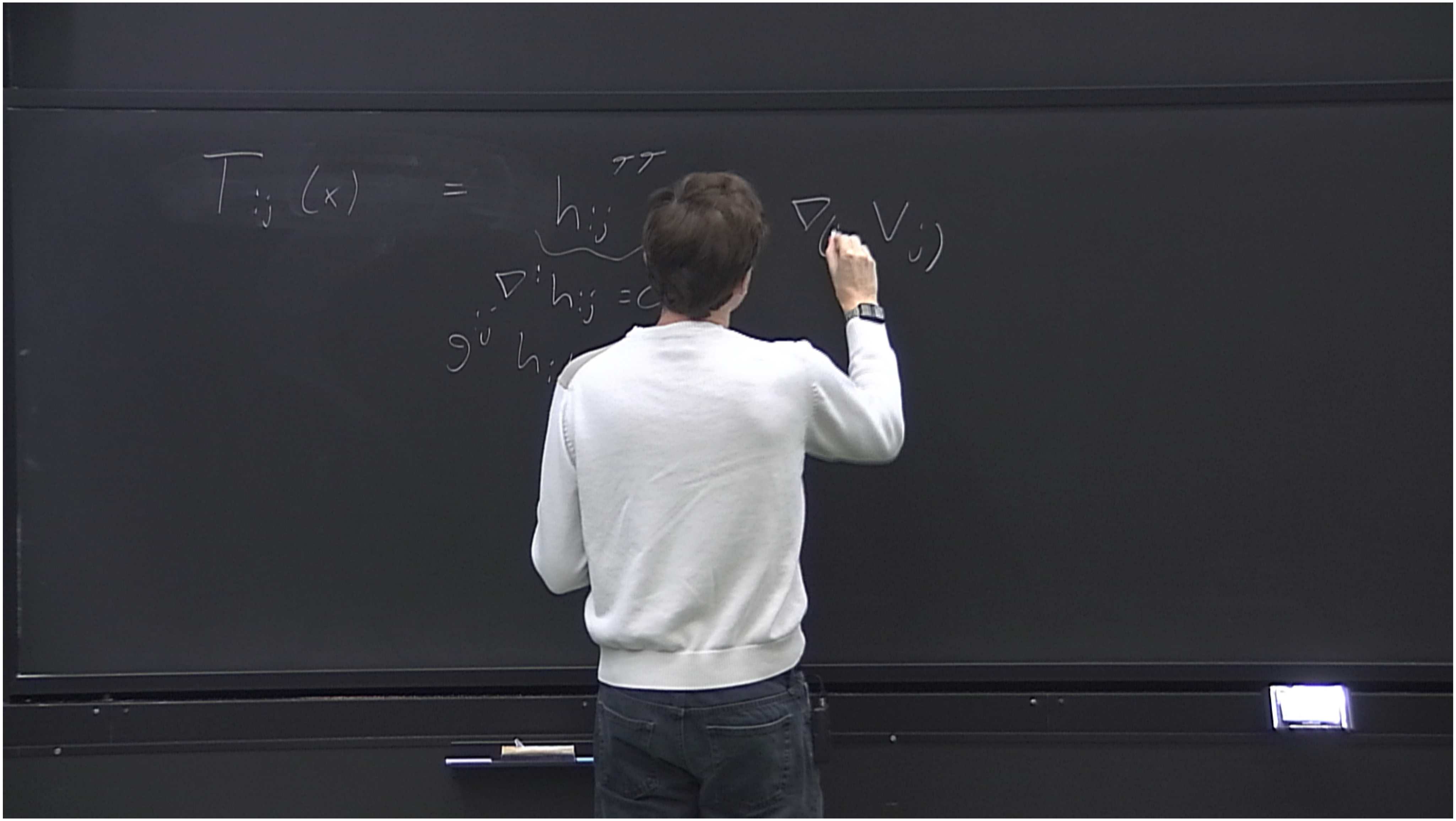
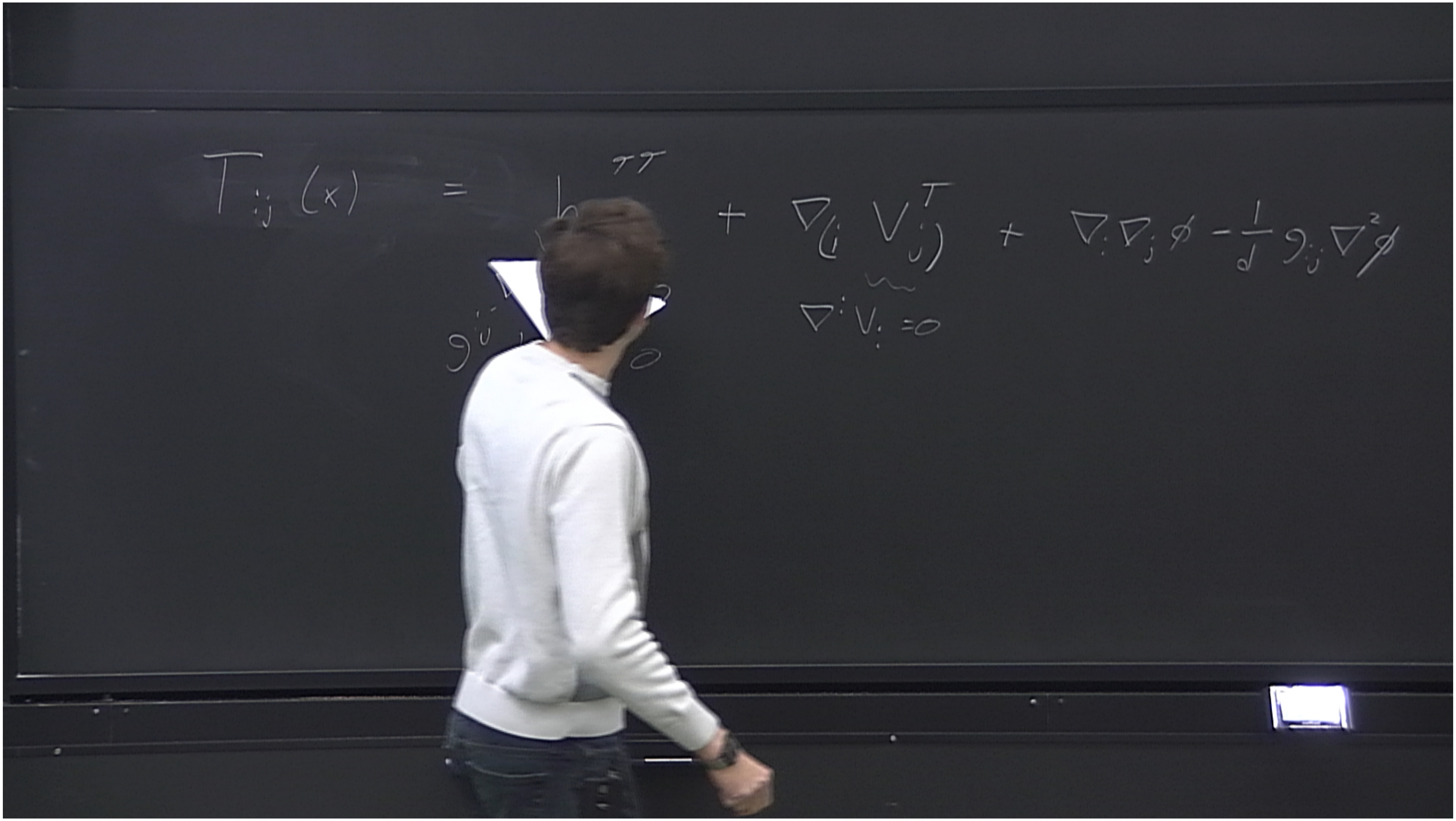
Title: Cosmology Review-9

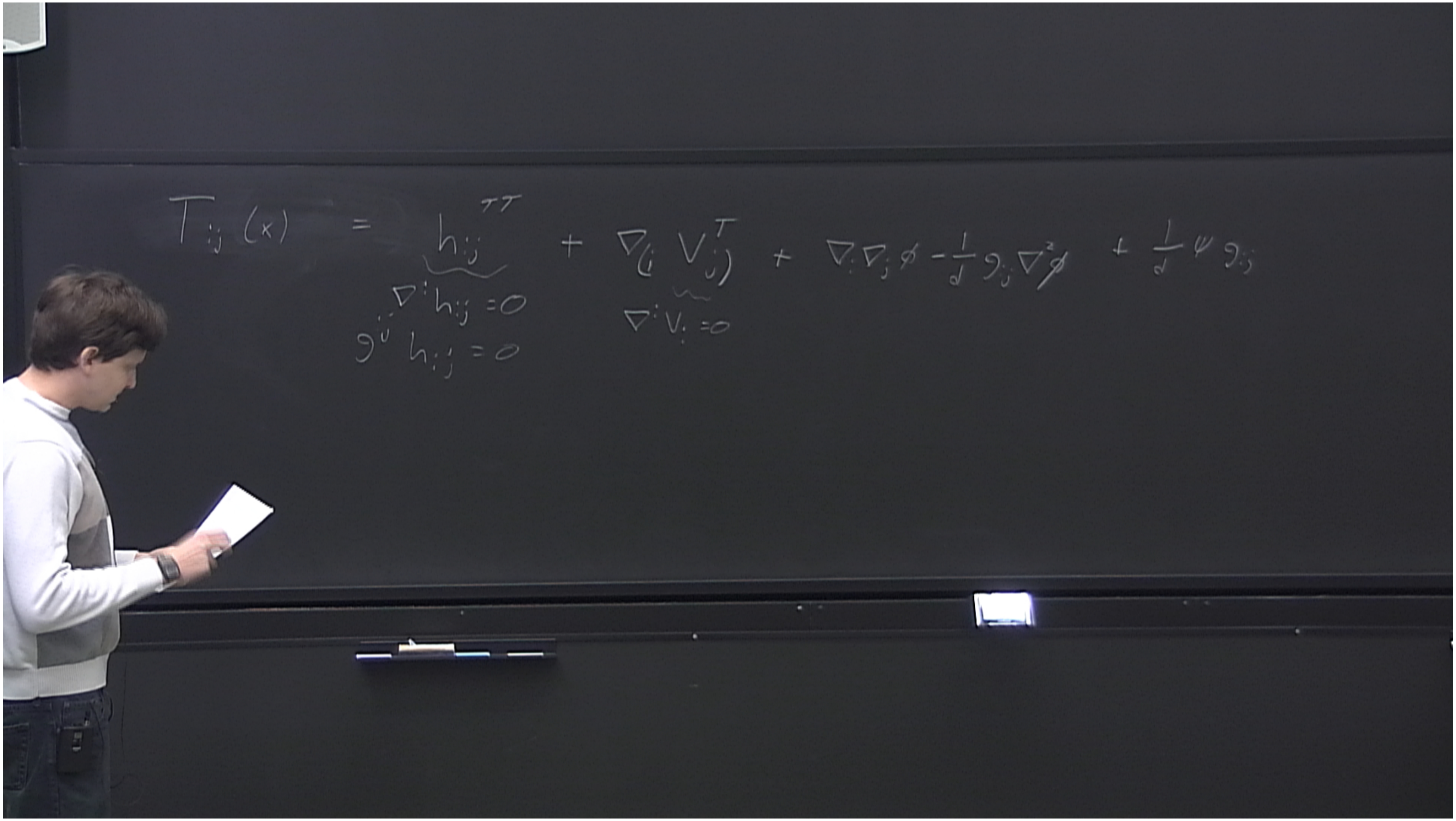
Date: Feb 05, 2015 11:30 AM

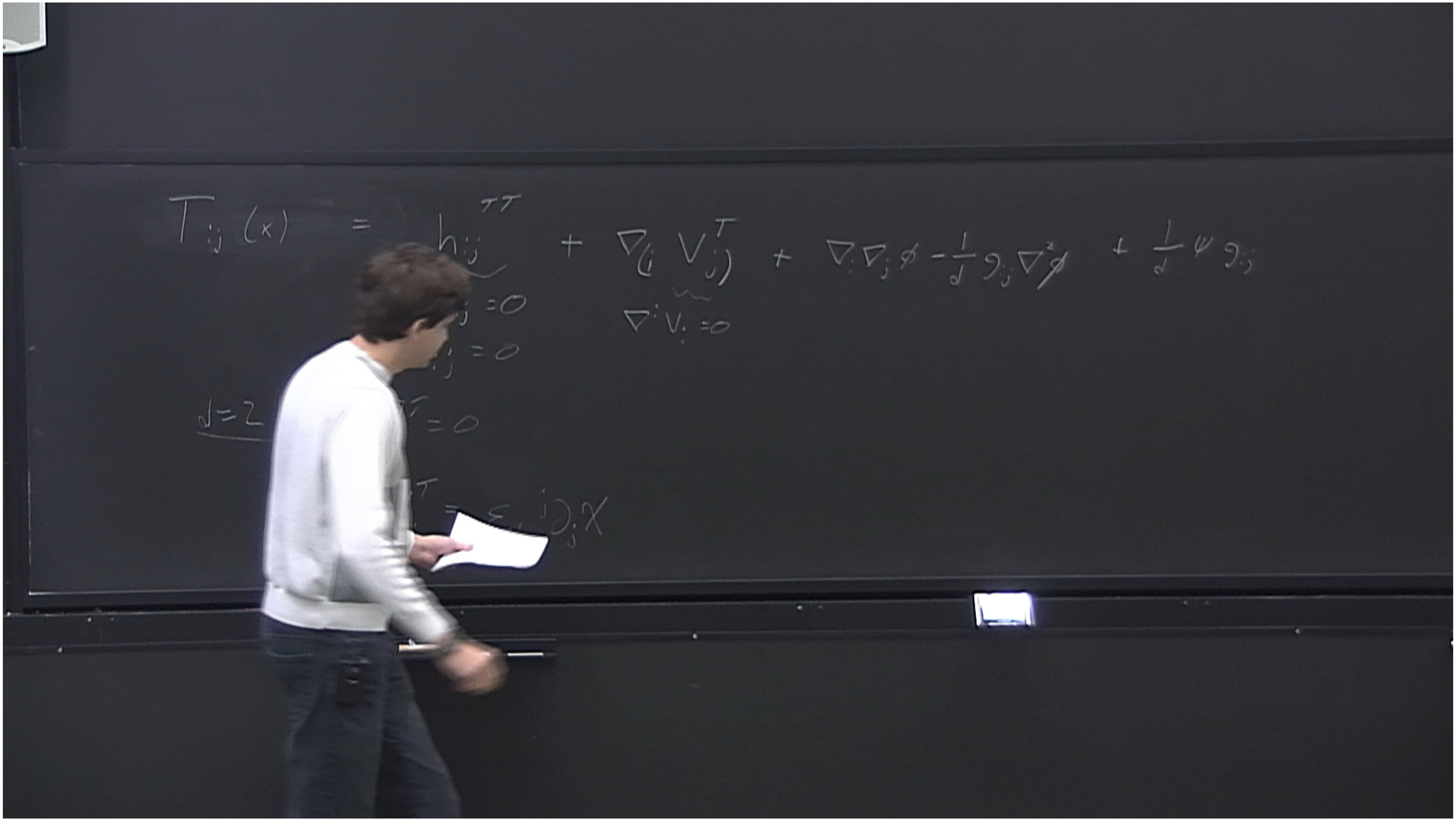
URL: http://pirsa.org/15020030

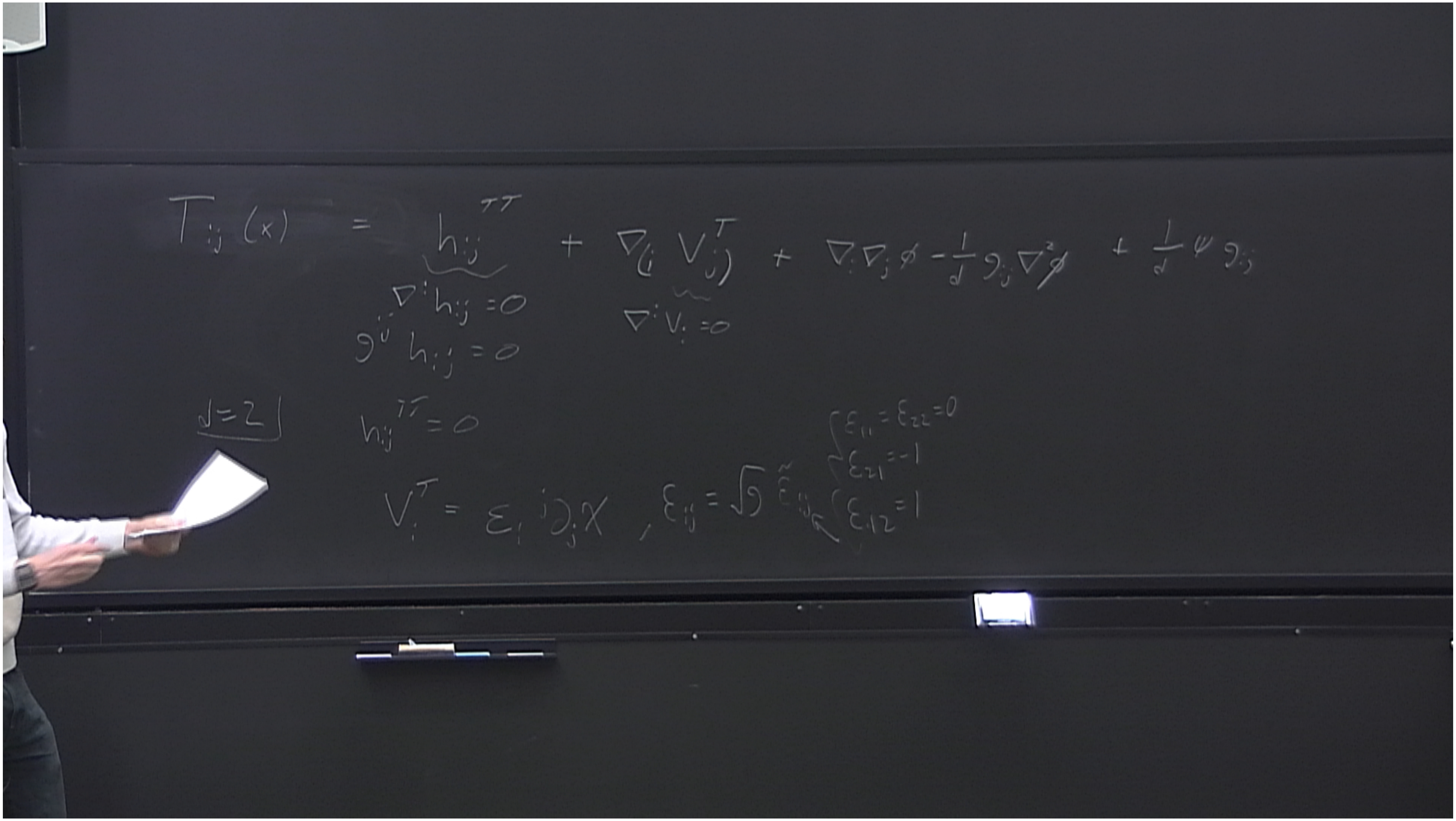
Abstract:

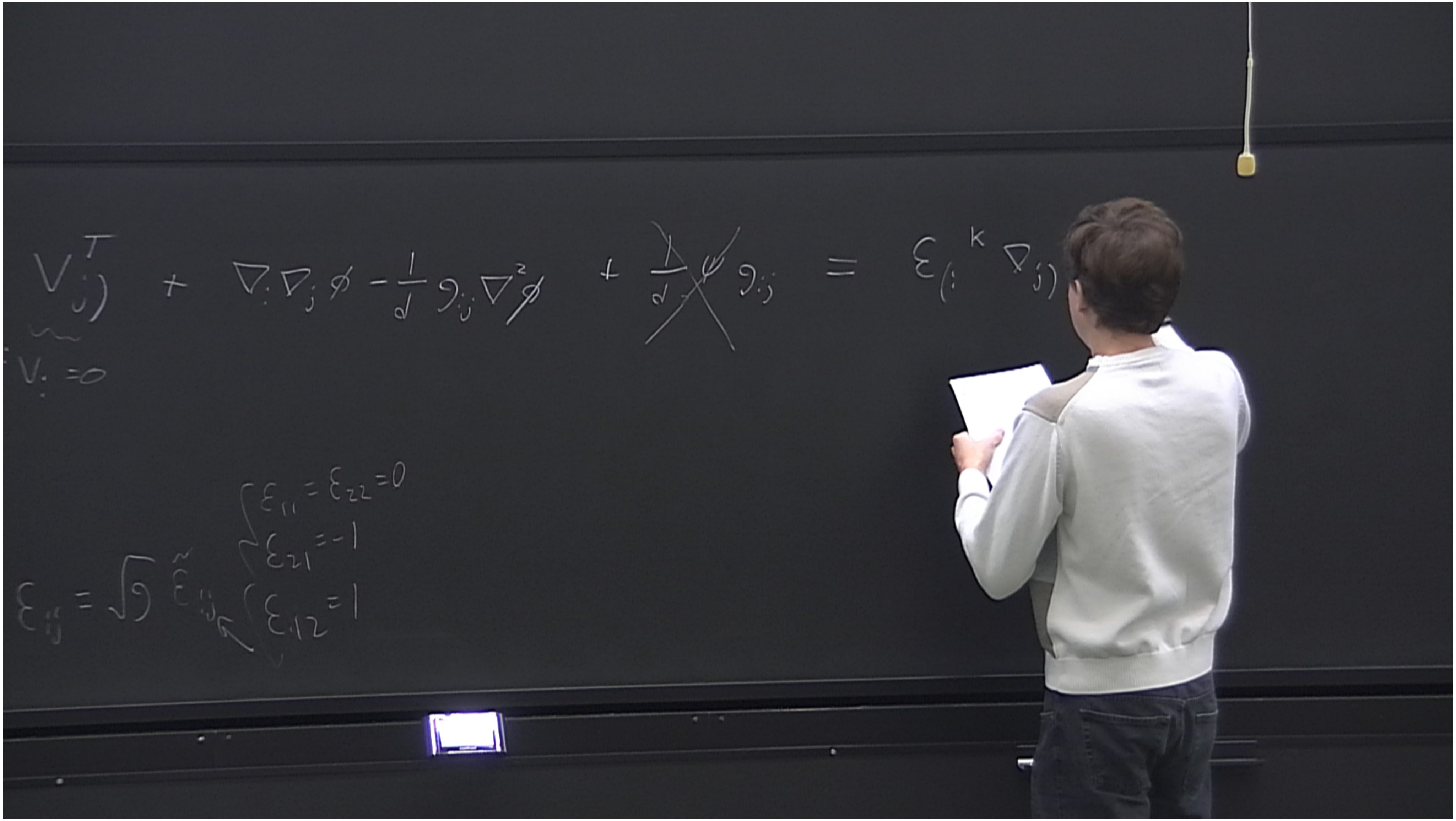

T_ij (x) = h_ij^TT ∇(i V_j)
∇^i h_ij = 0
g^ij h_ij

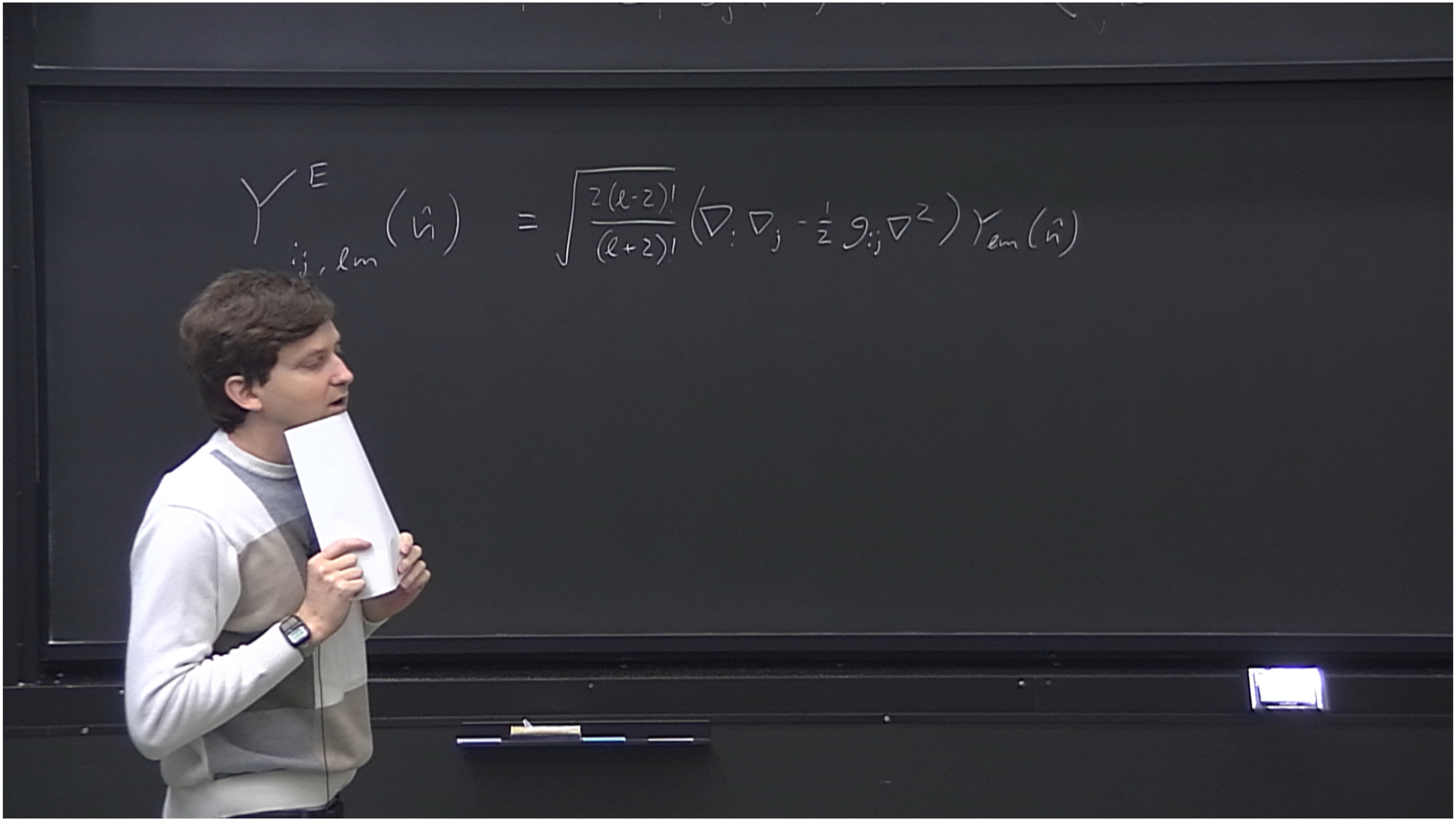

$T_{ij}(x) = h^{TT}_{ij} + \nabla_{(i} V^{T}_{j)} + \nabla_i \nabla_j \phi - \frac{1}{d} g_{ij} \nabla^2 \phi$
$\nabla^i V_i = 0$

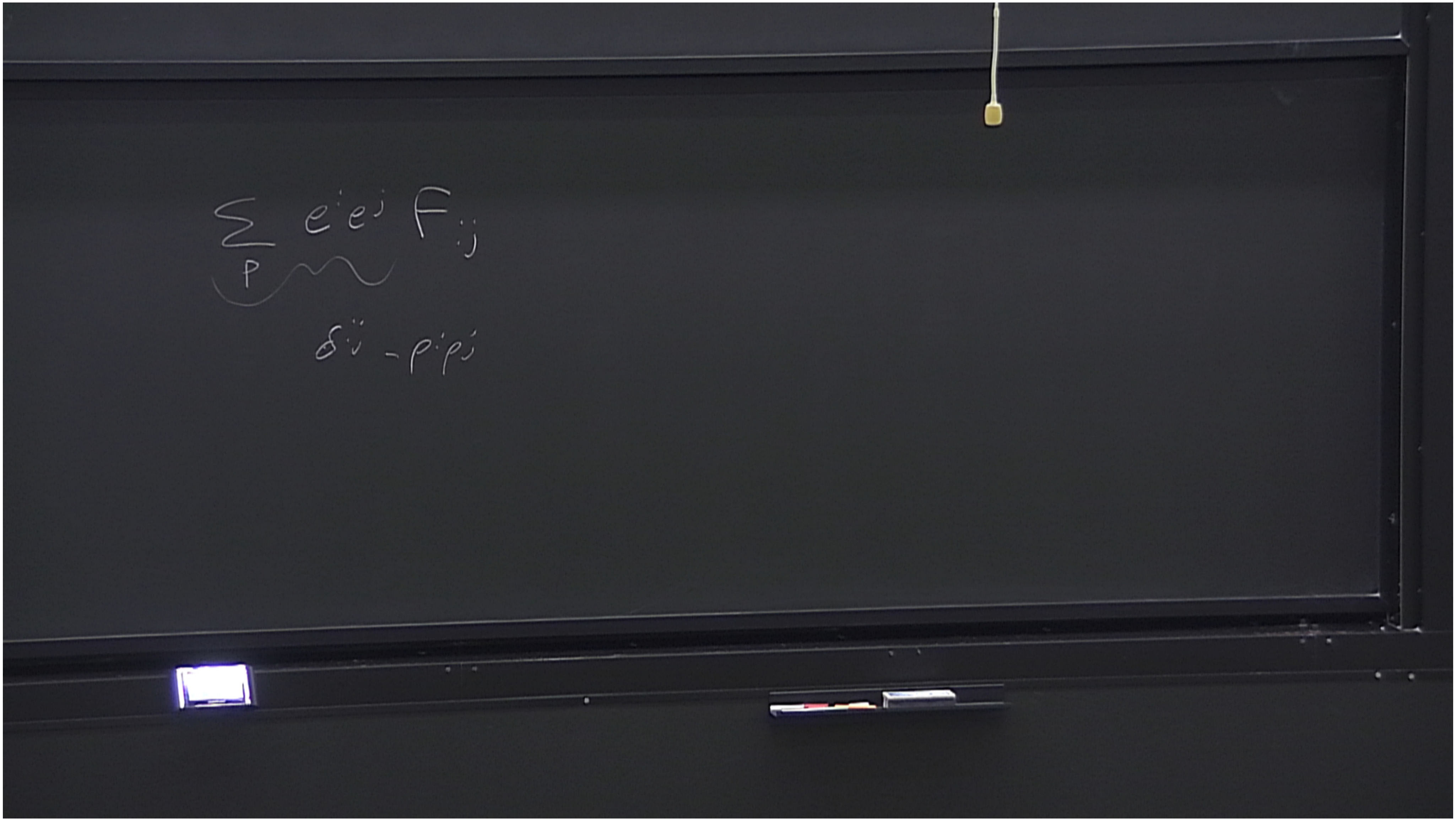

$T_{ij}(x) = h_{ij}^{TT} + \nabla_{(i} V_{j)}^{T} + \nabla_i \nabla_j \phi - \frac{1}{d} g_{ij} \nabla^2 \phi + \frac{1}{d} \psi g_{ij}$
$\nabla^i h_{ij} = 0$
$g^{ij} h_{ij} = 0$
$\nabla^i V_i = 0$

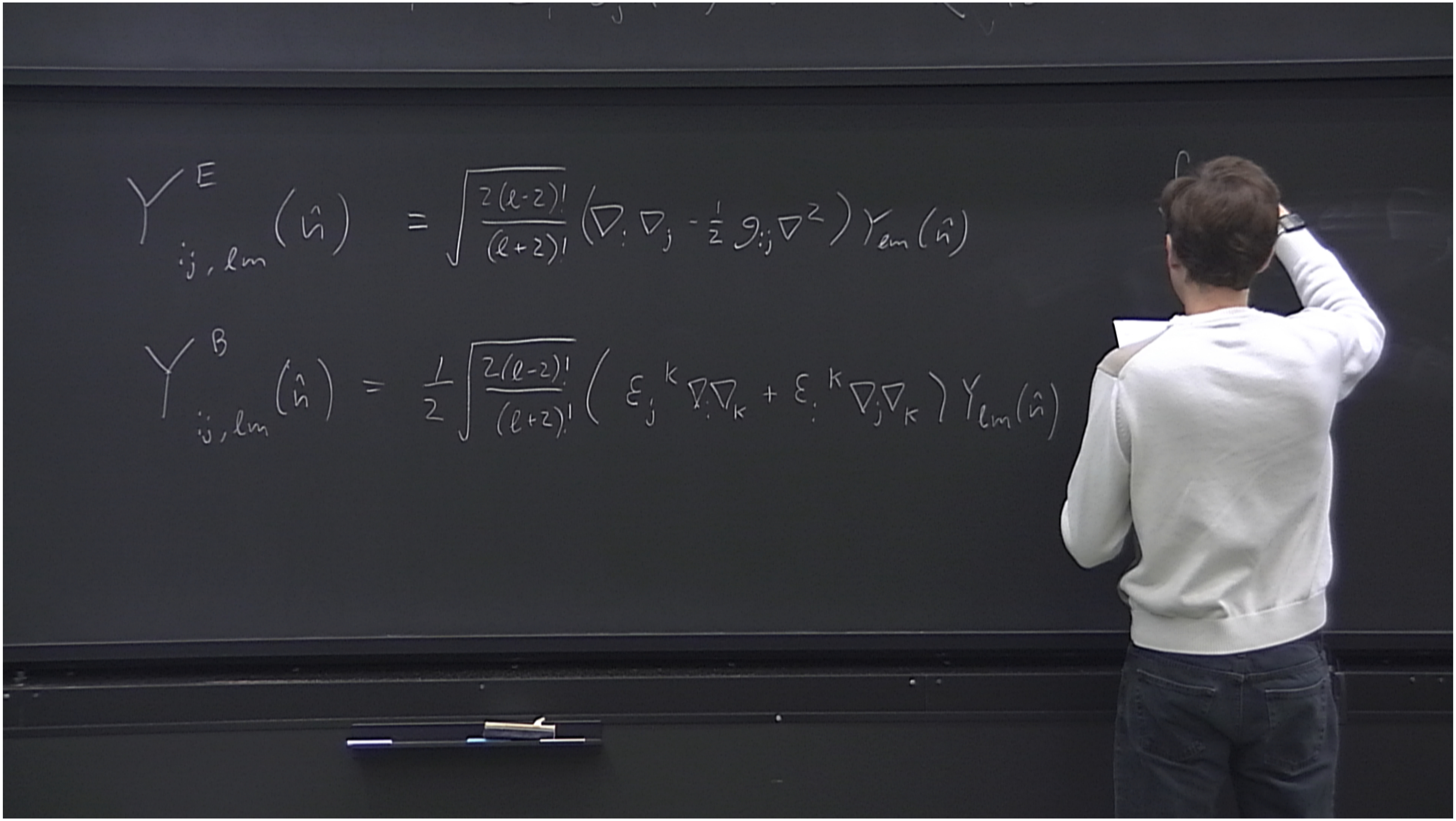

T_ij(x) = h_ij^TT + ∇_(i V_j)^T + ∇_i ∇_j φ − 1/d g_ij ∇²φ + 1/d ψ g_ij
∇^i V_i = 0
d=2


