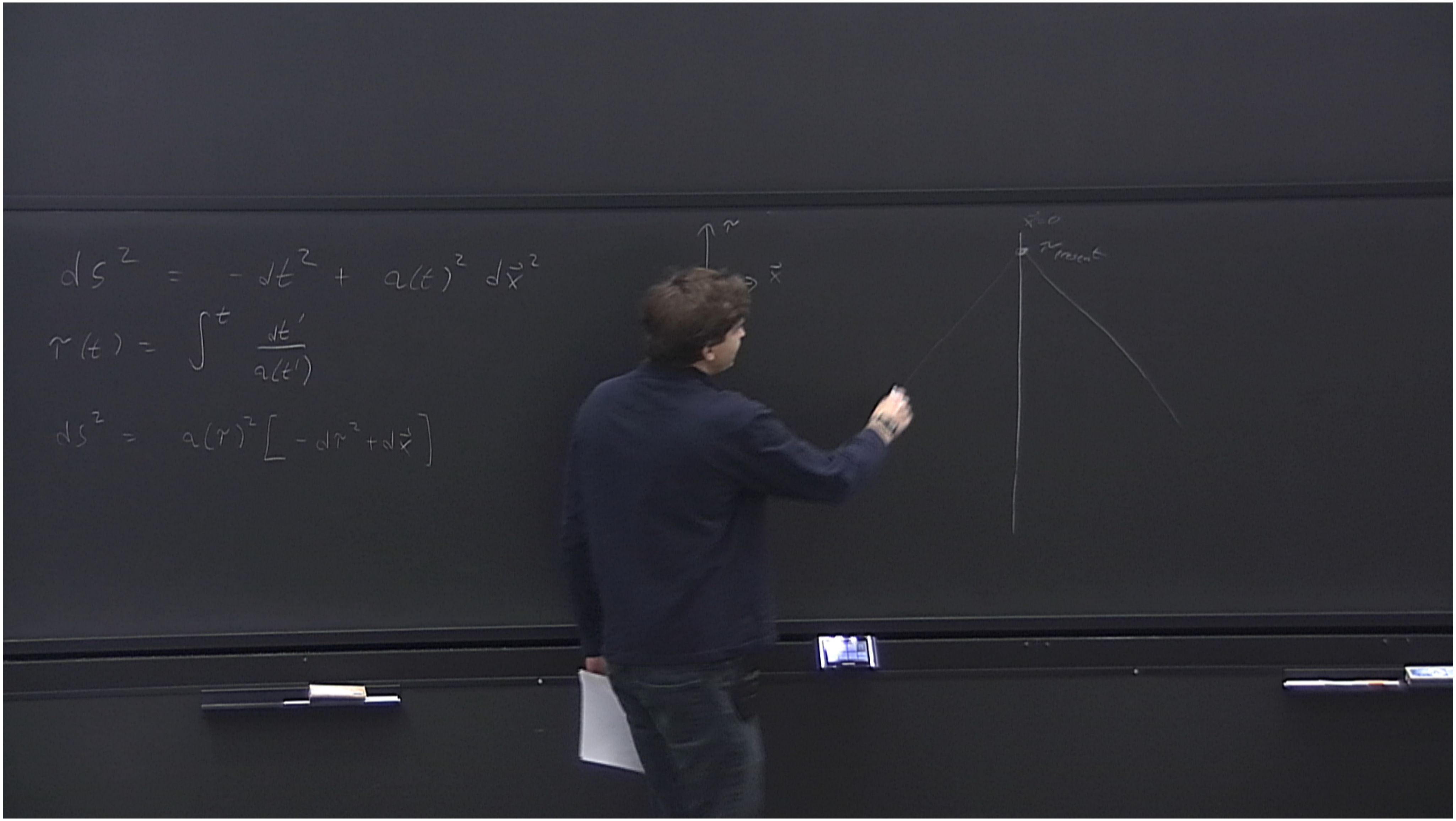
