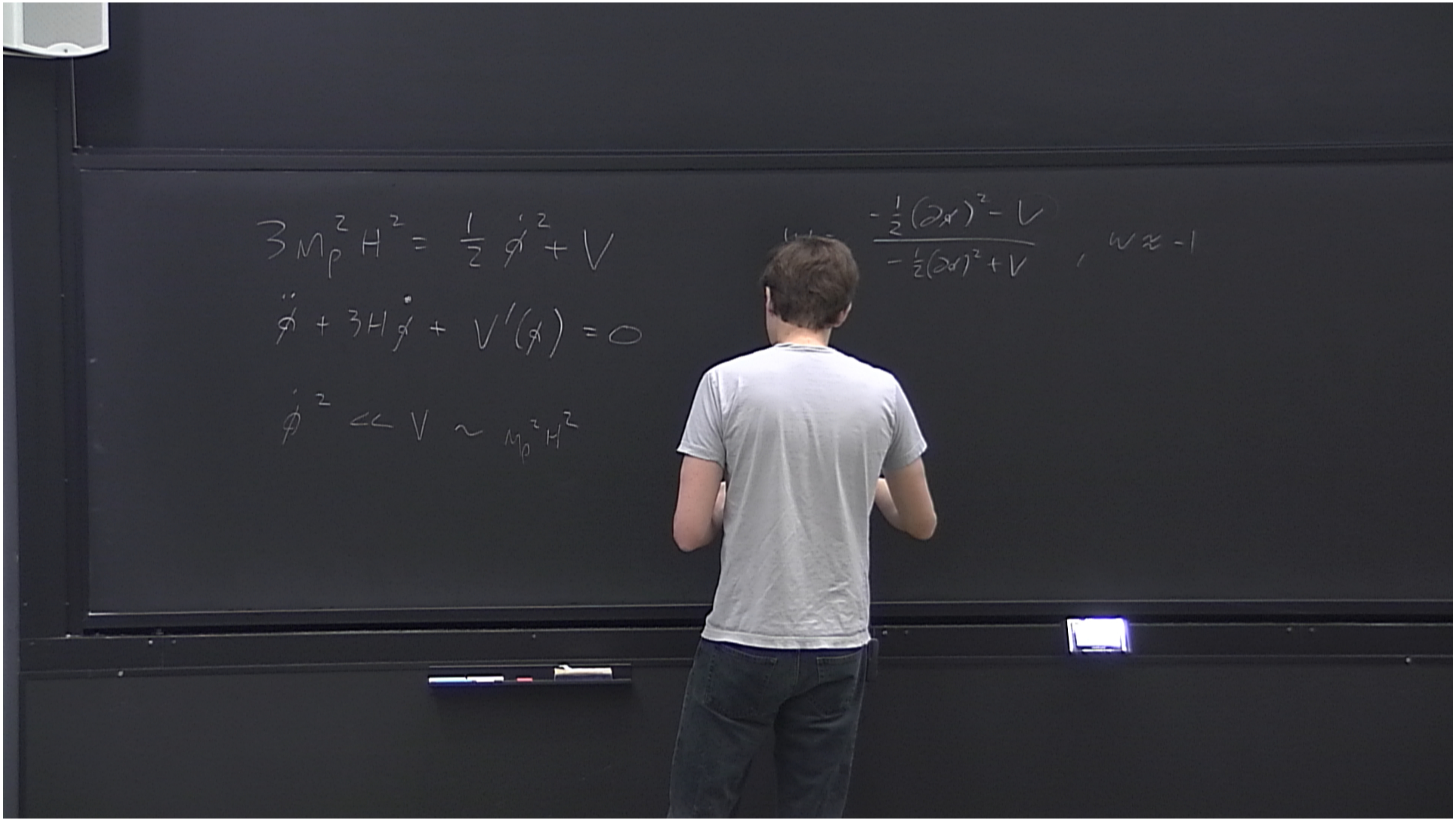
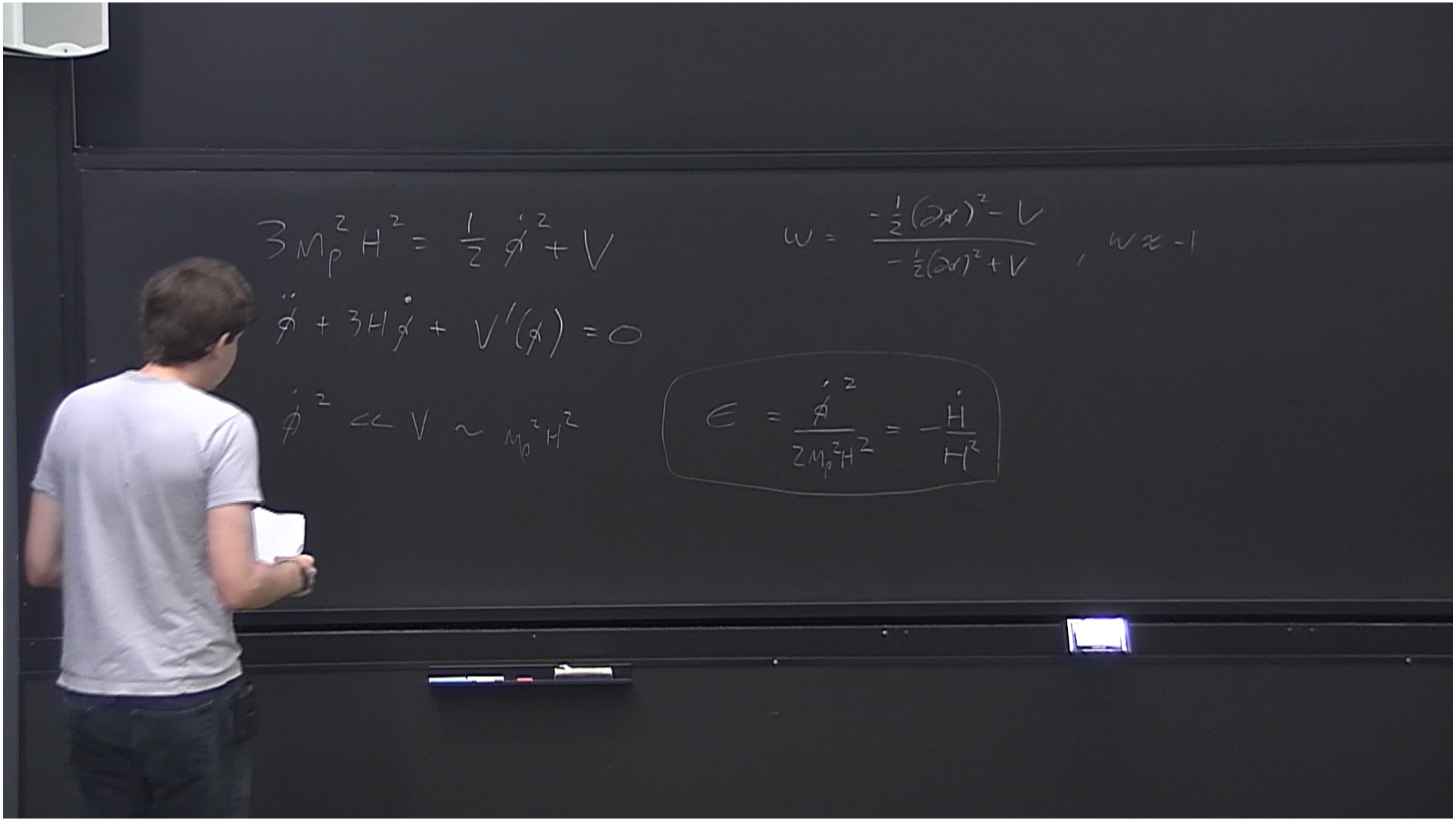
Title: Cosmology Review-10

Date: Feb 06, 2015 11:30 AM

URL: http://pirsa.org/15020031

Abstract:


$3M_p^2H^2 = \frac{1}{2}\dot{\phi}^2 + V$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$\dot{\phi}^2 \ll V \sim M_p^2H^2$
$\frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}, \quad w \approx -1$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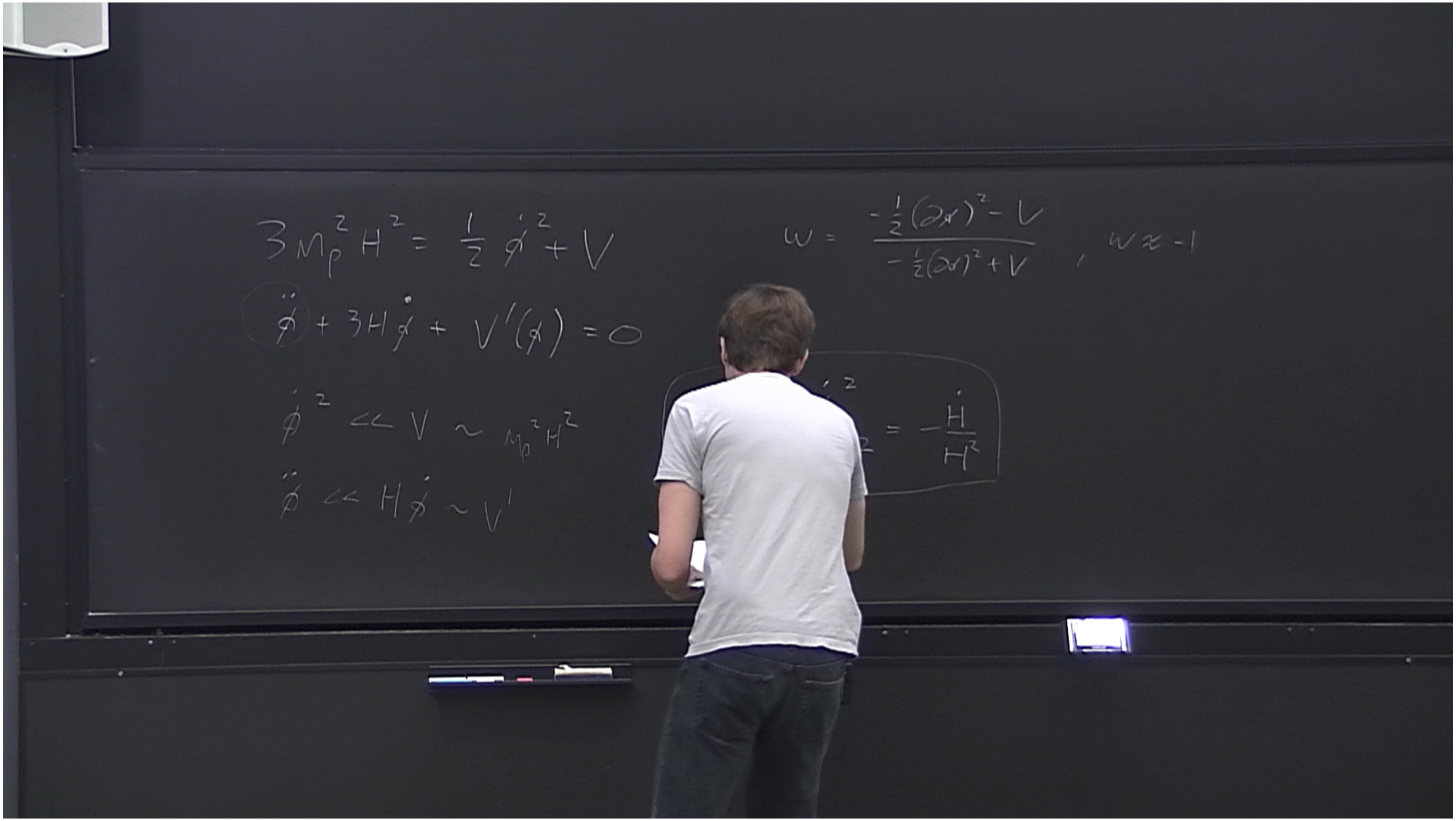
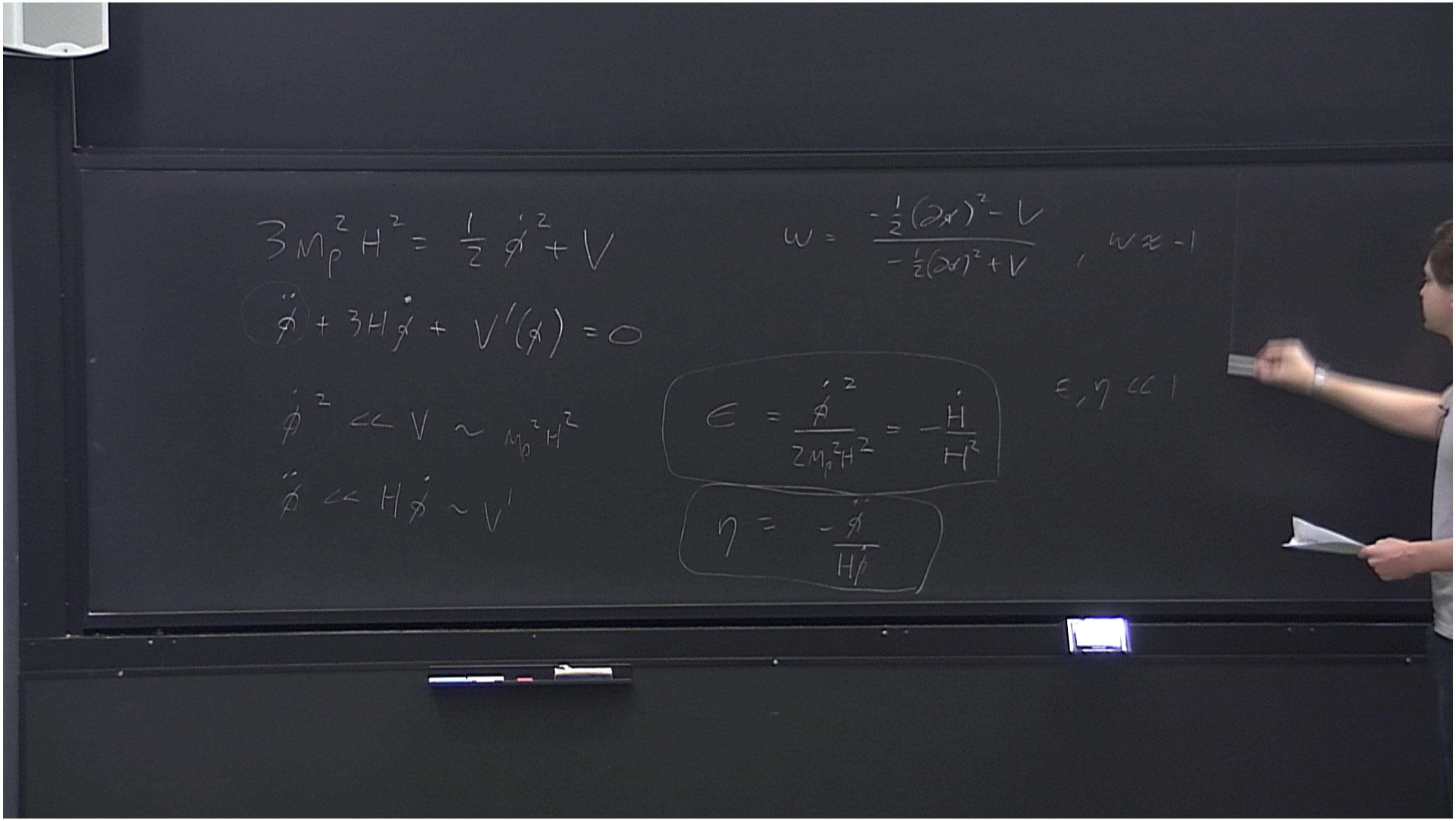
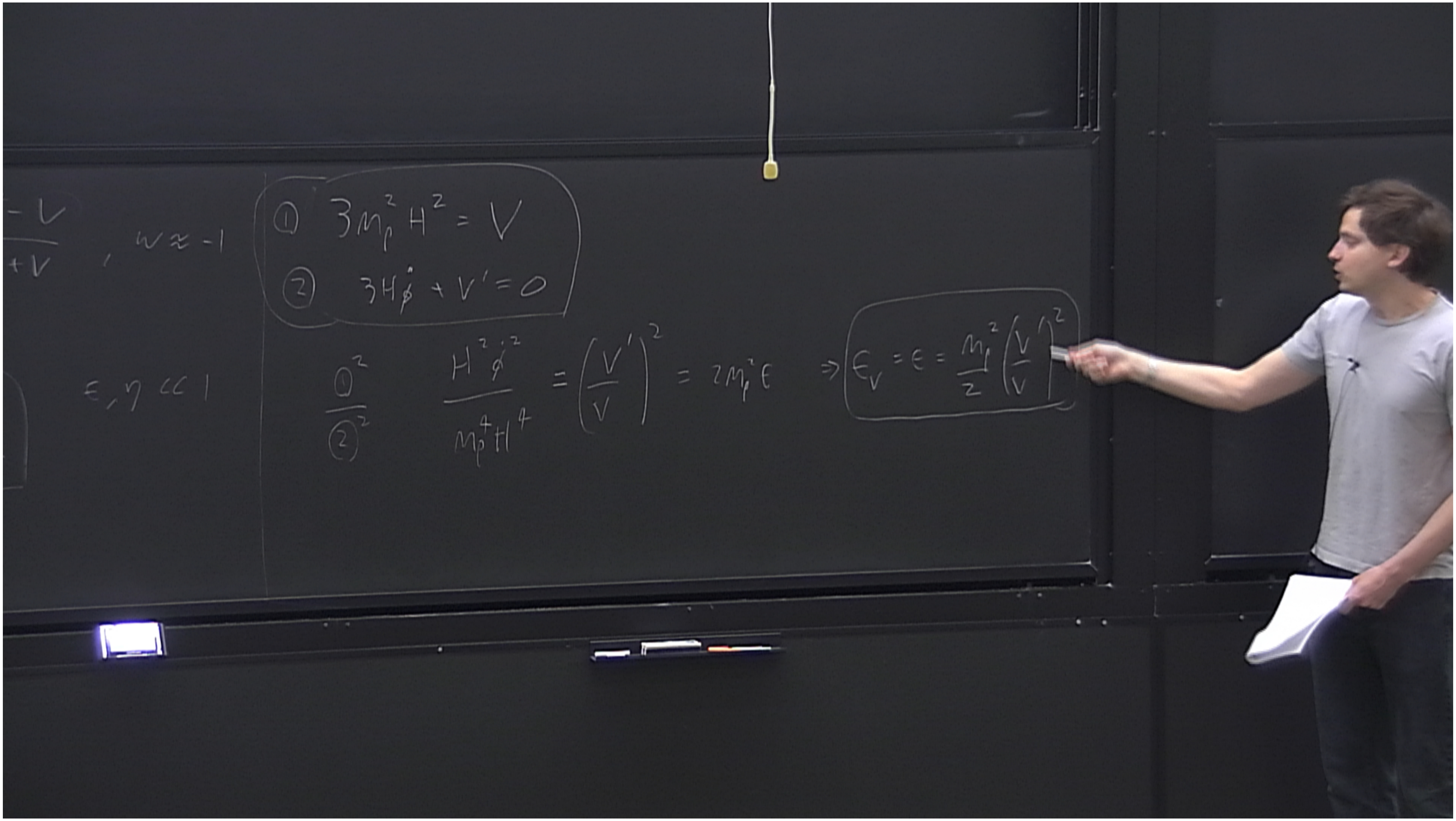
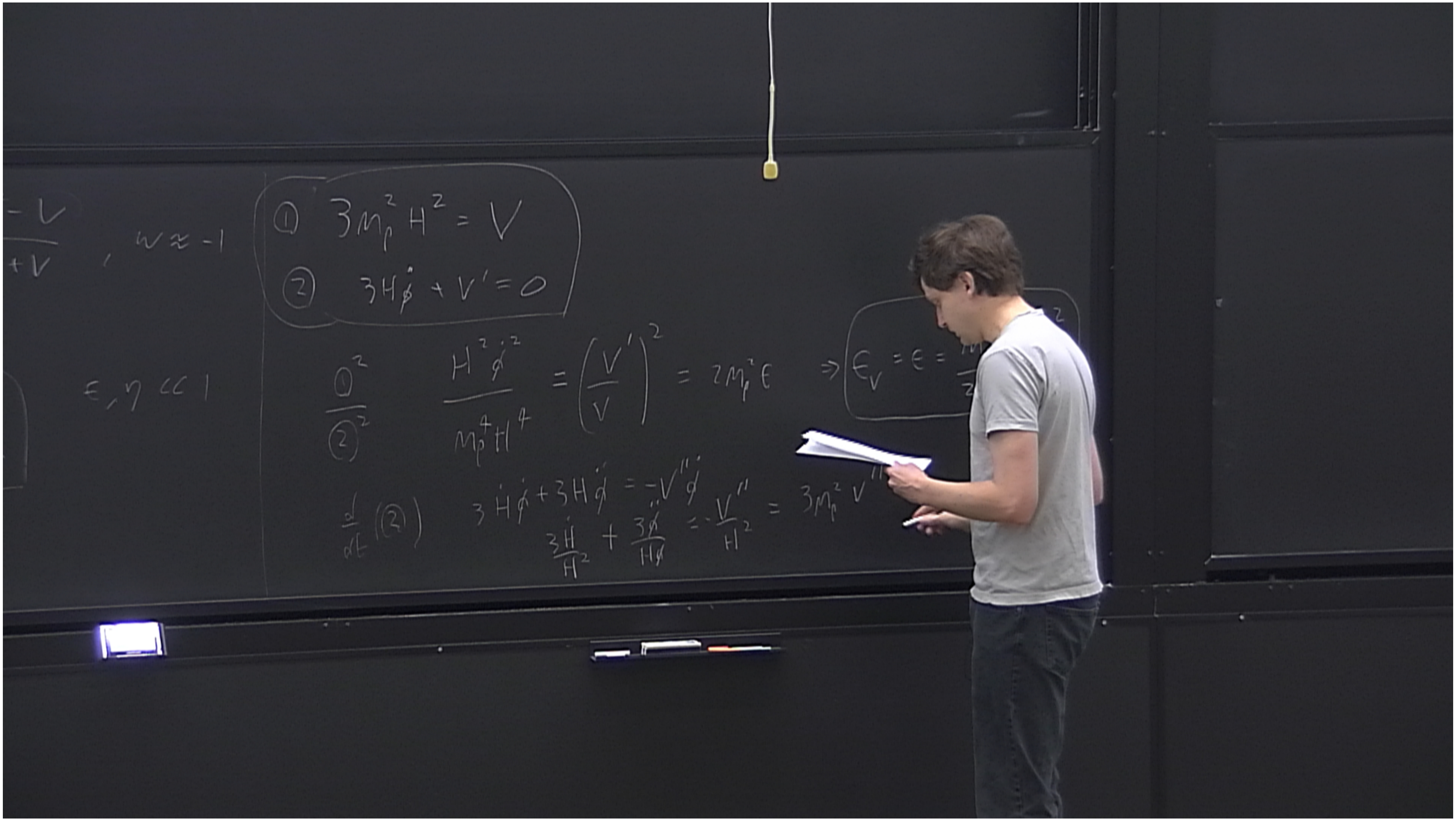
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$\dot{\phi}^2 \ll V \sim M_p^2H^2$
$w = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}, \quad w \approx -1$
$\epsilon = \frac{\dot{\phi}^2}{2M_p^2H^2} = -\frac{\dot{H}}{H^2}$

