

Title: Lie algebraic formulation of tt* equations and topological strings

Date: Jan 22, 2015 11:00 AM

URL: <http://pirsa.org/15010132>

Abstract: <p>The tt* equations define a flat connection on the moduli spaces of 2d, N=2 quantum field theories. For conformal theories with c=3d, which can be realized as nonlinear sigma models into Calabi-Yau d-folds, this flat connection is equivalent to special geometry for threefolds and to its analogs in other dimensions. I will show that the non-holomorphic content of the tt* equations in the cases d=1,2,3 is captured in terms of finitely many generators of special functions, which close under derivatives. The generators are understood as coordinates on a larger moduli space. This space parameterizes a freedom in choosing representatives of the chiral ring while preserving a constant topological metric. Geometrically, the freedom corresponds to a choice of forms on the target space respecting the Hodge filtration and having a constant pairing. Linear combinations of vector fields on that space are identified with generators of a Lie algebra. This Lie algebra replaces the anti-holomorphic derivatives of tt* and provides these with a finer and algebraic meaning. The generators of the differential rings of special functions are given by quasi-modular forms for d=1 and their generalizations in d=2,3. For d=3, this can be used to provide a purely Lie algebraic formulation of the higher genus topological string theory amplitudes and of the BCOV holomorphic anomaly equations which govern them.</p>



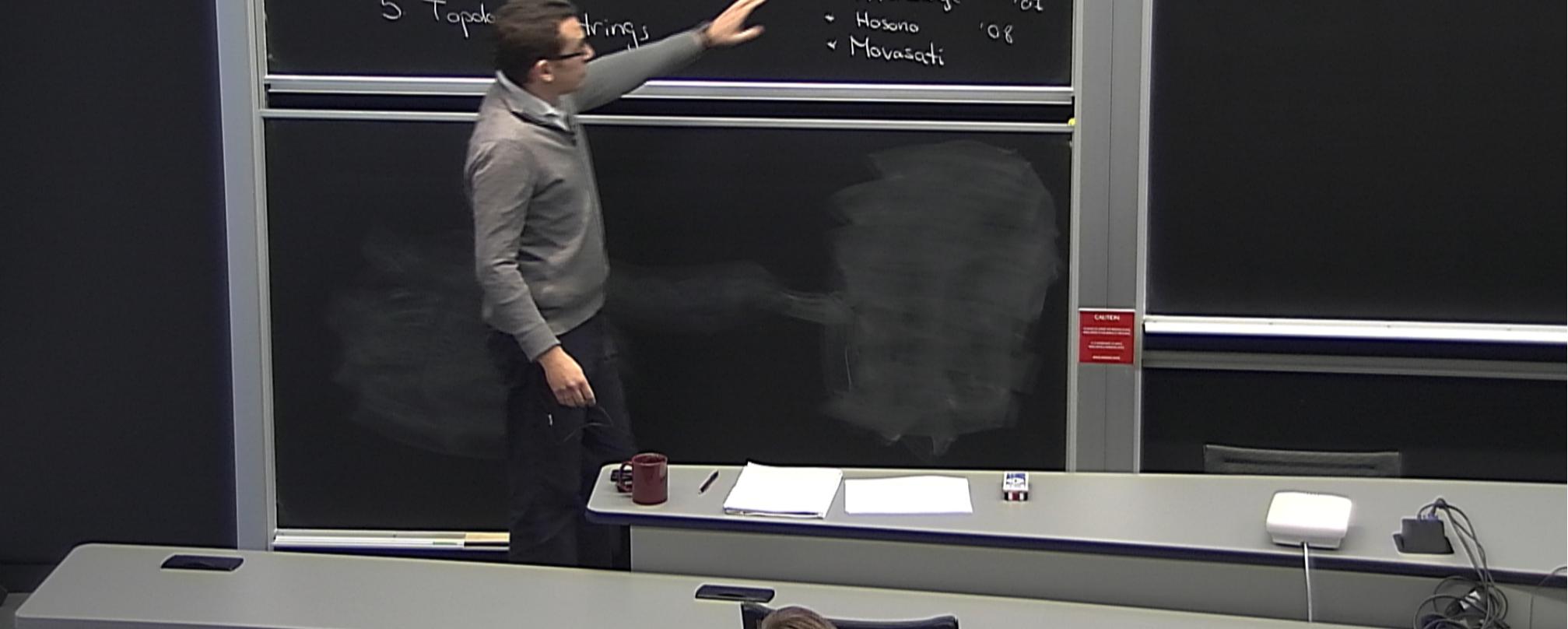
$t\bar{t}^*$ equations of topological strings

- Plan:
1. Introduction & Motivation
 - 2 Chiral ring & $t\bar{t}^*$ equation,
 3. Geometric realization
 4. Lie-algebraic structure
 5. Topological strings

also
1410.1889
with Mavasati,
Schedegger
& Yau

related

- Yamaguchi & Yau '04
- MA & Lüngu '02
- Hosono '08
- ✓ Mavasati



- Plan:
1. Introduction & motivation
 2. Chiral ring & H* equation,
 3. Geometric realization
 4. Lie-algebraic structure
 5. Topological strings

with
Movasati,
Scheidegger
& Yau

related

- Yamaguchi & Yau '04
- MA & Länge '07
- Hosono '08
- Movasati



MA 1412, 3454

also

1410.1889

with Movasati,
Scheidegger
& Yau

related

- Yamaguchi & Yau '04
- MA & Lüange '07
- Hosono '08
- Movasati

H^* equations of topological strings

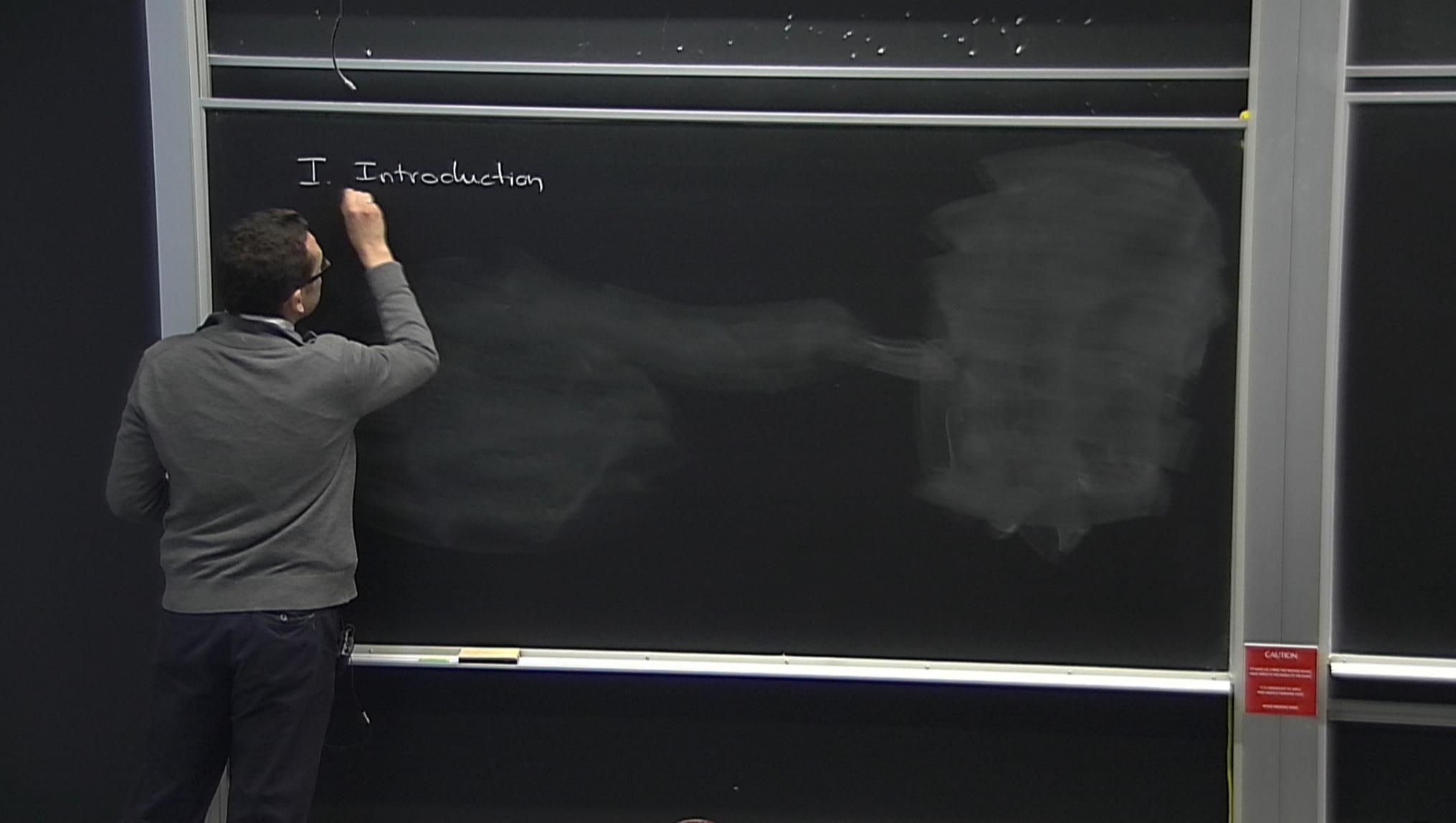
Plan. 1. Introduction & Motivation

2. Chiral ring & H^* equation,

3. Geometric realization

4. Lie-algebraic structure

5. Topological strings



A man with dark hair and glasses, wearing a grey sweater over a collared shirt, stands facing a chalkboard. He is writing the title "I. Introduction" in white chalk. The chalkboard is dark and shows some faint, illegible markings from previous use. A red caution sign is visible on the right side of the chalkboard frame. The background includes a window and a door.

I. Introduction

I. Introduction,

2d $N=(2,2)$ ~~CFT~~ & realization as NLSM
into CY ch-fields

X

CAUTION
DO NOT CLIMB THE SCREEN
TO PREVENT DAMAGE TO THE SCREEN
OR PERSONAL INJURY

I. Introduction,

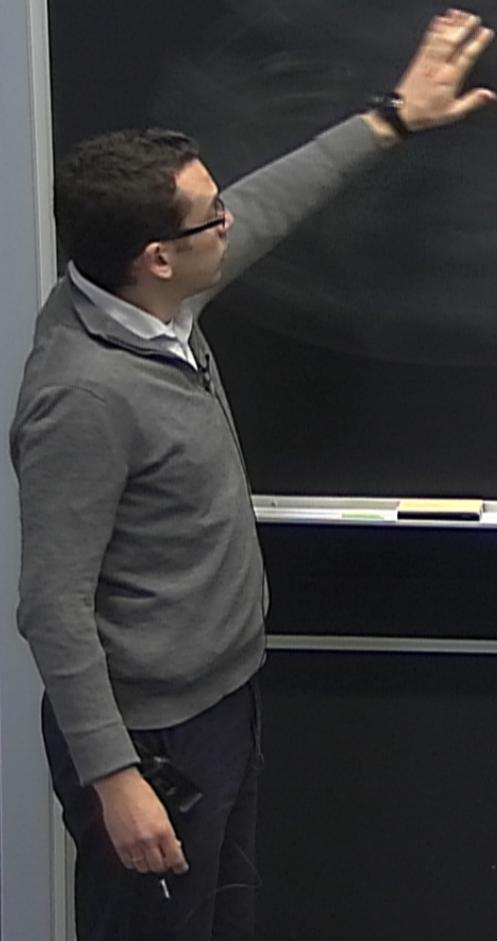
2d $N=(2,2)$ SCFT & realization as NLSM
into CY ch-fields
 X_t, X_z

I. Introduction,

- * 2d $N=(2,2)$ SCFT & realization as NLSM
into CY ch-fields
 X_t, X_z

I. Introduction,

* 2d $N=(2,2)$ SCFT & realization as NLSM
into CY d-fields
 X_t, X_z



CAUTION
DO NOT USE LASER LIGHT SOURCE
FOR THIS BOARD
DO NOT USE LASER LIGHT SOURCE
FOR THIS BOARD

I. Introduction,

* 2d $N=(2,2)$ SCFT & realization as NLSM

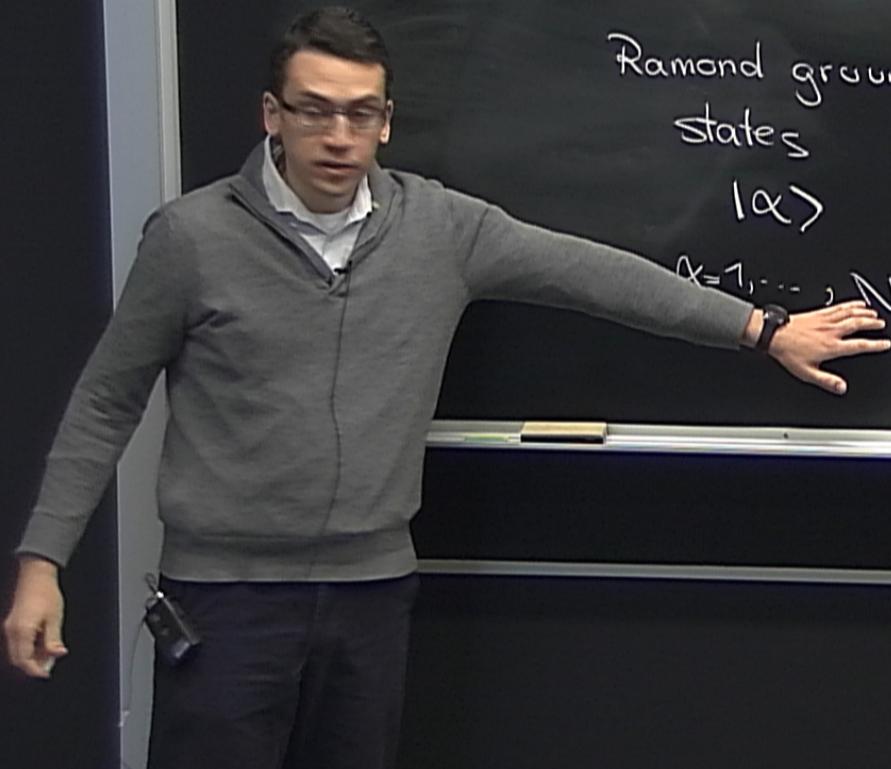
into CY 6-fields

$$X_t, X_z$$

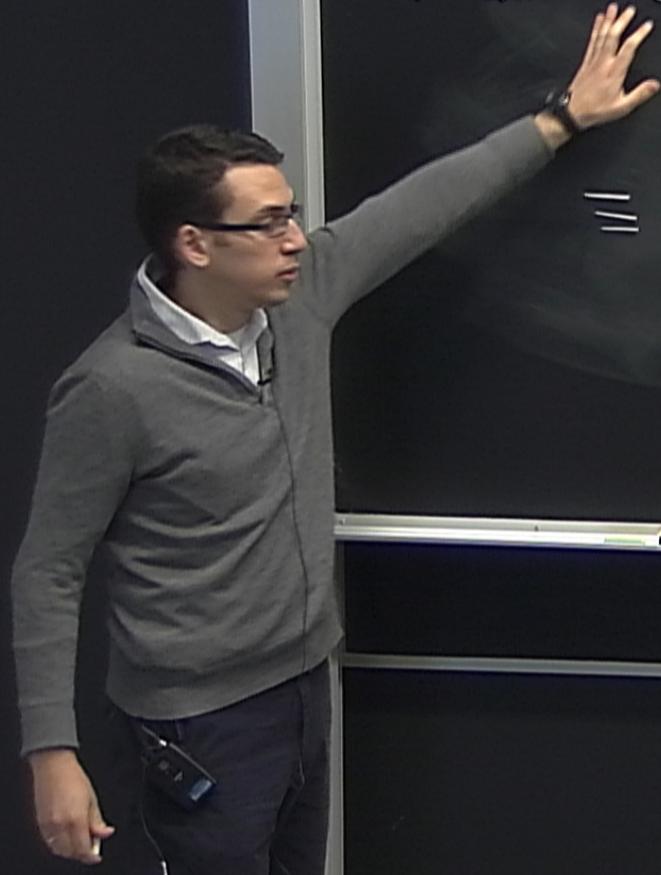
Ramond ground
states

$$|\alpha\rangle$$

$$\alpha=1, \dots, N$$



CAUTION



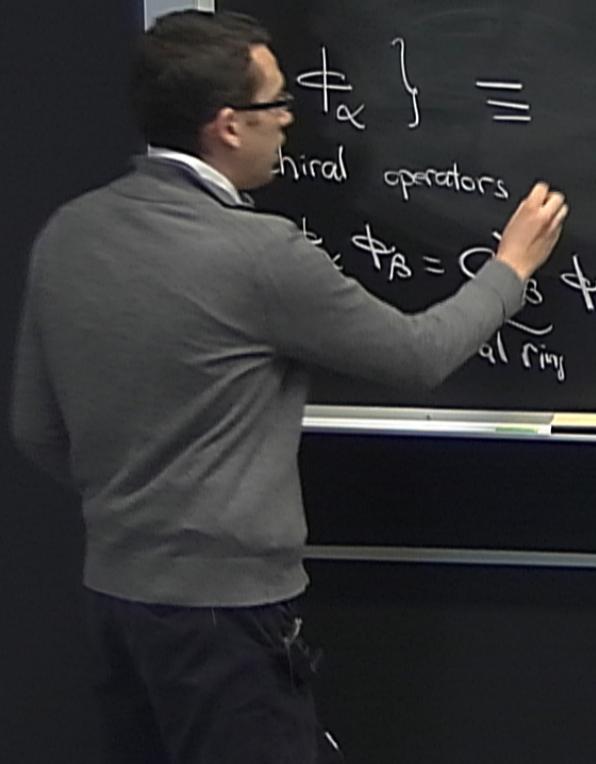
I. Introduction,
2d $N=(2,2)$ SCFT & realization as NLSM
into CY d-fields
 X_t, X_z

= Ramond ground
states
 $|\alpha\rangle$
 $\alpha=1, \dots, N$

I. Introduction

* 2d $N=(2,2)$ SCFT & realization as NLSM
into CY d-fields

$$\begin{aligned} \left\{ \phi_\alpha \right\} &= \text{Ramond ground states} \\ \text{chiral operators} & \quad | \alpha \rangle \quad \equiv \quad \left\{ \phi_{\bar{\alpha}} \right\} \\ \phi_B = C \sum_B \phi_B & \quad \alpha = 1, \dots, N \\ \text{anti-chiral } (\alpha) & \end{aligned}$$



* Deformation families



* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

(a)

folds

\approx

D

(a)

CAUTION
TO AVOID INJURY OR PROPERTY DAMAGE,
DO NOT STAND IN THE DIRECT LINE OF THE BEAM.
DO NOT STAND IN THE DIRECT LINE OF THE BEAM.
DO NOT STAND IN THE DIRECT LINE OF THE BEAM.

* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

—

SM

Y off-fields

, X_z

$\bar{\alpha}$ }
chiral (α)

CAUTION

CAUTION

* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

—

X_t, X_z

$$\{ \phi_{\bar{\alpha}} \}$$

anti-chiral (α)



* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

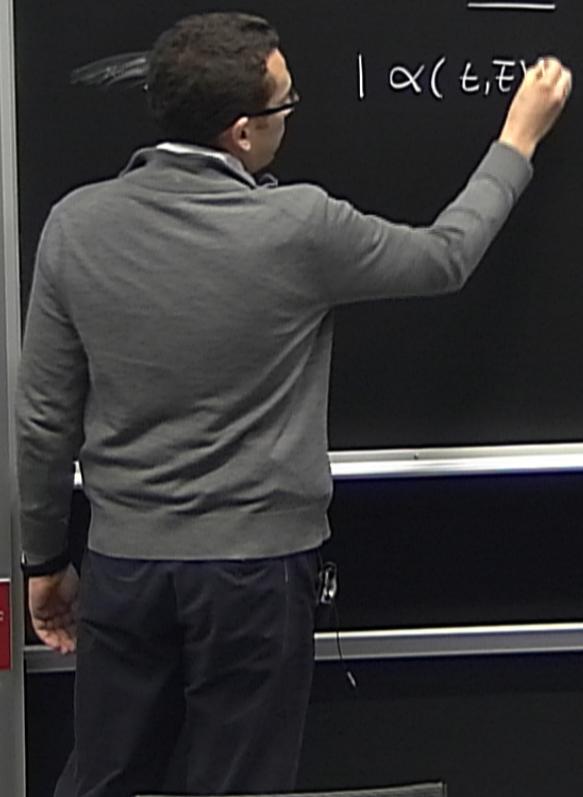
$$| \alpha(t, \bar{t})$$

as NLSM
into CY fields

$$\tilde{X}_t, X_{\bar{t}}$$

$$\{\phi_{\bar{\alpha}}\}$$

anti-chiral (α)



* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

$$| \alpha(t, \bar{t}) \rangle \quad \text{Berry connection}$$

$$D_i = \partial_i - A_i$$

as NLSM

into CY 0-fields

$$\tilde{X}_t, X_z$$

$$\{\phi_{\bar{\alpha}}\}$$

anti-chiral (α)



* Deformation families , marginal def. moduli space M

$$\phi_i \quad i=1, \dots, n \leq N$$

$$|\alpha(t, \bar{t})\rangle$$

Berry connection

$$D_i = \partial_i - A_i$$

as NLSM

into CY 0-fields

$$X_t, X_z$$

$$Y \begin{pmatrix} + \\ - \end{pmatrix}$$

anti-chiral (α)

* Deformation families , marginal def. moduli space M

$$+_i \quad i=1, \dots, n \leq N$$

* $|\alpha(t, \bar{t})\rangle$ Berry connection

$$D_i = \partial_i - A_i$$

* H^* equations.

$$[F_{ij}, \omega_{\bar{i}}] = - [C_{ij}, \omega_{\bar{i}}]$$

* Deformation families, marginal def. moduli space M

$$+_i \quad i=1, \dots, n \leq N$$

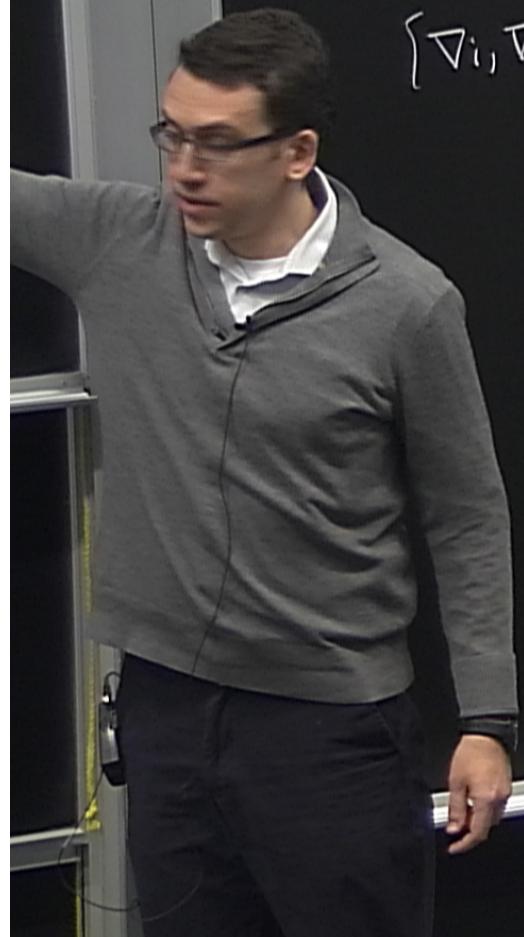
* $|\alpha(t, \bar{t})\rangle$ Berry connection

$$D_i = \partial_i - A_i$$

* H^* equations. $[D_i, D_{\bar{i}}] = -[C_i, C_{\bar{i}}]$

$$\nabla_i = D_i + \xi C_i, \quad \nabla_{\bar{i}} = D_{\bar{i}} + \frac{1}{\xi} C_{\bar{i}}$$

$$[\nabla_i, \nabla_j] = 0 \quad , \quad [\nabla_i, \nabla_{\bar{j}}] = 0, \quad [\nabla_{\bar{i}}, \nabla_{\bar{j}}] = 0$$



$$[\nabla_i, \nabla_j] = 0 \quad , \quad [\nabla_i, \nabla_j] = 0, \quad [\nabla_i, \nabla_j] = 0$$

Berry connection

$$D_i = \partial_i - A_i$$

$$\tilde{D}_j = -[C_i, G_i]$$

$$D_T + \frac{1}{\xi} C_T$$

G

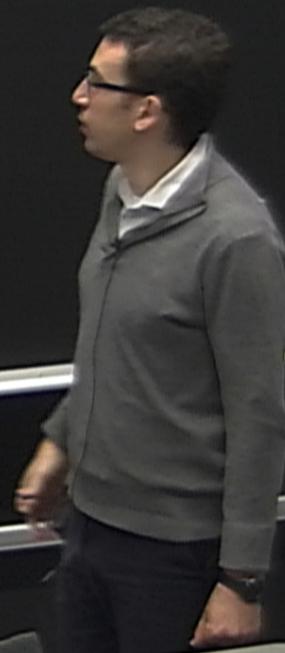
Berry connection

$$D_i = \partial_i - A_i$$

$$\bar{D}_j = - [C_i, G_i]$$
$$D_{\bar{i}} + \frac{1}{2} C_{\bar{i}}$$

$$[\nabla_i, \nabla_j] = 0 \quad , \quad [\nabla_i, \nabla_j] = 0, \quad [\nabla_i, \nabla_j] = 0$$

Geometric realization



$$[\nabla_i, \nabla_j] = 0 \quad , \quad [\nabla_i, \nabla_k] = 0, \quad [\nabla_i, \nabla_l] = 0$$

Berry connection

$$\omega_i = \partial_i - A_i$$

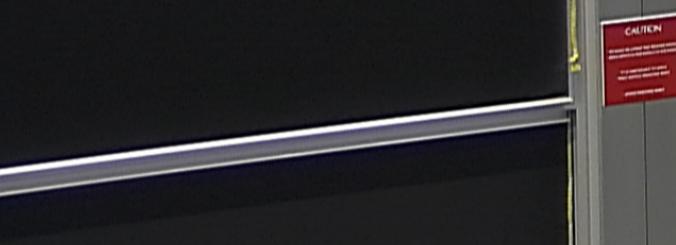
$$J = -[C_i, C_j]$$

$$D_T + \frac{1}{S} C_T$$

Geometric realization

chiral ring \Rightarrow VHS

flat



LSM

CY orbifolds

X_z

$P_{\bar{\alpha}}$

-chiral (α)

Answer: Consider a far moduli space

CAUTION
DANGER OF EXPLOSION
DO NOT SPARKLE OR SMOKE
IN THIS AREA

LSM

CY 6-folds

X_z

$\mathbb{P}_{\bar{\alpha}} \}$

-chiral (α)

Answer:

Consider a larger moduli space T

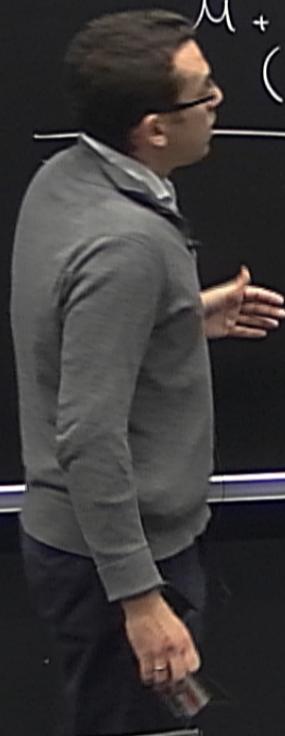
$= M + \text{extra data}$
(

LSM
CY 6-folds
 X_7
 $\mathbb{P} \bar{\alpha} \}$
-chiral (α)

Answer: Consider a larger moduli space \bar{T}

$M + \text{extra data}$

(choices of representative of chiral ring)



LSM
CY 6-folds

X_z

$\bar{\alpha}$

-chiral (α)

Answer: Consider a larger moduli space T

$= M + \text{extra data}$

(choices of orientation of chiral ring)

$$\nabla_{\bar{t}} \rightarrow \partial_{\bar{a}}$$



CAUTION

LSM
CY 6-folds

X_z

$\mathbb{P}^{\infty}_{\alpha}$

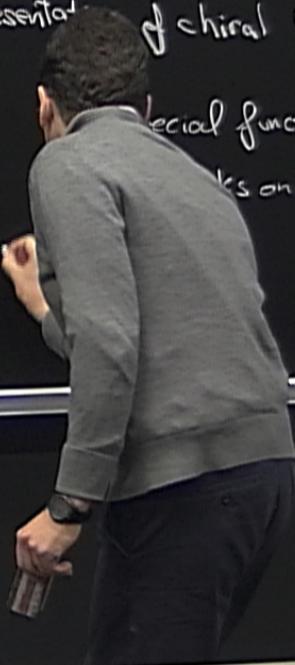
-chiral (α)

Answer: Consider a larger moduli space T

$= M + \text{extra data}$

(choices of representation of chiral ring)

$\nabla_{\bar{t}} \rightarrow \partial_g$, special functions
works on T



LSM

CY 4-folds

X_2

$\bar{\alpha}$

-chiral (α)

Answer: Consider a larger moduli space T

$= M + \text{extra data}$

(choices of representative of chiral ring)

$\nabla_{\bar{\alpha}} \rightarrow \partial_g$, g special functions
Gordmarks on T

$\{\partial_g\}$ form a Lie algebra

I. Introduction,

* 2d $N=(2,2)$ SCFT & realization as NLSM
into CY ch-fields

$$\begin{aligned} \{ \phi_\alpha \} &\equiv \text{Ramond ground states} & \equiv \{ \phi_{\bar{\alpha}} \} \\ \text{chiral operators } (\zeta) && | \alpha \rangle && \text{anti-chiral } (\alpha) \\ \phi_\alpha \phi_\beta = C_{\alpha\beta}^\gamma \phi_\gamma && \alpha, \beta, \gamma = 1, \dots, N && \text{chiral ring} \end{aligned}$$

Answer: Consider

$$= M +$$



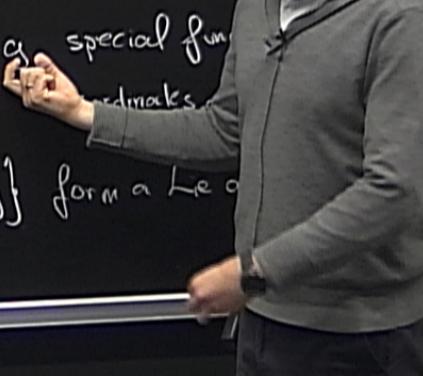
Answer: Consider a larger moduli space \bar{T}

$= M + \text{extra data}$

(choices of representative of chiral)

$$\nabla_{\bar{T}} \rightarrow \partial g , \quad \begin{matrix} \text{as special fun} \\ \text{cardmarks} \end{matrix}$$

$\{\partial g\}$ form a Lie o



Answer: Consider a larger moduli space \bar{T}

$= M + \text{extra data}$

(choices of representative of chiral ring)

$\nabla_{\bar{T}} \rightarrow \partial_g$, g special functions
coordinates on \bar{T}

$\{\partial_g\}$ form a Lie algebra



∞ space \overline{T}

(relative of chiral ring)

g special functions
coordinates on T

$\partial g_j \}$ form a Lie algebra

why?

1. Solutions to $\partial^* \partial$ equations
2. In



\mathbb{R} space \overline{T}

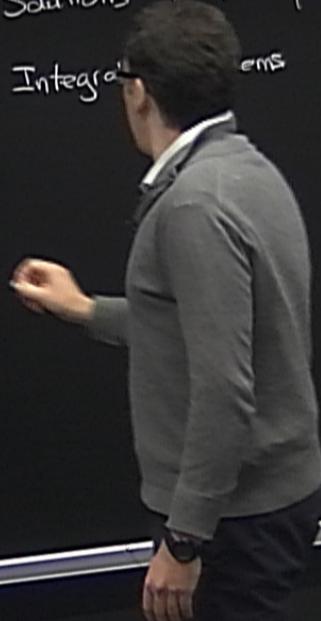
(relative of chiral ring)

as special functions
coordinates on T

$\partial_g \}$ form a Lie algebra

why?

1. Solutions to \mathfrak{t}^* equations
2. Integrable ^{ems}
- 3.



∞ space \overline{T}

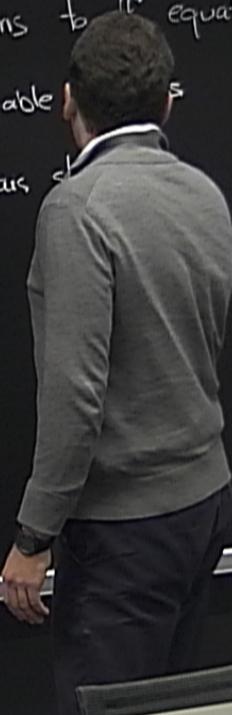
(relative of chiral ring)

as special functions
coordinates on T

$\partial_g \}$ form a Lie algebra

why?

1. Solutions to Lie^* equations
2. Integrable systems
3. Algebras of



CAUTION

∞ space \overline{T}

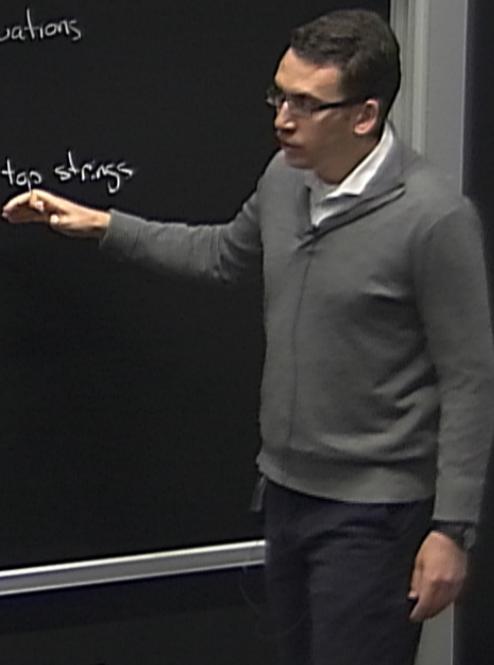
(relative of chiral ring)

as special functions
coordinates on T

$\partial g_j \}$ form a Lie algebra

why?

