Title: Condensed Matter Review-7

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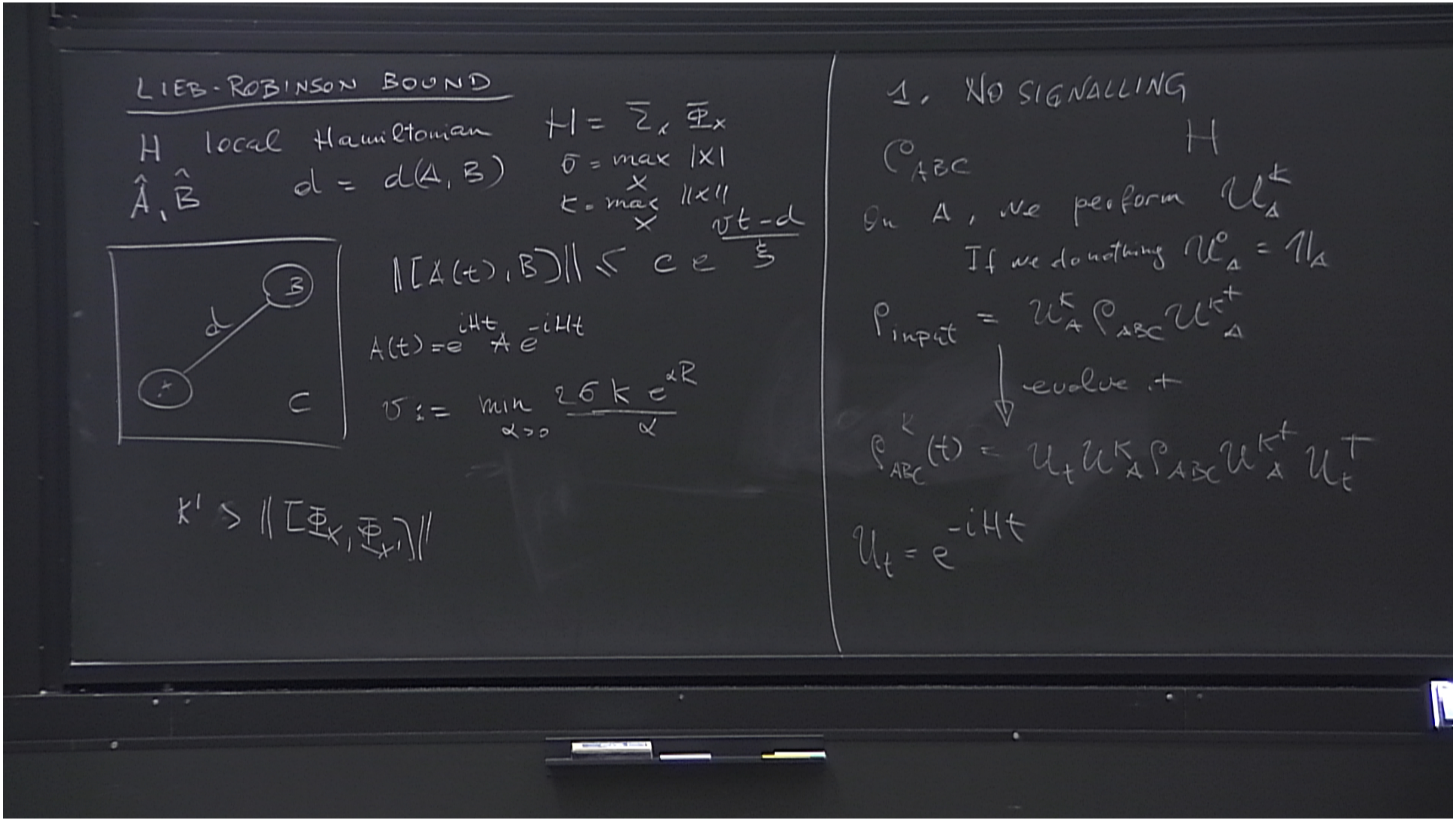
Abstract:


