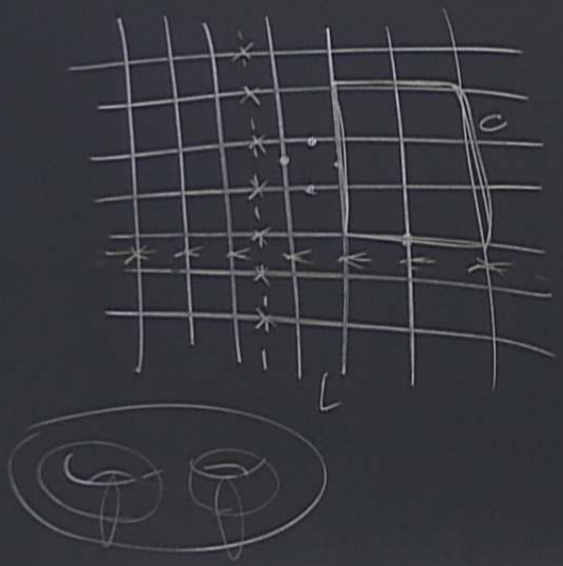
Title: Condensed Matter Review-10

Date: Feb 06, 2015 09:00 AM

URL: http://pirsa.org/15020005

Abstract:

Z_2 Quantum Lattice Gauge theory



$\mathcal{H}_{TOT} \simeq \mathbb{C}^{2 \otimes 2L^2}$

$A(n) = \prod_{i \in n} \hat{\sigma}_i^x$

$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid A(n)\psi = \psi \ \forall n \}$

$\hat{B}_p = \prod_{i \in p} \hat{\sigma}_i^z$

$H_{Z_2} = -J \sum_p B_p - g \sum_i \hat{\sigma}_i^x$

$g=0$ Gap $\Delta E = 4J$

degeneracy $= 4 = 2^{2g}$

$W(C) = \prod_{i \in C} \hat{\sigma}_i^z$ — small $g \sim e^{-\alpha C}$ / large $g \sim e^{-\beta A(C)}$

Gauge theory

$\mathcal{H}_{TOT} \simeq \Phi^{2 \otimes 2L^2}$

$A(n) = \prod_{i \in n} \hat{\sigma}^x_i$

$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid A(n)\psi = \psi \ \forall n \}$

$\hat{B}_p = \prod_{i \in p} \hat{\sigma}^z_i$

$H_{Z_2} = -J \sum_p B_p - g \sum_i \hat{\sigma}^x_i$

$g = 0$ Gap $\Delta E = 4J$

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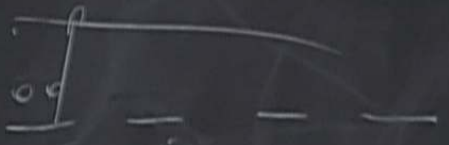
$W(C) = \prod_{i \in C} \hat{\sigma}^z_i$ / small $g \sim e^{-\alpha C}$

