

Title: Condensed Matter Review-11

Date: Feb 09, 2015 09:00 AM

URL: <http://pirsa.org/15020008>

Abstract:

$$H_{TC} = -U \sum_n A_n - J \sum_p B_p$$

$$GS = \mathcal{L} = \{ \psi \in \mathcal{H} \mid A_n \psi = B_p \psi = \psi \quad \forall n, p \}$$

$$= \text{span} \{ \psi_0, W_1^x \psi_0, W_2^x \psi_0, W_1^x W_2^x \psi_0 \}$$

$$|g\rangle = g |1 \dots 1\rangle$$

$$\psi_0 = \left\{ A_n = +1; B_p = +1; W_1^z = W_2^z = +1 \right\} = \frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$$

$$\{ W_i^x, W_i^z \} = 0$$

$$G = \langle A_1 \dots A_{L-1} \rangle; \quad G^* = \langle B_1 \dots B_{L-1} \rangle$$

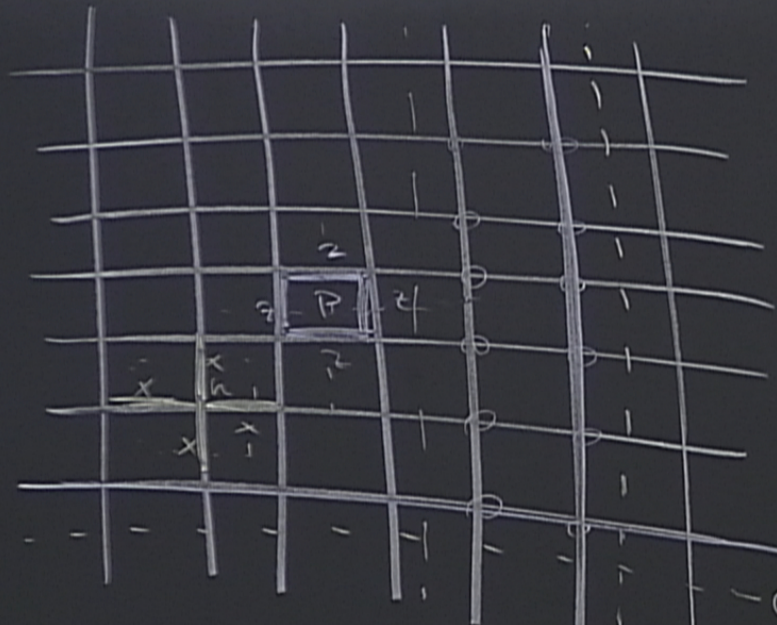
$$|G| = 2^{L^2 - 1}$$

$\gamma$  is closed and contractible

$$W(\gamma) = \prod_{j \in \gamma} \sigma_{i,j}^{x,z}$$

$$W(\gamma) \in G$$





$$\tilde{W}_c = \tilde{g} W_c$$

$$W_1^z, W_2^x$$

$$-c_1, W_2^x$$

$$H_{TC} = -U \sum_n A_n - J \sum_p B_p$$

$$GS = \mathcal{L} = \{ \psi \in \mathcal{H} \mid A_n \psi = B_p \psi = \psi \}$$

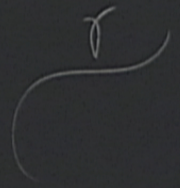
$$= \text{span} \{ \psi_0, W_1^x \psi_0, W_2^x \psi_0, W_1^x W_2^x \psi_0 \}$$

