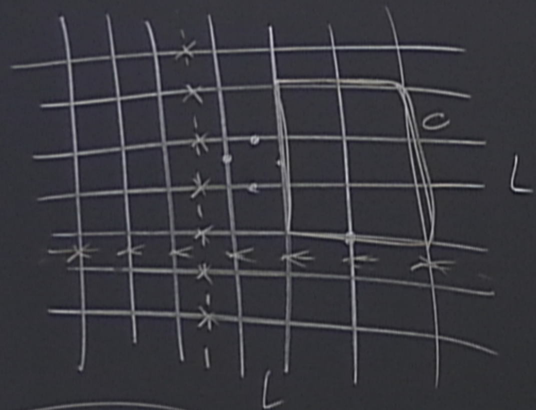
Title: Condensed Matter Review-10

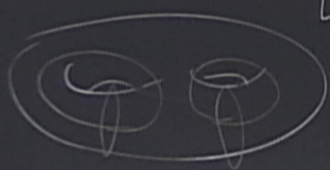
Date: Feb 06, 2015 09:00 AM

URL: http://pirsa.org/15020005

Abstract:

$\mathbb{Z}_2$ Quantum Lattice Gauge theory





$$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$$

$$A(n) = \prod_{i \in n} \hat{\sigma}_i^x$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid A(n)\psi = \psi \ \forall n \}$$

$$\hat{B}_p = \prod_{i \in p} \hat{\sigma}_i^z$$

$$H_{\mathbb{Z}_2} = -J \sum_p B_p - g \sum_i \hat{\sigma}_i^x$$

$\boxed{g=0}$ Gap $\Delta E = 4J$

degeneracy $= 4 = 2^{2g}$

$$W(C) = \prod_{i \in C} \hat{\sigma}_i^z$$

small g $\sim e^{-\alpha C}$

large g $\sim e^{-\beta A(C)}$

Gauge theory

$$\mathcal{H}_{TOT} \simeq (\mathbb{C}^2)^{\otimes 2L^2}$$

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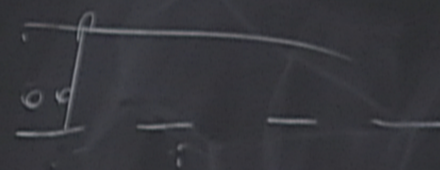


$g \ll J$

$$\langle \psi_{00} | \left(-g \sum_i \hat{\sigma}_i^x \right)^L | \psi_{01} \rangle = 0$$

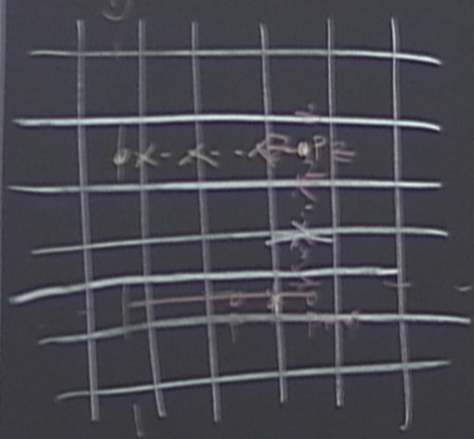
$$\delta E = \left(\frac{g}{J} \right)^L \sim e^{-L}$$

$\sim e^{-\alpha C}$

$\sim e^{-\beta A(C)}$

$|\psi_0\rangle = 0$

Duality



$$\left\{ \begin{array}{l} \hat\mu_1(p) \equiv \hat B_p \\ \hat\mu_3(p,\hat\nu) = \prod_{\ell=\sigma(\hat\nu)} \hat\sigma^x_{(\ell,\hat\nu)} \end{array} \right.$$