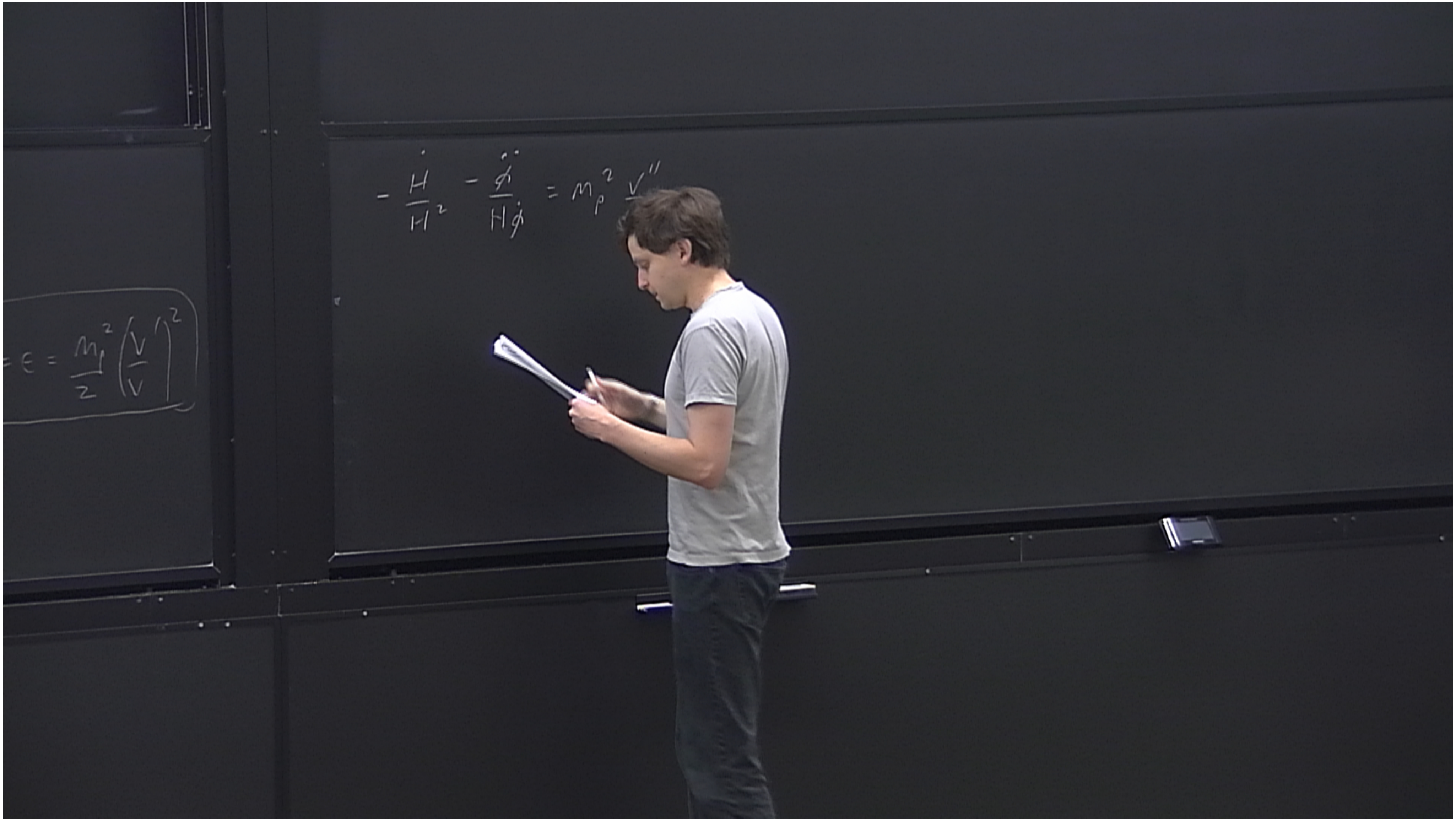

$3M_p^2H^2 = \frac{1}{2}\dot\phi^2 + V$
$w = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$, $w \approx -1$
$\ddot\phi + 3H\dot\phi + V'(\phi) = 0$
$\dot\phi^2 << V \sim M_p^2H^2$
$\ddot\phi << H\dot\phi \sim V'$
$= -\frac{\dot H}{H^2}$

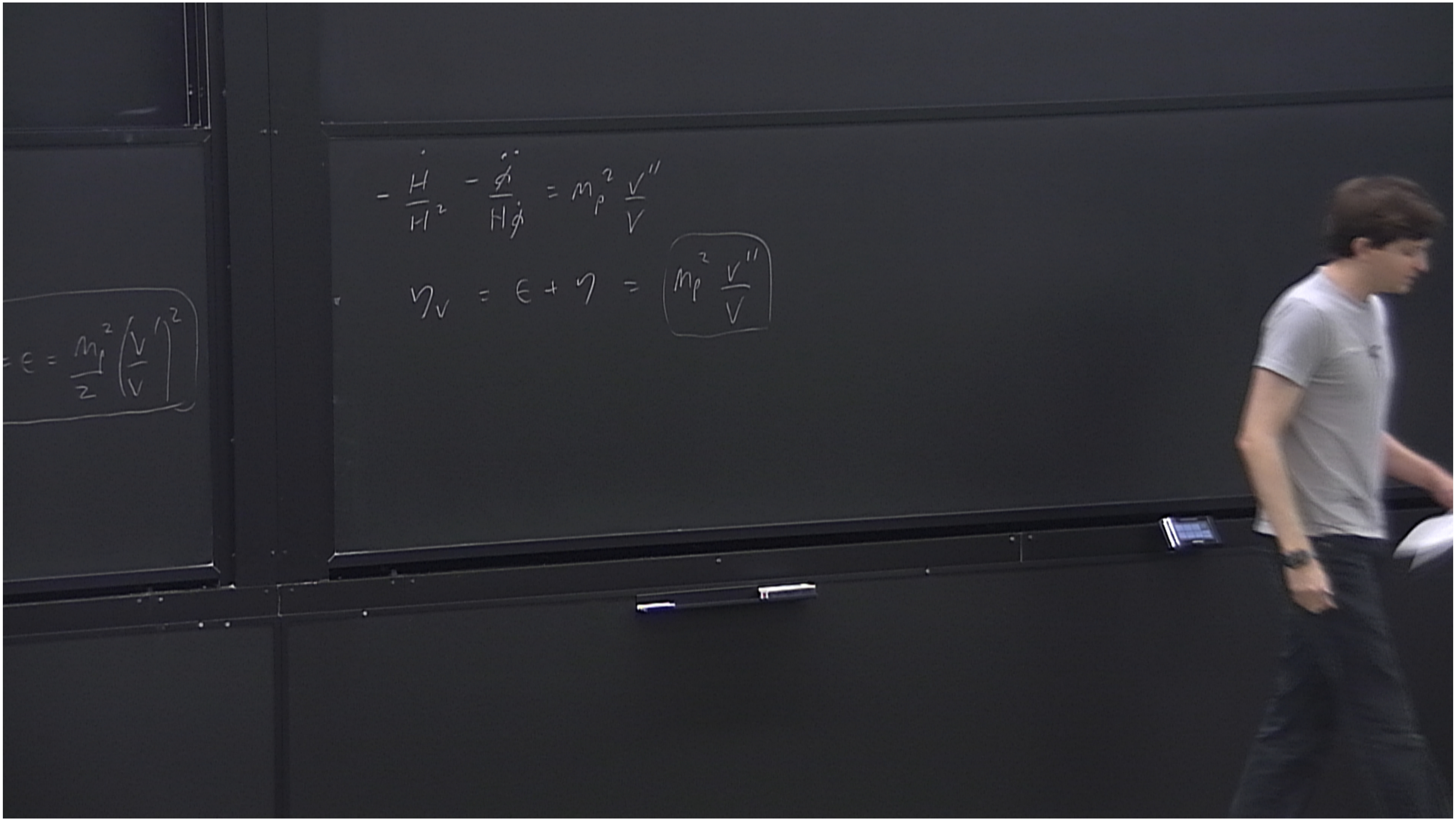

$3M_p^2H^2 = \frac{1}{2}\dot{\phi}^2 + V$
$w = \frac{-\frac{1}{2}(\partial\phi)^2 - V}{-\frac{1}{2}(\partial\phi)^2 + V}$, $w \approx -1$
$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$
$\dot{\phi}^2 << V \sim M_p^2H^2$
$\ddot{\phi} << H\dot{\phi} \sim V'$
$\boxed{\epsilon = \frac{\dot{\phi}^2}{2M_p^2H^2} = -\frac{\dot{H}}{H^2}}$
$\boxed{\eta = -\frac{\ddot{\phi}}{H\dot{\phi}}}$
$\epsilon, \eta << 1$

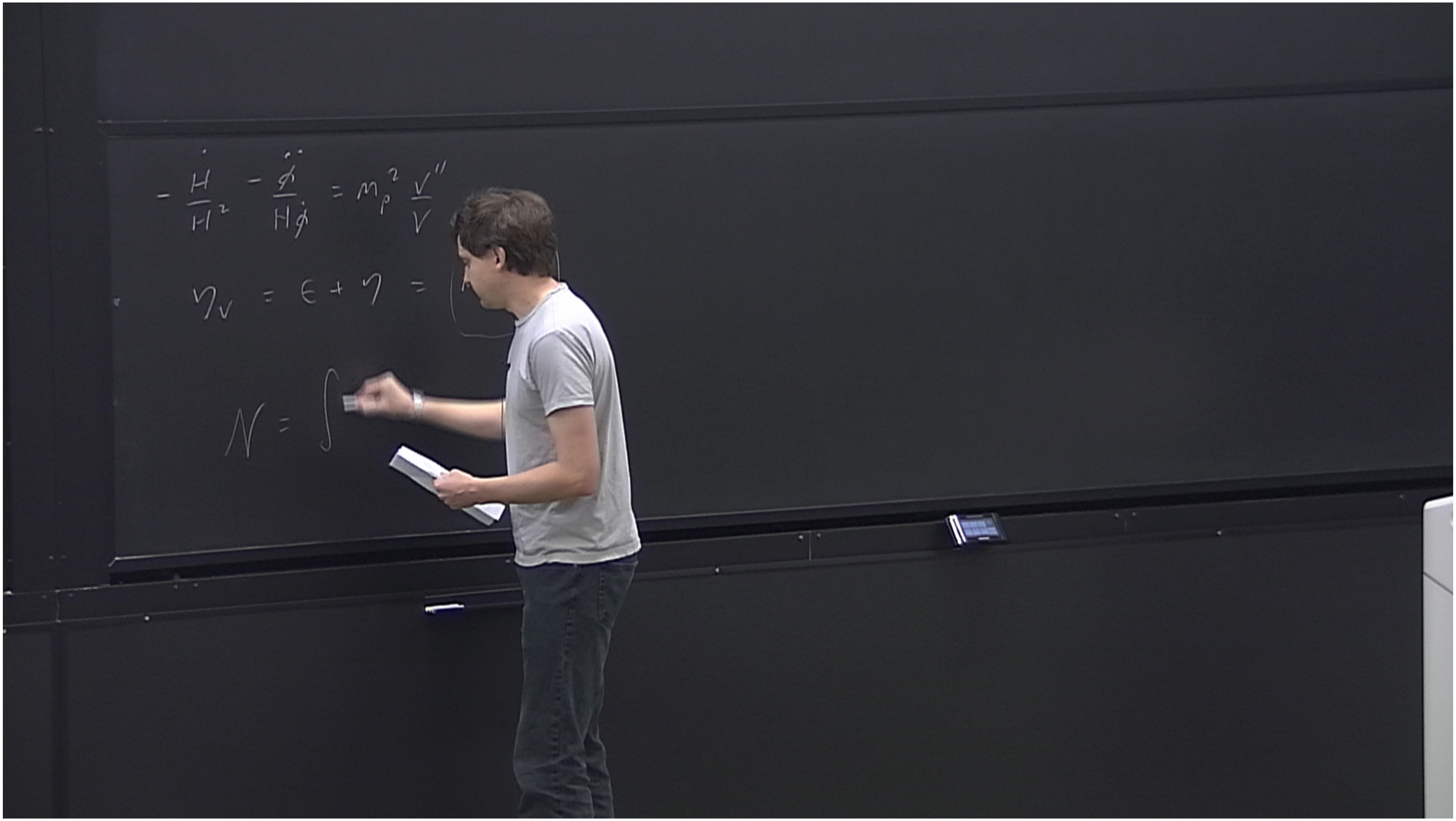
$3M_p^2 H^2 = V$
$3H\dot{\phi} + V' =$
$w \approx -1$
$\epsilon, \eta \ll 1$

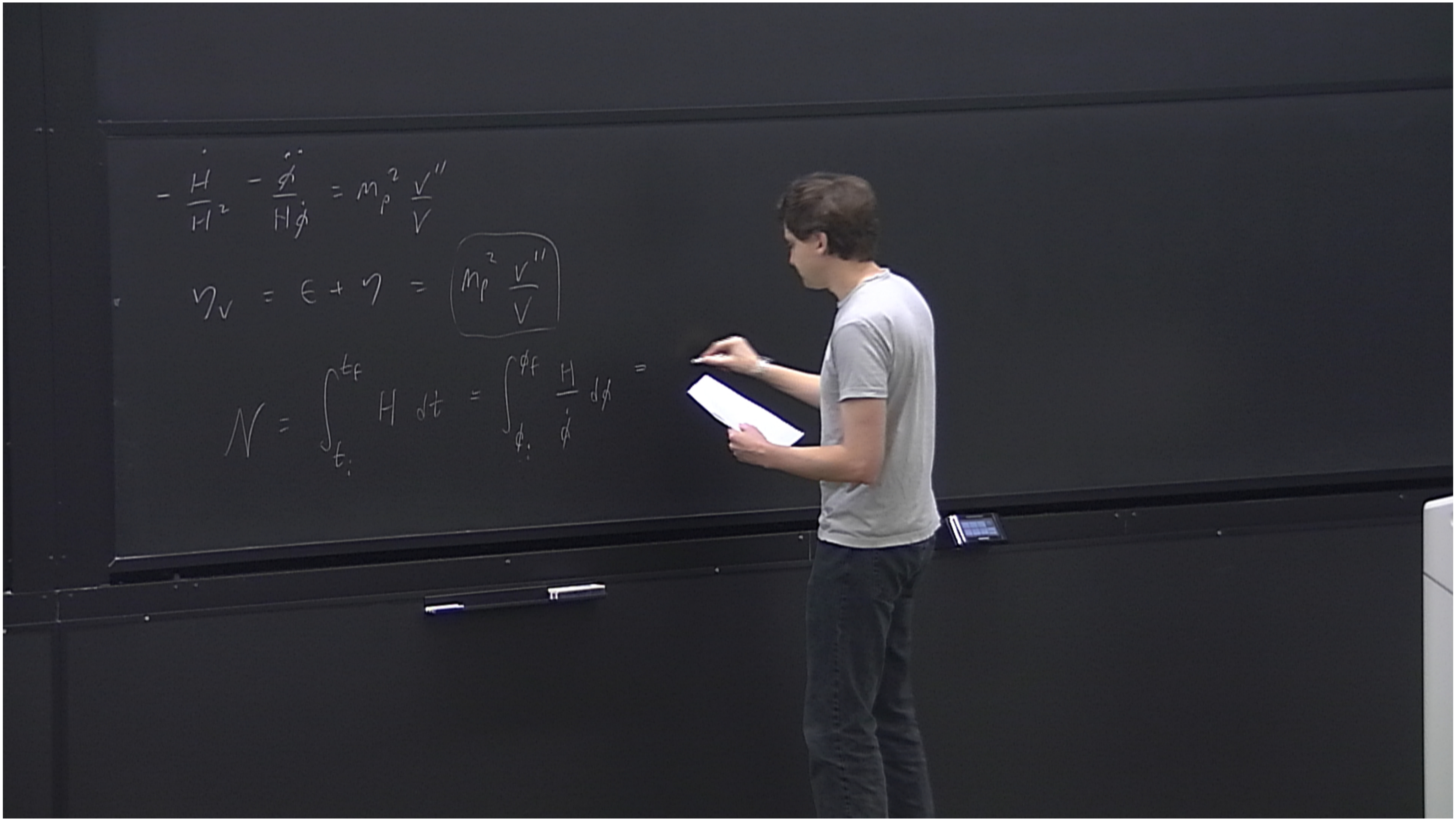

$w \approx -1$
① $3M_p^2 H^2 = V$
② $3H\dot\phi + V' = 0$
$\epsilon, \eta \ll 1$
$\frac{①^2}{②^2}$ $\frac{H^2 \dot\phi^2}{M_p^4 H^4} = \left(\frac{V'}{V}\right)^2 = 2M_p^2 \epsilon$
$\Rightarrow \epsilon_V = \epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$

