Title: Condensed Matter Review-13

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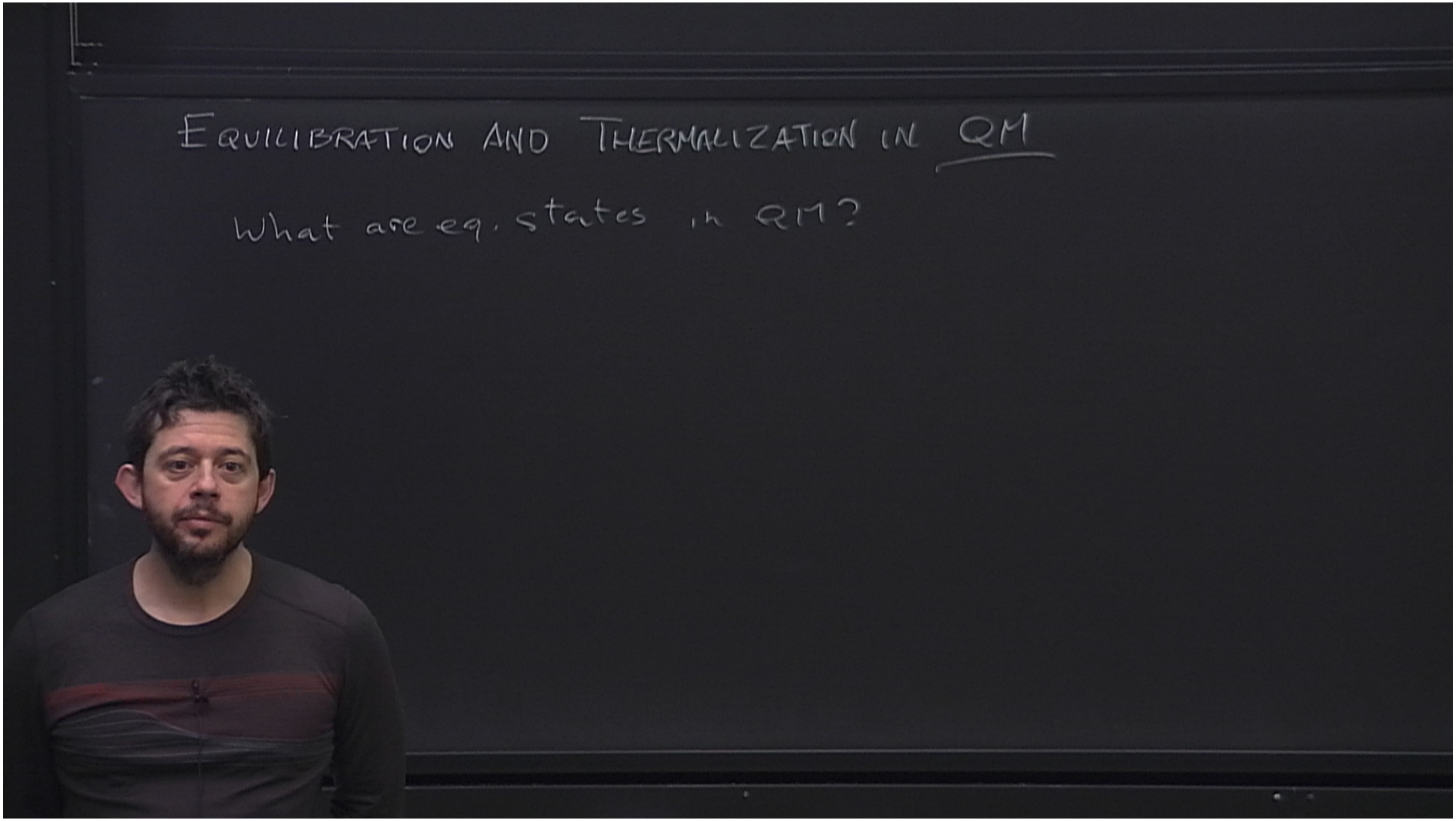
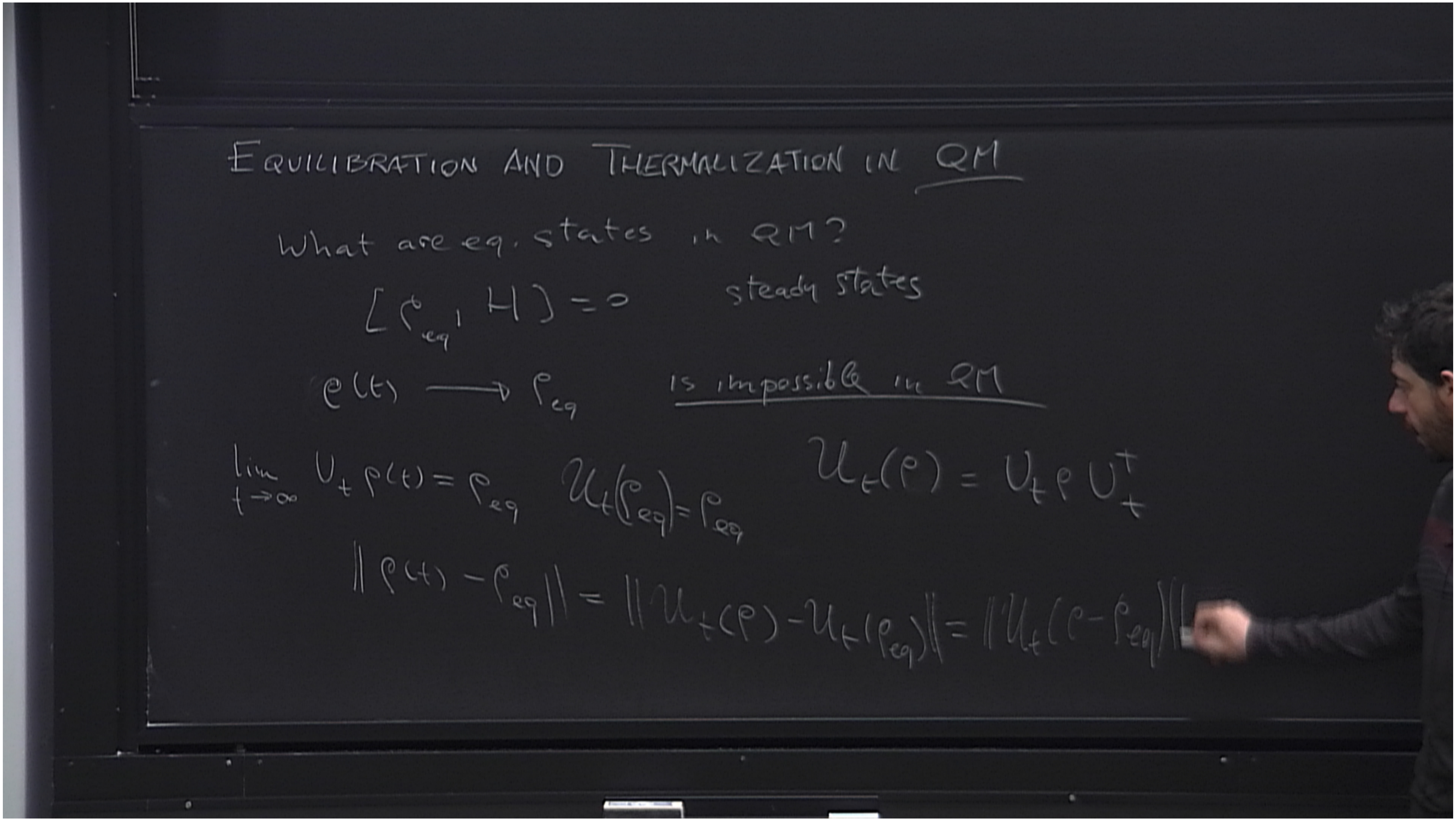
Abstract:


EQUILIBRATION AND THERMALIZATION IN QM


EQUILIBRATION AND THERMALIZATION IN QM
What are eq. states in QM?


EQUILIBRATION AND THERMALIZATION IN QM
What are eq. states in QM?
$[\rho_{eq}, H] = 0$ steady states
$\rho(t) \longrightarrow \rho_{eq}$ is impossible in QM
$\lim_{t\to\infty} U_t \rho(t) = \rho_{eq}$ $\mathcal{U}_t(\rho_{eq}) = \rho_{eq}$
$\mathcal{U}_t(\rho) = U_t \rho U_t^\dagger$
$\|\rho(t) - \rho_{eq}\| = \|\mathcal{U}_t(\rho) - \mathcal{U}_t(\rho_{eq})\| = \|\mathcal{U}_t(\rho - \rho_{eq})\|$

EQUILIBRATION AND THERMALIZATION IN QM

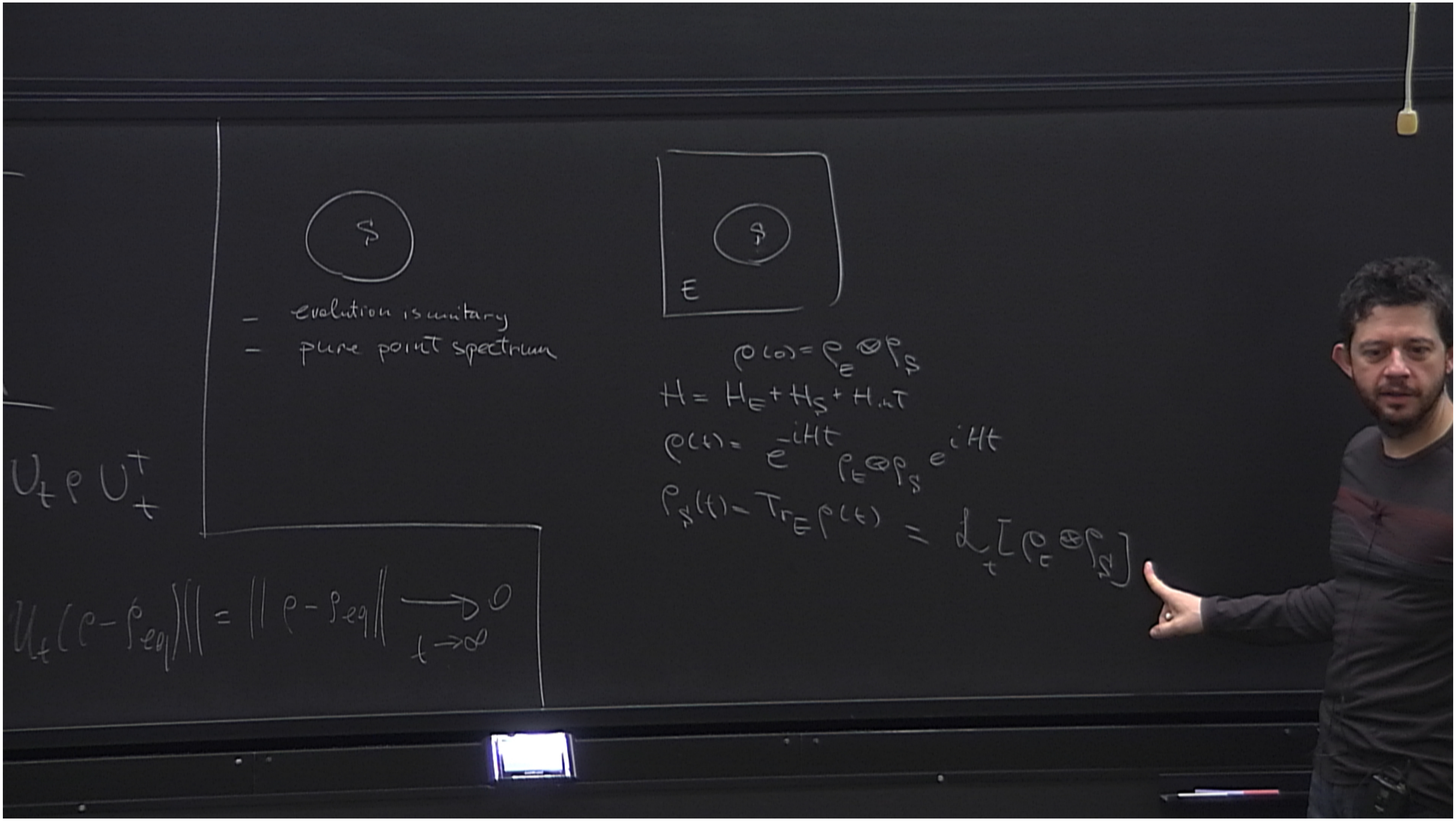
What are eq. states in QM?