1. Solutions to ff^* equations
2. Integrable systems
3. Algebraic structure of top strings



CAUTION

∞ space T

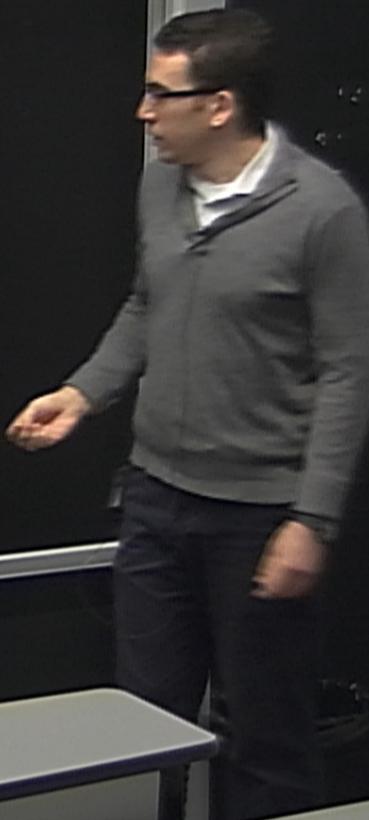
(relative of chiral ring)

g special functions
coordinates on T

$\partial g_j \}$ form a Lie algebra

why?

1. Solutions to fl^{*} equations
2. Integrable systems
3. Algebraic structure of top strings
(Definition)



CAUTION

∞ space \overline{T}

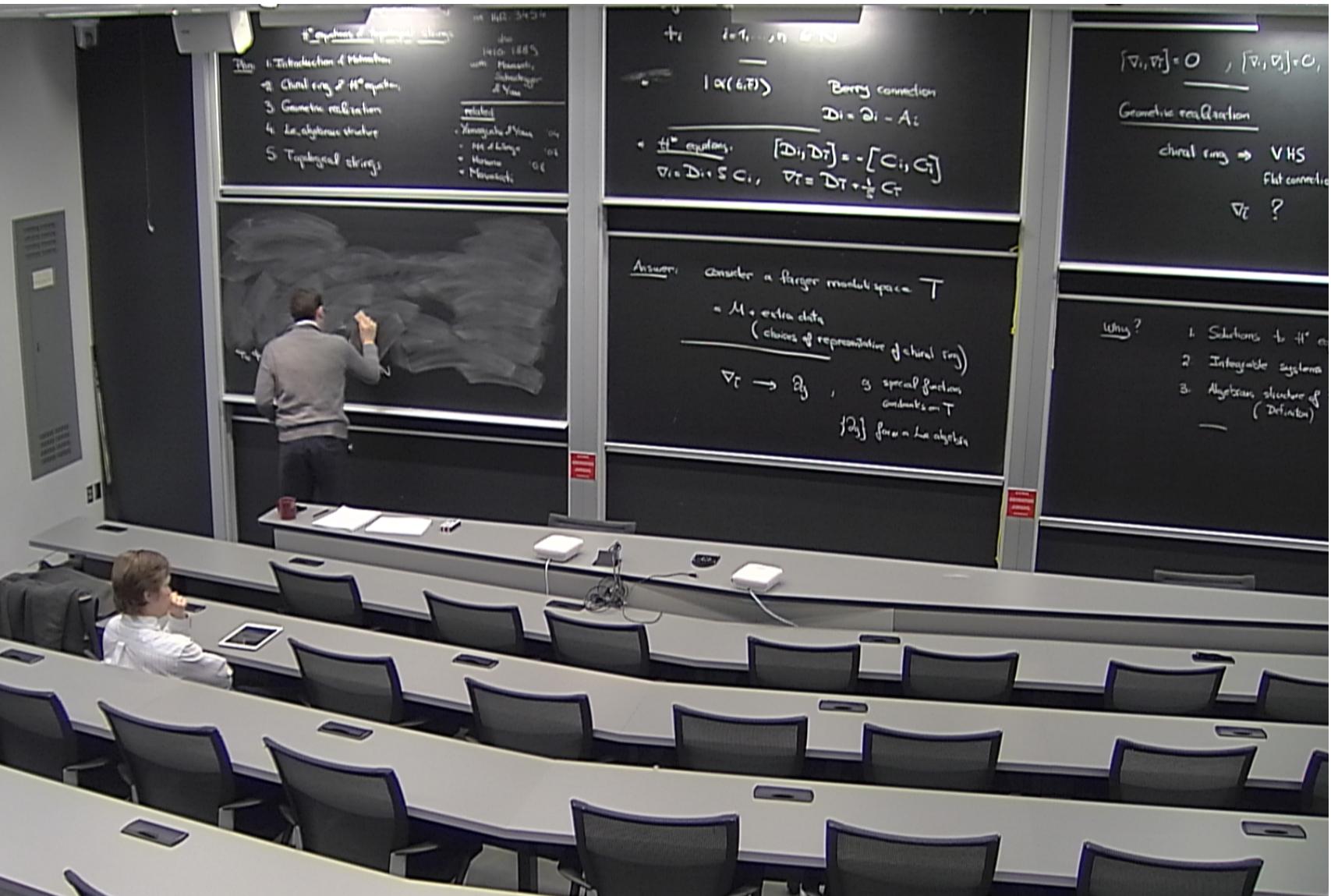
state of chiral

as special fun
coordinates

$\partial g_j \}$ form a L

why?

1. Solutions to \bar{f}^* equations
 2. Integrable systems
 3. Algebraic structure of top strings
(Definition)
-



Plan:

1. Introduction of Mathematics
2. Challenging of H¹ equation
3. Geometric realization
4. Lie-algebraic structure
5. Topological strings

on 14.10.2014
with
Mozaffari,
Schmidgasser
& Vass

Related:

- Yamagishi & Vass
- M. d'Leeuw
- Horava
- Mousavchi

$$\begin{aligned} & \text{+ } i \alpha(6, i) \quad \text{Berry connection} \\ & D_i = \partial_i - A_i \\ & + \text{ H}^1 \text{ eqns.} \quad [D_i, D_j] = -[C_i, C_j] \\ & \nabla_i = D_i + S_i C_i, \quad \nabla_i = D_i + \frac{1}{2} C_i \end{aligned}$$

Answer: consider a larger moduli space T
 $\approx M + \text{extra data}$
 (choices of representative of chiral ring)
 $\nabla_T \rightarrow \partial_T$, 3 special functions
 coordinates on T
 $\{\partial_T\}$ form a Lie algebra

$$[\nabla_i, \nabla_j] = 0, \quad [\nabla_i, D_j] = 0,$$

Geometric realization

chiral ring \rightarrow VHS

flat connection

∇_C ?

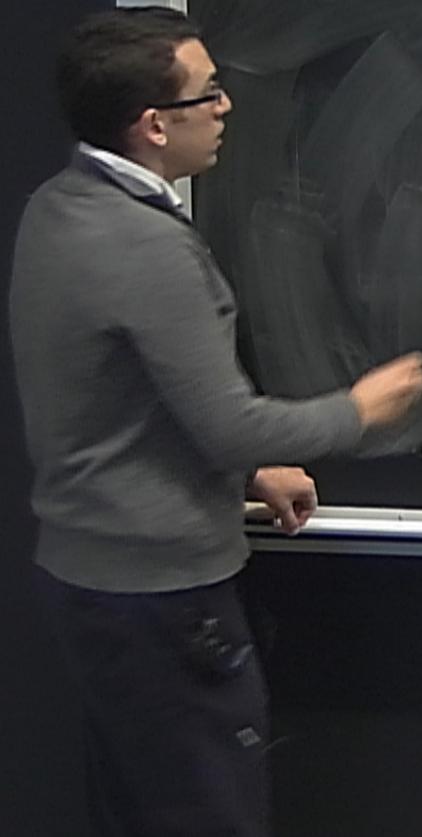
Why?

1. Solutions to H¹ eq
2. Integrable systems
3. Algebraic structure of (Definition)

2.1 Chiral ring

$N=2$

T G^+
 G^- J, c



2.1 Chiral rings

$N=2$

T

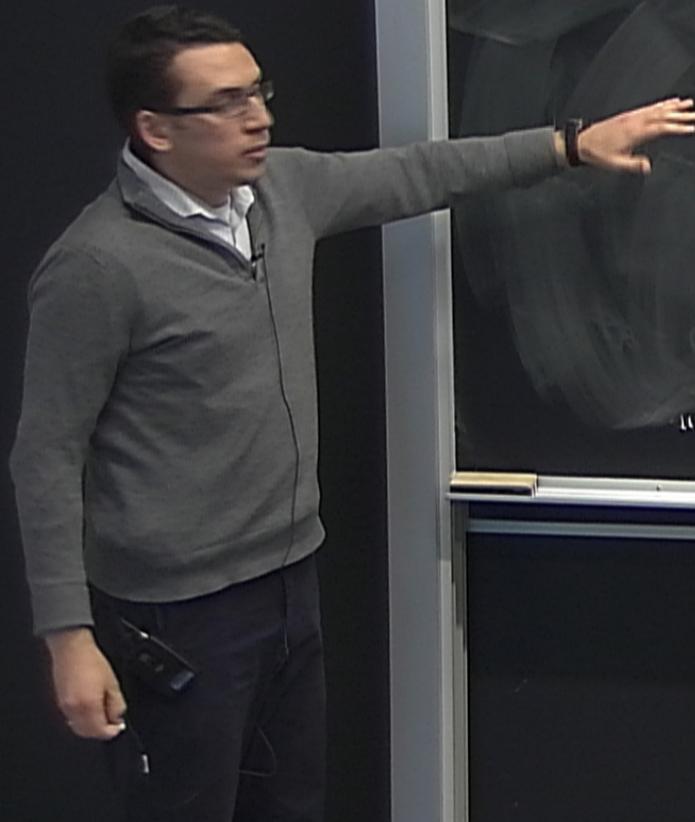
G^+

G^-

J, \subset

$h = 2, \frac{3}{2}$

I



CAUTION

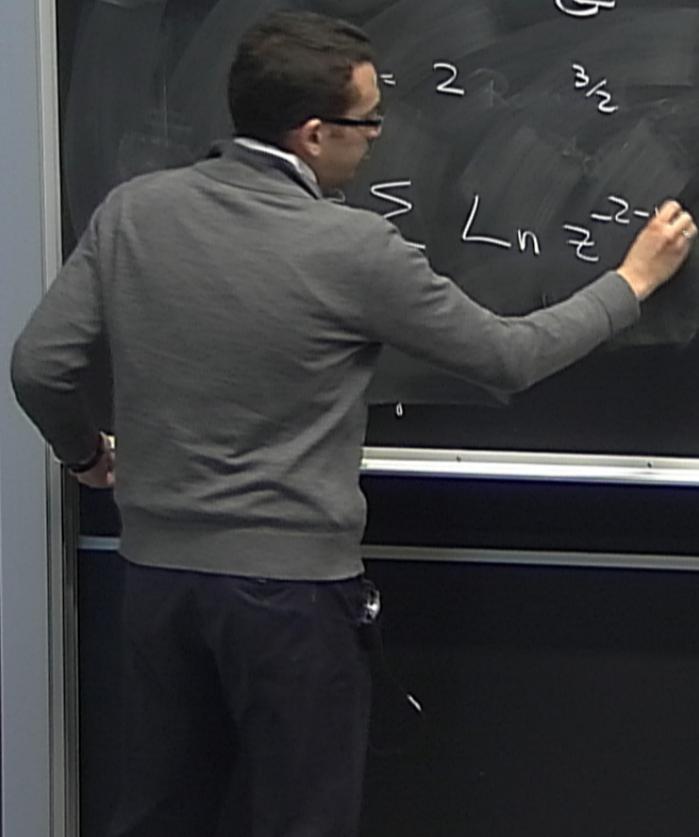
2.1 Chiral rings

$N=2$

T G^+
 G^- J, \subset

$= 2^{3/2}$

$$\sum L_n z^{-2-n}$$



CALUTION

2.1 Chiral rings

$N=2$

T

G^+

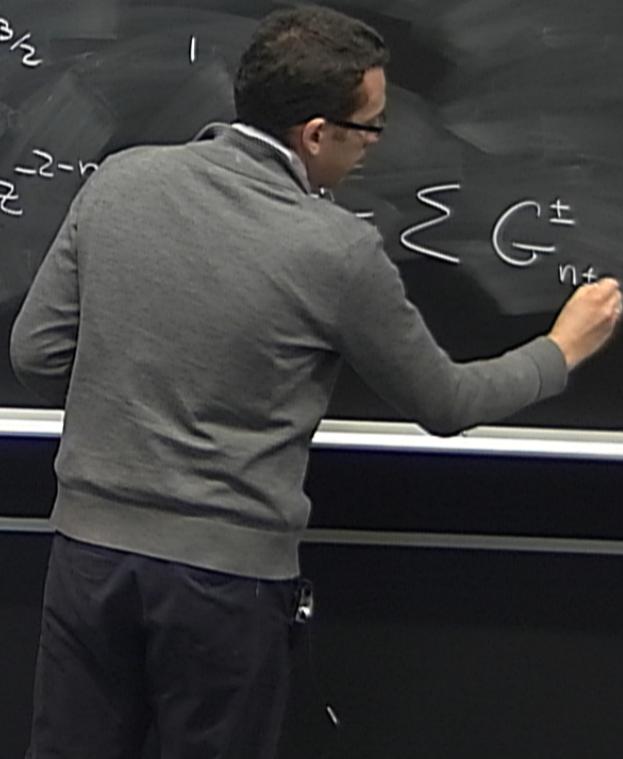
G^-

J, C

$h = 2 \frac{3}{2}$

$$T = \sum L_n z^{-2-n}$$

$$\sum G_{n+}^{\pm}$$



CAUTION

2.1 Chiral rings

$N=2$

T

G^+

G^-

J, <

$h = 2 \quad \frac{3}{2}$

I

$$J = \sum J_n z^{-n-1}$$

$$T = \sum L_n z^{-2-n} \quad , \quad G^\pm = \sum G_{(n \pm \alpha)}^\pm z^{-\frac{3}{2}-n}$$

2.15 Chiral rings

$N=2$

$T \quad G^+$

G^-

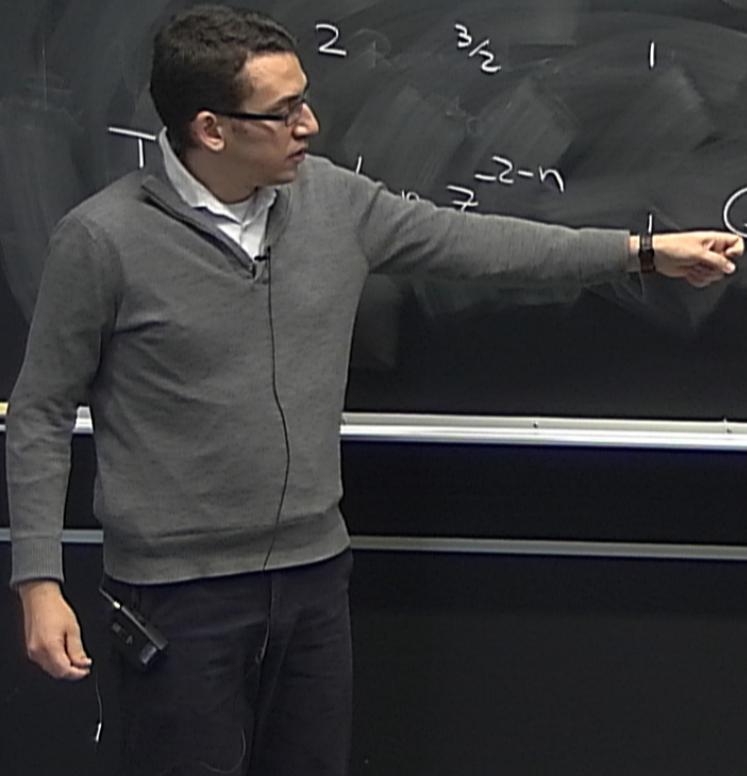
J, C

$2 \quad \frac{3}{2}$

1

$$J = \sum J_n z^{-n-1}$$

$$G^\pm = \sum G_{(n \pm \alpha)}^\pm z^{-\frac{3}{2} - n}$$



CAUTION
DO NOT CLIMB THE REAR SCREEN
TO PREVENT DAMAGE TO THE SCREEN
AND POSSIBLE INJURY

2.1) Chiral rings

$$N=2 \quad T \quad G^+ \quad G^- \quad J, C$$

$$h = 2 - \frac{3}{2} \quad 1$$

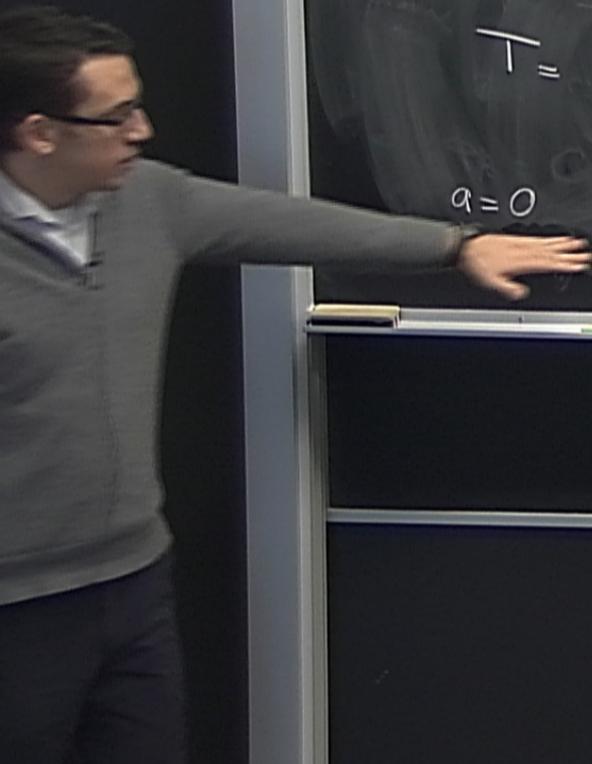
$$J = \sum J_n z^{-n-1}$$

$$T = \sum L_n z^{-2-n} \quad , \quad G^\pm = \sum G_{(n\pm\alpha)}^\pm z^{-\frac{3}{2}-n}$$

\downarrow

$$0 \leq \alpha < 1$$

$a=0$ Ramond sector



CAUTION
DO NOT OPERATE EQUIPMENT
WHILE DRIVING OR OPERATING A MOTOR VEHICLE.
DO NOT OPERATE EQUIPMENT
WHILE DRIVING OR OPERATING A MOTOR VEHICLE.