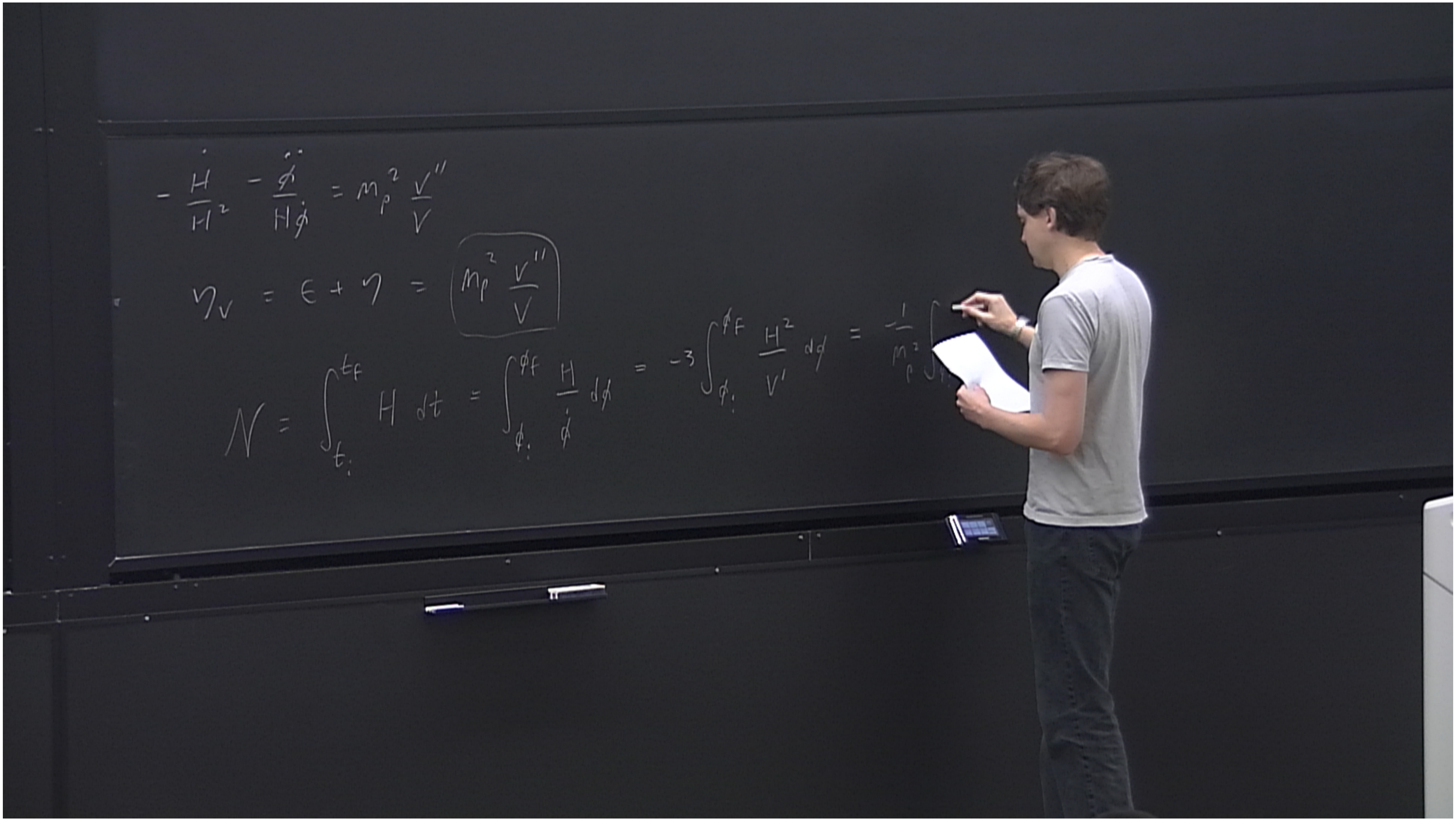

w ≈ -1
① 3M_p^2 H^2 = V
② 3Hφ̇ + V' = 0
ε, η << 1
H^2 φ̇^2 / M_p^4 H^4 = (V'/V)^2 = 2M_p^2 ε
⇒ ε_V = ε
d/dt (②) 3Ḣφ̇ + 3Hφ̈ = -V''φ̇
3Ḣ/H^2 + 3φ̈/Hφ̇ = -V''/H^2 = 3M_p^2 V''


$-\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}} = m_p^2 \frac{V''}{}$
$\epsilon = \frac{m_p^2}{2}\left(\frac{V'}{V}\right)^2$


$-\frac{\dot H}{H^2} - \frac{\ddot\phi}{H\dot\phi} = M_p^2 \frac{V''}{V}$
$\eta_V = \epsilon + \eta = \boxed{M_p^2 \frac{V''}{V}}$
$= \epsilon = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2$


$-\frac{\dot H}{H^2} - \frac{\ddot\phi}{H\dot\phi} = M_p^2 \frac{V''}{V}$
$\eta_V = \epsilon + \eta =$
$N = \int$

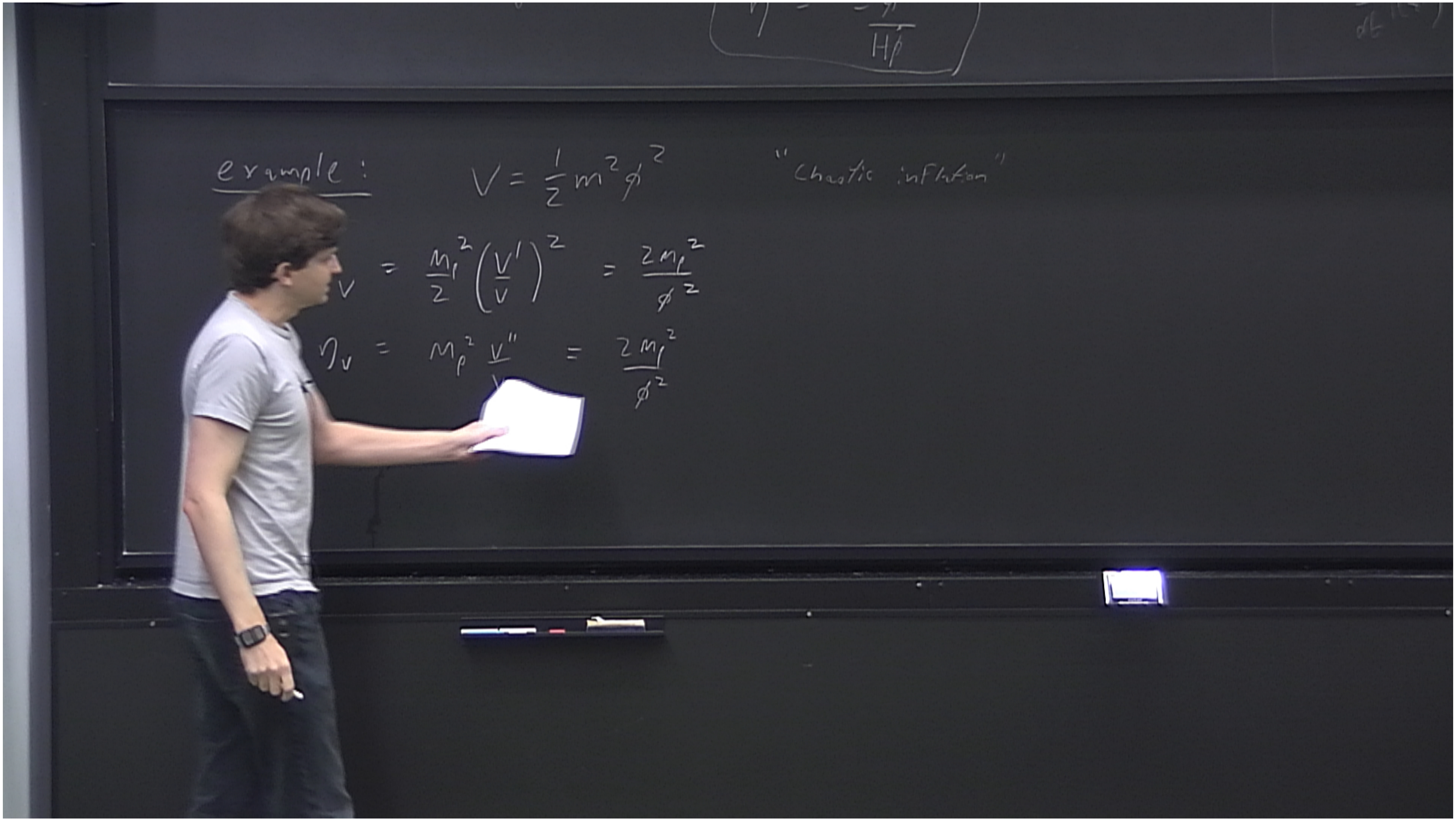

$-\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}} = m_p^2 \frac{V''}{V}$
$\eta_V = \epsilon + \eta = \boxed{m_p^2 \frac{V''}{V}}$
$N = \int_{t_i}^{t_f} H \, dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi =$

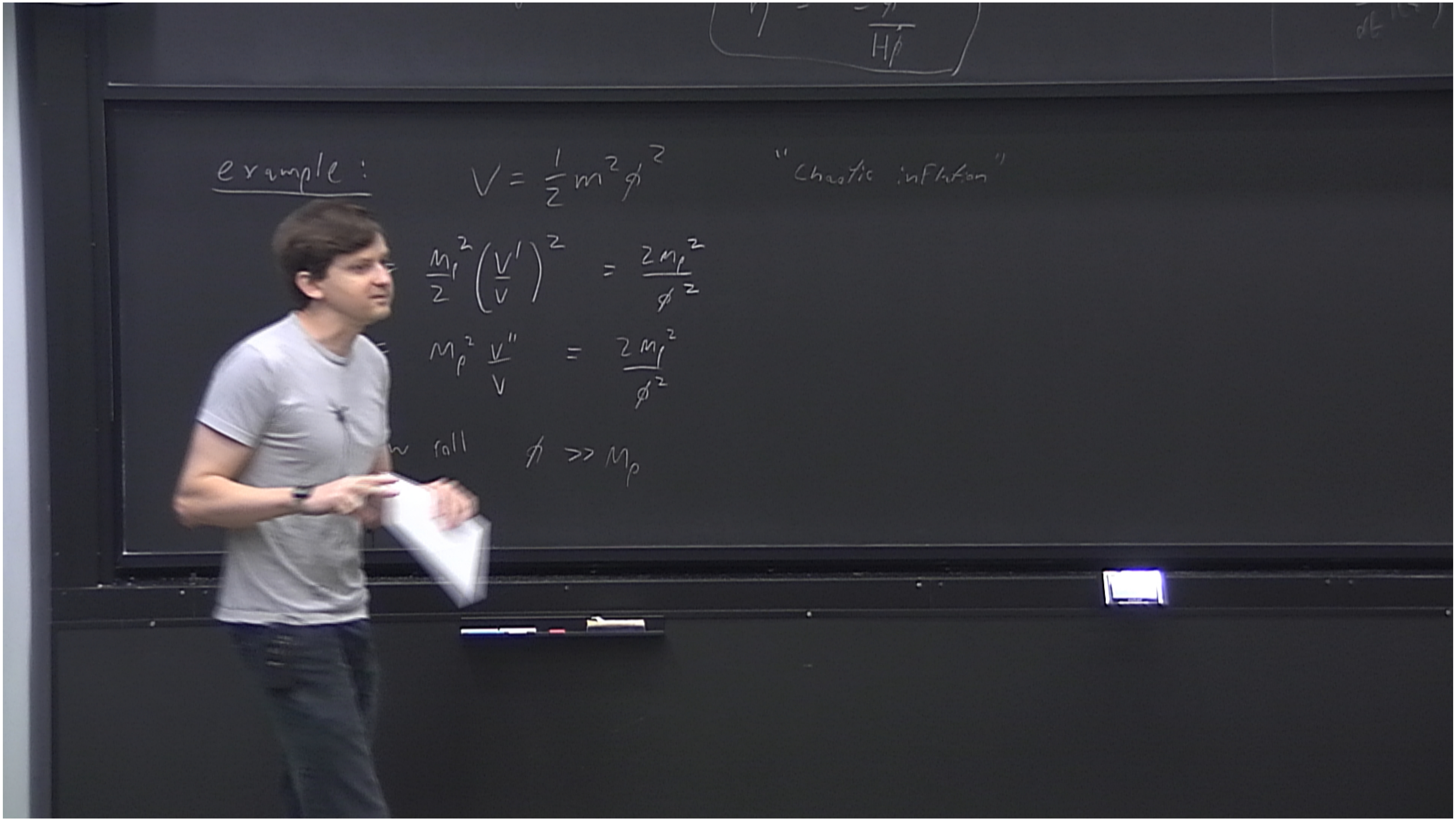

$-\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}} = M_p^2 \frac{V''}{V}$
$\eta_V = \epsilon + \eta = \boxed{M_p^2 \frac{V''}{V}}$
$N = \int_{t_i}^{t_f} H \, dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi = -3 \int_{\phi_i}^{\phi_f} \frac{H^2}{V'} d\phi = -\frac{1}{M_p^2} \int_{\phi_i}$

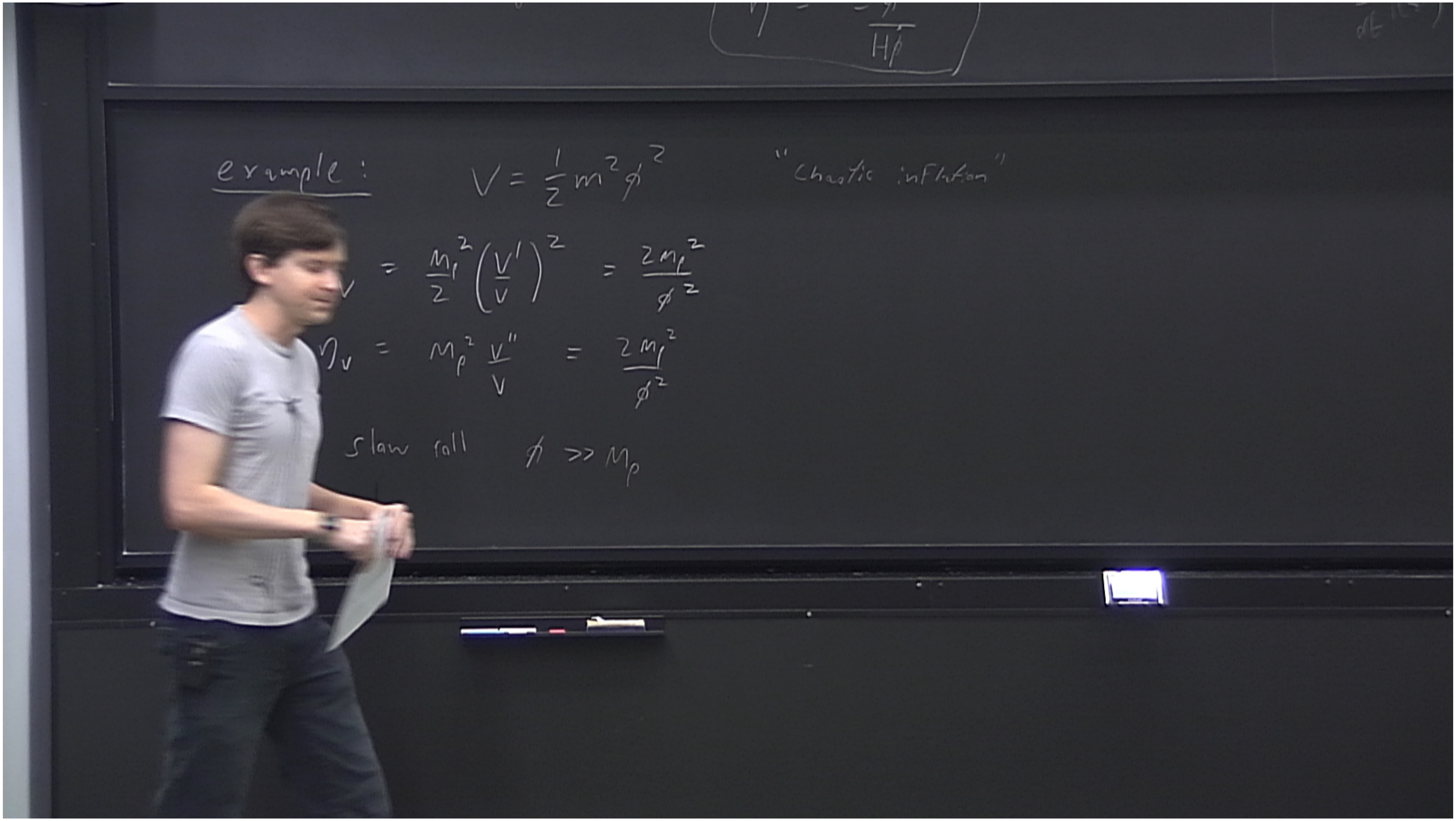
$$-\frac{\dot{H}}{H^2} - \frac{\ddot{\phi}}{H\dot{\phi}} = m_p^2 \frac{V''}{V}$$

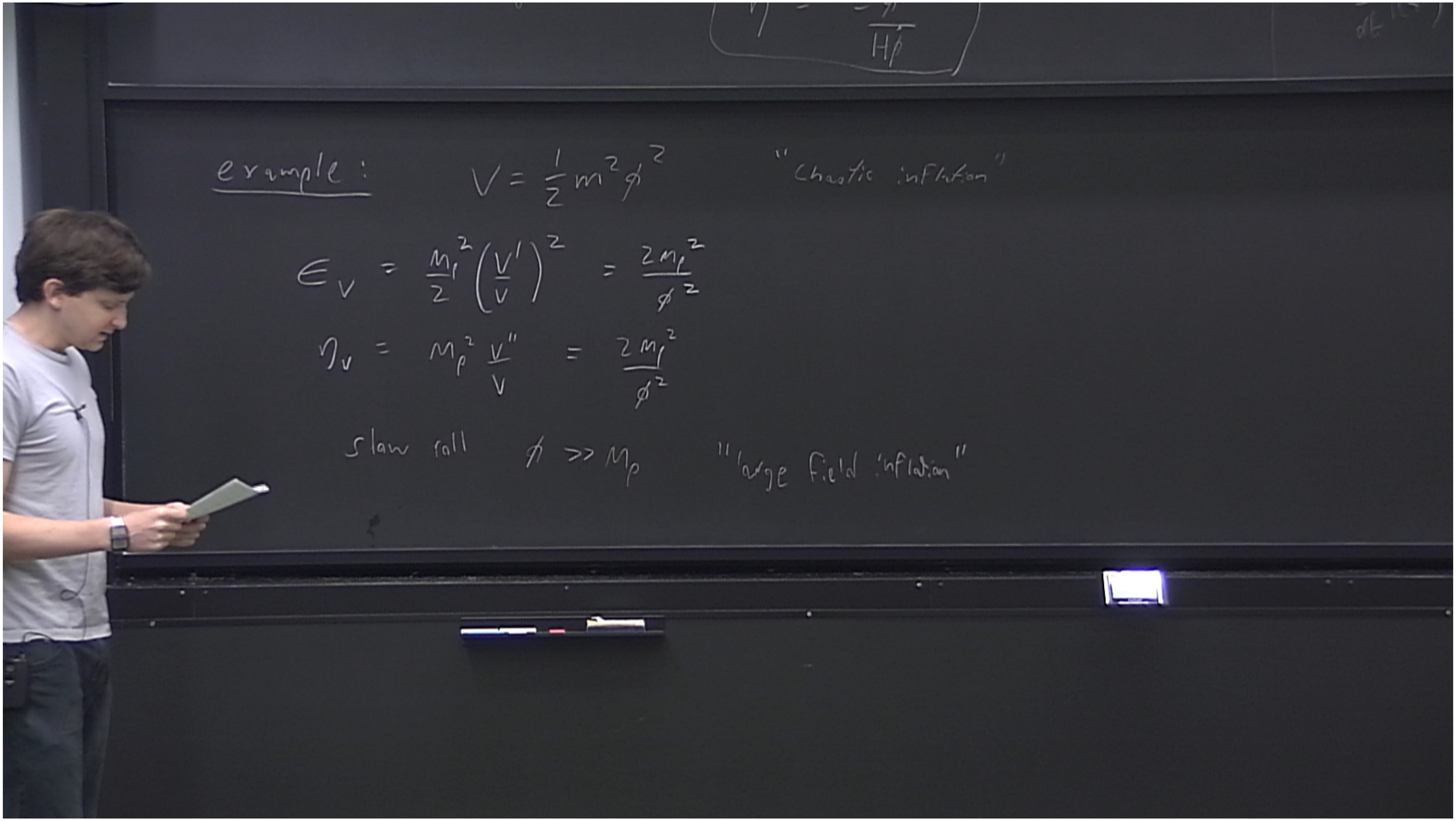
$$\eta_V = \epsilon + \eta = \boxed{m_p^2 \frac{V''}{V}}$$

