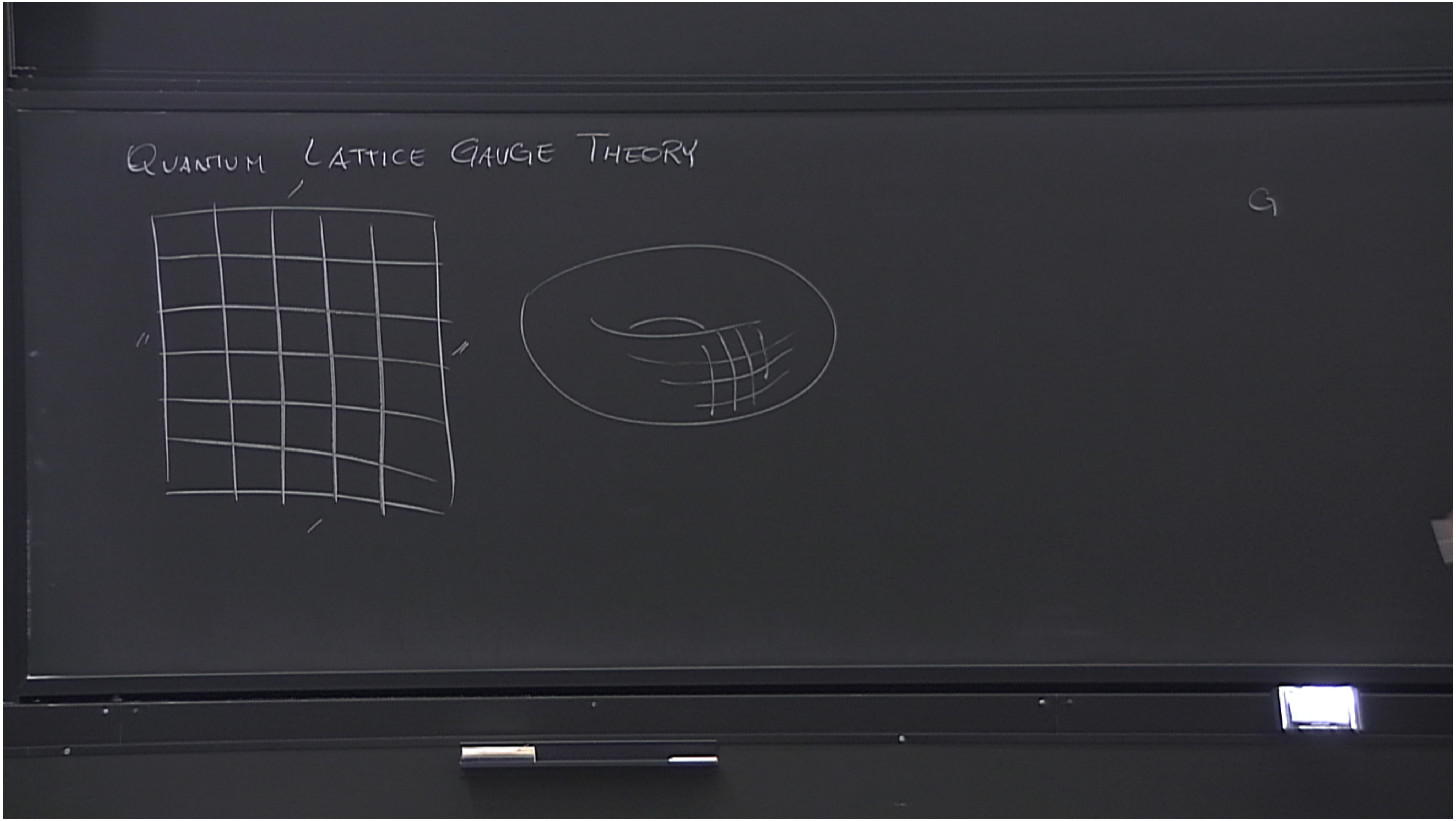
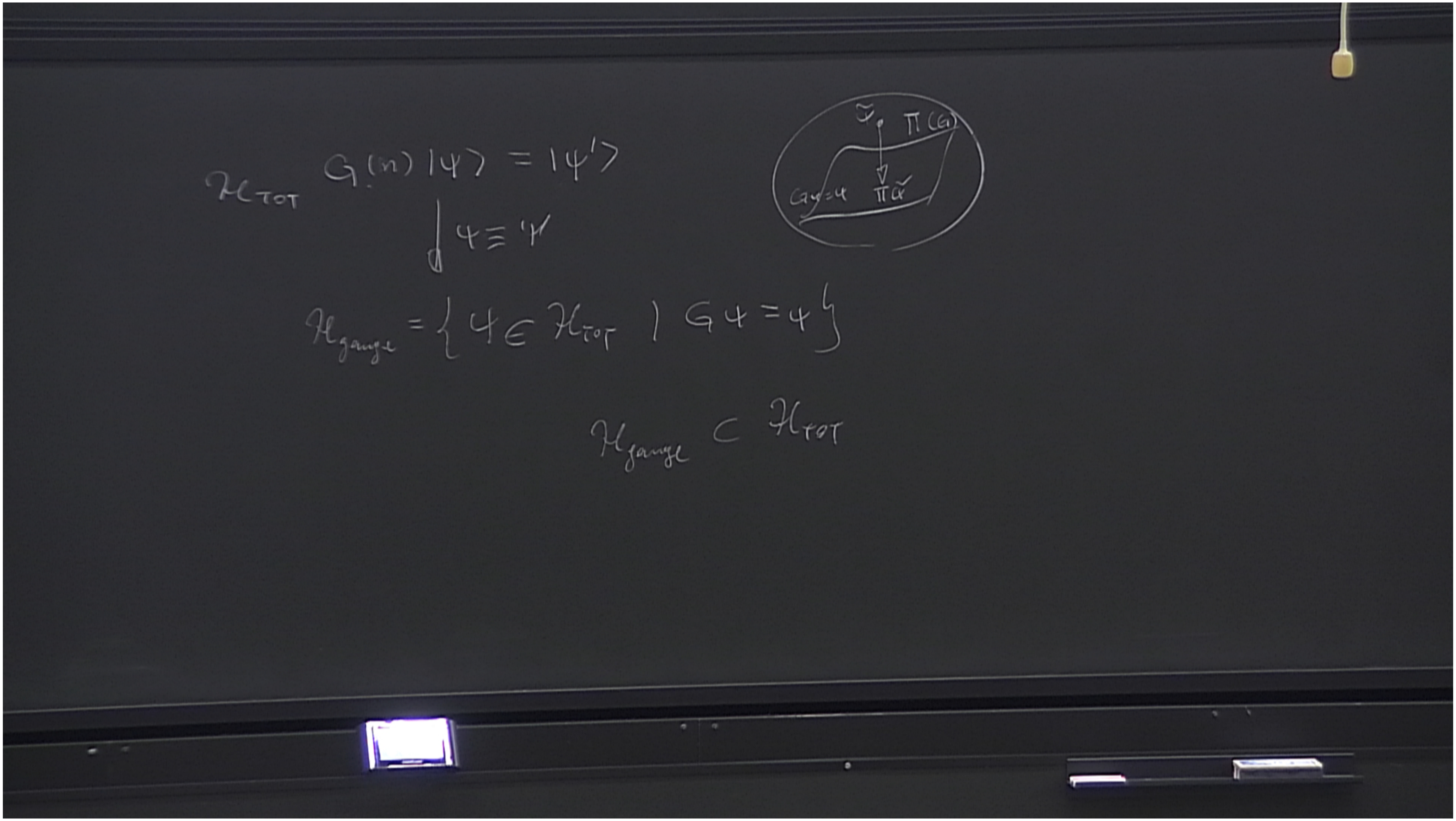
Title: Condensed Matter Review-9

