Title: Condensed Matter Review-6

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URL: http://pirsa.org/15020001

Abstract:

Disordered Phase
(paramagnet)

GS unique

no local order parameter ≡ disordered

$$\langle \sigma_i^z ; \sigma_j^z \rangle_c = \langle \sigma_i^z \sigma_j^z \rangle - \langle \sigma_i^z \rangle \langle \sigma_j^z \rangle \sim e^{-\frac{|i-j|}{\xi}}$$

$\xi \sim \Delta^{-1}$

Ordered Phase

ferromagnet

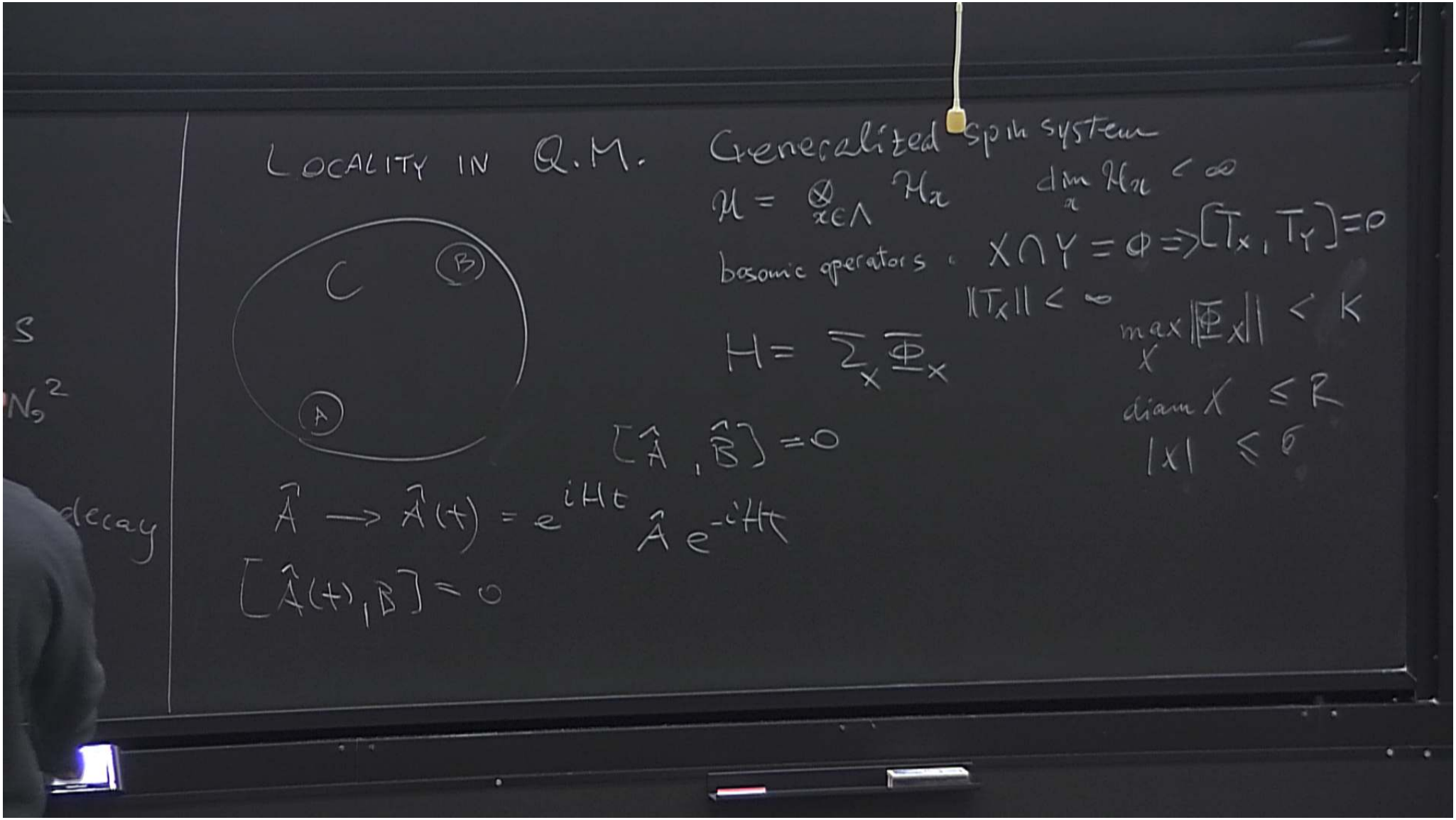
GS degenerate — — } $\Delta$

There is local order par.