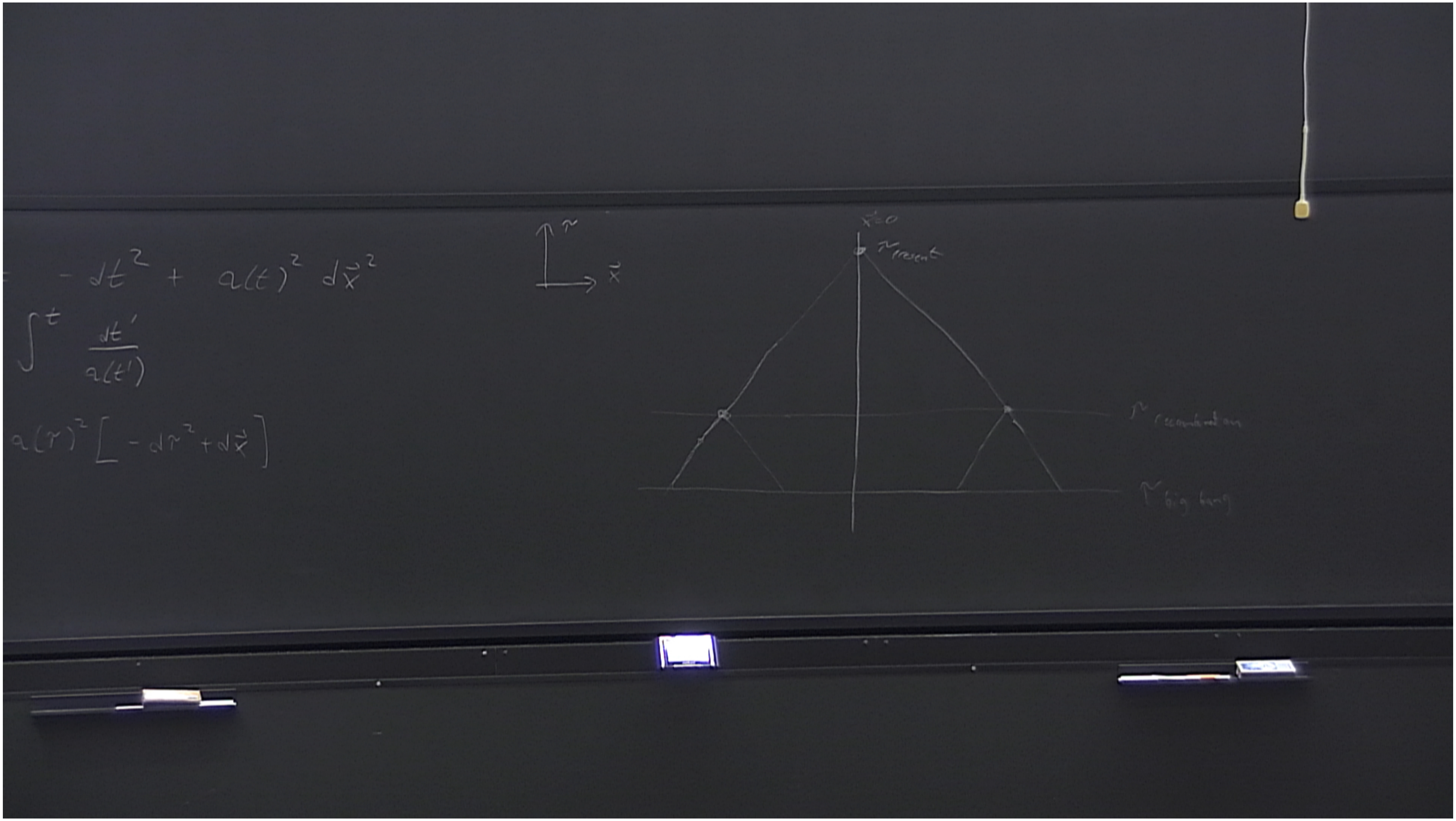
Title: Cosmology Review-8