$$\psi_0 = \{ A_n = +1, B_p = +1, W_1^z = W_2^z = +1 \}$$

$$\{ W_i^x, W_i^z \} = 0$$

$$G = \langle A_0, \dots, A_{2L-1} \rangle$$

$$|G| = 2^{L^2 - 1}$$



$$W(\sigma) = \prod_{j \in \sigma} \sigma_{j,j}^{x,z}$$



When  $g$  is open

$$W_g^{1,2} |\psi_0\rangle =$$

$$= \int_{\Sigma} \frac{1}{|\mathcal{K}|} \bar{z}_s |\psi\rangle$$



$$\psi^{in} = W_{PQ} \psi_0$$

$$\psi^{out} = W_C W_{PQ} \psi_0$$

$$= W_{PQ} W_C \psi_0$$

$$\psi_0 = \psi^{in}$$

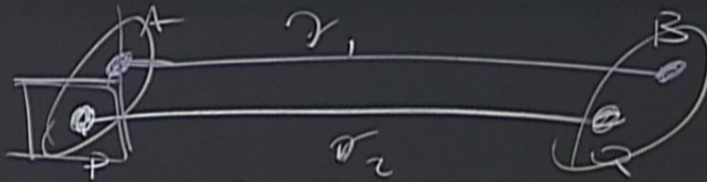
$$\psi^{in} = W_{\alpha_1} W_{\alpha_2} \psi_0$$

$$\psi^{out} = W_C W_{\alpha_1} W_{\alpha_2} \psi_0$$

$$= - W_{\alpha_1} W_{\alpha_2} \psi_0 = -\psi^{in}$$



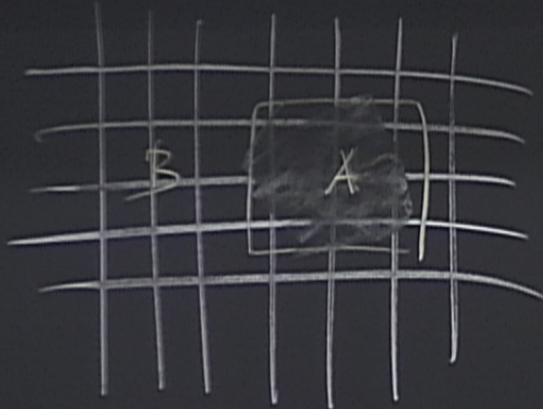
# EXERCISE



$$\psi^{\text{in}} = W_{\sigma_1(A)B}^z W_{\sigma_2(A)B}^x \psi_0$$

$$\psi^{\text{out}} \text{ (after swap) } = -\psi^{\text{in}}$$

$$\langle \psi_0 | \sigma_j^z | \psi_0 \rangle = 0$$



$$\phi \in \mathcal{L} \rightarrow \phi_A = \text{Tr}_B \phi$$

$$\phi = \sum_{i,j=0,1} \alpha_{ij} W_1^i W_2^j |\psi_0\rangle \rightarrow \phi_A = \text{Tr}_B \phi$$