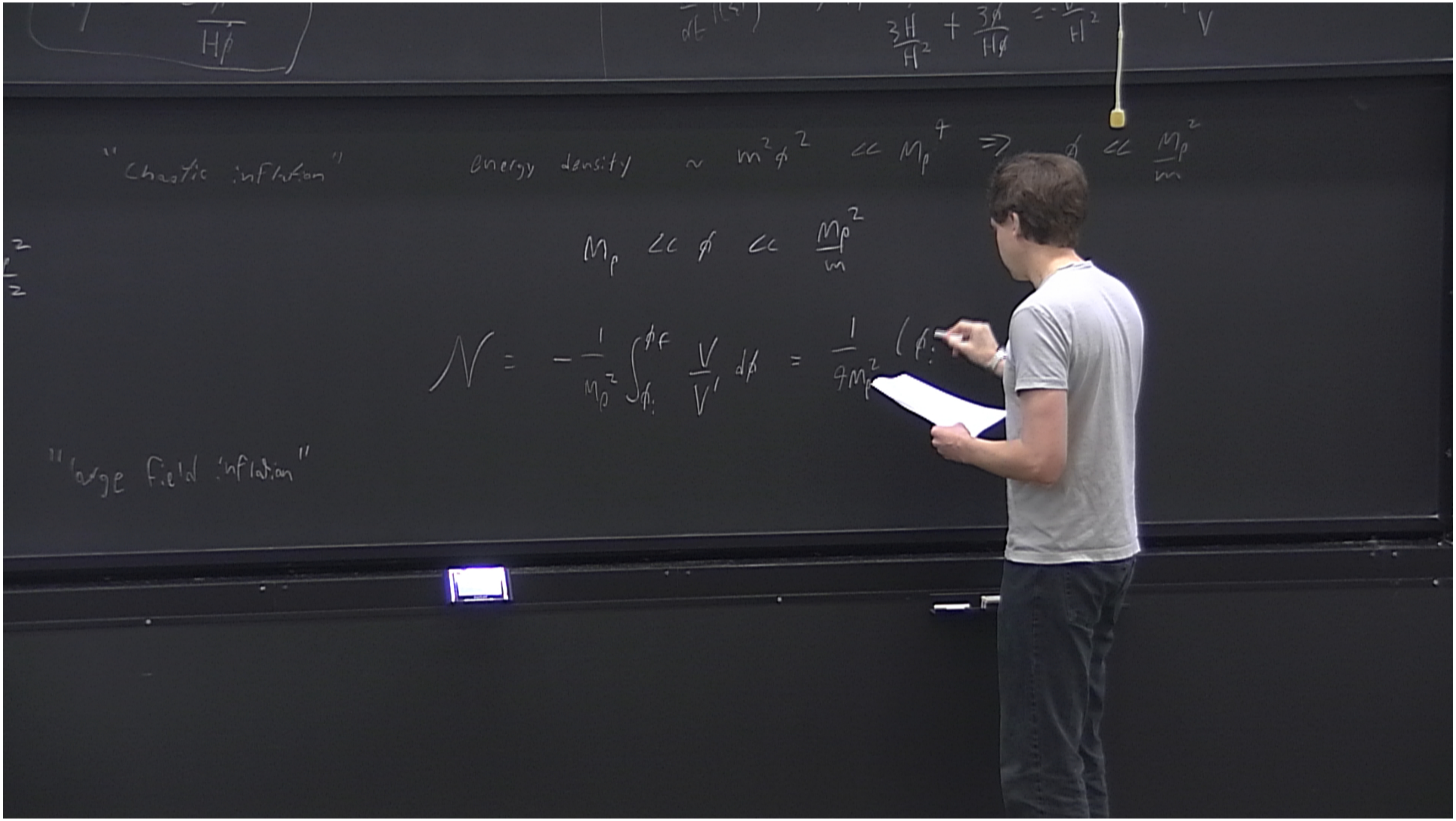
$$N = \int_{t_i}^{t_f} H \, dt = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} \, d\phi = -3 \int_{\phi_i}^{\phi_f} \frac{H^2}{V'} \, d\phi = \boxed{-\frac{1}{m_p^2} \int_{\phi_i}^{\phi_f} \frac{V}{V'} \, d\phi}$$

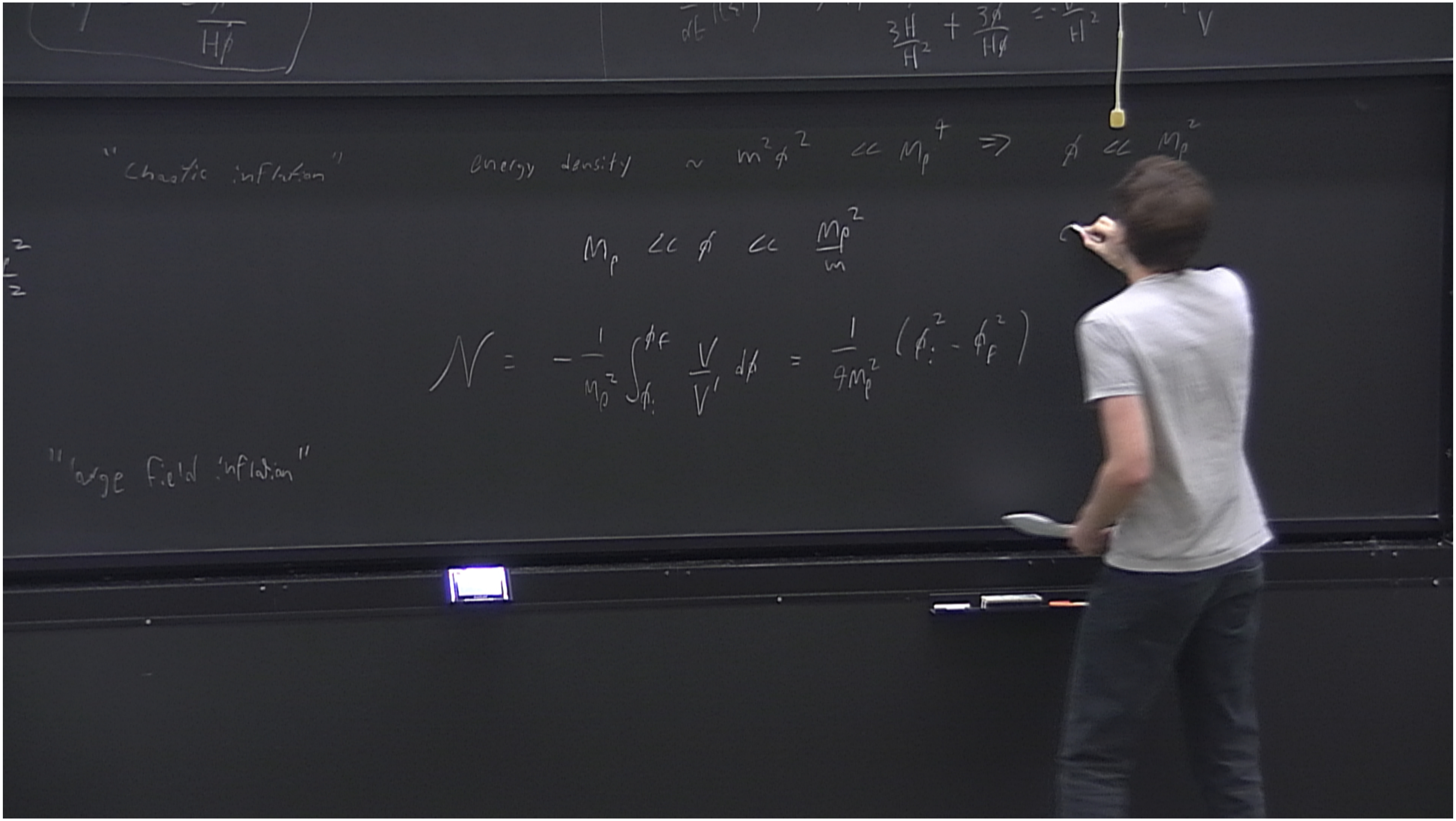

example:
$V = \frac{1}{2} m^2 \phi^2$
"Chaotic inflation"
$= \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{2M_{pl}^2}{\phi^2}$
$\eta_V = M_{pl}^2 \frac{V''}{V} = \frac{2M_{pl}^2}{\phi^2}$

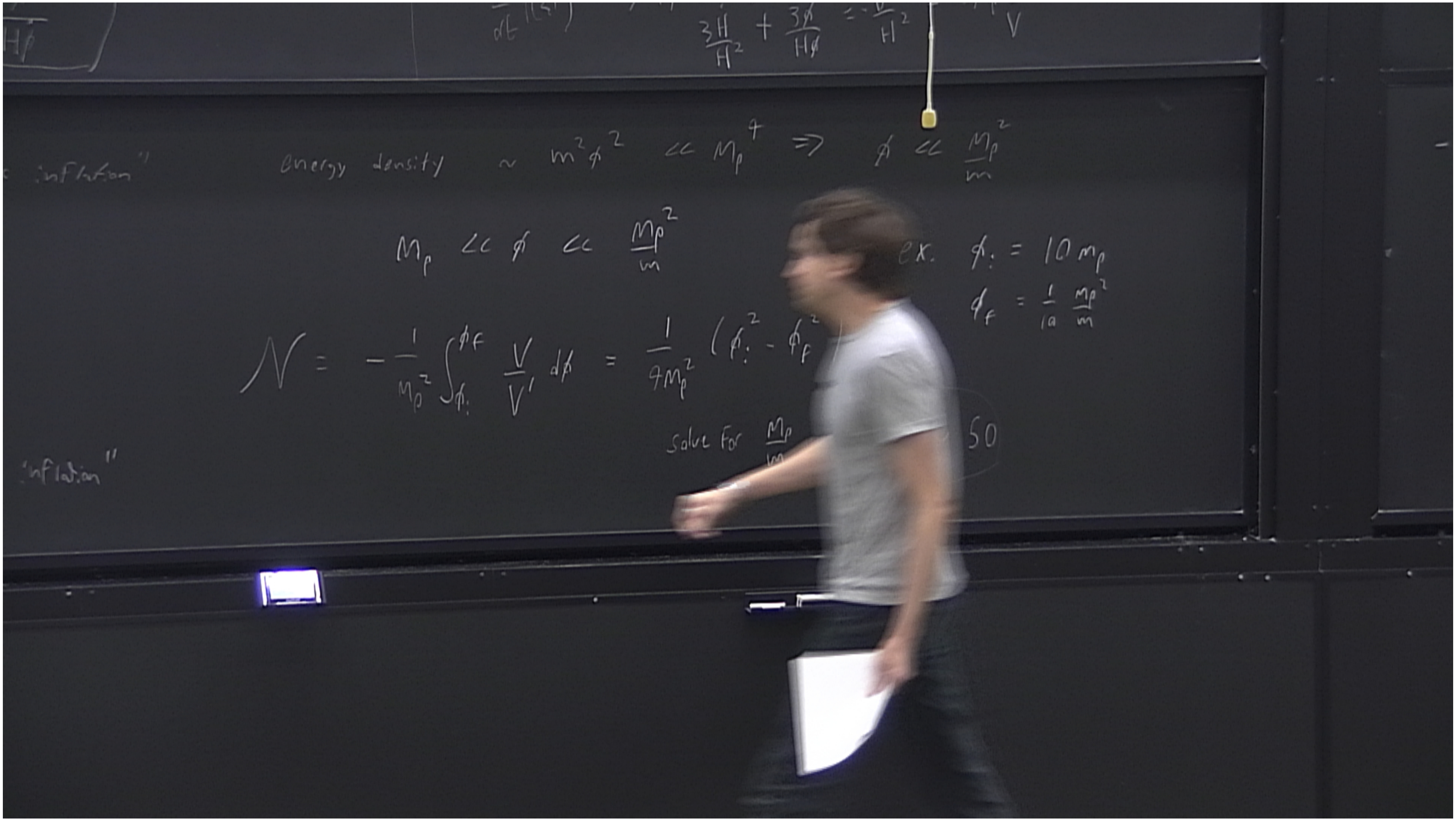

example:
$V = \frac{1}{2} m^2 \phi^2$
"Chaotic inflation"
$= \frac{M_p^2}{2} \left(\frac{V'}{V}\right)^2 = \frac{2M_p^2}{\phi^2}$
$= M_p^2 \frac{V''}{V} = \frac{2M_p^2}{\phi^2}$
roll
$\phi \gg M_p$

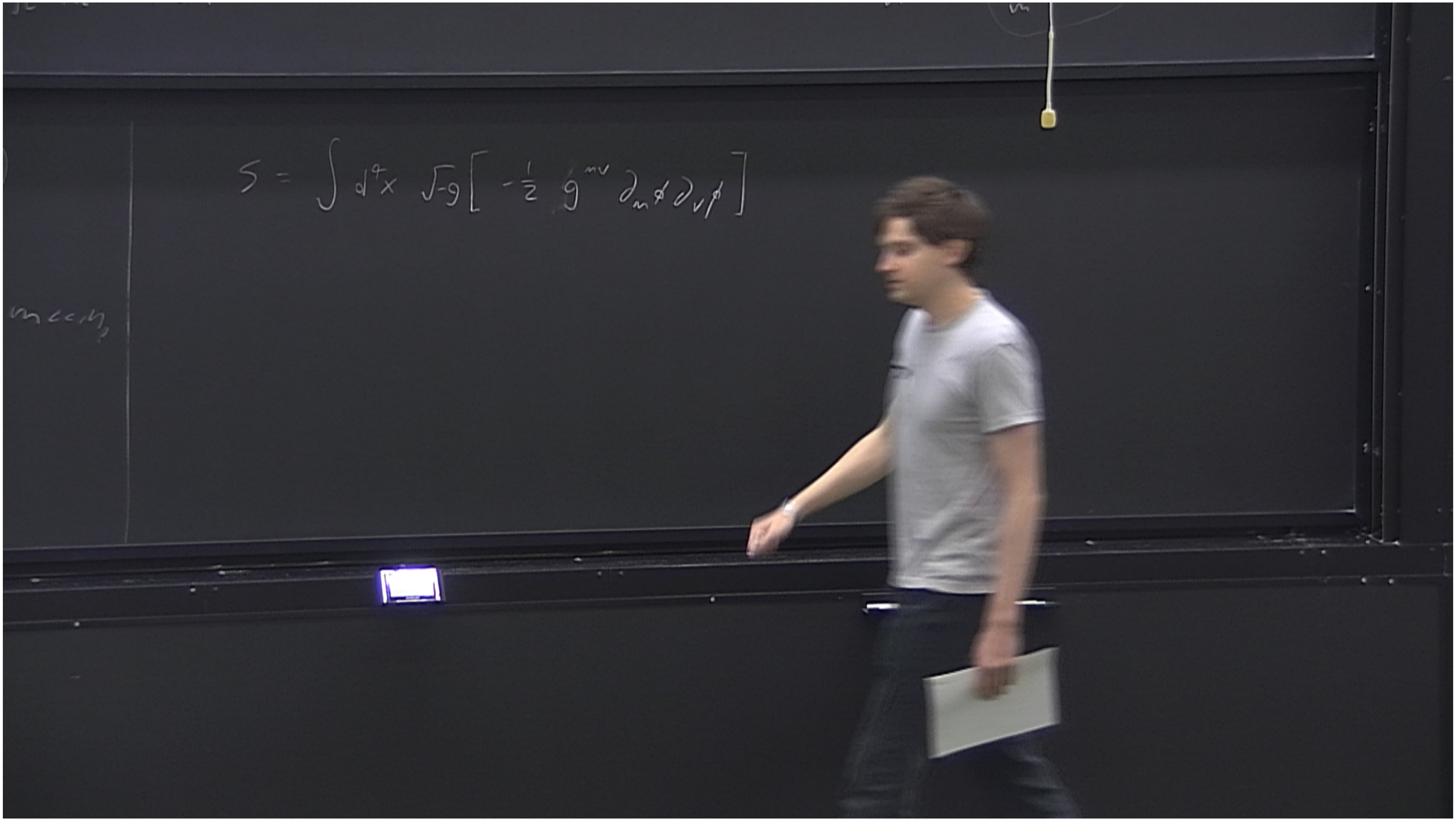

example:
V = 1/2 m^2 φ^2
"Chaotic inflation"
= M_p^2/2 (V'/V)^2 = 2M_p^2/φ^2
η_V = M_p^2 V''/V = 2M_p^2/φ^2
slow roll φ >> M_p

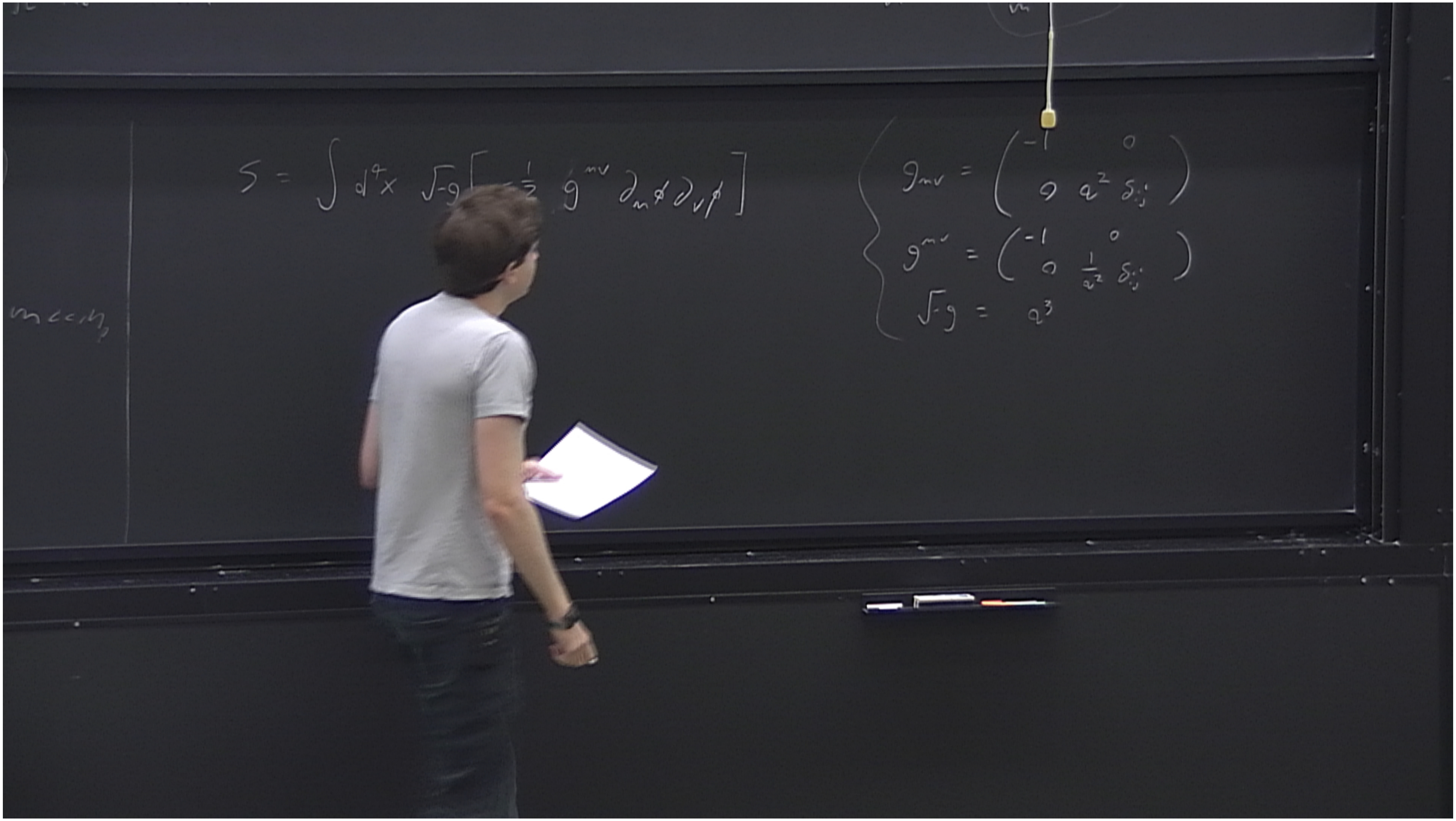

example: $V = \frac{1}{2} m^2 \phi^2$
"Chaotic inflation"
$\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 = \frac{2M_p^2}{\phi^2}$
$\eta_V = M_p^2 \frac{V''}{V} = \frac{2M_p^2}{\phi^2}$
slow roll $\phi \gg M_p$
"large field inflation"