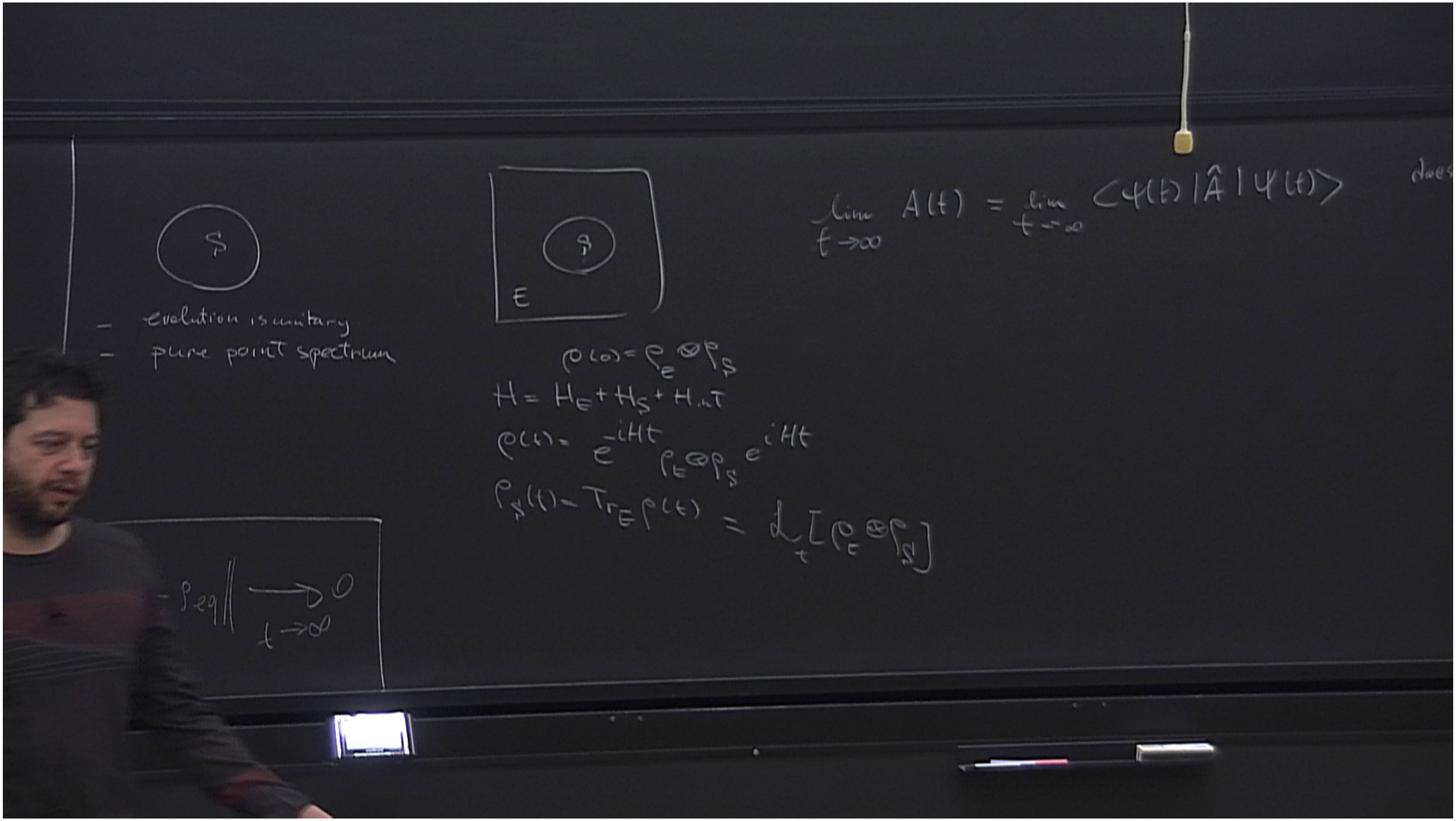
$$[\rho_{eq}, H] = 0 \qquad \text{steady states}$$

$\rho(t) \longrightarrow \rho_{eq}$ is impossible in QM

$$\lim_{t\to\infty} U_t\, \rho(t) = \rho_{eq} \qquad \mathcal{U}_t(\rho_{eq}) = \rho_{eq} \qquad \mathcal{U}_t(\rho) = U_t\, \rho\, U_t^\dagger$$

$$\|\rho(t) - \rho_{eq}\| = \|\mathcal{U}_t(\rho) - \mathcal{U}_t(\rho_{eq})\| = \|\mathcal{U}_t(\rho - \rho_{eq})\| = \|\rho - \rho_{eq}\| \xrightarrow[t\to\infty]{} 0$$


S
- evolution is unitary
- pure point spectrum
S
E
ρ(0) = ρ_E ⊗ ρ_S
H = H_E + H_S + H_int
ρ(t) = e^{-iHt} ρ_E ⊗ ρ_S e^{iHt}
ρ_S(t) = Tr_E ρ(t) = L_t[ρ_E ⊗ ρ_S]
U_t ρ U_t^†
U_t(ρ − ρ_eq)|| = ||ρ − ρ_eq|| → 0
t → ∞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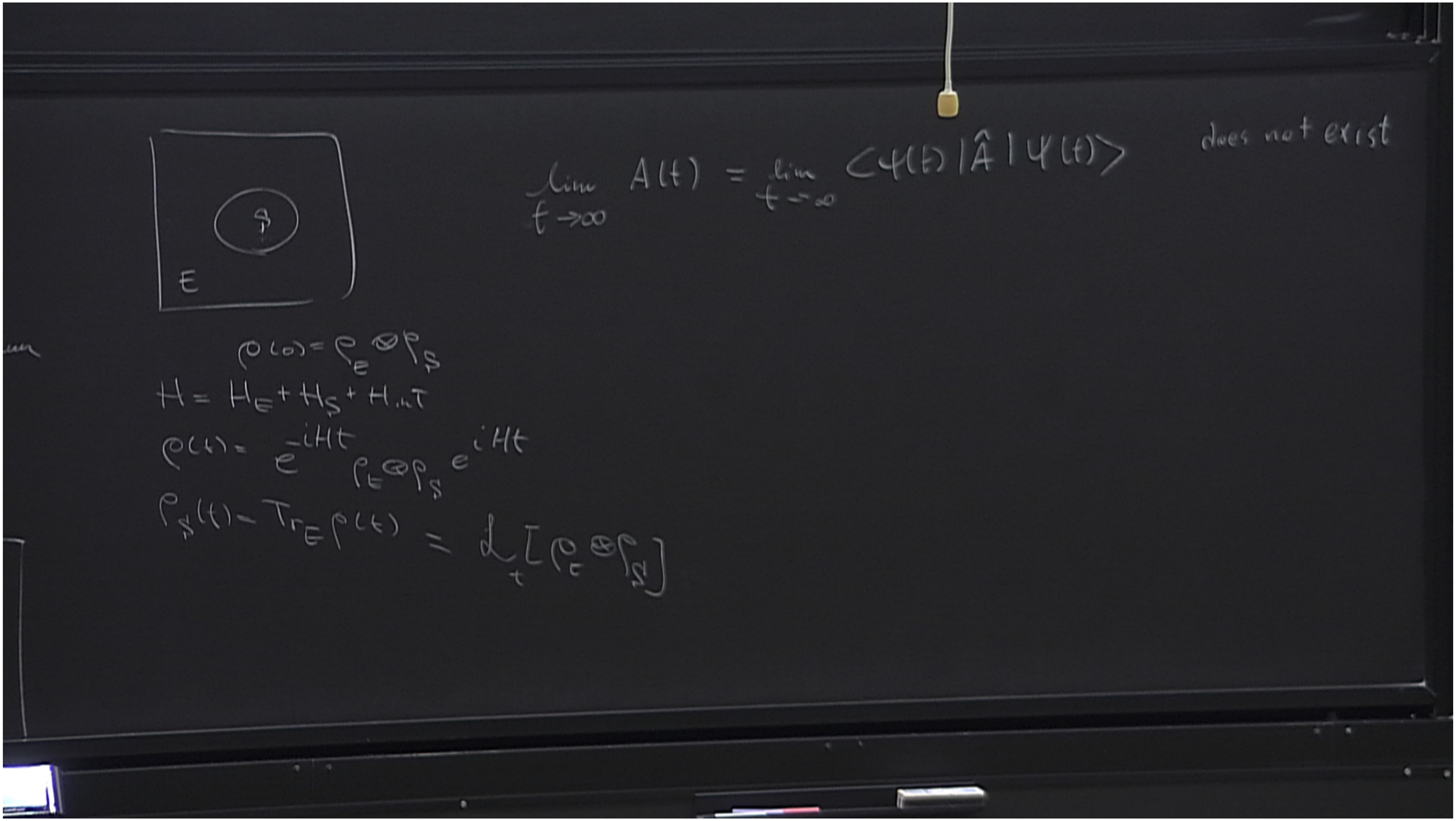
ρ_S(t) = Tr_E ρ(t) = L_t[ρ_E ⊗ ρ_S]
lim_{t→∞} A(t) = lim_{t→∞} <ψ(t)|Â|ψ(t)>
t→∞


