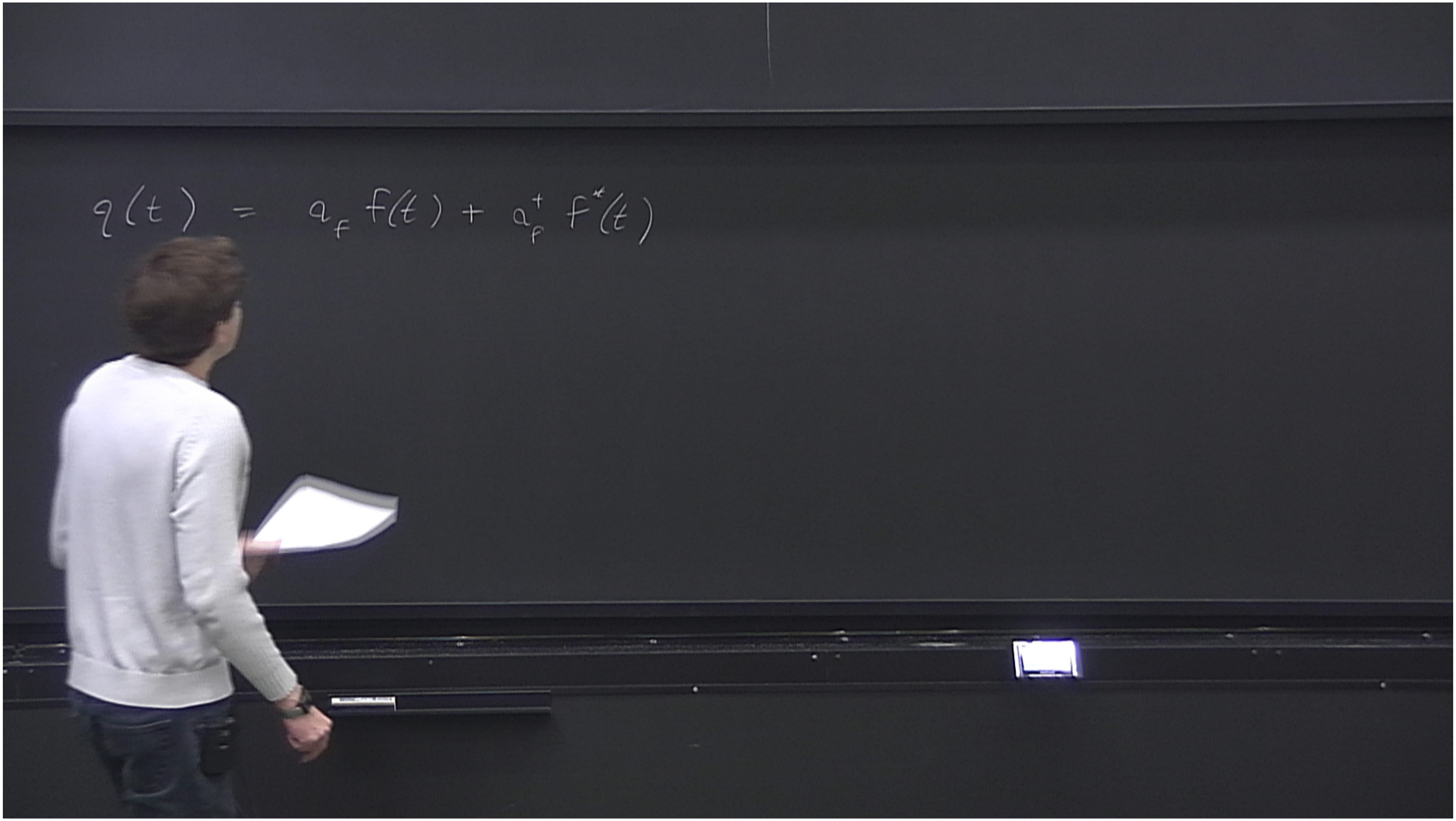

$q(t) = a_f f(t$
$a_f^+ f^*$


$q(t) = a_f f(t) + a_f^{+} f^{*}(t)$

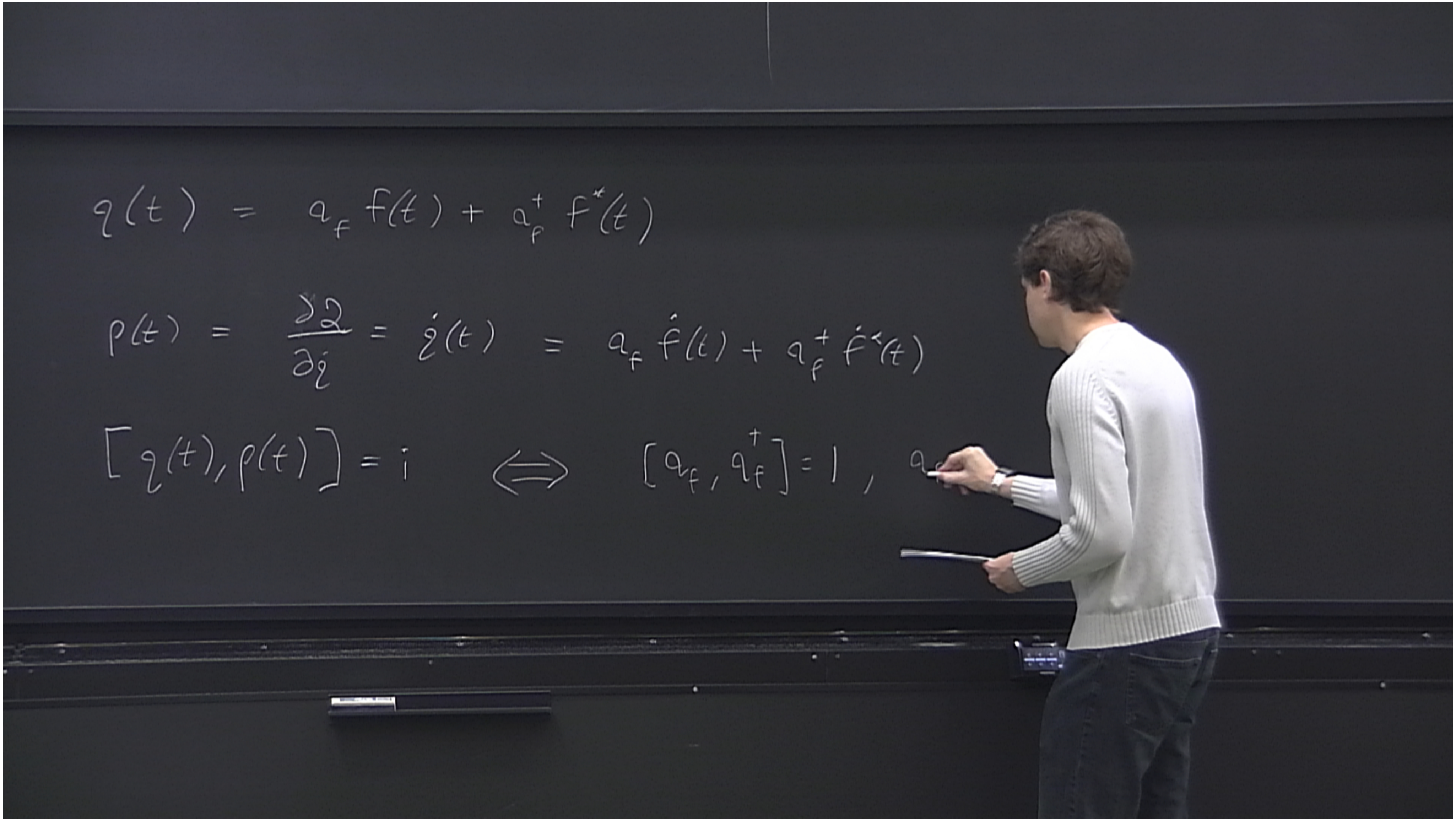
$$q(t) = a_f f(t) + a_f^\dagger f^*(t)$$

$$p(t) = \frac{\partial \mathcal{L}}{\partial \dot{q}} = \dot{q}(t) = a_f \dot{f}(t) + a_f^\dagger \dot{f}^*(t)$$

$$q(t) = a_f f(t) + a_f^{+} f^{*}(t)$$

$$p(t) = \frac{\partial L}{\partial \dot{q}} = \dot{q}(t) = a_f \dot{f}(t) + a_f^{+} \dot{f}^{*}(t)$$

$$[q(t), p(t)] = i$$


$q(t) = a_f f(t) + a_f^{\dagger} f^{*}(t)$
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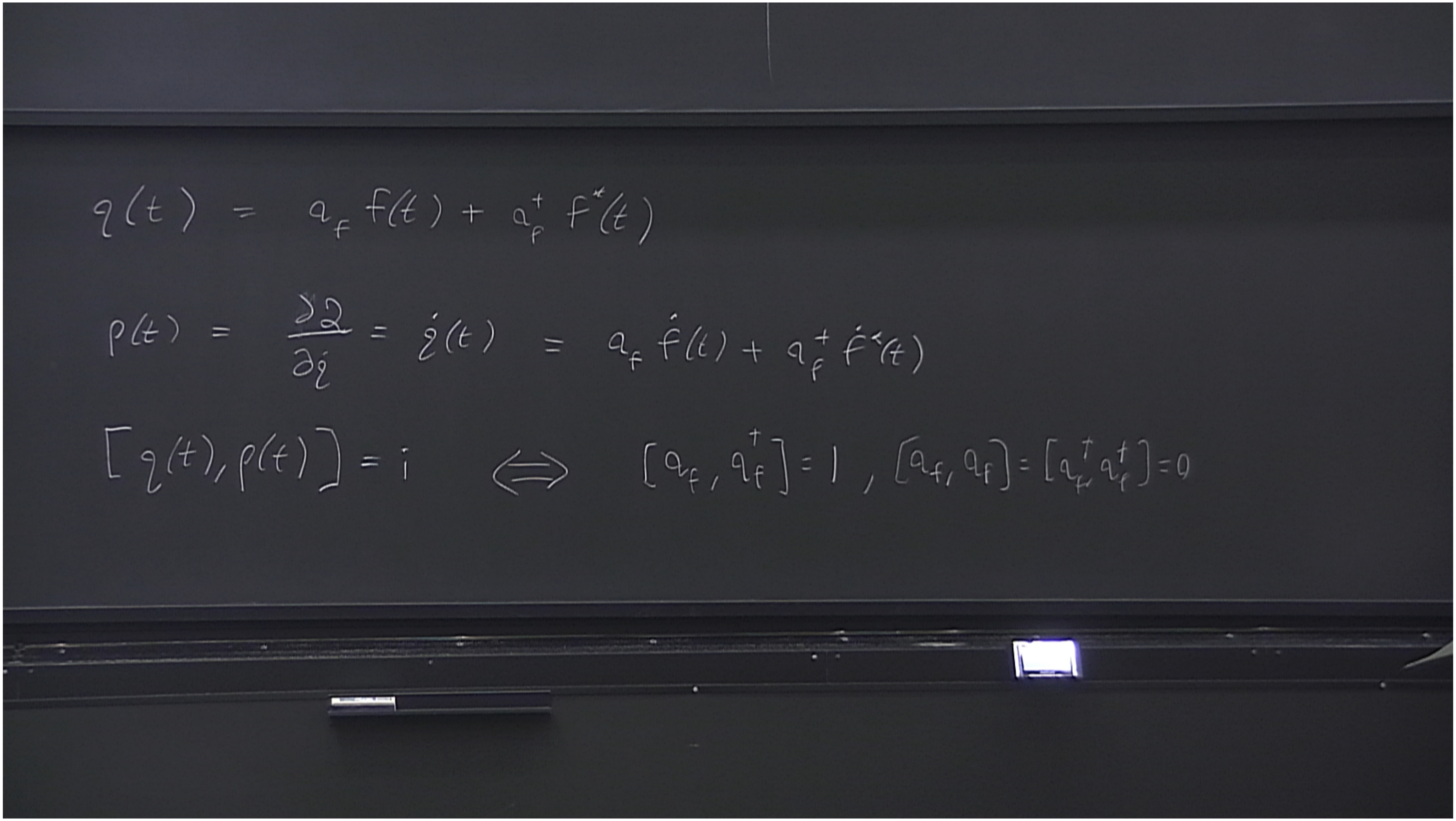
$[q(t), p(t)] = i \iff [a_f, a_f^{\dagger}] = 1 ,$