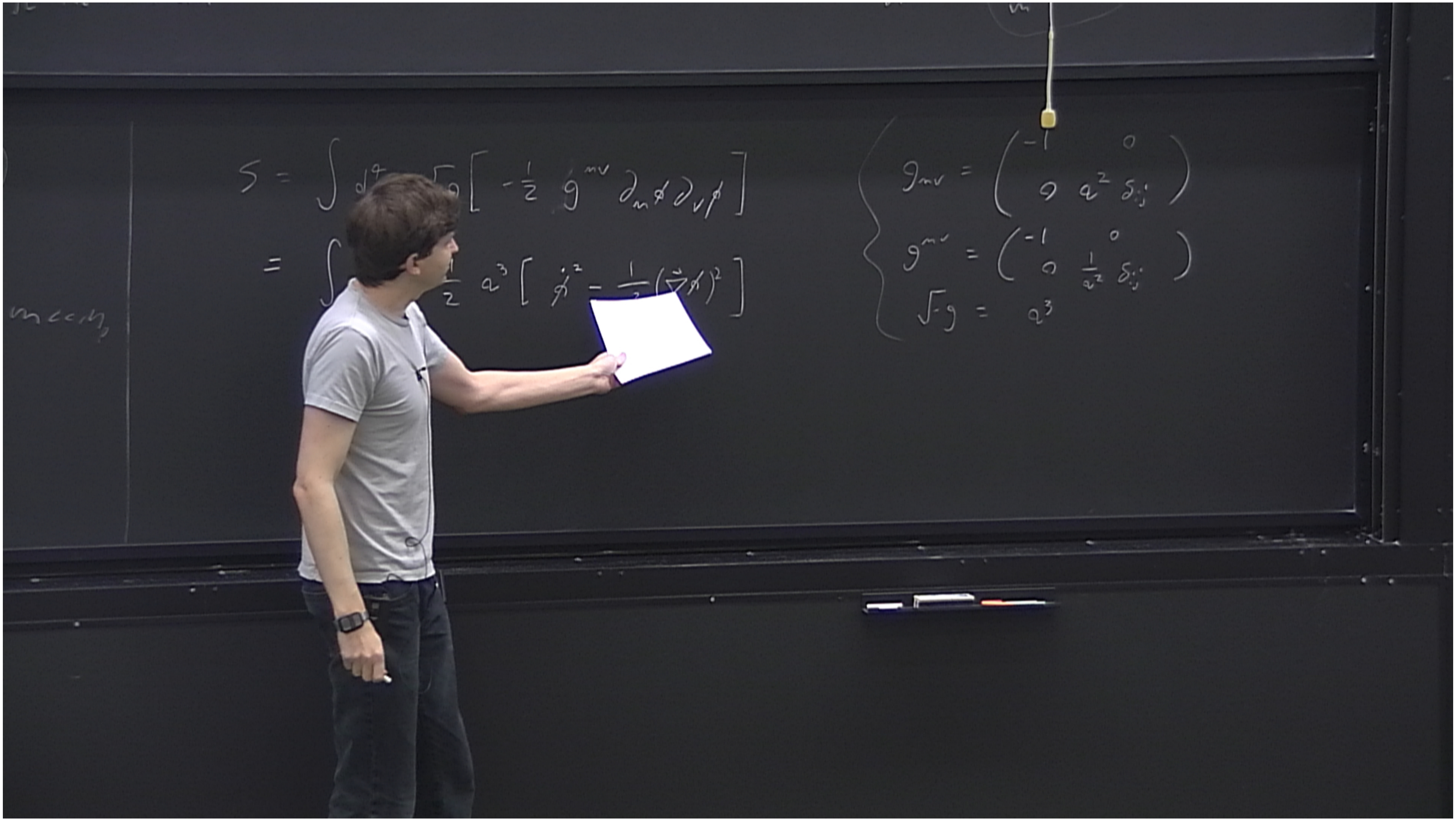

"Chaotic inflation"
energy density ~ m²φ² << Mp⁴ => φ << Mp²/m
Mp << φ << Mp²/m
N = − 1/Mp² ∫ V/V' dφ = 1/4Mp² (φi
"large field inflation"

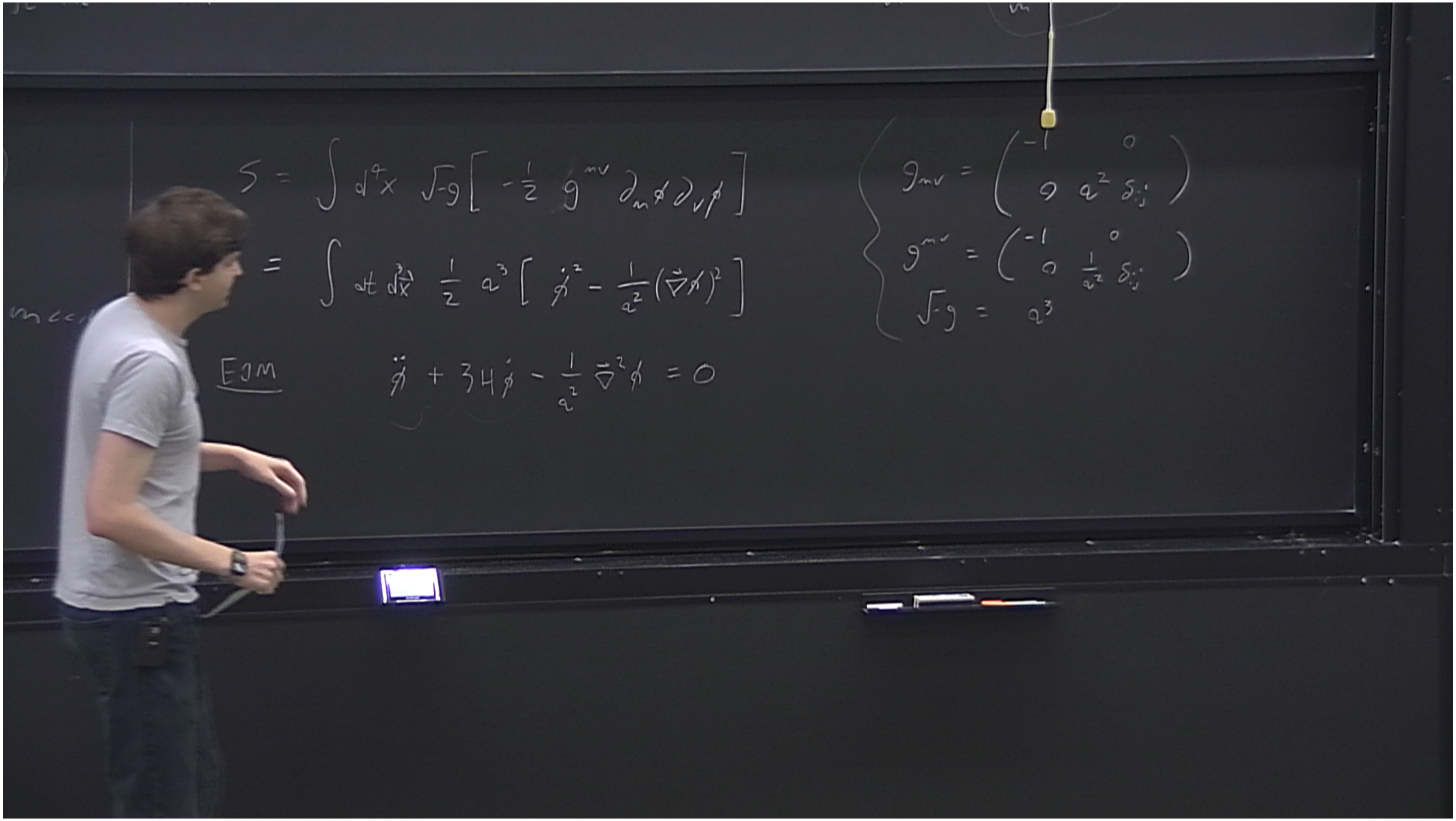

"Chaotic inflation"
energy density $\sim m^2\phi^2 \ll M_p^4 \Rightarrow \phi \ll M_p^2$
$M_p \ll \phi \ll \frac{M_p^2}{m}$
$N = -\frac{1}{M_p^2}\int_{\phi_i}^{\phi_f} \frac{V}{V'}\,d\phi = \frac{1}{4M_p^2}(\phi_i^2 - \phi_f^2)$
"large field inflation"


"inflation"
energy density ~ m²φ² << Mp⁴ => φ << Mp²/m
Mp << φ << Mp²/m
ex. φi = 10 mp
φf = 1/10 Mp²/m
N = − 1/Mp² ∫ V/V' dφ = 1/(4Mp²) (φi² − φf²
solve for Mp/m
50
"inflation"

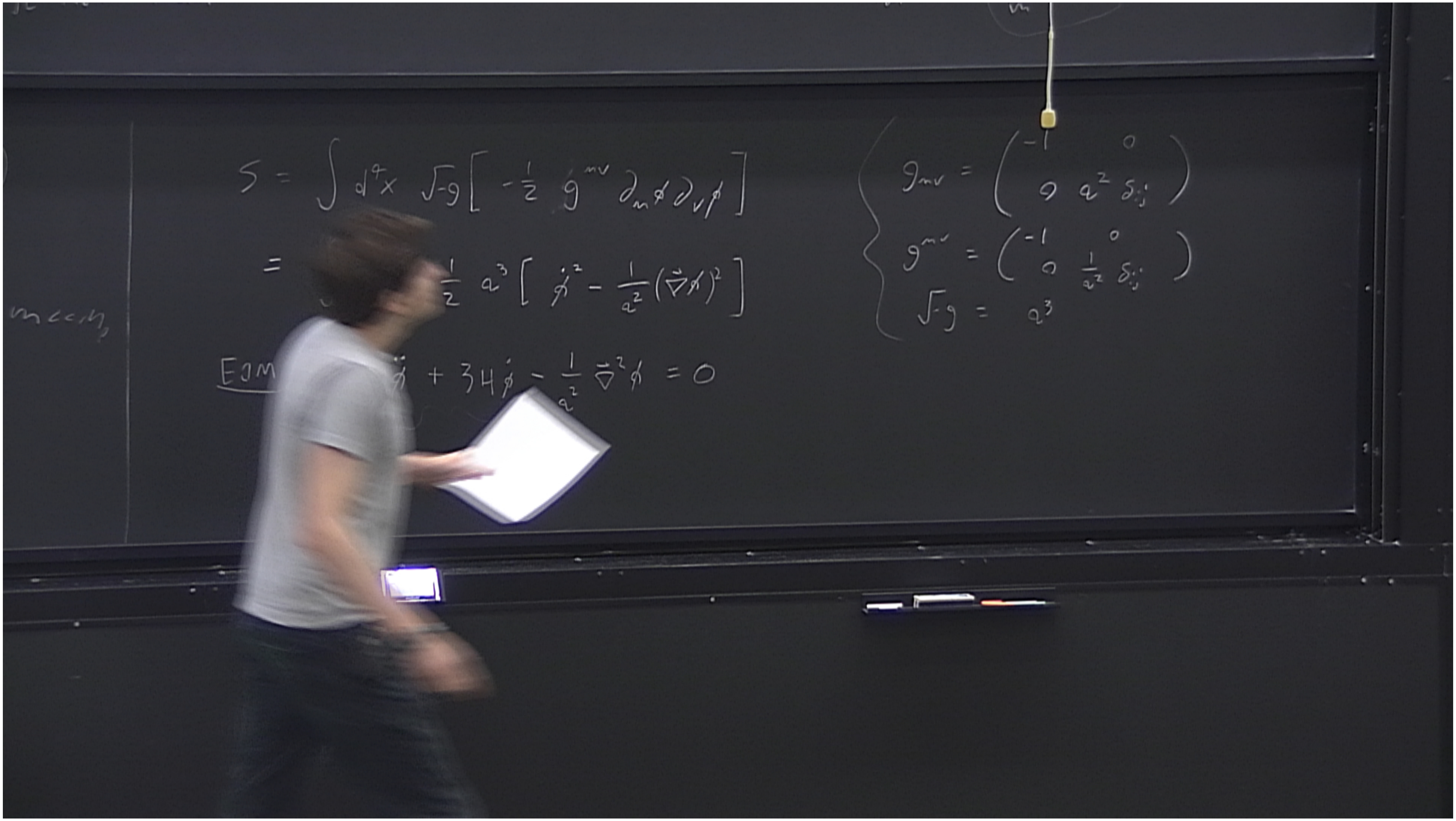

$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$


$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2 \delta_{ij} \end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{a^2} \delta_{ij} \end{pmatrix}$
$\sqrt{-g} = a^3$


$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
$= \int \ldots \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2} (\vec{\nabla}\phi)^2 \right]$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2 \delta_{ij} \end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{a^2} \delta_{ij} \end{pmatrix}$
$\sqrt{-g} = a^3$



$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$
$= \int dt\, d^3\vec{x}\, \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2} (\vec{\nabla}\phi)^2 \right]$
EOM
$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2} \vec{\nabla}^2 \phi = 0$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2 \delta_{ij} \end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{a^2} \delta_{ij} \end{pmatrix}$
$\sqrt{-g} = a^3$


$S = \int d^4x \sqrt{-g}\left[-\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi\right]$
$= \ldots \frac{1}{2} a^3\left[\dot\phi^2 - \frac{1}{a^2}(\vec\nabla\phi)^2\right]$
EOM $\ldots \phi + 3H\dot\phi - \frac{1}{a^2}\vec\nabla^2\phi = 0$
$g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2\delta_{ij}\end{pmatrix}$
$g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{a^2}\delta_{ij}\end{pmatrix}$
$\sqrt{-g} = a^3$

$m \ll M_p$

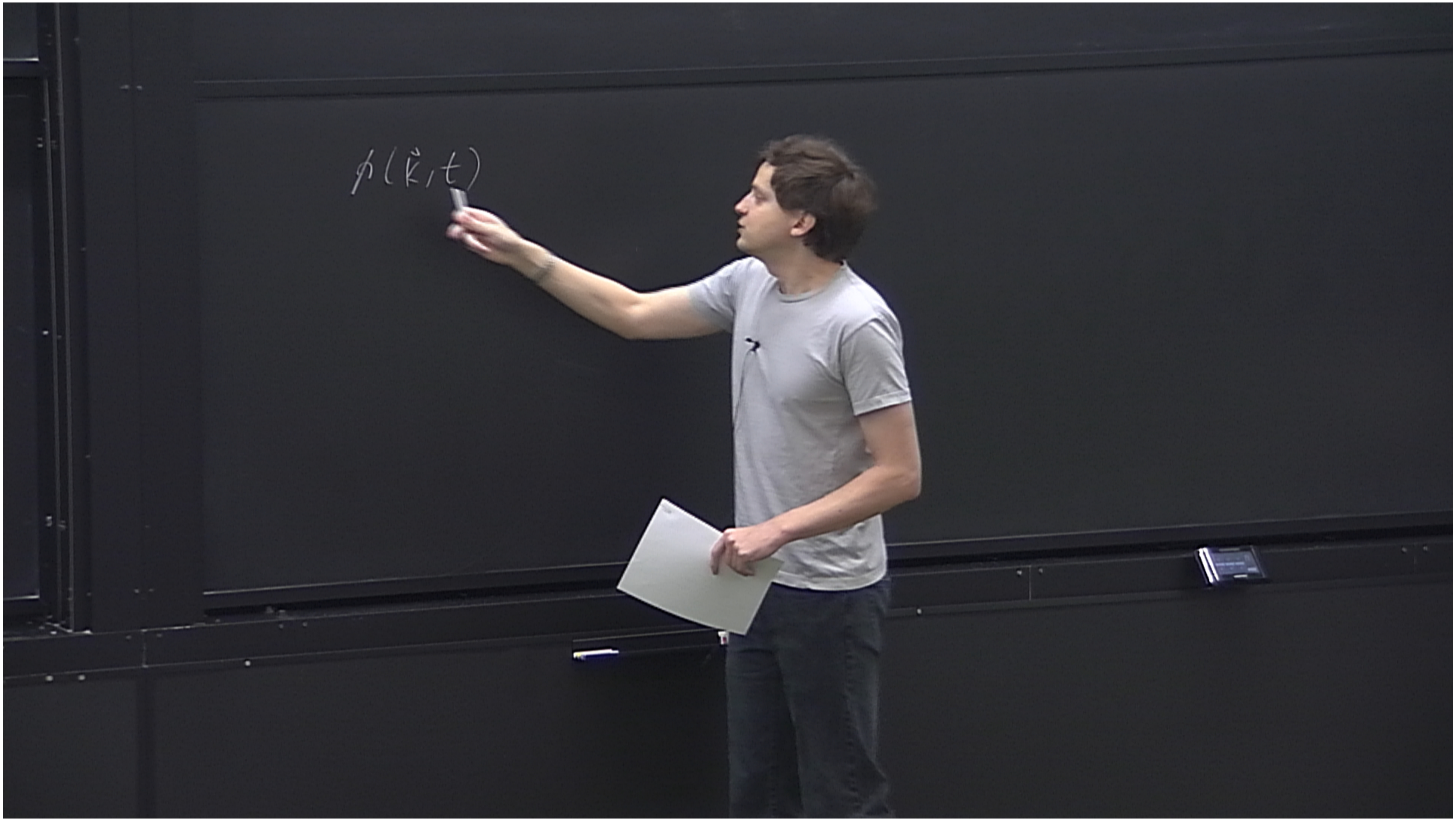
$$S = \int d^4x \, \sqrt{-g} \left[-\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi \right]$$