$\exists$ $\psi_1, \psi_2$ basis in GS

s.t. $\langle \sigma_i^z ; \sigma_j^z \rangle_c^{\psi_1} \sim N_0^2$

$\langle \sigma_i^z ; \sigma_j^z \rangle_c^{\psi^\perp} \sim$ exp decay


LOCALITY IN Q.M.
Generalized spin system
$\mathcal{H} = \bigotimes_{x \in \Lambda} \mathcal{H}_x$
$\dim_x \mathcal{H}_x < \infty$
basonic operators
$X \cap Y = \emptyset \Rightarrow [T_X, T_Y] = 0$
$\|T_X\| < \infty$
$H = \sum_x \Phi_x$
$\max_x \|\Phi_x\| < K$
$\operatorname{diam} X \leq R$
$|X| \leq \sigma$
C
B
A
$[\hat{A}, \hat{B}] = 0$
$\hat{A} \longrightarrow \hat{A}(t) = e^{iHt} \hat{A} e^{-iHt}$
$[\hat{A}(t), B] = 0$
decay

LIEB-ROBINSON BOUNDS

$f(t) = [A(t), B]$ $\quad \|f(t)\| < ?$

$f'(t) = -i[[A(t), H], B] = -i[U_t[A, H]U_t^\dagger, B] = -i\sum_X [U_t[A, \Phi_X]U_t^\dagger, B]$

$= -i\sum_{X_1 \in Z_1} [[A(t), \Phi_X(t)], B] = -i\sum_{X_1 \in Z_1} \{[A(t), [\Phi_X(t), B]$

$[H, e^{-iHt}] = 0$

$\|$

$U_t$

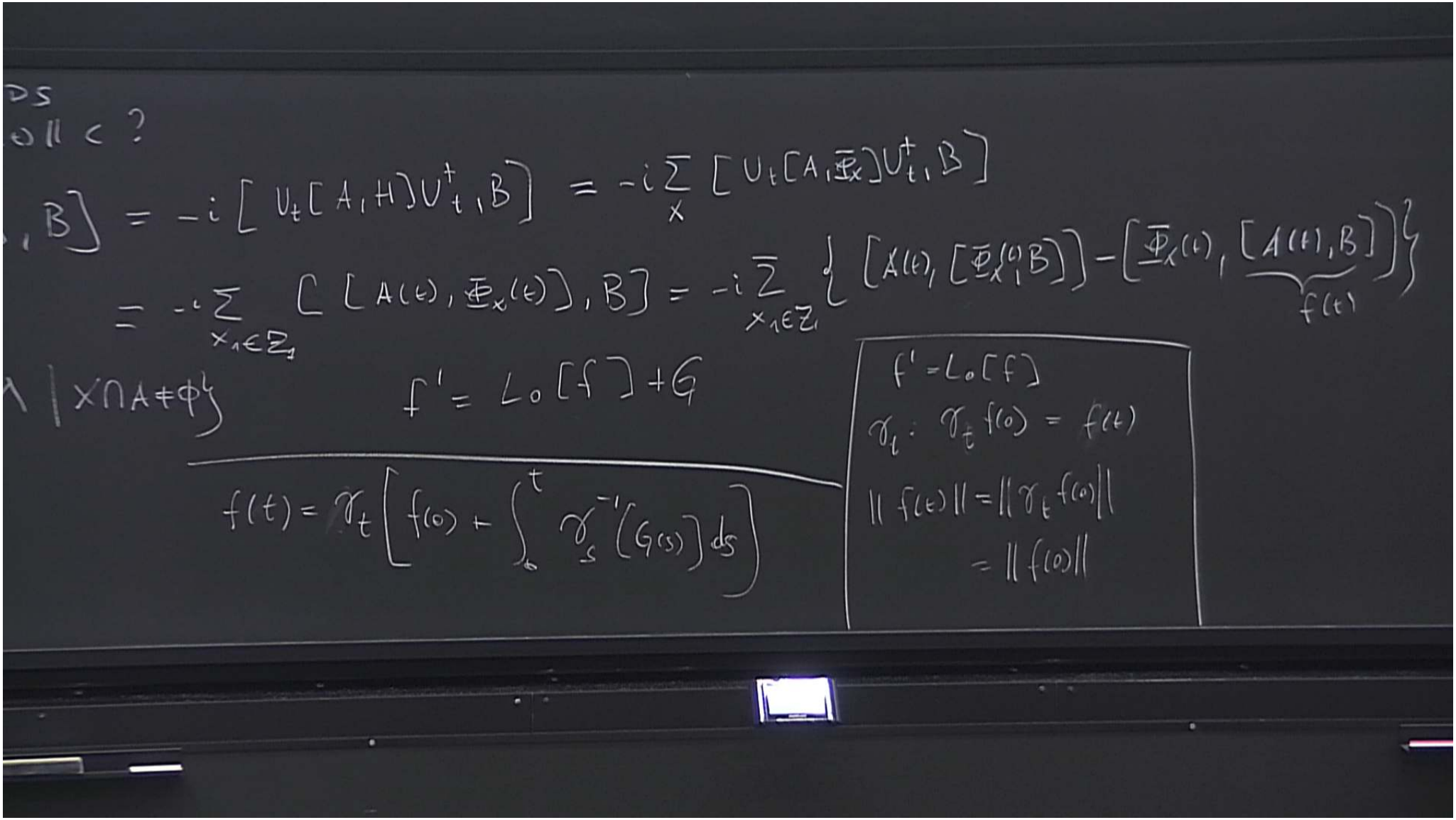
$Z_1 : \{X \subset \Lambda \mid X \cap A \neq \phi\}$