$q(t) = a_f f(t) + a_f^\dagger f^*(t)$
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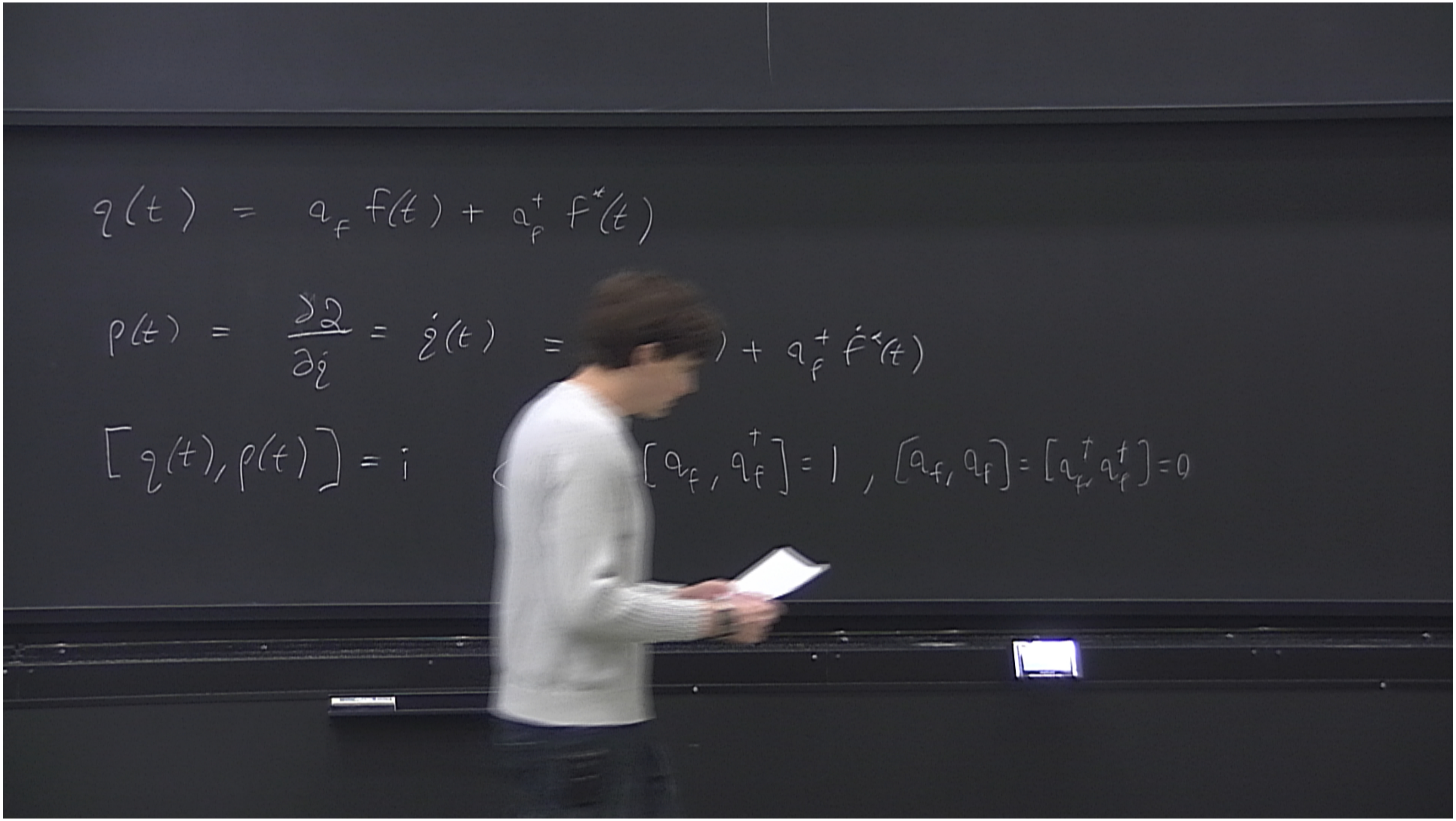
$q(t) = a_f f(t) + a_f^\dagger f^*(t)$
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$) + a_f^\dagger \dot{f}^*(t)$
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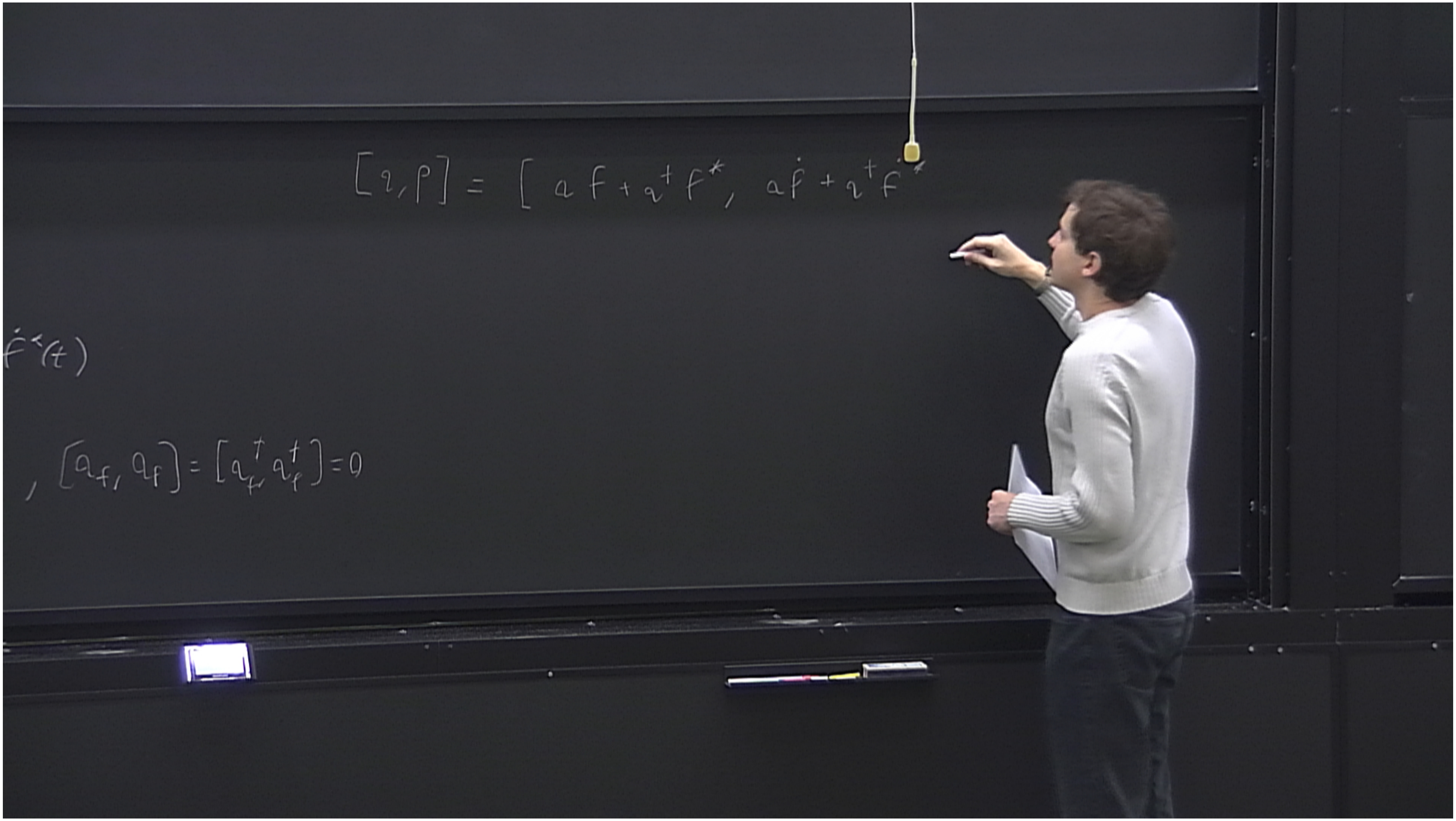
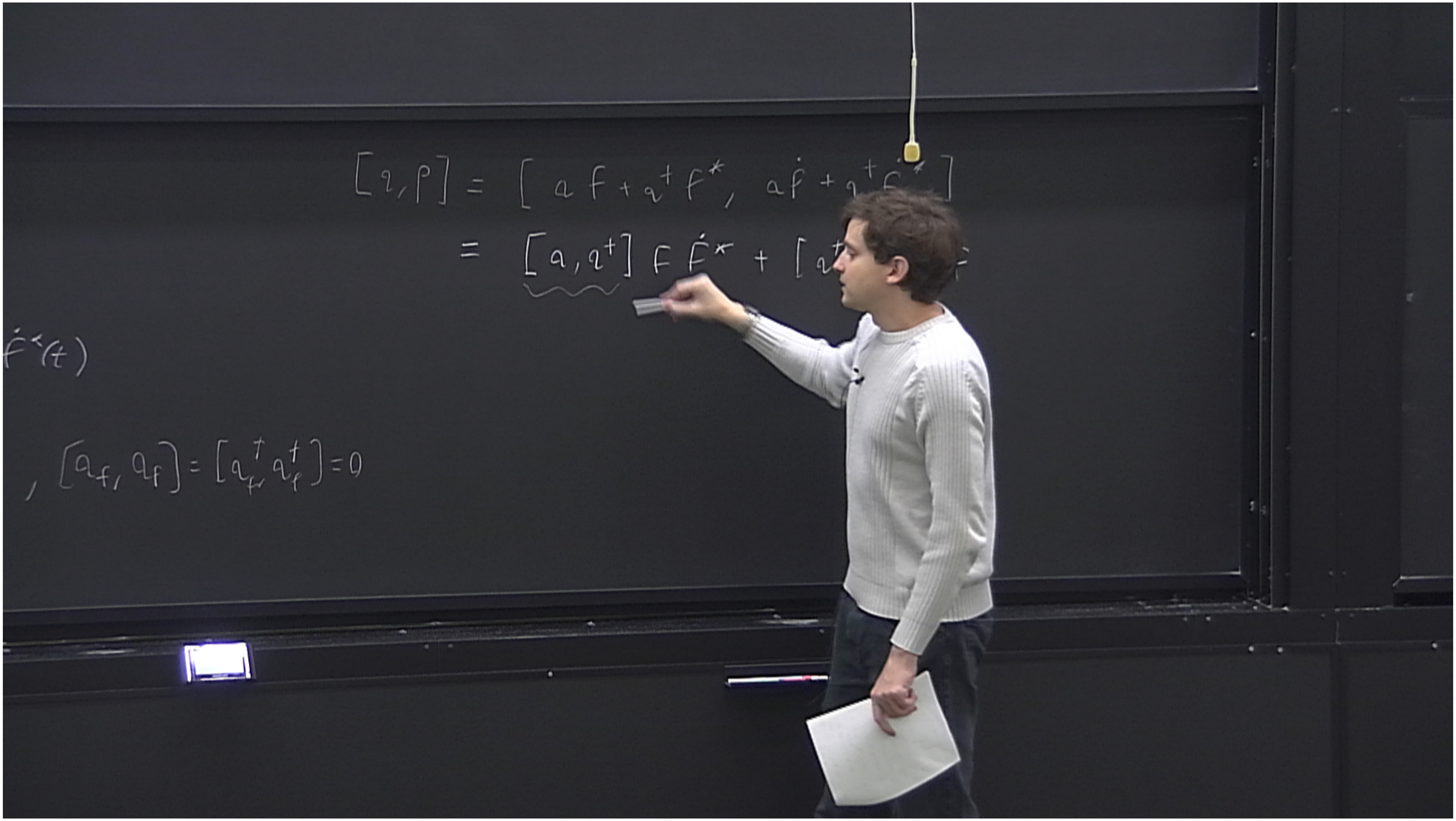
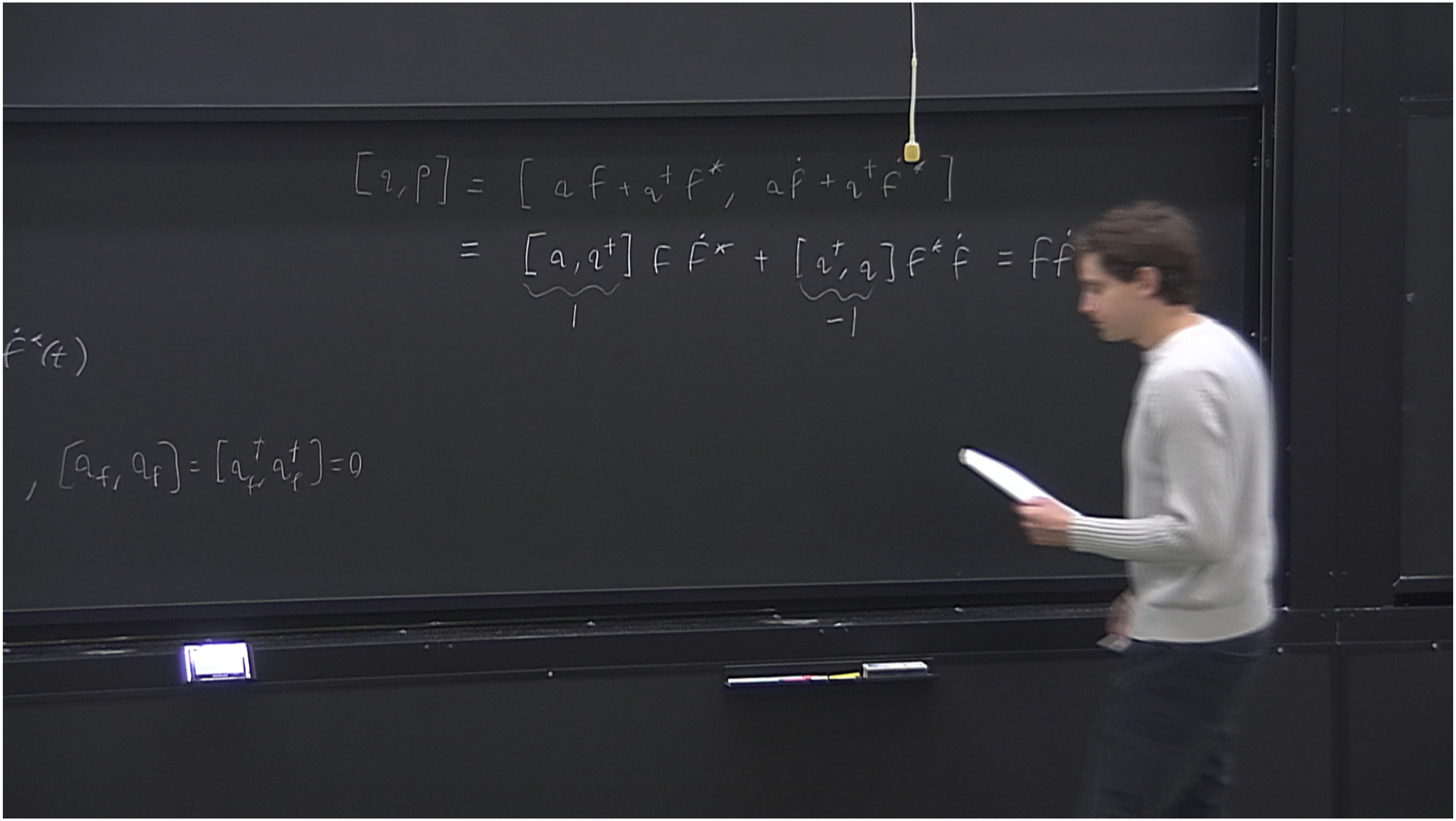
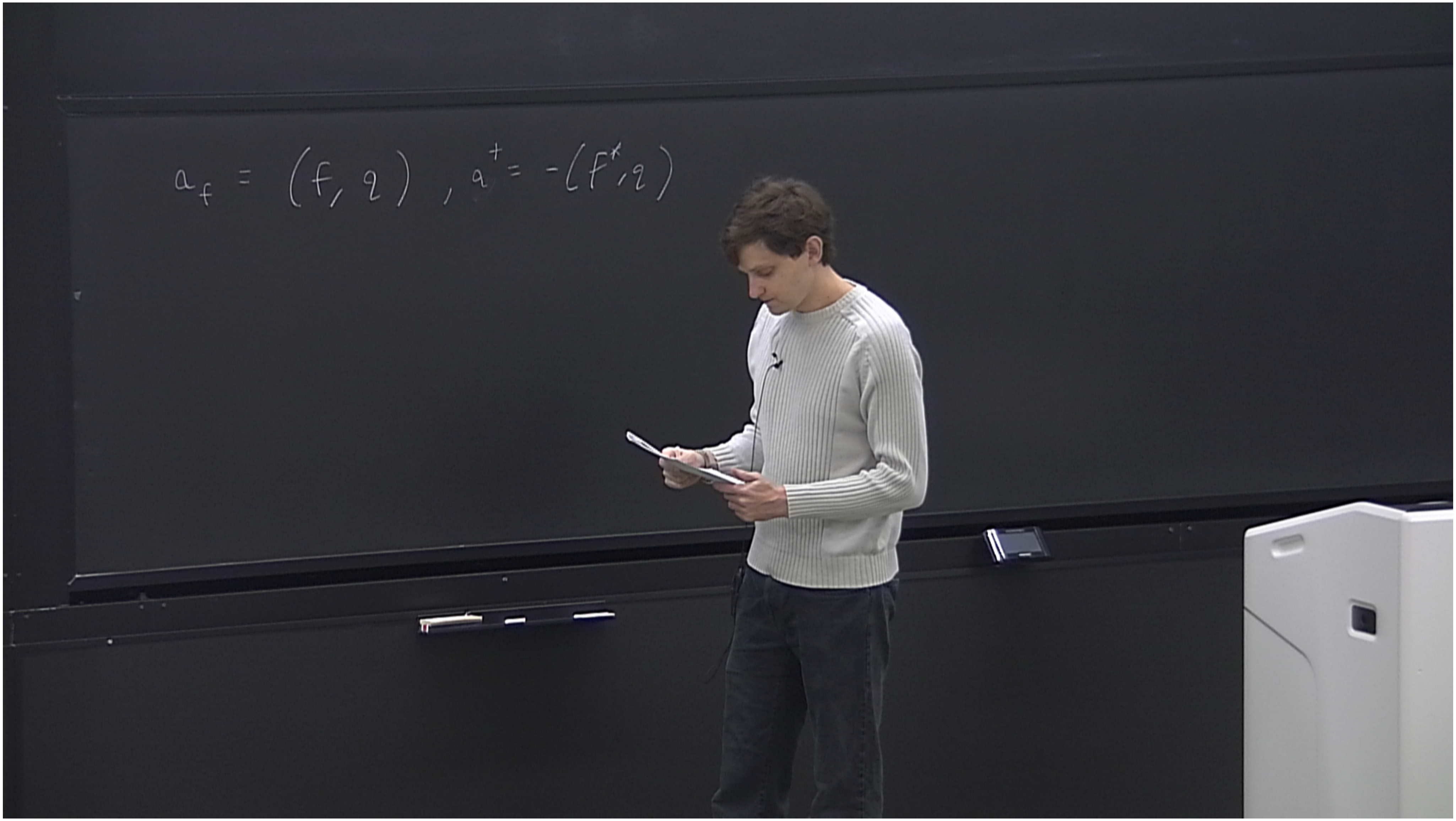
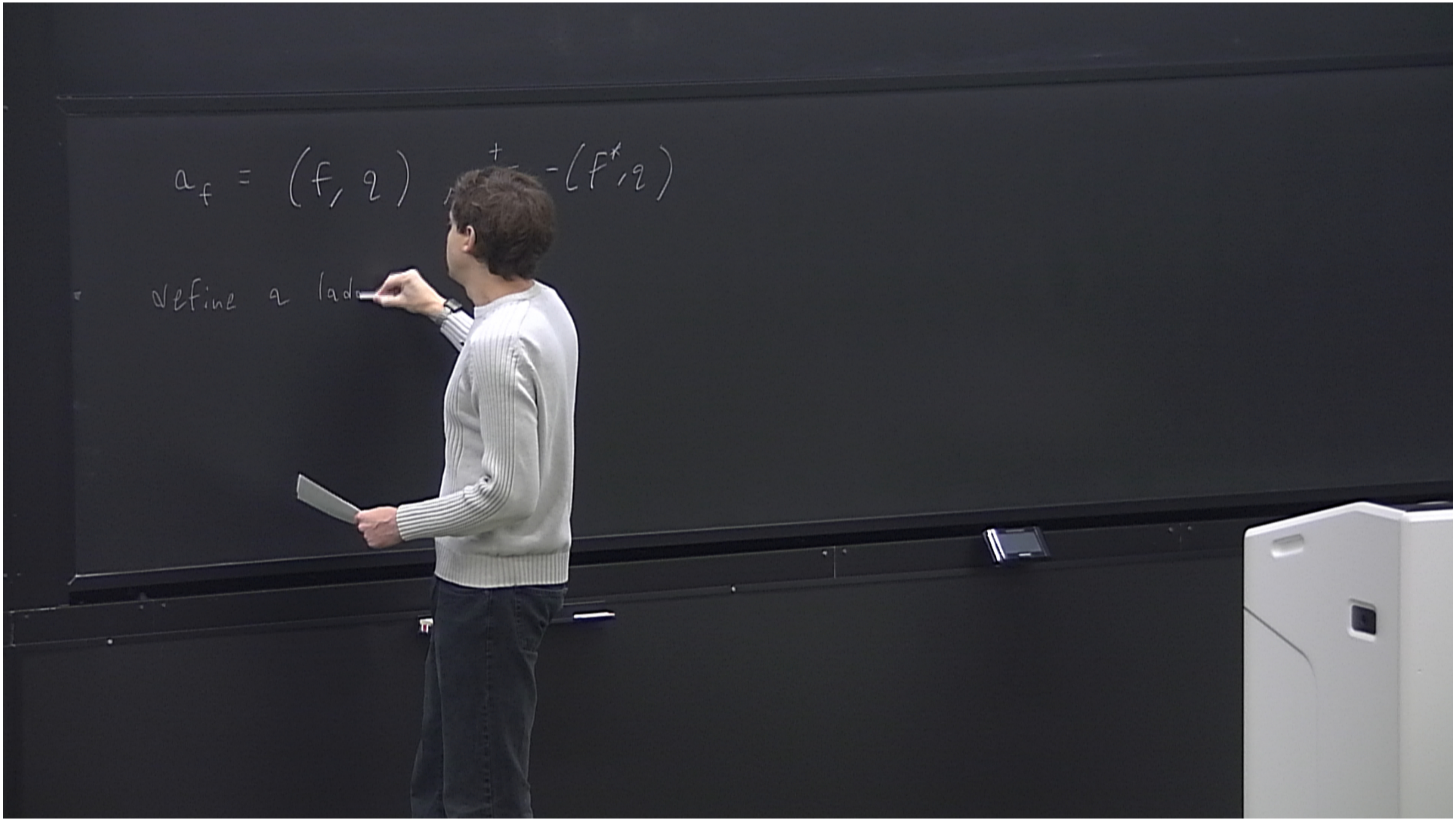


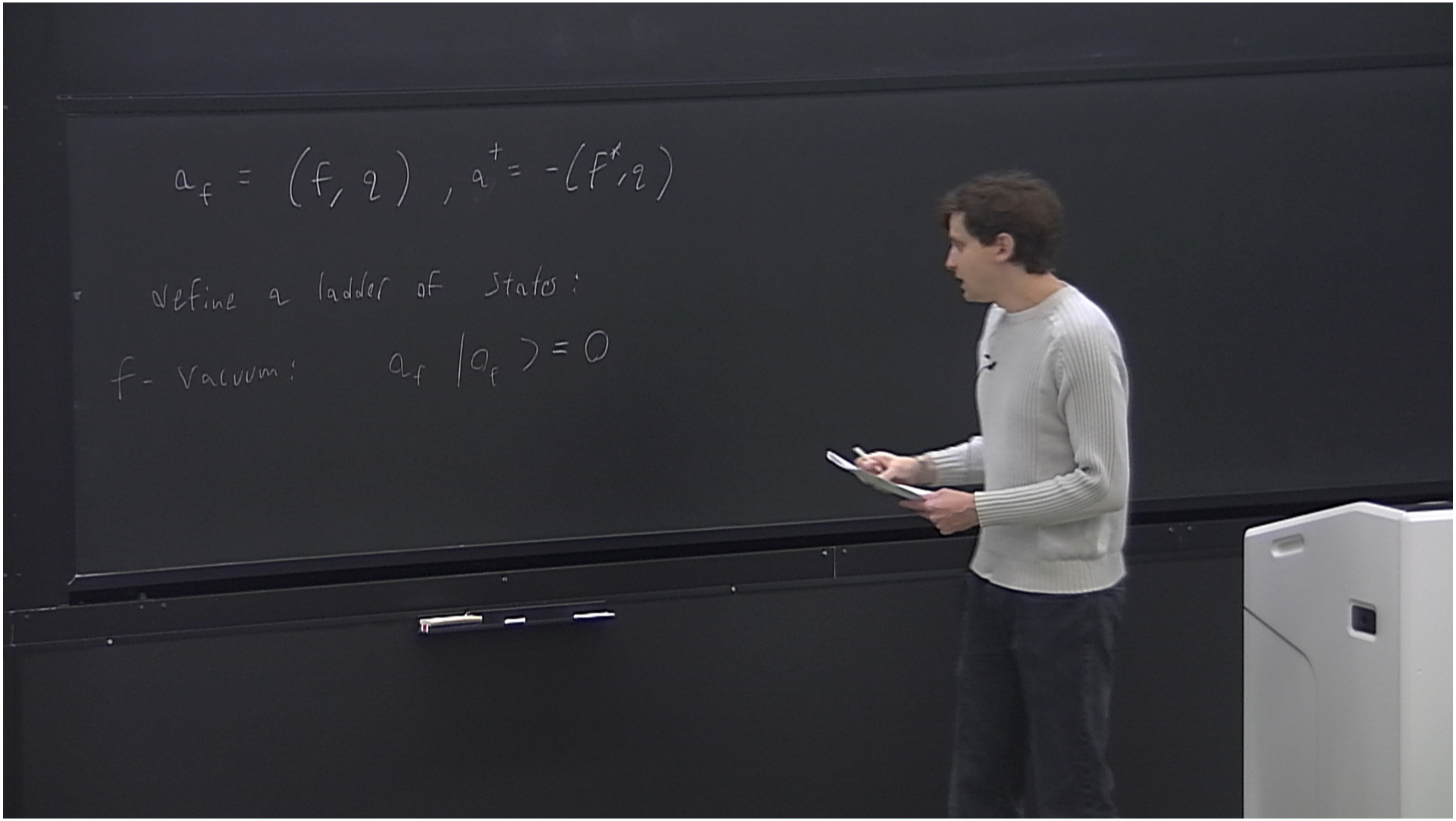
$[q,p] = [\, a f + a^\dagger f^*,\ a\dot f + a^\dagger \dot f^*$
$\dot f^*(t))$
$,\ [a_f, a_f] = [a_f^\dagger, a_f^\dagger] = 0$