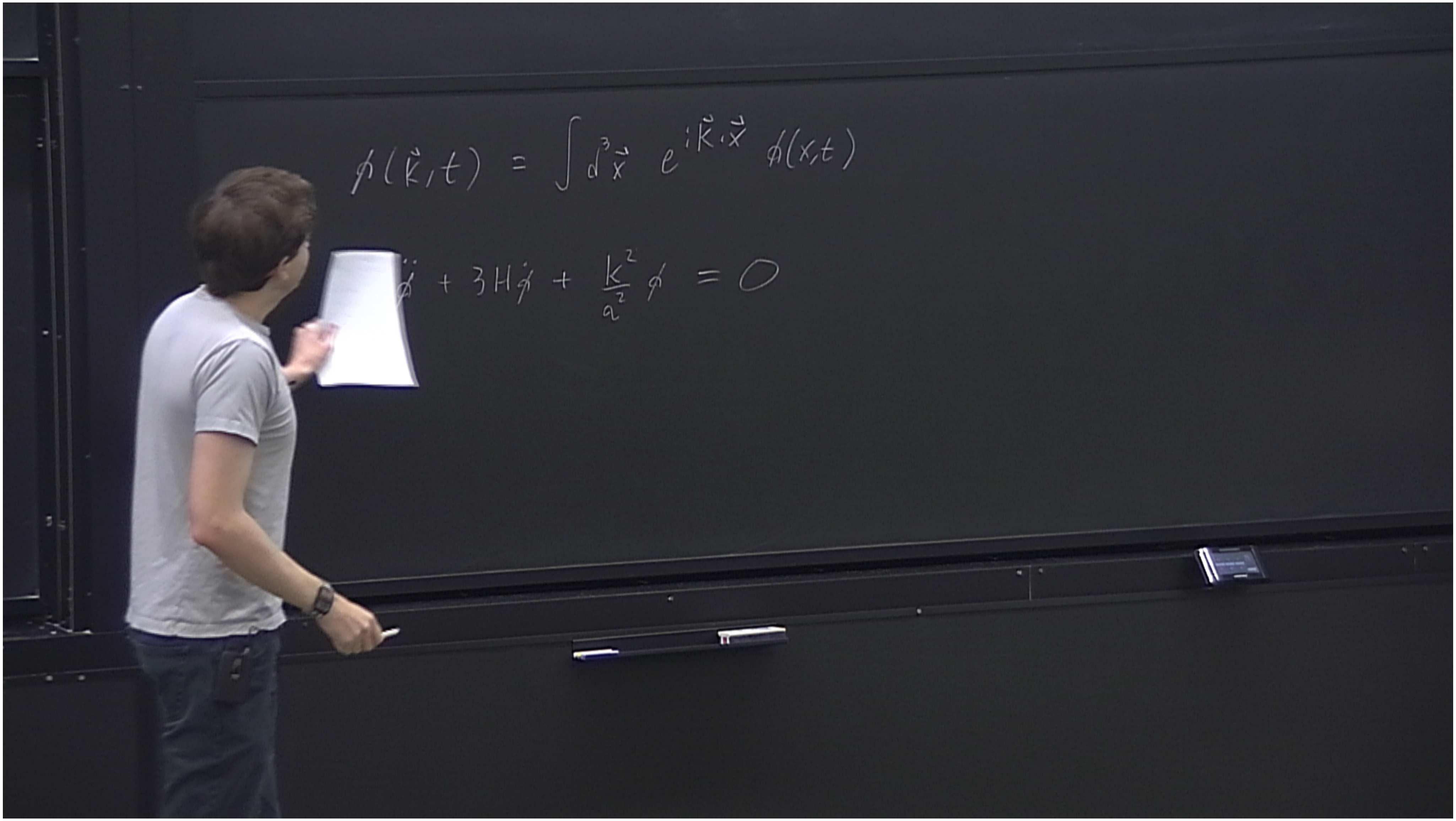
$$= \int dt \, d^3\vec{x} \, \frac{1}{2} a^3 \left[\dot{\phi}^2 - \frac{1}{a^2} (\vec{\nabla} \phi)^2 \right]$$

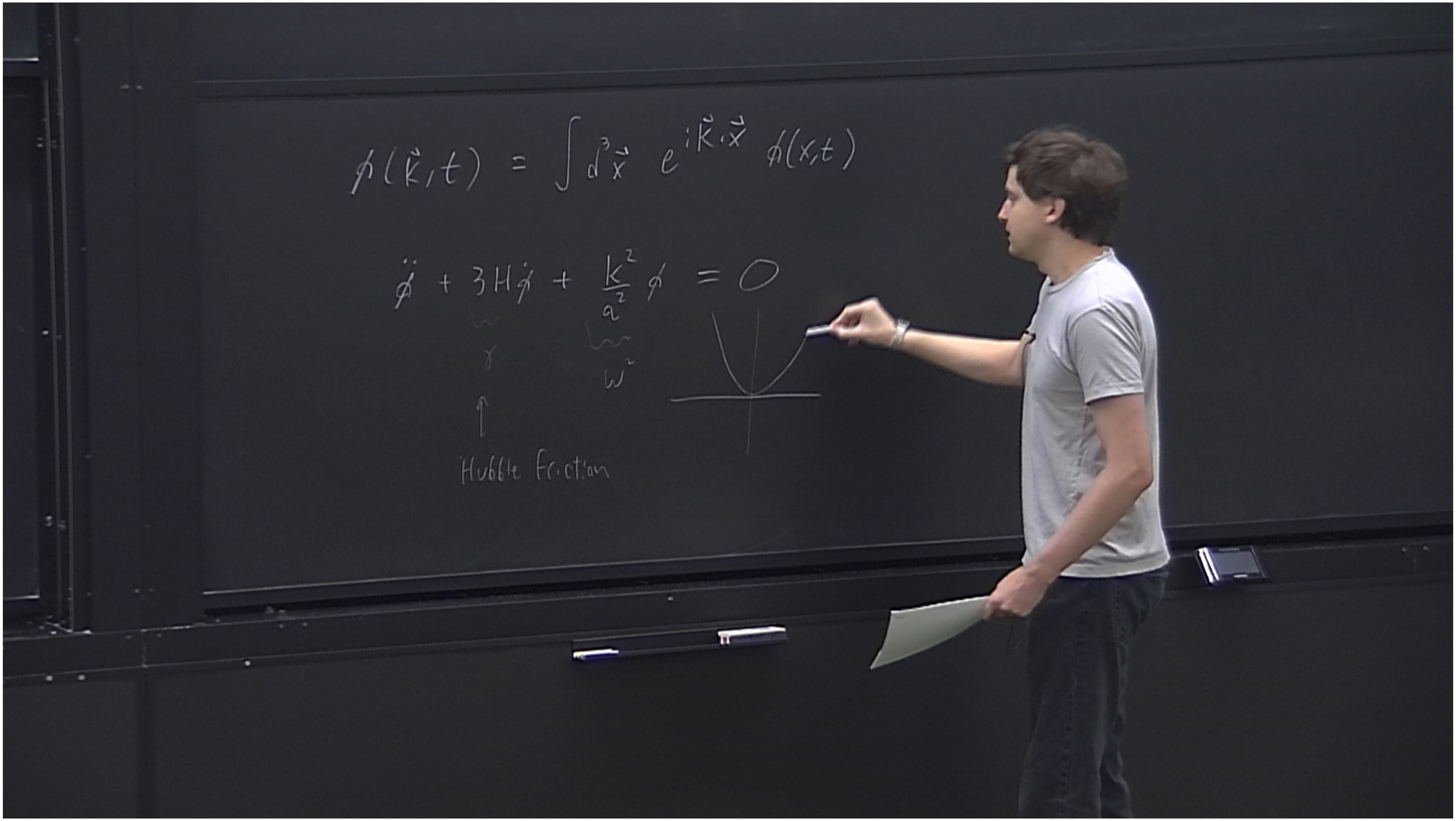
$$\left\{ \begin{array}{l} g_{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & a^2 \delta_{ij} \end{pmatrix} \\ g^{\mu\nu} = \begin{pmatrix} -1 & 0 \\ 0 & \frac{1}{a^2} \delta_{ij} \end{pmatrix} \\ \sqrt{-g} = a^3 \end{array} \right.$$

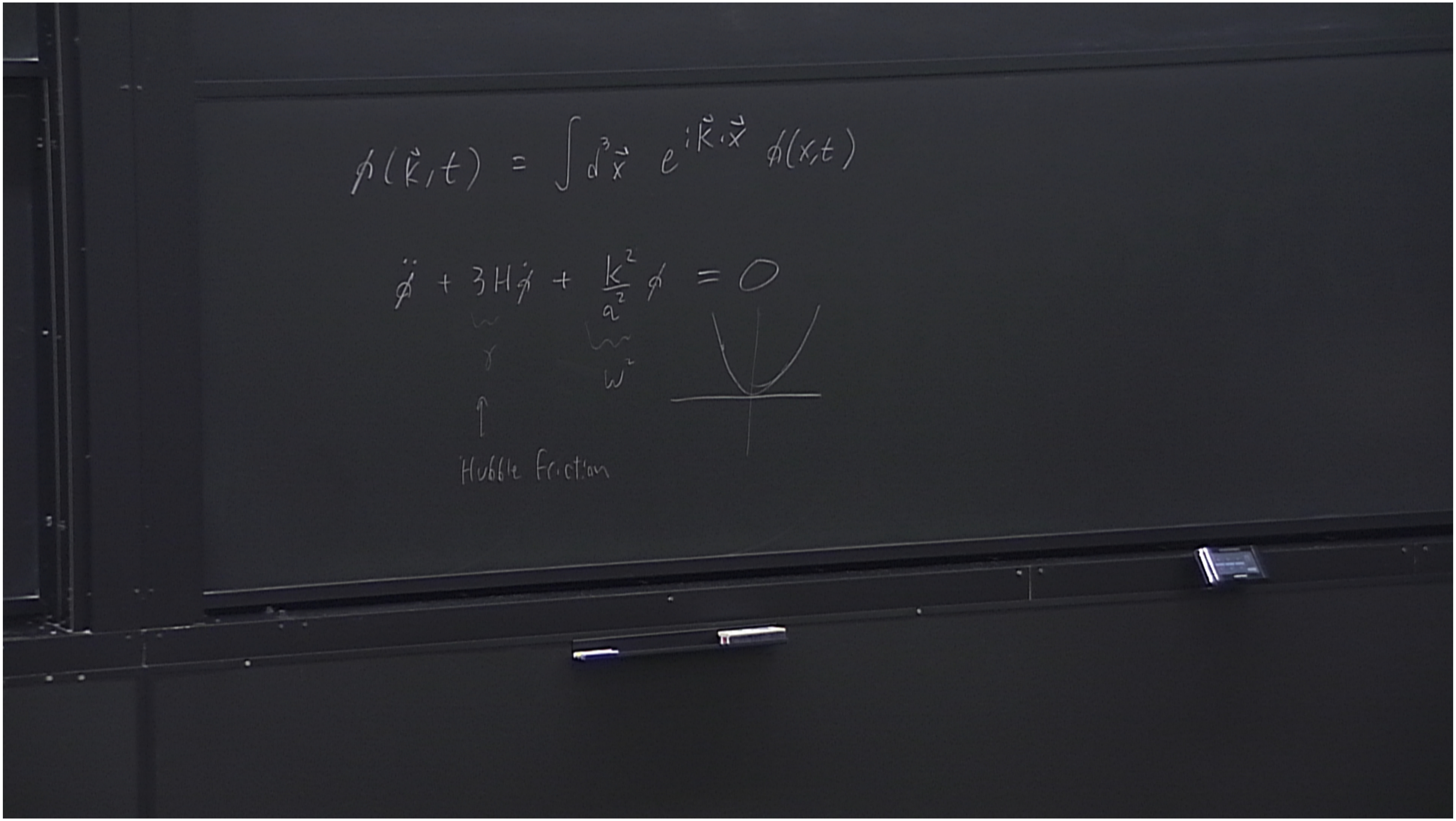
EOM

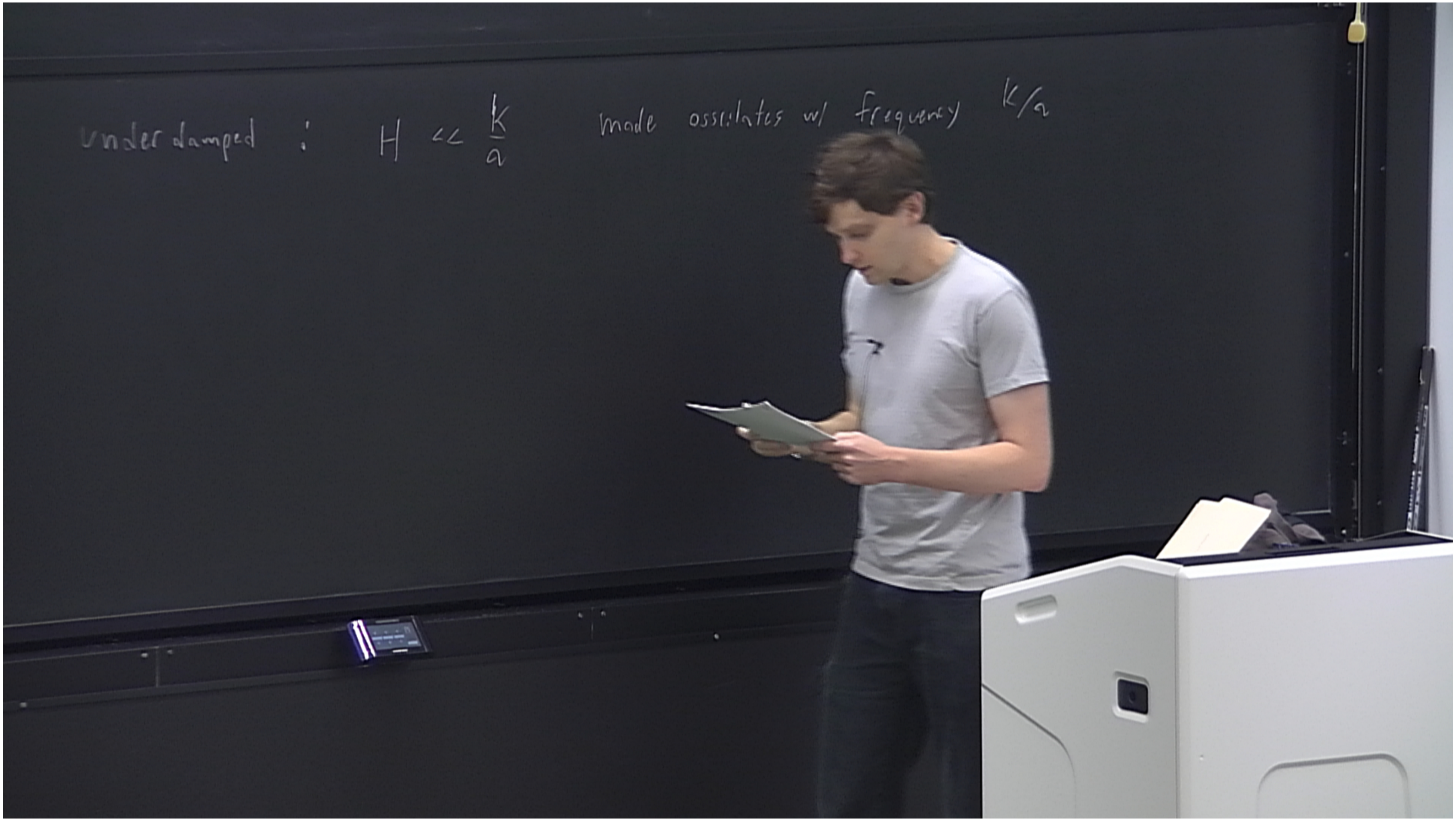
$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{a^2} \vec{\nabla}^2 \phi = 0$$

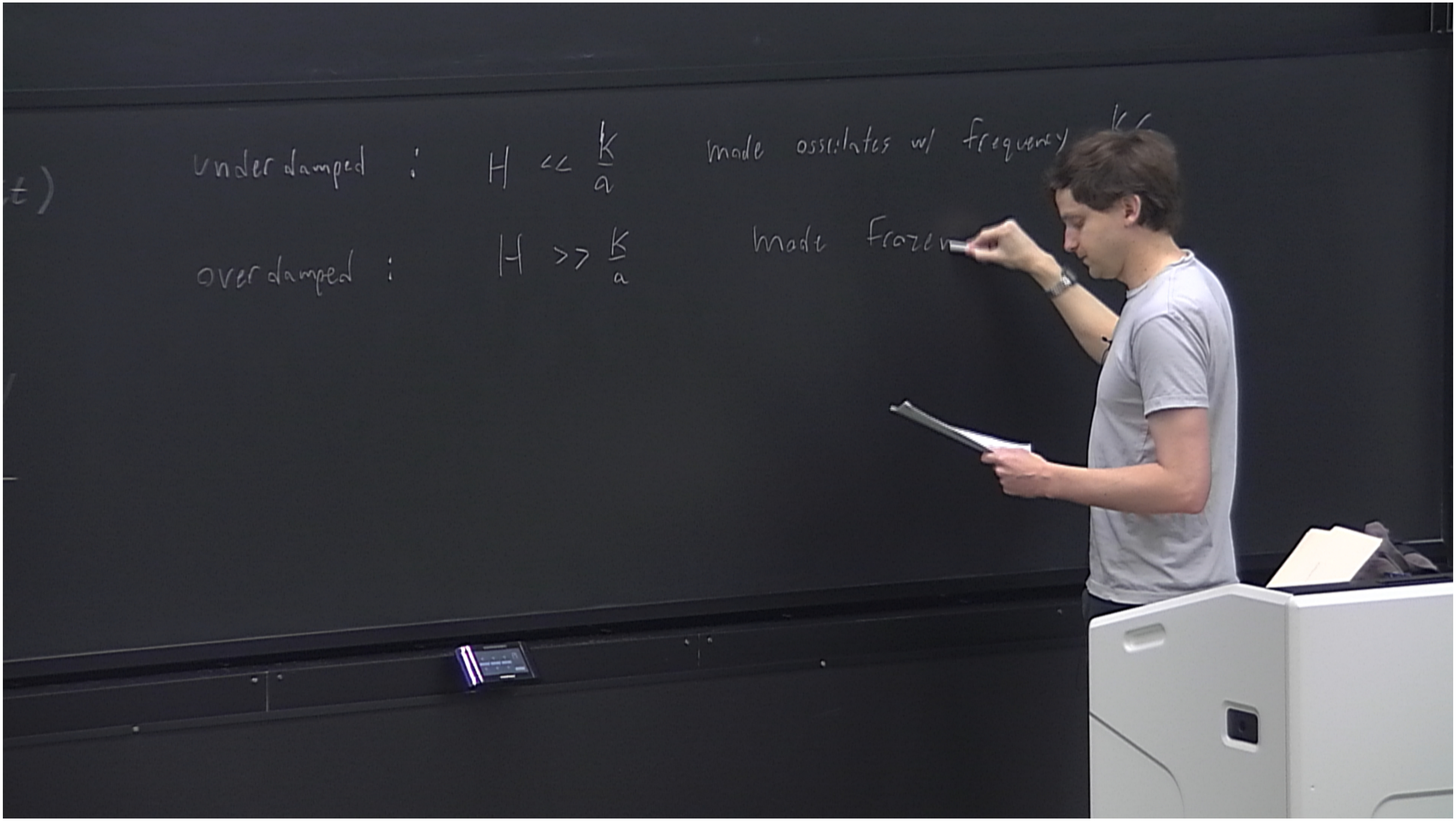

$\phi(\vec{k}, t)$

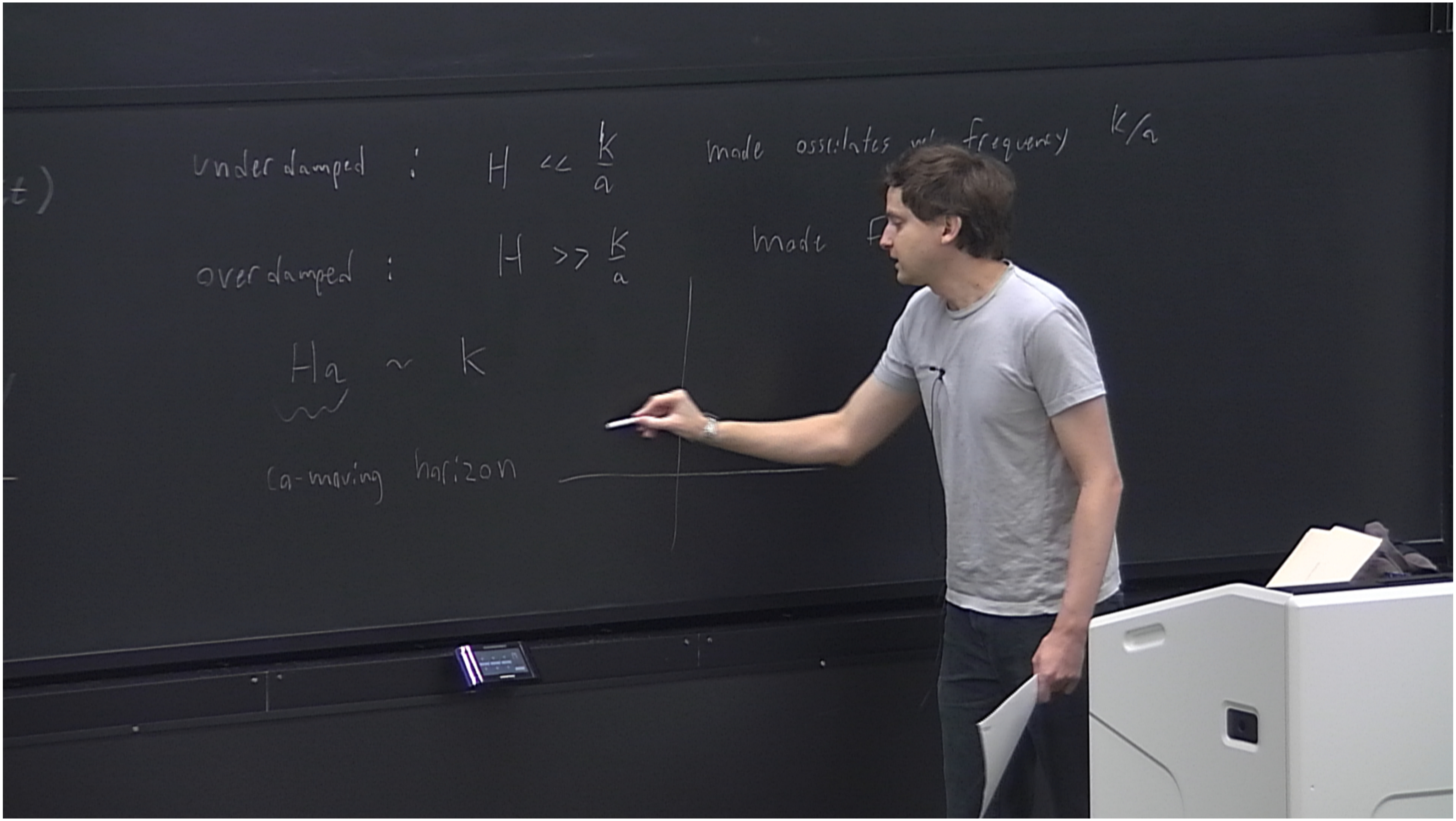

$\phi(\vec{k},t) = \int d^3\vec{x}\ e^{i\vec{k}\cdot\vec{x}}\ \phi(x,t)$
$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0$