$T_{ij}(x) = h_{ij}^{TT} + \nabla_{(i} V^{T}_{j)} + \nabla_i \nabla_j \phi - \frac{1}{d} g_{ij} \nabla^2 \phi + \frac{1}{d} \psi g_{ij}$
$\nabla^i h_{ij} = 0$
$g^{ij} h_{ij} = 0$
$\nabla^i V_i = 0$
$d=2$
$h_{ij}^{TT} = 0$
$V_i^T = \varepsilon_i{}^j \partial_j \chi$, $\varepsilon_{ij} = \sqrt{g}\, \tilde{\varepsilon}_{ij}$
$\tilde{\varepsilon}_{11} = \tilde{\varepsilon}_{22} = 0$
$\tilde{\varepsilon}_{21} = -1$
$\tilde{\varepsilon}_{12} = 1$


V_i = 0
ε_11 = ε_22 = 0
ε_21 = -1
ε_12 = 1
ε_ij = √g ε̃_ij

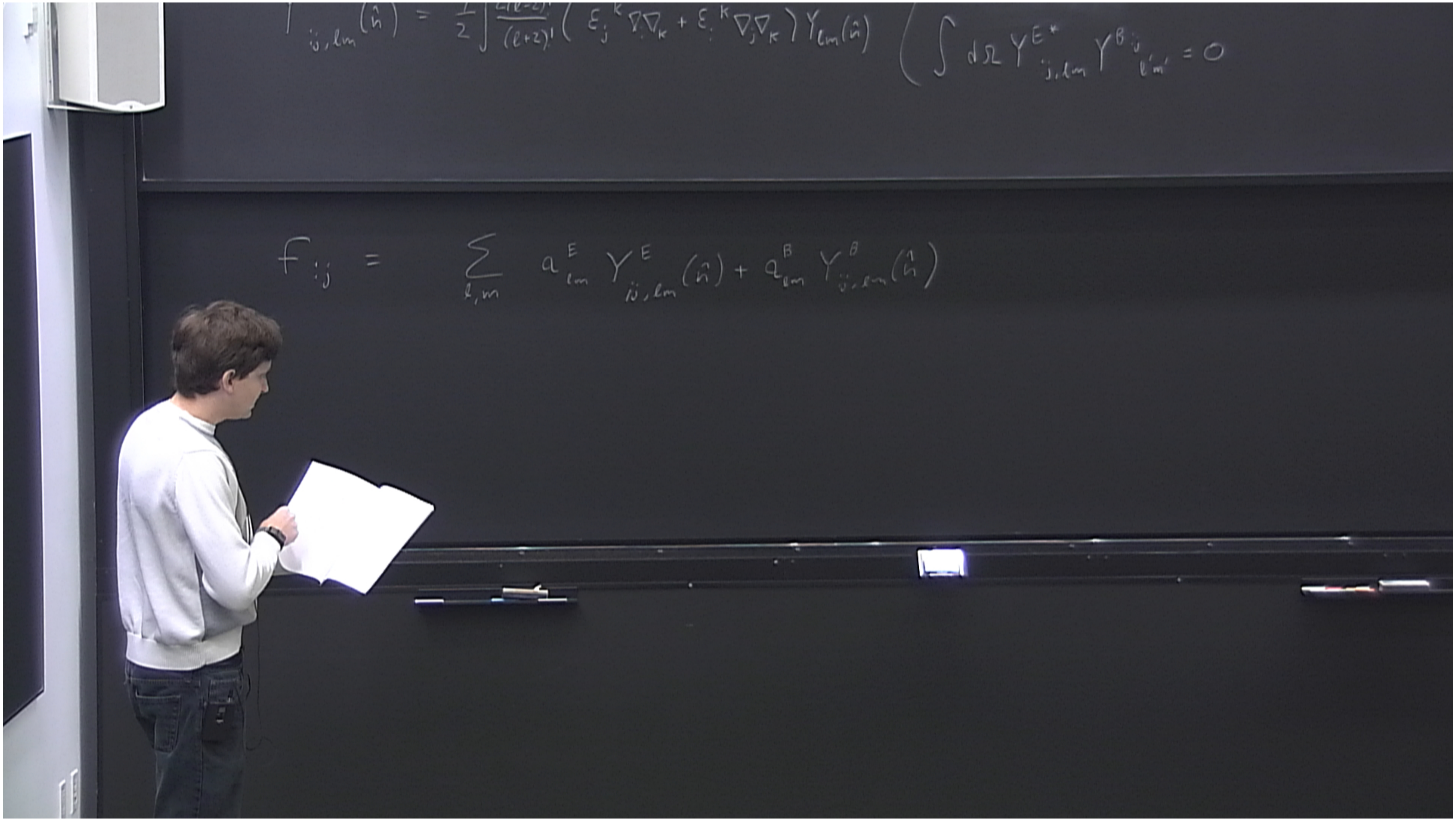

$Y^{E}_{ij,\ell m}(\hat{n}) = \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}} \left(\nabla_i \nabla_j - \frac{1}{2} g_{ij} \nabla^2\right) Y_{\ell m}(\hat{n})$

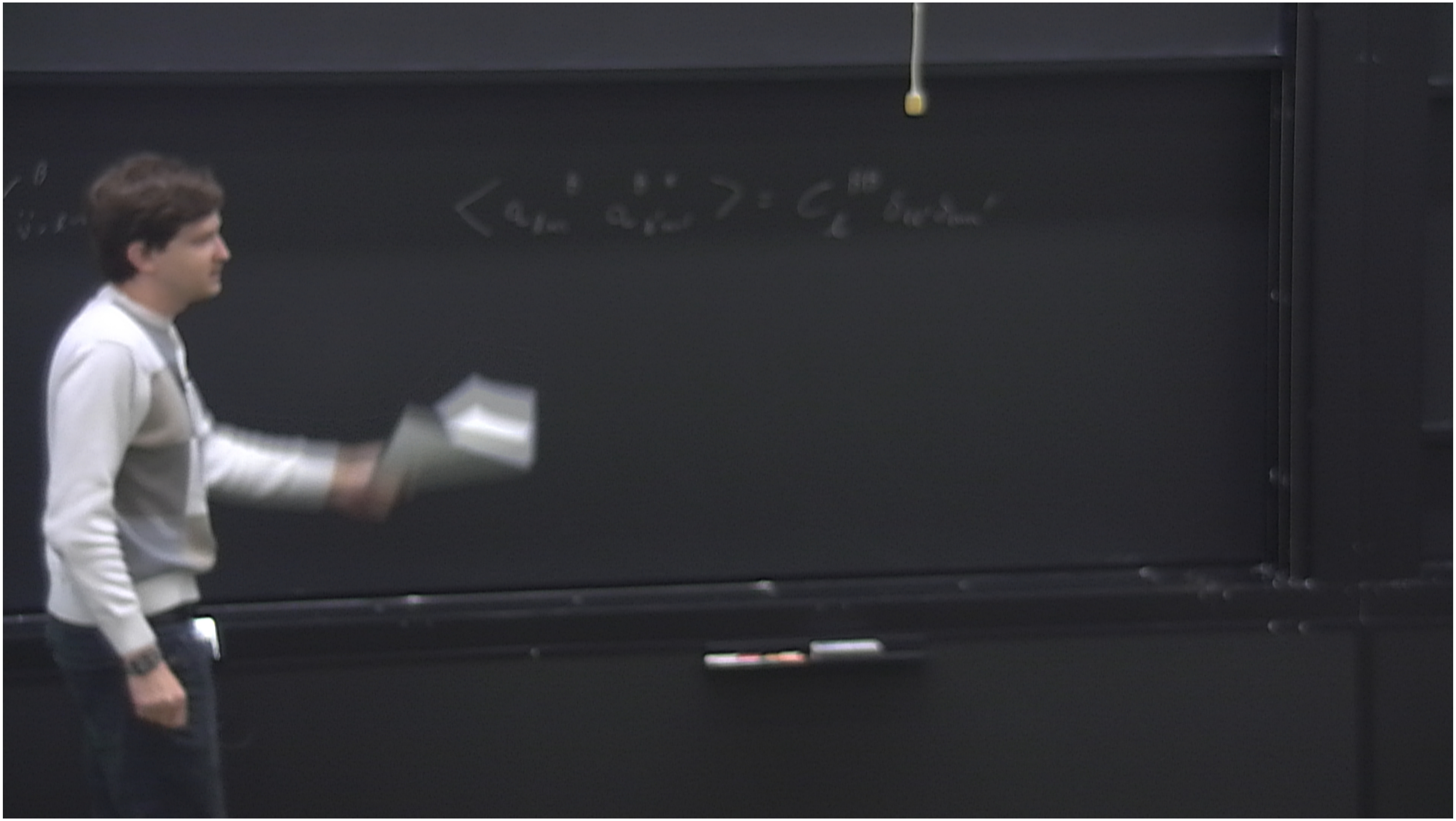

Σ_p e^i e^j F_ij
δ^ij − p^i p^j

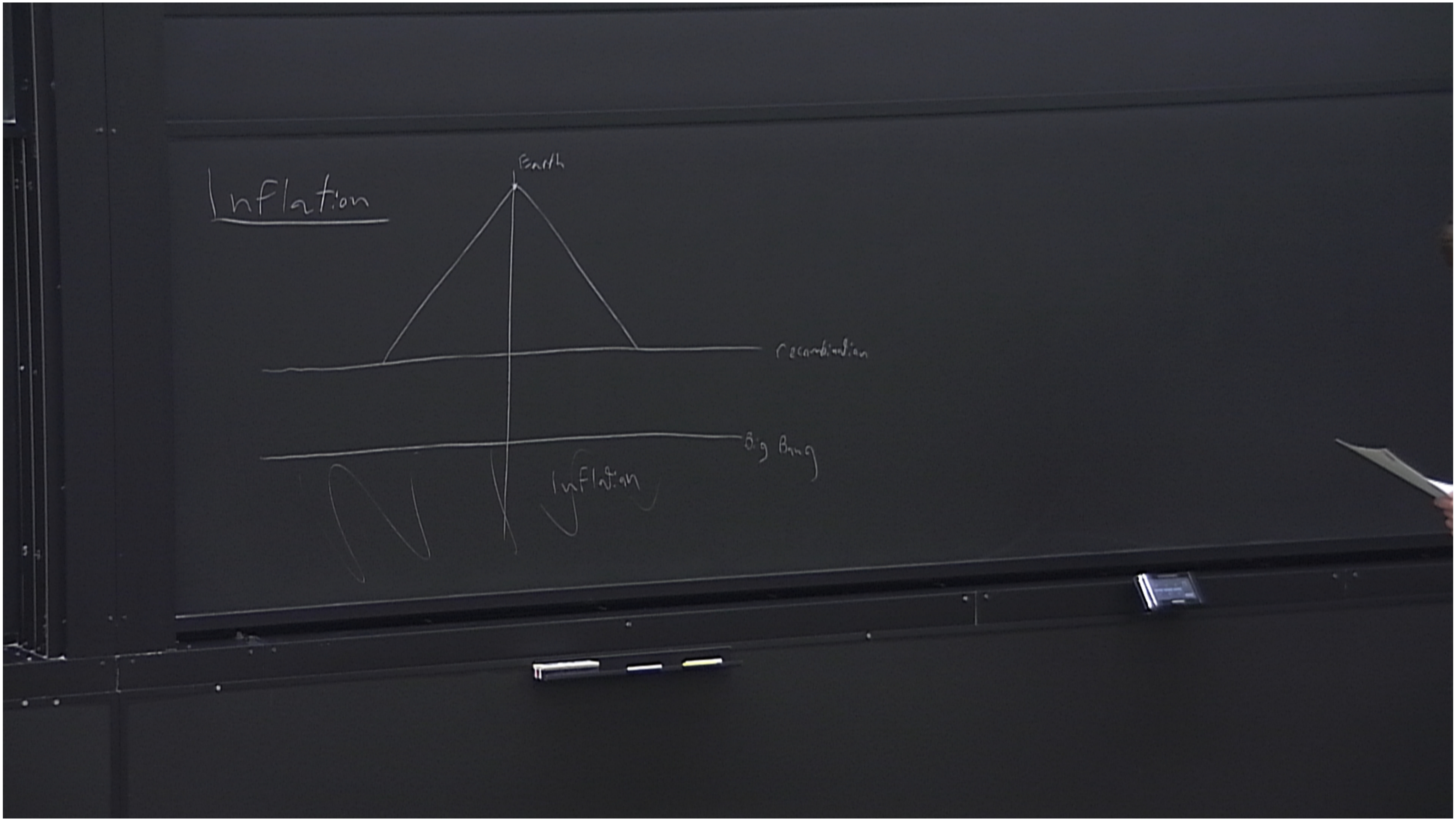

$Y^{E}_{ij,\ell m}(\hat{n}) = \sqrt{\frac{2(\ell-2)!}{(\ell+2)!}} \left(\nabla_i \nabla_j - \frac{1}{2} g_{ij} \nabla^2\right) Y_{\ell m}(\hat{n})$
$Y^{B}_{ij,\ell m}(\hat{n}) = \frac{1}{2}\sqrt{\frac{2(\ell-2)!}{(\ell+2)!}} \left(\varepsilon_j{}^{k} \nabla_i \nabla_k + \varepsilon_i{}^{k} \nabla_j \nabla_k\right) Y_{\ell m}(\hat{n})$

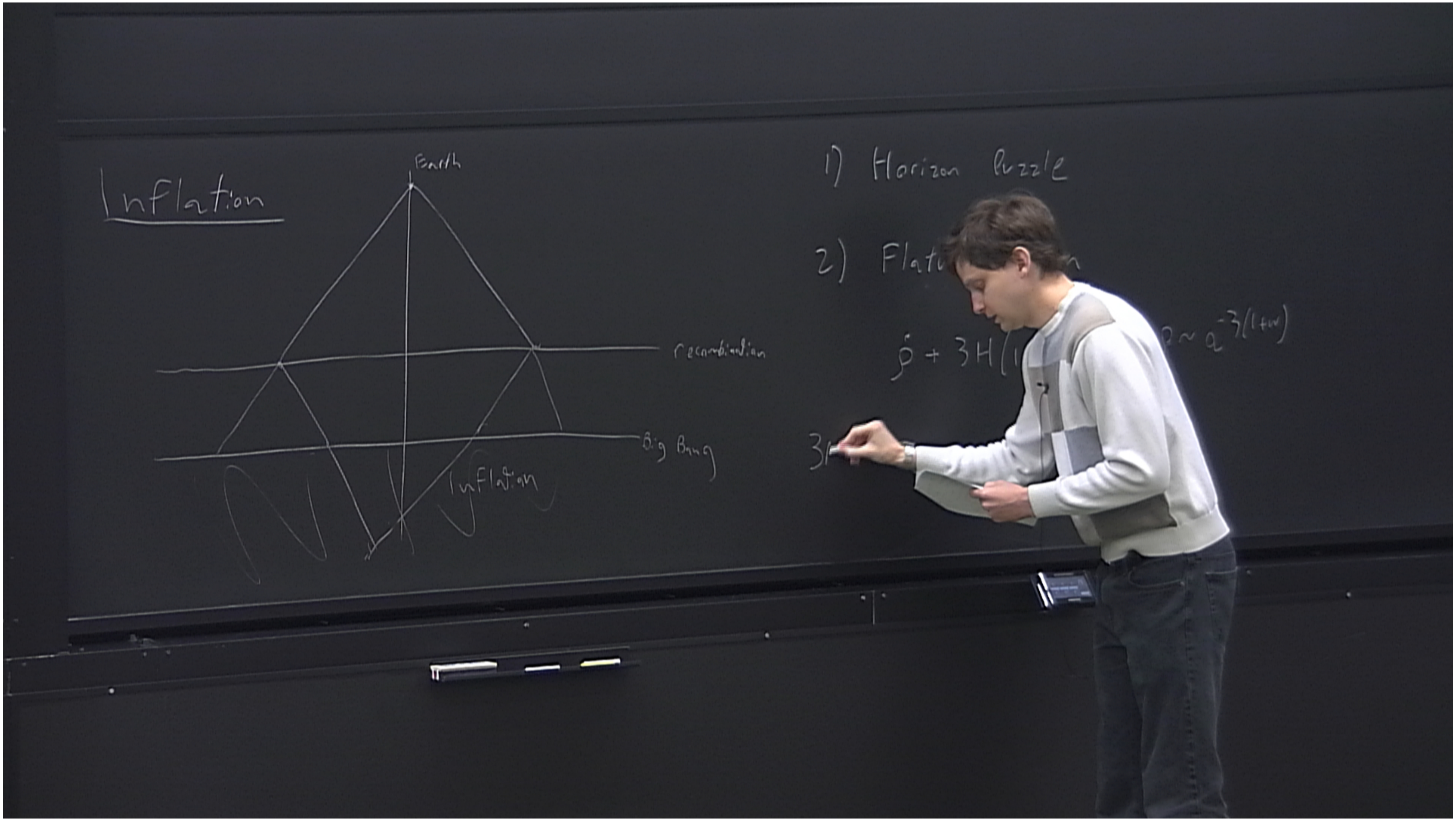
$$\left\{\begin{aligned} &\int d\Omega \, Y^{E\,*}_{ij,\ell m} \, Y^{E\,ij}_{\ell' m'} = \delta_{\ell\ell'}\delta_{mm'} \\ &\int d\Omega \, Y^{B\,*}_{ij,\ell m} \, Y^{B\,ij}_{\ell' m'} = \delta_{\ell\ell'}\delta_{mm'} \\ &\int d\Omega \, Y^{E\,*}_{ij,\ell m} \, Y^{B\,ij}_{\ell' m'} = 0 \end{aligned}\right.$$

