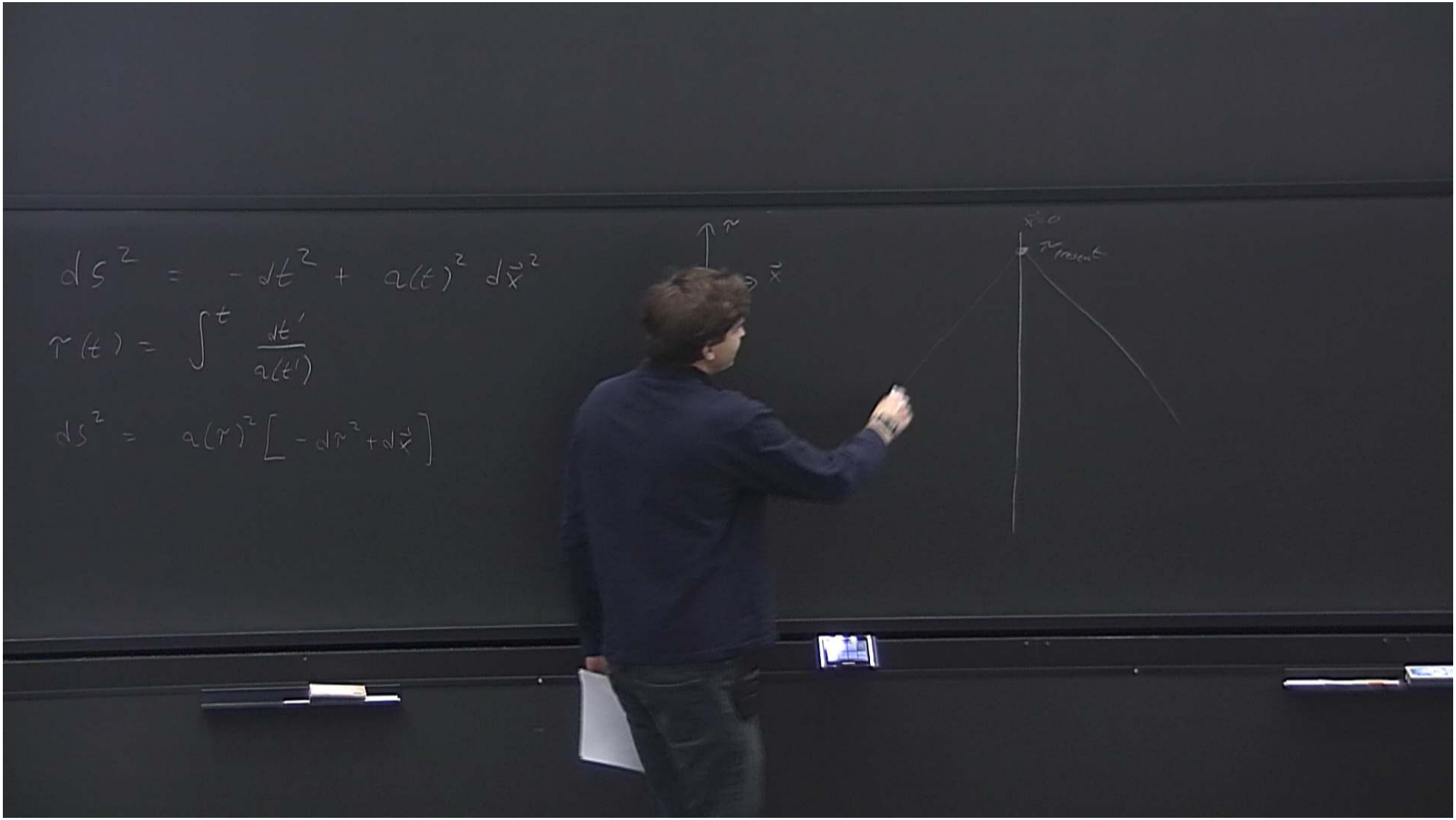
Title: Cosmology Review-8

Date: Feb 04, 2015 11:30 AM

URL: http://pirsa.org/15020029

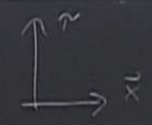
Abstract:

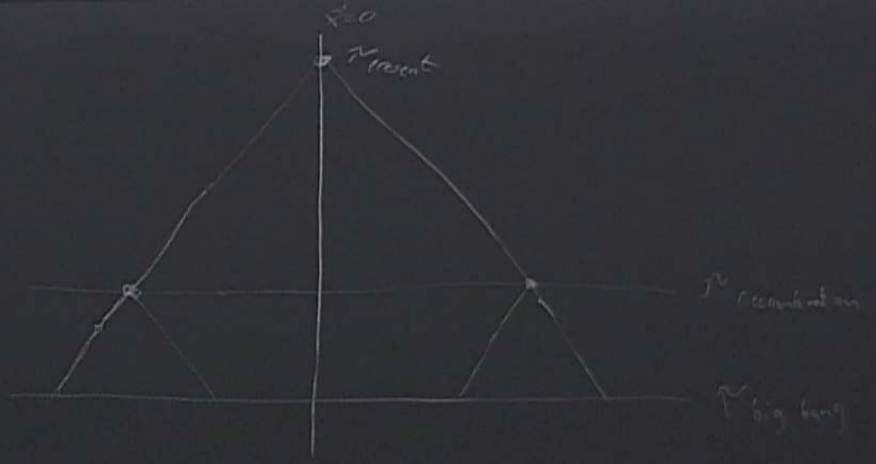

$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$
$\tau(t) = \int^t \frac{dt'}{a(t')}$
$ds^2 = a(\tau)^2 [-d\tau^2 + d\vec{x}^2]$
τ
$\vec{x}$
$x^i = 0$
$\tau_{present}$