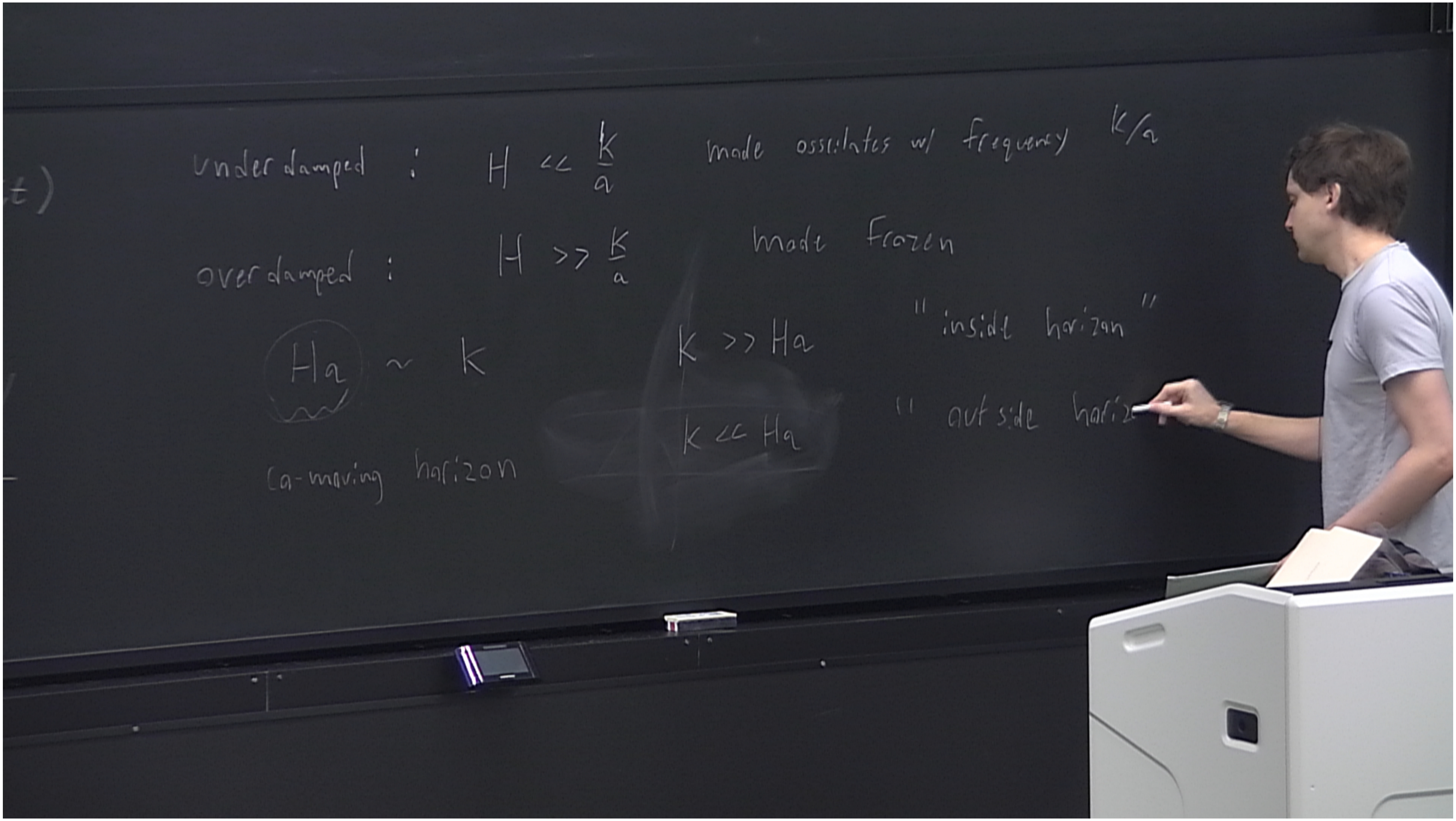

$\phi(\vec{k},t) = \int d^3\vec{x}\, e^{i\vec{k}\cdot\vec{x}}\, \phi(x,t)$
$\ddot{\phi} + 3H\dot{\phi} + \frac{k^2}{a^2}\phi = 0$
γ
ω^2
Hubble friction

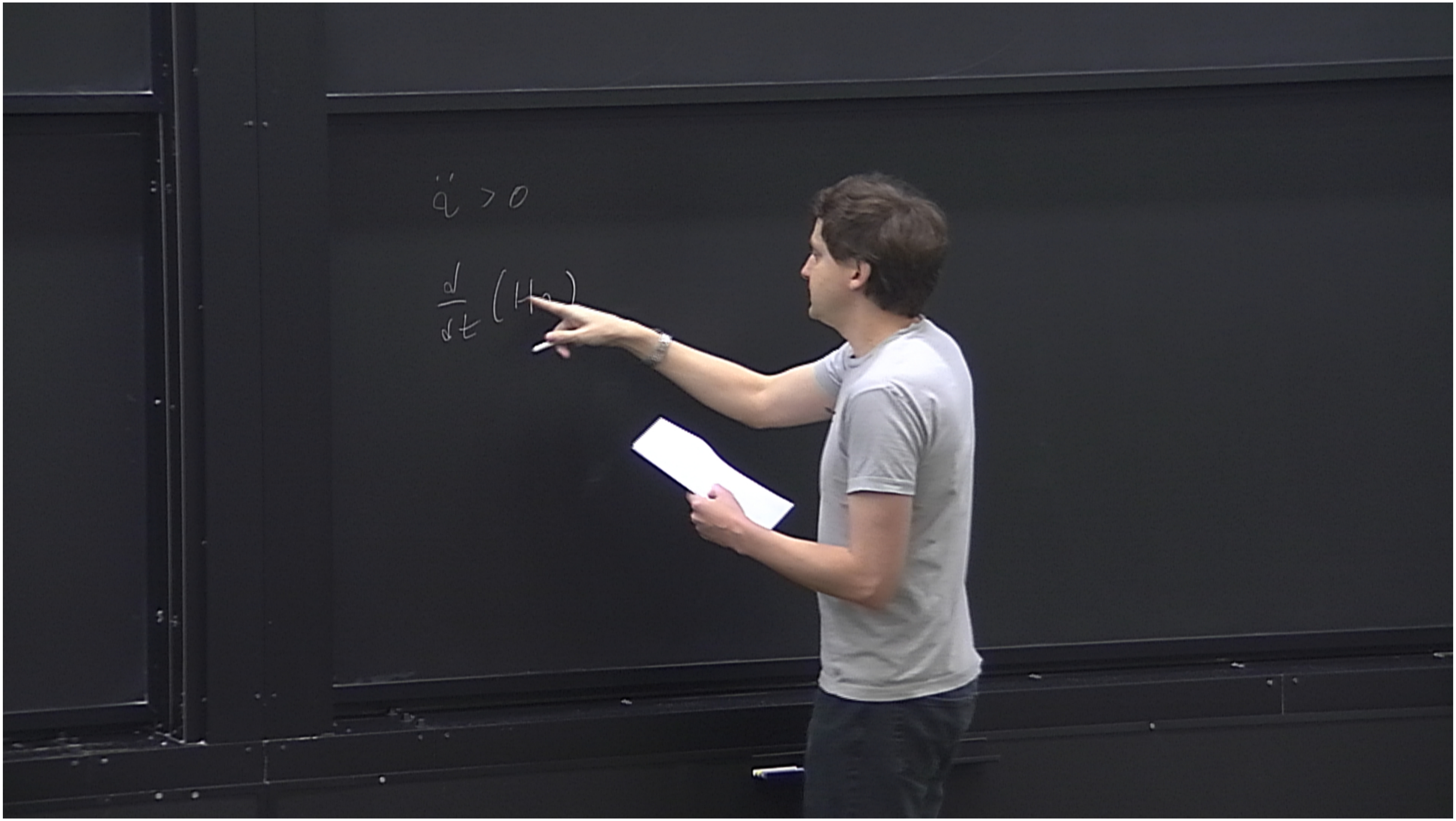

$\phi(\vec{k},t) = \int d^3\vec{x}\ e^{i\vec{k}\cdot\vec{x}}\ \phi(x,t)$
$\ddot{\phi} + \underbrace{3H}_{\gamma}\dot{\phi} + \underbrace{\frac{k^2}{a^2}}_{\omega^2}\phi = 0$
Hubble Friction

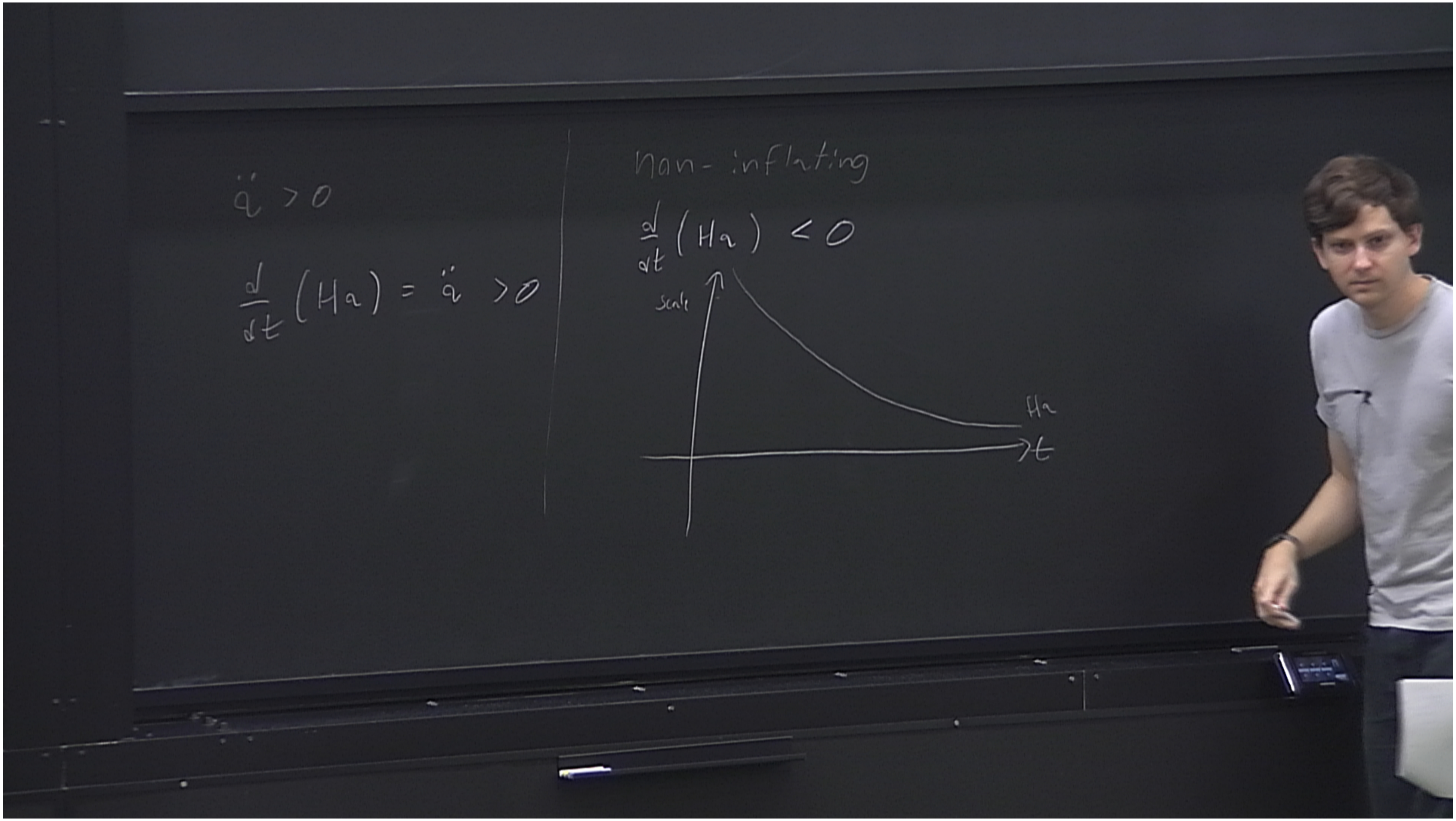

underdamped : $H \ll \frac{k}{a}$
mode ossilates w/ frequency k/a

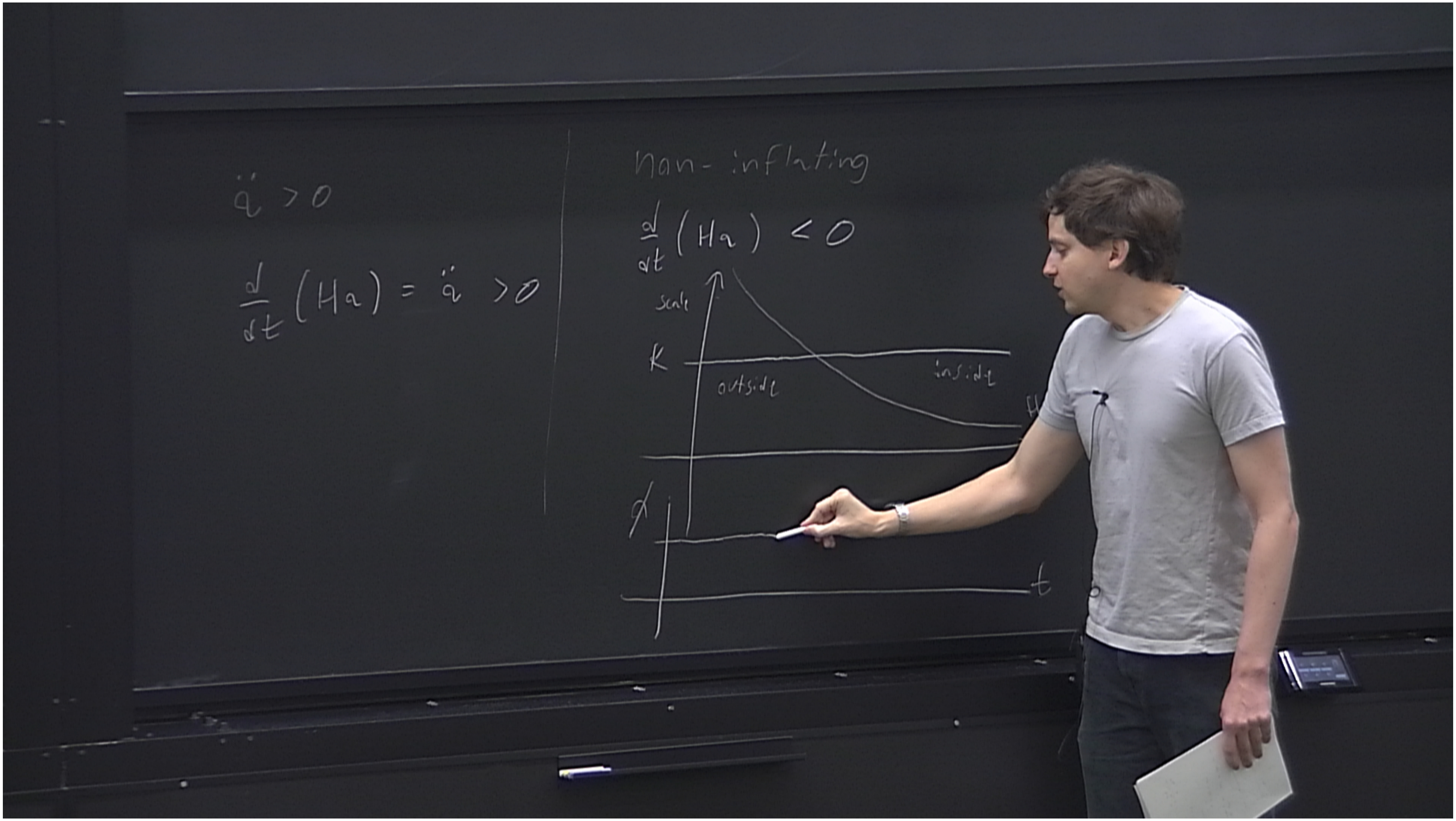

underdamped : H << k/a
mode ossilates w/ frequency
overdamped : H >> k/a
mode frozen