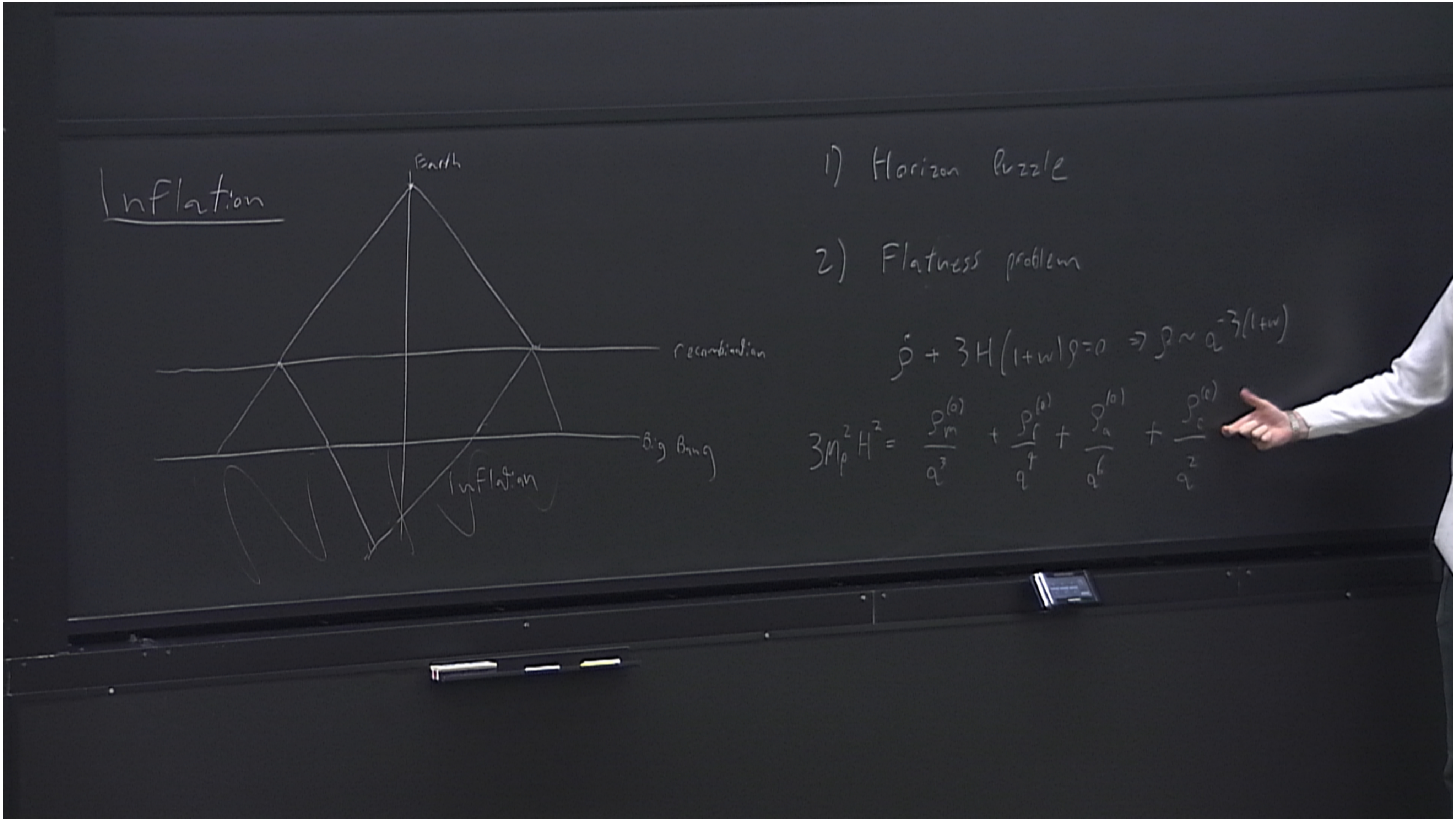
$Y_{\ell m}(\hat{n})$

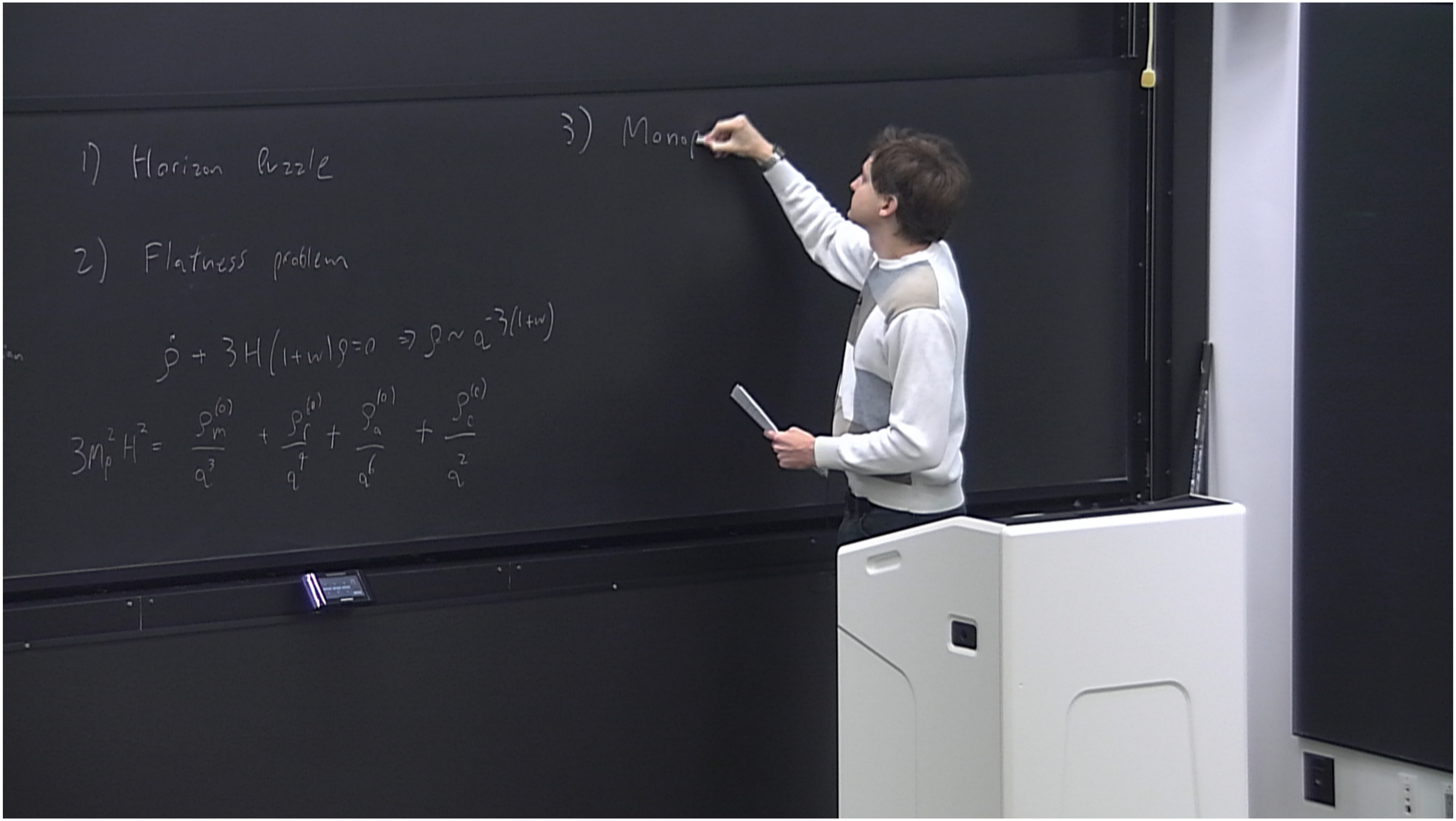

$\int d\Omega \, Y^{E*}_{ij,lm} Y^{B\,ij}_{l'm'} = 0$
$f_{ij} = \sum_{l,m} a^E_{lm} Y^E_{ij,lm}(\hat{n}) + a^B_{lm} Y^B_{ij,lm}(\hat{n})$