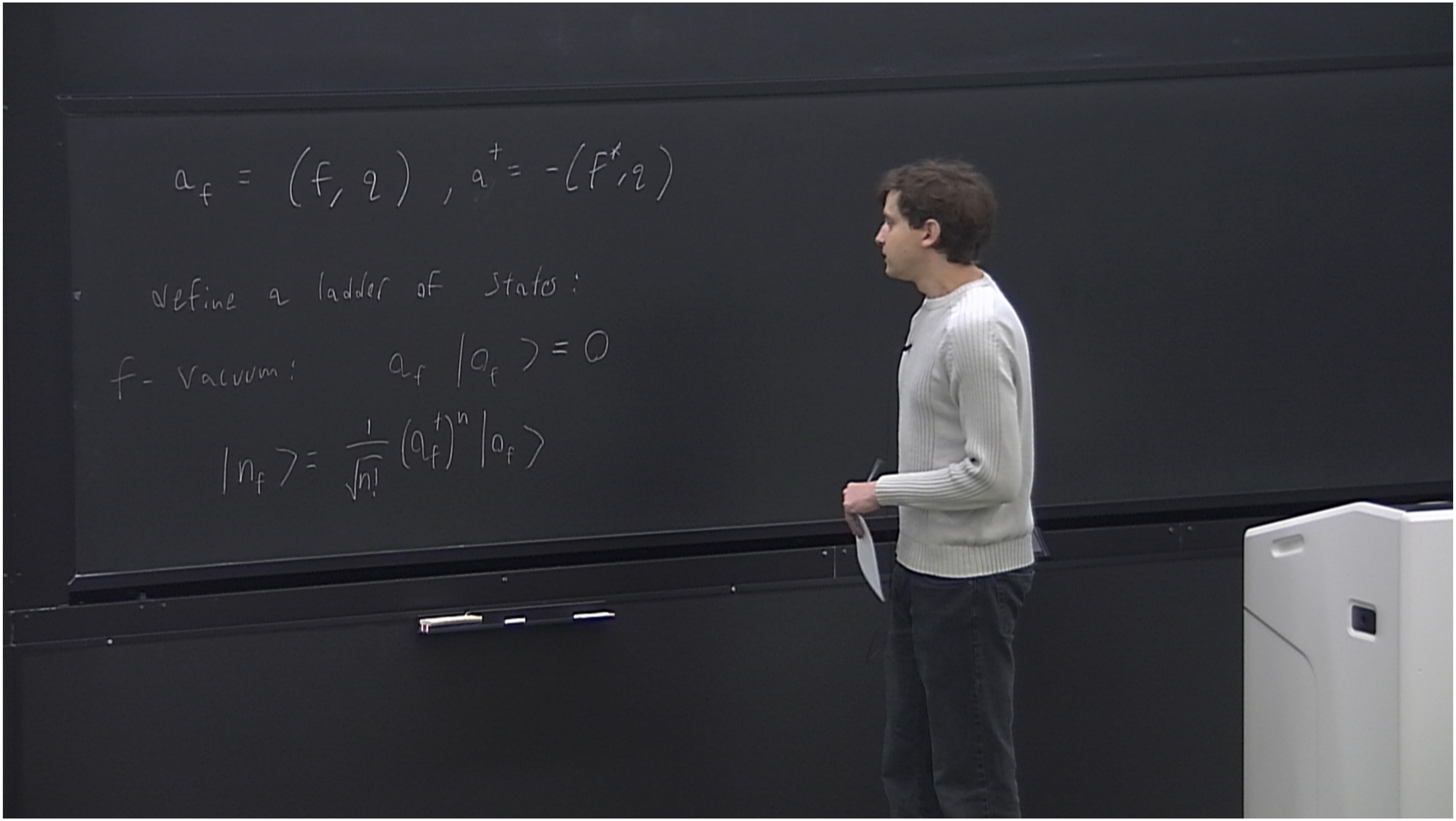

[q, p] = [a f + a† f*, a ḟ + a† ḟ*]
= [a, a†] f ḟ* + [a†
ḟ*(t)
[a_f, a_f] = [a_f†, a_f†] = 0


[q, p] = [a f + a† f*, a ḟ + a† ḟ*]
= [a, a†] f ḟ* + [a†, a] f* ḟ = f f
1
−1
ḟ*(t)
, [a_f, a_f] = [a_f†, a_f†] = 0


$a_f = (f, q)$, $a^+ = -(f^*, q)$


$a_f = (f, a)$
$-(f^*, a)$
define a lad


$a_f = (f, q)$, $a^\dagger = -(f^*, q)$
define a ladder of states:
f- vacuum: $a_f |0_f\rangle = 0$


$a_f = (f, q)$, $a^\dagger = -(f^*, q)$
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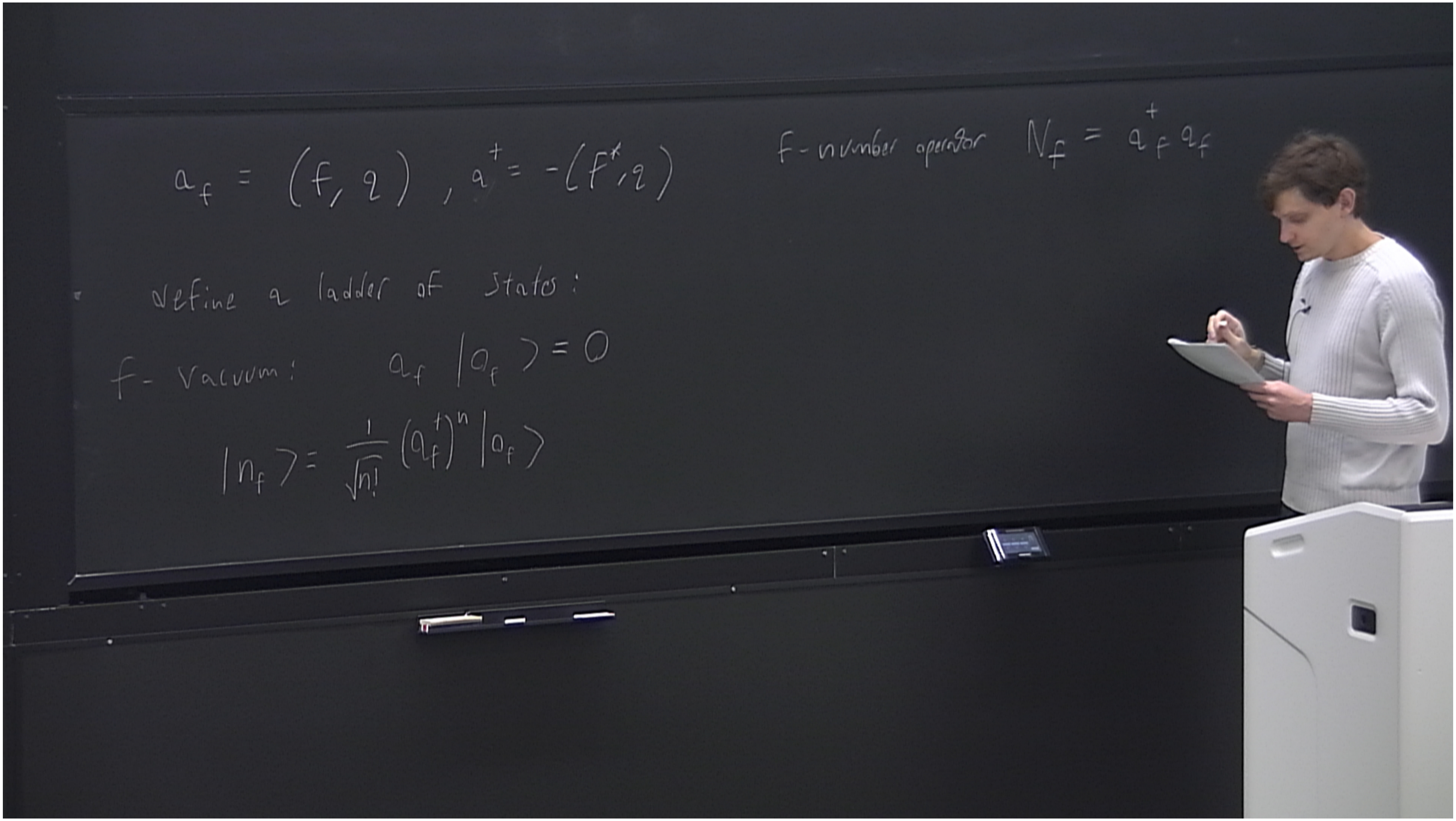
$|n_f\rangle = \frac{1}{\sqrt{n!}} (a_f^\dagger)^n |0_f\rangle$