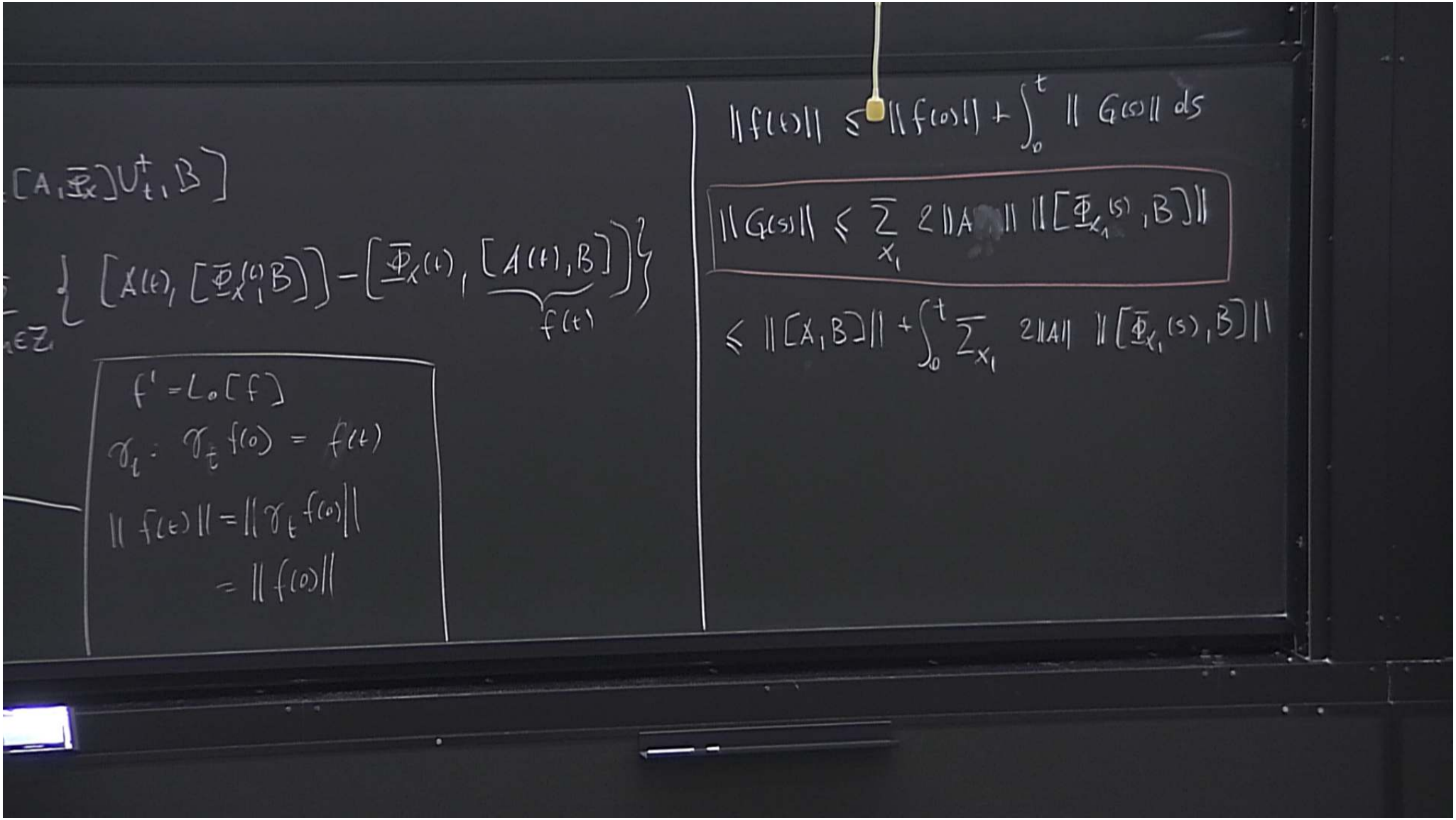
$< ?$

$$= -i\left[U_t[A, H]U_{t,1}^{\dagger}, B\right] = -i\sum_{X}\left[U_t[A, \Phi_X]U_{t,1}^{\dagger}, B\right]$$

$$= -i\sum_{X_1 \in \mathbb{Z}_1}\left[\left[A(t), \Phi_X(t)\right], B\right] = -i\sum_{X_1 \in \mathbb{Z}_1}\left\{\left[A(t), \left[\Phi_X(t), B\right]\right] - \left[\Phi_X(t), \underbrace{\left[A(t), B\right]}_{f(t)}\right]\right\}$$

$X \cap A \neq \phi\}$


DS
$\|\cdot\| <$ ?
$\dot{\tau}_t(A) = -i\,[H, \tau_t(A)]$
$\dot{\tau}_t(A) = \sum_{X \cap A \neq \emptyset} \dot{\tau}_t^{X}(A)$
$\dot{\tau}_t^{X}(A) = -i\,[H_X, \tau_t^{X}(A)]$
$\dot{\tau}_t(A) = \sum_{X \cap A \neq \emptyset} -i\,[H_X, \tau_t(A)]$
$f' = L_0[f] + G$
$f(t) = \gamma_t\left[f(0) + \int_0^t \gamma_s^{-1}[G(s)]\,ds\right]$