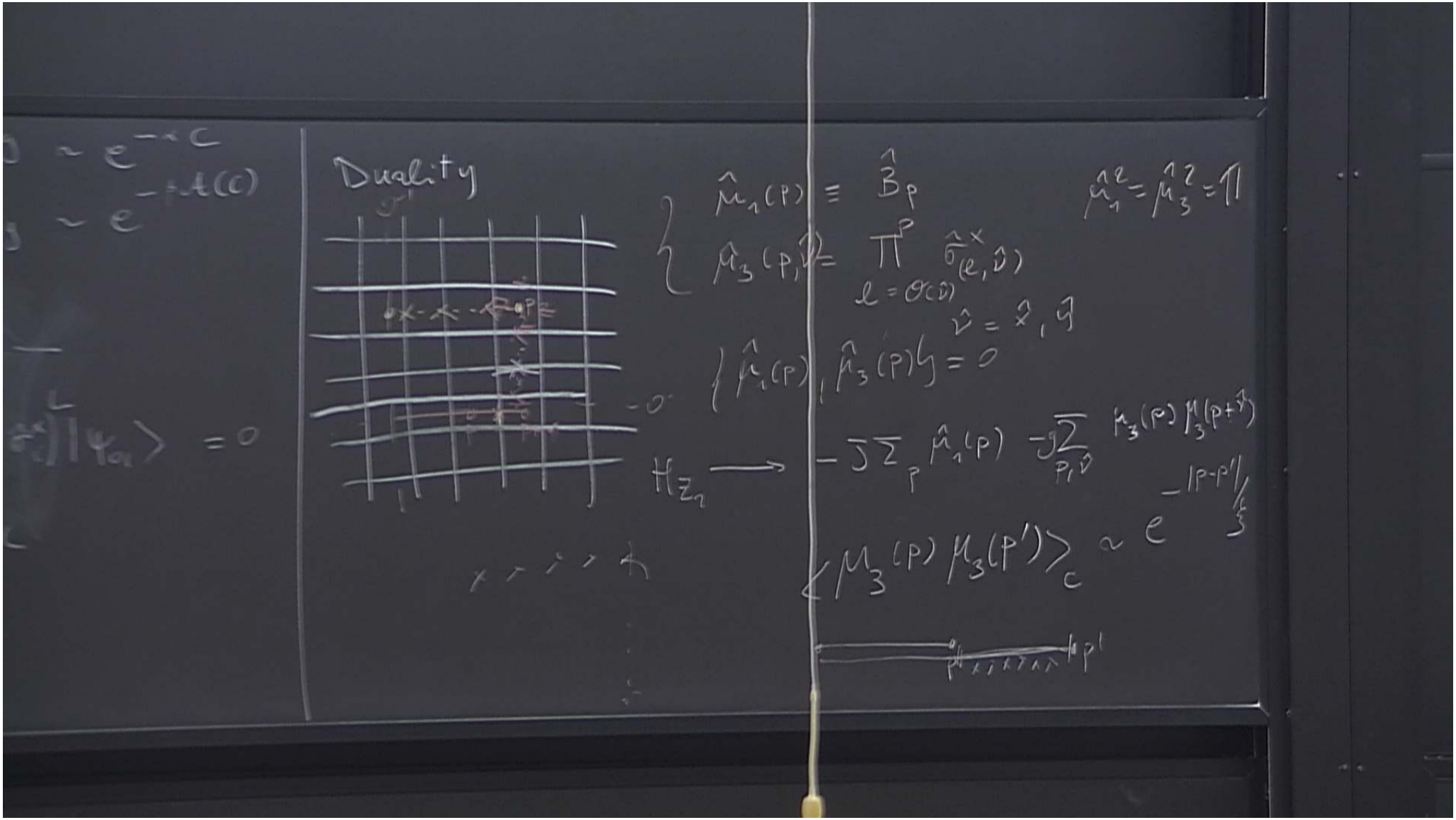
\ large $g \sim e^{-\beta A(C)}$

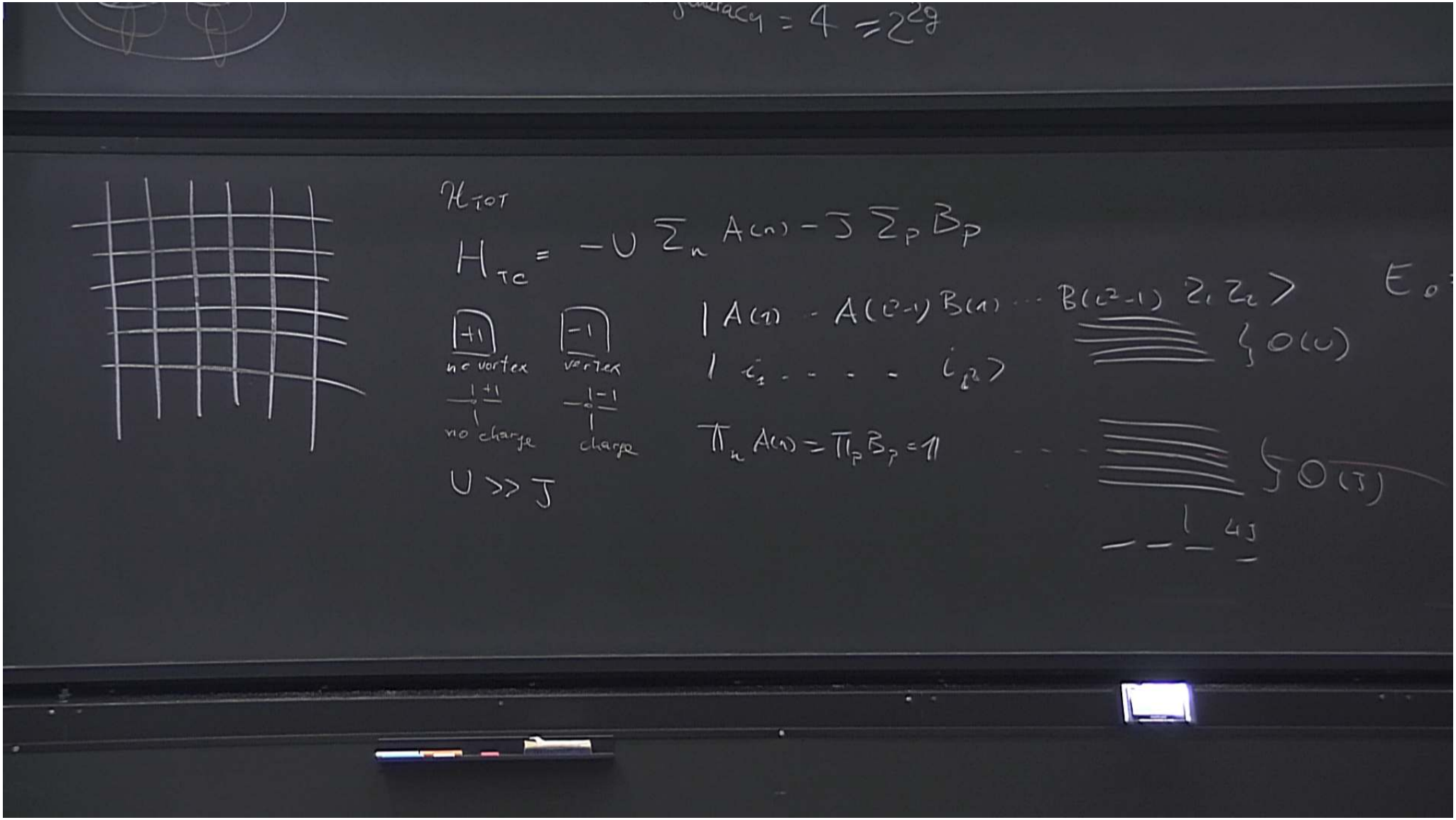


$g \ll J$

$\langle \psi_{00} | \left(-g \sum_i \hat{\sigma}^x_i \right)^L | \psi_{01} \rangle = 0$

$\delta E = \left(\frac{g}{J} \right)^L \sim e^{-L}$


Duality
$\hat{\mu}_1(p) \equiv \hat{B}_p$
$\hat{\mu}_3(p,\vec{\nu}) = \prod^{p} \hat{\sigma}^x_{(e,\vec{\nu})}$
$e = \sigma(\vec{\nu})$
$\vec{\nu} = x, y$
$\{\hat{\mu}_1(p), \hat{\mu}_3(p)\} = 0$
$H_{Z_2} \longrightarrow -J\sum_p \hat{\mu}_1(p) - \overline{J}\sum_{p,\vec{\nu}} \hat{\mu}_3(p)\hat{\mu}_3(p+\vec{\nu})$
$\langle \mu_3(p)\mu_3(p')\rangle_c \sim e^{-|p-p'|/\xi}$
$\hat{\mu}_1^2 = \hat{\mu}_3^2 = \mathbb{1}$


$= 4 = 2^{2g}$
$\mathcal{H}_{TOT}$
$H_{TC} = -U \sum_n A(n) - J \sum_p B_p$
$|A(1) \cdots A(L^2-1) B(1) \cdots B(L^2-1) Z_1 Z_2\rangle$
$|i_1 \cdots i_{L^2}\rangle$
$\prod_n A(n) = \prod_p B_p = \mathbb{1}$
+1
no vortex
-1
vortex
+1
no charge
-1
charge
$U \gg J$
$O(U)$
$O(J)$
$4J$