$a_f = (f, a)$, $a^\dagger = -(f^*, a)$
f-number operator $N_f = a^\dagger_f a_f$
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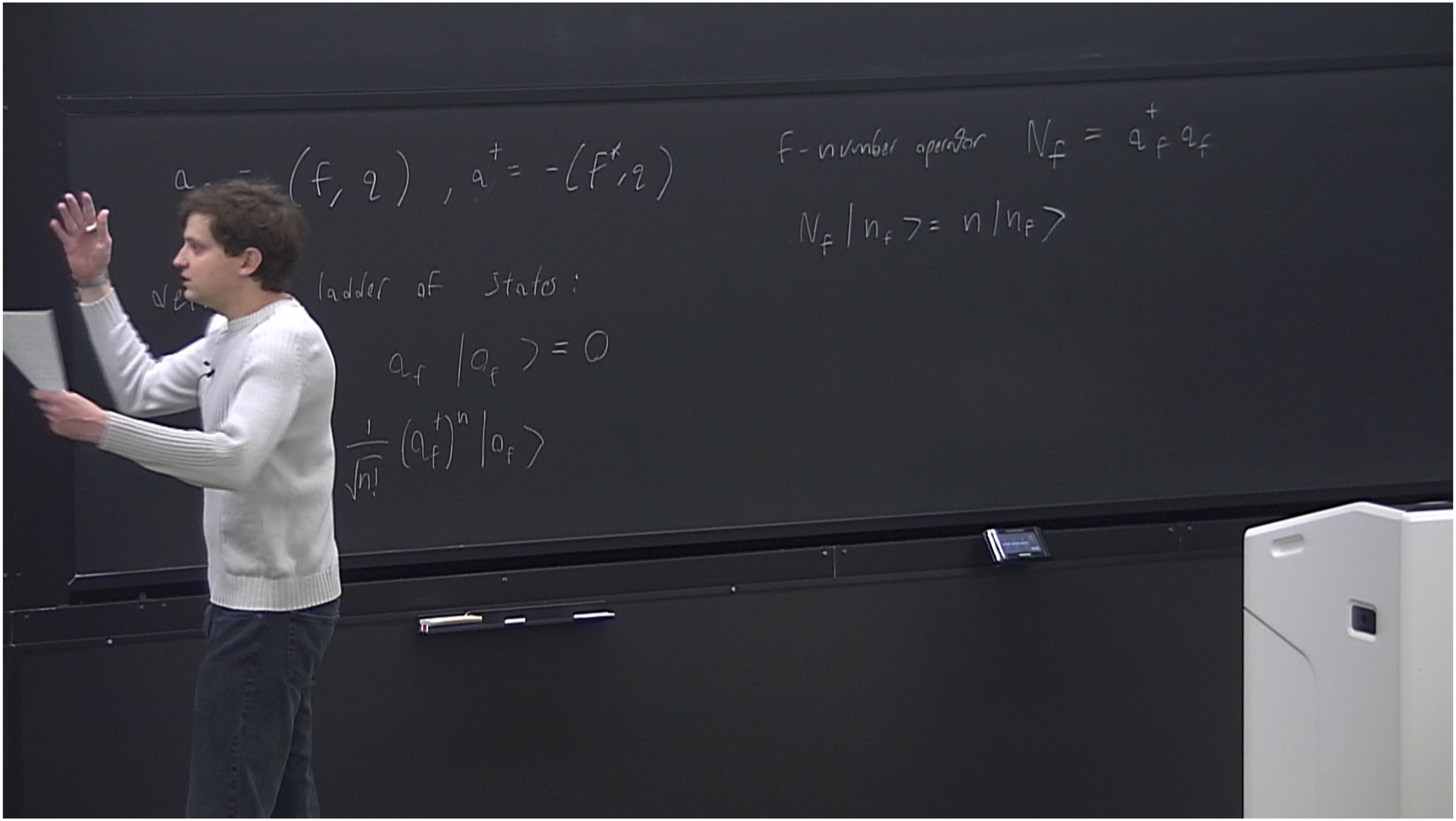
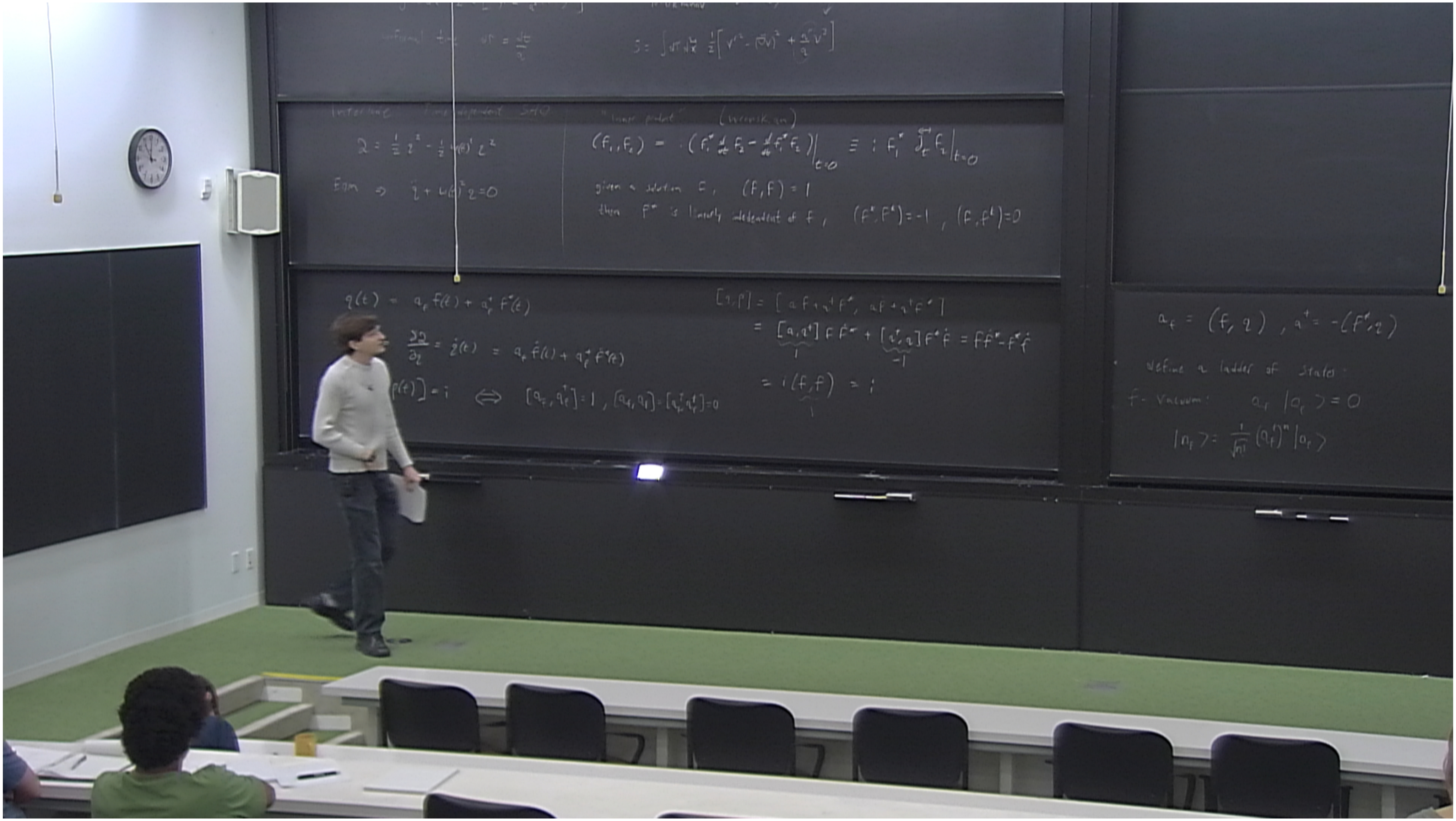
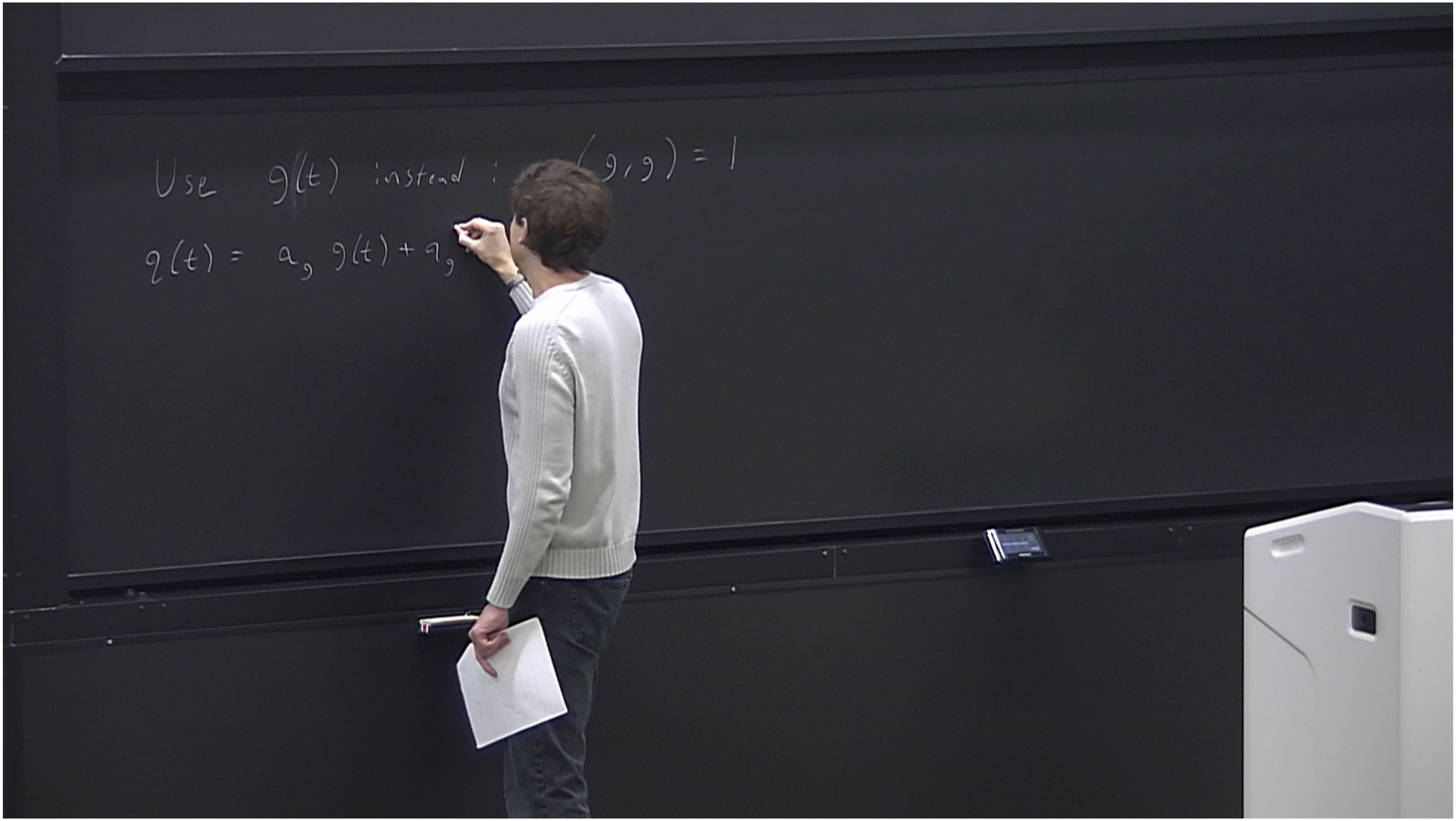
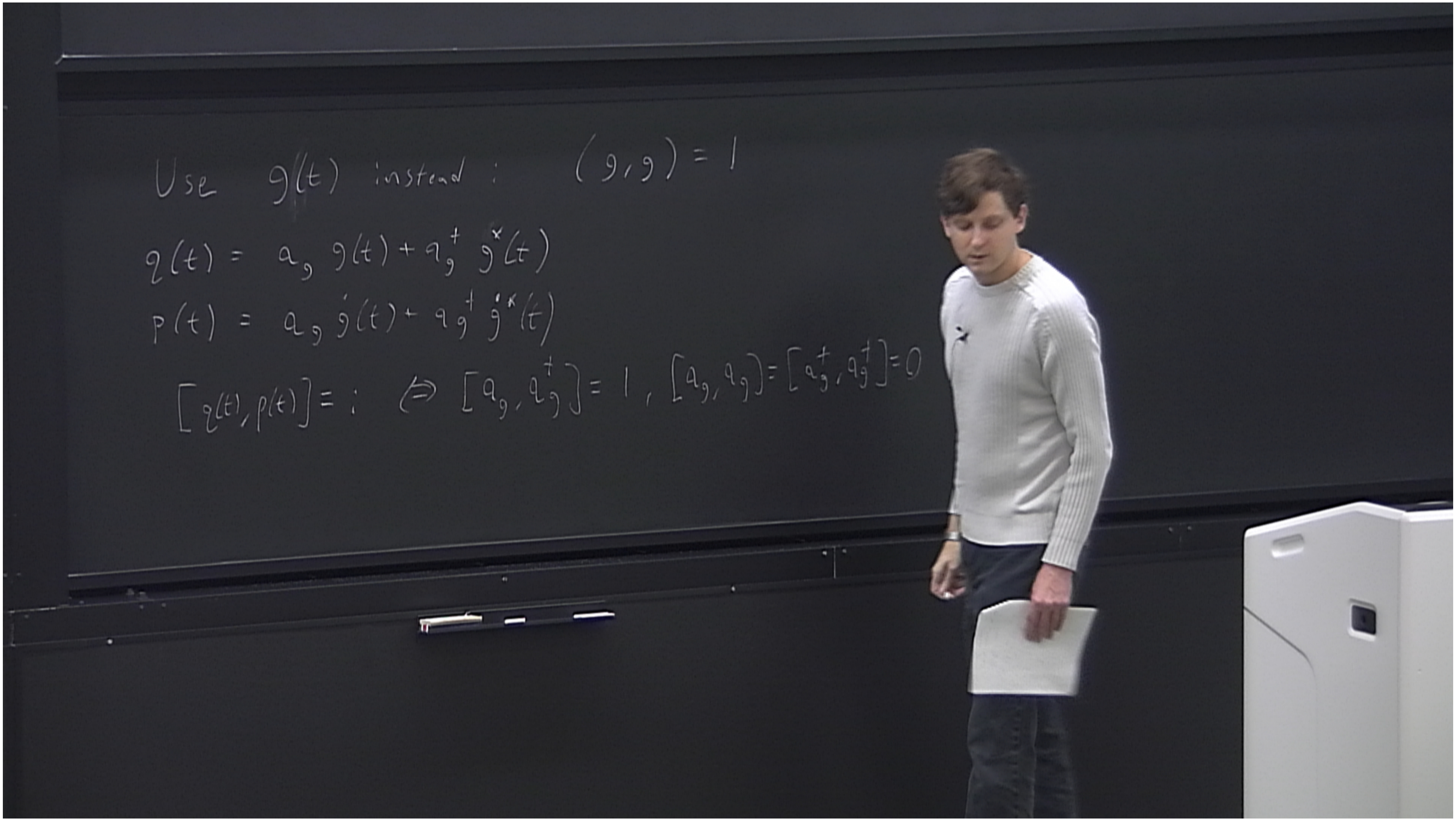
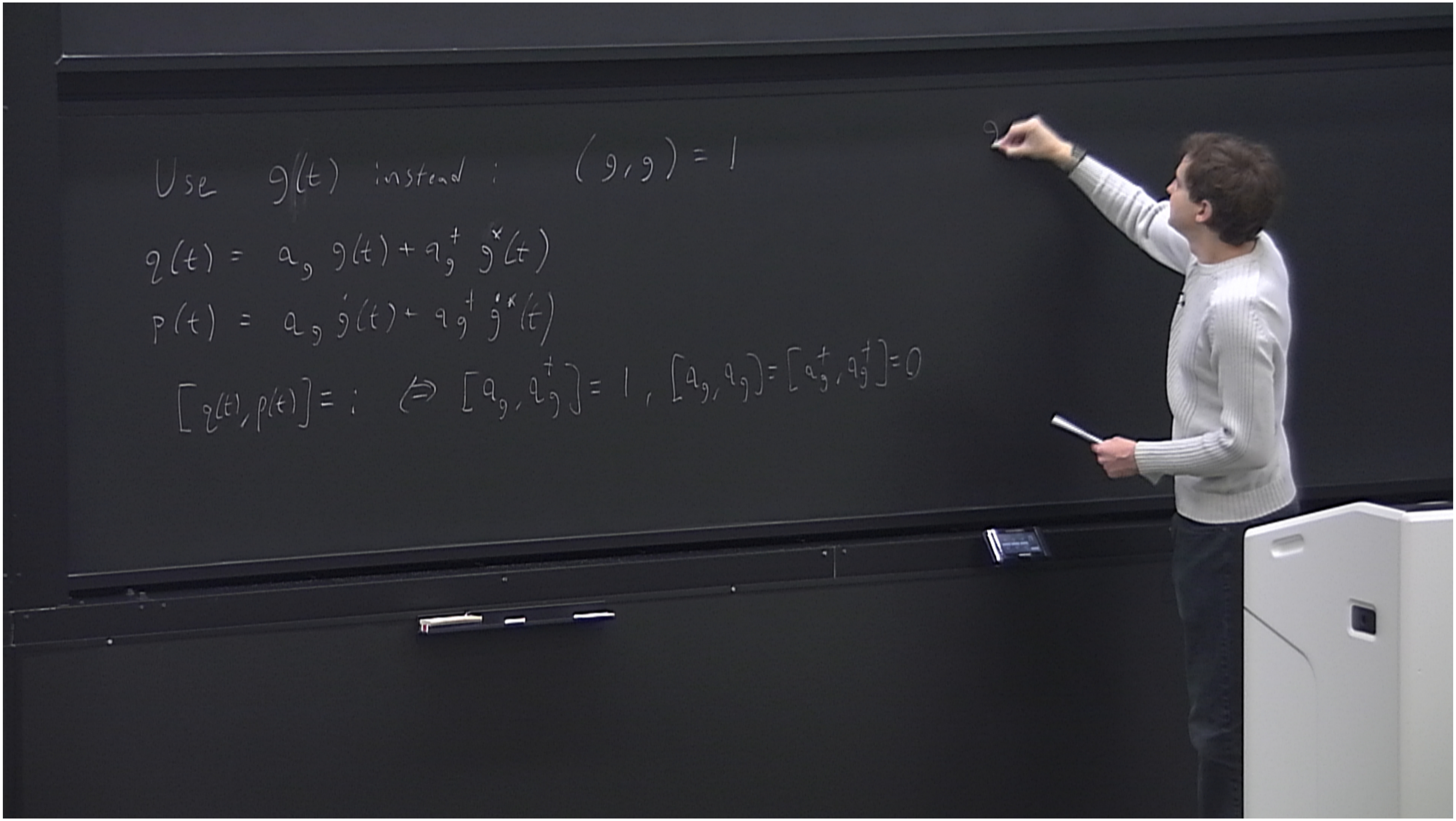

, $a^\dagger = -(f^*, a)$
f-number operator $N_f = a_f^\dagger a_f$
$N_f | n_f \rangle = n | n_f \rangle$
ladder of states:
$a_f | 0_f \rangle = 0$
$\frac{1}{\sqrt{n!}} (a_f^\dagger)^n | 0_f \rangle$




Use g(t) instead : (g,g) = 1
q(t) = a_g g(t) + a_g


Use $g(t)$ instead : $(g,g) = 1$
$q(t) = a_g\, g(t) + a_g^\dagger\, g^*(t)$
$p(t) = a_g\, \dot g(t) + a_g^\dagger\, \dot g^*(t)$
$[q(t), p(t)] = i \iff [a_g, a_g^\dagger] = 1,\ [a_g, a_g] = [a_g^\dagger, a_g^\dagger] = 0$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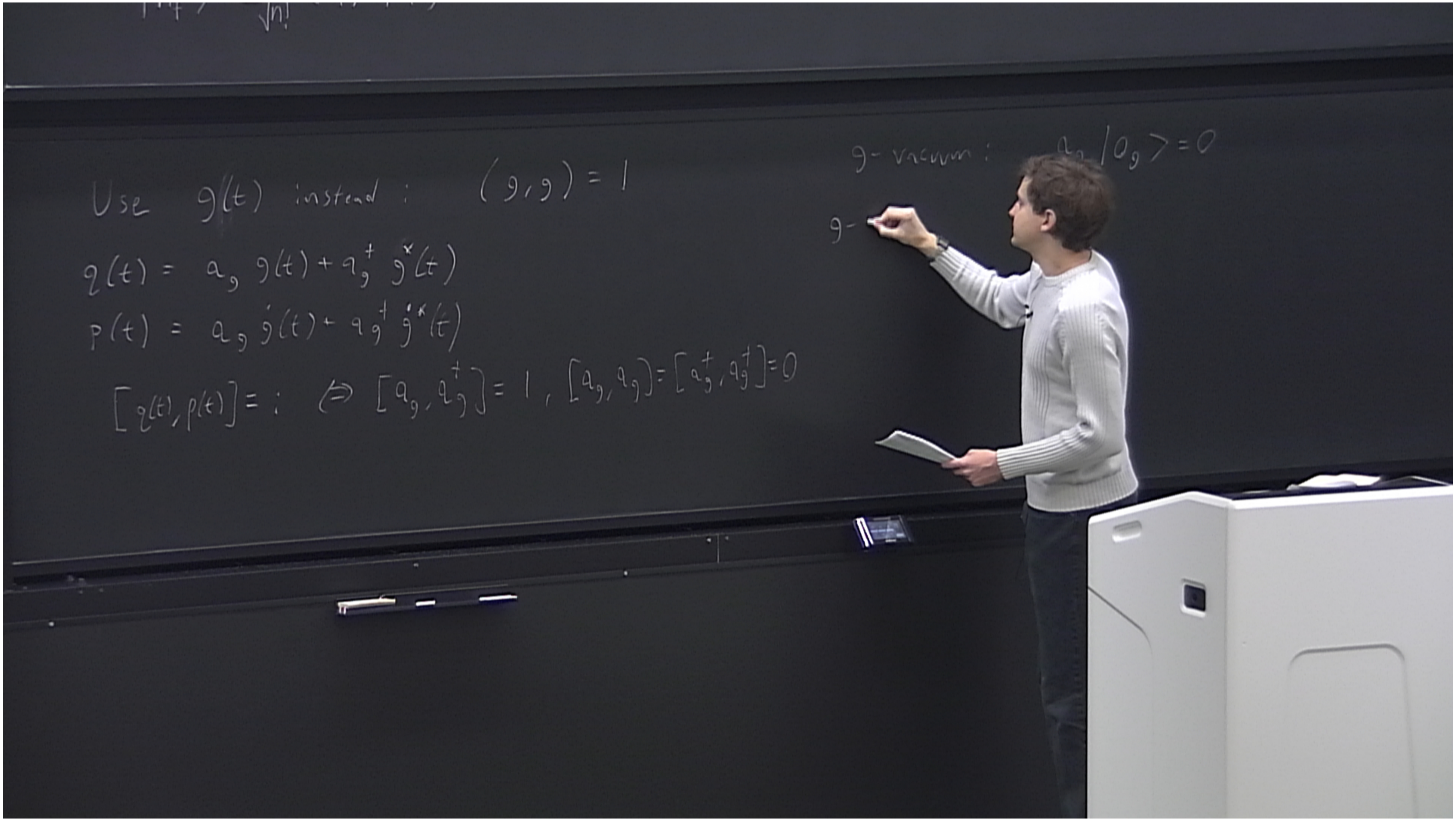
$[q(t), p(t)] = i \Leftrightarrow [a_g, a_g^\dagger] = 1 , [a_g, a_g] = [a_g^\dagger, a_g^\dagger] = 0$



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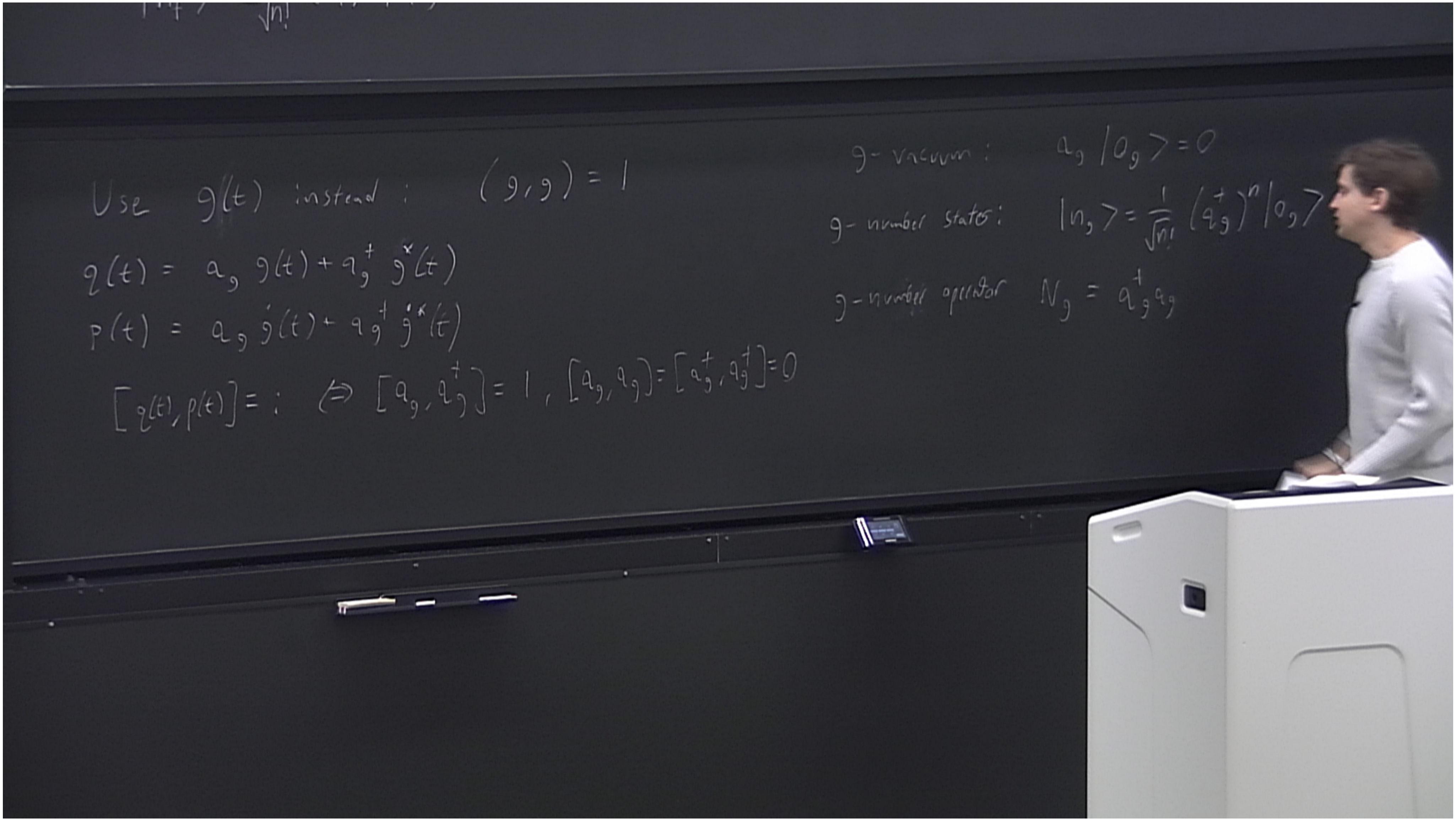
g-vacuum : $a_g |0_g\rangle = 0$
g-