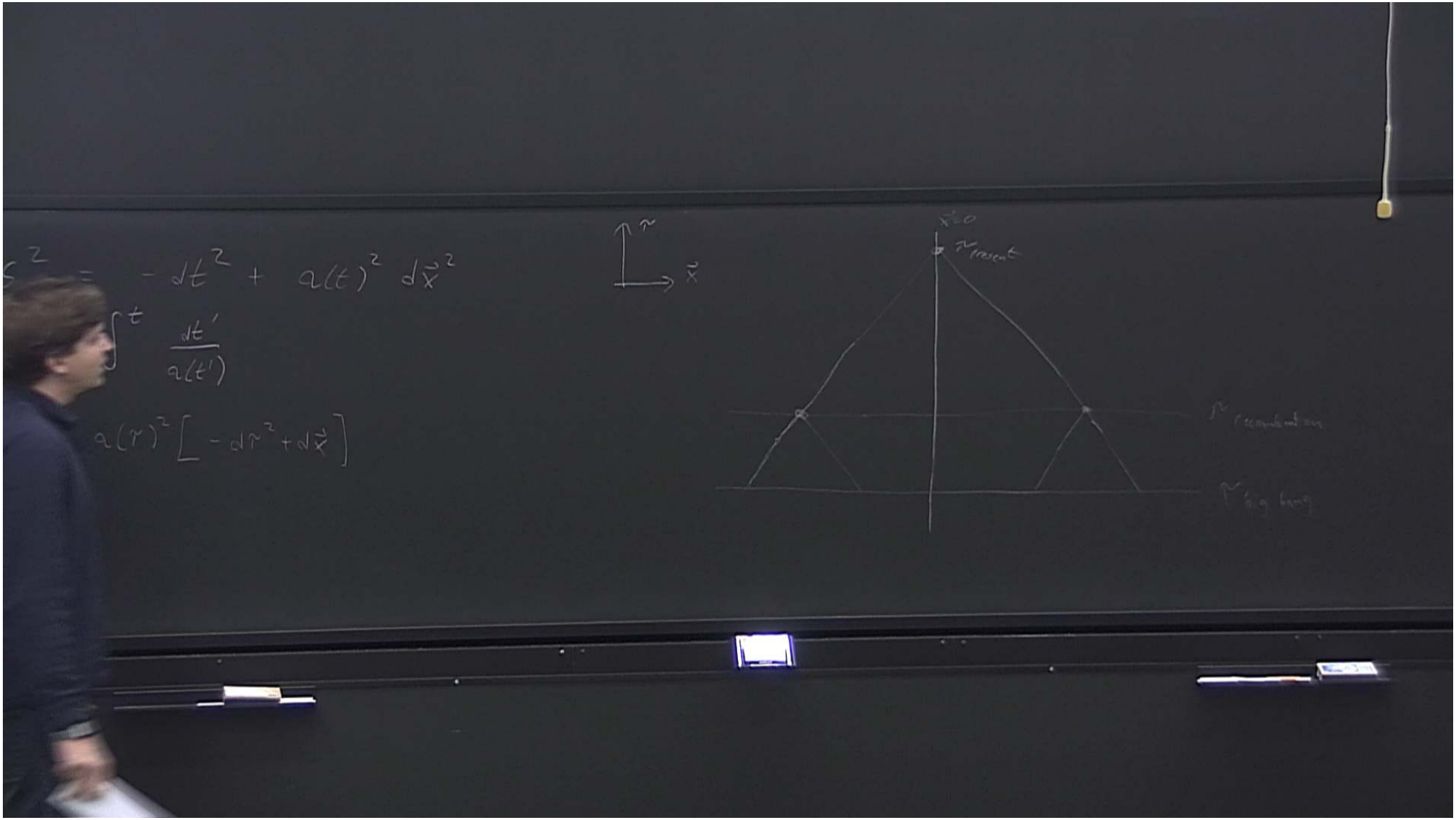
$$= -dt^2 + a(t)^2\, d\vec{x}^2$$

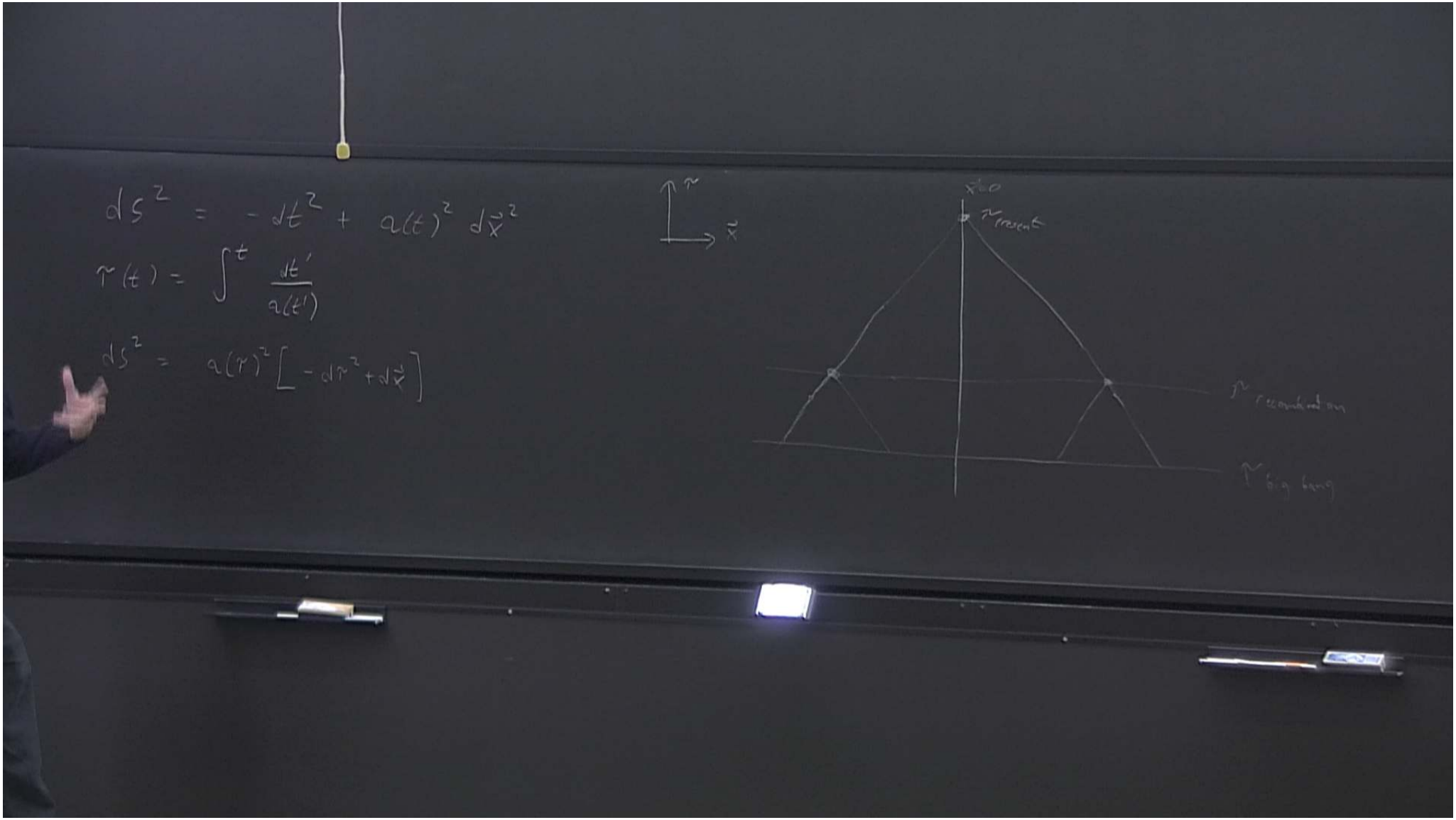