$$\mathcal{L}_{GS} = \{ \psi \in \mathcal{H}_{TOT} \mid A_v \psi = \psi = B_p \psi \quad \forall v, p \}$$

$$|0\rangle \equiv |\uparrow\uparrow\uparrow \cdots \uparrow\rangle$$

$$B_p |0\rangle = |0\rangle$$

$$G = \langle A(1), \cdots, A(2N) \rangle$$

$$g \in G \qquad |G| = 2^{2N-1}$$

$$P = \prod_v \frac{(\mathbb{1} + A(v))}{2}$$

$$P|0\rangle = \prod_v \frac{(\mathbb{1} + A(v))}{2} |0\rangle \equiv \psi_{00}$$

$$= \frac{1}{N} \sum (\mathbb{1} + A(1) + A(1)A(1') + \cdots) |0\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} g|0\rangle = \frac{1}{\sqrt{|G|}} \sum_{g \in G} |g\rangle$$

$$g|0\rangle \equiv |g\rangle$$

$$\Psi_{00} = \frac{1}{|G|} \sum_{g} |g 0\rangle$$

$\Psi_{01} =$

$\Psi_{10} -$

$\Psi_{11} =$

$$H(\lambda) = H_{TC} + \lambda \bar{V}$$

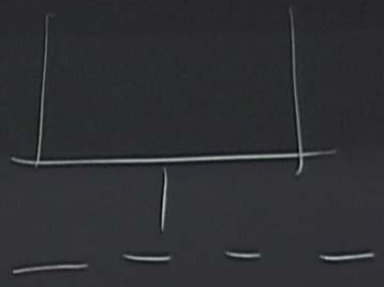
$$\bar{V} = \sum_{\alpha} \bar{V}_{\alpha} \qquad \text{diam } V_{\alpha} \leq R$$

$$\langle \Psi_{00} | (\lambda V)^{n} | \Psi_{01} \rangle \neq 0 \Rightarrow n R = L$$