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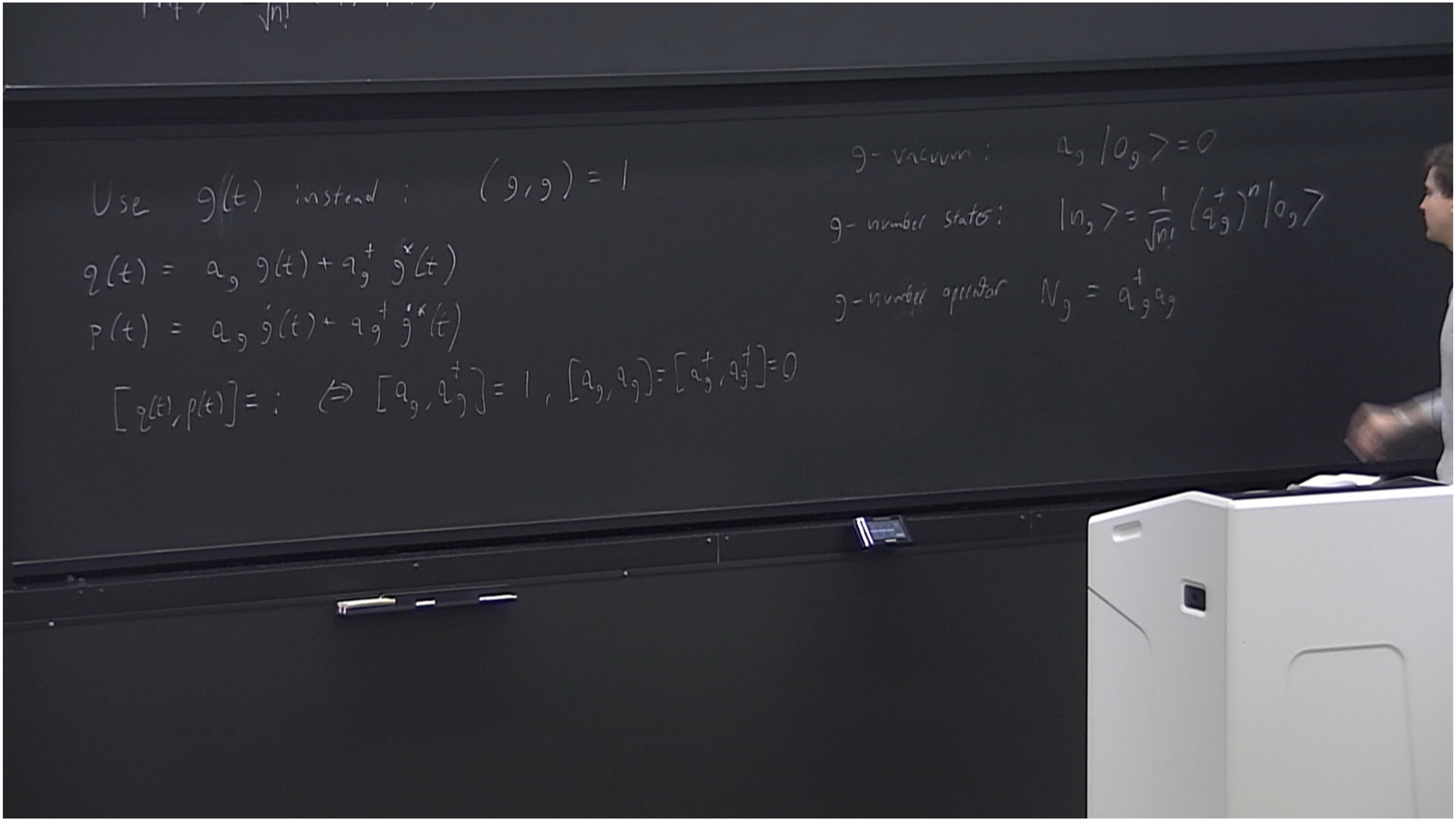
$q(t) = a_g\, g(t) + a_g^\dagger\, g^*(t)$
$p(t) = a_g\, \dot g(t) + a_g^\dagger\, \dot g^*(t)$
$[q(t), p(t)] = i \iff [a_g, a_g^\dagger] = 1,\ [a_g, a_g] = [a_g^\dagger, a_g^\dagger] = 0$
g-vacuum : $a_g |0_g\rangle = 0$
g-number states : $|n_g\rangle = \frac{1}{\sqrt{n!}} (a_g^\dagger)^n |0_g\rangle$
g-number operator $N_g = a_g^\dagger a_g$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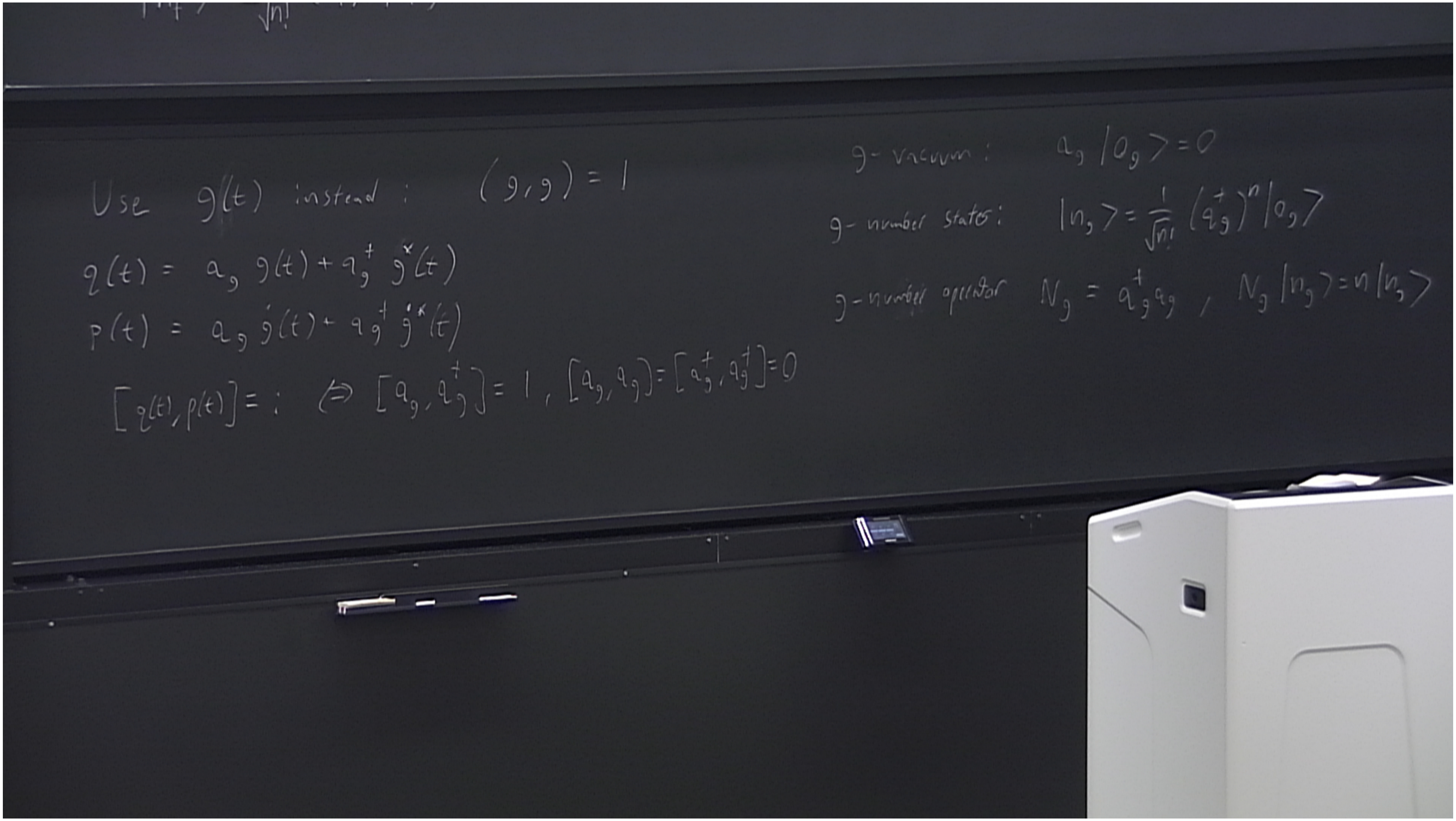
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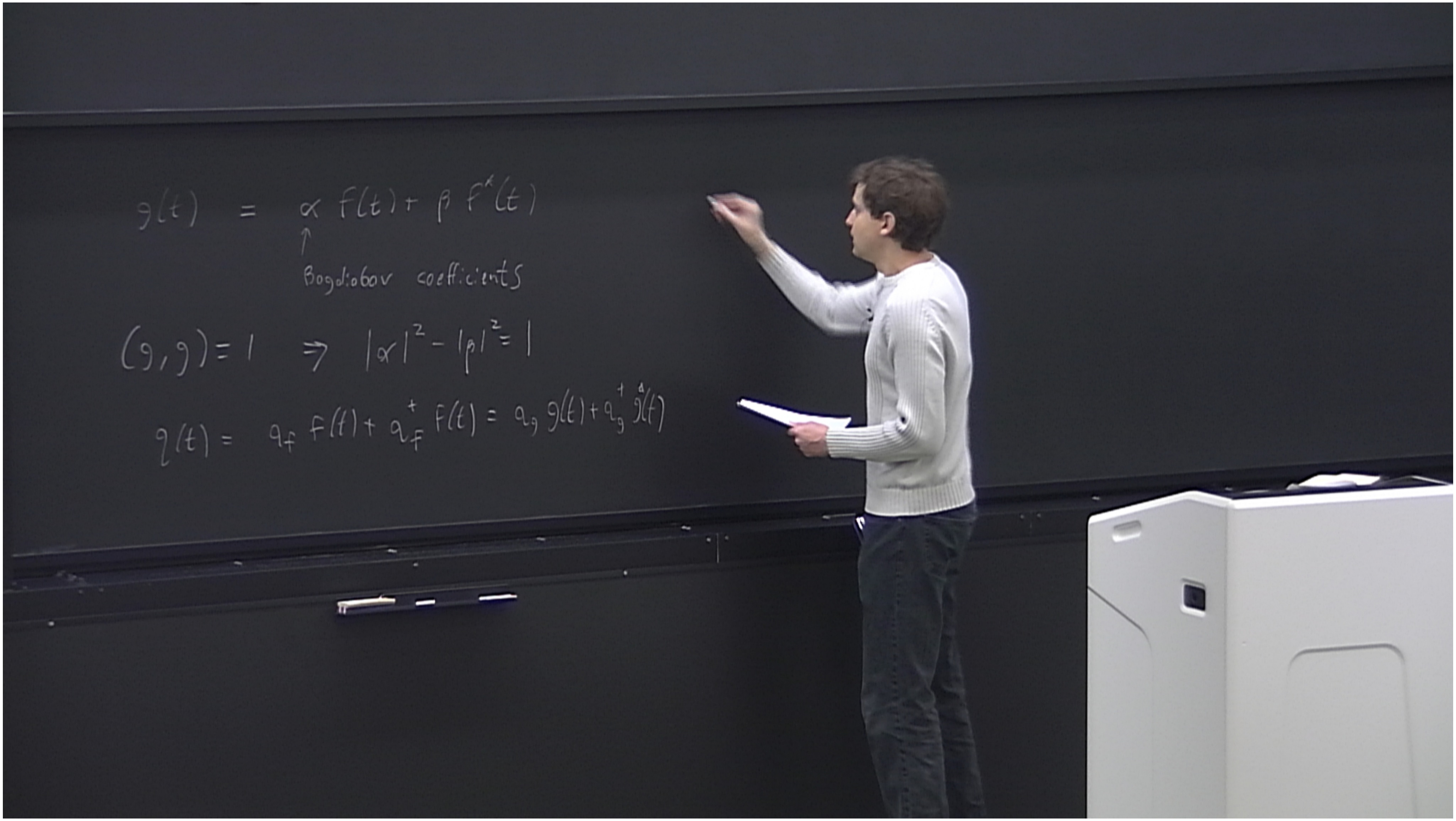
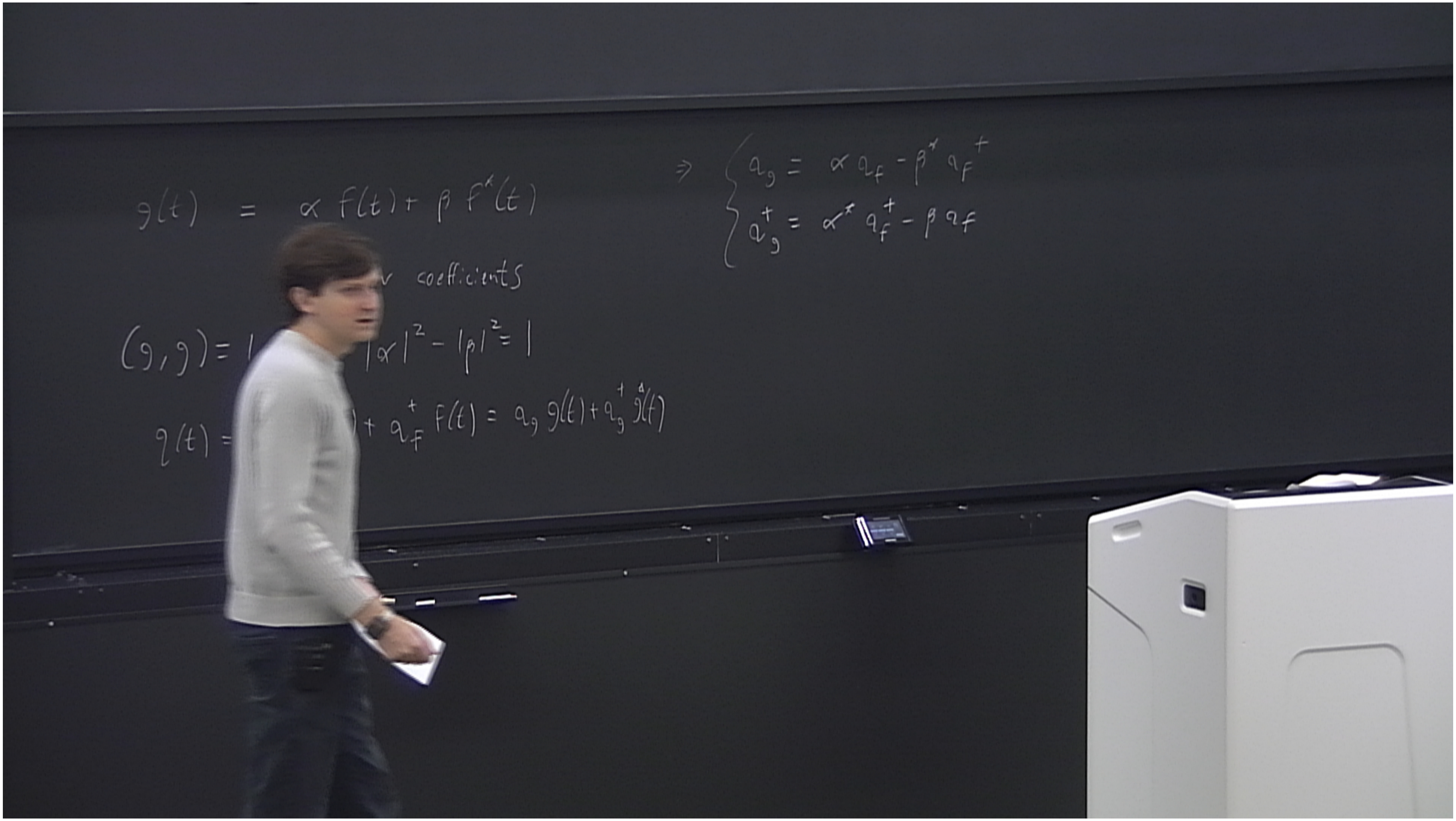
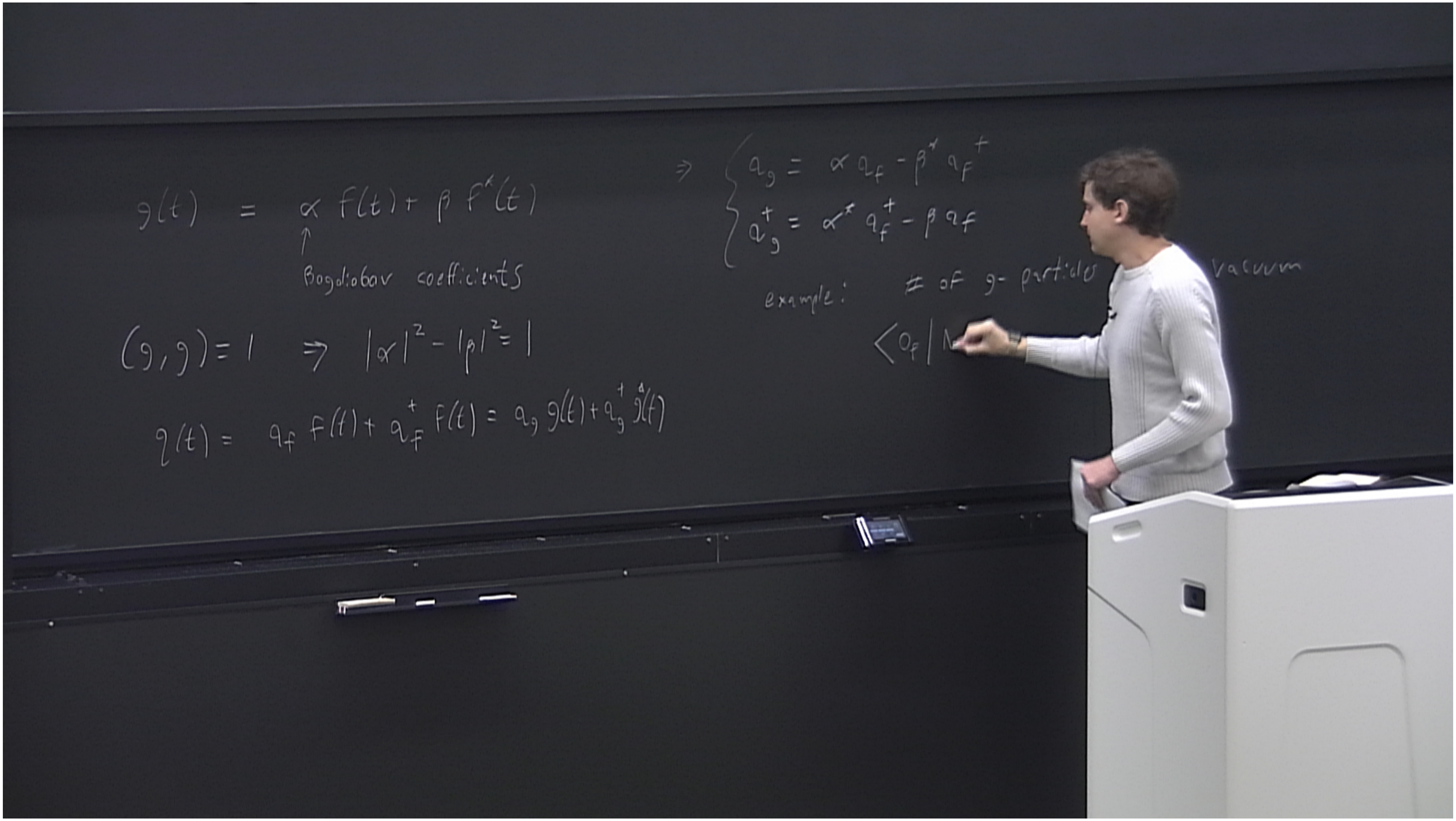
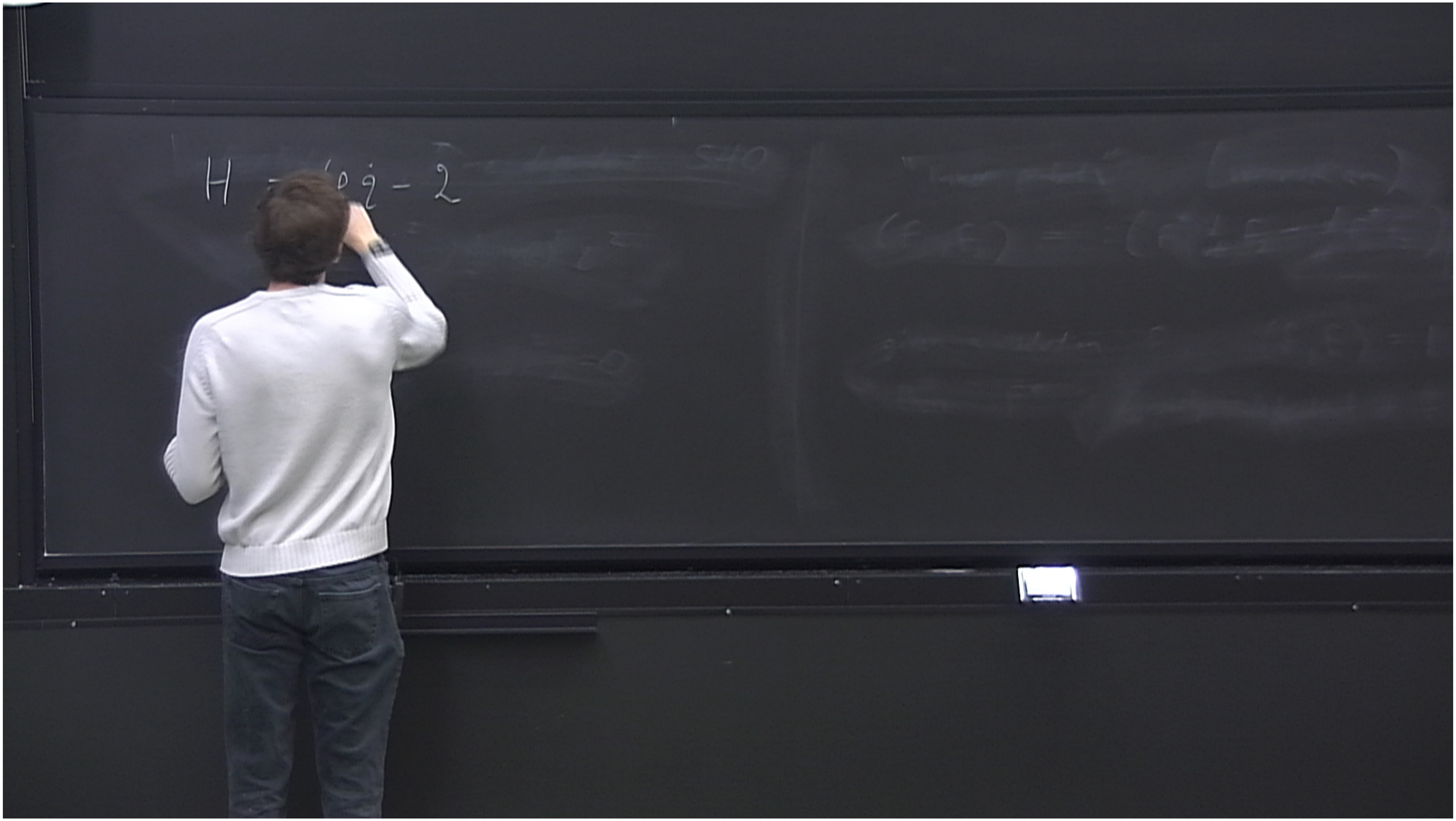
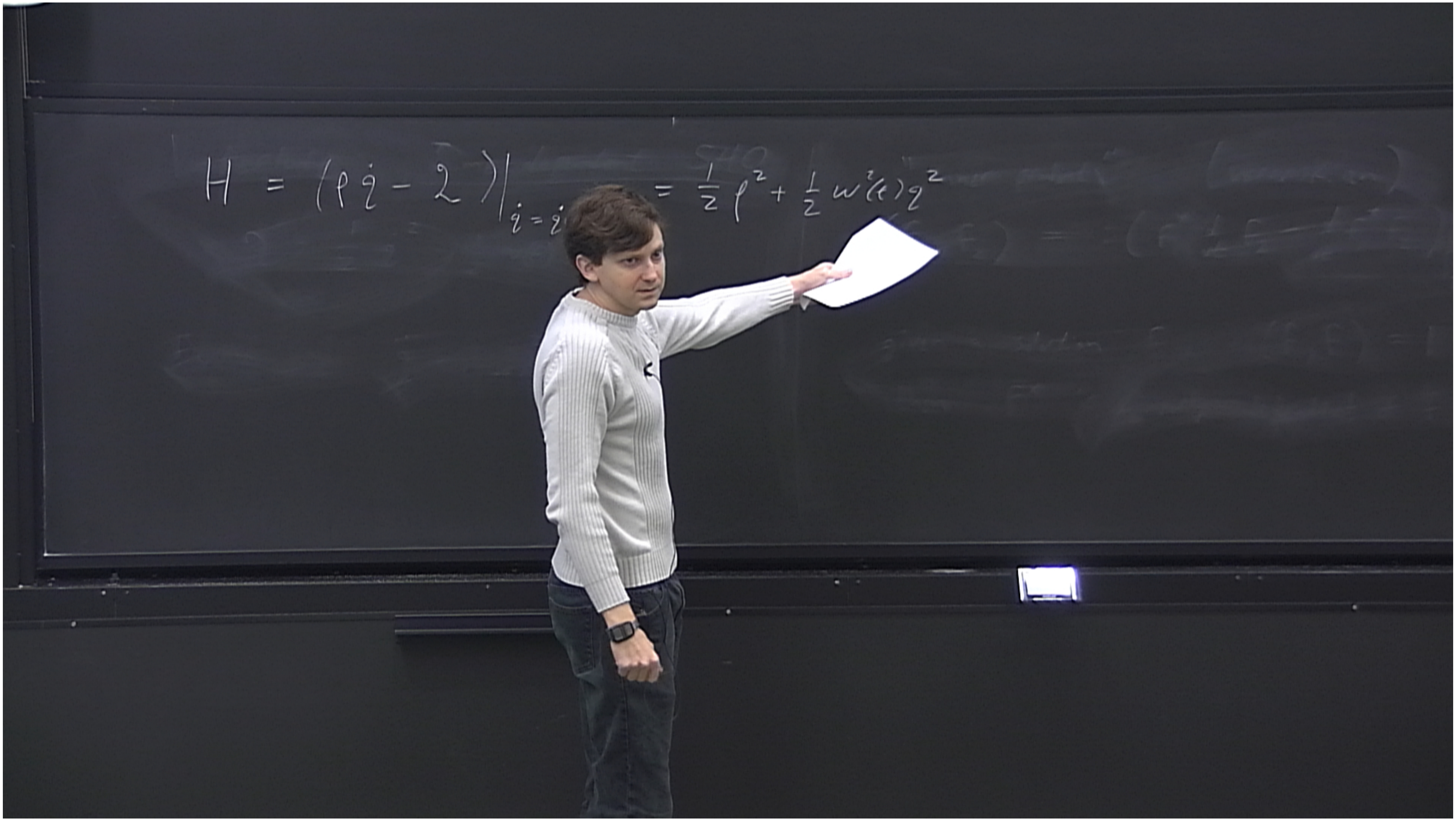