$$\int^t \frac{dt'}{a(t')}$$

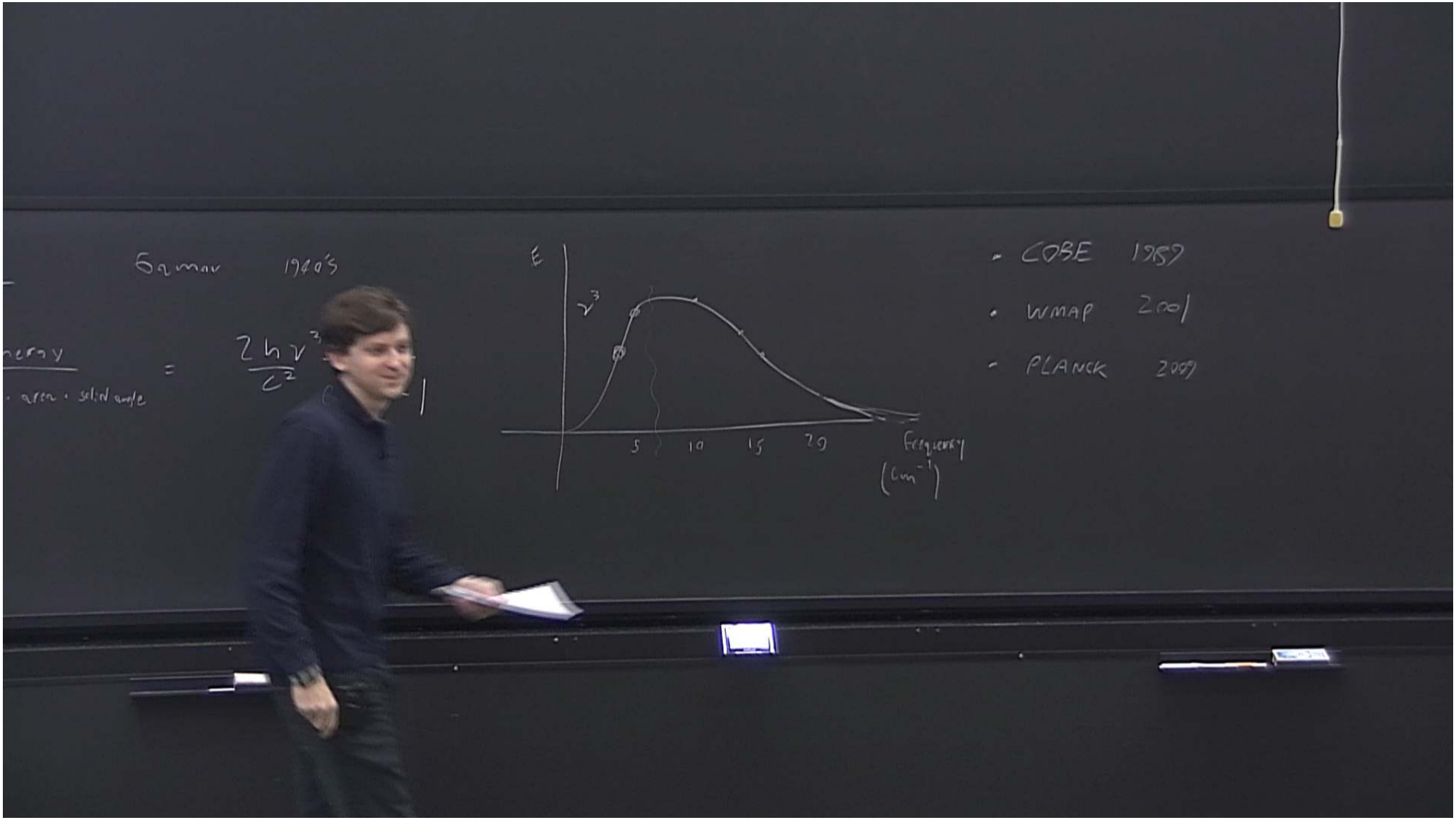
$$a(\tau)^2\left[-d\tau^2 + d\vec{x}^2\right]$$