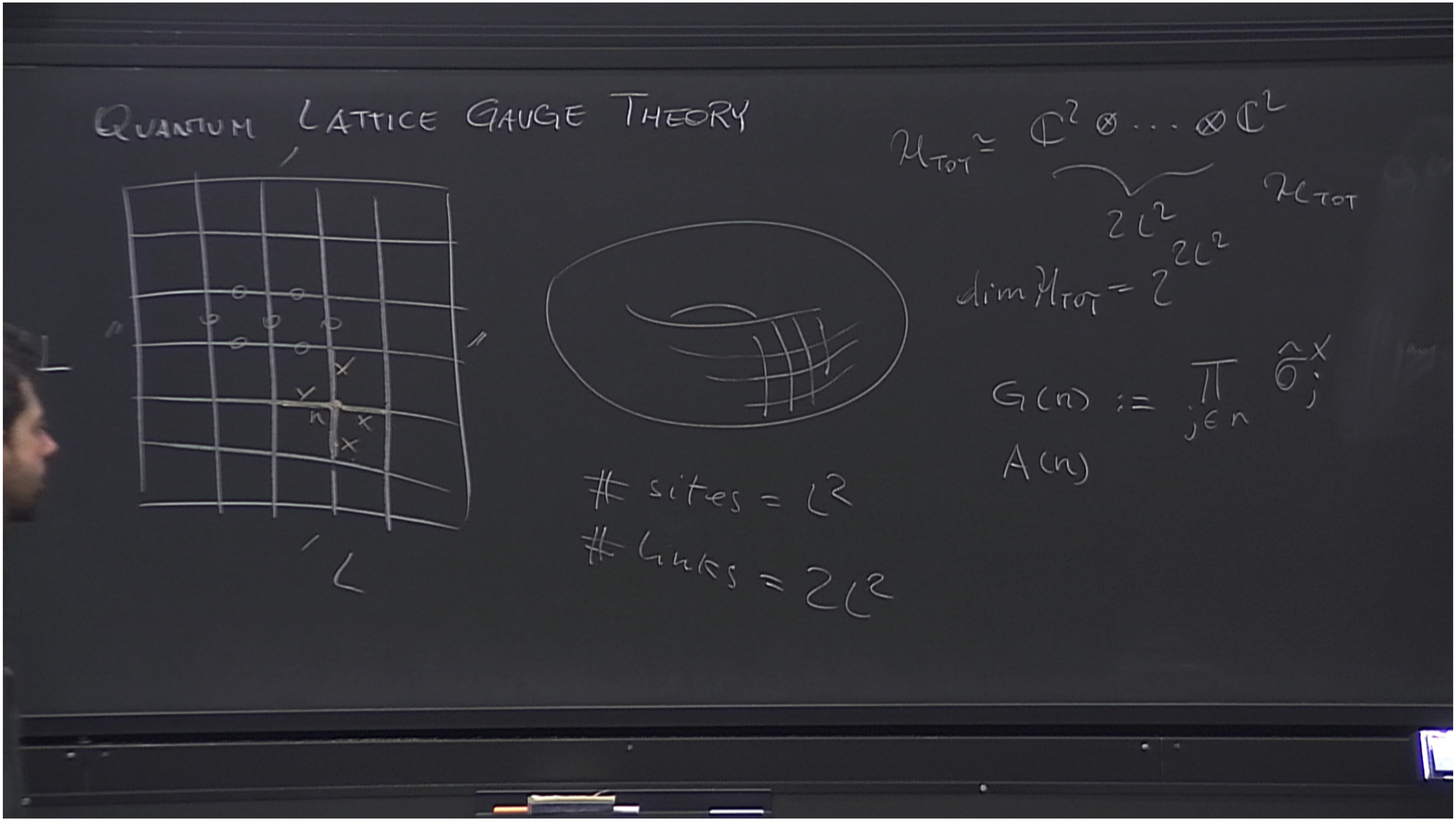
Date: Feb 05, 2015 09:00 AM

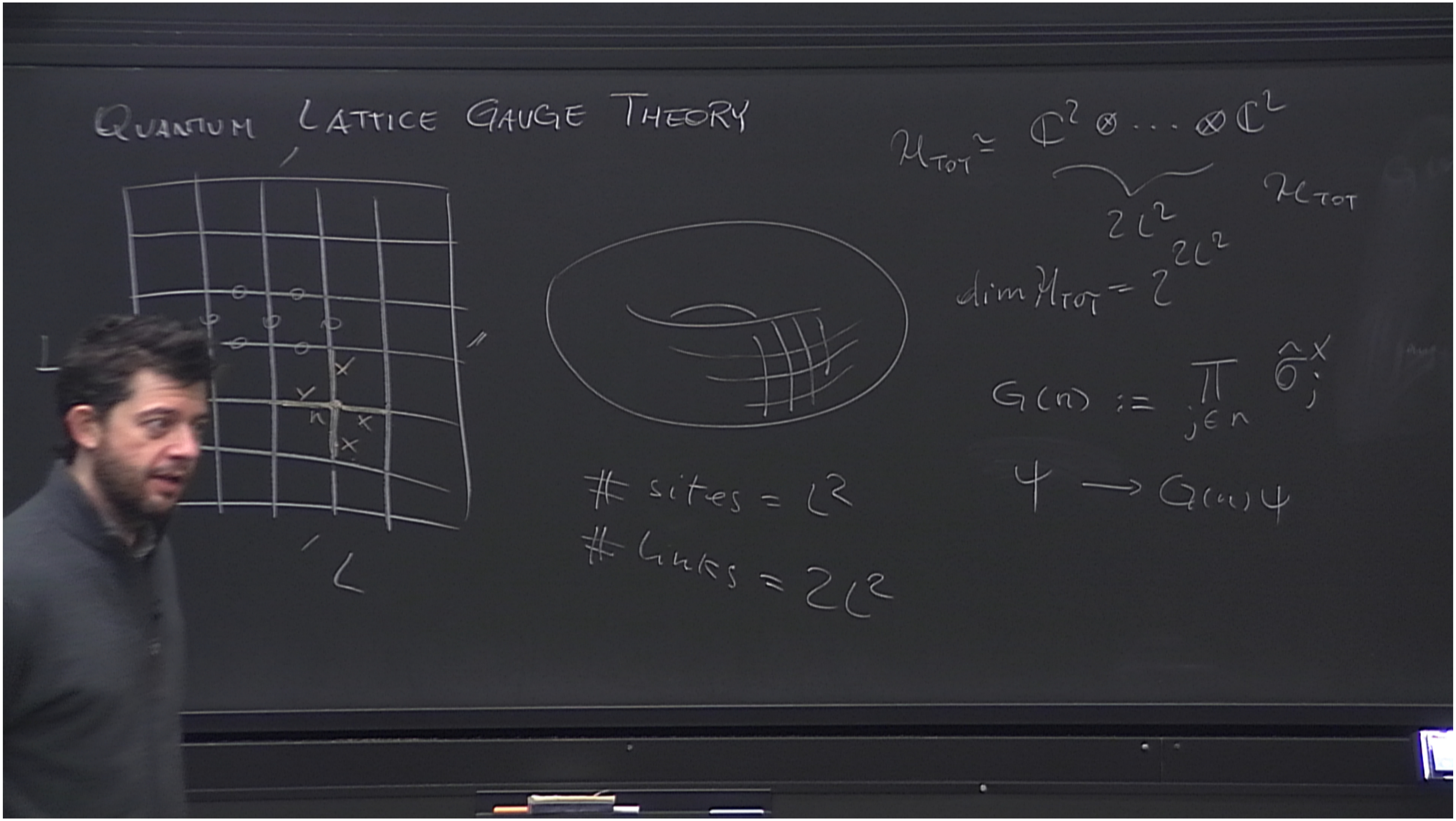
URL: http://pirsa.org/15020004

Abstract:

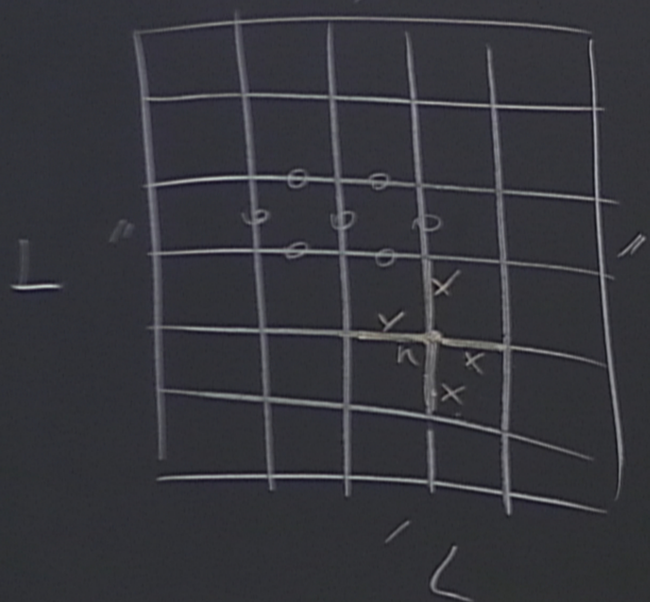

QUANTUM LATTICE GAUGE THEORY
G

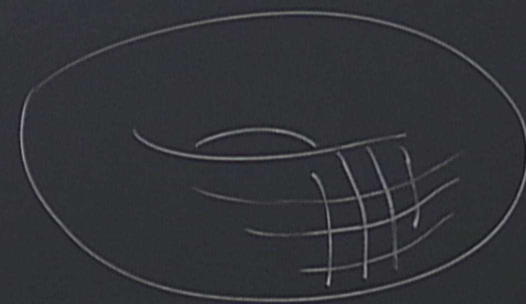

$\mathcal{H}_{TOT}$ $G(n)|\psi\rangle = |\psi'\rangle$
$\psi \equiv \psi'$
$\mathcal{H}_{gauge} = \{\psi \in \mathcal{H}_{TOT} \mid G\psi = \psi\}$
$\mathcal{H}_{gauge} \subset \mathcal{H}_{TOT}$
$\Pi(G)$
$G\psi = \psi$


QUANTUM LATTICE GAUGE THEORY
L
L
sites = L^2
links = $2L^2$
$\mathcal{H}_{TOT} \cong \underbrace{\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{2L^2}$
$\mathcal{H}_{TOT}$
$\dim \mathcal{H}_{TOT} = 2^{2L^2}$
$G(n) := \prod_{j \in n} \hat{\sigma}_j^x$
$A(n)$


QUANTUM LATTICE GAUGE THEORY
L
L
sites = L^2
links = 2L^2
H_TOT ≅ C^2 ⊗ . . . ⊗ C^2
2L^2
H_TOT
dim H_TOT = 2^{2L^2}
G(n) := Π_{j∈n} σ̂_j^x
ψ → G(n)ψ

QUANTUM LATTICE GAUGE THEORY





$\# \text{ sites} = L^2$

$\# \text{ links} = 2L^2$

$$\mathcal{H}_{TOT} = \underbrace{\mathbb{C}^2 \otimes \cdots \otimes \mathbb{C}^2}_{2L^2}$$

$\mathcal{H}_{TOT}$

$$\dim \mathcal{H}_{TOT} = 2^{2L^2}$$

$$G(n) := \prod_{j \in n} \hat{\sigma}_j^x$$

$$\psi \longrightarrow G(n)\psi$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

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$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{tor} \mid G(n)\psi = \psi \quad \forall n \}$$

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$$\mathbb{Z}_2 = \{ \mathbb{1}, G(n) \}$$

$$\underline{P} = \prod_n P_n$$

$$P_n \longrightarrow G(n)\psi = \psi$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$$

$$\mathbb{Z}_2 = \{ 1\!\!1, G(n) \}$$

$$\underline{P} = \prod_n P_n$$

$$P_n \longrightarrow G(n)\psi = \psi$$

$$P_n = \frac{1\!\!1 + G(n)}{2}$$

$$P_n^2 = P_n$$

$$\psi \longrightarrow P\psi = \psi'$$

$$G\psi' = GP\psi = G\frac{(1\!\!1 + G)}{2}\psi = \frac{G + 1\!\!1}{2}\psi = P\psi = \psi$$

$$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{tor} \mid G(n)\psi = \psi \quad \forall n \}$$

$$= \{ 1, G(n) \}$$

$$= \prod_n P_n$$

$$P_n \longrightarrow G(n)\psi = \psi$$

$$P_n = \frac{1 + G(n)}{2}$$

$$- P_n$$

$$\psi \longrightarrow P\psi = \psi'$$

$$G\psi' = GP\psi = G\frac{(1+G)}{2}\psi = \frac{G+1}{2}\psi = P\psi = \psi'$$

$$P\psi = \prod_n \left(\frac{1+G(n)}{2}\right)\psi = \frac{1}{N}\Big(1 + \sum_n G(n) + \sum_{n n'} G(n)G(n') + \sum_{n n' n''} G(n)G(n')G(n'')\Big)\psi$$

$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{tor} \mid G(n)\psi = \psi \quad \forall n \}$