$$\sum_{ij} |\alpha_{ij}|^2 = 1$$

$$\text{Tr}[\mathcal{O}_A \phi] = \text{Tr}[\mathcal{O}_A \phi_A]$$

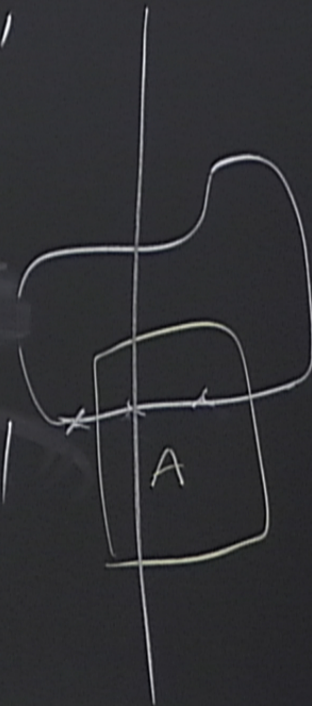


$$= \text{Tr}_B \phi$$

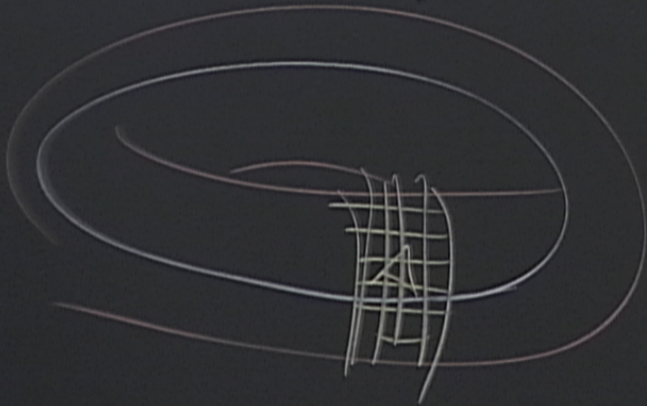
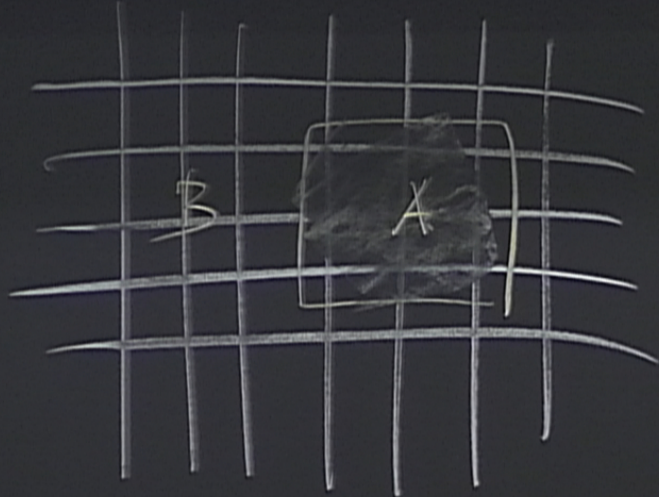
$$\psi_0 \rightarrow \phi_A = \text{Tr}_B \phi = \text{Tr}_B \sum_{i,j,i',j'} d_{ij} d_{i'j'}^* W_{i,j} W_{i',j'} \chi_{\psi_0} | \chi_{\psi_0} | W_{i,j} W_{i',j'}$$

$$= \frac{1}{|G|} \sum_{i,j,i',j'} \sum_{g,g' \in G} \langle i,j | g \rangle \langle g' | i',j' \rangle W_{i,j} W_{i',j'}$$

$$|g_A \rangle \langle g'_A| \otimes |g_B \rangle \langle g'_B|$$







$$\phi \in \mathcal{L} \rightarrow \phi_A = \text{Tr}_B \phi$$

$$|\phi\rangle = \sum_{i,j=0,1} \alpha_{ij} w_1^i w_2^j |\psi_0\rangle \rightarrow$$

$$\sum_{i,j} |\alpha_{ij}|^2 = 1$$

$$\text{Tr}_B = \sum_{i,j} \frac{1}{|Q|}$$

$$|\phi\rangle = \tilde{g}|\phi\rangle$$



$$\phi \in \mathcal{L} \rightarrow \phi_A = \text{Tr}_B \phi$$

$$|\phi\rangle = \sum_{i,j=0,1} \alpha_{ij} W_1^i W_2^j |4_0\rangle$$

$$\sum_{ij} |\alpha_{ij}|^2 = 1$$

$$\begin{aligned} \text{Tr}_B &= \frac{1}{|G|} \sum_{i,j} \sum_{g,g' \in G} \langle i,j | \alpha_{ij}^* \dots W_1^i W_2^j |g\rangle \langle g' | \dots \end{aligned}$$

$$|\phi\rangle = \tilde{g}|\phi\rangle$$

$$\text{Tr}_B \frac{1}{|G|} \sum \sum \langle \alpha | g_A \rangle \langle g'_A |$$