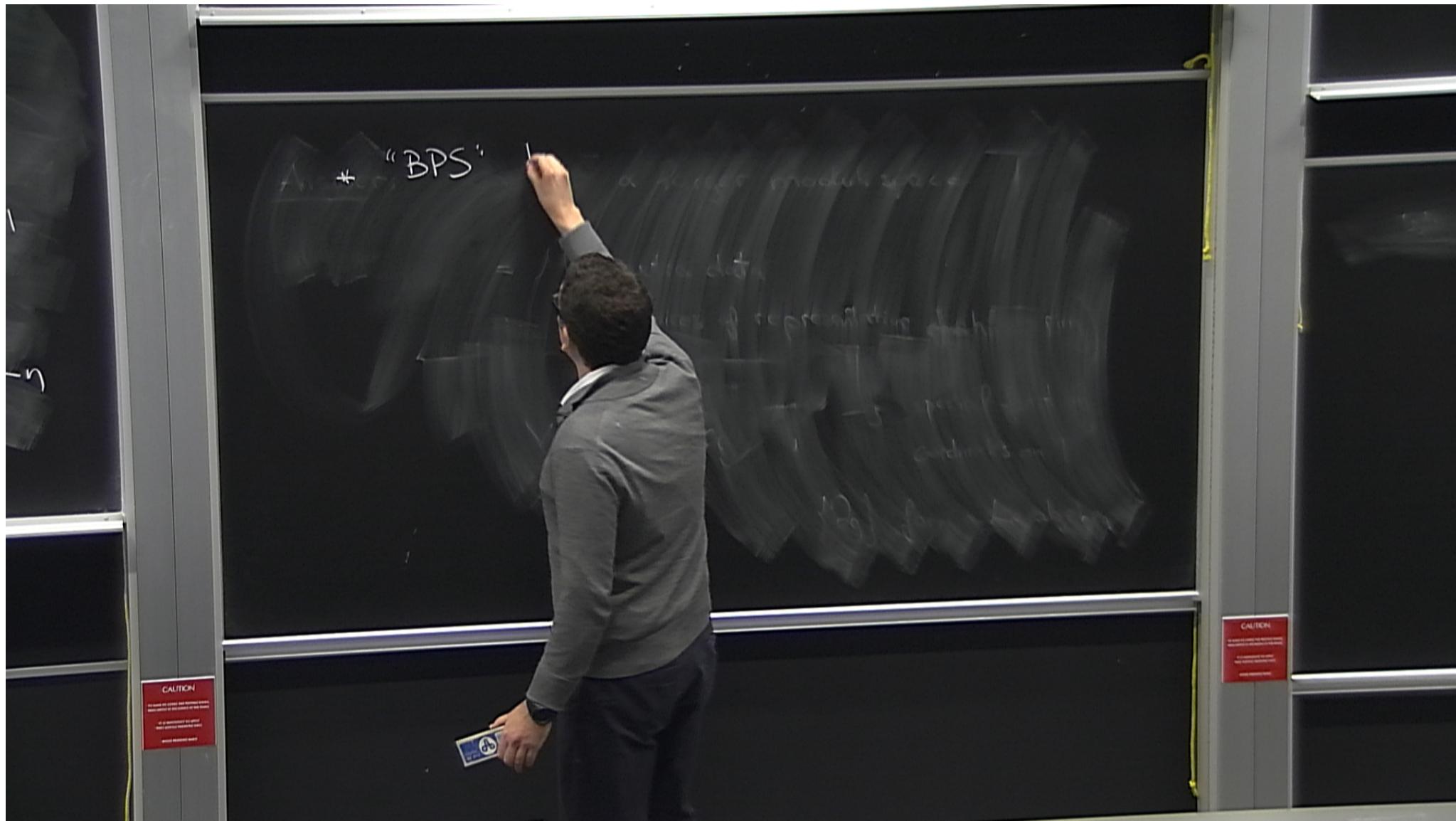
2.1 Chiral rings

$$N=2 \quad T \quad G^+ \quad G^- \quad J, c$$

$$h = 2 - \frac{3}{2} \quad 1$$

$$J = \sum J_n z^{-n-1}$$

$$T = \sum_{\alpha=0}^1 I \quad z^{-2-\eta} \quad , \quad G^\pm = \sum_{0 \leq \alpha < 1} G_{(n \pm \alpha)}^\pm z^{-\frac{3}{2}-\eta}$$



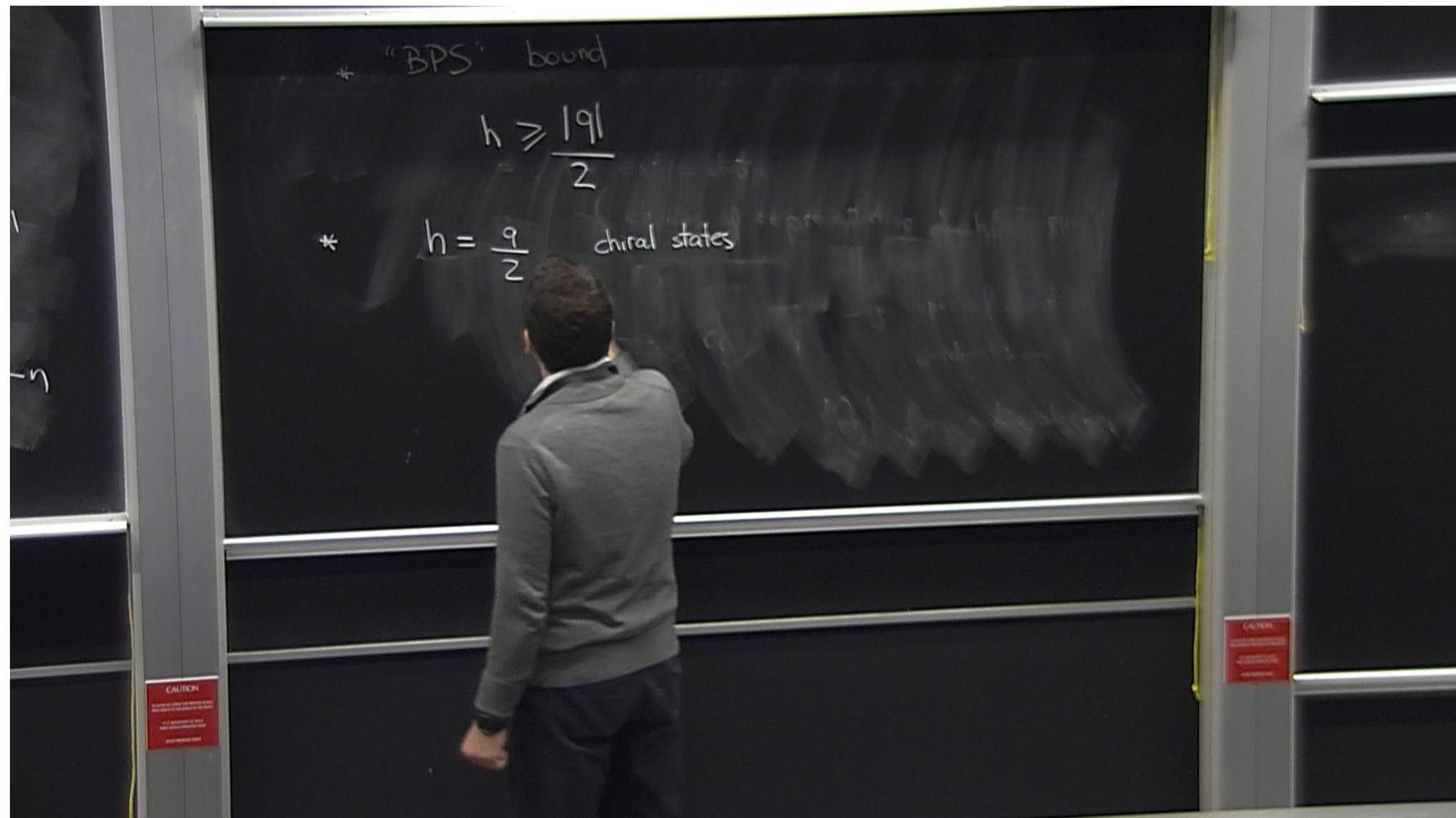
* "BPS" bound

$$h \geq \frac{|q|}{2}$$

* "BPS" bound

$$h \geq \frac{19}{2}$$

* $h = \frac{9}{2}$ chiral states



$a = \nu_2$ NS

* "BPS" bound

$$h \geq \frac{|q|}{2}$$

* $h = \frac{q}{2}$ chiral states

$$\Leftrightarrow G_{-\nu_2}^+ |\phi\rangle = 0$$



CAUTION
TO CLIMB OR SIT ON CHALKBOARD
CABINET OR ANY PART OF THE STAND
IS DANGEROUS AND
CAN CAUSE SERIOUS INJURY
AND POSSIBLE DEATH

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TO CLIMB OR SIT ON CHALKBOARD
CABINET OR ANY PART OF THE STAND
IS DANGEROUS AND
CAN CAUSE SERIOUS INJURY
AND POSSIBLE DEATH

$$\phi_\alpha \phi_\beta = C^\gamma \phi_\gamma$$

(chiral)

$$h = \frac{-q}{2}$$

$$G_{-\gamma_2}^- |+\rangle = 0$$

$$\phi_\alpha \phi_\beta = C_{\alpha\beta}^\gamma \phi_\gamma$$

(chiral ring)

$$h = \frac{-g}{2}$$

$$G_{-\gamma_2}^{-1} |+\rangle = 0$$

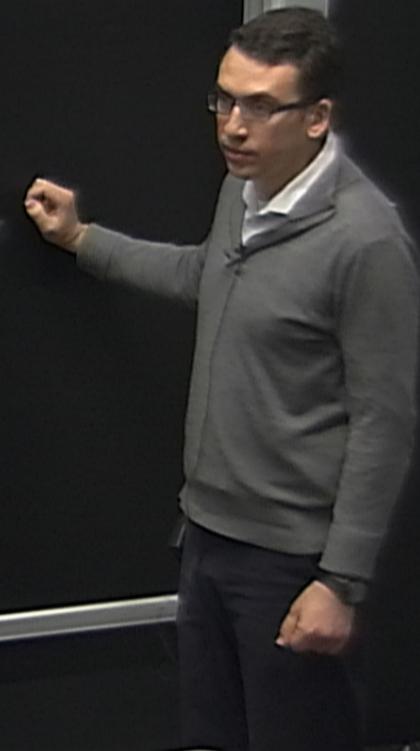
$$h = \frac{-q}{2}$$

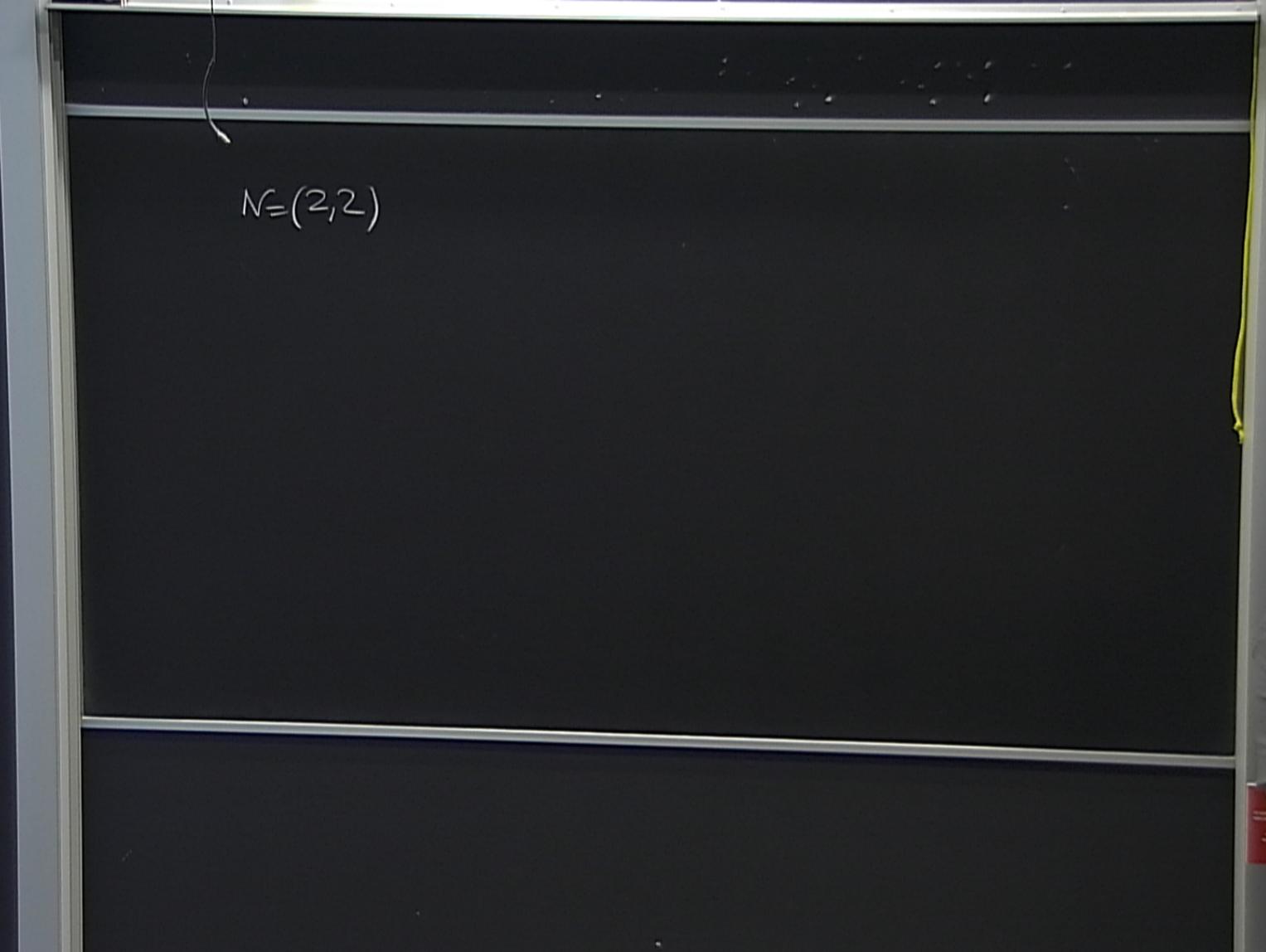
$$G_{-\gamma_2}^{-1} |+\rangle = 0$$

$$\phi_\alpha \phi_\beta = C_{\alpha\beta}^\gamma \phi_\gamma$$

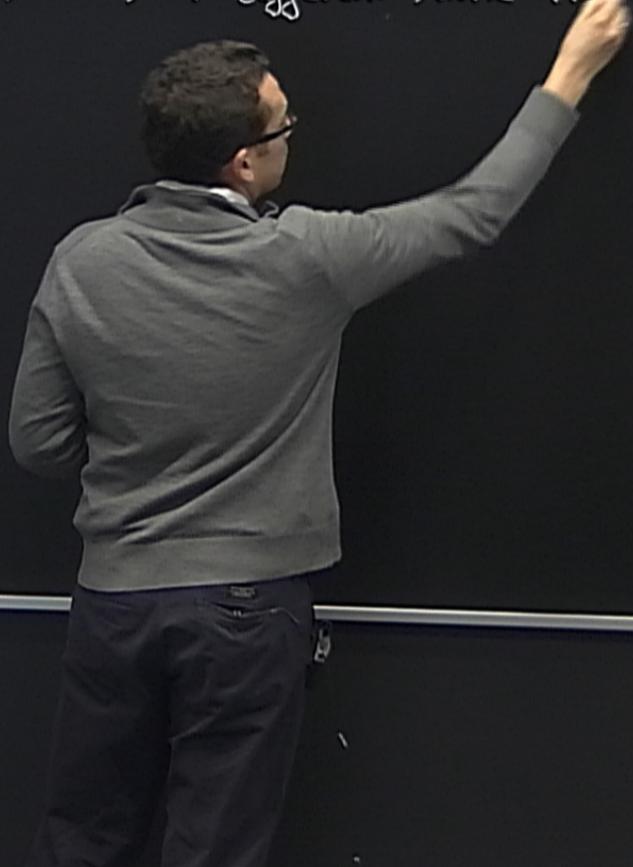
(chiral ring)