$\dim \mathcal{H}_{gauge} = 2^{2L^2 - (L^2-1)} = 2^{L^2+1}$

$\mathbb{Z}_2 = \{ 1\!\!1, G(n) \}$

$\prod_n G(n) = 1\!\!1 \; ; \; \prod_p B_p = 1\!\!1$

$P = \prod_n P_n$

$L^2 - 1$ indep. projections

$L^2 - 1$ indep B_p's

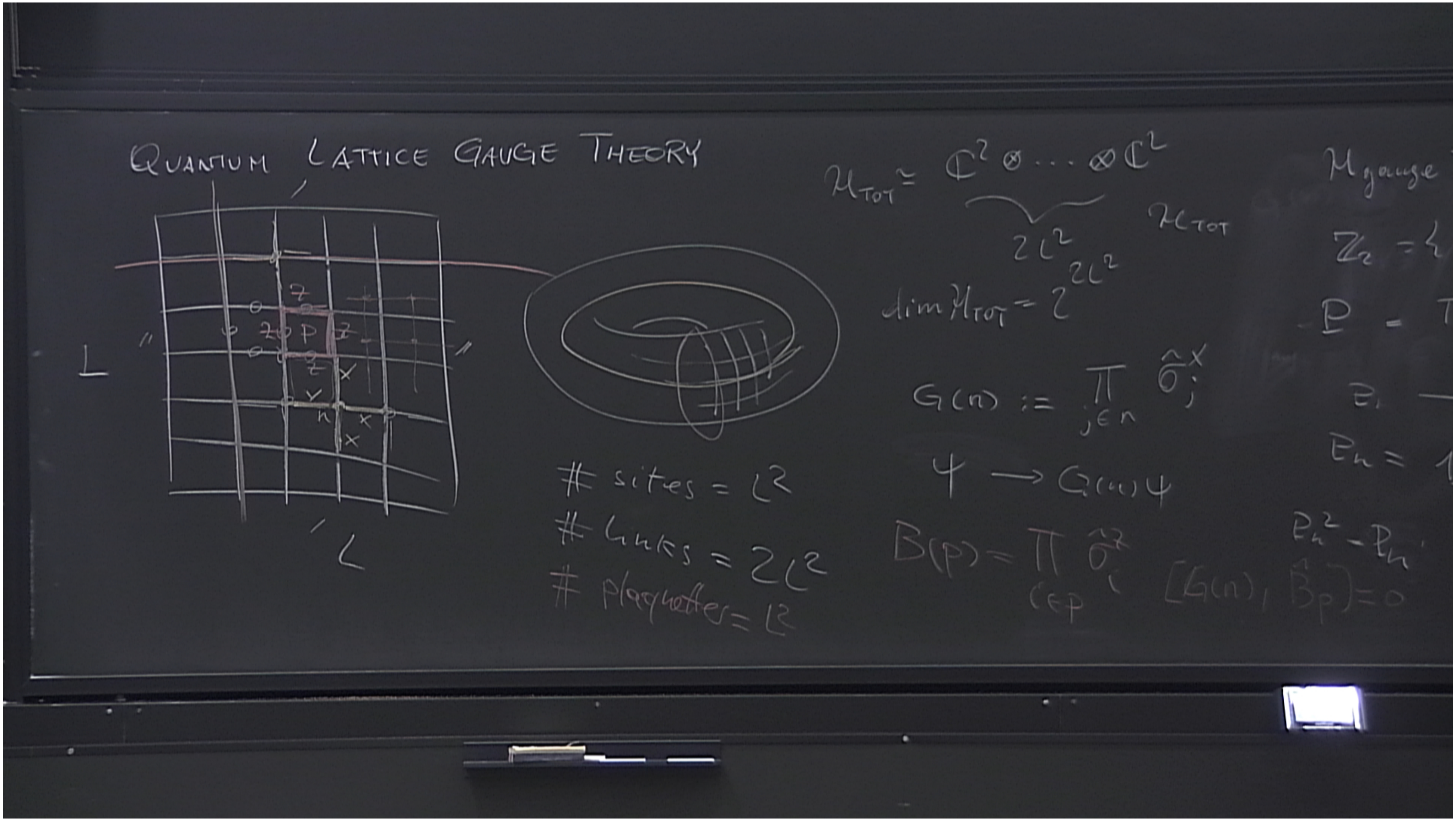
$P_n \longrightarrow G(n)\psi = \psi$

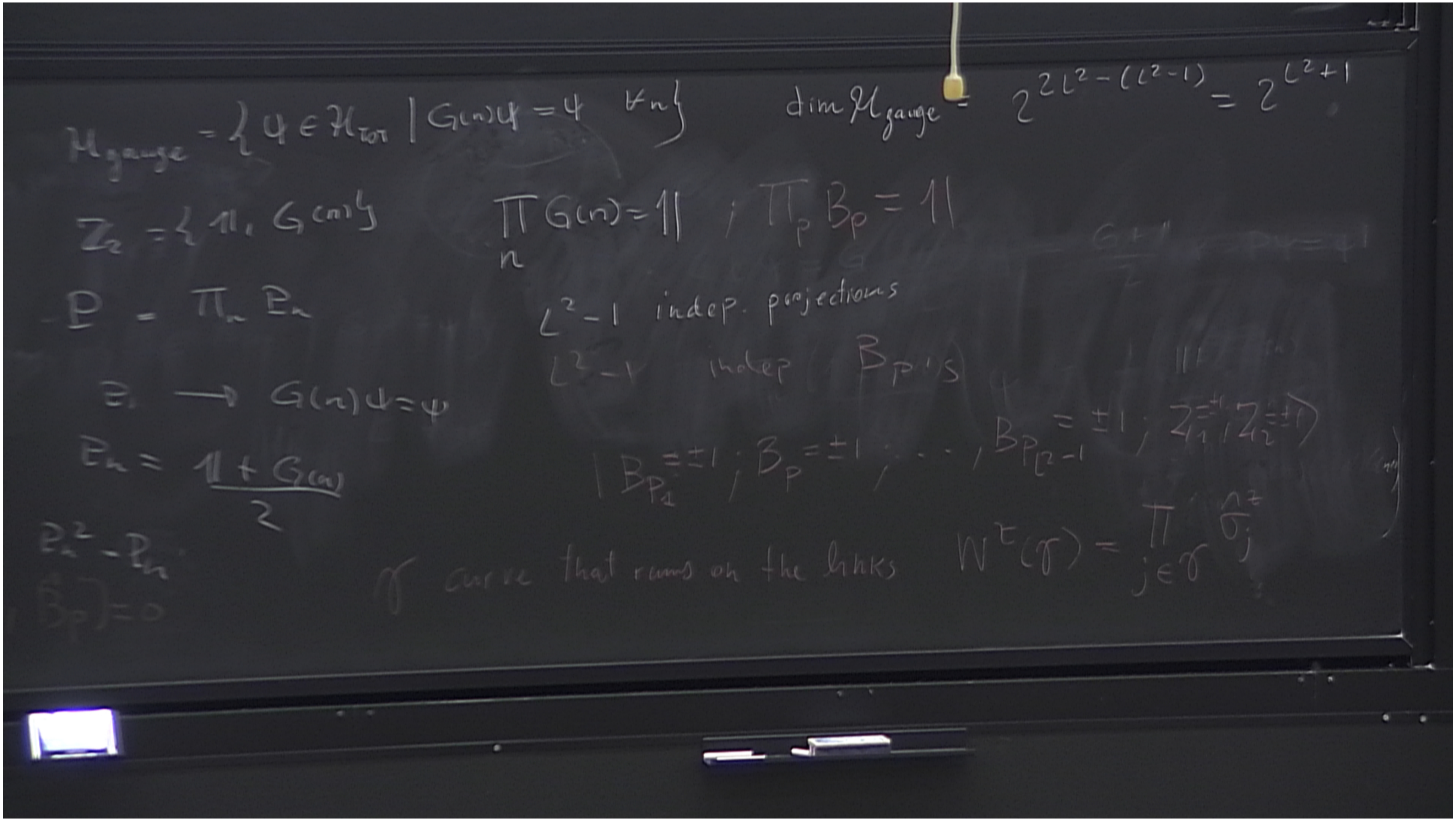
$| B_{p_1} = \pm 1 ; B_p = \pm 1 ; \ldots ; B_{p_{L^2-1}} = \pm 1 ; Z_{\ell_1} = \pm 1 ; Z_{\ell_2} = \pm 1 \rangle$