$f' = L_0[f]$
$\gamma_t : \gamma_t f(0) = f(t)$
$\|f(t)\| = \|\gamma_t f(0)\|$
$= \|f(0)\|$


$\|f(t)\| \leq \|f(0)\| + \int_0^t \|G(s)\|\,ds$
$\|G(s)\| \leq \sum_{x_1} 2\|A\|\,\|[\Phi_{x_1}(s), B]\|$
$\leq \|[A,B]\| + \int_0^t \sum_{x_1} 2\|A\|\,\|[\Phi_{x_1}(s), B]\|$
$[A, \Phi_x] U_{t,}^{\dagger} B]$
$\{[A(t), [\Phi_x(t) B]] - [\Phi_x(t), \underbrace{[A(t), B]}_{f(t)}]\}$
$\in \mathbb{Z}$
$f' = L_0[f]$
$\gamma_t : \gamma_t f(0) = f(t)$
$\|f(t)\| = \|\gamma_t f(0)\|$
$= \|f(0)\|$

$$f_1 = [\Phi_{X_1}(s), B]$$

$$f_1' = i \sum_{X_2 \in \mathcal{Z}_2} [[\Phi_{X_1}(s), \Phi_{X_2}(s)], B] = -i \sum_{X_2} \Big[ \Phi_{X_1}(s), [\Phi_{X_2}(s), B] \Big] - \Big[ \Phi_{X_2}(s), \underbrace{[\Phi_{X_1}(s), B]}_{f_1} \Big]$$