$$|q| \leq \frac{\epsilon}{3} = \epsilon$$

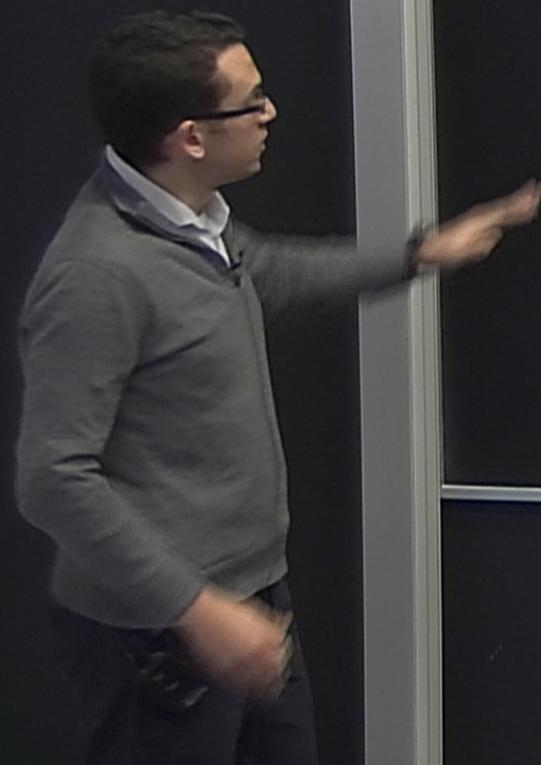




$N=(2,2)$

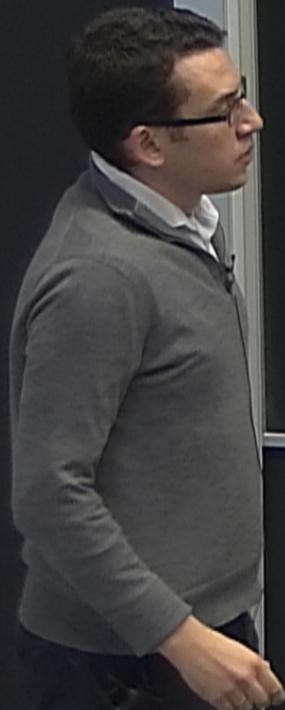
$$N=(2,2) \Rightarrow 4 \text{ different finite rings}$$


$N=(2,2)$ \Rightarrow 4 different finite rings of operators



$N=(2,2)$ \Rightarrow 4 different finite rings of operators

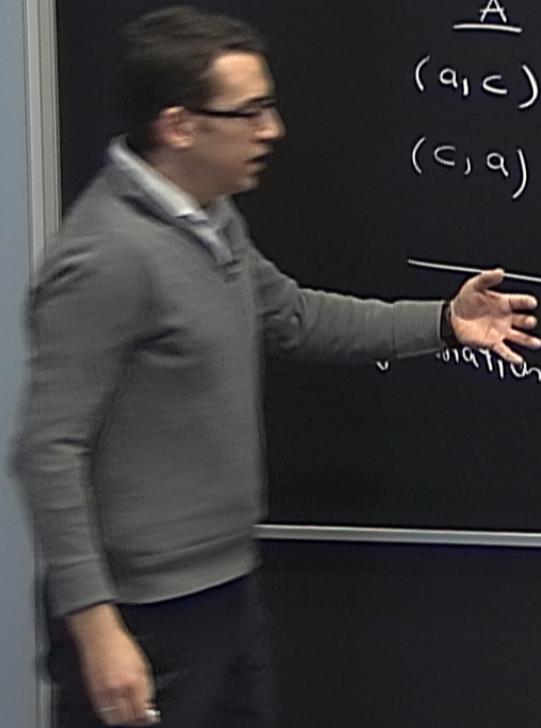
$$\begin{array}{c|c} \text{A} & \text{B} \\ \left(\alpha, c\right) & \left(c, c\right) \\ \left(c, \alpha\right) & \left(\alpha, \alpha\right) \end{array}$$



CAUTION
DO NOT USE SPONGE TO WET THE BOARD
DO NOT SCRATCH THE BOARD
DO NOT USE SHARP OBJECTS ON THE BOARD
DO NOT SPILL LIQUIDS ON THE BOARD

$N=(2,2) \Rightarrow$ 4 different finite rings of operators

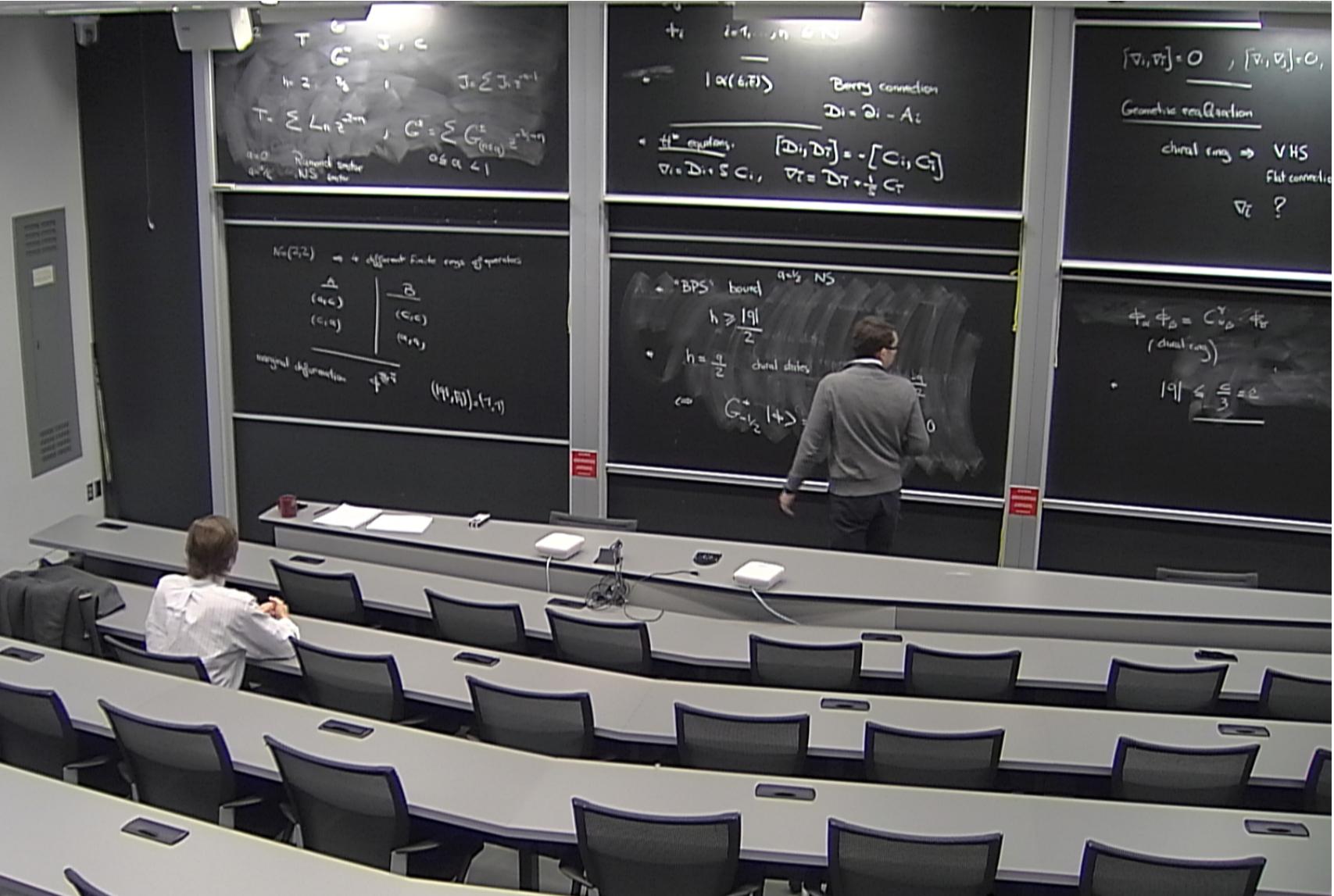
$$\begin{array}{c|c} \text{A} & \text{B} \\ \begin{array}{l} (\alpha, \zeta) \\ (\zeta, \alpha) \end{array} & \begin{array}{l} (\zeta, \zeta) \\ (\alpha, \alpha) \end{array} \\ \hline & \phi_{\alpha, \zeta} \\ & (\bar{\alpha}, \bar{\zeta}) \\ & ((\bar{\alpha}, \bar{\zeta})) = (1, 1) \end{array}$$

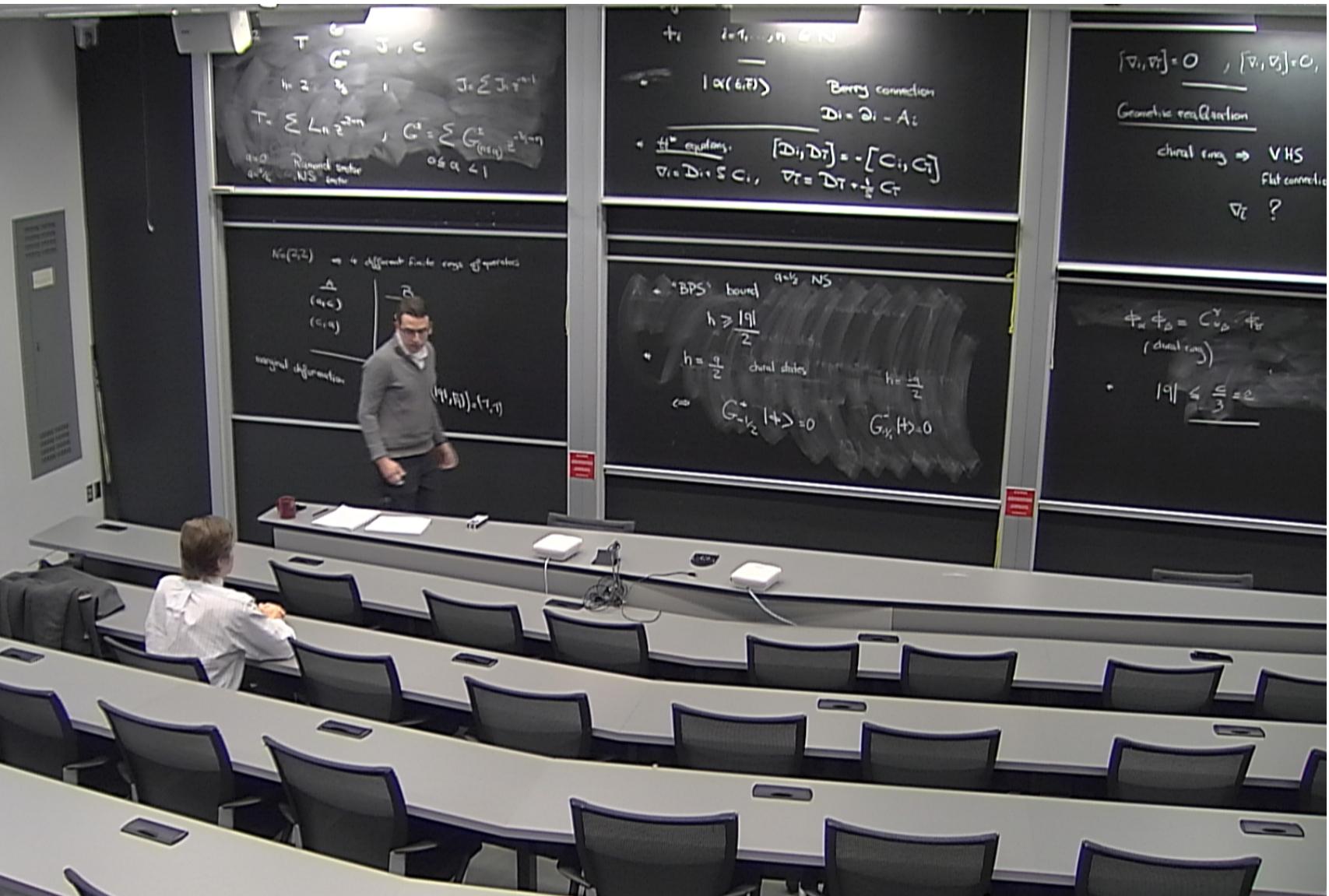


$N=(2,2) \Rightarrow$ 4 different finite rings of operators

$$\begin{array}{c|c} \overline{A} & \overline{B} \\ (\alpha, \zeta) & (\zeta, \zeta) \\ (\zeta, \alpha) & (\alpha, \alpha) \\ \hline \text{marginal deformation} & \phi^{\alpha, \bar{\alpha}} \\ & (|\alpha|, |\bar{\alpha}|) = (1, 1) \end{array}$$







For example

CAUTION
DO NOT OPERATE THIS EQUIPMENT
IF YOU ARE WEARING A
HEART MONITOR OR
IMPLANTABLE MEDICAL DEVICE.

For example

$$Q_B = G^+ - \gamma_2$$

$$h = \frac{-g}{2}$$

$$\gamma_2 |+\rangle = 0$$

CAUTION
WARNING: TO PREVENT PERSONAL INJURY
DO NOT STAND ON THE CHALKBOARD
DO NOT BUMP INTO THE CHALKBOARD
DO NOT SPILL LIQUIDS ON THE CHALKBOARD

For example

$$Q_B = G_{-\gamma_2}^+ + G_{-\gamma_2}^-$$

$$h = \frac{-q}{2}$$

$$\bar{\psi}_{-\gamma_2} |+\rangle = 0$$

CAUTION

For example

$$Q_B = G_{-\gamma_2}^+ + \bar{G}_{-\gamma_2}^+$$

Q

$$h = \frac{-g}{2}$$

$$\bar{\psi}_{-\gamma_2} |+\rangle = 0$$

CAUTION

For example

$$Q_B = G_{-\gamma_2}^+ + \bar{G}_{-\gamma_2}^+$$

$$Q_B^z = 0$$

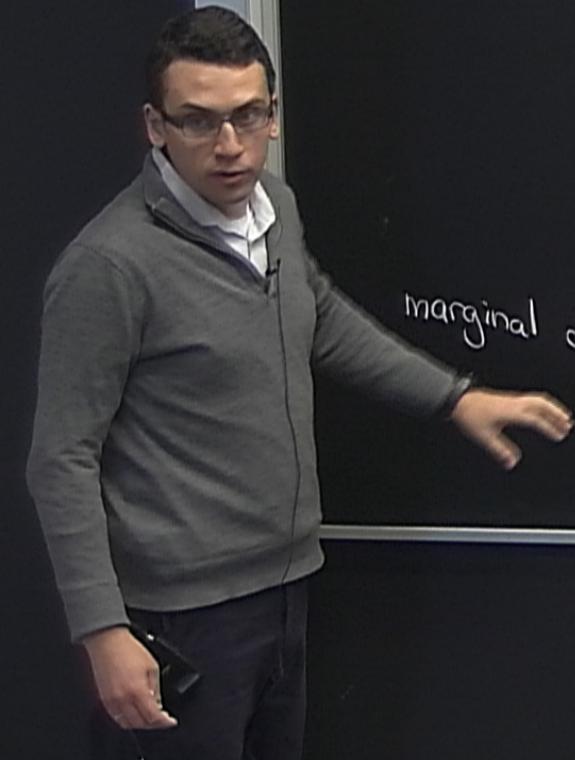
CAUTION

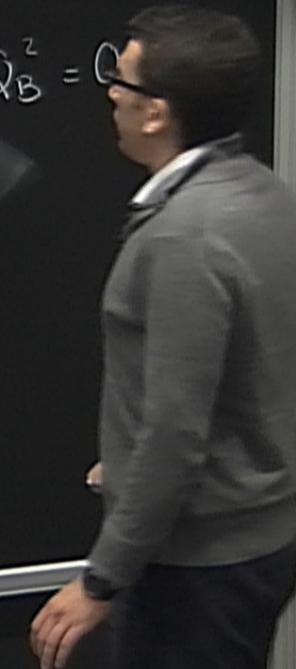
$N=(2,2) \Rightarrow$ 4 different finite rings of operators

$$\begin{array}{c|c} \text{A} & \text{B} \\ \begin{array}{l} (\alpha, c) \\ \rightarrow q \end{array} & \begin{array}{l} (c, c) \\ (\alpha, q) \end{array} \\ \hline \text{arbitrary deformation} & \phi^{q, \bar{q}} \\ & ((\bar{q}, \bar{q})) = (1, 1) \end{array}$$