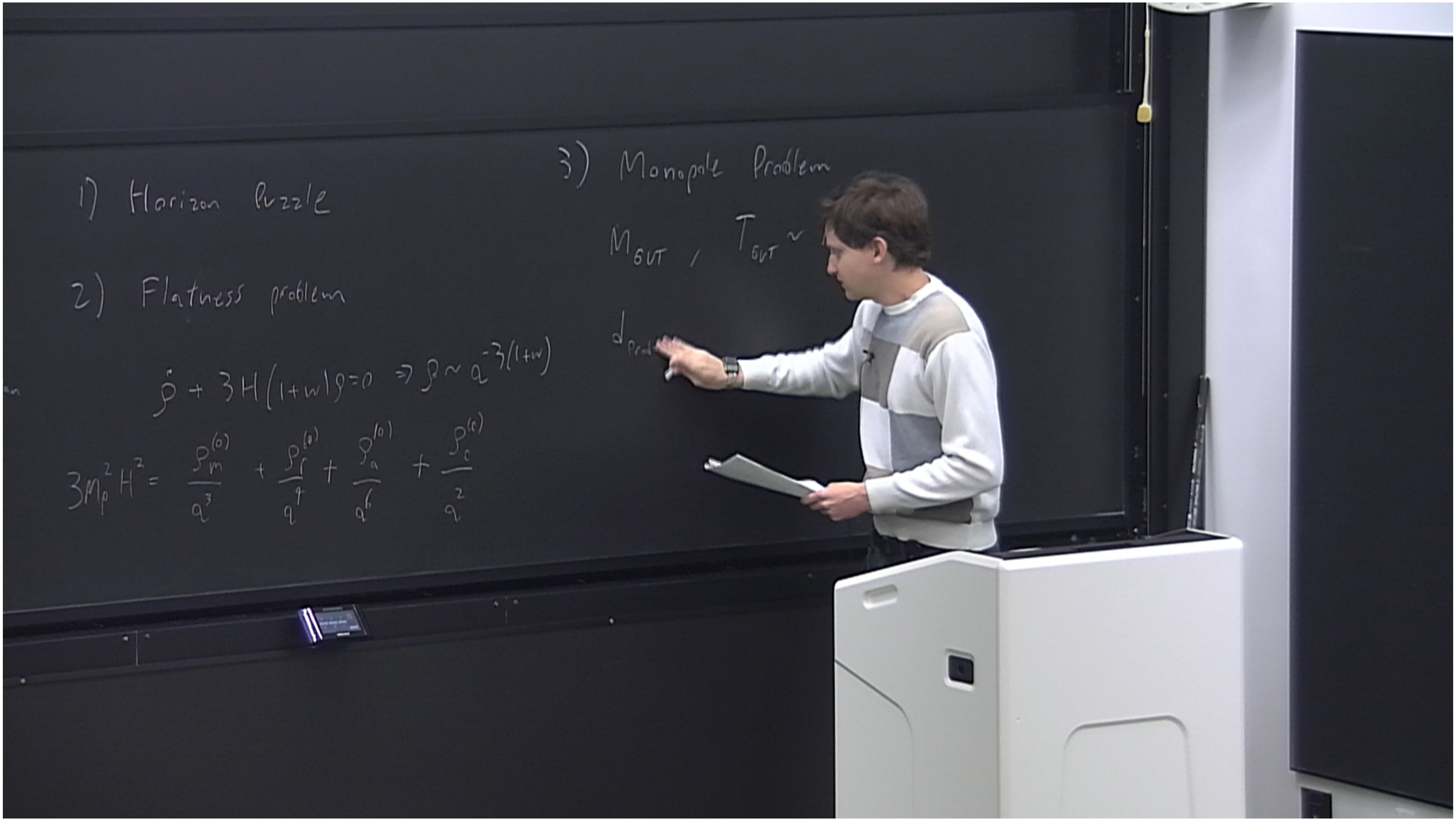


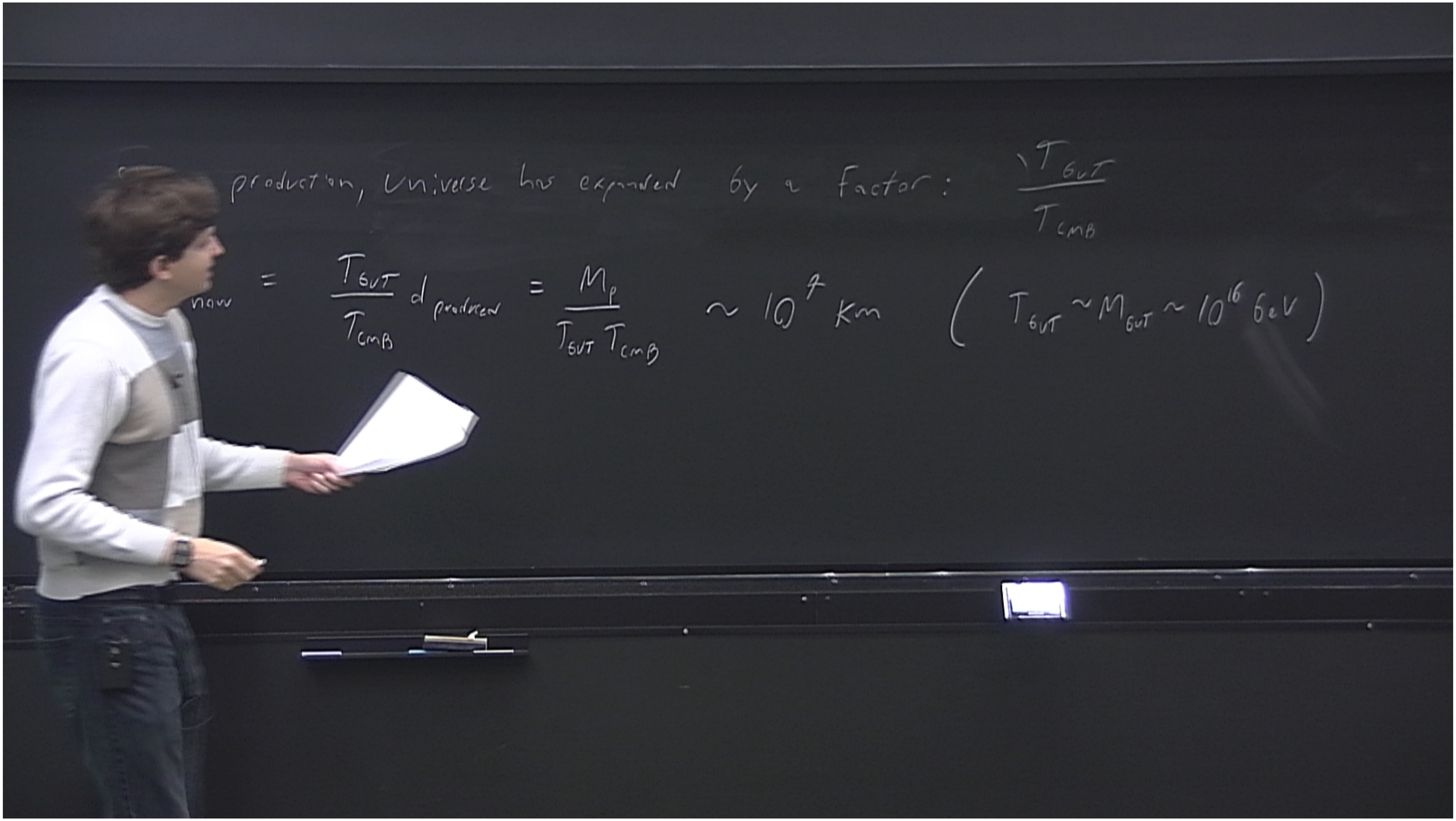

Inflation
Earth
recombination
Big Bang
Inflation

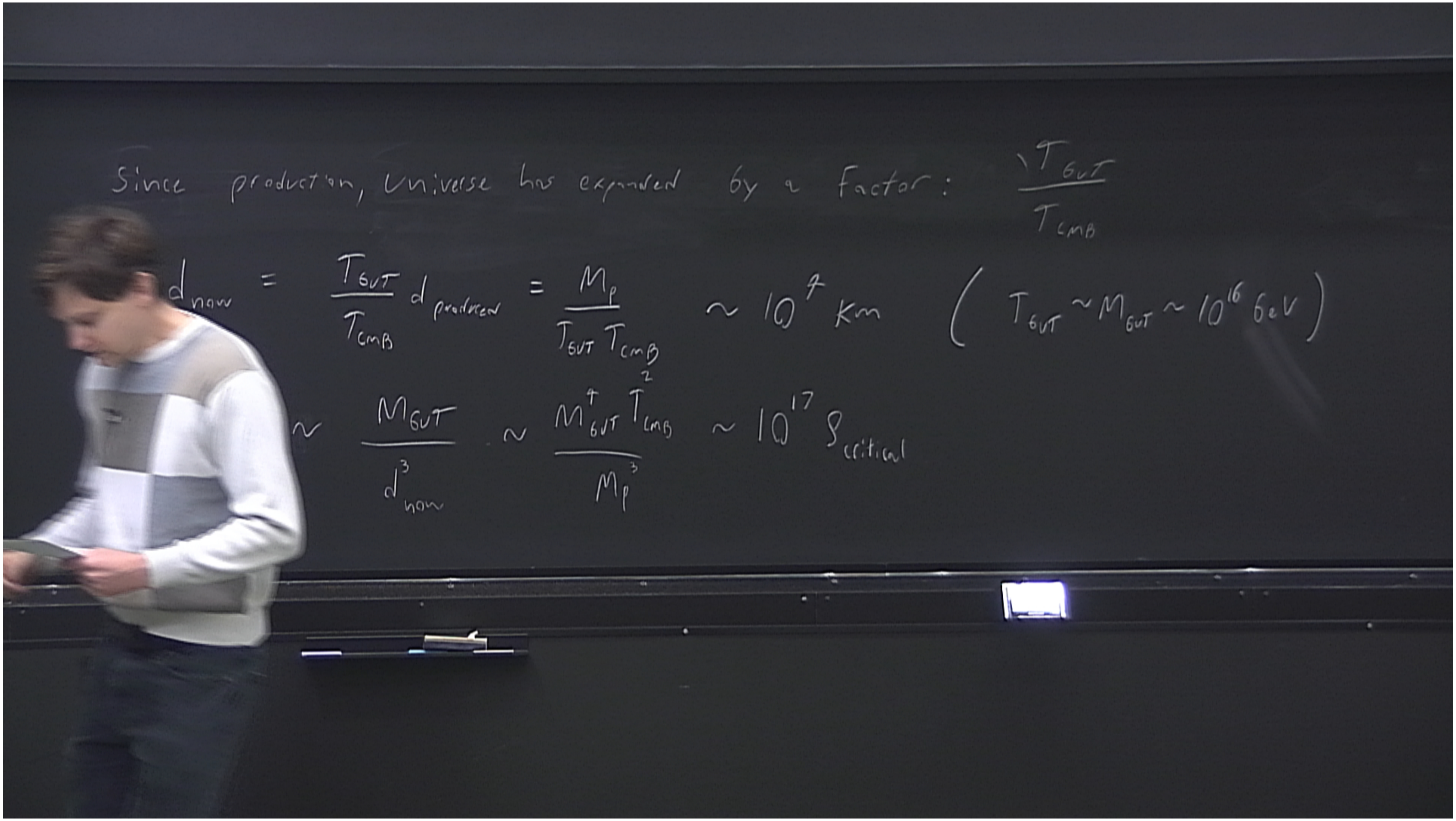

Inflation
Earth
recombination
Big Bang
Inflation
1) Horizon Puzzle
2) Flat

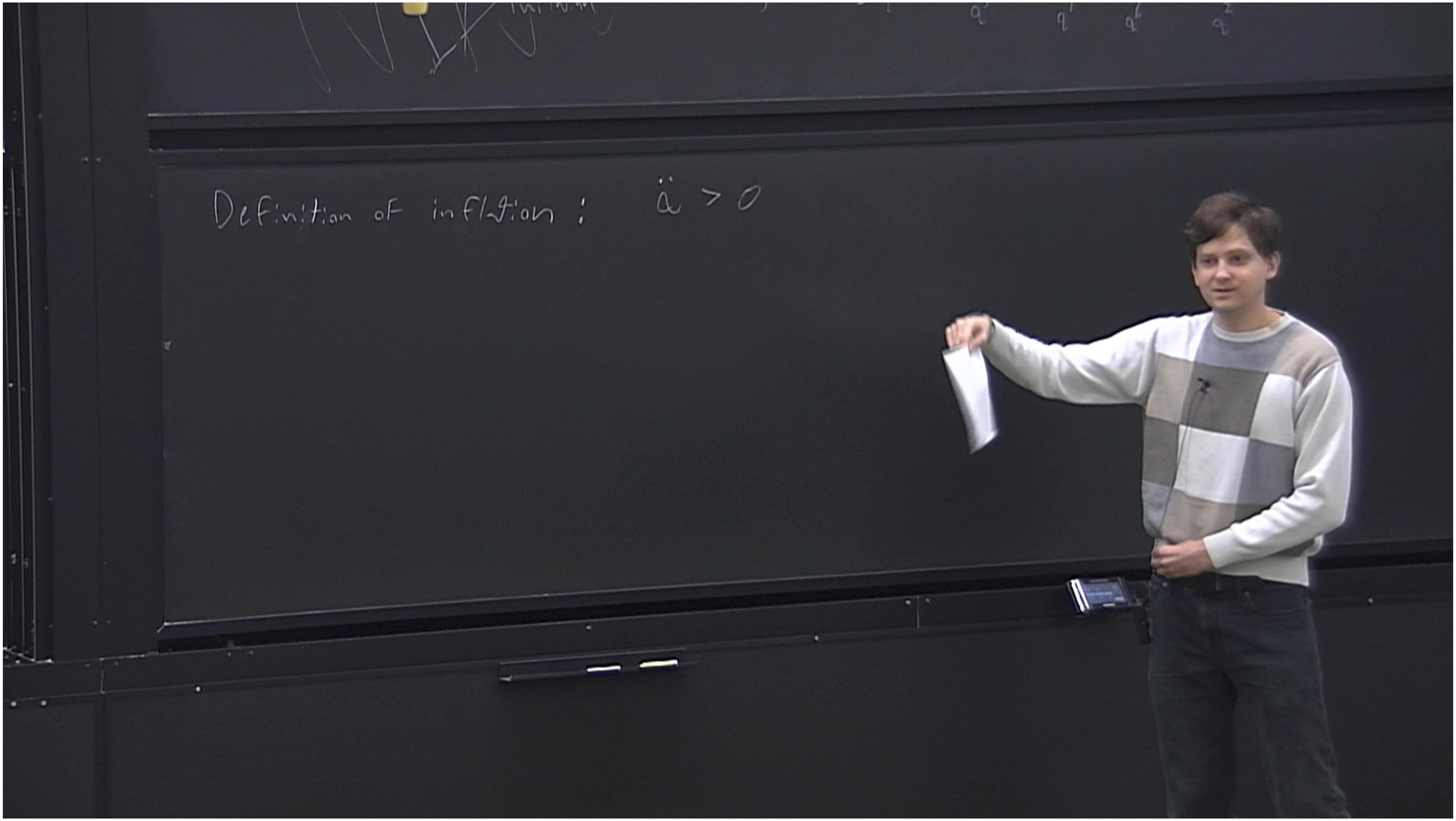

Inflation
Earth
recombination
Big Bang
Inflation
1) Horizon Puzzle
2) Flatness problem
$\dot{\rho} + 3H(1+w)\rho = 0 \Rightarrow \rho \sim a^{-3(1+w)}$
$3M_p^2 H^2 = \frac{\rho_m^{(0)}}{a^3} + \frac{\rho_r^{(0)}}{a^4} + \frac{\rho_a^{(0)}}{a^6} + \frac{\rho_c^{(0)}}{a^2}$

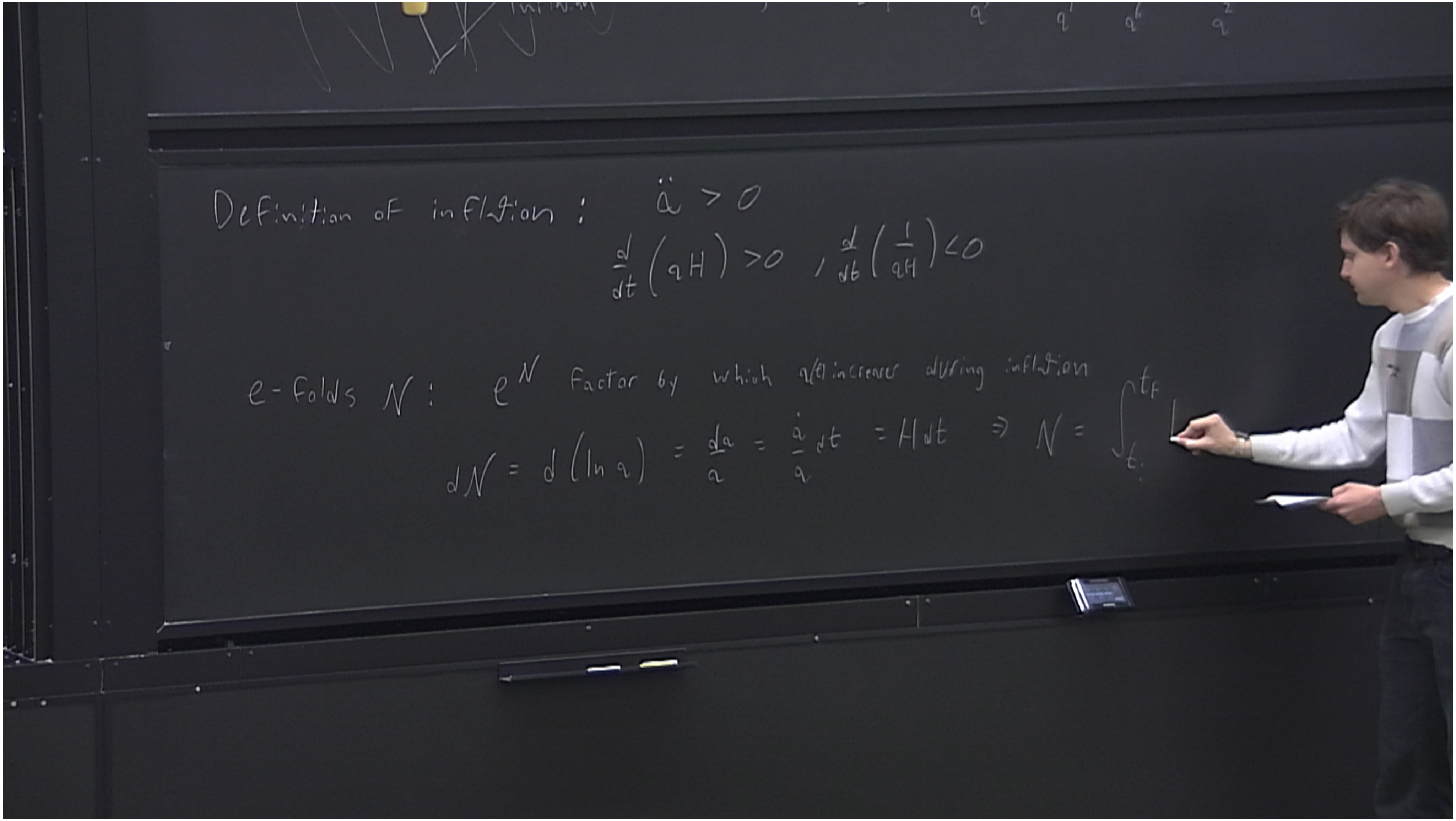

1) Horizon Puzzle
2) Flatness problem
$\dot{\rho} + 3H(1+w)\rho = 0 \Rightarrow \rho \sim a^{-3(1+w)}$
$3M_p^2 H^2 = \frac{\rho_m^{(0)}}{a^3} + \frac{\rho_r^{(0)}}{a^4} + \frac{\rho_a^{(0)}}{a^6} + \frac{\rho_c^{(0)}}{a^2}$
3) Monop

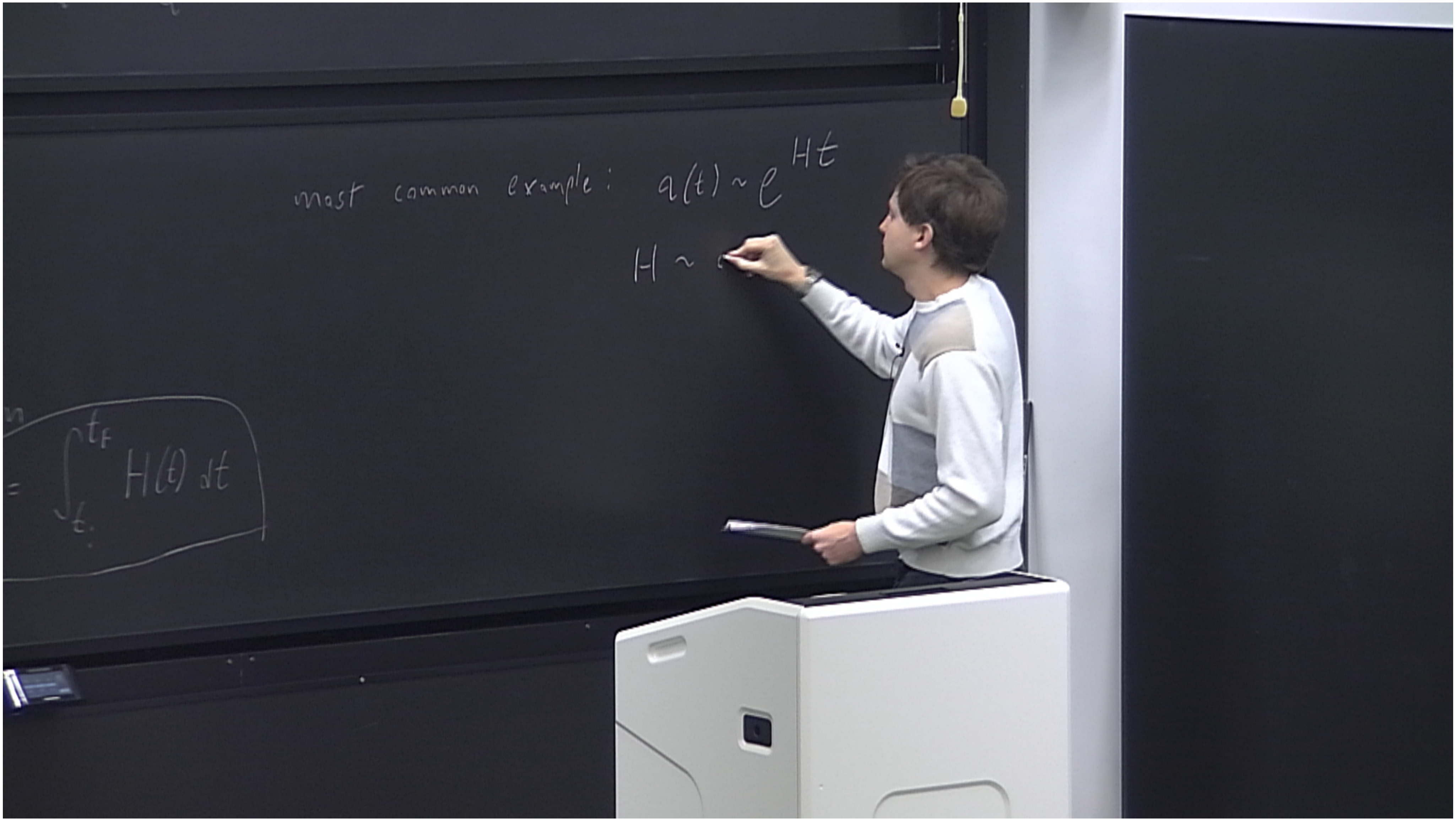

1) Horizon Puzzle
2) Flatness problem
3) Monopole Problem

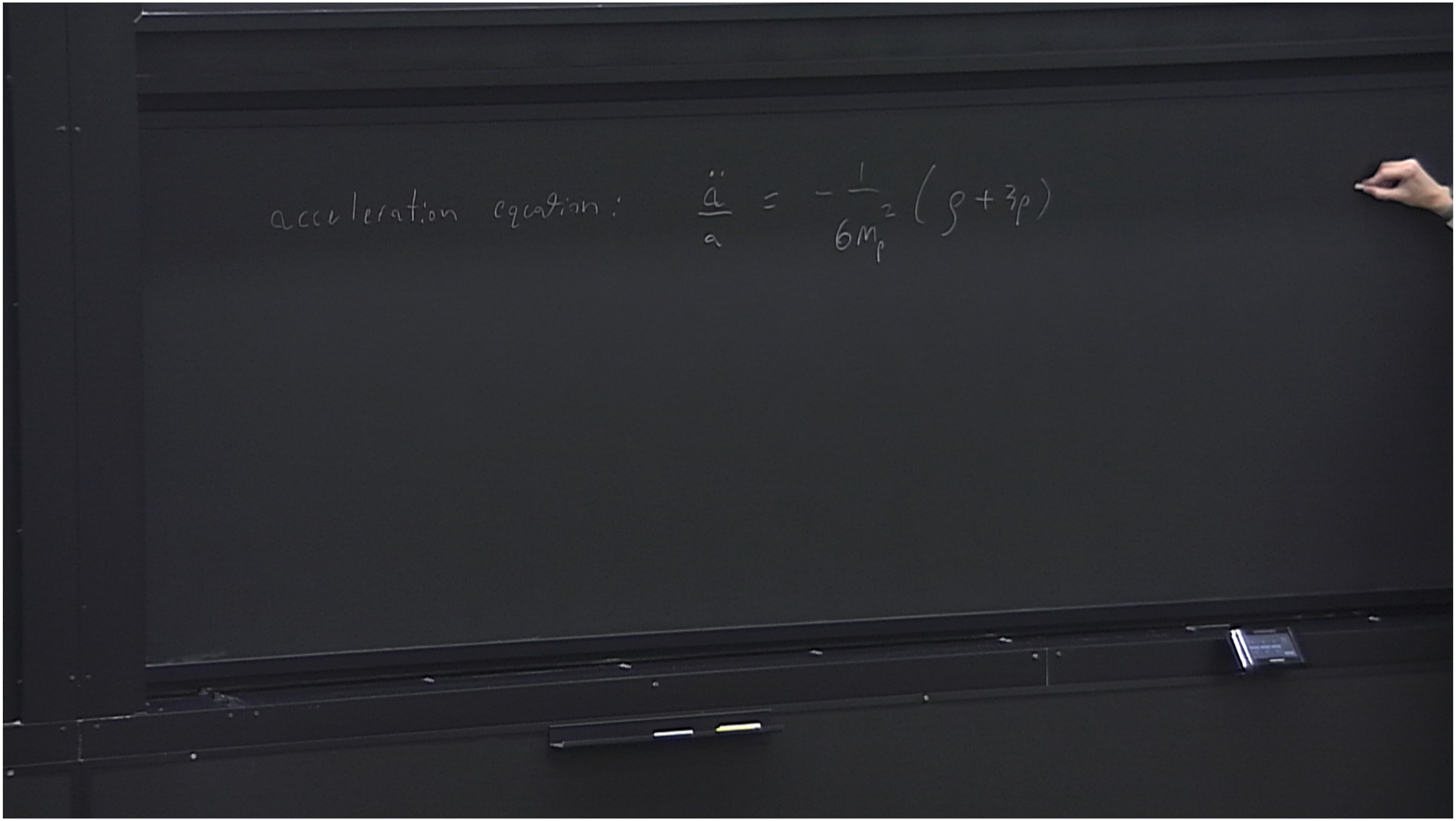

production, Universe has expanded by a factor: $\frac{T_{GUT}}{T_{CMB}}$
now $= \frac{T_{GUT}}{T_{CMB}} d_{produced} = \frac{M_p}{T_{GUT} T_{CMB}} \sim 10^{7}$ km $\left(T_{GUT} \sim M_{GUT} \sim 10^{16}\ \text{GeV} \right)$