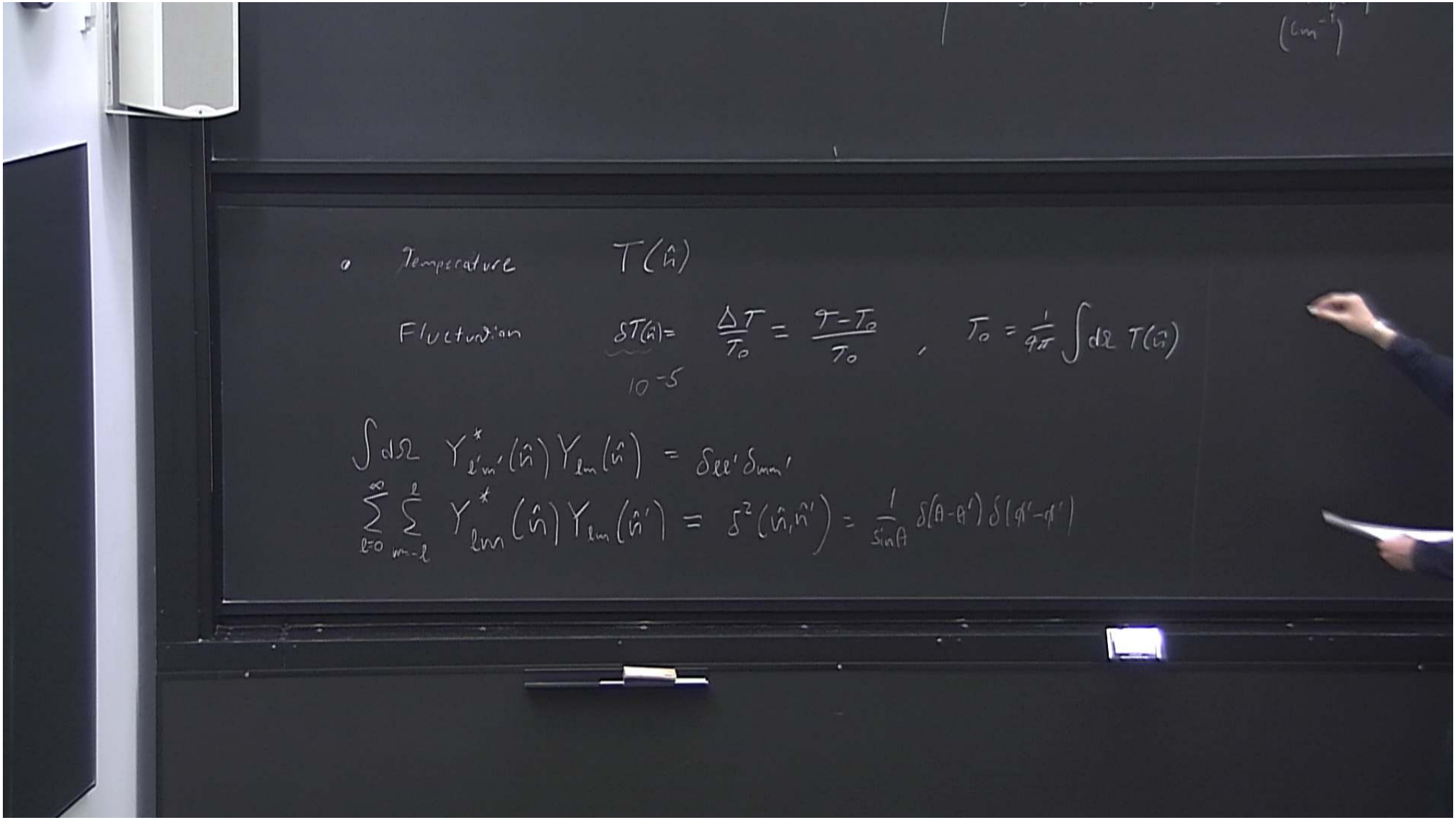


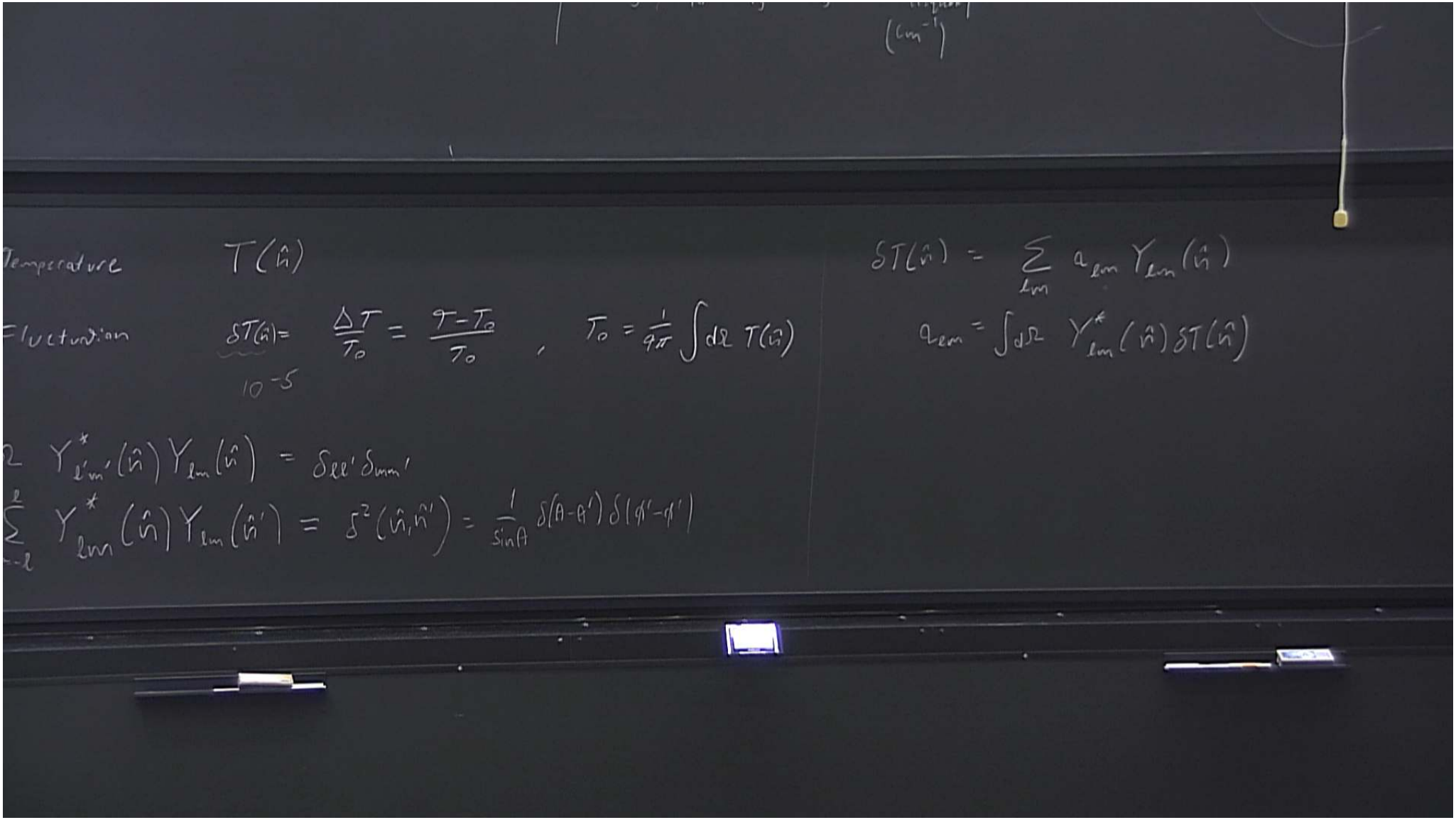


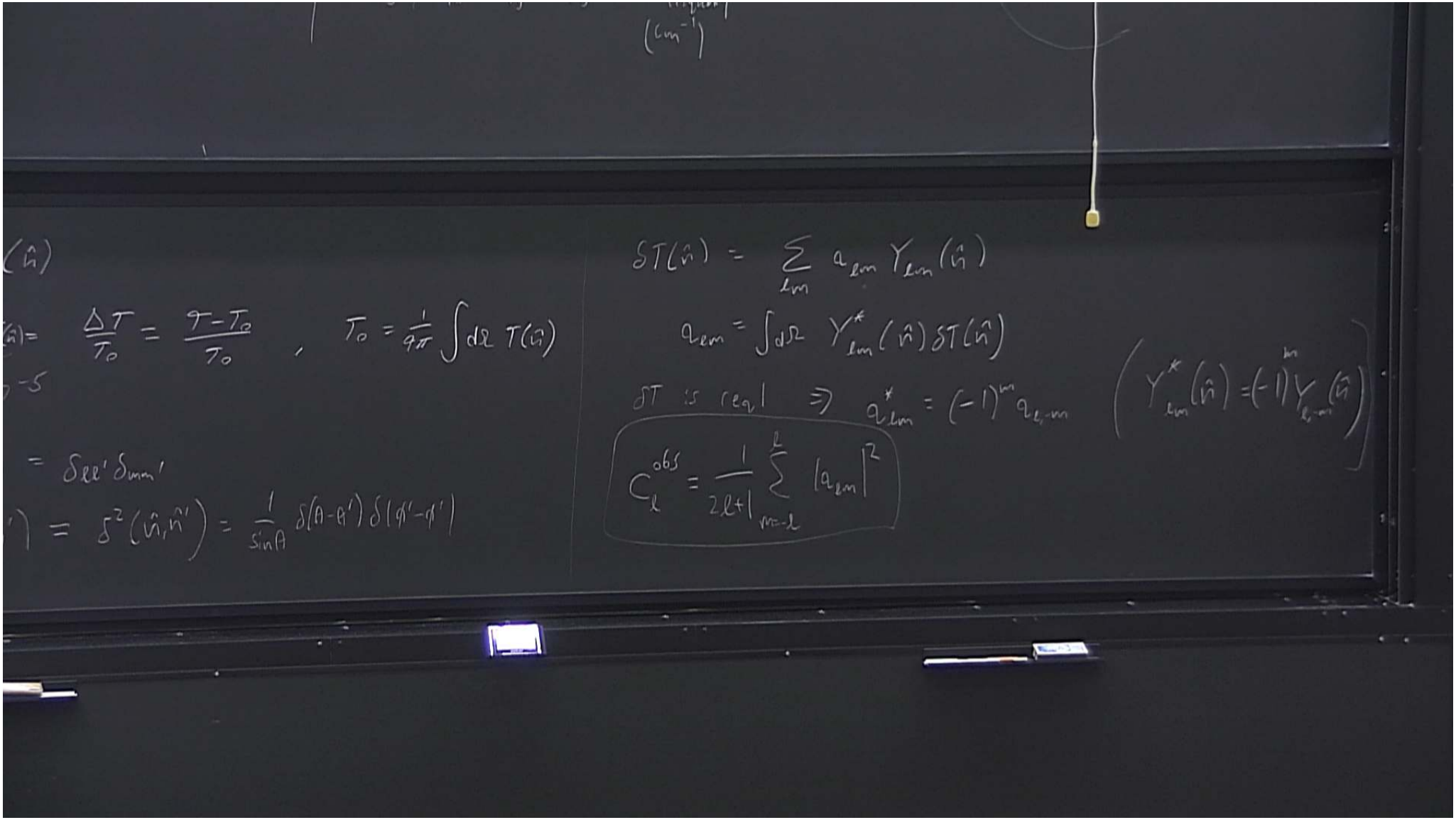

$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$
$\int_0^t \frac{dt'}{a(t')}$
$a(\tau)^2 \left[-d\tau^2 + d\vec{x}^2 \right]$
τ
$\vec{x}$
$\tau = 0$
$\tau_{recombination}$
$\tau_{big\ bang}$