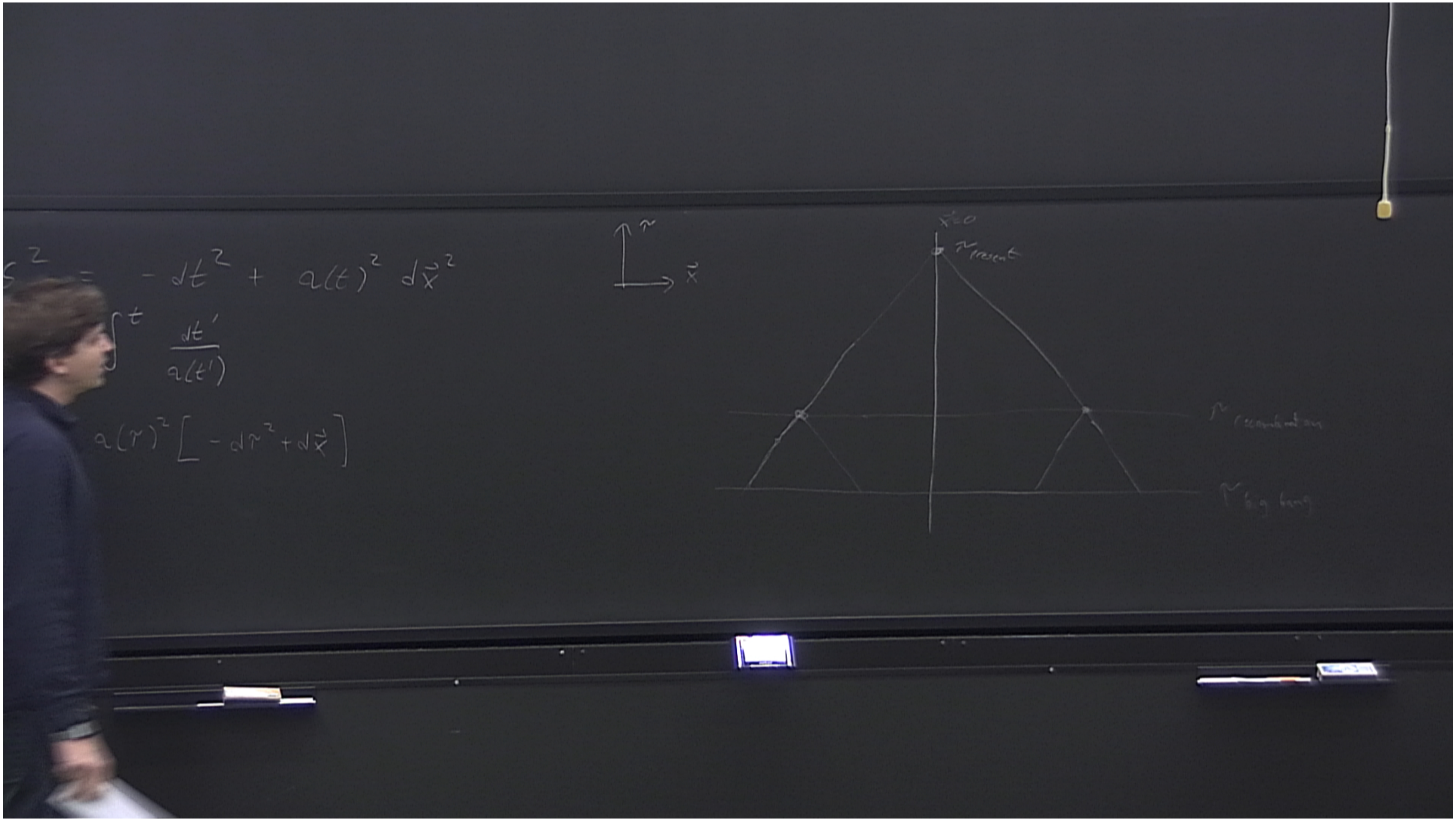
Date: Feb 04, 2015 11:30 AM

URL: http://pirsa.org/15020029

Abstract:


$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$
$\tau(t) = \int^t \frac{dt'}{a(t')}$
$ds^2 = a(\tau)^2 [-d\tau^2 + d\vec{x}]$
τ
$\vec{x}$
$\vec{x}=0$
$\tau_{present}$