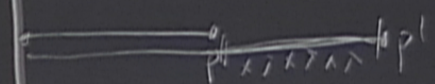
$\hat\nu = \hat x, \hat y$

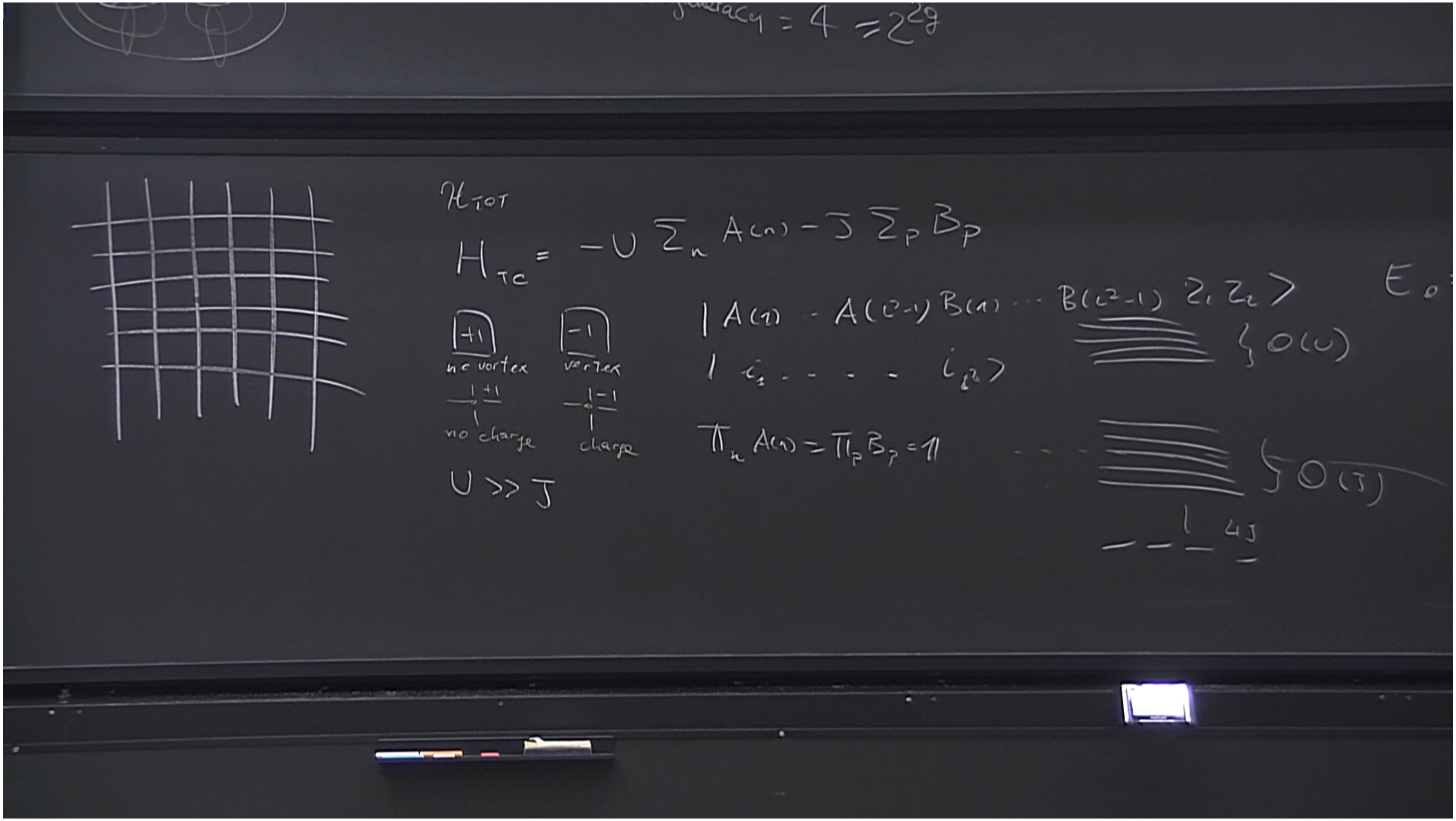
$\hat\mu_1^2 = \hat\mu_3^2 = 1$

$\{\hat\mu_1(p), \hat\mu_3(p)\} = 0$

$$H_{Z_2} \longrightarrow -J\sum_p \hat\mu_1(p) - g\sum_{p,\hat\nu} \mu_3(p)\,\mu_3(p+\hat\nu)$$

$$\langle \mu_3(p)\,\mu_3(p')\rangle_c \sim e^{-|p-p'|/\xi}$$





acy = 4 = 2^2g
H_TOT
H_TC = −U Σ_n A(n) − J Σ_p B_p
+1
−1
no vortex
vertex
no charge
charge
U >> J
| A(1) · A(L²−1) B(1) ··· B(L²−1) Z1 Z2 >
| i1 . . . . . i_L² >
Π_n A(n) = Π_p B_p = 1
E0
O(U)
O(J)
4J

$$\mathcal{L}_{GS} = \{ \psi \in \mathcal{H}_{TOT} \mid A_n\psi = \psi = B_p\psi \quad \forall n, p \}$$