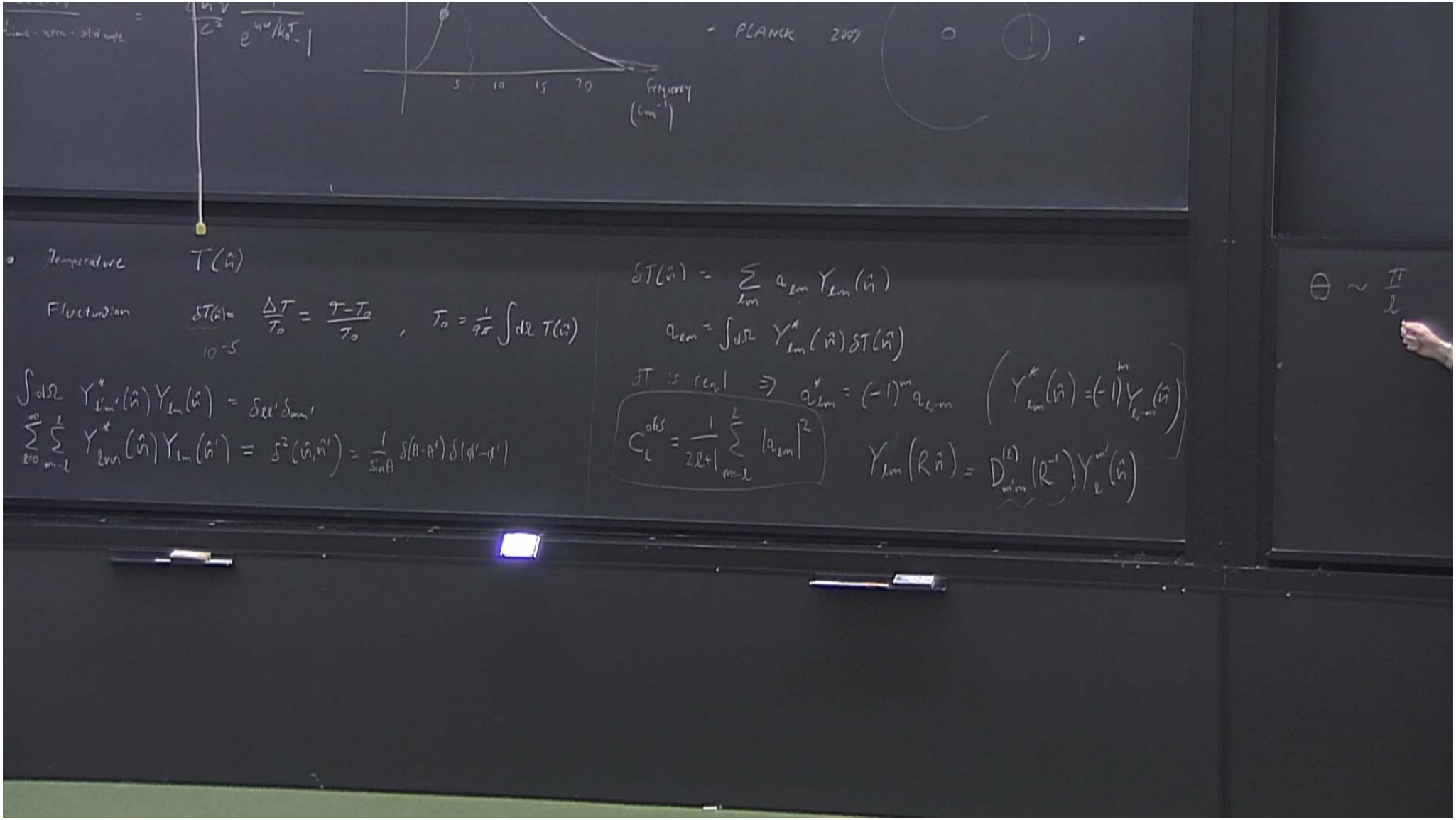

$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$
$\tau(t) = \int^t \frac{dt'}{a(t')}$
$ds^2 = a(\tau)^2 [-d\tau^2 + d\vec{x}^2]$
τ
$\vec{x}$
$\vec{x}=0$
$\tau_{present}$
$\tau_{recombination}$
$\tau_{big\ bang}$


Gamow 1940's
E
ν^3
5
10
15
20
Frequency
(cm^{-1})
- COBE 1989
- WMAP 2001
- PLANCK 2009


Temperature $T(\hat{n})$
Fluctuation $\delta T(\hat{n}) = \frac{\Delta T}{T_0} = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$
10^{-5}
$\int d\Omega\, Y^*_{l'm'}(\hat{n}) Y_{lm}(\hat{n}) = \delta_{ll'}\delta_{mm'}$
$\sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y^*_{lm}(\hat{n}) Y_{lm}(\hat{n}') = \delta^2(\hat{n},\hat{n}') = \frac{1}{\sin\theta}\delta(\theta-\theta')\delta(\varphi-\varphi')$


(cm^{-1})
Temperature $T(\hat{n})$
$\delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$
Fluctuation $\delta T(\hat{n}) = \frac{\Delta T}{T_0} = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$
10^{-5}
$a_{lm} = \int d\Omega\, Y^*_{lm}(\hat{n})\, \delta T(\hat{n})$
$\int d\Omega\, Y^*_{l'm'}(\hat{n}) Y_{lm}(\hat{n}) = \delta_{ll'}\delta_{mm'}$
$\sum_{m=-l}^{l} Y^*_{lm}(\hat{n}) Y_{lm}(\hat{n}') = \delta^2(\hat{n},\hat{n}') = \frac{1}{\sin\theta}\delta(\theta-\theta')\delta(\varphi-\varphi')$


(cm^{-1})
$(\hat{n})$
$\frac{\Delta T}{T_0} = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$
$= \delta_{ll'}\delta_{mm'}$
$= \delta^2(\hat{n},\hat{n}') = \frac{1}{\sin\theta}\delta(\theta-\theta')\delta(\varphi-\varphi')$
$\delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$
$a_{lm} = \int d\Omega\, Y^*_{lm}(\hat{n})\,\delta T(\hat{n})$
δT is real $\Rightarrow a^*_{lm} = (-1)^m a_{l,-m}$
$\left(Y^*_{lm}(\hat{n}) = (-1)^m Y_{l,-m}(\hat{n})\right)$
$C_l^{obs} = \frac{1}{2l+1}\sum_{m=-l}^{l} |a_{lm}|^2$