$$\mathcal{Z}_2 = \{ X \subset \Lambda \mid X \cap X_1 \neq \emptyset \text{ with } x_1 \in z_1 \}$$

$$\| f_1(s) \| \leq \| f_1(0) \| + \int_0^s \sum_{X_2} 2 \| \Phi_{X_1} \| \, \| [\Phi_{X_2}(s_1), B] \| \, ds_1$$

$$\| f_1(0) \| \leq 2 \| \Phi_{X_1} \| \, \| B \| \, s_1$$

$$\left[ \int_0^s \gamma_s(G(s))\, ds \right]$$

$$= \|f(0)\|$$

$$\|f(0)\| \leq \|[A,B]\| + \int_0^t \Sigma_{x_1} 2\|A\| \left\{ 2\|\Phi_{x_1}\|\|B\| + \int_0^s \Sigma_{x_2} 2\|\Phi_{x_1}\| \|[\Phi_{x_2}(s_1), B]\|\, ds_1 \right\}$$

$$\leq 2\|A\|\|B\|\Sigma_0 + 2\|A\|\|B\|\Sigma_1 t K + 2\|A\|\|B\|\Sigma_2 \int_0^t \int_0^s 2^2 K^2 + \dots$$

$$\left[\int_0^t \gamma_s(G(s))\,ds\right] \qquad = \|f(0)\|$$

$$\|f(0)\| \le \|[A,B]\| + \int_0^t \Sigma_{x_1} 2\|A\| \left\{ 2\|\Phi_{x_1}\|\,\|B\| + \int_0^s \Sigma_{x_2} 2\|\Phi_{x_1}\|\,\|[\Phi_{x_2}(s_1),B]\|\,ds_1 \right\}$$

$$\le 2\|A\|\,\|B\|\Sigma_0 + 2\|A\|\,\|B\|\Sigma_1 2t\,K + 2\|A\|\,\|B\|\,\Sigma_2 \int_0^t \int_0^s 2^2K^2 + \cdots$$

$$= 2\|A\|\,\|B\| \sum_{n=0}^{\infty} 2^n t^n \Sigma_n \int_0^t \int_0^{s_1} \cdots \int_0^{s_{n-1}} = 2\|A\|\,\|B\| \sum_{n=0}^{\infty} \frac{2^n K^n t^n}{n!} \Sigma_n$$

$\Sigma_n$ ↑ # paths of length $n$ from $A$ to $B$

Dyson series

$$\int_0^t \int_0^{s_1} \cdots \int_0^{s_{n-1}} = \frac{1}{n!} \int_0^t$$

$\| \phi_{x_1} \| \| [\phi_{x_2}(s_1), B] \| ds_1 \}$

$d(A,B) \equiv d$

$d \overset{?}{\gtrless} nR$

$\leq$

$\leq c^2 K^2 \ldots$

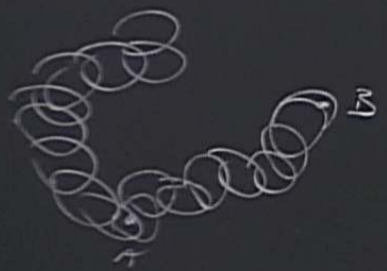
$\sum_{n=0}^{\infty} \frac{c^n K^n t^n}{n!} \Sigma_n$

- $d > nR \Rightarrow \Sigma_n = 0$
- $d < nR$ $\quad \Sigma_n \leq c \bar{\Sigma}_{tot} \leq c \sigma^n$

$\sigma < 1$

$c = e^{\alpha(nR-d)} > 1$

$\forall \alpha > 0$



$$\|[A(t), B]\| \leq 2\|A\|\,\|B\| \sum_{n=0}^{\infty} \frac{2^n K^n \sigma^n t^n e^{\mu n R}}{n!} e^{-\mu d}$$

$$= 2\|A\|\,\|B\| \; e^{\mu(2K\sigma t e^{\mu R} - d)} = 2\|A\|\,\|B\| \, e^{\frac{\mu}{a}(vt - d)}$$

$$v \equiv 2K\sigma \frac{e^{\mu R}}{\mu}$$

$$d \to \frac{d}{a}$$

$$v \to \frac{v}{a}$$