does not exist
lim A(t) = lim ⟨ψ(t)|Â|ψ(t)⟩
t→∞
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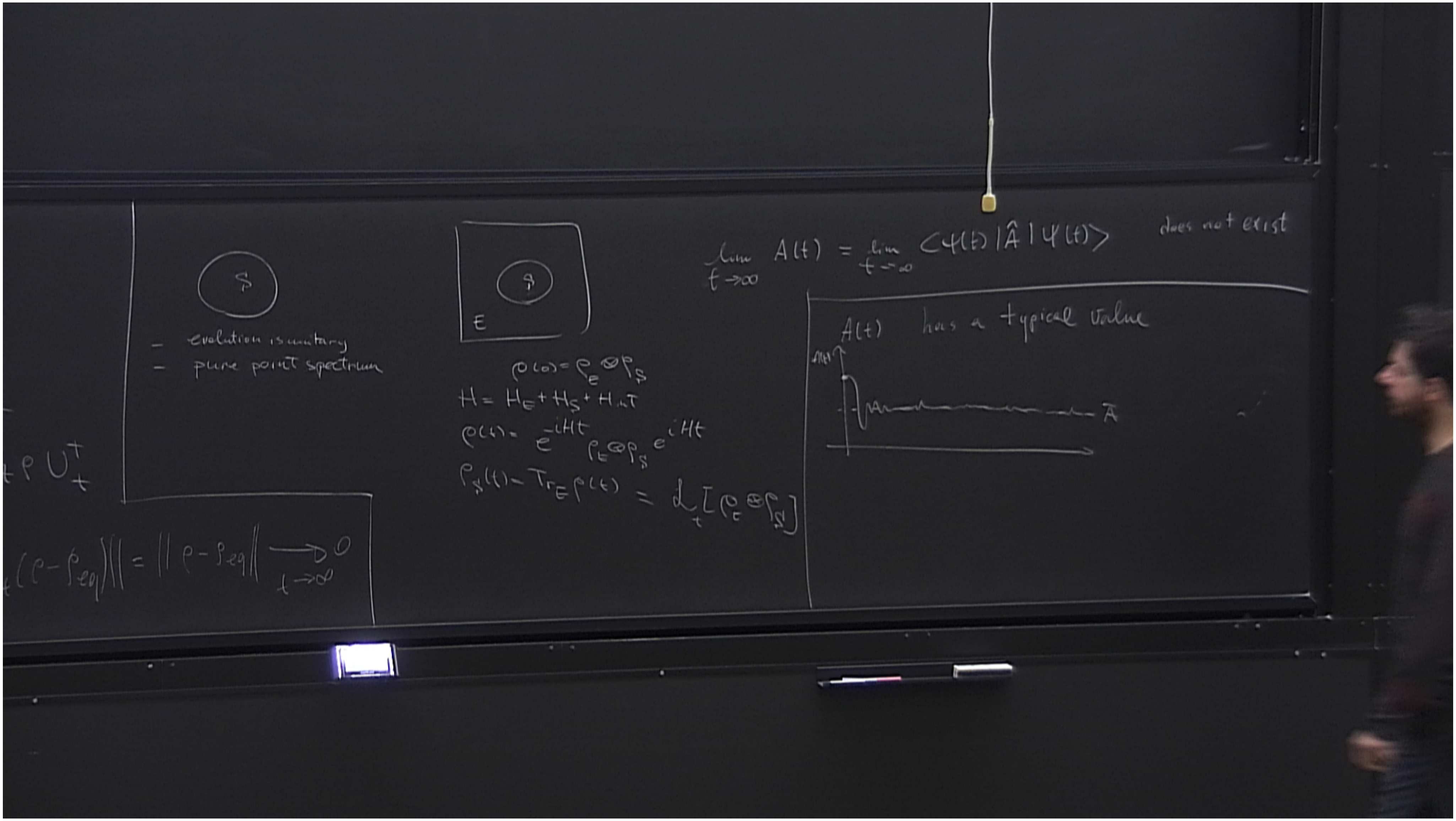
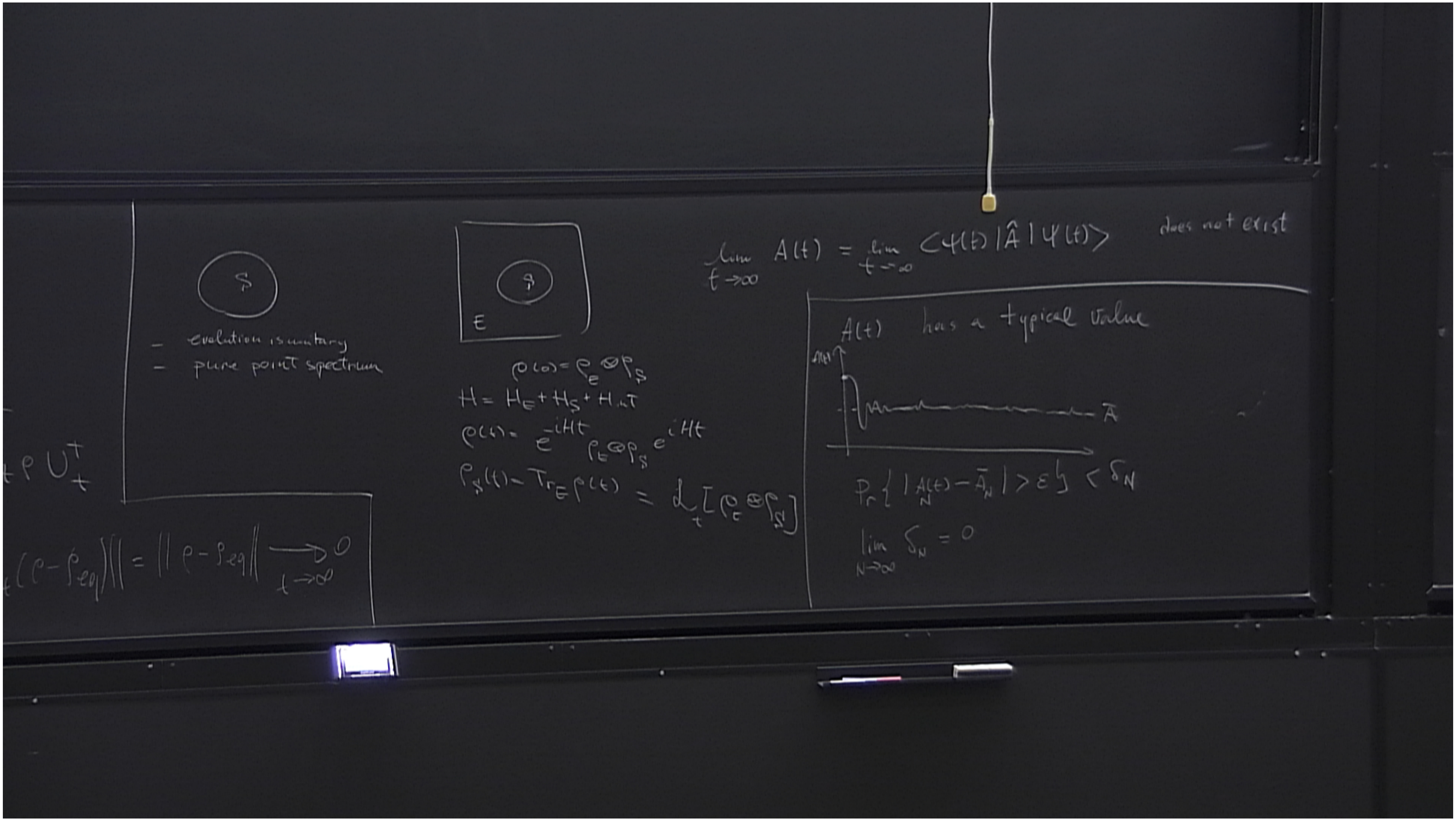