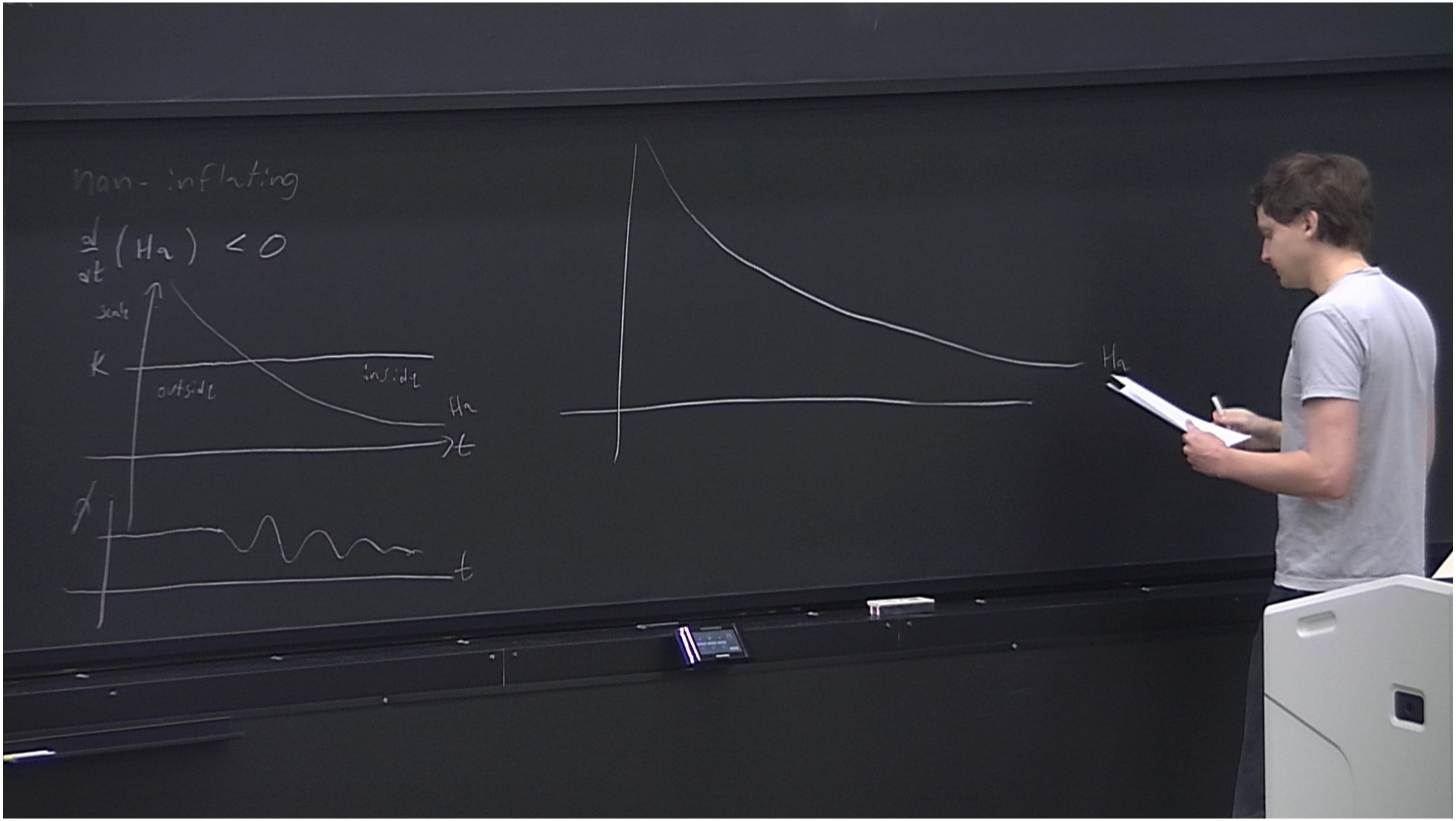

underdamped : $H \ll \frac{k}{a}$
mode oscillates w/ frequency k/a
overdamped : $H \gg \frac{k}{a}$
mode
$Ha \sim k$
co-moving horizon


underdamped : $H \ll \frac{k}{a}$
mode ossilates w/ frequency k/a
overdamped : $H \gg \frac{k}{a}$
mode frozen
$Ha \sim k$
$k \gg Ha$ "inside horizon"
$k \ll Ha$ " outside horiz
co-moving horizon

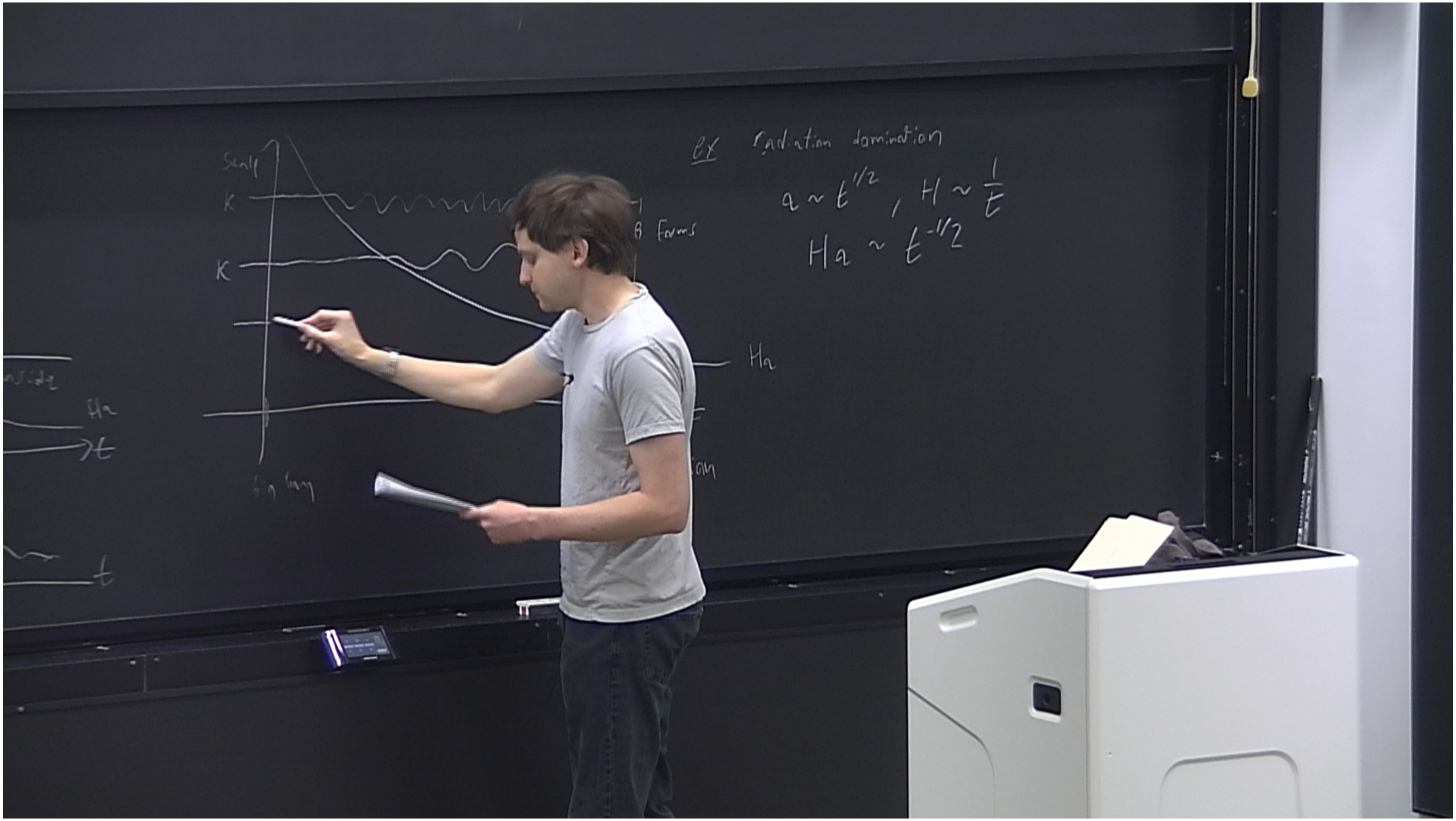

ä > 0

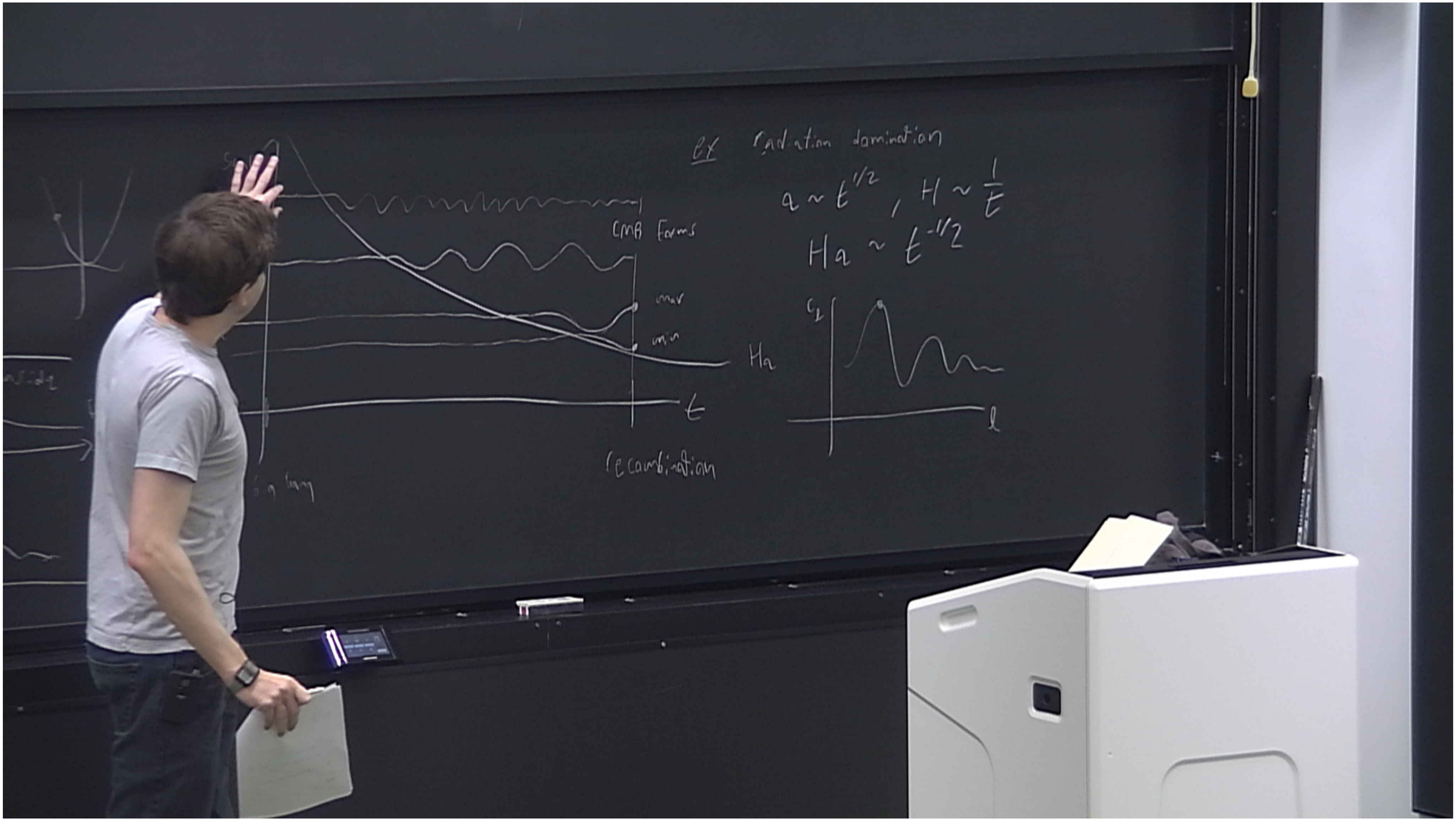

$\ddot{a} > 0$
$\frac{d}{dt}(Ha) = \ddot{a} > 0$
non-inflating
$\frac{d}{dt}(Ha) < 0$
scale
Ha
t

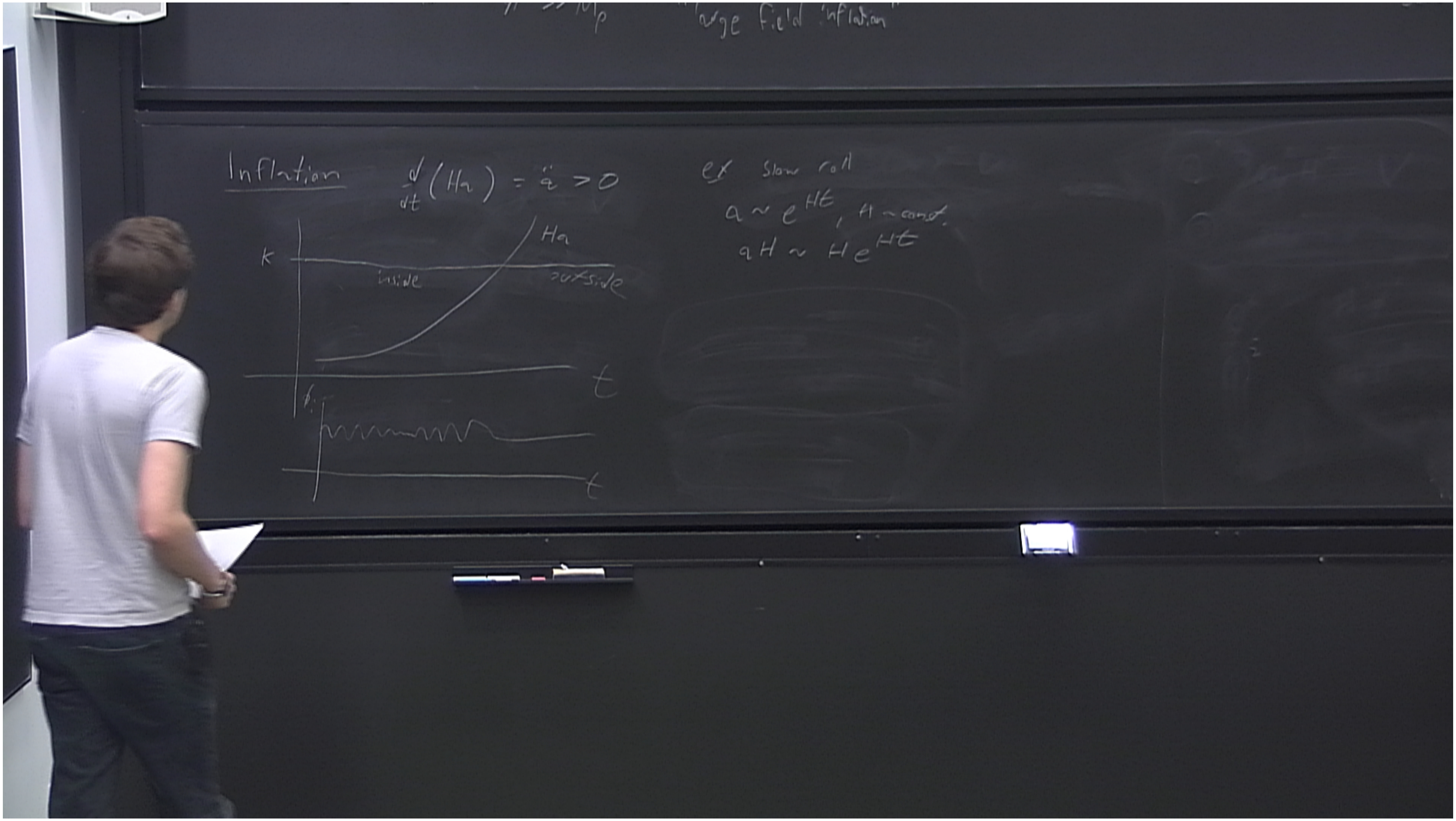

$\ddot{a} > 0$
$\frac{d}{dt}(Ha) = \ddot{a} > 0$
non-inflating
$\frac{d}{dt}(Ha) < 0$
scale
k
outside
inside
H
ϕ
t

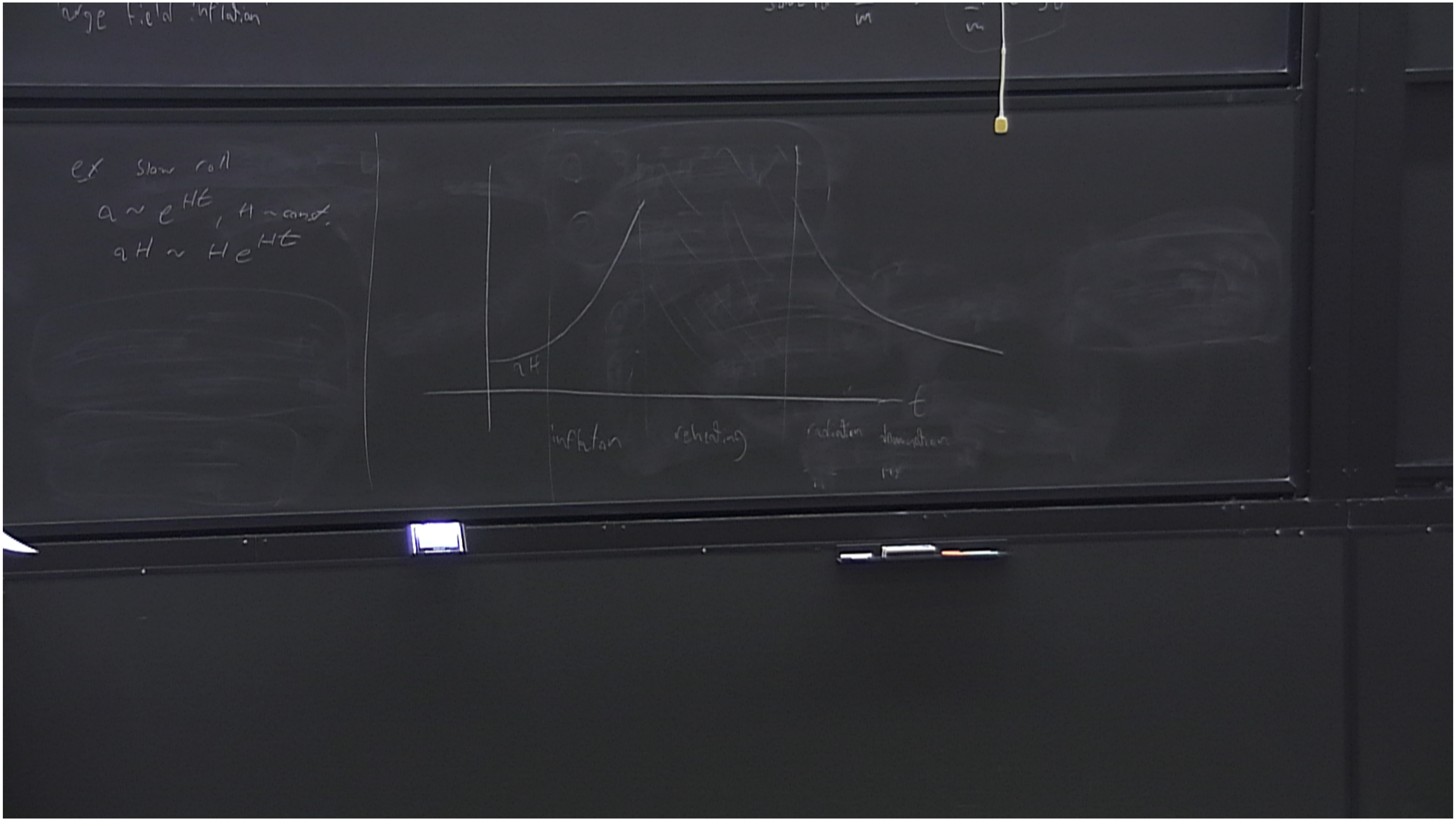

non-inflating
$\frac{d}{dt}(Ha) < 0$
scale
k
outside
inside
Ha
t
ϕ
t
Ha


scale
k
CMB forms
Ha
t
recombination
big bang
ex radiation domination
$a \sim t^{1/2}$


ex radiation domination
$a \sim t^{1/2}$, $H \sim \frac{1}{t}$
$Ha \sim t^{-1/2}$
Scale
K
K
Ha
Forms
big bang
t


ex radiation domination
$a \sim t^{1/2}$, $H \sim \frac{1}{t}$
$Ha \sim t^{-1/2}$
CMB forms
max
min
Ha
t
recombination
C_ℓ
ℓ


"large field inflation"
Inflation
$\frac{d}{dt}(Ha) = \ddot{a} > 0$
ex slow roll
$a \sim e^{Ht}$, $H \sim$ const.
$aH \sim He^{Ht}$
Ha
k
inside
outside
t
t


ex slow roll
a ~ e^{Ht}, H ~ const.
aH ~ He^{Ht}
aH
t
inflation
reheating
radiation domination