$N=(2,2) \Rightarrow$ 4 different finite rings of operators

$$\begin{array}{c|c} \overline{A} & \overline{B} \\ \begin{matrix} (\alpha, c) \\ (c, \alpha) \end{matrix} & \begin{matrix} (c, c) \\ (\alpha, \alpha) \end{matrix} \\ \hline & \phi_{\tilde{q}, \tilde{q}} \\ \text{marginal deformation} & \begin{pmatrix} |\tilde{q}|, |\tilde{q}| \end{pmatrix} = (1, 1) \end{array}$$





For example

(G_c)

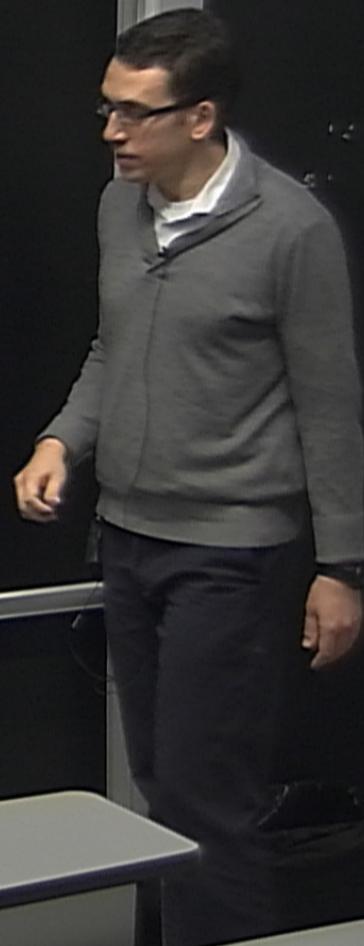
$$Q_B = G_{-\frac{1}{2}}^+ + \overline{G}_{\frac{1}{2}}^+$$

$$Q_B^2 = 0$$

$$\frac{-g}{2}$$

$$|+\rangle = 0$$

CAUTION



For example

(G_c)

$$Q_B = G_{-\frac{1}{2}}^+ + \overline{G}_{-\frac{1}{2}}^+$$

$$Q_B^2 = 0$$

$$T_{top} = T + \frac{1}{2} \partial J$$

$$\frac{-g}{2}$$

$$|+\rangle = 0$$

CAUTION
DO NOT PLACE ANY OBJECTS ON THE CHALKBOARD
IT IS EXPENSIVE TO REPLACE IT
www.prairielearning.ca

For example

(G_c)

$$Q_B = G^+ - \gamma_2 + \bar{G}$$

$$Q_B^2 = 0$$

$$T_c = T + \frac{1}{\gamma_2}$$

1

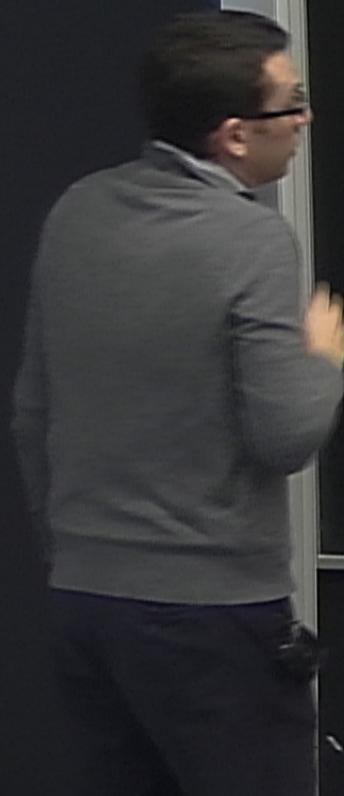
$$\frac{-g}{2}$$

$$|+\rangle = 0$$

CAUTION

3. Geometric realization:

3.1) Generalities



CAUTION
DO NOT USE CHALK
ON THIS SURFACE.
THIS SURFACE IS
NOT FOR SCRIBBLES
OR DRAWINGS.
DO NOT SCRIBBLE
OR DRAW ON THE
SURFACE.

3. Geometric realizations:

3.1) General

$$\sum_{2d} \rightarrow X$$

CAUTION
DO NOT USE CHALK ON THE PROJECTOR SCREEN.
PROJECTOR SCREEN IS MADE OF POLYCARBONATE WHICH
IS HIGHLY SUSCEPTIBLE TO SCRATCHES.
KEEP CHALK DUST AWAY FROM THE PROJECTOR SCREEN.
PROJECTOR SCREEN IS MADE OF POLYCARBONATE WHICH
IS HIGHLY SUSCEPTIBLE TO SCRATCHES.

3. Geometric realizations:

3.1] Generalities

$$\phi, \psi : \sum_{\text{2d}} \rightarrow X \quad \text{CY-d-field}$$



3. Geometric realization:

3.1] Generalities

$$\phi, \psi : \sum_{\text{2d}} \rightarrow X \quad \text{CY-d-field}$$
$$\phi^{q, \bar{q}} \equiv H^{0, \bar{q}}(X, \wedge^q TX)$$

CAUTION
DO NOT USE CHALK ON THIS SURFACE.
IT WILL DAMAGE THE SURFACE.
USE MARKERS ONLY.

3. Geometric realization:

3.1] Generations

Unique hol

$$\Omega^{d,0}$$

CY-d-field

$$\rightarrow X$$

$$H^{0,\overline{q}}(X, \Lambda^q TX)$$



3. Geometric realizations:

3.1] Generalities

$$\begin{array}{ccc} \phi, \psi : & \longrightarrow & X \\ \phi^{q,\gamma} & \sim & \Omega^{0,\bar{q}}(X, \wedge^q T_X) \end{array}$$

unique hol

$\Omega^{d,0}$

CY-d-field

3. Geometric realization:

3.1] Generalities

Unique hol

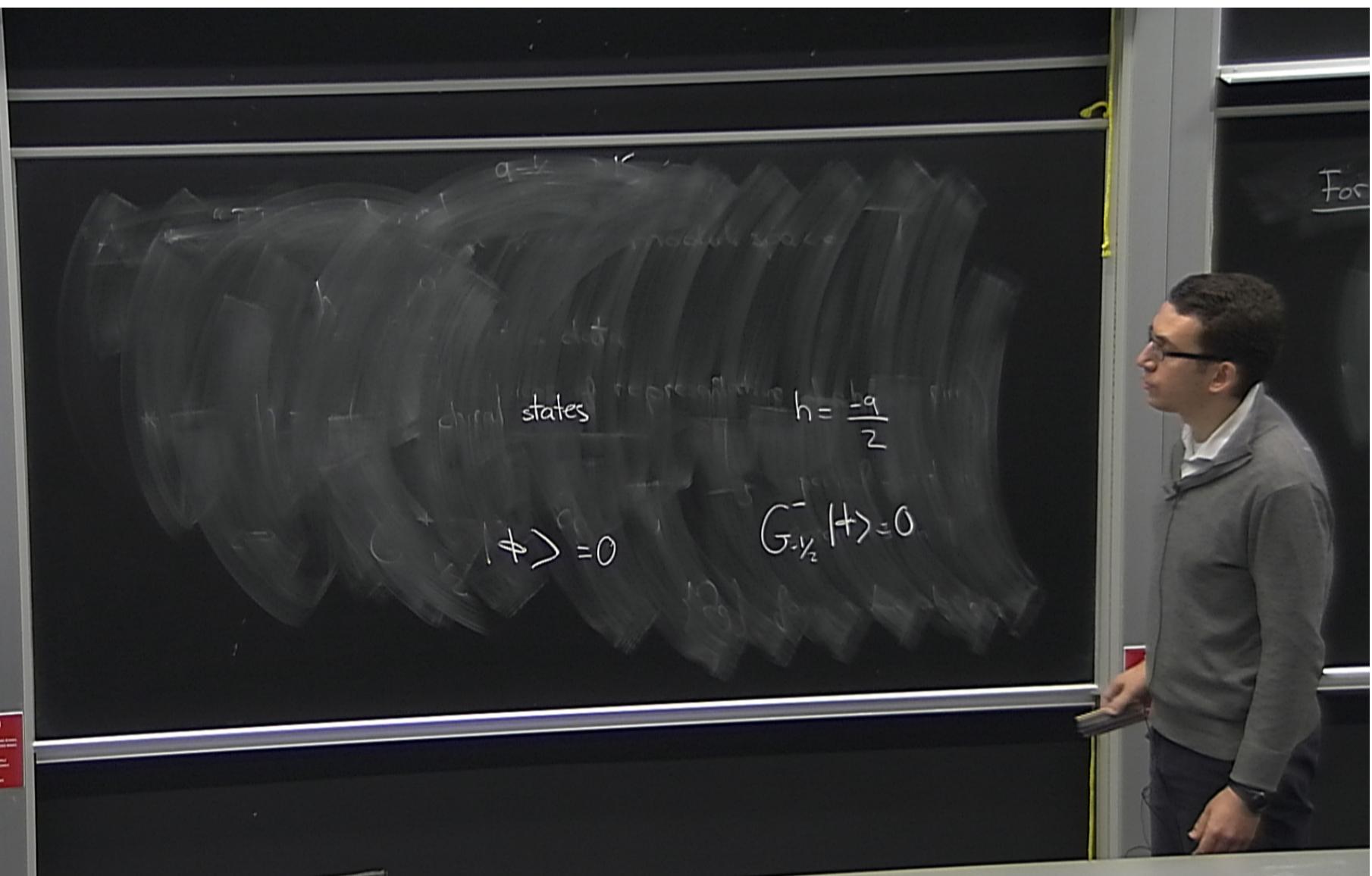
$$\Omega^{d,0}$$

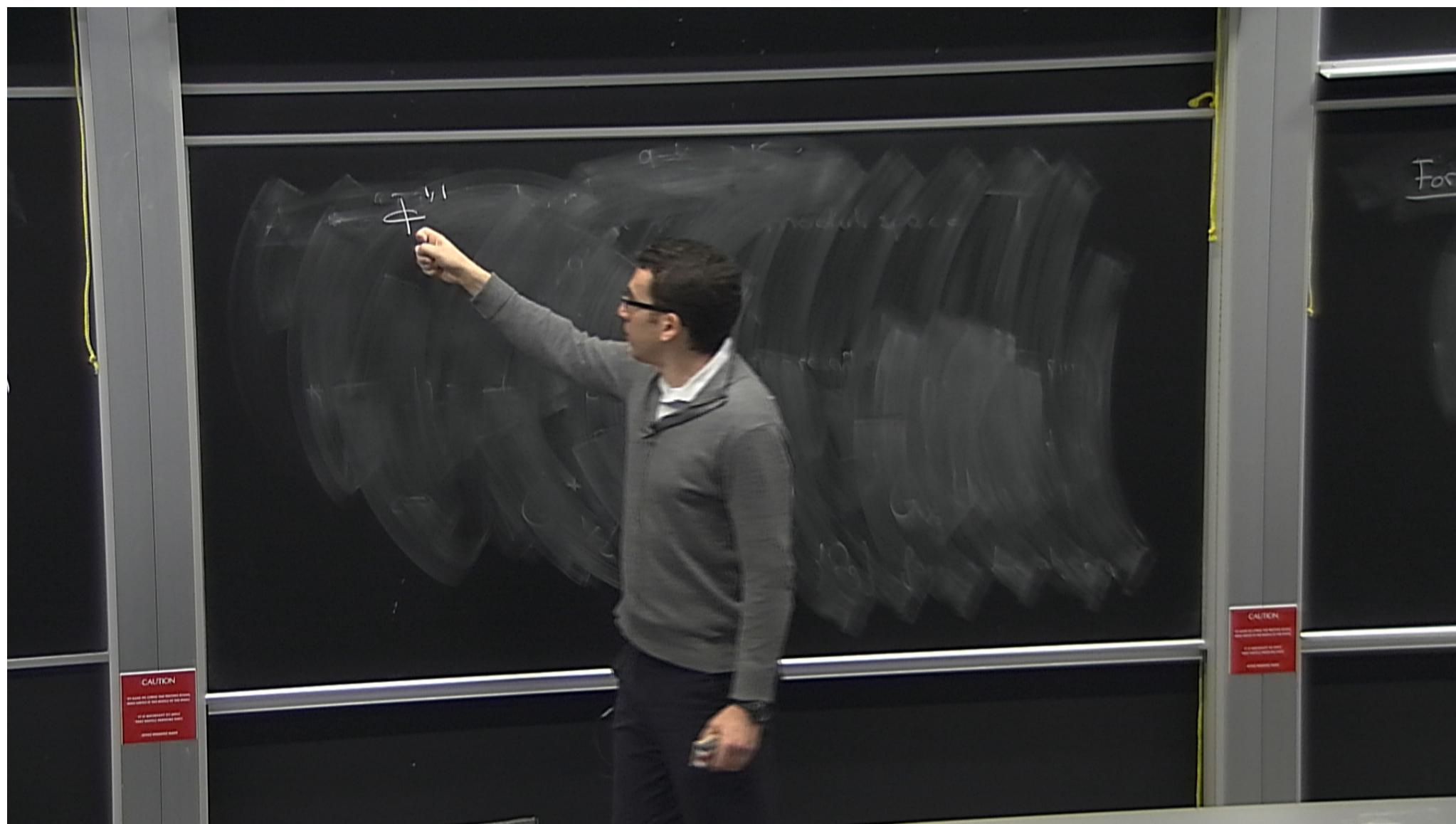
CY-d-fold

$$\phi, \psi : \sum_{\text{2d}} \rightarrow X$$

$$\begin{aligned}\phi^{q, \bar{q}} &\equiv H^{0, \bar{q}}(X, \wedge^q TX) \\ &\cong H^{d-q, \bar{q}}(X)\end{aligned}$$







$$\mathcal{F}^{2,1} \equiv H^{2,1}(X)$$

M = moduli space of
complex structures of X



For

CAUTION

$$\mathcal{F}^{1,1} \equiv H^{2,1}(X)$$

M = moduli space of
complex structures of X

3.2)

For

CAUTION

CAUTION

$$\mathcal{F}^{1,1} \equiv H^{1,1}(X)$$

M = moduli space of
complex structures of X

3.2)

Elliptic \hookrightarrow :

$$\mathcal{F}^{1,1} \equiv H^{1,1}(X)$$

M = moduli space of
complex structures of X

3.2)

Elliptic curves

$\dim M = 1$,

real

For

$$\mathcal{F}^{1,1} \equiv H^{1,1}(X)$$

M = moduli space of
complex structures of X

3.2)

Elliptic curve: $\dim M = 1,$

z local coordinate on M

$$\mathcal{F}^{1,1} \equiv H^{1,1}(X)$$

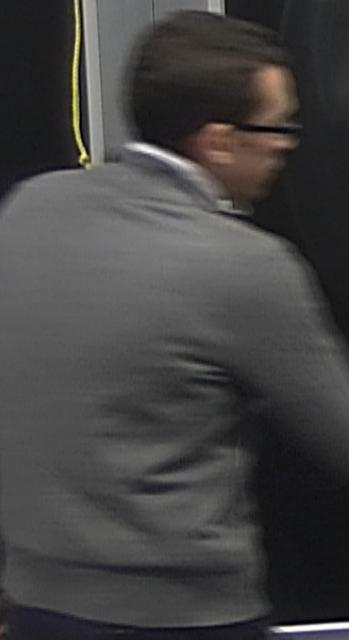
M = moduli space of
complex structures of X

3.2)

Elliptic curve: $\dim M = 1,$

z local coordinate on M

$\mathcal{D}^{1,0}$



$$\mathcal{F}^{2,1} \equiv H^{2,1}(X)$$

M = moduli space of
complex structures of X

3.2) Elliptic curve: $\dim M = 1$,

z local coordinate on M

$\Omega^{1,0} \in \Gamma(X)$, $\mathcal{L} \rightarrow M$ line bundle

$$e^{-K} := i \int \Omega \wedge \bar{\Omega}$$

CAUTION

$$e^{-K} := \int_X \Omega \wedge \bar{\Omega}$$

$$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K, \quad \text{K\"ahler metric on } M$$

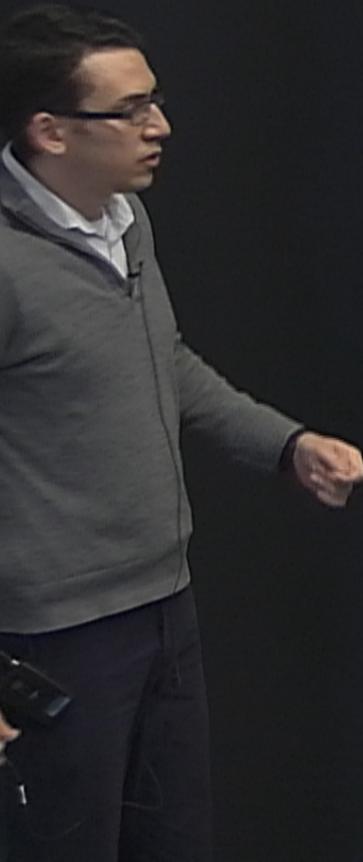
CAUTION

$$e^{-K} := \int_X \Omega \wedge \bar{\Omega}$$

$$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K,$$

$$G_z$$

Kähler metric on M



$$\Phi^{2,1} \equiv H^{2,1}(X)$$

M = moduli space of
Complex structures of X

3.2)