does not exist
A(t) has a typical value
evolution is unitary
pure point spectrum
E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– evolution is unitary
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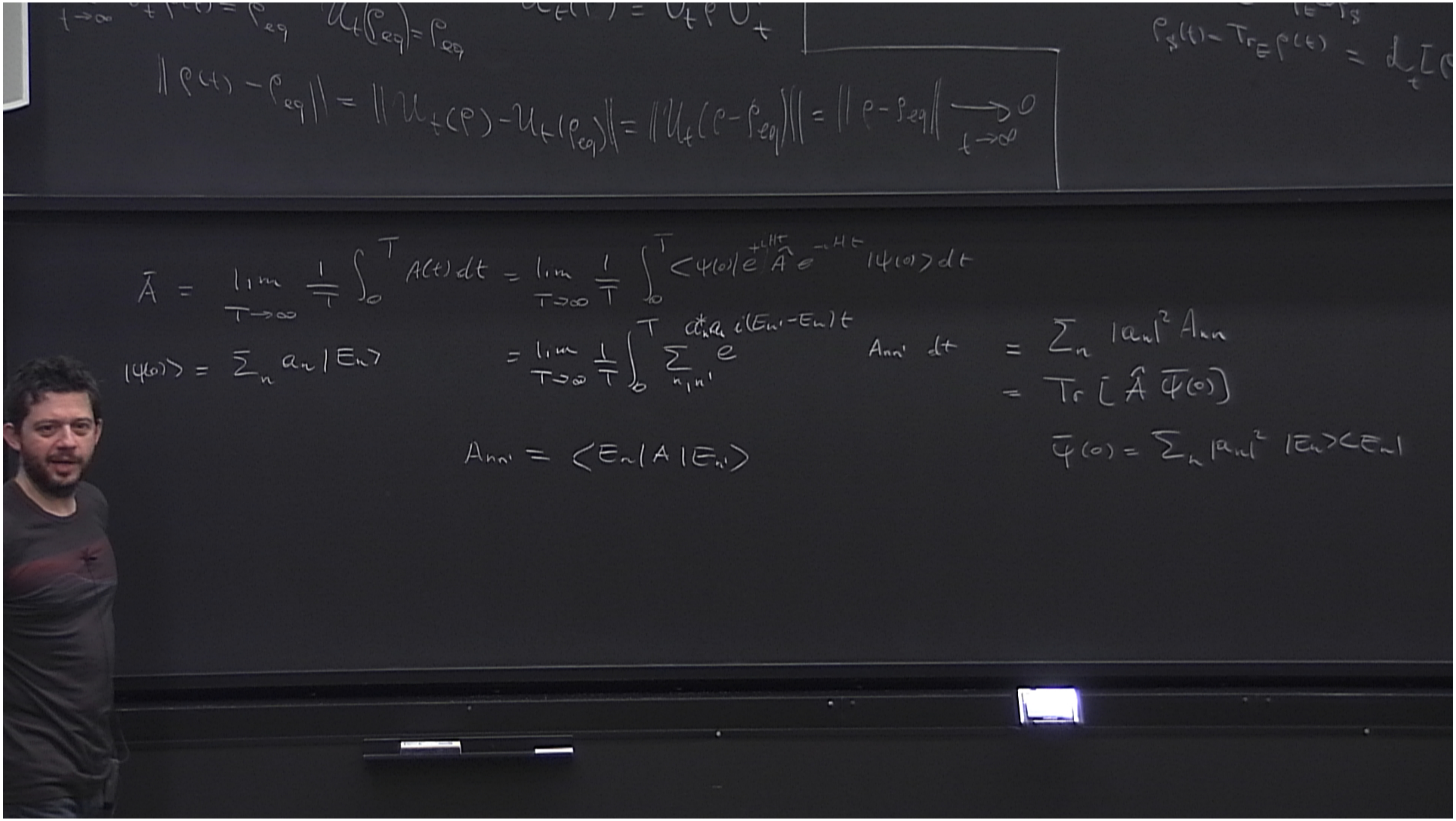
H = H_E + H_S + H_INT
ρ(t) = e^{-iHt} ρ_E ⊗ ρ_S e^{iHt}
ρ_S(t) = Tr_E ρ(t) = L_t [ρ_E ⊗ ρ_S]
lim A(t) = lim ⟨ψ(t)|Â|ψ(t)⟩
t→∞
does not exist
A(t) has a typical value
Ā
Pr{ |A_N(t) − Ā_N| > ε } < δ_N
lim δ_N = 0
N→∞
|| ρ − ρ_eq || → 0
t→∞