$g(t) = \alpha f(t) + \beta f^*(t)$
Bogoliobov coefficients
$(g,g)=1 \Rightarrow |\alpha|^2 - |\beta|^2 = 1$
$q(t) = a_f f(t) + a_f^+ f(t) = a_g g(t) + a_g^+ \hat{g}(t)$

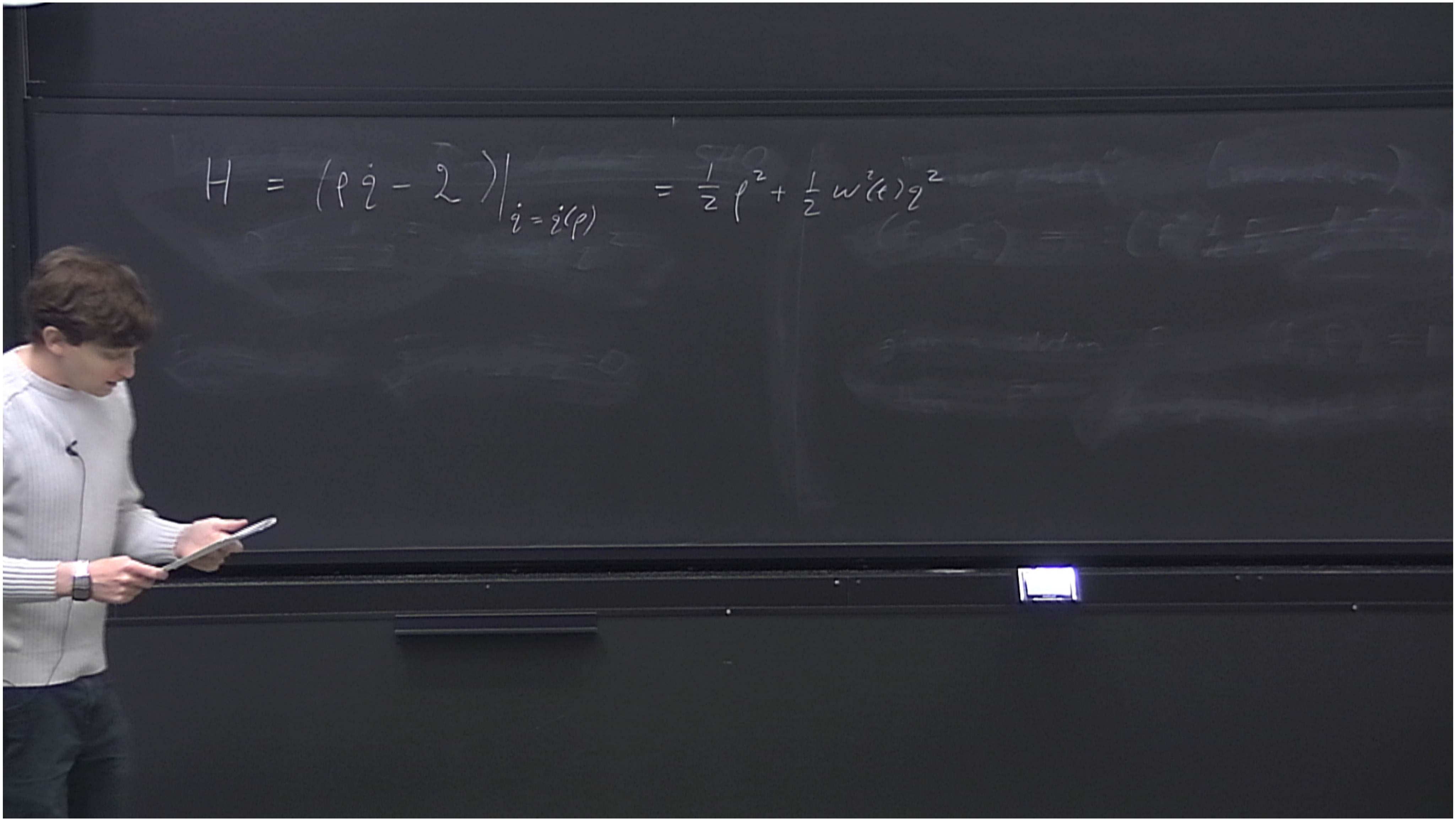

$g(t) = \alpha f(t) + \beta f^*(t)$
coefficients
$(g,g) = 1$
$|\alpha|^2 - |\beta|^2 = 1$
$a_g = \alpha a_f - \beta^* a_f^+$
$a_g^+ = \alpha^* a_f^+ - \beta a_f$

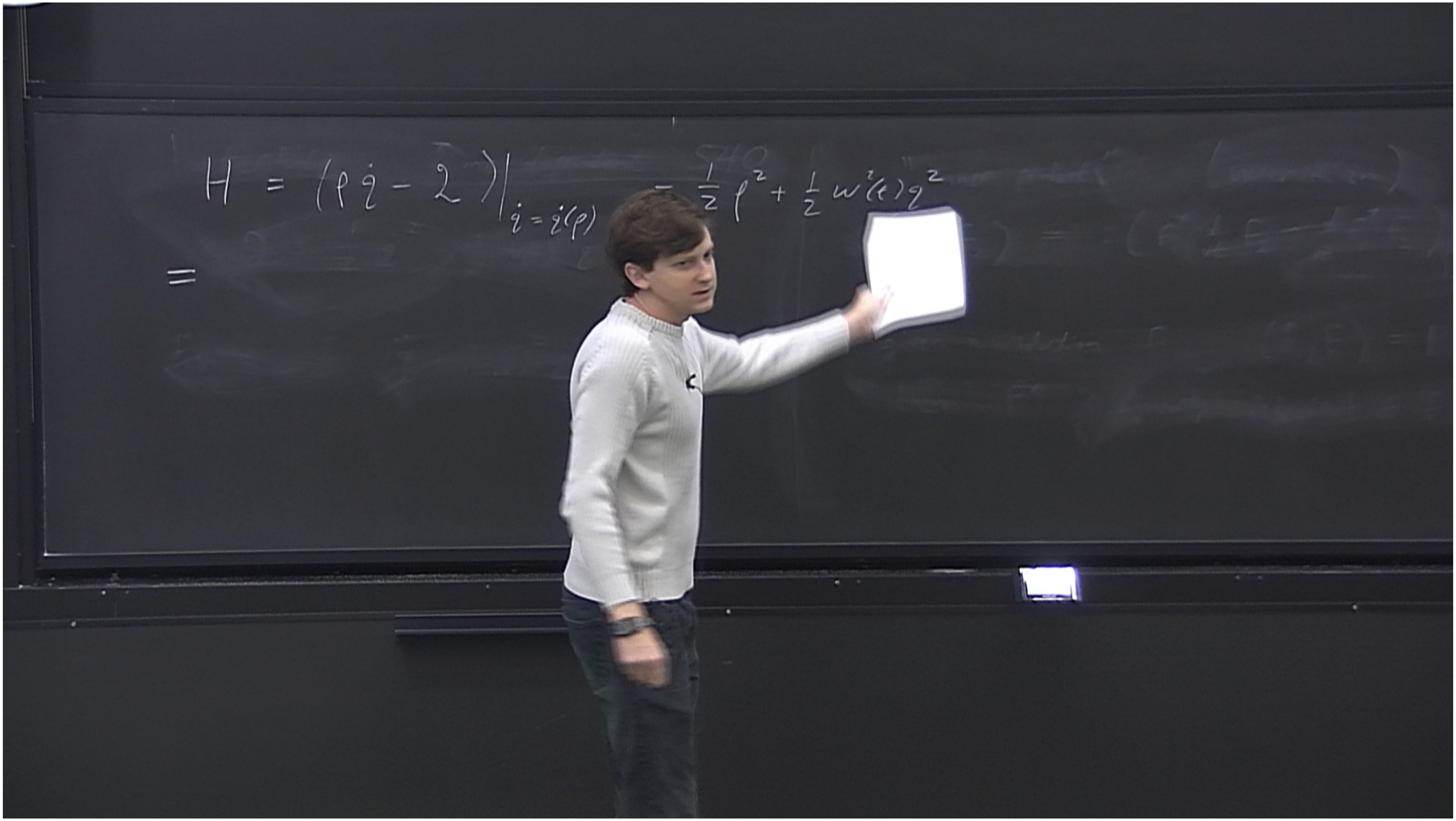

$g(t) = \alpha f(t) + \beta f^*(t)$
Bogoliobov coefficients
$(g,g)=1 \Rightarrow |\alpha|^2 - |\beta|^2 = 1$
$q(t) = a_f f(t) + a_f^+ f(t) = a_g g(t) + a_g^+ \hat{g}(t)$
$\Rightarrow \begin{cases} a_g = \alpha a_f - \beta^* a_f^+ \\ a_g^+ = \alpha^* a_f^+ - \beta a_f \end{cases}$
example: # of g-particles
vacuum
$\langle 0_f |$

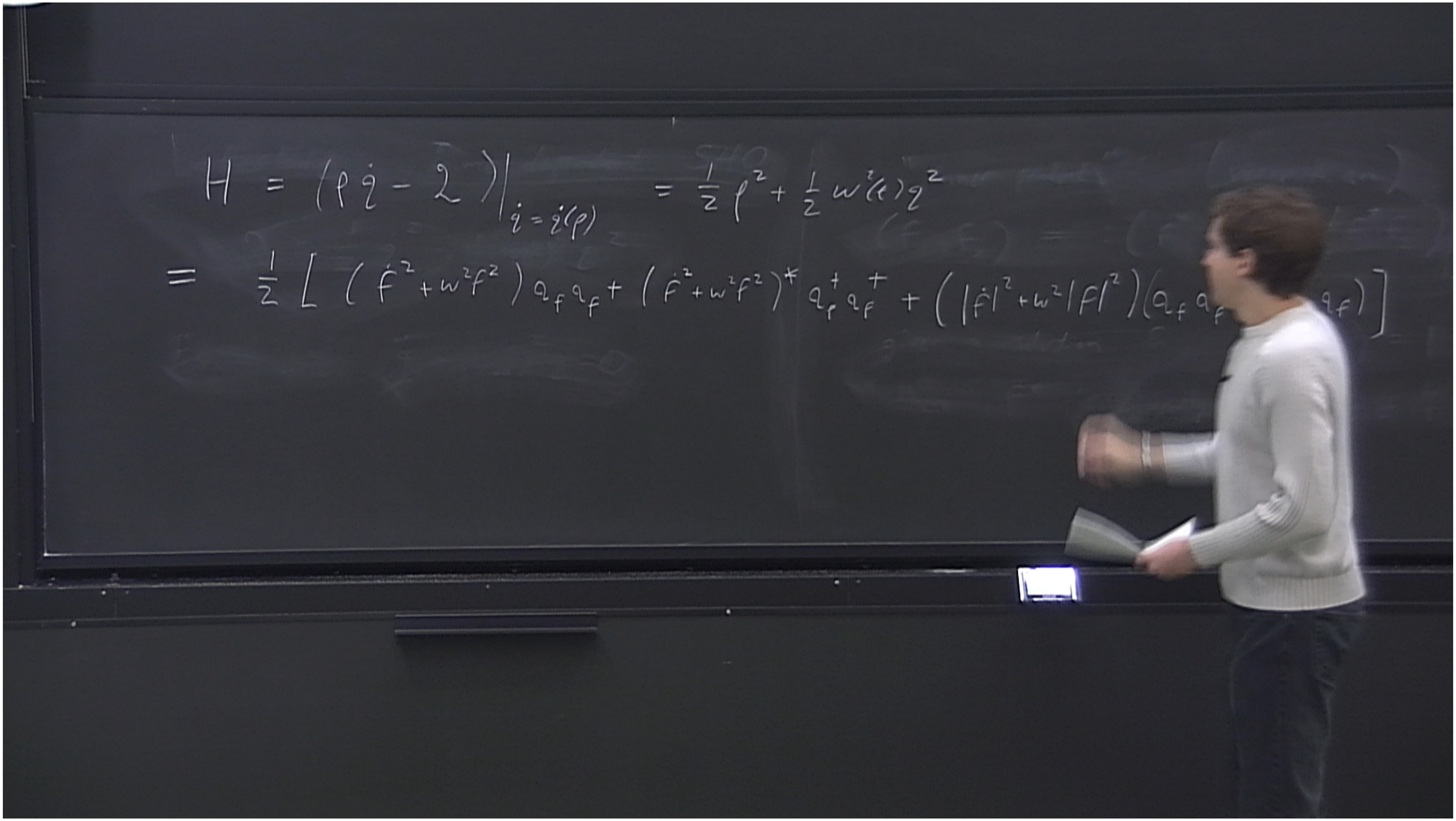
$$= |\beta|^2 \langle 0_f | \underbrace{a_f a_f^\dagger}_{\underbrace{a_f^\dagger a_f}_{=0} + 1} | 0_f \rangle = \boxed{|\beta|^2}$$