LIEB-ROBINSON BOUND
H local Hamiltonian
$\hat{A}, \hat{B}$
$d = d(A, B)$
$H = \sum_x \Phi_x$
$\sigma = \max_x |X|$
$k = \max_x \|X\|$
$\|[A(t), B]\| \le c\, e^{\frac{vt-d}{\xi}}$
$A(t) = e^{iHt} A e^{-iHt}$
$v := \min_{\alpha>0} \frac{2\sigma k e^{\alpha R}}{\alpha}$
B
d
A
C
$k' \ge \|[\Phi_X, \Phi_{X'}]\|$
1. NO SIGNALLING
ρ_{ABC}
H
On A, we perform $\mathcal{U}_A^k$
If we do nothing $\mathcal{U}_A^0 = \mathbb{1}_A$
$\rho_{input} = \mathcal{U}_A^k \rho_{ABC} \mathcal{U}_A^{k\dagger}$
evolve it
$\rho_{ABC}^k(t) = \mathcal{U}_t \mathcal{U}_A^k \rho_{ABC} \mathcal{U}_A^{k\dagger} \mathcal{U}_t^\dagger$
$\mathcal{U}_t = e^{-iHt}$

1. NO SIGNALLING

H

ρ_{ABC}

On A, we perform $\mathcal{U}_A^k$

If we do nothing $\mathcal{U}_A^0 = \mathbb{1}_A$

$$\rho_{input} = \mathcal{U}_A^k \rho_{ABC} \mathcal{U}_A^{k\dagger}$$

$\Big\downarrow$ evolve it

$$\rho^k_{ABC}(t) = \mathcal{U}_t \mathcal{U}^k_A \rho_{ABC} \mathcal{U}^{k\dagger}_A \mathcal{U}_t^\dagger$$

$$\mathcal{U}_t = e^{-iHt}$$

$$\boxed{\begin{array}{l} \hat{O}_B \\ Tr(\hat{O}_B \rho) \\ = Tr(\hat{O}_B \rho_B) \end{array}}$$

$$\sigma^k_B(t) = Tr_{AC}\, \sigma^k_{ABC}(t)$$

$$\hat{O}_B \to O_B^k = Tr[\hat{O}_B \sigma^k_B(t)]$$

$$|O_B^0 - O_B^k| = |Tr[\hat{O}_B(\sigma^k_B(t) - \sigma^0_B(t))]|$$

$$= |Tr[\hat{O}_B(t)(\mathcal{U}^k_A \rho^0 \mathcal{U}^{k\dagger}_A - \rho^0 \dots)]|$$

$$= |Tr[\mathcal{U}^k_A \rho^0 [\hat{O}_B(t), \mathcal{U}^{k\dagger}_A]]|$$

$$\leq c\, e^{-\frac{d - vt}{\xi}}$$

$$= \mathrm{Tr}_{AC}\, \sigma^k_{ABC}(t)$$

$$O^k_B = \mathrm{Tr}[\hat{O}_B\, \sigma^k_B(t)]$$

$$|O^k_B| = |\mathrm{Tr}[\hat{O}_B(\sigma^k_B(t) - \sigma^0_B(t))]|$$

$$= |\mathrm{Tr}[\hat{O}_B(t)(\mathcal{U}^k_A \rho^0 \mathcal{U}^{k\dagger}_A - \rho^0)]|$$

$$= |\mathrm{Tr}[\mathcal{U}^k_A \rho^0 [\hat{O}_B(t), \mathcal{U}^{k\dagger}_A]]|$$

$$\leq C e^{-\frac{d - vt}{\xi}}$$

2. Truncation

Lemma $d\mu(\mathcal{U})$

1a. $\int d\mu(\mathcal{U})\, \mathcal{U}|\psi\rangle\langle\psi|\mathcal{U}^\dagger = \frac{\mathbb{1}}{d_{\mathcal{H}}}$

1b. $\int d\mu(\mathcal{U}_A)\, \mathcal{U}_A \hat{O}\, \mathcal{U}^\dagger_A = \hat{O}_{\setminus A} \otimes \frac{\mathbb{1}_A}{d_A}$

$$O^\ell_A(t) := \mathrm{Tr}_S\, O_A(t) \otimes \frac{\mathbb{1}_S}{d_S}$$

$$S := \{x \in \Lambda \mid d(x, A) > \ell\}$$

$$\| O^{\ell}_A(t) - O_A(t) \| \le \int d\mu(\mathcal{U}_S)\, \| \mathcal{U}_S O_A(t)\, \mathcal{U}_S^{\dagger} - O_A(t) \|$$

$$\le \int d\mu(\mathcal{U}_S)\, \|\mathcal{U}_S\| \; \| [\mathcal{U}_S^{\dagger}, O_A(t)] \|$$

$$\le \tilde{c}\; e^{\frac{vt-\ell}{\xi}}$$

3. Spreading of Correlations

$$|\langle O_A O_B \rangle_c| \leq c_1 e^{-d_{AB}/\chi}$$

$$|\langle O_A(t) O_B(t) \rangle_c| \leq |\langle O_A^{\ell}(t) O_B^{\ell}(t) \rangle_c| + c_2 e^{\frac{vt-\ell}{\xi}} \leq (c_1+c_2) e^{\frac{vt}{\xi} - \frac{\chi vt + d_{AB}\xi}{\xi(2\xi+\chi)}}$$