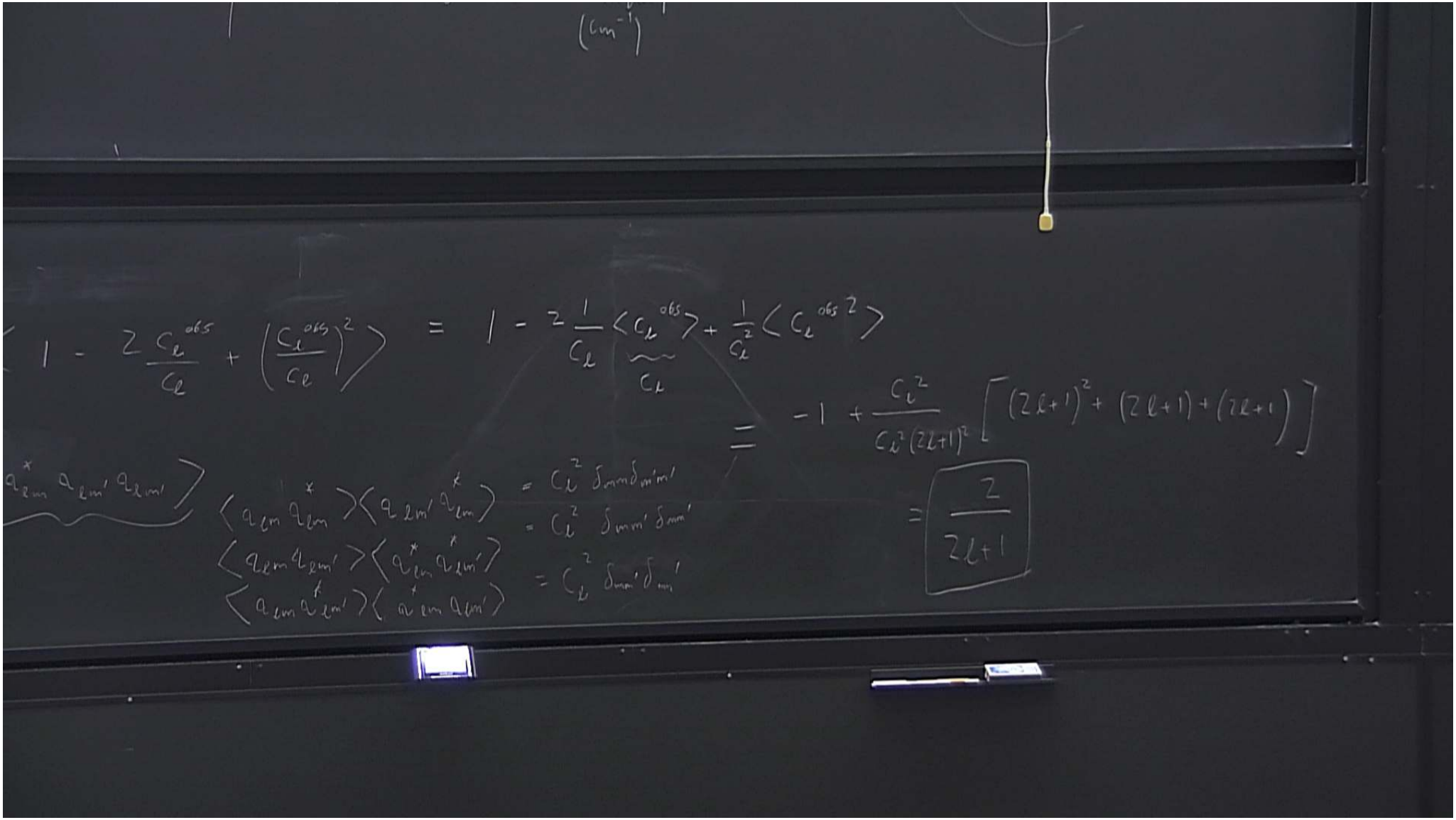

PLANCK 2009
5
10
15
20
Frequency
(cm⁻¹)
Temperature T(n̂)
Fluctuation δT(n̂) = ΔT/T₀ = (T − T₀)/T₀ , T₀ = 1/4π ∫ dΩ T(n̂)
10⁻⁵
∫ dΩ Y*_{l'm'}(n̂) Y_{lm}(n̂) = δ_{ll'} δ_{mm'}
Σ_{l=0}^{∞} Σ_{m=−l}^{l} Y*_{lm}(n̂) Y_{lm}(n̂') = δ²(n̂, n̂') = 1/sinθ δ(θ − θ') δ(φ − φ')
δT(n̂) = Σ_{lm} a_{lm} Y_{lm}(n̂)
a_{lm} = ∫ dΩ Y*_{lm}(n̂) δT(n̂)
δT is real ⇒ a*_{lm} = (−1)^m a_{l,−m}
(Y*_{lm}(n̂) = (−1)^m Y_{l,−m}(n̂))
C_l^{obs} = 1/(2l+1) Σ_{m=−l}^{l} |a_{lm}|²
Y_{lm}(Rn̂) = D^{(l)}_{m'm}(R⁻¹) Y^{m'}_l(n̂)
θ ~ π/l