$$|0\rangle \equiv |\uparrow \uparrow \cdots \uparrow\rangle$$

$$B_p|0\rangle = |0\rangle$$

$$G = \langle A(1), \dots, A(L^2) \rangle$$

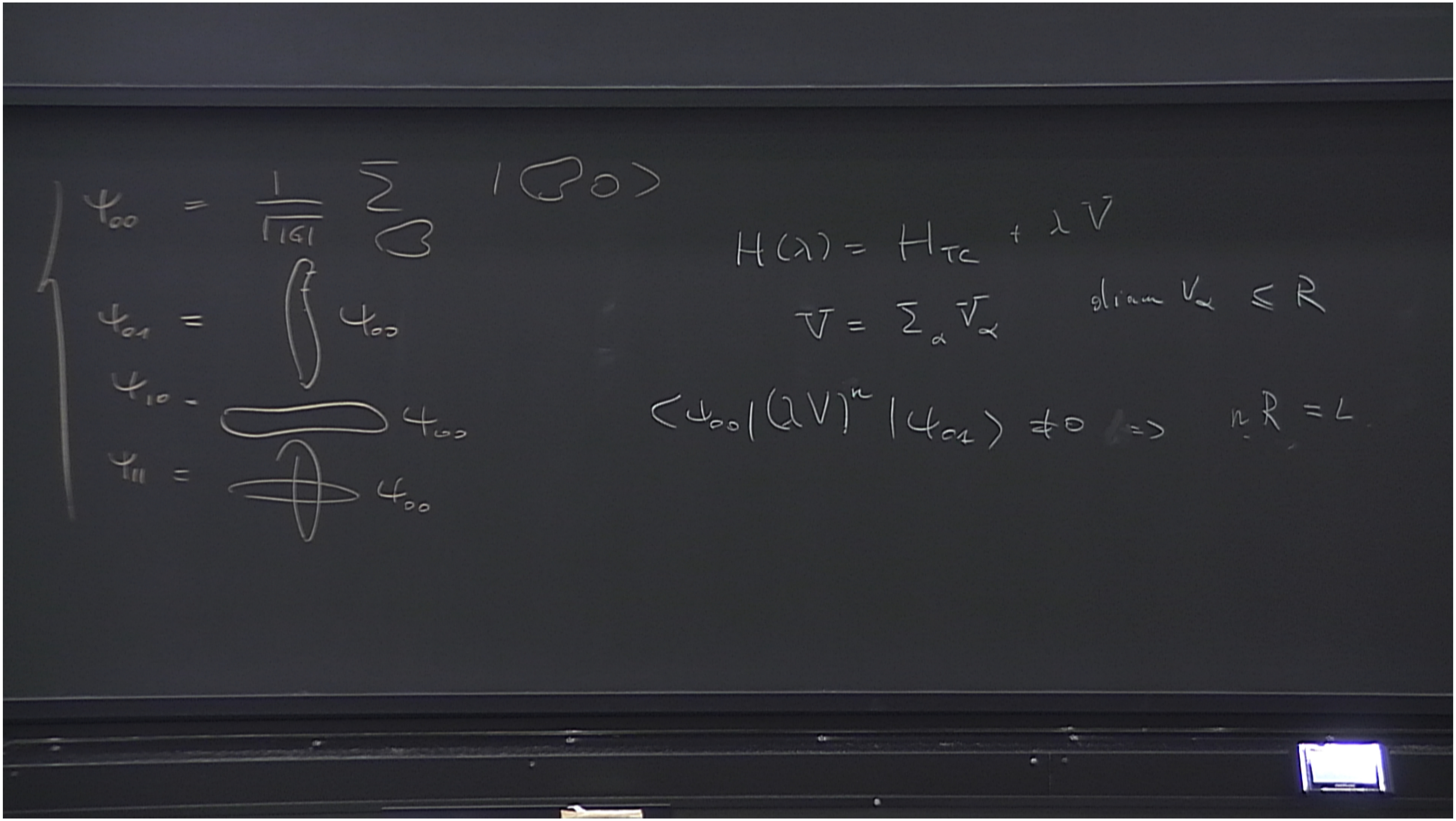
$$g \in G \qquad |G| = 2^{L^2-1}$$

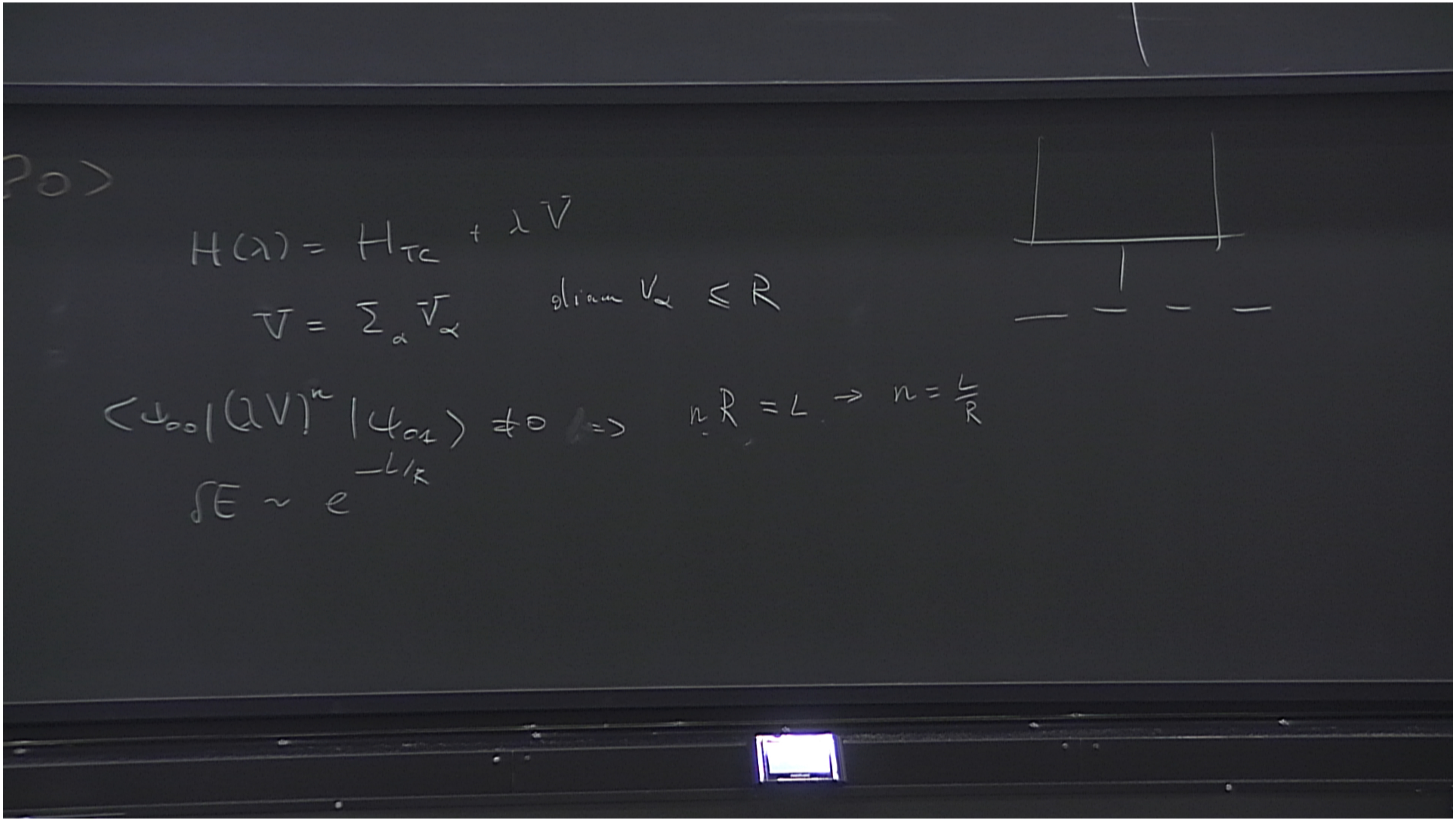
$$P = \prod_n \left(\frac{1\!\!1 + A(n)}{2} \right)$$