||ρ(t) − ρ_eq|| = ||U_t(ρ) − U_t(ρ_eq)|| = ||U_t(ρ − ρ_eq)|| = ||ρ − ρ_eq|| → 0 (t→∞)
Ā = lim_{T→∞} 1/T ∫_0^T A(t) dt = lim_{T→∞} 1/T ∫_0^T <ψ(0)| e^{+iHt} Â e^{−iHt} |ψ(0)> dt
|ψ(0)> = Σ_n a_n |E_n>
= lim_{T→∞} 1/T ∫_0^T Σ_{n,n'} a*_n a_{n'} e^{i(E_n' − E_n)t} A_{nn'} dt = Σ_n |a_n|^2 A_nn
= Tr [Â Ψ̄(0)]
A_{nn'} = <E_n| A |E_n'>
Ψ̄(0) = Σ_n |a_n|^2 |E_n><E_n|

$$\lim_{t\to\infty} U_t\,\rho(t) = \rho_{eq} \qquad \mathcal{U}_t(\rho_{eq}) = \rho_{eq} \qquad \mathcal{U}_t(\rho) = U_t\,\rho\,U_t^\dagger$$

$$\|\rho(t) - \rho_{eq}\| = \|\mathcal{U}_t(\rho) - \mathcal{U}_t(\rho_{eq})\| = \|\mathcal{U}_t(\rho - \rho_{eq})\| = \|\rho - \rho_{eq}\| \xrightarrow[t\to\infty]{} 0$$

$$H = H_E + H_S + H_{int}$$

$$\rho(t) = e^{-iHt}\,\rho_E \otimes \rho_S\, e^{iHt}$$

$$\rho_S(t) = \mathrm{Tr}_E\,\rho(t) = \mathcal{L}_t[\rho_E \otimes \rho_S]$$

$$\bar{A} = \lim_{T\to\infty} \frac{1}{T}\int_0^T A(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_0^T \langle\psi(0)|e^{+iHt}\hat{A}\,e^{-iHt}|\psi(0)\rangle\,dt$$

$$|\psi(0)\rangle = \sum_n a_n |E_n\rangle$$

$$= \lim_{T\to\infty}\frac{1}{T}\int_0^T \sum_{n,n'} a_n^* a_{n'}\, e^{i(E_{n'}-E_n)t} A_{nn'}\,dt = \sum_n |a_n|^2 A_{nn}$$

$$= \mathrm{Tr}[\hat{A}\,\bar{\Psi}(0)]$$

$$|\psi(0)\rangle\langle\psi(0)| = \begin{pmatrix} |a_1|^2 & a_1^* a_2 & \cdots \\ a_2^* a_1 & \ddots & \\ \vdots & & \\ \vdots & & |a_n|^2 \end{pmatrix}$$

$$A_{nn'} = \langle E_n | A | E_{n'}\rangle$$

$$\bar{\Psi}(0) = \sum_n |a_n|^2\, |E_n\rangle\langle E_n|$$

$$\bar{\Psi}(0) = \begin{pmatrix} |a_1|^2 & & 0 \\ & \ddots & \\ 0 & & |a_N|^2 \end{pmatrix}$$ ← completely dephased state

$$\lim_{N\to\infty} \delta_N = 0$$

$$= \sum_n |a_n|^2 A_{nn}$$

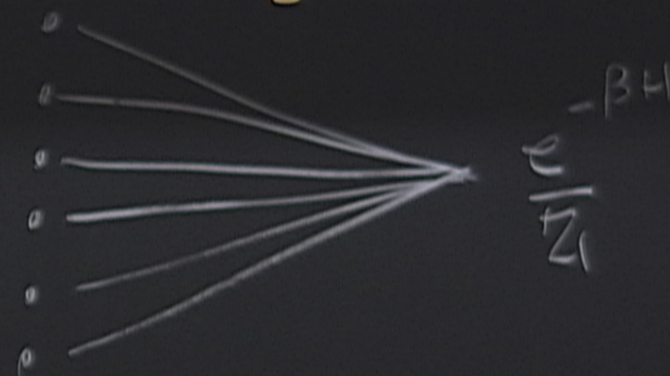
$$= \mathrm{Tr}[\hat{A}\, \bar{\Psi}(0)]$$