$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}\ldots} = \frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$


$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$


H = (p q̇ − L)|q̇ = q̇(p) = ½ p² + ½ ω²(t) q²
=


$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$
$= \frac{1}{2}\Big[(\dot{f}^2 + \omega^2 f^2)\, a_f a_f + (\dot{f}^2 + \omega^2 f^2)^* a_f^\dagger a_f^\dagger + \big(|\dot{f}|^2 + \omega^2 |f|^2\big)(a_f a_f$

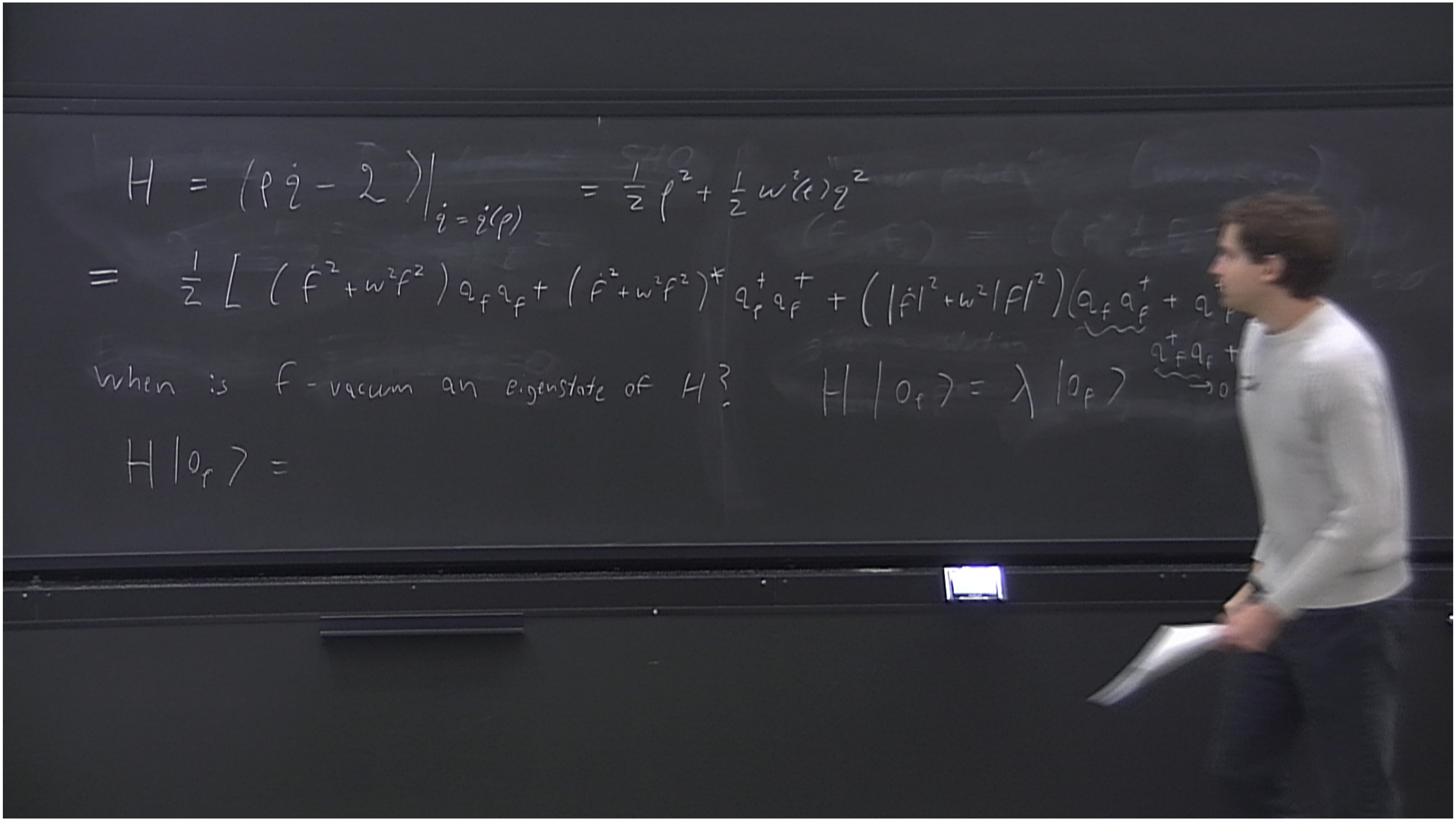
$$H = \left(p\dot{q} - \mathcal{L}\right)\Big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$$

$$= \frac{1}{2}\left[\left(\dot{f}^2 + \omega^2 f^2\right)a_f a_f + \left(\dot{f}^2 + \omega^2 f^2\right)^* a_f^\dagger a_f^\dagger + \left(|\dot{f}|^2 + \omega^2 |f|^2\right)\left(a_f a_f^\dagger + a_f^\dagger a_f\right)\right]$$

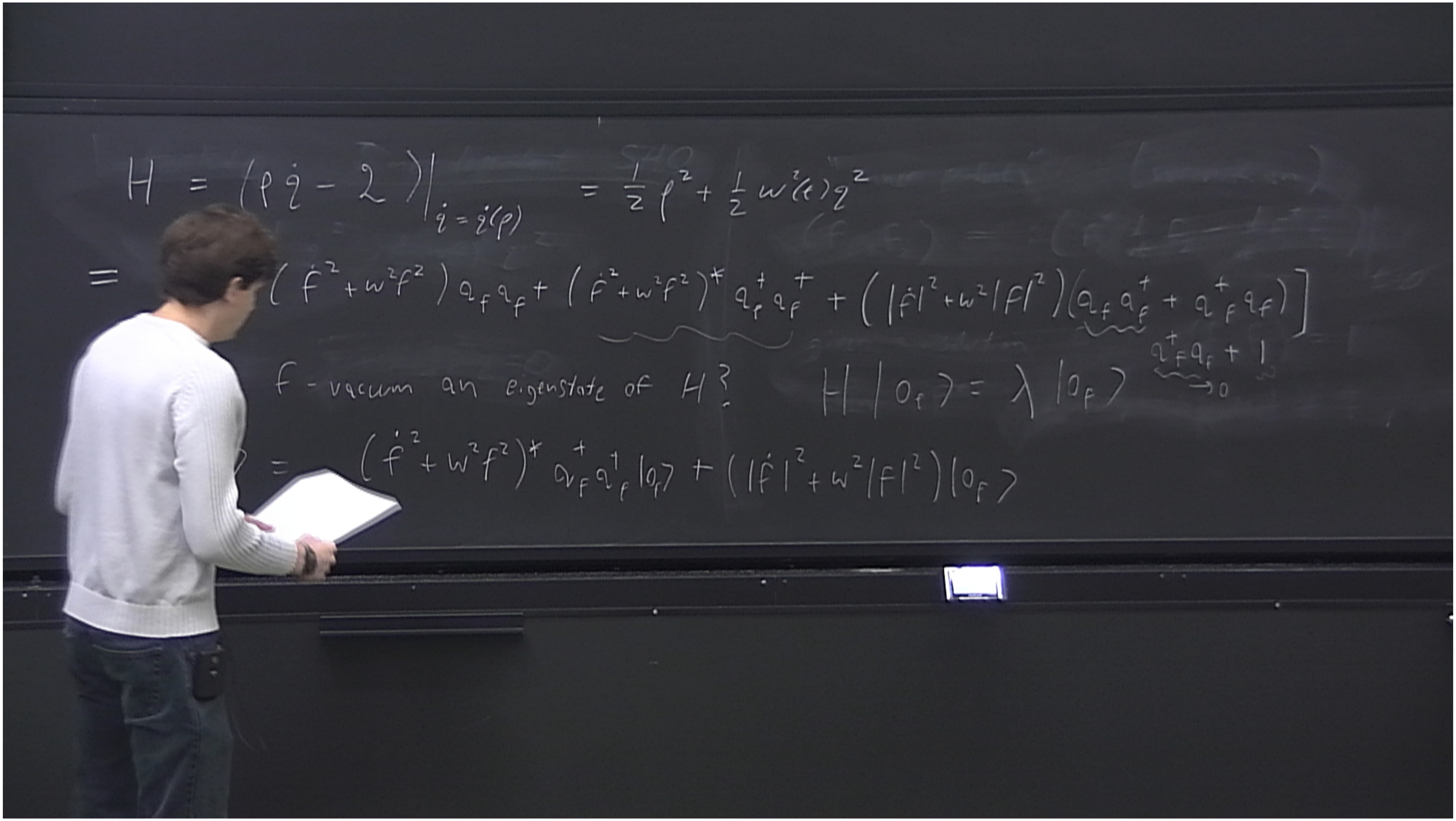
$$H = (p\dot{q} - L)\Big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}\omega^2(t)q^2$$

$$= \frac{1}{2}\Big[(\dot{f}^2 + \omega^2 f^2)\, a_f a_f + (\dot{f}^2 + \omega^2 f^2)^*\, a_f^\dagger a_f^\dagger + \left(|\dot{f}|^2 + \omega^2 |f|^2\right)\left(a_f a_f^\dagger + a_f^\dagger a_f\right)\Big]$$

When is f-vacuum an eigenstate of H? $\quad H|0_f\rangle = \lambda |0_f\rangle$



$H = (p\dot{q} - L)\big|_{\dot{q}=\dot{q}(p)} = \frac{1}{2}p^2 + \frac{1}{2}w^2(t)q^2$
$= \frac{1}{2}\Big[(\dot{f}^2 + w^2 f^2)\,a_f a_f + (\dot{f}^2 + w^2 f^2)^*\,a_f^\dagger a_f^\dagger + (|\dot{f}|^2 + w^2|f|^2)(a_f a_f^\dagger + a_f^\dagger a_f$
when is f-vacuum an eigenstate of H?
$H|0_f\rangle = \lambda|0_f\rangle$
$H|0_f\rangle =$


H = (pq̇ − L)|q̇ = q̇(p) = ½ p² + ½ w²(t) q²
= (ḟ² + w²f²) af af + (ḟ² + w²f²)* af† af† + (|ḟ|² + w²|f|²)(af af† + af† af)]
af† af + 1
→ 0
f - vacuum an eigenstate of H?
H |0f⟩ = λ |0f⟩
= (ḟ² + w²f²)* af† af† |0f⟩ + (|ḟ|² + w²|f|²) |0f⟩

ne

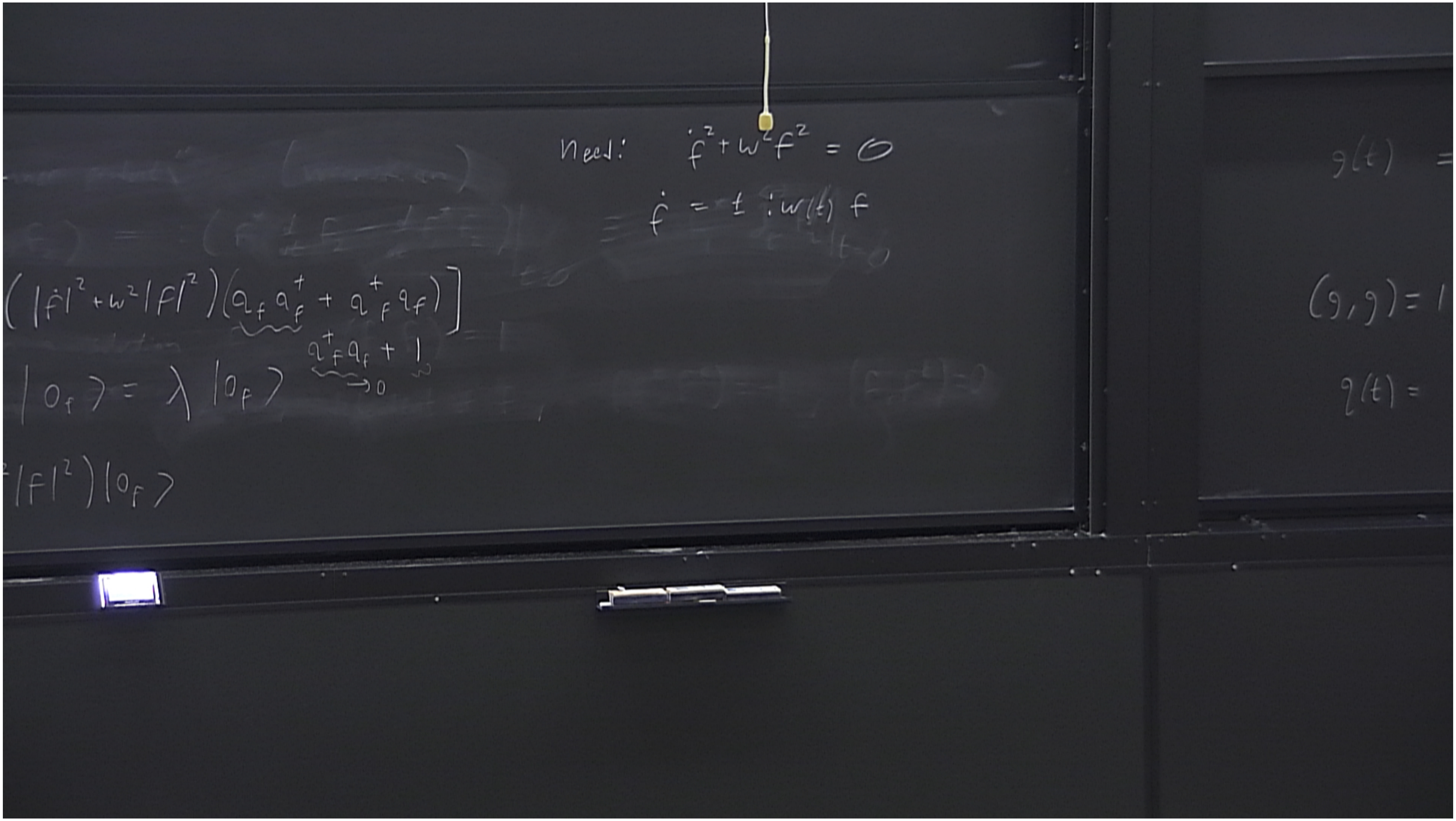
$$H = (p\dot{q} - L)\big|_{\dot{q} = \dot{q}(p)} = \frac{1}{2} p^2 + \frac{1}{2}\omega^2(t) q^2$$

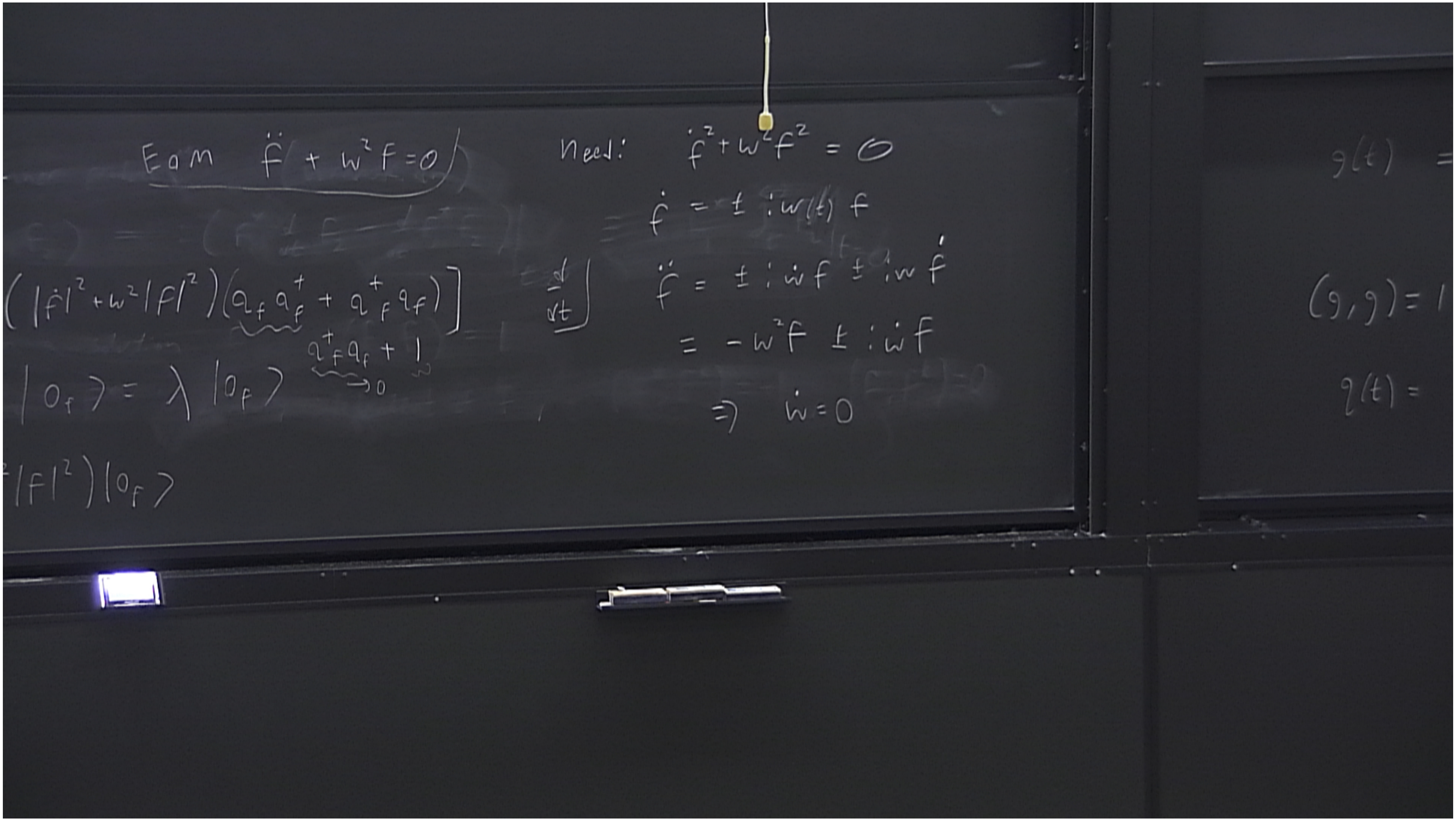
$$= \frac{1}{2}\Big[(\dot{f}^2 + \omega^2 f^2)\, a_f a_f + (\dot{f}^2 + \omega^2 f^2)^* a_f^\dagger a_f^\dagger + (|\dot{f}|^2 + \omega^2 |f|^2)(a_f a_f^\dagger + a_f^\dagger a_f)\Big]$$