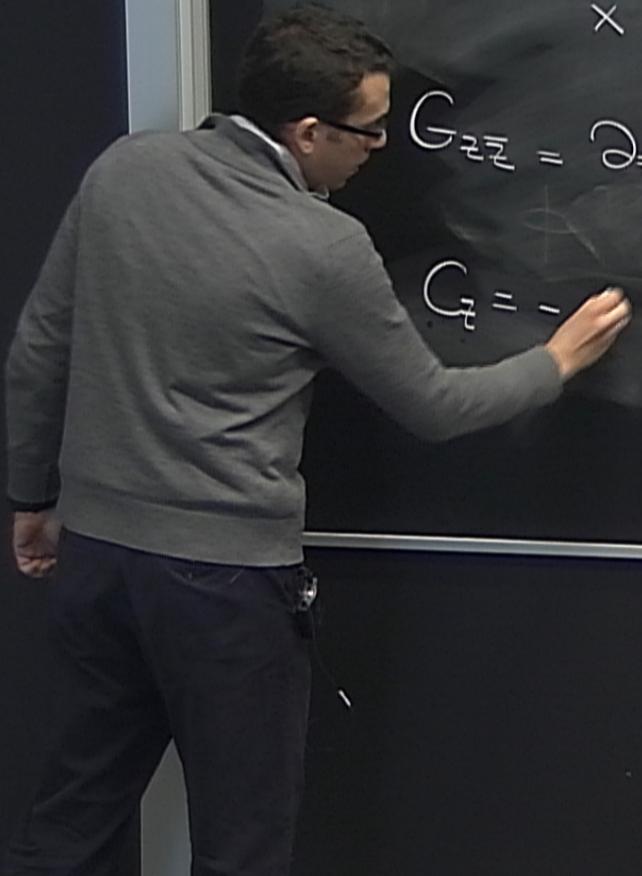
Elliptic

$$\dim M = 1, \quad q \leq \tilde{c} = 1$$

z local coordinate on M

$$\Omega^{1,0}$$

$\mathcal{L} \rightarrow M$ line bundle



$$e^{-K} := \int_X \Omega \wedge \bar{\Omega}$$

$$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K, \quad \text{K\"ahler metric on } M$$

$$G_{\bar{z}} = -$$

$$e^{-K} := \int_X \Omega \wedge \bar{\Omega}$$

$$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K, \quad \text{K\"ahler metric}$$

$$G_t = - \int_X \Omega \wedge \partial_{\bar{z}} \Omega$$

$\phi^{0,0}$ $\phi^{1,1}$

3.3

Variation of Hodge structure

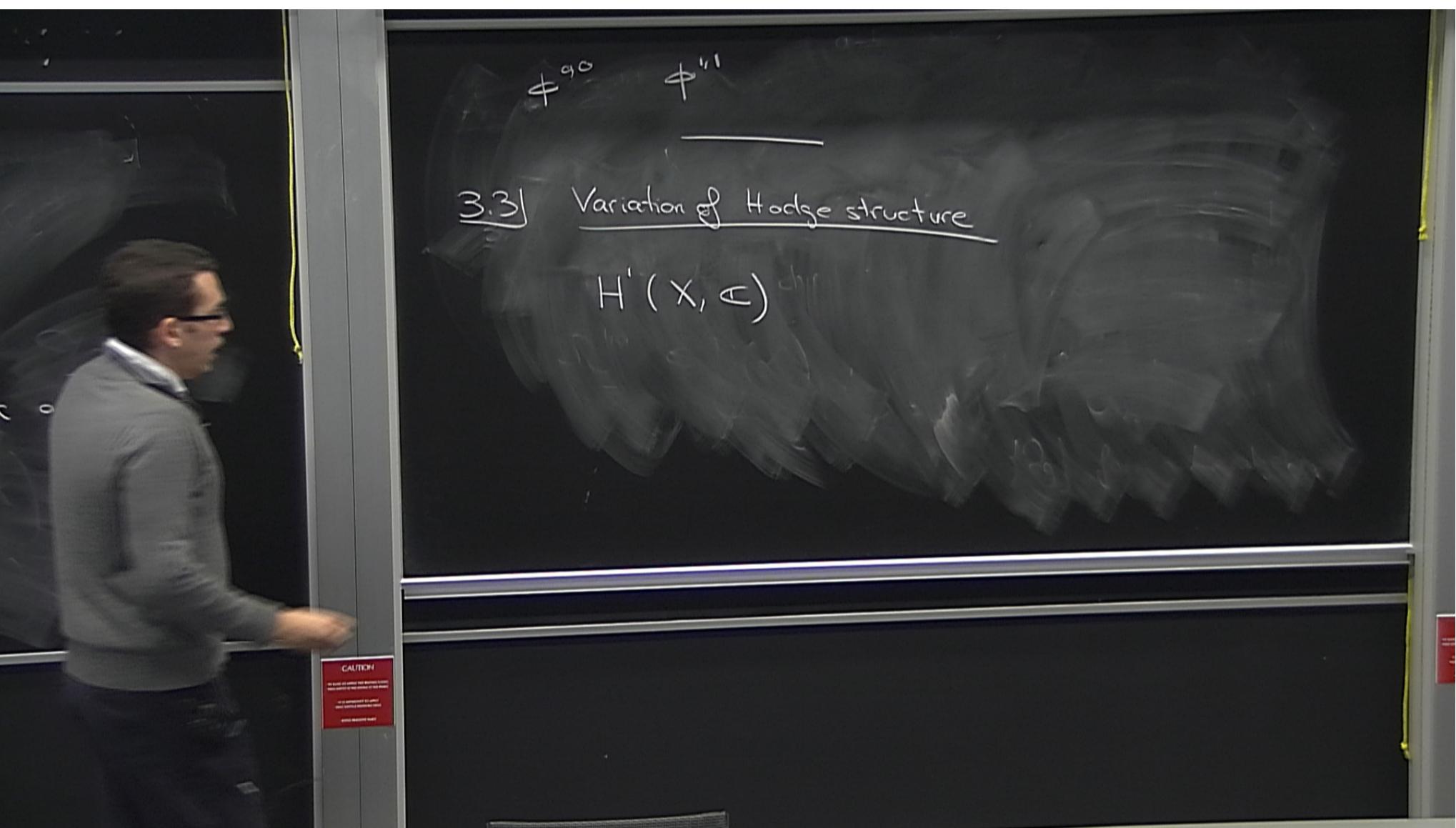
CAUTION
DO NOT USE CHALK OR MARKERS.
THIS SURFACE IS MADE OF POLYCARBONATE
AND IS VULNERABLE TO SCRATCHES AND
SHATTERING IF HIT WITH A HARSH OBJECT.

CAUTION
DO NOT USE CHALK OR MARKERS.
THIS SURFACE IS MADE OF POLYCARBONATE
AND IS VULNERABLE TO SCRATCHES AND
SHATTERING IF HIT WITH A HARSH OBJECT.

$$\phi^{0,0} \quad \phi^{1,1}$$

3.3] Variation of Hodge structure

$$H^*(X, \mathbb{C})$$



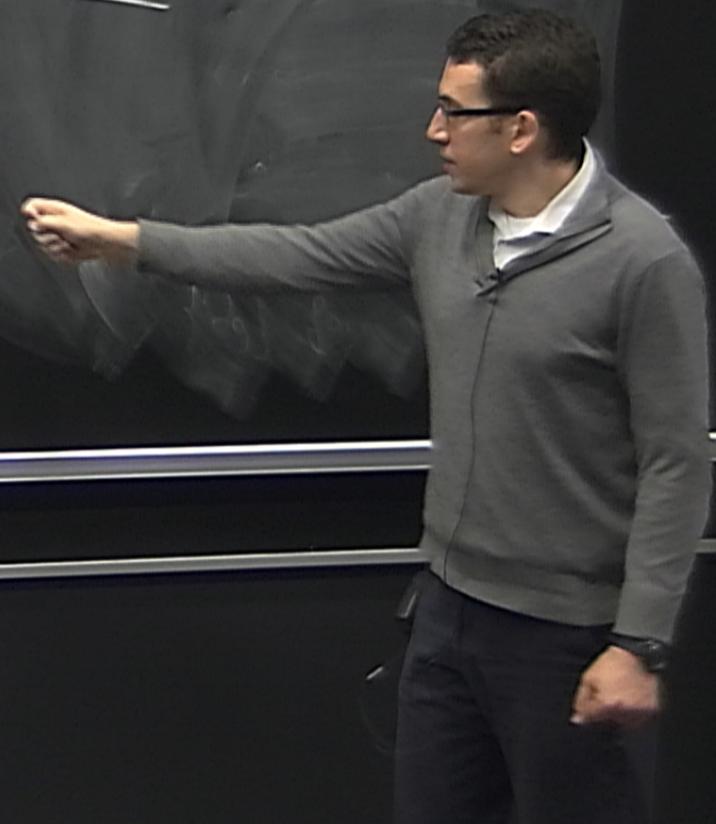
$$\phi^{0,0} \quad \phi^{1,1}$$

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\Omega^{1,0} \in H^{1,0}(X) = \mathcal{F}^1$$

on M



$$\phi^{0,0} \quad \phi^{1,1}$$

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\Omega^{1,0} \in H^{1,0}(X) = \mathcal{F}^1 \subset \mathcal{F}^0 = H^i(X, \mathbb{C}) =$$

on M



CAUTION
DO NOT USE SPONGES THAT CONTAIN
AMMONIA OR OTHER ACIDIC CHEMICALS.
IT IS DANGEROUS TO SPONGE
OVER POLISHED SURFACES.
KEEP SPONGES AWAY FROM EYES AND SKIN.

$$\phi^{0,0} \quad \phi^{1,1}$$

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\Omega^{1,0} \in H^{1,0}(X) = \mathcal{F}^1 \subset \mathcal{F}^0 = H^i(X, \mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$$

on M

CAUTION

$$\phi^{0,0} \quad \phi^{1,1}$$

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\Omega^{1,0} \in H^{1,0}(X) = \mathcal{F}^1 \subset \mathcal{F}^0 = H^i(X, \mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$$

on M

CAUTION

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\mathcal{L}^{1,0} \in H^{1,0}(X) = \mathcal{F}^1 \subset \mathcal{F}^0 \quad (X, \mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$$

$$\mathcal{F}^1 \xrightarrow{\nabla} \mathcal{F}^0$$

on M

CAUTION
DO NOT CLIMB THIS POSITION BOARD
DO NOT SWING FROM THE ARMS OF THE BOARD
IT IS DANGEROUS TO SIT
ON THE SWING POSITION BOARD
ALWAYS PROTECT YOUR BACK

3.3] Variation of Hodge structure

$$H^i(X, \mathbb{C})$$

$$\mathcal{L}^{1,0} \in H^{1,0}(X) = \mathcal{F}^1 \subset \mathcal{F}^\circ = H^i(X, \mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$$

$$\mathcal{F}^1 \xrightarrow{\nabla} \mathcal{F}^\circ$$

Gauss-Manin co

on M

CAUTION

$$H^{1,0}(X) \oplus H^{0,1}(X)$$

connection

$$\begin{pmatrix} \Omega \\ \partial_{\bar{z}}\Omega \end{pmatrix} =$$

\mathcal{F}'



$$H^{1,0}(X) \oplus H^{0,1}(X)$$

connection

$$\begin{pmatrix} \Omega \\ \partial_{\bar{z}}\Omega \end{pmatrix} \subset \begin{matrix} \mathcal{F}' \\ \mathcal{F}^{\circ} \end{matrix}$$

$$\partial_z \begin{pmatrix} \Omega \\ \partial_{\bar{z}}\Omega \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \# & \star \end{pmatrix} \begin{pmatrix} \Omega \\ \partial_{\bar{z}}\Omega \end{pmatrix}$$



$$H^{1,0}(X) \oplus H^{0,1}(X)$$

connection

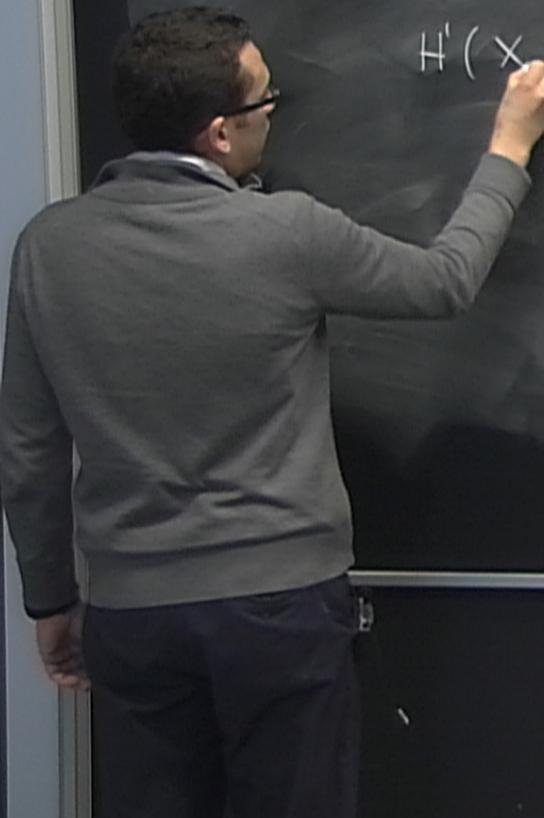
$$\begin{pmatrix} \Omega \\ \partial_z \Omega \end{pmatrix} \subset \begin{pmatrix} \mathcal{F}' \\ \mathcal{F}^{\circ} \end{pmatrix}$$

$$\partial_z \begin{pmatrix} \Omega \\ \partial_z \Omega \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \# & +1 \end{pmatrix} \begin{pmatrix} \Omega \\ \partial_z \Omega \end{pmatrix}, \quad \text{Gauss-Manin connection}$$

CAUTION

3.4 $t\bar{t}$ equations

$$H'(x)$$



3.4 \mathbb{H}^* equations

$$H^*(X, \mathbb{C}) = \langle \rangle \oplus H^{*,*}(X)$$

3.3 Σ^*

CAUTION
DO NOT USE THIS MATERIAL WHILE DRIVING.
THIS IS A PROTECTED AREA.
DO NOT DISTURB THE AREA.
KEEP YOUR DISTANCE.

3.4 $t\bar{t}$ equations

$$H^i(X, \mathbb{C}) = H^{i,0}(X) \oplus H^{0,i}(X)$$
$$\mathcal{R}^{\psi} \quad \mathcal{R}^{\psi}$$

3.3 $\Sigma^{i,0}$

$$D\bar{z} \begin{pmatrix} z \\ \bar{z} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ ie^{\bar{z}} \bar{G} & 0 \end{pmatrix}$$



$$D_{\bar{z}} \begin{pmatrix} z \\ \bar{z} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 0 \\ ie^{ik\bar{z}} G_{\bar{z}} & 0 \end{pmatrix}}_{C_{\bar{z}}} \begin{pmatrix} z \\ \bar{z} \end{pmatrix}$$

$$[D_z, D_{\bar{z}}] = - [C_z, C_{\bar{z}}]$$

$$H^{1,0} \xrightarrow{D_{\bar{z}}}$$

CAUTION
DO NOT SPIT ON THE FLOOR
DO NOT SPIT ON THE FLOOR

CAUTION

* Deformation families

$$\dot{\phi}_i \quad i = 1, \dots, n$$

$$* |\alpha(t, \tau)$$

* H* equations.