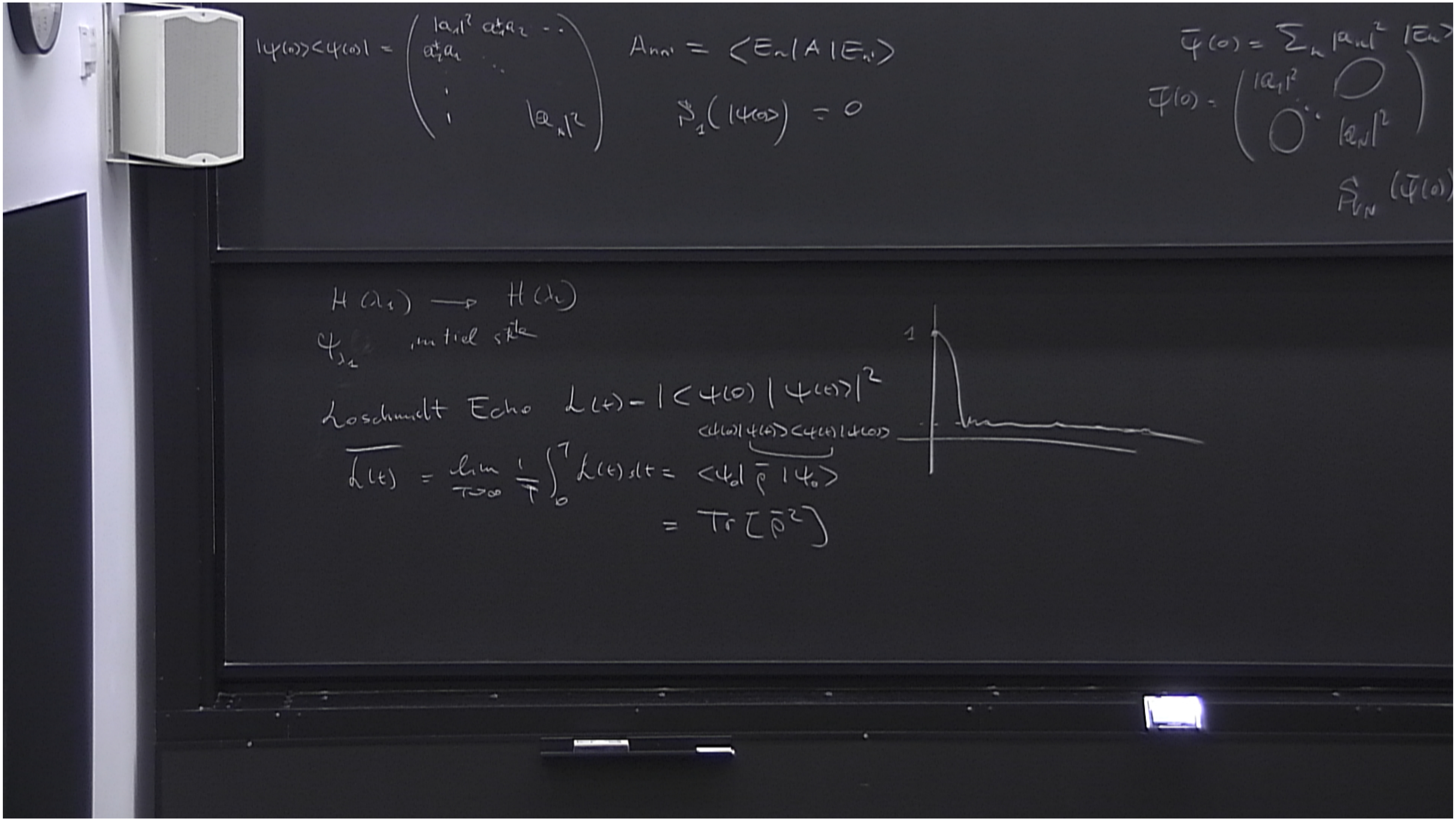
$$\bar{\Psi}(0) = \sum_n |a_n|^2\, |E_n\rangle\langle E_n|$$

$$\bar{\Psi}(0) = \begin{pmatrix} |a_1|^2 & & 0 \\ & \ddots & \\ 0 & & |a_N|^2 \end{pmatrix}$$