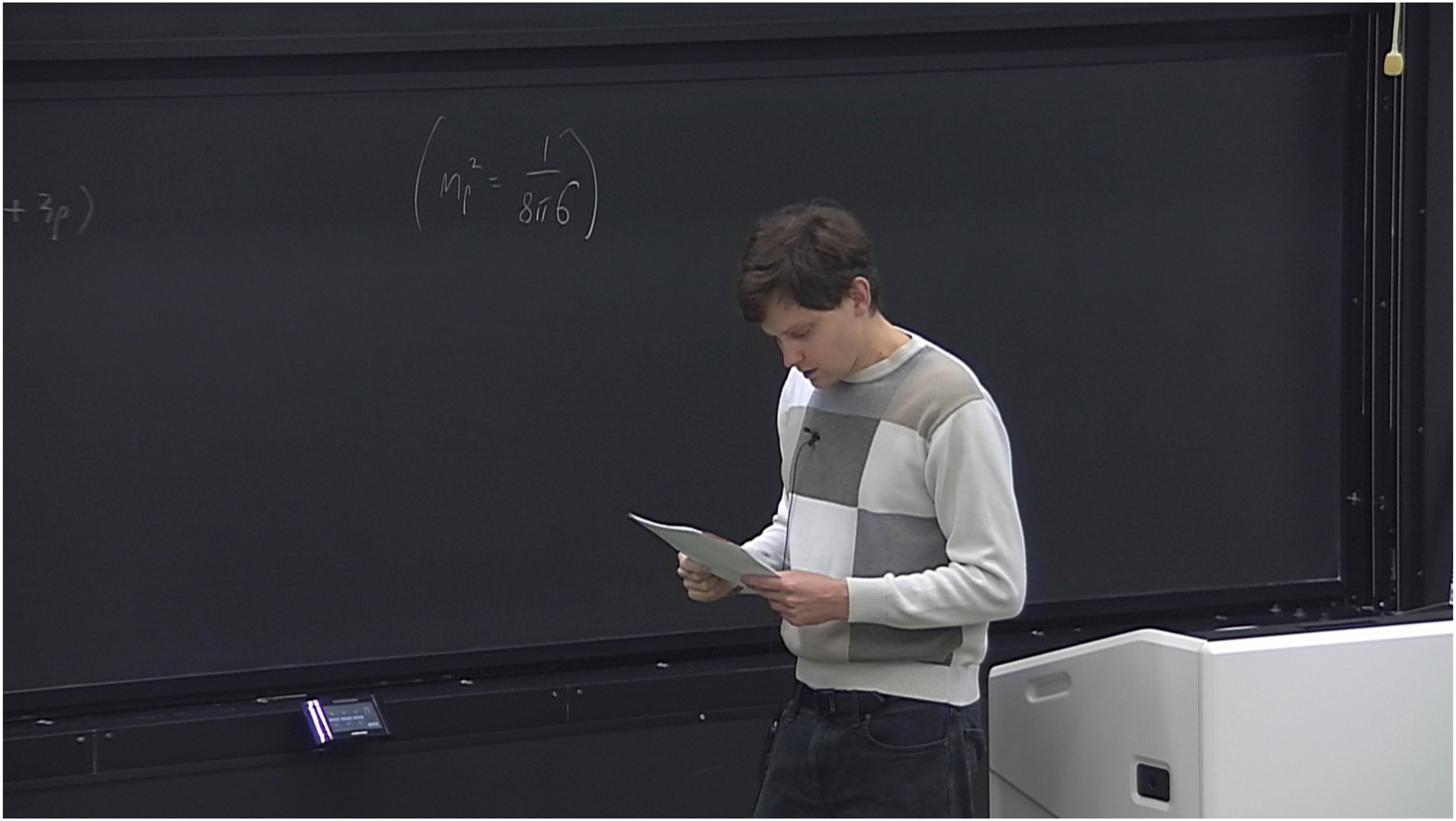

Since production, Universe has expanded by a factor: $\frac{T_{GUT}}{T_{CMB}}$
$d_{now} = \frac{T_{GUT}}{T_{CMB}} d_{produced} = \frac{M_p}{T_{GUT} T_{CMB}} \sim 10^{4}$ km
$(T_{GUT} \sim M_{GUT} \sim 10^{16}$ GeV $)$
$\sim \frac{M_{GUT}}{d_{now}^3} \sim \frac{M_{GUT}^4 T_{CMB}^2}{M_p^3} \sim 10^{17} \rho_{critical}$

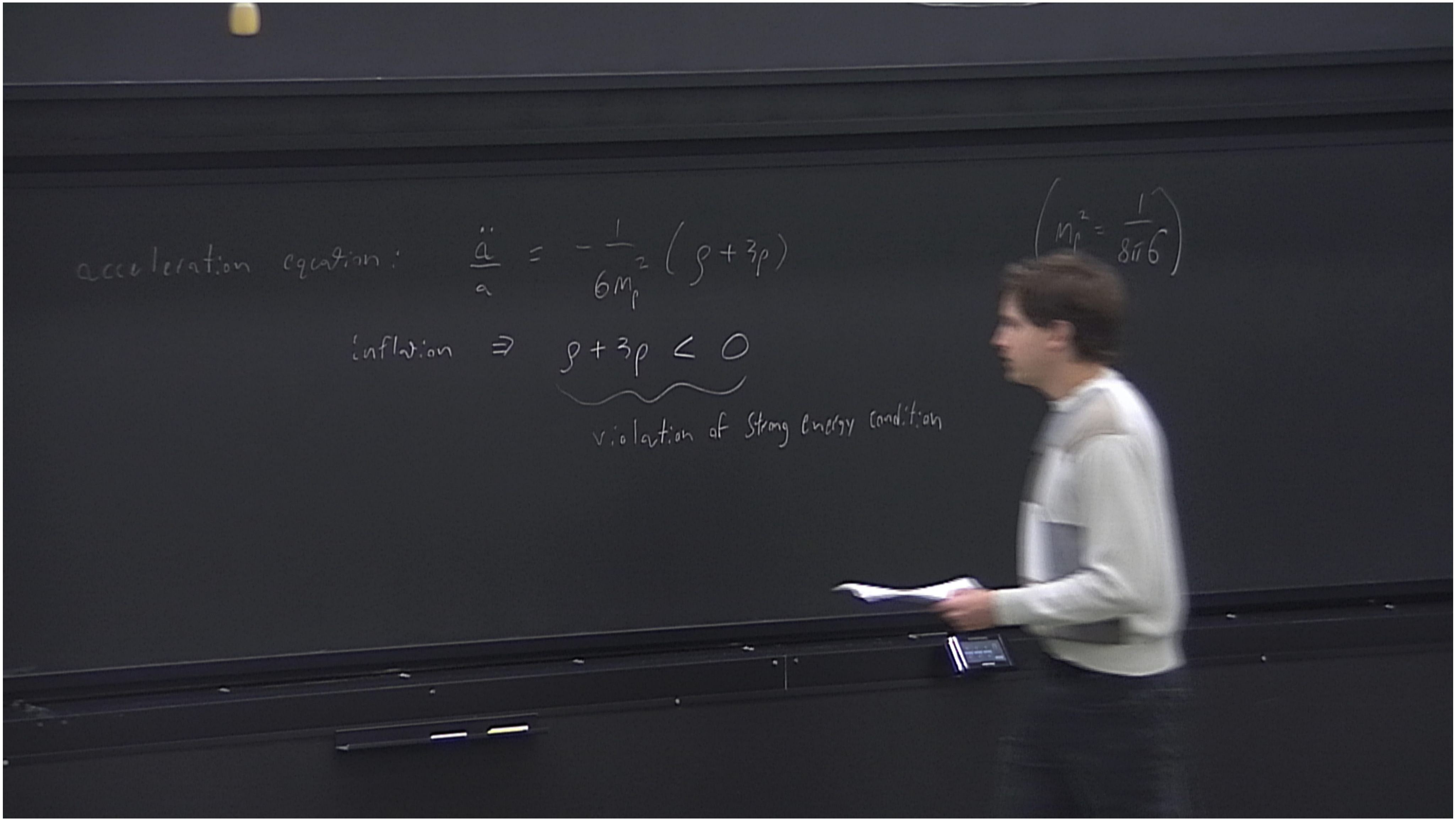

Definition of inflation : $\ddot{a} > 0$

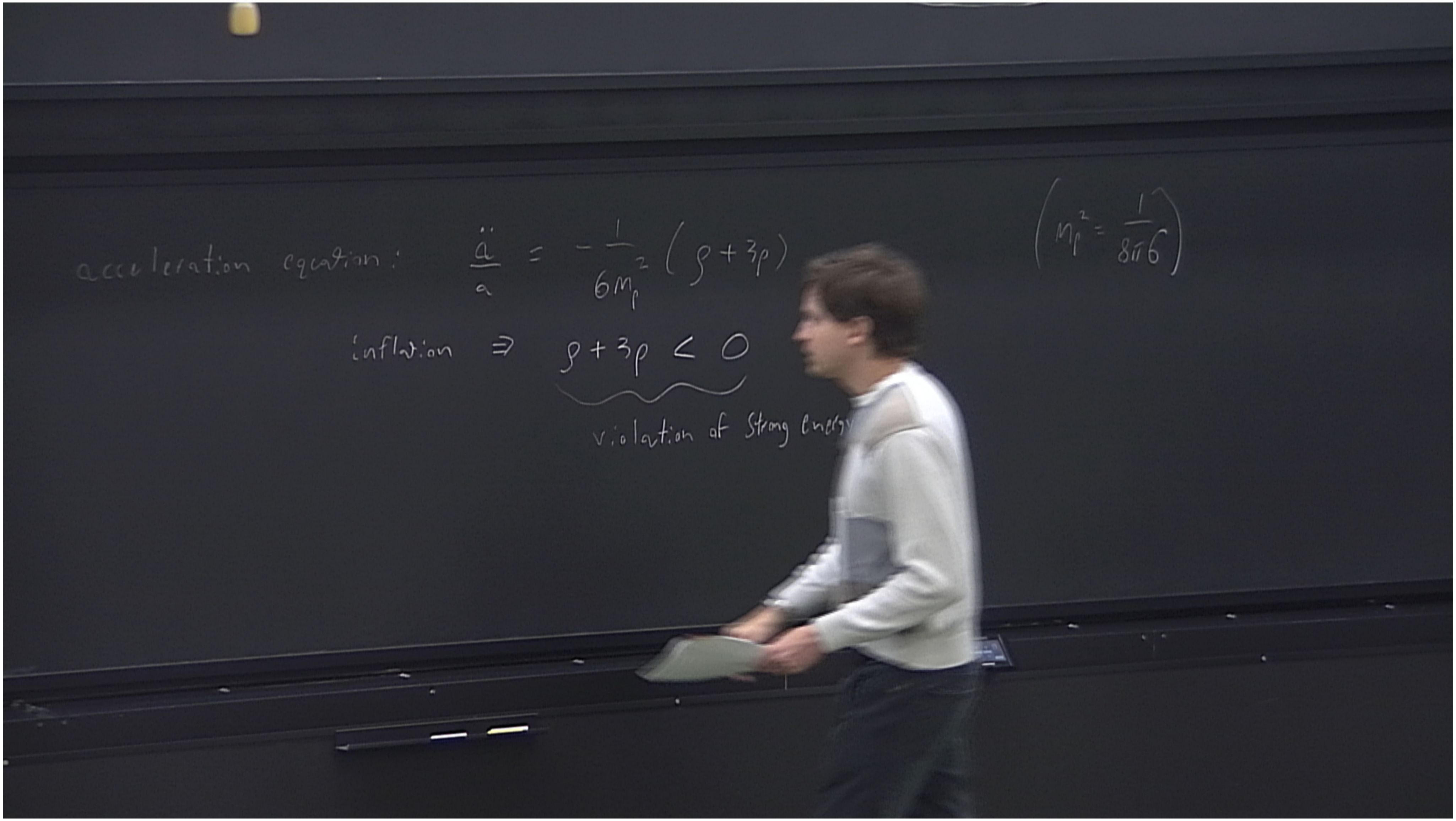

Definition of inflation: $\ddot{a} > 0$
$\frac{d}{dt}(aH) > 0$, $\frac{d}{dt}\left(\frac{1}{aH}\right) < 0$
e-folds N: e^N factor by which a(t) increases during inflation
$dN = d(\ln a) = \frac{da}{a} = \frac{\dot{a}}{a} dt = H dt \Rightarrow N = \int_{t_i}^{t_f}$