$= -dt^2 + a(t)^2 d\vec{x}^2$
$\int^t \frac{dt'}{a(t')}$
$a(\tau)^2 [-d\tau^2 + d\vec{x}]$
τ
$\vec{x}$
$\vec{x}=0$
$\tau_{present}$
$\tau_{recombination}$
$\tau_{big\ bang}$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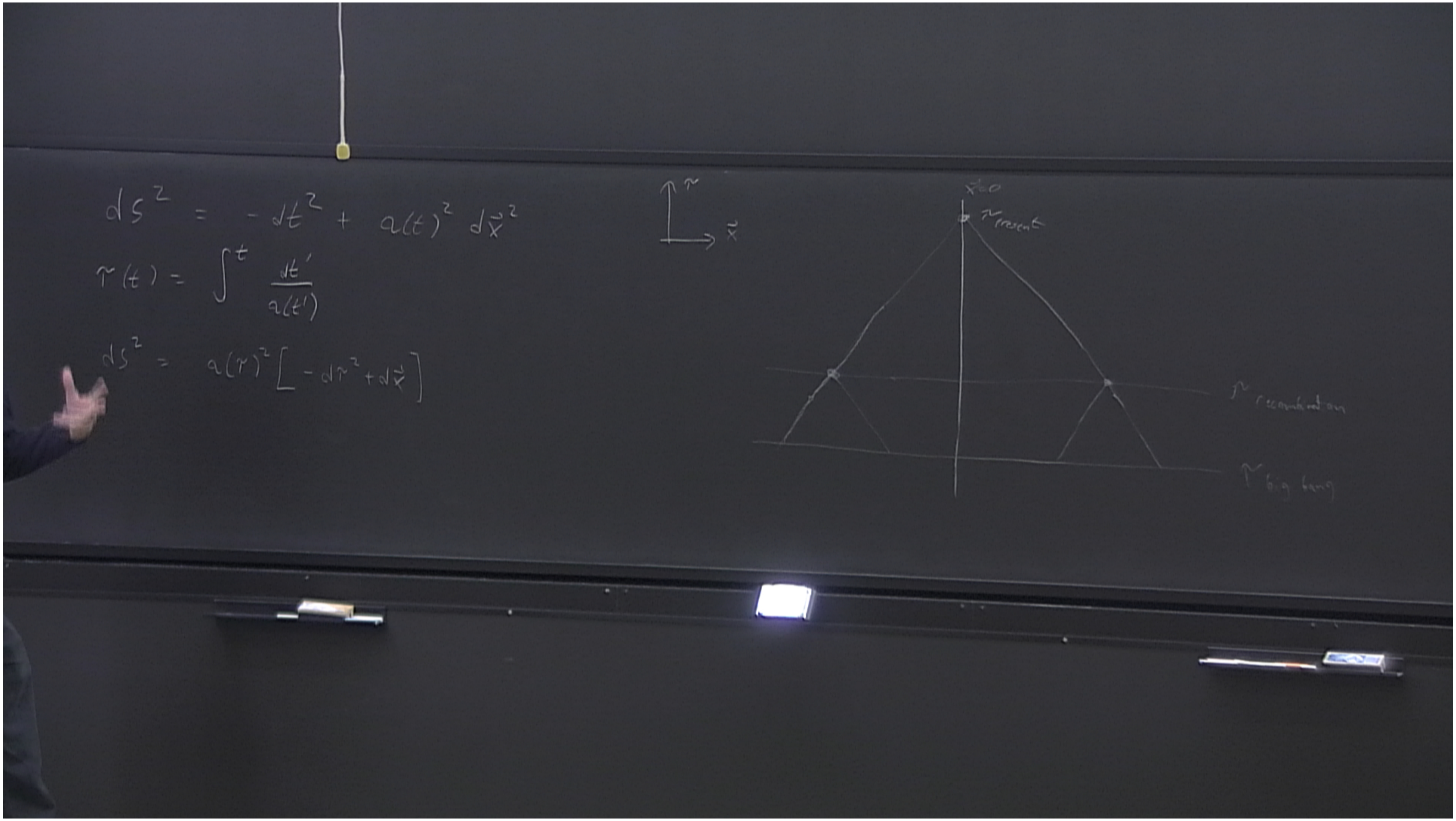