$$\langle g'_B | W_1^i W_2^j W_1^i W_2^j |g_B\rangle$$

$$|g_A\rangle \langle g'_A| \otimes |g_B\rangle \langle g'_B|$$



$$\phi \in \mathcal{L} \rightarrow \phi_A = \text{Tr}_B \phi$$

$$|\phi\rangle = \sum_{i,j=0,1} \alpha_{ij} w_1^i w_2^j | \psi_0 \rangle \rightarrow \phi_A = \text{Tr}_B \phi = \text{Tr}_B \sum_{i,j} \alpha_{ij} | \psi_0 \rangle \langle \psi_0 |$$

$$\sum_{ij} |\alpha_{ij}|^2 = 1$$

$$\begin{aligned} & \text{Tr}_B \sum_{i,j} \sum_{g,g' \in G} \alpha_{ij} \alpha_{i'j'}^* w_1^i w_2^j |g\rangle \langle g'| \\ &= \frac{1}{|G|} \sum_{i,j} \sum_{g,g' \in G} \alpha_{ij} \alpha_{i'j'}^* w_1^i w_2^j |g\rangle \langle g'| \end{aligned}$$

$$\phi_A = \frac{1}{|G|} \sum_g |g_A\rangle \langle g_A| \langle g_B | g_B \rangle$$

$$\langle g_B | w_1 w_2 w_1 w_2 | g_B \rangle$$



$$\frac{1}{\sqrt{2}} ( | \uparrow \uparrow \uparrow \uparrow \rangle + | \downarrow \downarrow \downarrow \downarrow \rangle )$$

$$\frac{1}{\sqrt{2}} ( | \uparrow \uparrow \downarrow \downarrow \rangle - | \downarrow \downarrow \uparrow \uparrow \rangle )$$

$$| \uparrow \uparrow \uparrow \uparrow \rangle \quad | \downarrow \downarrow \downarrow \downarrow \rangle$$

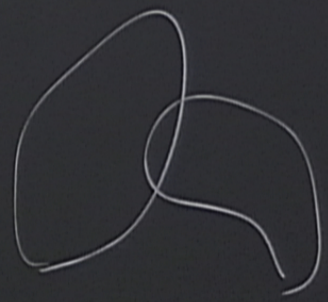
$$\frac{1}{|G|} \sum_{g \in G} \sum_{g' \in G_A} |B_n \rangle \langle g' |$$

$$|B_n \rangle \langle g' |$$

$$G_A = \{g \in G \mid g = g_A \otimes 1_B\}$$

$$\langle 0 | g'_B | 10 \rangle$$

$$g'_B = 1_B$$





$$\phi_A = \frac{1}{|G|} \sum_{\substack{g \in G \\ g' \in G_A}} |g_A\rangle \langle g'_A|$$

$$\phi_A^2 = \frac{1}{|G|^2} \sum_{\substack{g, \tilde{g} \in G \\ g', \tilde{g}' \in G_A}} |g_A\rangle \langle g'_A | \tilde{g}_A \rangle \langle \tilde{g}'_A |$$

$$G_A = \{g\}$$

$$\langle 0 | g'_B | 10 \rangle$$

$$g'_B =$$



$$1g_A \rangle \langle \tilde{g}'_A |$$

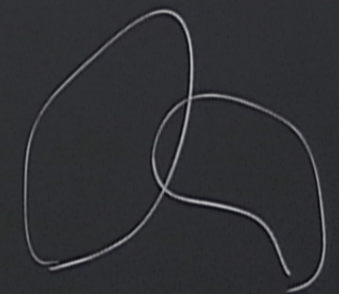
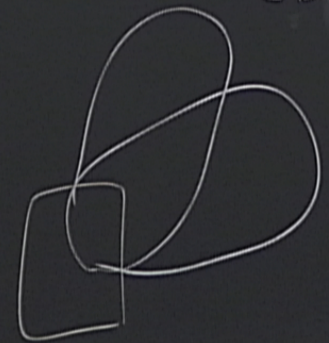
$$G_A = \{g \in G \mid g = g_A \otimes 1_B\}$$