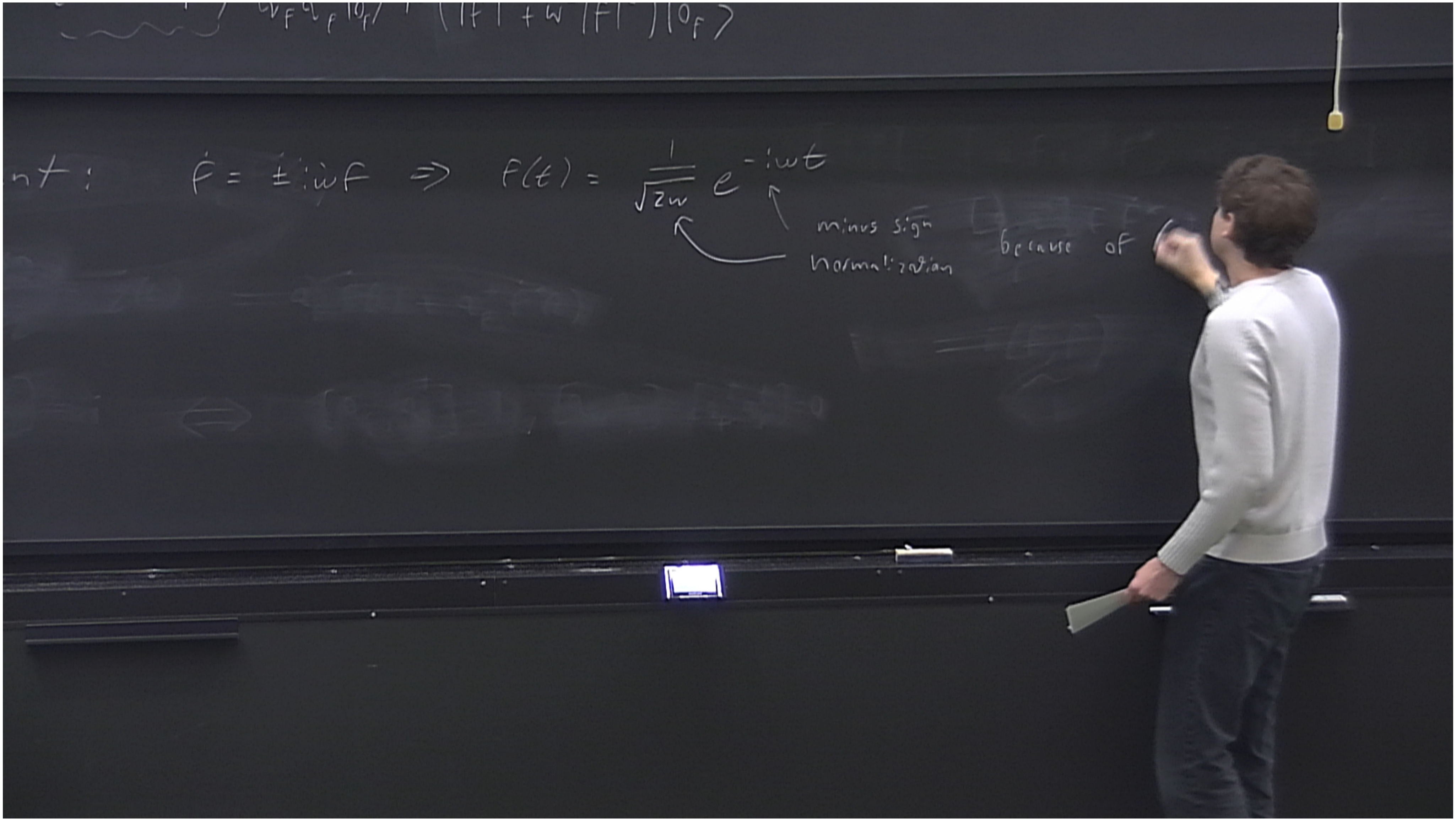
$$a_f^\dagger a_f + 1 \qquad \underbrace{a_f^\dagger a_f}_{\to 0}$$

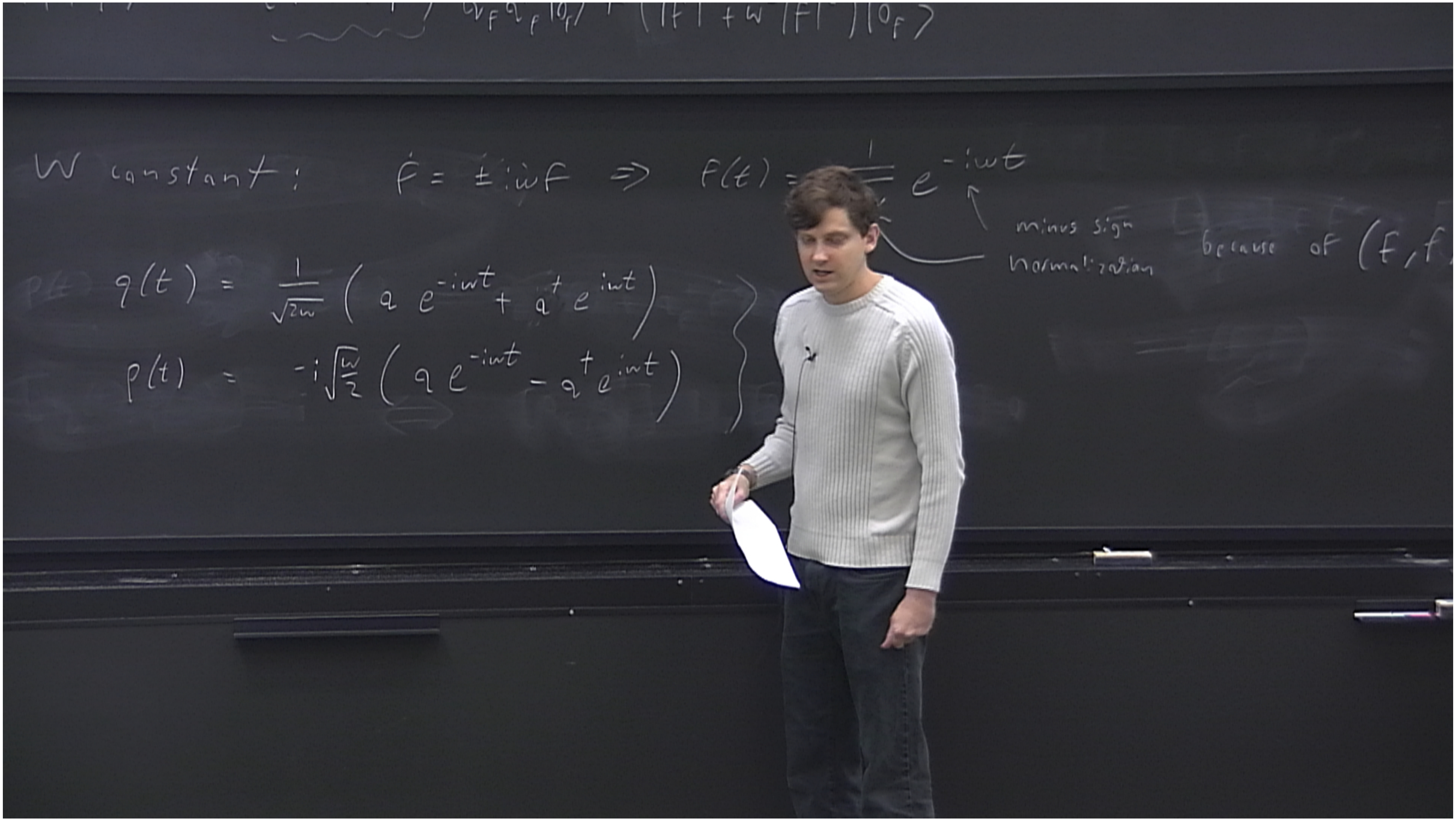
When is f-vacuum an eigenstate of H? $\quad H|0_f\rangle = \lambda |0_f\rangle$

$$H|0_f\rangle = (\dot{f}^2 + \omega^2 f^2)^* a_f^\dagger a_f^\dagger |0_f\rangle + (|\dot{f}|^2 + \omega^2 |f|^2)|0_f\rangle$$

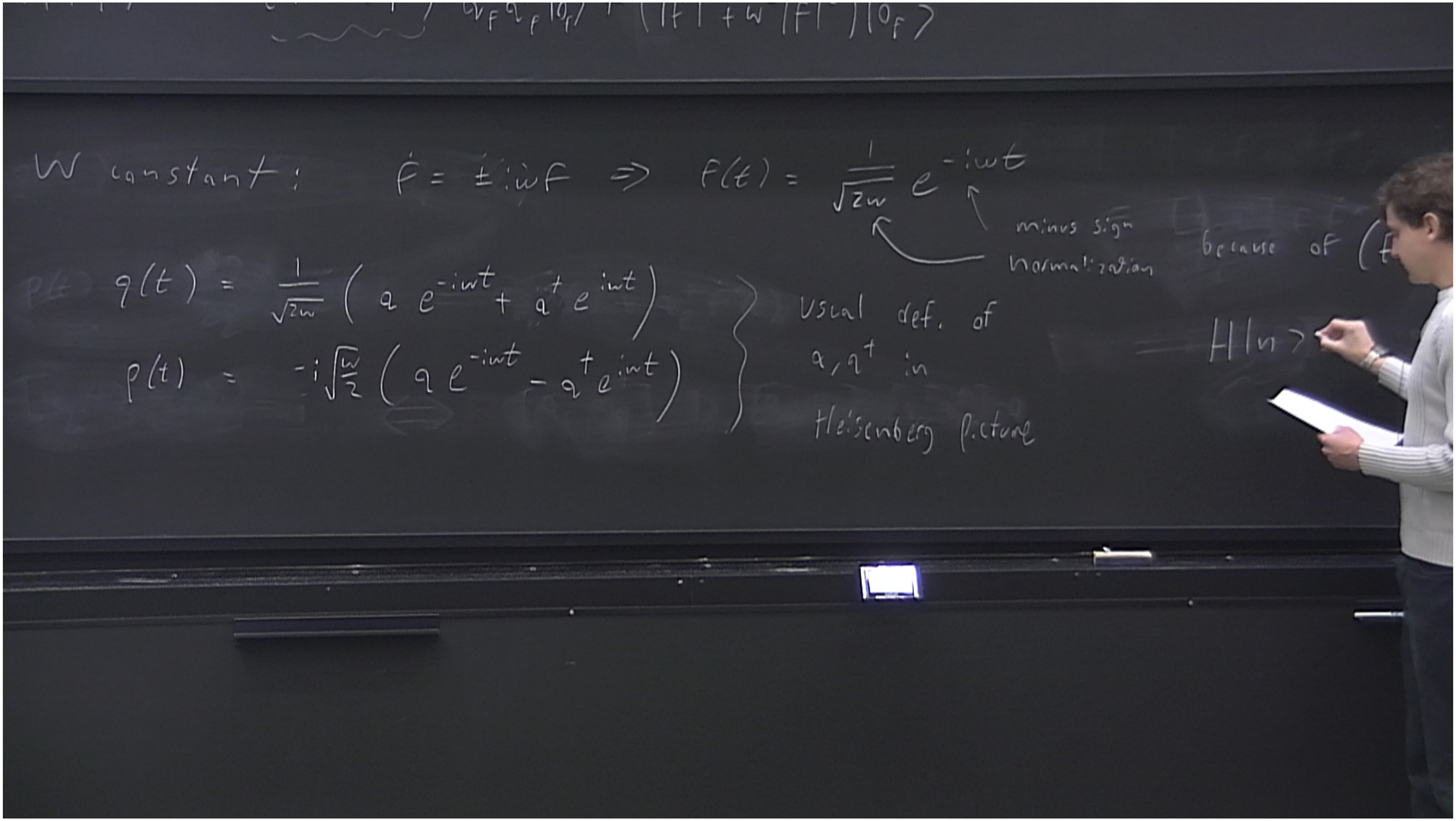

Need: $\dot{f}^2 + w^2 f^2 = 0$
$\dot{f} = \pm i w(t) f$
$(|\dot{f}|^2 + w^2 |f|^2)(a_f a_f^\dagger + a_f^\dagger a_f)]$
$a_f^\dagger a_f + 1$
$\to 0$
$|0_f\rangle = \lambda |0_f\rangle$
$|f|^2)|0_f\rangle$
$g(t) =$
$(g,g) = 1$
$q(t) =$

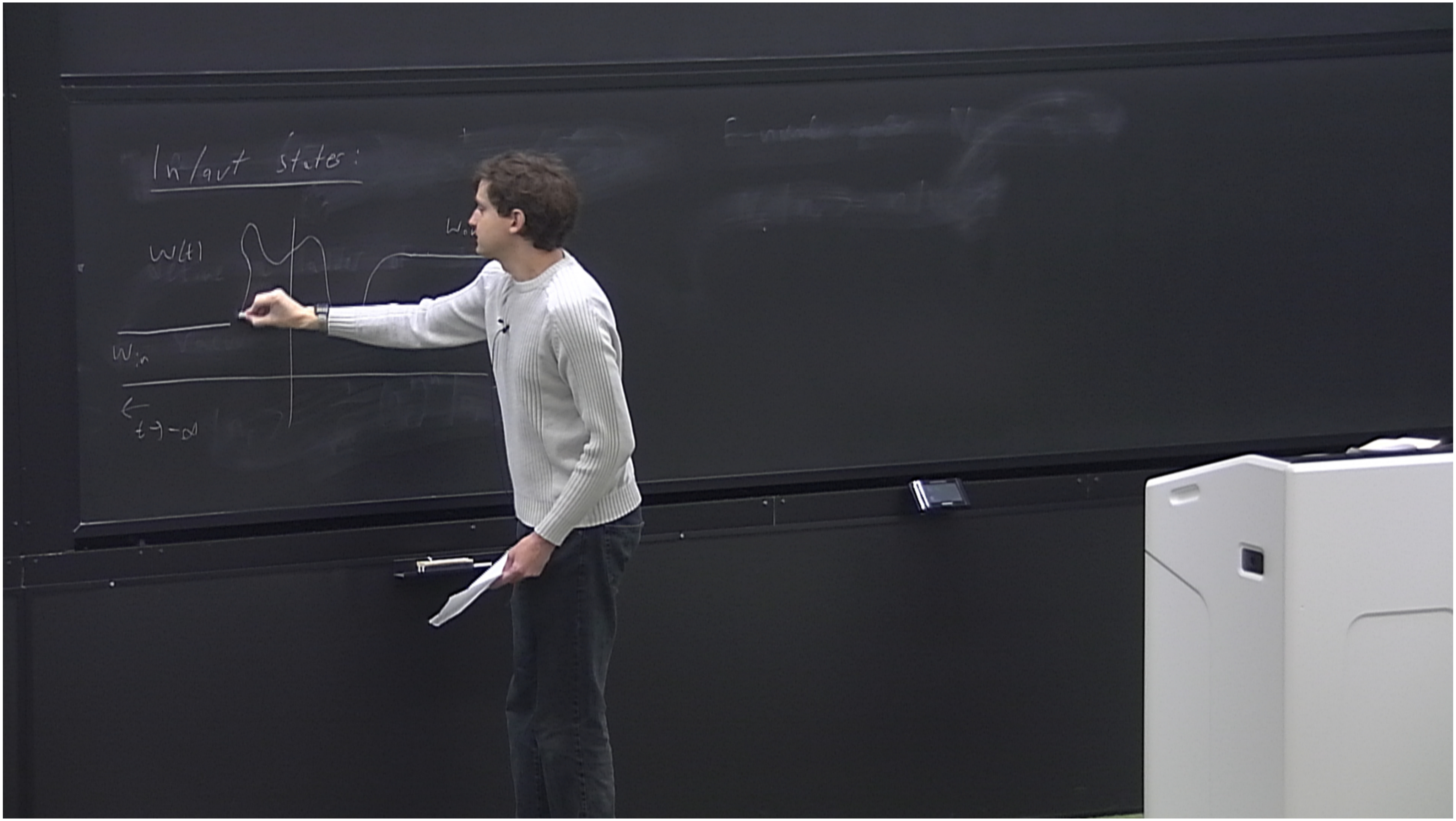

Eqm $\ddot{f} + w^2 f = 0$
need: $\dot{f}^2 + w^2 f^2 = 0$
$\dot{f} = \pm i w(t) f$
$\ddot{f} = \pm i \dot{w} f \pm i w \dot{f}$
$= -w^2 f \pm i \dot{w} f$
$\Rightarrow \dot{w} = 0$
$\frac{d}{dt}$
$(|\dot{f}|^2 + w^2 |f|^2)(a_f a_f^+ + a_f^+ a_f)]$
$a_f^+ a_f + 1$
$|0_f\rangle = \lambda |0_f\rangle$
$|f|^2)|0_f\rangle$
$g(t) =$
$(g,g) = 1$
$q(t) =$


nt :
ḟ = ± iωf ⇒ f(t) = 1/√(2w) e^(-iωt)
minus sign
normalization
because of



W constant :
minus sign
normalization
because of (f, f


W constant: $\dot{f} = \pm i\omega f \Rightarrow f(t) = \frac{1}{\sqrt{2\omega}} e^{-i\omega t}$
minus sign
normalization
because of (f
$q(t) = \frac{1}{\sqrt{2\omega}}\left(a e^{-i\omega t} + a^\dagger e^{i\omega t}\right)$
$p(t) = -i\sqrt{\frac{\omega}{2}}\left(a e^{-i\omega t} - a^\dagger e^{i\omega t}\right)$
Usual def. of $a, a^\dagger$ in Heisenberg picture
$H|n\rangle$


In/out states:
W(t)
Win
t→-∞