$$\leq \quad c_1 e^{-\frac{(d_{AB}-2\ell)}{\chi}}$$

$$\frac{2\ell - d_{AB}}{\chi} = \frac{vt-\ell}{\xi}$$

$$2\ell\xi - d_{AB}\xi + \ell\chi - \chi vt = 0$$

$$\ell = \frac{\chi vt + d_{AB}\xi}{2\xi+\chi}$$

ations

$$e^{-d_{AB}/\chi}$$

$$\langle O_A^{\ell}(t)\, O_B^{\ell}(t)\rangle_c \,| + c_2\, e^{\frac{vt-\ell}{\xi}} \leq (c_1+c_2)\, e^{\frac{vt}{\xi} - \frac{\chi vt + d_{AB}\xi}{\xi(2\xi+\chi)}}$$

$$\leq (c_1+c_2)\, e^{\frac{2vt - d_{AB}}{2\xi+\chi}}$$

$$\wedge$$

$$c_1\, e^{-\frac{(d_{AB}-2\ell)}{\chi}}$$

Clustering of equal time correlations

ΔE, ψ

$$\langle O_i O_j\rangle_\psi^c \leq c\, e^{-|i-j|/\xi}$$

$$\xi^{-1} \sim \Delta E\, v$$

$2\xi + x$

$|\psi\rangle \to S(|\psi\rangle) = 0$

$S(U|\psi\rangle) = 0$

$(\mathrm{Tr}_B\, \rho) \equiv \rho_A$

$S(\rho_A)$

$\psi(0) = \psi_A \otimes \psi_B$

$S(\rho_A(t)) < ?$

$= -\mathrm{Tr}\, \rho_A \log \rho_A$

$2\xi + x$

$) = 0$

$\psi(0) = \psi_A \otimes \psi_B$

$S(\rho_A(t)) < ?$

$= -\mathrm{Tr}\, \rho_A \log \rho_A$

Renyi entropies

$S_\alpha(\rho_A) = \frac{1}{1-\alpha} \log \mathrm{Tr}\, \rho_A^\alpha$

$\alpha = 2 \qquad S_2 = -\log \underbrace{\mathrm{Tr}[\rho_A^2]}_{\wp} =$

entropies

$$S_\alpha(\rho_A) = \frac{1}{1-\alpha} \log \mathrm{Tr}\, \rho_A^\alpha$$

$$S_2 = -\log \underbrace{\mathrm{Tr}[\rho_A^2]}_{\infty} =$$

$$\rho \to \rho \otimes \rho$$

$$S_A \, |i_A\rangle |j_B\rangle |i'_A\rangle |j'_B\rangle$$

$$= |i'_A\rangle |j_B\rangle |i_A\rangle |j'_B\rangle$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \to \mathcal{H} \otimes \mathcal{H}' = (\mathcal{H}_A \otimes \mathcal{H}_B) \otimes (\mathcal{H}_{A'} \otimes \mathcal{H}_{B'})$$

$$|i_1 \cdots i_{n_A}\rangle \otimes |j_1 \otimes \cdots \otimes j_{n_B}\rangle \otimes |i'_1 \cdots i'_{n_A}\rangle |j'_1 \cdots j'_{n_B}\rangle$$

i entropies

$$(\rho_A) = \frac{1}{1-\alpha} \log \operatorname{Tr} \rho_A^{\alpha}$$

$$\boxed{S_2 = -\log \underbrace{\operatorname{Tr}[\rho_A^2]} = -\log \operatorname{Tr}[S_A\, \rho\otimes\rho]}$$

S_A

$$|i_A\rangle|j_B\rangle|i'_A\rangle|j'_B\rangle$$

$$= |i'_A\rangle|j_B\rangle|i_A\rangle|j'_B\rangle$$

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B \to \mathcal{H}\otimes\mathcal{H}' = (\mathcal{H}_A\otimes\mathcal{H}_B)\otimes(\mathcal{H}_{A'}\otimes\mathcal{H}_{B'})$$

$$\rho \to \rho\otimes\rho$$

$$|i_1 \cdots i_{n_A}\rangle \otimes |j_1 \otimes \cdots \otimes j_{n_B}\rangle \otimes |i'_1 \cdots i'_{n_A}\rangle |j'_1 \cdots j'_{n_B}\rangle$$