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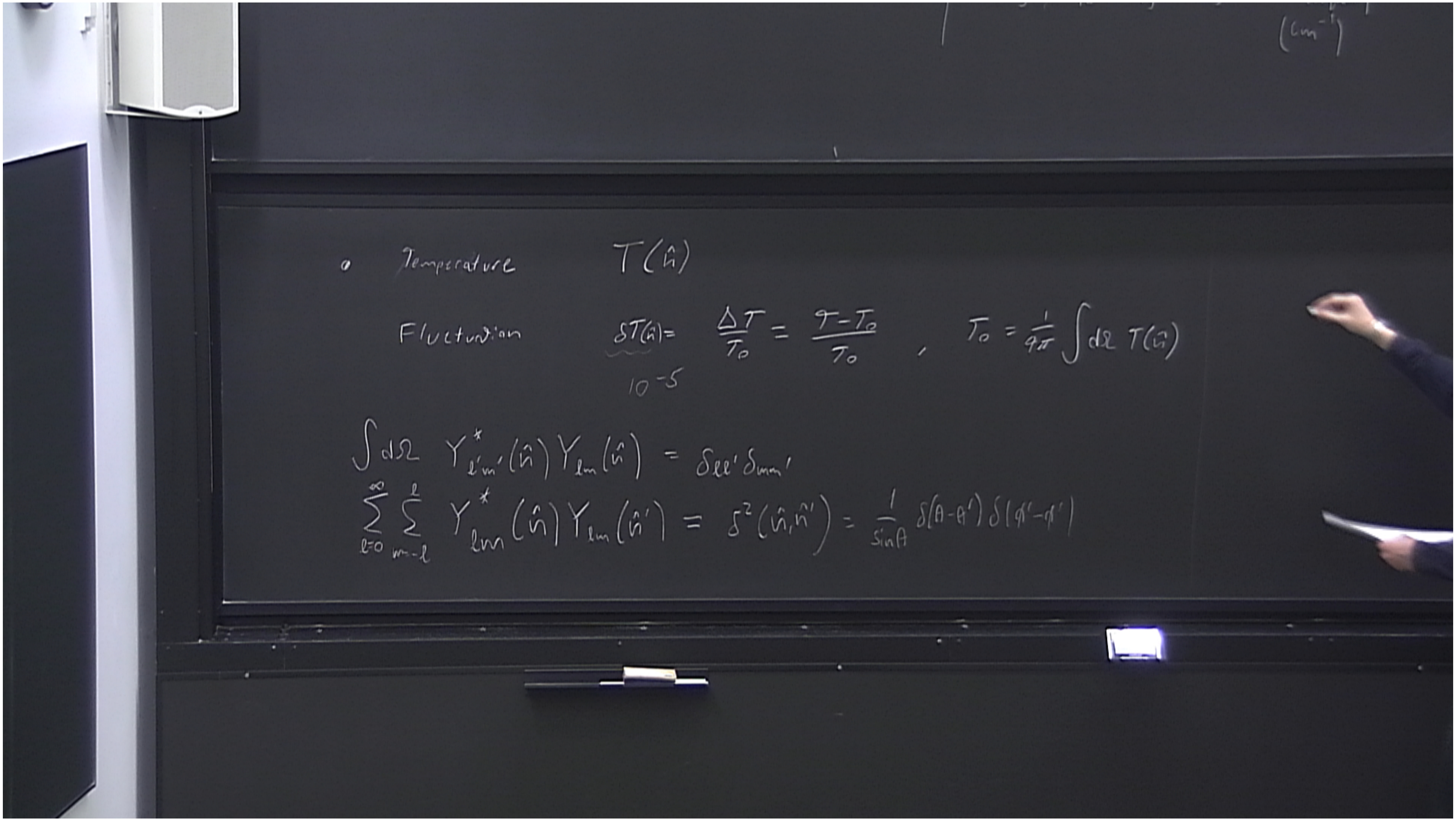
$\tau(t) = \int^t \frac{dt'}{a(t')}$
$ds^2 = a(\tau)^2 \left[-d\tau^2 + d\vec{x}^2 \right]$
τ
$\vec{x}$
$\vec{x}=0$
$\tau_{present}$
$\tau_{recombination}$
$\tau_{big\ bang}$