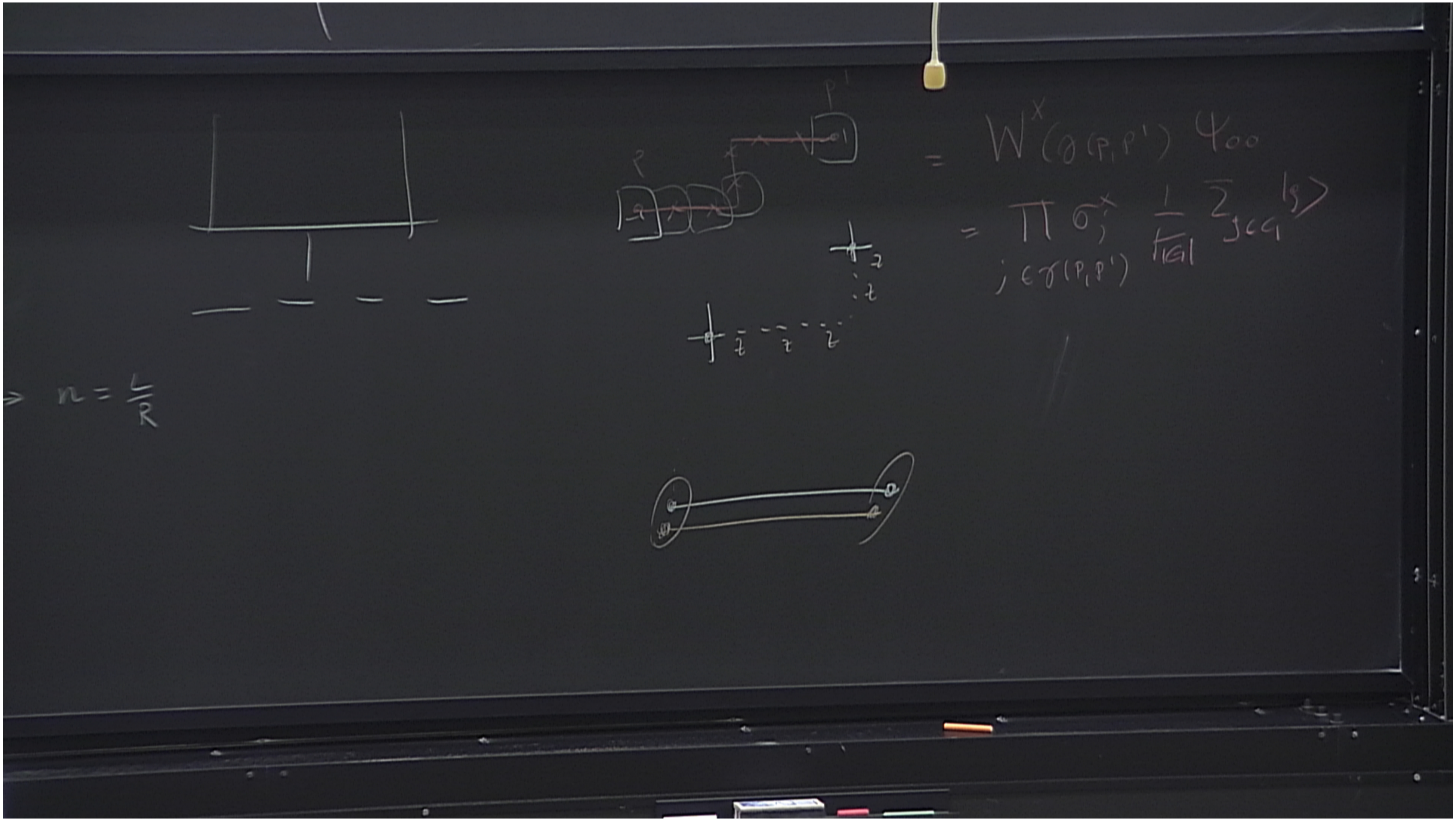
$$P|0\rangle = \prod_n \left(\frac{1\!\!1 + A(n)}{2} \right)|0\rangle \equiv \psi_{00}$$

$$= \frac{1}{N} \sum \left(1\!\!1 + A(n) + A(n)A(n') + \dots \right)|0\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g|0\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$$

$$g|0\rangle \equiv |g\rangle$$


$\Psi_{00} = \frac{1}{\sqrt{|G|}} \sum |\ 0\rangle$
$H(\lambda) = H_{TC} + \lambda V$
$V = \sum_\alpha V_\alpha$
diam $V_\alpha \leq R$
$\Psi_{01} = \Psi_{00}$
$\Psi_{10} = \Psi_{00}$
$\Psi_{11} = \Psi_{00}$
$\langle \Psi_{00} | (\lambda V)^n | \Psi_{01} \rangle \neq 0 \Rightarrow nR = L$


$H(\lambda) = H_{TC} + \lambda V$
$V = \sum_\alpha V_\alpha$
diam $V_\alpha \leq R$
$\langle \psi_{00} | (\lambda V)^n | \psi_{01} \rangle \neq 0 \Rightarrow nR = L \rightarrow n = \frac{L}{R}$
$\delta E \sim e^{-L/R}$



n = L/R
= W^x (γ(p,p')) Ψ_00
= Π σ_j^x 1/√|G| Σ_{g∈G} |g⟩
j ∈ γ(p,p')