$P_n = \frac{1\!\!1 + G(n)}{2}$

$1 - P_n$

$) = 0$


QUANTUM LATTICE GAUGE THEORY
L
L
P
n
# sites = L²
# links = 2L²
# plaquettes = L²
H_TOT = ℂ² ⊗ . . . ⊗ ℂ²
2L²
H_TOT
dim H_TOT = 2^(2L²)
G(n) := ∏_{j∈n} σ̂_j^x
ψ → G(n)ψ
B(p) = ∏_{i∈p} σ̂_i^z
[G(n), B̂_p] = 0
H_gauge
Z₂ = {
P_n² = P_n


$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{TOT} \mid G(n)\psi = \psi \quad \forall n \}$
$\dim \mathcal{H}_{gauge} = 2^{2L^2-(L^2-1)} = 2^{L^2+1}$
$\mathbb{Z}_2 = \{ 1\!\!1, G(n) \}$
$\prod_n G(n) = 1\!\!1 \; ; \; \prod_p B_p = 1\!\!1$
$P = \prod_n P_n$
L^2-1 indep. projections
L^2-1 indep B_p's
$P_n \longrightarrow G(n)\psi = \psi$
$B_{p_{L^2-1}} = \pm 1 ; Z_1 = \pm 1 ; Z_2 = \pm 1$
$P_n = \frac{1\!\!1 + G(n)}{2}$
$B_{p_1} = \pm 1 ; B_p = \pm 1 ; \ldots ,$
$P_n^2 = P_n$
γ curve that runs on the links $W^z(\gamma) = \prod_{j \in \gamma} \hat{\sigma}_j^z$

$\mathcal{H}_{gauge} = \{ \psi \in \mathcal{H}_{tot} \mid G(n)\psi = \psi \ \forall n \}$ $\quad \dim \mathcal{H}_{gauge} = 2^{2L^2 - (L^2-1)} = 2^{L^2+1}$

$Z_2 = \{ 1\!\!1, G(n) \}$ $\quad \prod_n G(n) = 1\!\!1 \ ; \ \prod_p B_p = 1\!\!1$ $\quad Z_1 = W^z(c_1)$