Gamow 1940's
E
ν^3
5 10 15 20
Frequency
(cm^{-1})
• COBE 1989
• WMAP 2001
• PLANCK 2009


Temperature $T(\hat{n})$
Fluctuation $\delta T(\hat{n}) = \frac{\Delta T}{T_0} = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$
10^{-5}
$\int d\Omega\, Y^*_{l'm'}(\hat{n})\, Y_{lm}(\hat{n}) = \delta_{ll'}\delta_{mm'}$
$\sum_{l=0}^{\infty}\sum_{m=-l}^{l} Y^*_{lm}(\hat{n})\, Y_{lm}(\hat{n}') = \delta^2(\hat{n},\hat{n}') = \frac{1}{\sin\theta}\delta(\theta-\theta')\delta(\phi-\phi')$

(cm^{-1})

Temperature $T(\hat{n})$

Fluctuation $\underbrace{\delta T(\hat{n})}_{10^{-5}} = \frac{\Delta T}{T_0} = \frac{T - T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat{n})$

$$\int d\Omega\, Y^*_{l'm'}(\hat{n})\, Y_{lm}(\hat{n}) = \delta_{ll'}\delta_{mm'}$$

$$\sum_{m=-l}^{l} Y^*_{lm}(\hat{n})\, Y_{lm}(\hat{n}') = \delta^2(\hat{n}, \hat{n}') = \frac{1}{\sin\theta}\delta(\theta-\theta')\delta(\phi'-\phi')$$

$$\delta T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$a_{lm} = \int d\Omega\, Y^*_{lm}(\hat{n})\, \delta T(\hat{n})$$

(cm^{-1})

$(\hat{n})$

$(\hat{n}) = \dfrac{\Delta T}{T_0} = \dfrac{T - T_0}{T_0}$, $T_0 = \dfrac{1}{4\pi}\int d\Omega\, T(\hat{n})$

-5

$= \delta_{\ell\ell'}\delta_{mm'}$

$') = \delta^2(\hat{n},\hat{n}') = \dfrac{1}{\sin\theta}\,\delta(\theta-\theta')\,\delta(\phi'-\phi')$

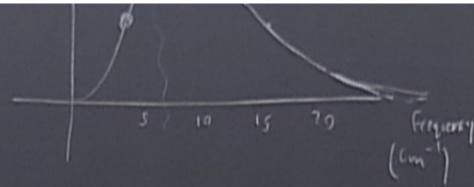
$$\delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

$$a_{\ell m} = \int d\Omega\, Y^*_{\ell m}(\hat{n})\,\delta T(\hat{n})$$

δT is real $\Rightarrow a^*_{\ell m} = (-1)^m a_{\ell,-m}$ $\left(Y^*_{\ell m}(\hat{n}) = (-1)^m Y_{\ell,-m}(\hat{n}) \right)$

$$\boxed{C_\ell^{obs} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2}$$

$\frac{\ldots}{\text{time} \cdot \text{area} \cdot \text{solid angle}} = \frac{4\pi\nu}{c^2}\,\frac{1}{e^{h\nu/k_BT}-1}$



- PLANCK 2009

• Temperature $T(\hat n)$

Fluctuation $\delta T(\hat n) = \frac{\Delta T}{T_0} = \frac{T-T_0}{T_0}$, $T_0 = \frac{1}{4\pi}\int d\Omega\, T(\hat n)$