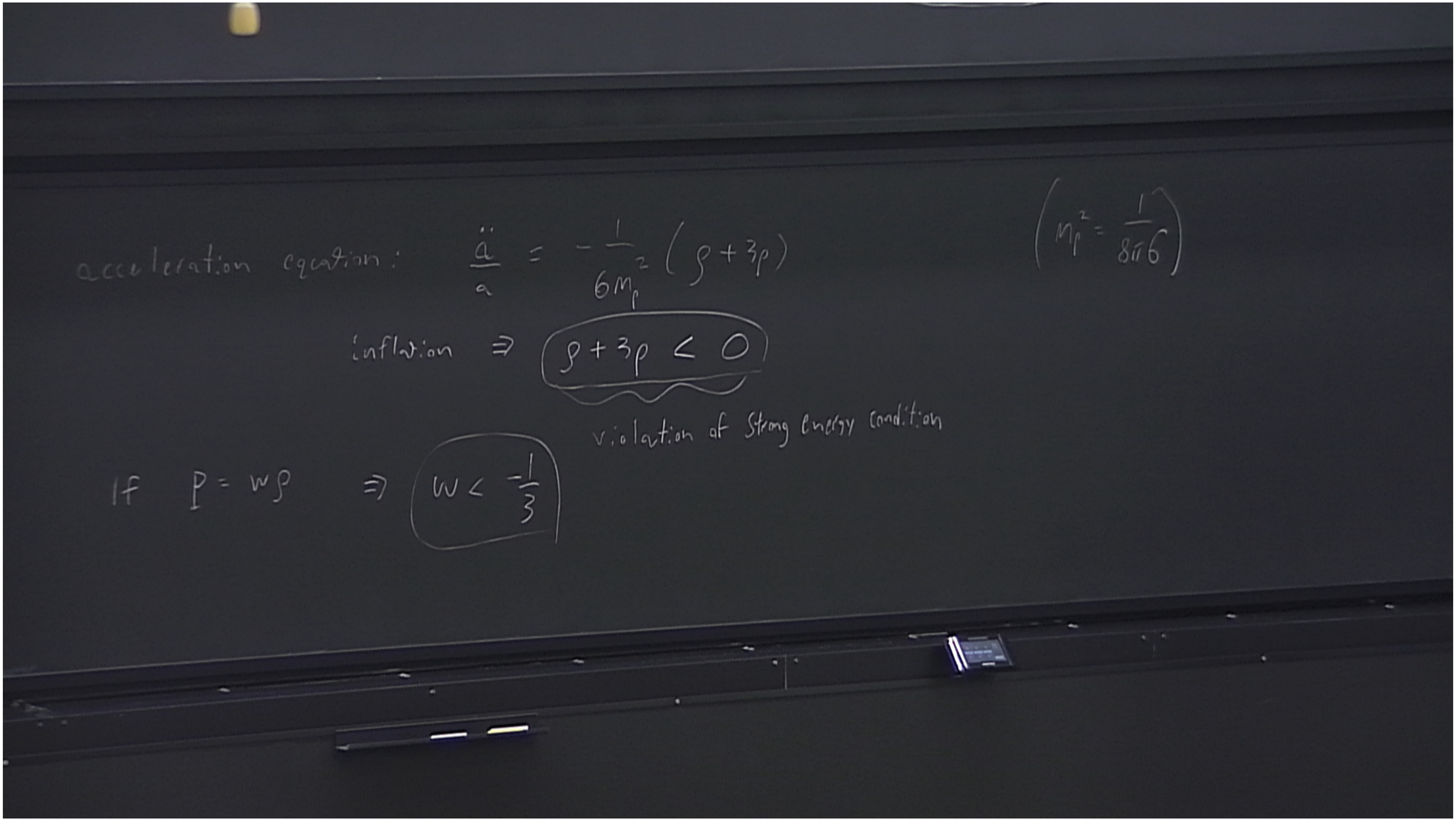

most common example: a(t) ~ e^{Ht}
H ~
= ∫_{t_i}^{t_f} H(t) dt

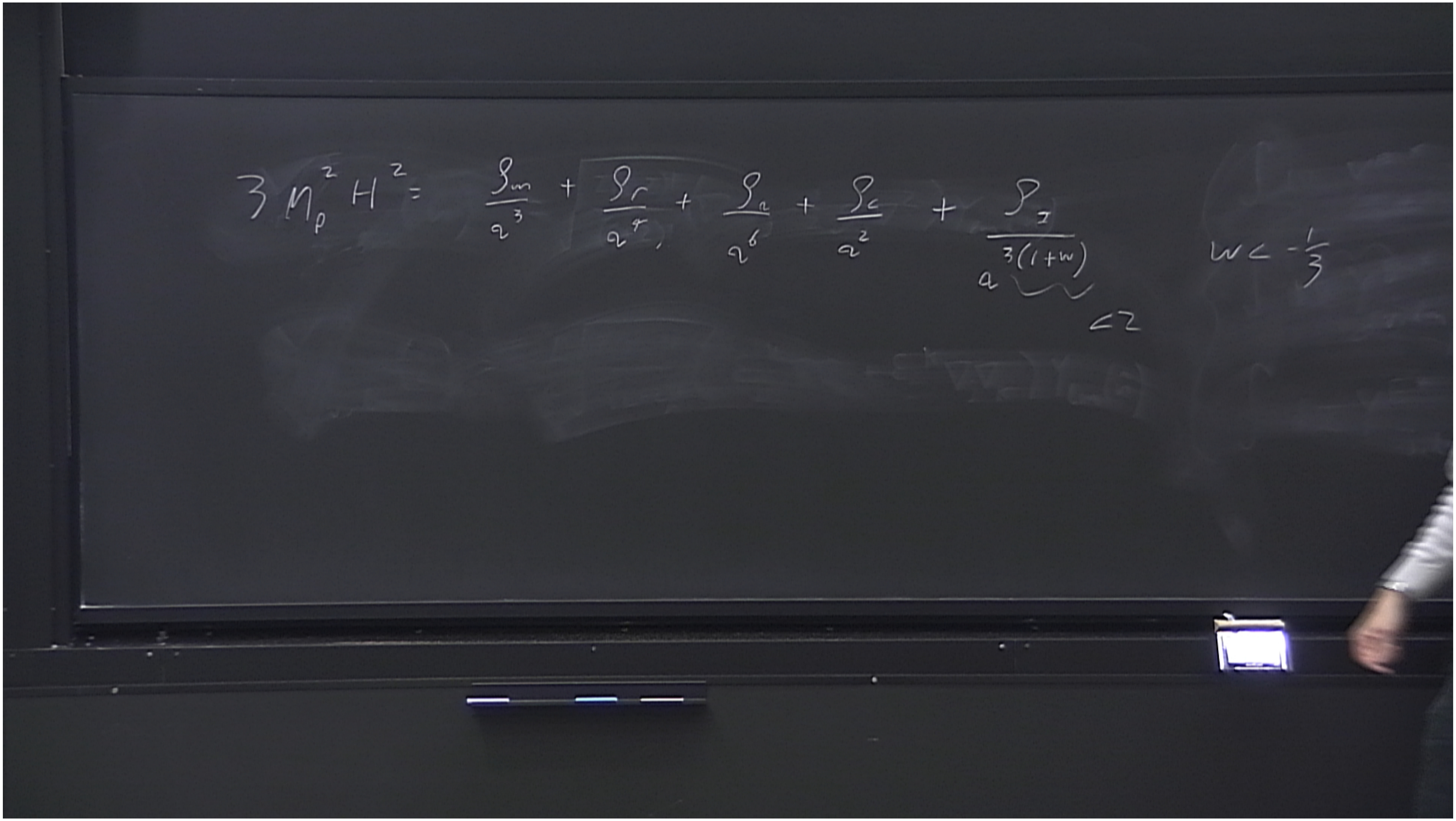

acceleration equation: $\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p)$

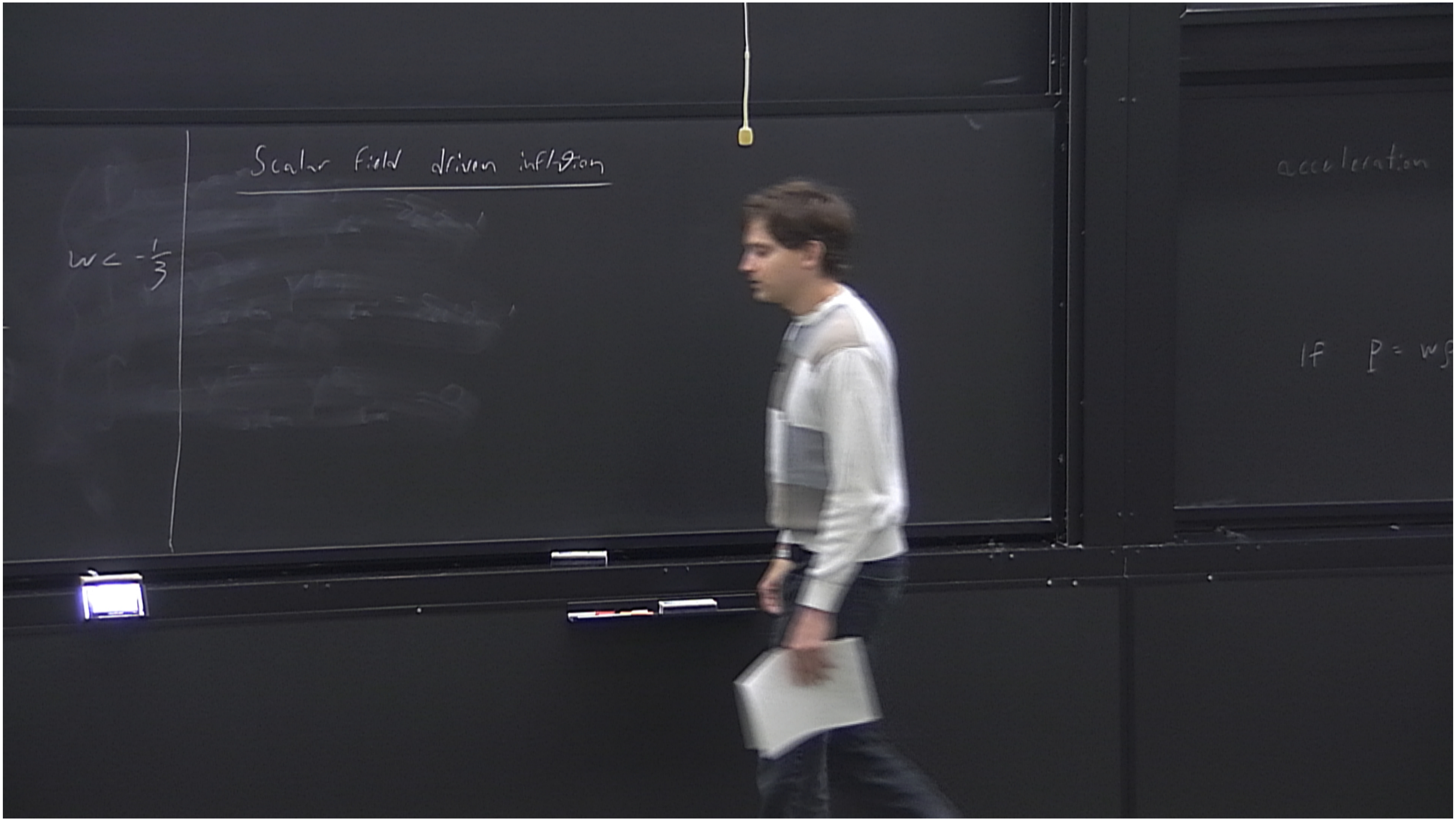

+ 3p)
(m_p^2 = 1/(8πG))

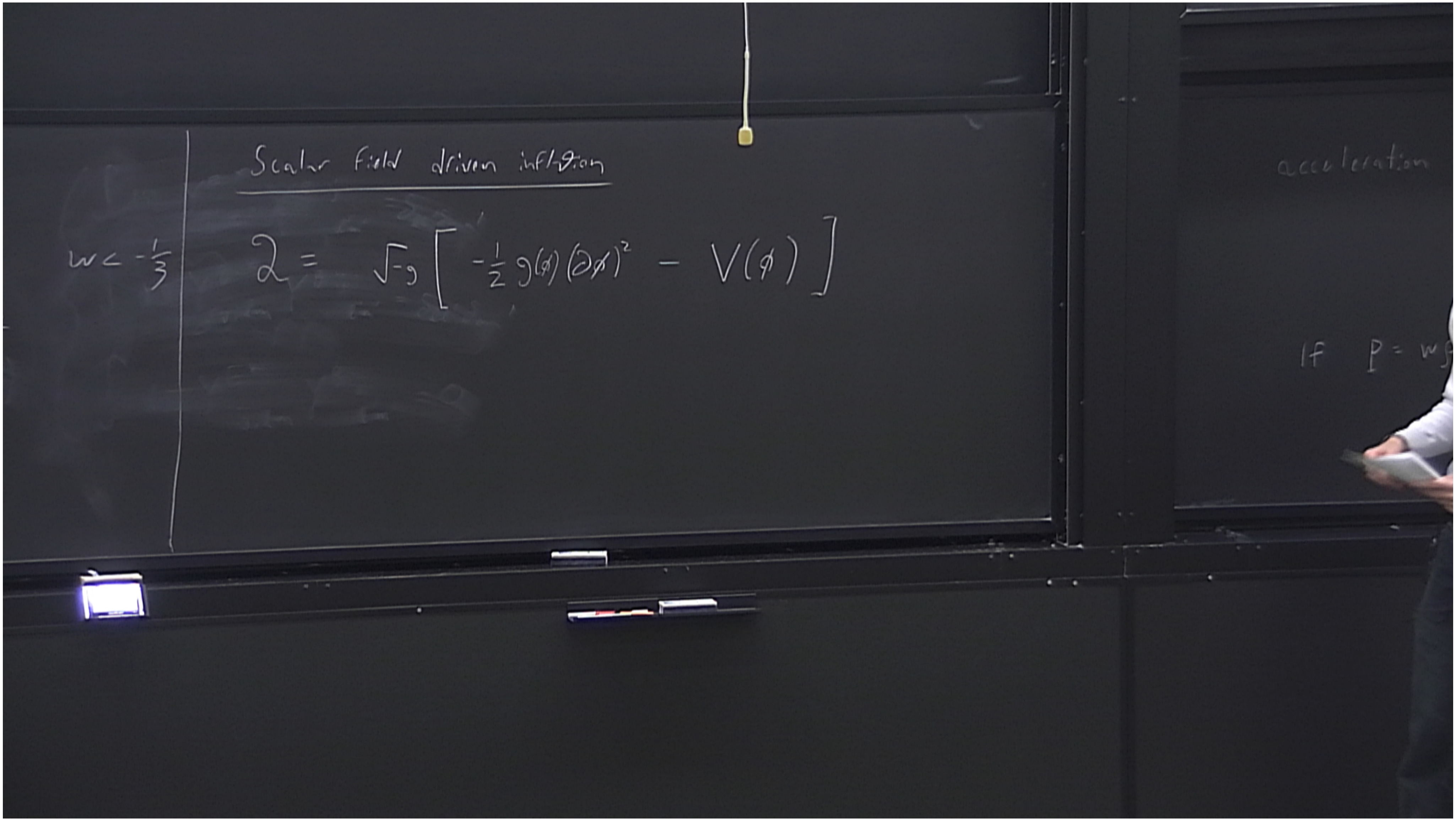

acceleration equation: $\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p)$
$\left(M_p^2 = \frac{1}{8\pi G}\right)$
inflation $\Rightarrow$ $\rho + 3p < 0$
violation of strong energy condition

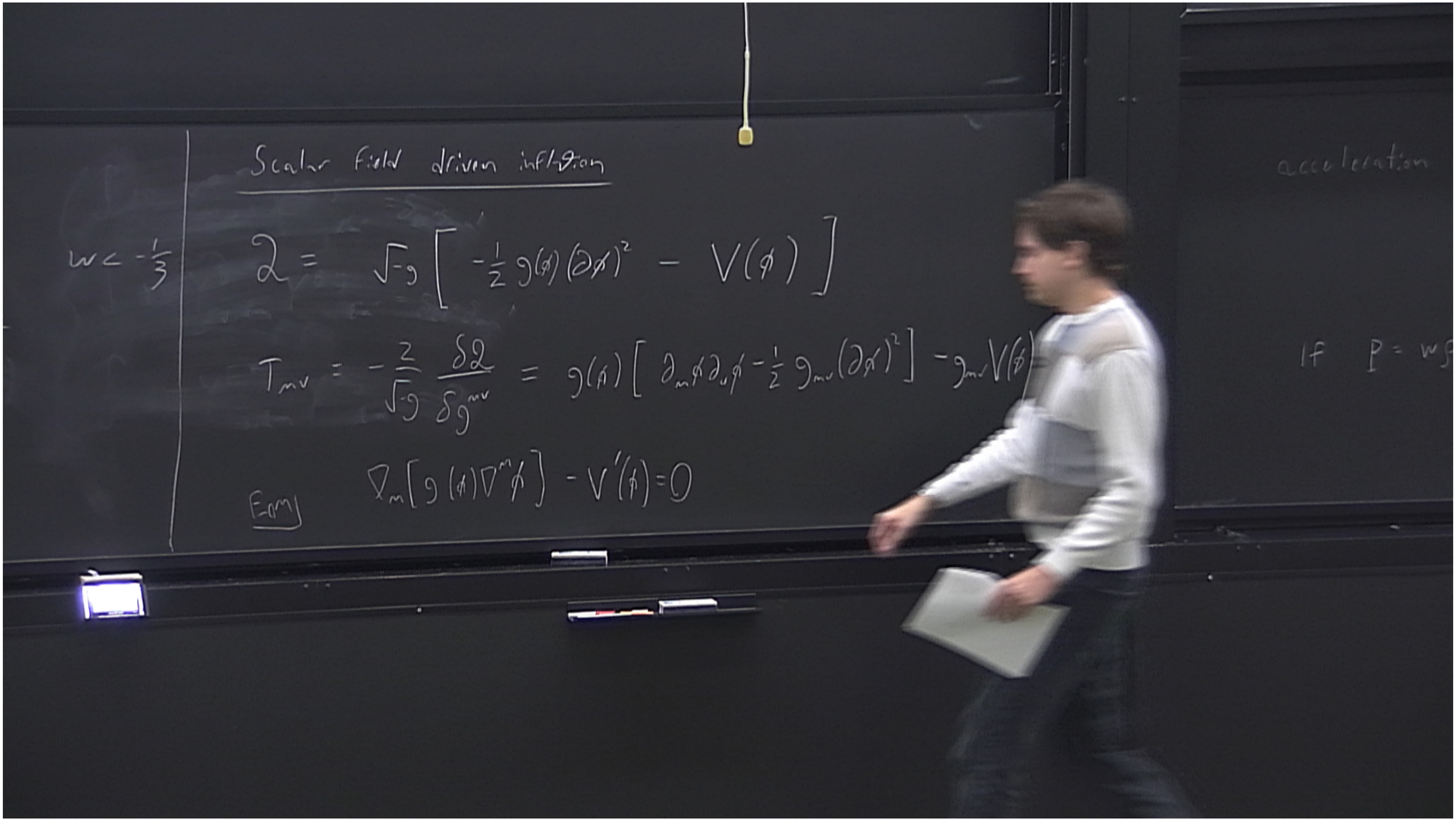

acceleration equation: $\frac{\ddot{a}}{a} = -\frac{1}{6M_p^2}(\rho + 3p)$
$\left(M_p^2 = \frac{1}{8\pi G}\right)$
inflation $\Rightarrow$ $\rho + 3p < 0$
violation of strong