$|\psi_0\rangle$

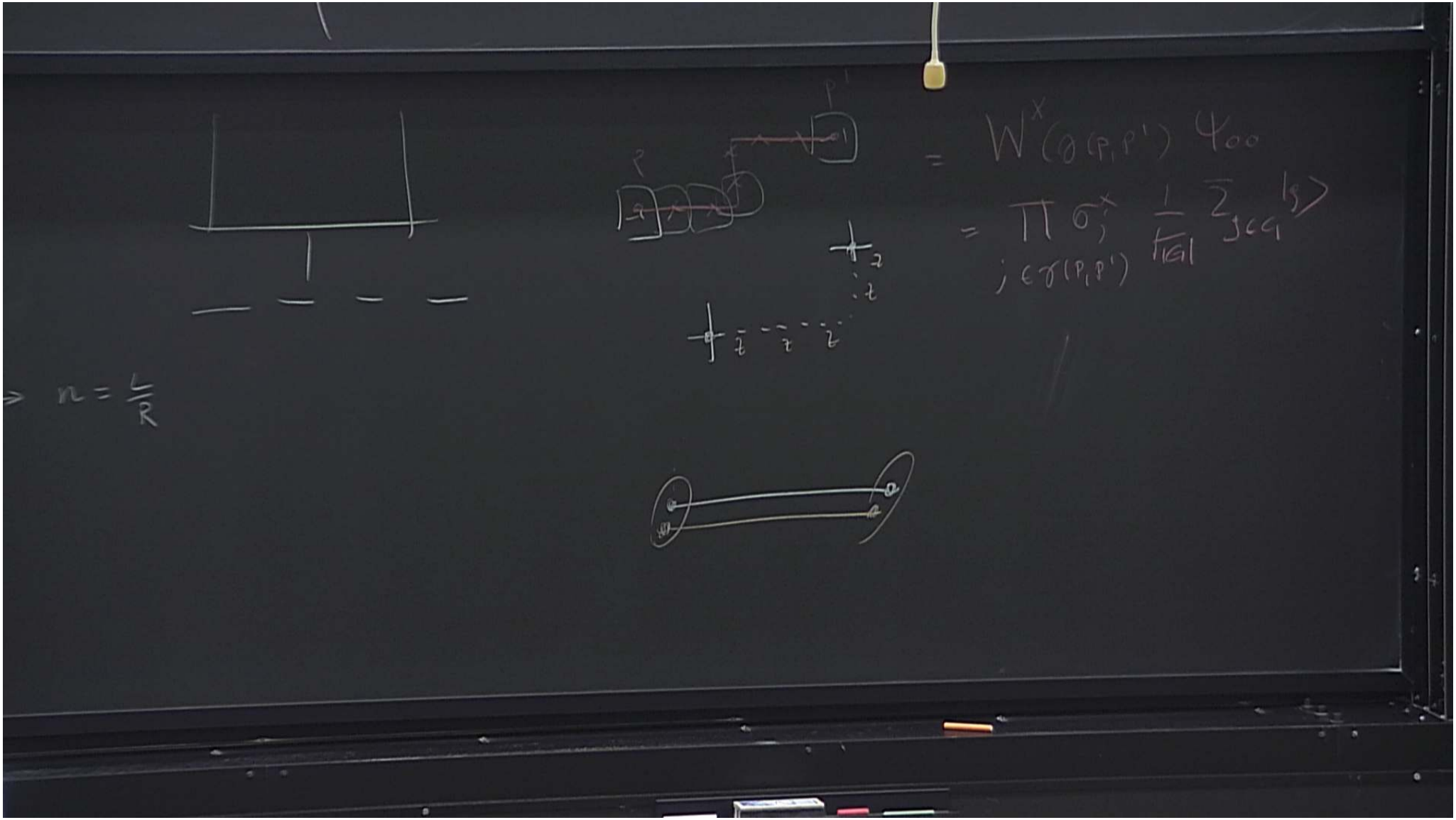
$$H(\lambda) = H_{TC} + \lambda V$$

$$V = \sum_\alpha V_\alpha \qquad \text{diam } V_\alpha \leq R$$



$$\langle \psi_{00} | (\lambda V)^n | \psi_{01} \rangle \neq 0 \Rightarrow nR = L \rightarrow n = \frac{L}{R}$$

$$\delta E \sim e^{-L/k}$$


$= W^{x}(\gamma(p,p')\,\Psi_{00}$
$= \prod_{j\in\gamma(p,p')} \sigma_j^x \frac{1}{\sqrt{|G|}} \sum_{g\in G} |g\rangle$
$\Rightarrow n = \frac{L}{R}$