10^{-5}

$$\int d\Omega\; Y^*_{\ell' m'}(\hat n)\, Y_{\ell m}(\hat n) = \delta_{\ell\ell'}\delta_{mm'}$$

$$\sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} Y^*_{\ell m}(\hat n)\, Y_{\ell m}(\hat n') = \delta^2(\hat n,\hat n') = \frac{1}{\sin\theta}\,\delta(\theta-\theta')\,\delta(\phi-\phi')$$

$$\delta T(\hat n) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat n)$$

$$a_{\ell m} = \int d\Omega\; Y^*_{\ell m}(\hat n)\,\delta T(\hat n)$$

δT is real $\Rightarrow$ $a^*_{\ell m} = (-1)^m a_{\ell,-m}$ $\left(Y^*_{\ell m}(\hat n) = (-1)^m Y_{\ell,-m}(\hat n) \right)$

$$\boxed{C_\ell^{obs} = \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2}$$

$$Y_{\ell m}(R\hat n) = D^{(\ell)}_{m' m}(R^{-1})\, Y_\ell^{m'}(\hat n)$$

$$\Theta \sim \frac{\pi}{\ell}$$

$$\langle a_{lm} a^*_{l'm'} \rangle = \delta_{ll'} \delta_{mm'} C_l$$

$$\frac{4\pi}{2l+1} \sum_m Y_{lm}(\hat{n}) Y^*_{lm}(\hat{n})$$

relate C_l^{obs}, C_l

$$\langle C_l^{obs} \rangle = \left\langle \frac{1}{2l+1} \sum_m a^*_{lm} a_{lm} \right\rangle = \frac{1}{2l+1} \sum_m \underbrace{\langle a_{lm} a^*_{lm} \rangle}_{\delta_{ll} \delta_{mm} C_l} = C_l$$

(cm^{-1})

$$\left\langle 1 - 2\frac{C_\ell^{obs}}{C_\ell} + \left(\frac{C_\ell^{obs}}{C_\ell}\right)^2 \right\rangle = 1 - 2\frac{1}{C_\ell}\underbrace{\langle C_\ell^{obs}\rangle}_{C_\ell} + \frac{1}{C_\ell^2}\langle C_\ell^{obs\,2}\rangle$$

$$= -1 + \frac{C_\ell^2}{C_\ell^2(2\ell+1)^2}\left[(2\ell+1)^2 + (2\ell+1) + (2\ell+1)\right]$$

$$= \boxed{\frac{2}{2\ell+1}}$$

$$\underbrace{\langle a^*_{\ell m}\, a_{\ell m'}\, a_{\ell m'} \rangle}$$

$$\langle a_{\ell m} a^*_{\ell m}\rangle\langle a_{\ell m'} a^*_{\ell m'}\rangle = C_\ell^2\, \delta_{mm}\delta_{m'm'}$$

$$\langle a_{\ell m} a_{\ell m'}\rangle\langle a^*_{\ell m} a^*_{\ell m'}\rangle = C_\ell^2\, \delta_{mm'}\delta_{mm'}$$

$$\langle a_{\ell m} a^*_{\ell m'}\rangle\langle a^*_{\ell m} a_{\ell m'}\rangle = C_\ell^2\, \delta_{mm'}\delta_{mm'}$$

(C_m^{-1})

$$\left\langle 1 - 2\frac{C_\ell^{obs}}{C_\ell} + \left(\frac{C_\ell^{obs}}{C_\ell}\right)^2 \right\rangle = 1 - 2\frac{1}{C_\ell}\underbrace{\langle C_\ell^{obs}\rangle}_{C_\ell} + \frac{1}{C_\ell^2}\langle C_\ell^{obs\,2}\rangle$$

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$$\langle a_{\ell m} a^*_{\ell m'}\rangle\langle a^*_{\ell m} a_{\ell m'}\rangle = C_\ell^2\,\delta_{mm'}\delta_{mm'}$$

$\rangle = \delta_{\ell\ell'}\delta_{mm'} C_\ell$

$\frac{4\pi}{2\ell+1} \sum_m Y_{\ell m}(\hat{n}) Y_{\ell m}(\hat{n}')$

$=$ # of photons in $d^3p\, d^3q$, w/ prob of e^i = $e^i e^j f_{ij}$

$$e^i = \begin{cases} \delta^{ij} e_i e_j = 1 \\ \delta_{ij} e^i p^j = 0 \end{cases}$$

$f_{ij}(\vec{q} = \text{earth}, -E\hat{n}, t = \text{now})$ $\quad p_j f^{ij} = 0$, f_{ij} is a symmetric tensor on sky