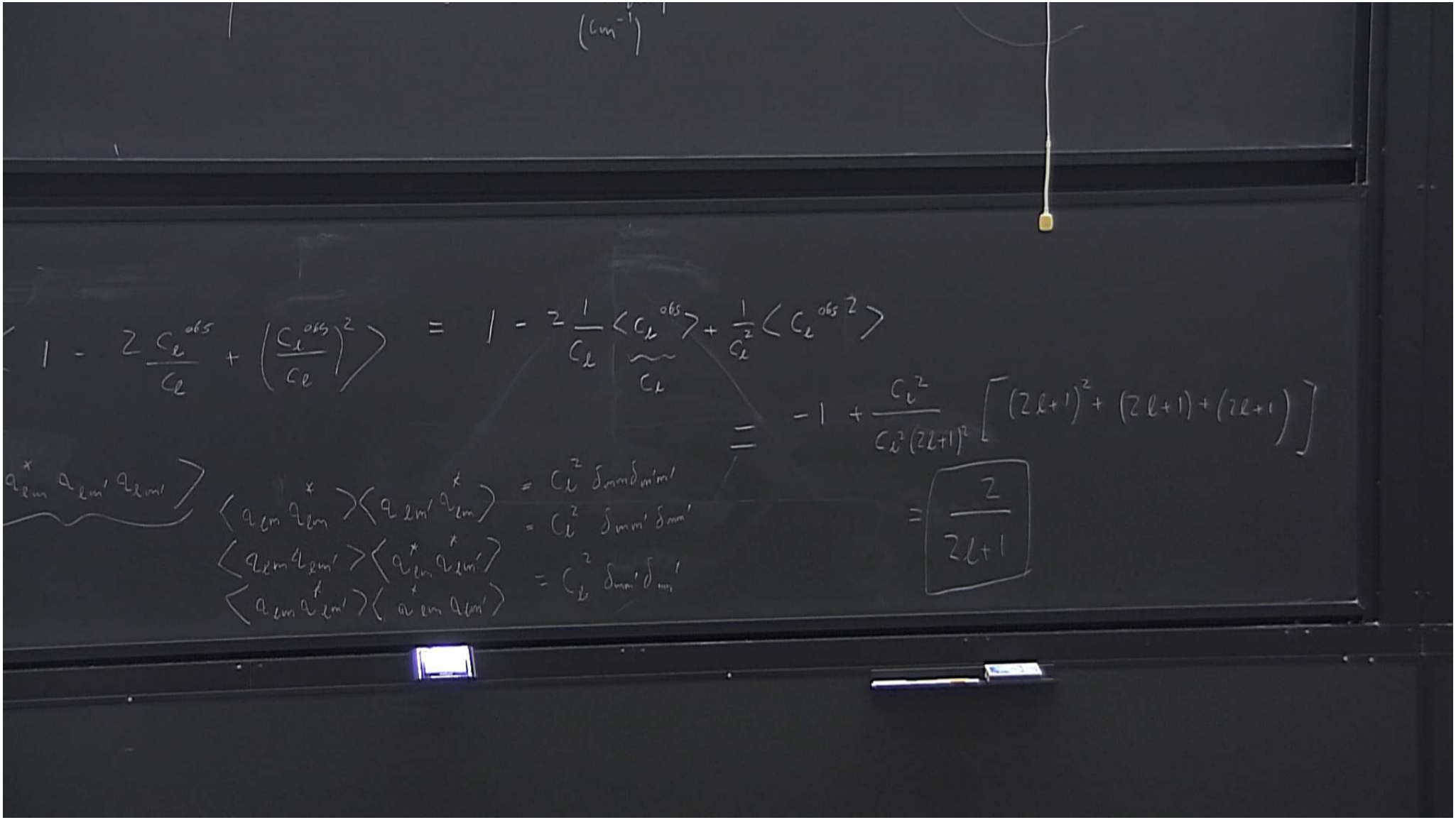
$\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$

$\frac{1}{2\ell+1} \sum_m Y_{\ell m}(\hat{n}) Y^*_{\ell m}(\hat{n}')$

relate C_ℓ^{obs}, C_ℓ

$$\langle C_\ell^{obs} \rangle = \left\langle \frac{1}{2\ell+1} \sum_m a^*_{\ell m} a_{\ell m} \right\rangle = \frac{1}{2\ell+1} \sum_m \underbrace{\langle a_{\ell m} a^*_{\ell m} \rangle}_{\delta_{\ell\ell} \delta_{mm} C_\ell} = C_\ell$$


$\left\langle 1 - 2\frac{C_\ell^{obs}}{C_\ell} + \left(\frac{C_\ell^{obs}}{C_\ell}\right)^2 \right\rangle = 1 - 2\frac{1}{C_\ell}\langle C_\ell^{obs}\rangle + \frac{1}{C_\ell^2}\langle C_\ell^{obs\,2}\rangle$
$= -1 + \frac{C_\ell^2}{C_\ell^2 (2\ell+1)^2}\left[(2\ell+1)^2 + (2\ell+1) + (2\ell+1)\right]$
$= \frac{2}{2\ell+1}$
$\langle a_{\ell m} a^*_{\ell m} \rangle \langle a_{\ell m'} a^*_{\ell m'} \rangle = C_\ell^2 \delta_{mm} \delta_{m'm'}$
$\langle a_{\ell m} a^*_{\ell m} \rangle \langle a_{\ell m'} a^*_{\ell m'} \rangle = C_\ell^2 \delta_{mm'} \delta_{mm'}$
$\langle a_{\ell m} a^*_{\ell m'} \rangle \langle a^*_{\ell m} a_{\ell m'} \rangle = C_\ell^2 \delta_{mm'} \delta_{m'm}$


$\left\langle 1 - 2\frac{C_\ell^{obs}}{C_\ell} + \left(\frac{C_\ell^{obs}}{C_\ell}\right)^2 \right\rangle = 1 - 2\frac{1}{C_\ell}\langle C_\ell^{obs}\rangle + \frac{1}{C_\ell^2}\langle C_\ell^{obs\,2}\rangle$
$= -1 + \frac{C_\ell^2}{C_\ell^2 (2\ell+1)^2}\left[(2\ell+1)^2 + (2\ell+1) + (2\ell+1)\right]$
$= \frac{2}{2\ell+1}$
$\langle a_{\ell m} a^*_{\ell m} \rangle \langle a_{\ell m'} a^*_{\ell m'} \rangle = C_\ell^2 \delta_{mm} \delta_{m'm'}$
$\langle a_{\ell m} a^*_{\ell m} \rangle \langle a_{\ell m'} a^*_{\ell m'} \rangle = C_\ell^2 \delta_{mm'} \delta_{mm'}$
$\langle a_{\ell m} a^*_{\ell m'} \rangle \langle a^*_{\ell m} a_{\ell m'} \rangle = C_\ell^2 \delta_{mm'} \delta_{m'm}$

$\langle \quad \rangle = \delta_{\ell\ell'} \delta_{mm'} C_\ell$

$\frac{1}{2\ell+1} \sum_m Y_{\ell m}(\hat{n}) Y_{\ell m}(\hat{n})$

$=$ # of photons in $d^3p\, d^3q$, w/ prob of $e^i = e^i e^j f_{ij}$

$e^i = \begin{cases} \delta^{ij} e_i e_j = 1 \\ \delta_{ij} e^i p^j = 0 \end{cases}$

$f_{ij}\left(\vec{q} = \text{earth}, -E\hat{n}, t = \text{now}\right)$ $\quad p_j f^{ij} = 0$, f_{ij} is a symmetric tensor on sky