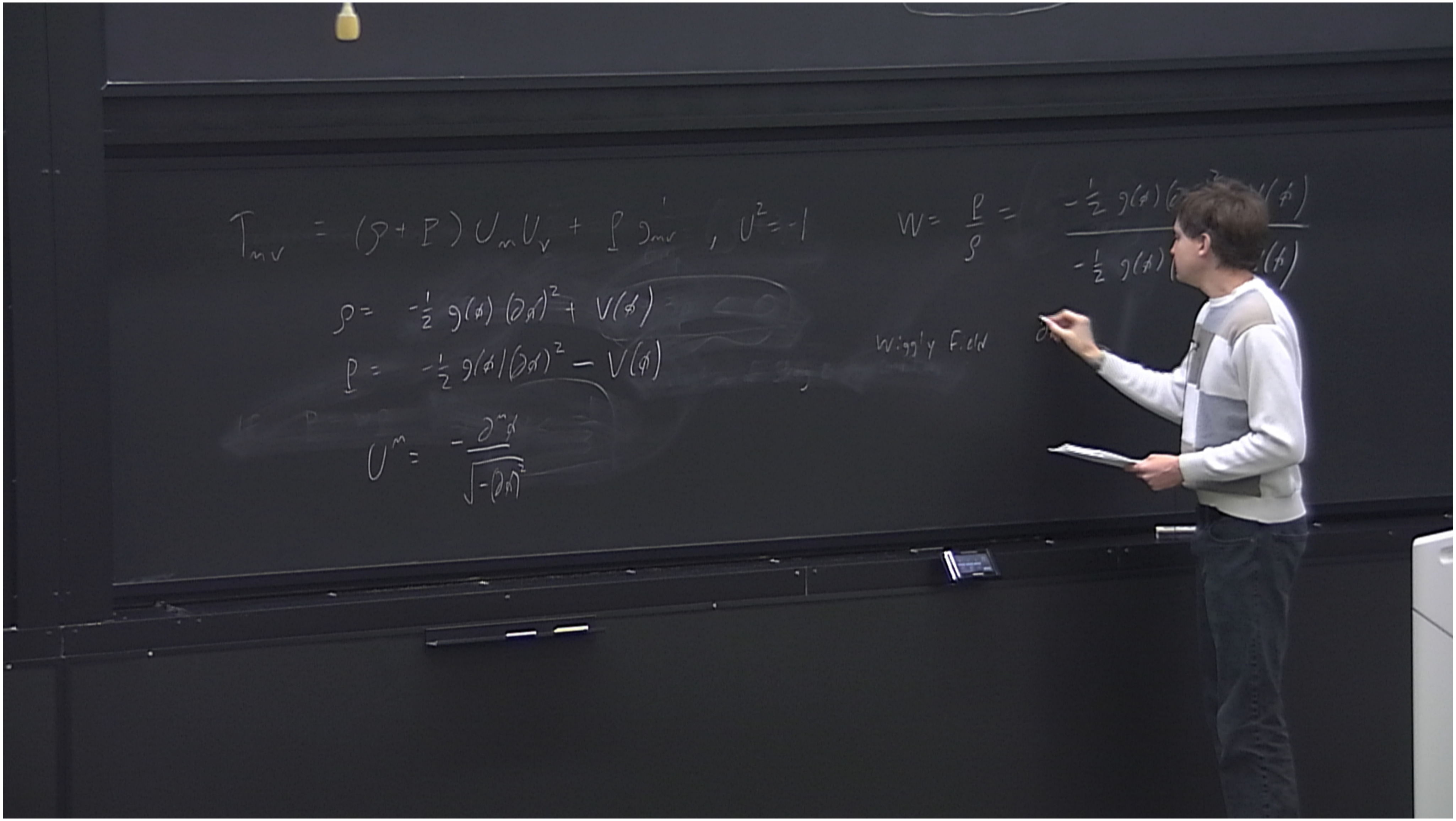

acceleration equation: $\frac{\ddot{a}}{a} = -\frac{1}{6m_p^2}(\rho + 3p)$
$\left(m_p^2 = \frac{1}{8\pi G}\right)$
inflation $\Rightarrow$ $\rho + 3p < 0$
violation of Strong energy condition
If $p = w\rho$ $\Rightarrow$ $w < -\frac{1}{3}$

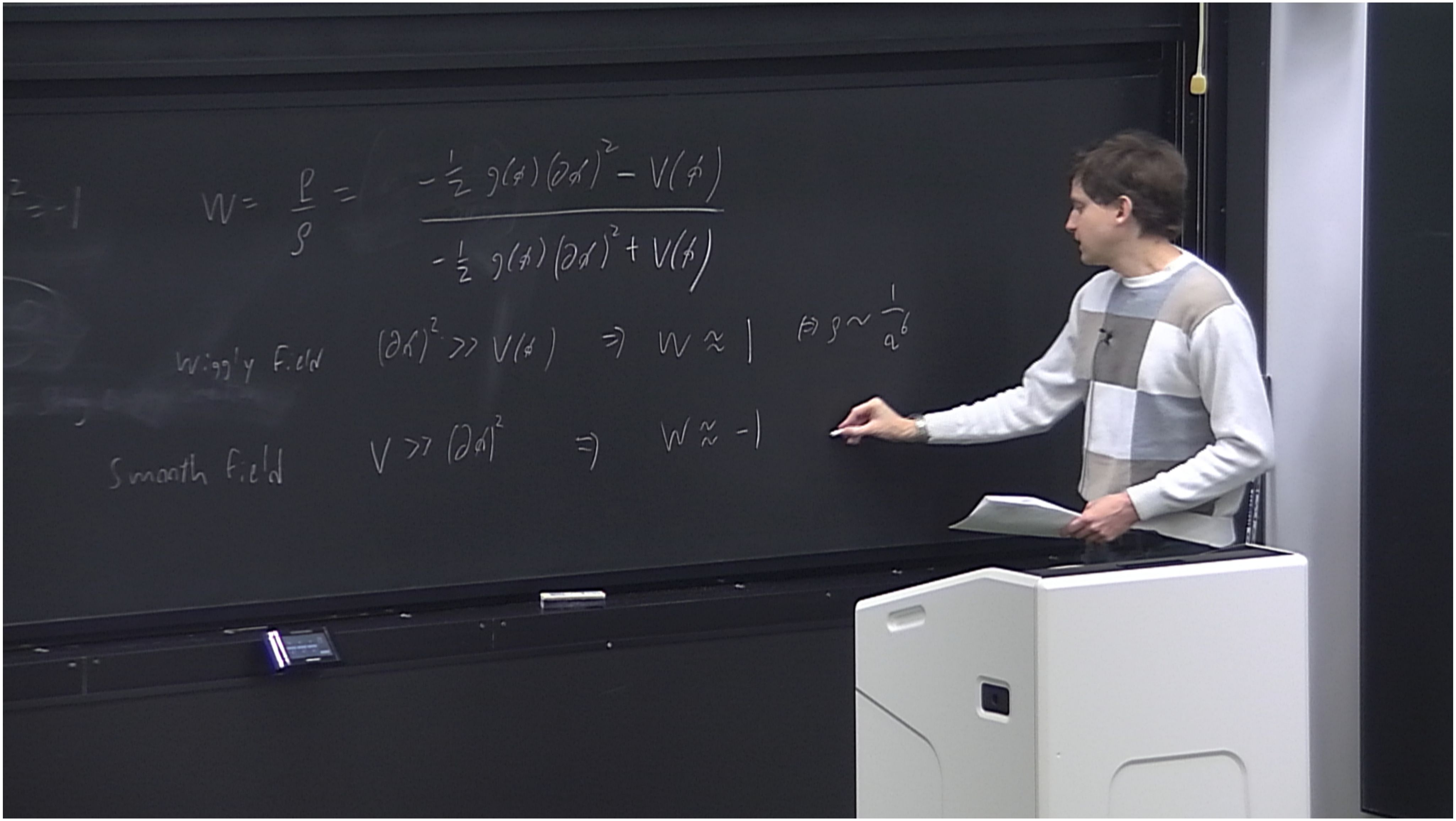

$3 M_p^2 H^2 = \frac{\rho_m}{a^3} + \frac{\rho_r}{a^4} + \frac{\rho_a}{a^6} + \frac{\rho_c}{a^2} + \frac{\rho_I}{a^{3(1+w)}}$
< 2
$w < -\frac{1}{3}$

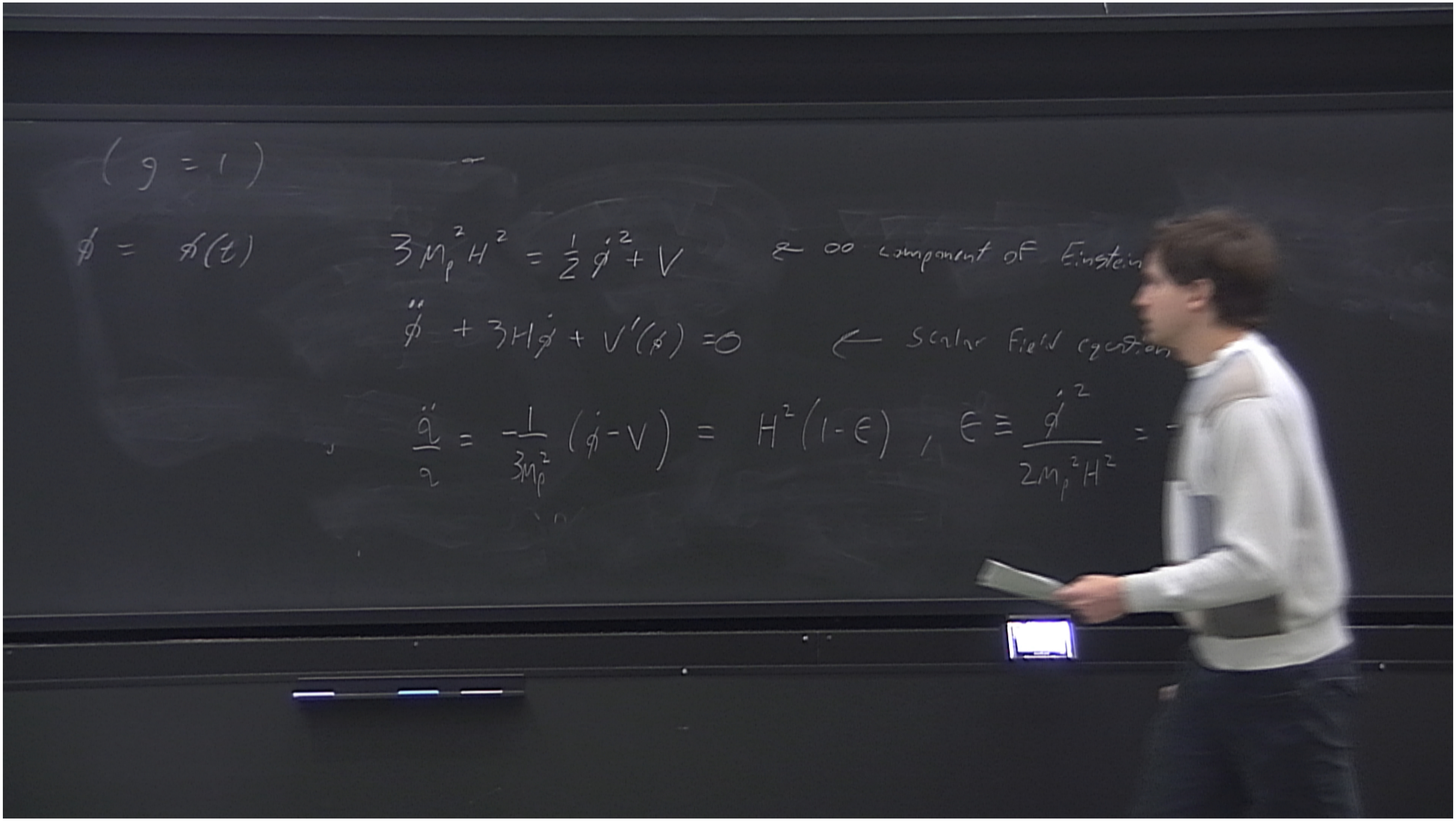

Scalar field driven inflation
$w < -\frac{1}{3}$
acceleration
If $p = w\rho$

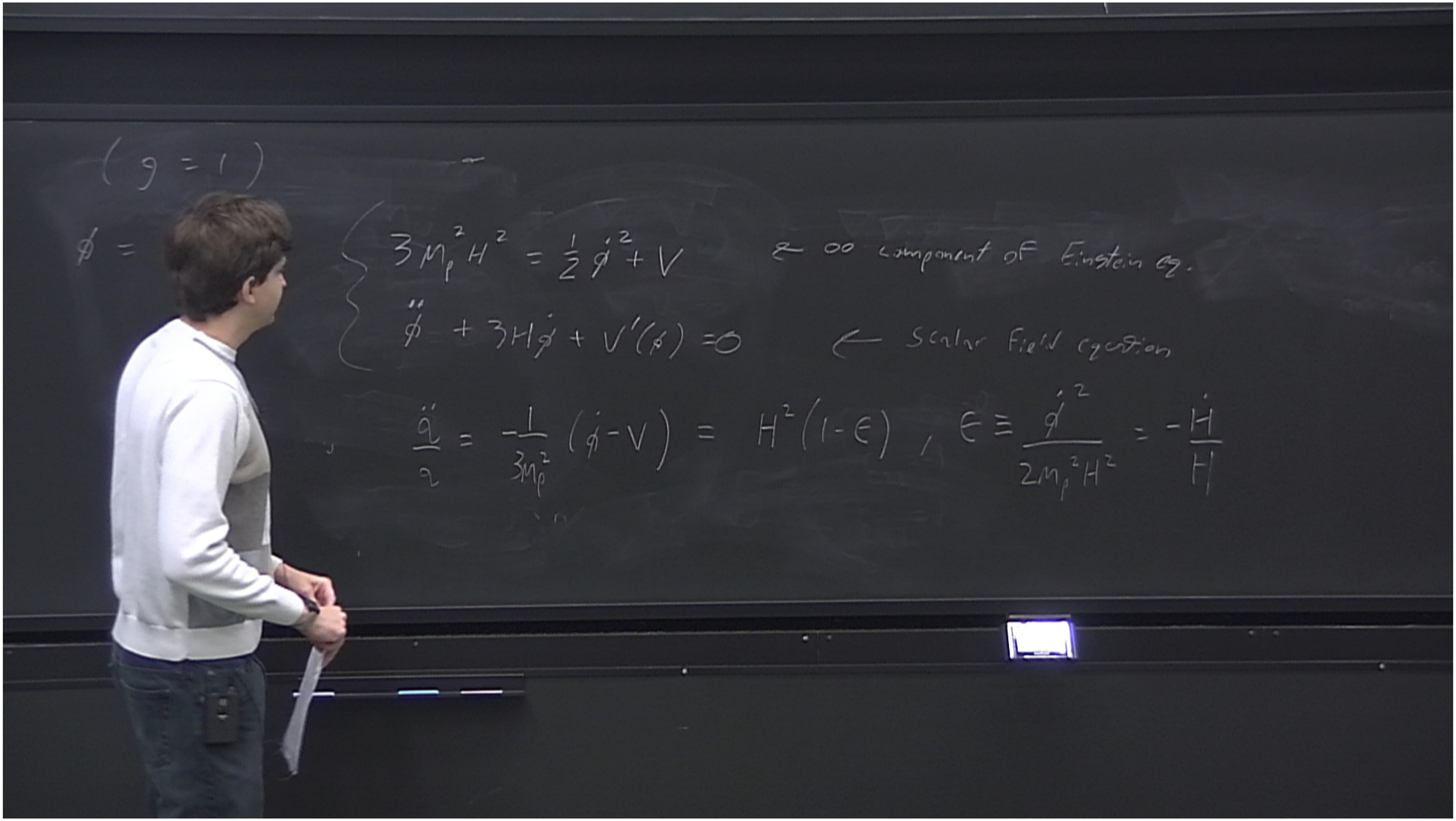

Scalar field driven inflation
w < -1/3
L = √-g [-1/2 g(φ)(∂φ)² - V(φ)]
acceleration
If


Scalar field driven inflation
acceleration
w < -1/3
If p = wρ
Eqm


$T_{\mu\nu} = (\rho + P) U_\mu U_\nu + P g_{\mu\nu}$, $U^2 = -1$
$\rho = -\frac{1}{2} g(\phi) (\partial\phi)^2 + V(\phi)$
$P = -\frac{1}{2} g(\phi) (\partial\phi)^2 - V(\phi)$
$U^\mu = -\frac{\partial^\mu \phi}{\sqrt{-(\partial\phi)^2}}$
$w = \frac{P}{\rho} =$
Wiggly Field


$w = \frac{p}{\rho} = \frac{-\frac{1}{2} g(\phi)(\partial\phi)^2 - V(\phi)}{-\frac{1}{2} g(\phi)(\partial\phi)^2 + V(\phi)}$
Wiggly Field $(\partial\phi)^2 \gg V(\phi) \Rightarrow w \approx 1 \Rightarrow \rho \sim \frac{1}{a^6}$
Smooth Field $V \gg (\partial\phi)^2 \Rightarrow w \approx -1$


(g = 1)
φ = φ(t)
3M_p^2 H^2 = 1/2 φ̇^2 + V
← 00 component of Einstein
φ̈ + 3Hφ̇ + V′(φ) = 0
← Scalar Field equation
ä/a = −1/(3M_p^2) (φ̇ − V) = H^2(1 − ε) , ε ≡ φ̇^2/(2M_p^2 H^2) =


(g = 1)
φ =
3 M_p^2 H^2 = 1/2 φ̇^2 + V ← 00 component of Einstein eq.
φ̈ + 3Hφ̇ + V′(φ) = 0 ← Scalar Field equation
ä/a = −1/(3M_p^2) (φ̇ − V) = H^2 (1 − ε) , ε ≡ φ̇^2/(2M_p^2 H^2) = −Ḣ/H