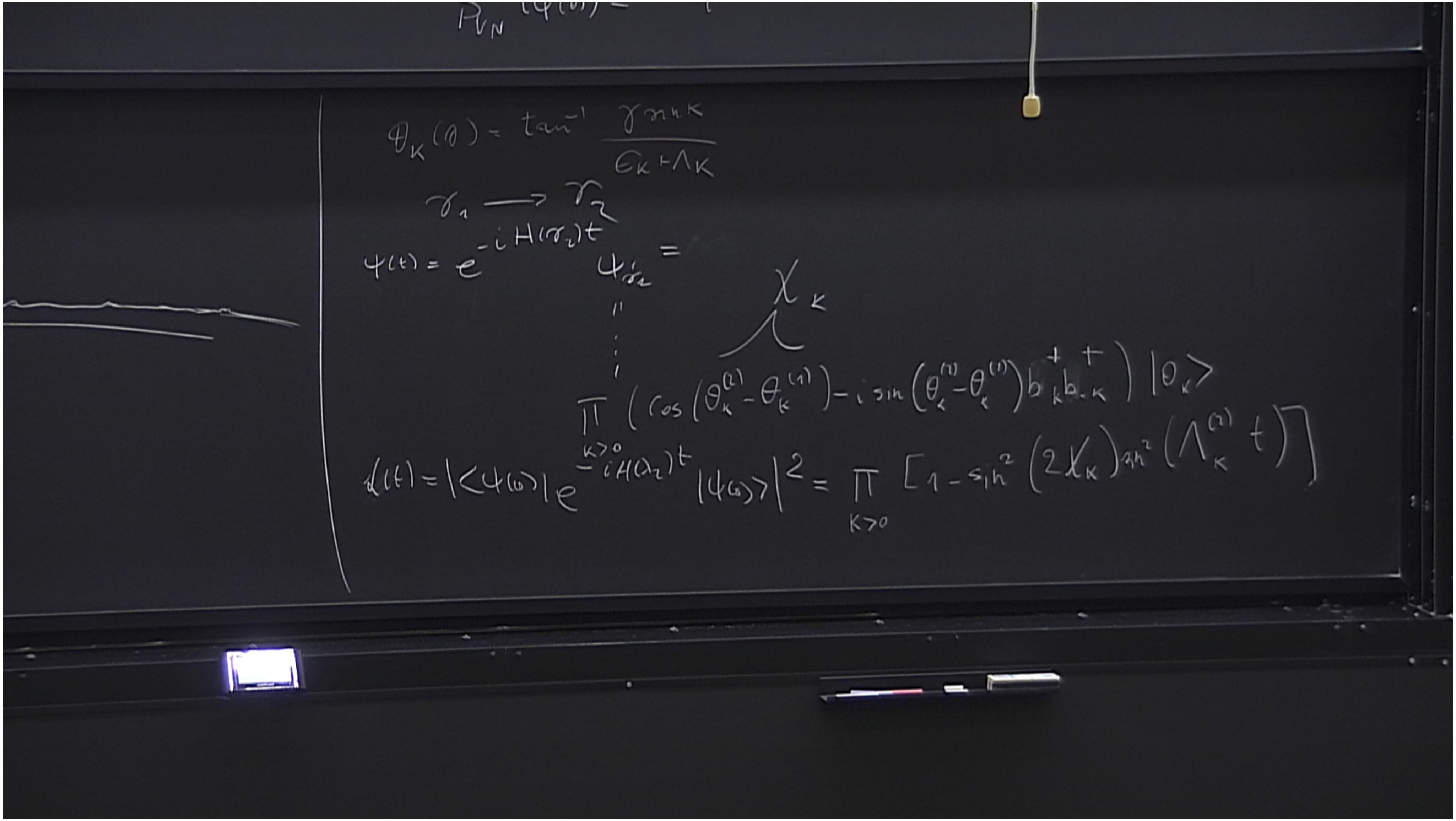
completely dephased state

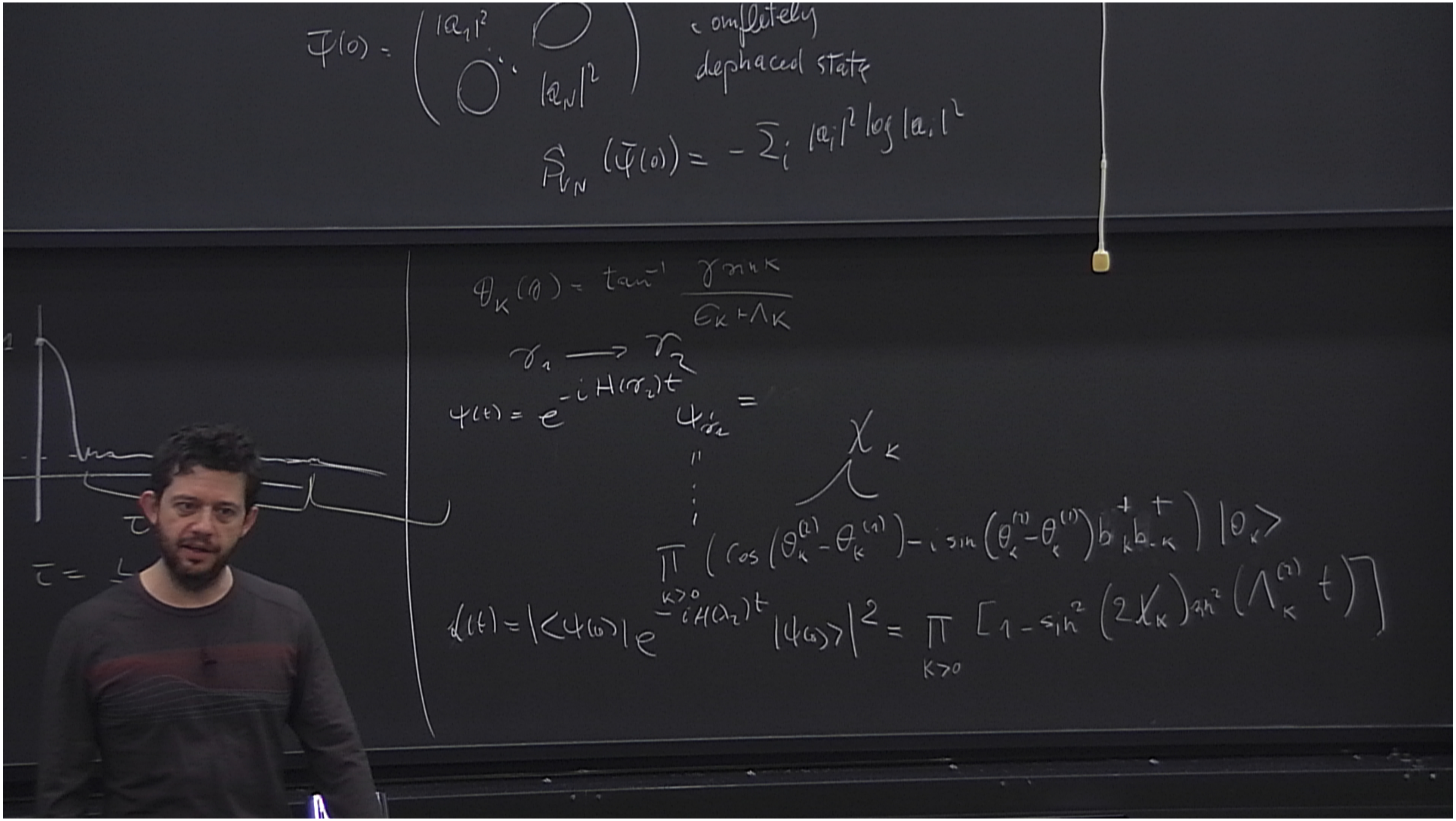
$$S_{VN}(\bar{\Psi}(0)) = -\sum_i |a_i|^2 \log |a_i|^2$$





$|\psi(0)\rangle\langle\psi(0)| = \begin{pmatrix} |a_1|^2 & a_1^* a_2 & \cdots \\ a_2^* a_1 & \ddots & \\ \vdots & & \\ \vdots & & |a_n|^2 \end{pmatrix}$
$A_{nn'} = \langle E_n | A | E_{n'} \rangle$
$S_1(|\psi(0)\rangle) = 0$
$\bar{\psi}(0) = \sum_n |a_n|^2 \, |E_n\rangle$
$\bar{\psi}(0) = \begin{pmatrix} |a_1|^2 & 0 \\ 0 & |a_N|^2 \end{pmatrix}$
$S_{VN}(\bar{\psi}(0))$
$H(\lambda_1) \longrightarrow H(\lambda_2)$
ψ_{λ_1} initial state
Loschmidt Echo $\mathcal{L}(t) = |\langle\psi(0)|\psi(t)\rangle|^2$
$\langle\psi(0)|\psi(t)\rangle\langle\psi(t)|\psi(0)\rangle$
$\overline{\mathcal{L}(t)} = \lim_{T\to\infty} \frac{1}{T}\int_0^T \mathcal{L}(t)\,dt = \langle\psi_0|\bar{\rho}|\psi_0\rangle$
$= \mathrm{Tr}[\bar{\rho}^2]$
1


θ_K(γ) = tan⁻¹ γ sin K / (ε_K + Λ_K)
γ₁ ⟶ γ₂
ψ(t) = e^{-iH(γ₂)t} ψ_{γ₂} =
Π_{K>0} (cos(θ_K^{(2)} − θ_K^{(1)}) − i sin(θ_K^{(2)} − θ_K^{(1)}) b†_K b†_{−K}) |0_K⟩
χ_K
L(t) = |⟨ψ(0)| e^{−iH(λ₂)t} |ψ(0)⟩|² = Π_{K>0} [1 − sin²(2χ_K) sin²(Λ_K^{(2)} t)]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