$P = \prod_n P_n$

L^2-1 indep. projections

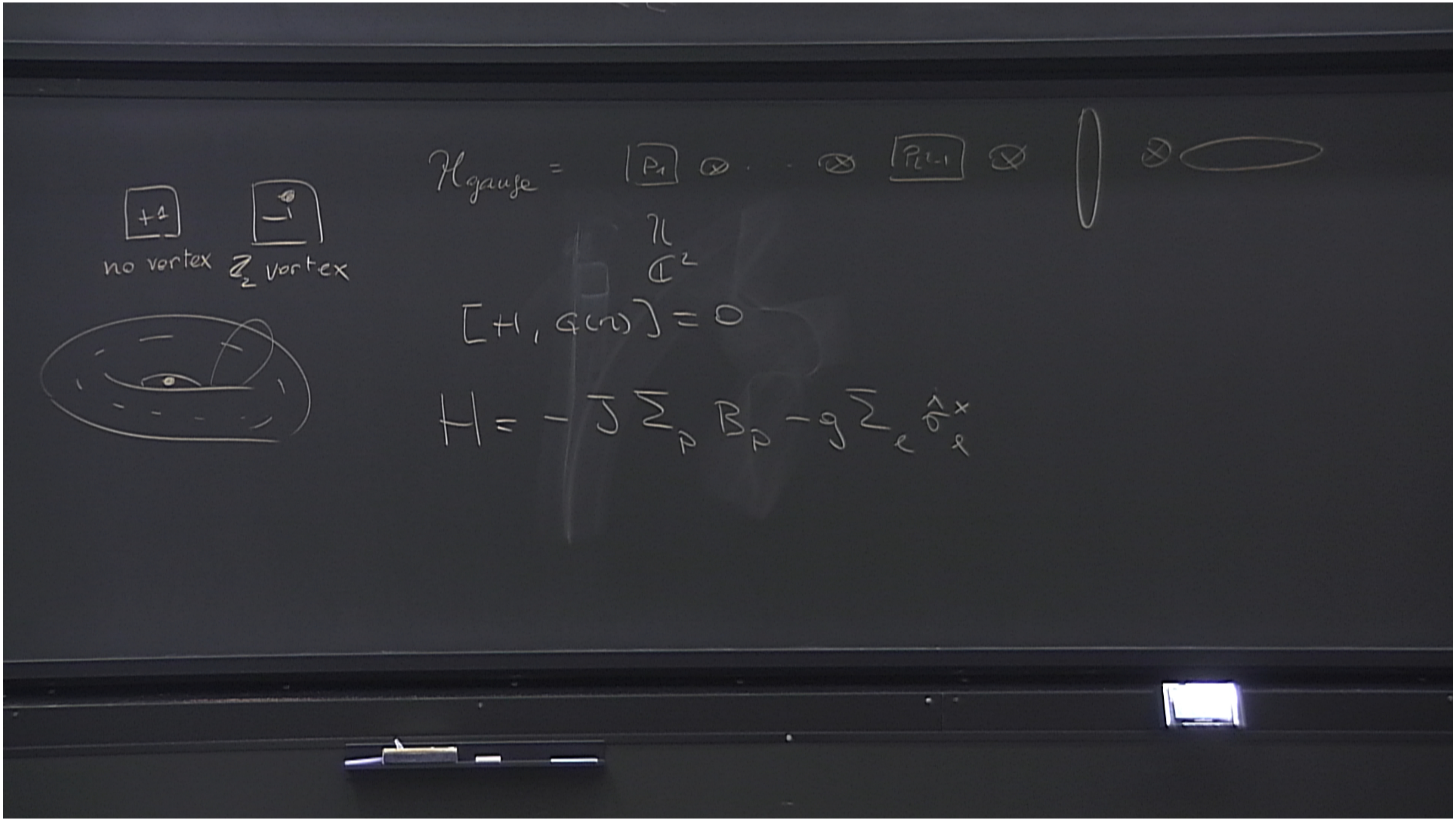
L^2-1 indep B_p's

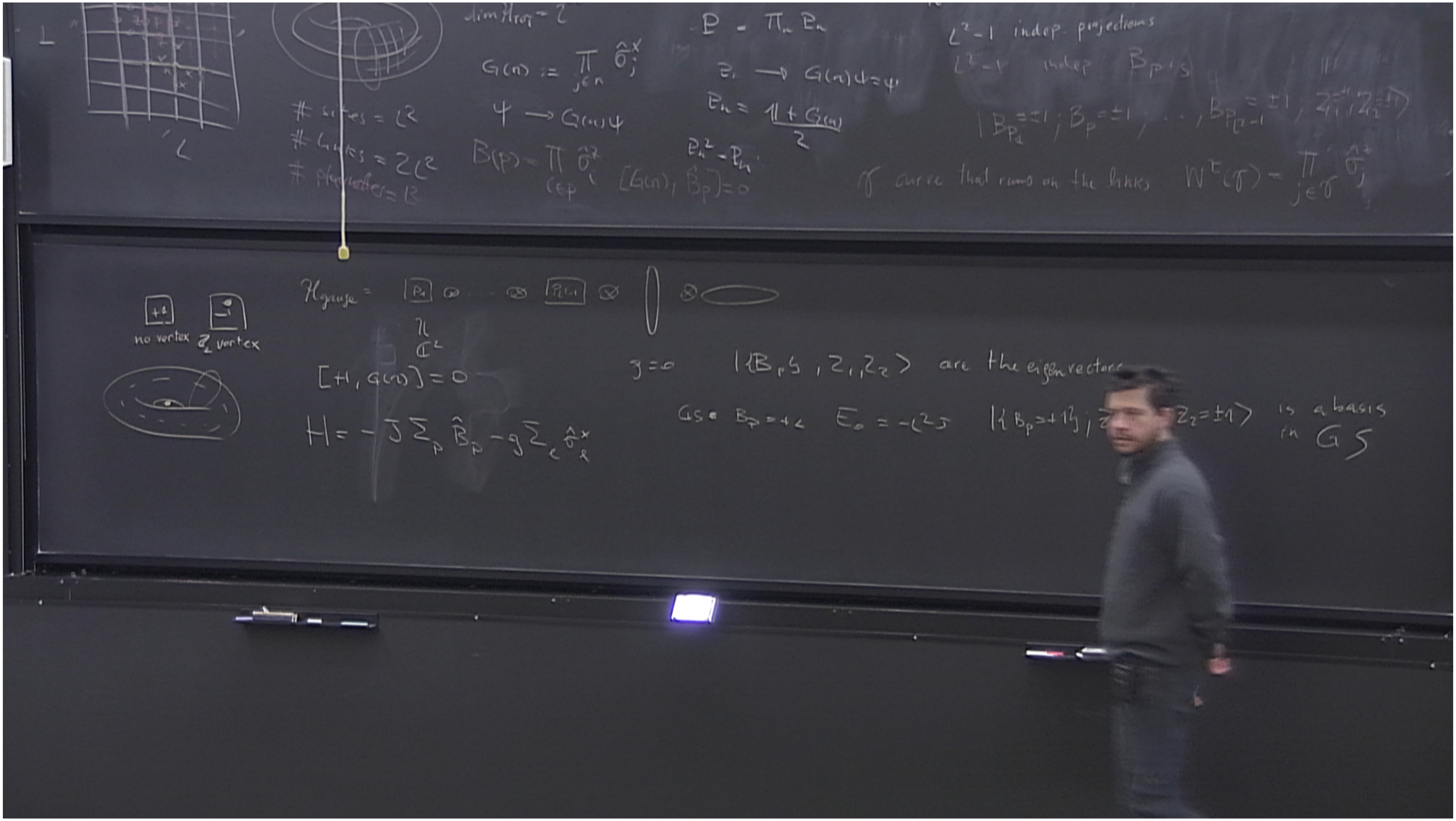
$P_n \longrightarrow G(n)\psi = \psi$

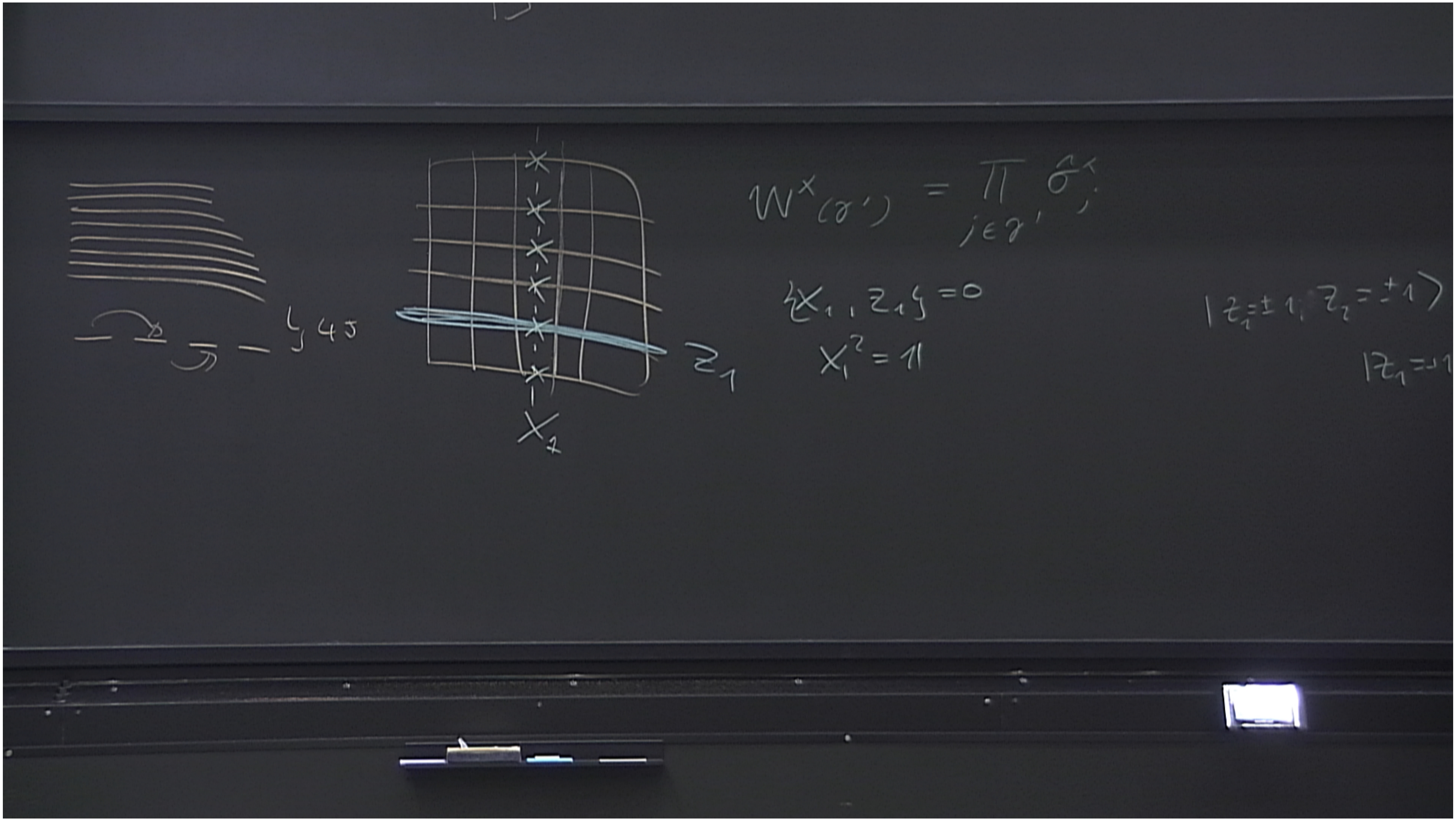
$P_n = \frac{1\!\!1 + G(n)}{2}$ $\quad |B_{p_1} = \pm 1 ; B_p = \pm 1 ; \dots ; B_{p_{L^2-1}} = \pm 1 ; Z_1 = \pm 1 ; Z_2 = \pm 1 \rangle$

$P_n^2 = P_n$

γ curve that runs on the links $\quad W^z(\gamma) = \prod_{j \in \gamma} \hat{\sigma}^z_j$


$+1$
-1
no vertex
$\mathbb{Z}_2$ vortex
$\mathcal{H}_{gauge} = P_1 \otimes \cdots \otimes P_{L-1} \otimes \otimes$
$\mathcal{H}$
$\mathbb{C}^2$
$[H, G(n)] = 0$
$H = -J \sum_p B_p - g \sum_e \hat{\sigma}^x_e$


indep. projections
curve that runs on the links
no vertex
vertex
are the eigenvectors
is a basis
in GS



{X1 , Z1} = 0
X1² = 1
Z1

$$\langle GS|\,\hat{\sigma}_i^z\,|GS\rangle = \langle GS|\,G^\dagger(n)\,\hat{\sigma}_i^z\,G(n)\,|GS\rangle$$

$$i \in n \qquad = -\langle GS|\,\cancel{G^\dagger G}\,\hat{\sigma}^z\,|GS\rangle = 0$$

$$|z_1 = \pm 1, z_2 = \pm 1\rangle$$

$$X_1^i X_2^j\,|z_1 = +1\; z_2 = +1\rangle = \psi_{ij}$$

$$i, j = 0, 1$$