$$G_B = \{g \in G \mid g = 1_A \otimes g_B\}$$

$$\langle \tilde{g}_A g'_A | \tilde{g}_A \rangle \langle \tilde{g}'_A |$$

$$\langle 0 | \tilde{g}'_B | 0 \rangle$$

$$\tilde{g}'_B = 1_B$$





$$\phi_A = \frac{|G_B|}{|G|} \sum_{\substack{g \in G/G_B \\ g' \in G_A}} |g_A\rangle \langle g'_A|$$

$$\phi_A^2 = \frac{|G_B|^2}{|G|^2} \sum_{\substack{g, \tilde{g} \in G_B \\ g', \tilde{g}' \in G_A}} |g_A\rangle \langle g_A g'_A | \tilde{g}_A \rangle \langle \tilde{g}_A \tilde{g}'_A |$$

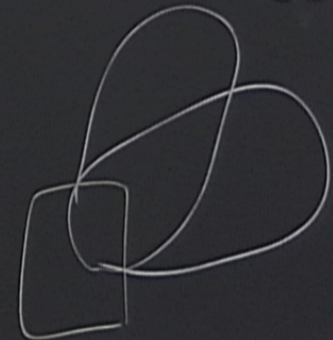
$$|g_A\rangle \langle g_A g'_A \tilde{g}'_A|$$

$$G_A = \{g \in G\}$$

$$G_B = \{g \in G\}$$

$$\langle 0 | \rho'_B | 0 \rangle$$

$$\rho'_B = \rho_B$$





$$\phi_A = \frac{|G_B|}{|G|} \sum_{\substack{g \in G/G_B \\ g' \in G_A}} |g_A\rangle \langle g'_A|$$

$$G_A = \{g \in G\}$$

$$G_B = \{g \in G\}$$

$$\phi_A^2 = \frac{|G_B|^2}{|G|^2} \sum_{\substack{g, \tilde{g} \in G_B \\ g^*, \tilde{g}' \in G_A}} |g_A\rangle \langle g_A g'_A | \tilde{g}_A \rangle \langle \tilde{g}_A \tilde{g}'_A |$$

$$\langle 0 | g'_B | 10 \rangle$$

$$g'_B | 10 \rangle = | 10 \rangle$$

$$\frac{|G| |G_B|^2 |G_A|}{|G_B| |G|^2} \sum_{\substack{g, \\ g^*}} |g_A\rangle \langle g_A g'_A | \tilde{g}'_A \rangle = \frac{|G_A| |G_B|}{|G|} \phi_A$$



$$|s_a\rangle \langle s'_a|$$

$G_B$

$$\langle g_A g'_A | \tilde{g}_A \rangle \langle \tilde{g}_A \tilde{g}'_A |$$

$$|g_A\rangle \langle g_A g'_A \tilde{g}_A | = \frac{|G_A| |G_B|}{|G|} \phi_A$$

$$G_A = \{g \in G \mid g = g_A \otimes 1_B\}$$

$$G_B = \{g \in G \mid g = 1_A \otimes g_B\}$$

$$\langle 0 | s'_B | 0 \rangle$$

$$s'_B = 1_B$$

$$\phi_A = K \begin{pmatrix} 1 & & & & & & & & \\ & 1 & & & & & & & \\ & & \ddots & & & & & & \\ & & & 1 & & & & & \\ & & & & 0 & & & & \\ & & & & & \ddots & & & \\ & & & & & & 0 & & \\ & & & & & & & & 0 \end{pmatrix}$$

$Kr = 1$



$$\sum_{T_\alpha} (\phi_A) = \frac{1}{1-\alpha} \log \kappa^{\alpha+1} = -\log \kappa = \log \frac{|G|}{|G_A| |G_B|}$$



$$\sum_{T_\alpha} (\psi_A) = \frac{1}{1-\alpha} \log \left( \kappa^{\alpha-1} = -\log \kappa = \log \frac{|G|}{|G_A| |G_B|} \right)$$