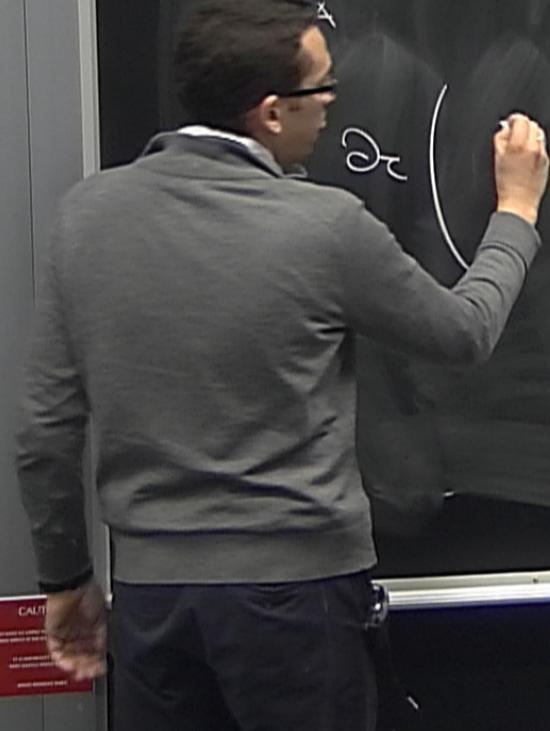
$$\nabla_i = D_i + \sum C_i,$$

Berry connection

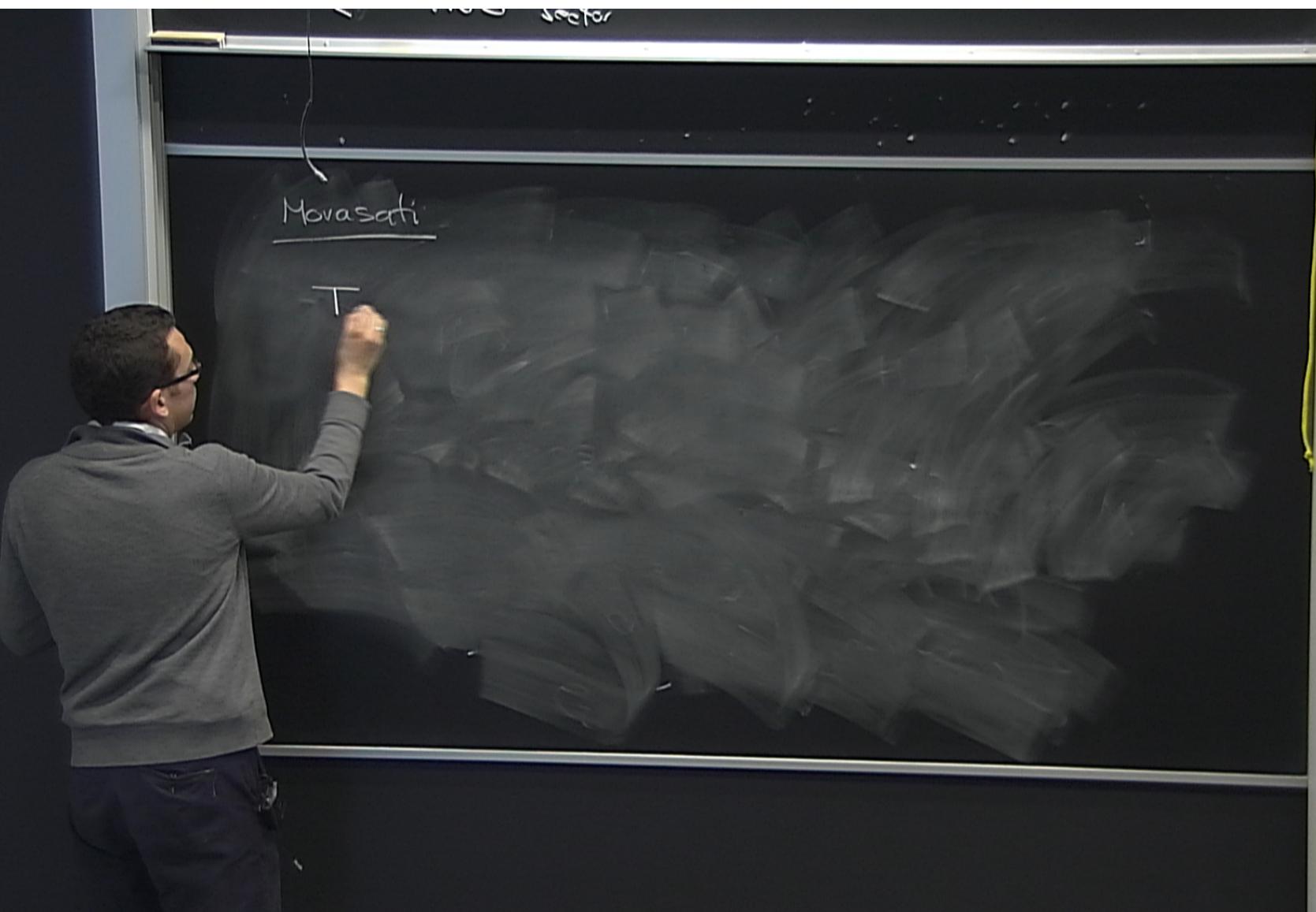
$$= \partial_i - A_i$$

$$= -[C_i, C_j] \\ + \frac{1}{S} C_T$$

$$\pi^o = \int_{\Omega}, \quad \pi' = \int_{\Omega_B}, \quad \tau = \frac{\pi'}{\pi^o}$$



CAUTION
TEACHING STAFF AND STUDENTS ARE REMINDED THAT THE REPRODUCTION OF ANY PART OF THIS DOCUMENT IS PROHIBITED UNLESS EXPRESSLY AUTHORIZED BY THE UNIVERSITY



Movasati

$$T = \{ X, \vec{v} \}$$
$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

s.t.

$$\langle \vec{v}, \vec{v} \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
$$\alpha \in \mathbb{F}^1, \quad \beta \in \mathbb{F}^0$$



Movasati

$$T = \left\{ X, \vec{N} \right\}$$

$\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$

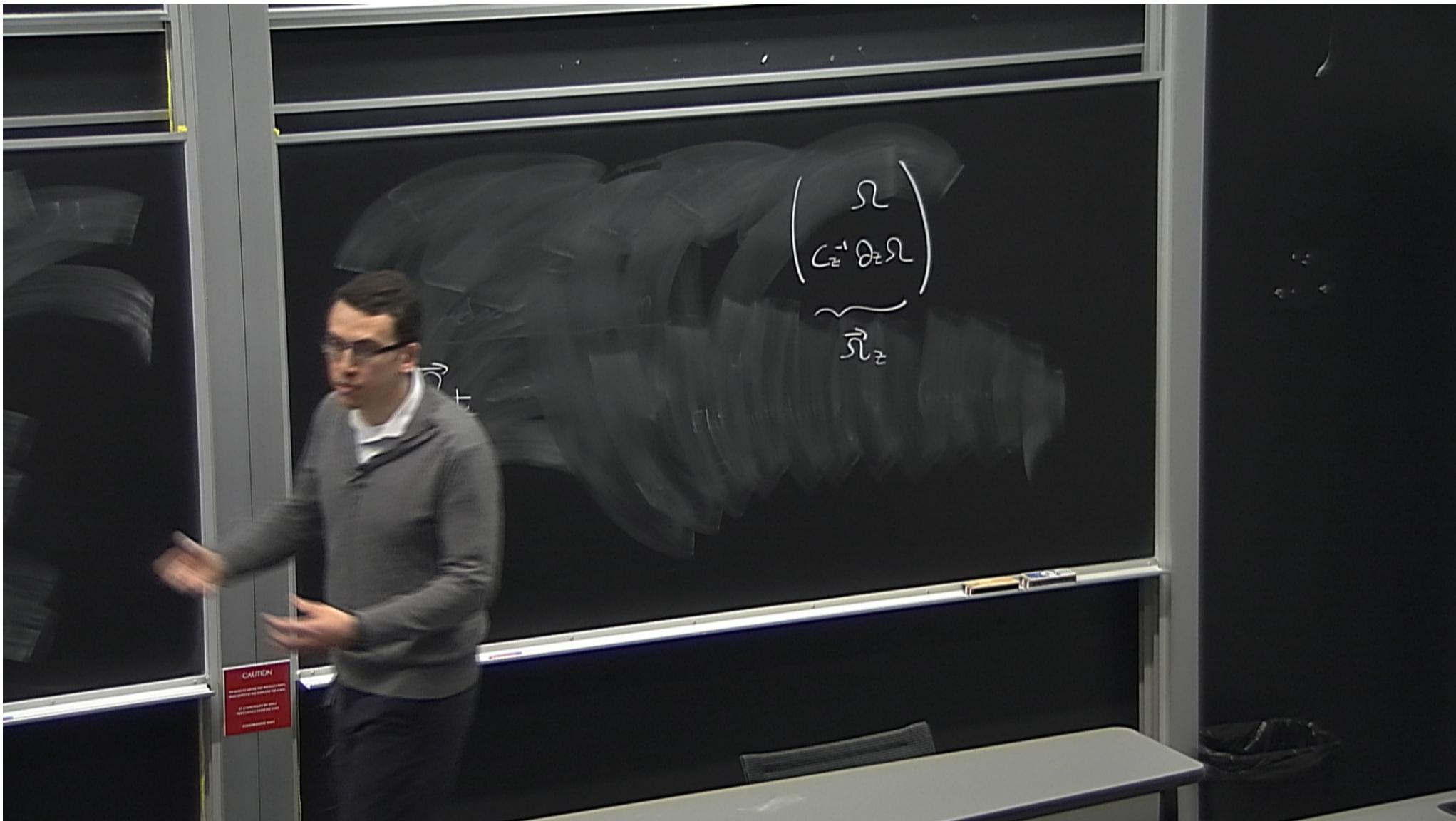
s.t.

$$\langle \vec{N}, \vec{N} \rangle = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\alpha \in \mathbb{F}^1, \quad \beta \in \mathbb{F}^0$$

CAUTION
DO NOT USE LIQUID INK MARKERS
DO NOT USE LIQUID ERASERS
DO NOT USE LIQUID GLUE





$$\begin{pmatrix} \tilde{\mathcal{L}} \\ \partial_t \tilde{\mathcal{L}} \\ \vec{\mathcal{R}}_t \end{pmatrix} = \left(\quad \right) \begin{pmatrix} \mathcal{L} \\ \mathcal{L}_z^{-1} \partial_z \mathcal{L} \\ \sim \vec{\mathcal{R}}_z \end{pmatrix}$$

$$\tilde{\Omega} = g_0^{-1} \Omega$$

$$g_0 = e^{-K}$$

$$e^{-K} \Big|_{hol} = \pi^\circ$$

CAUTION

$$\begin{pmatrix} \tilde{\Omega} \\ \partial_t \tilde{\Omega} \\ \vec{\Omega}_t \end{pmatrix} = \begin{pmatrix} g_0^{-1} & & \\ -\alpha_0^{-1} K_t & g_0 & \\ & & \end{pmatrix} \begin{pmatrix} \Omega \\ C_z^{-1} \partial_z \Omega \\ \tilde{\Omega}_z \end{pmatrix}$$

CAUTION
DO NOT SWEEP THE DUST FROM THE CHALKBOARD
DO NOT SWEEP THE DUST FROM THE CHALKBOARD

$$\tilde{\Omega} = g_0^{-1} \Omega$$

$$g_0 = e^{-K}$$

$$t =$$

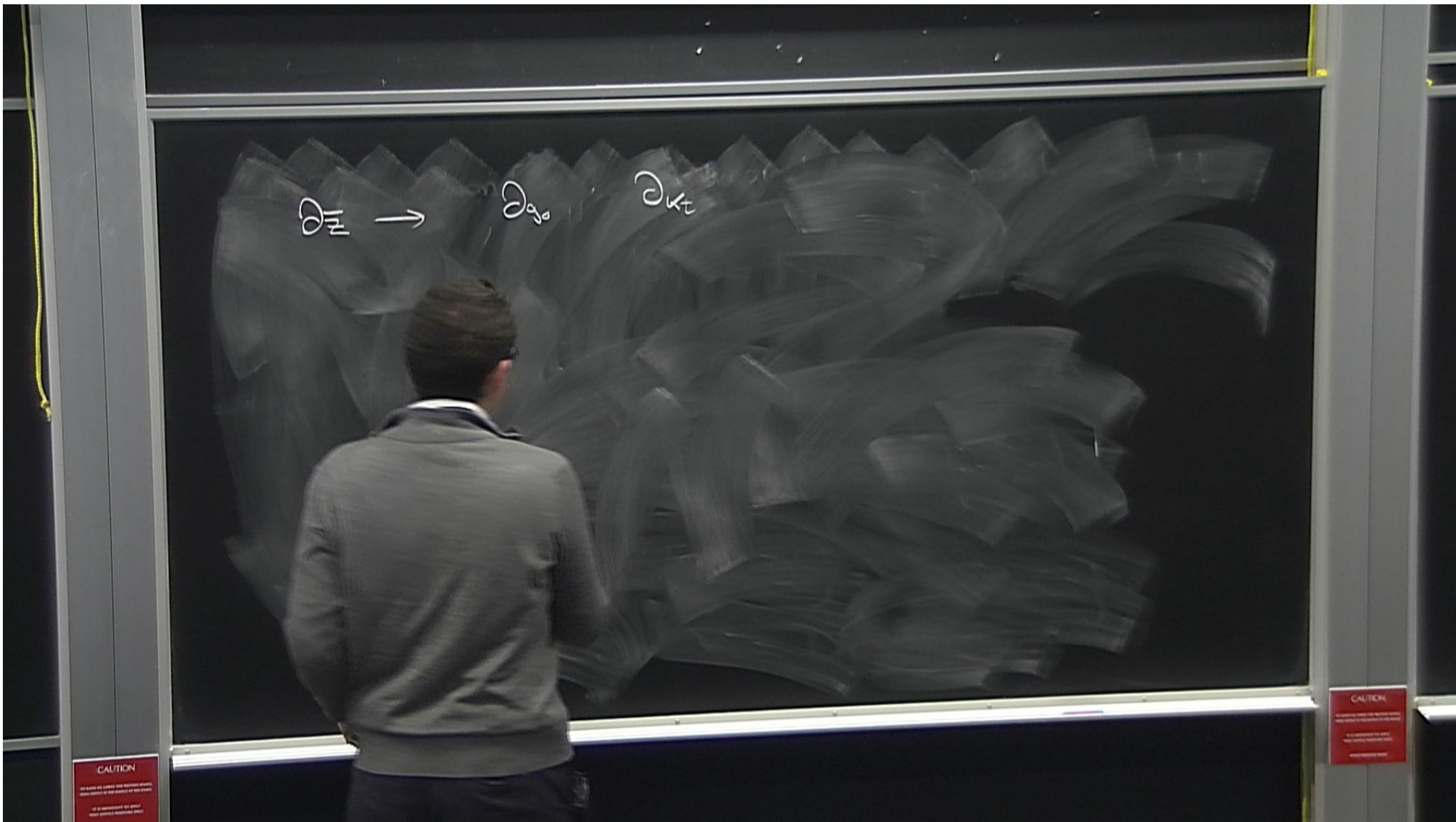
$$|_{h_0} = \pi^\circ$$

$$|_{h_0} = \tau$$

Differential
ring

$$\partial_t g_0 = -g_0 K_t$$

$$\partial_t K_t = -K_t^2 + h_0$$



$$\partial \Xi \rightarrow \partial g_0$$

$$\partial k_t$$

$$\partial g_0 \vec{n}_t = M_t \vec{n}_t$$

CAUTION

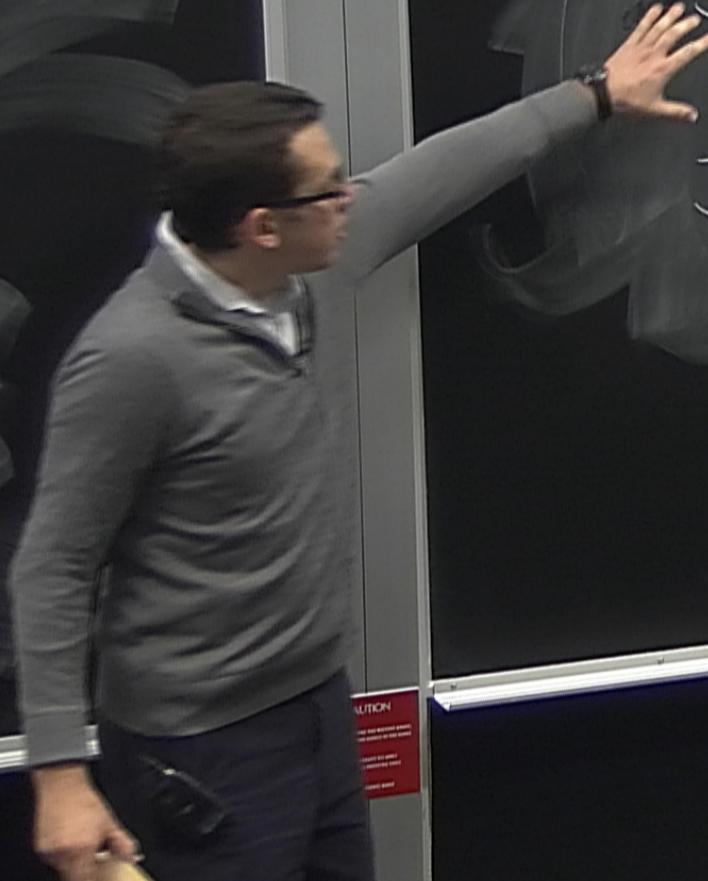
DO NOT USE DIRECTLY FROM THE SOURCE
DANGER OF EXPLOSION
DO NOT USE DIRECTLY FROM THE SOURCE
DANGER OF EXPLOSION

CAUTION

DO NOT USE DIRECTLY FROM THE SOURCE
DANGER OF EXPLOSION
DO NOT USE DIRECTLY FROM THE SOURCE
DANGER OF EXPLOSION

$$J_0 = -g_0 M_{g0} + 2 M_{Kt} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$J_- = -M_{Kt} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$



$$\partial_t \vec{N}_t = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \vec{N}_t$$

\bar{J}_+

$t\mathbb{H}^*$

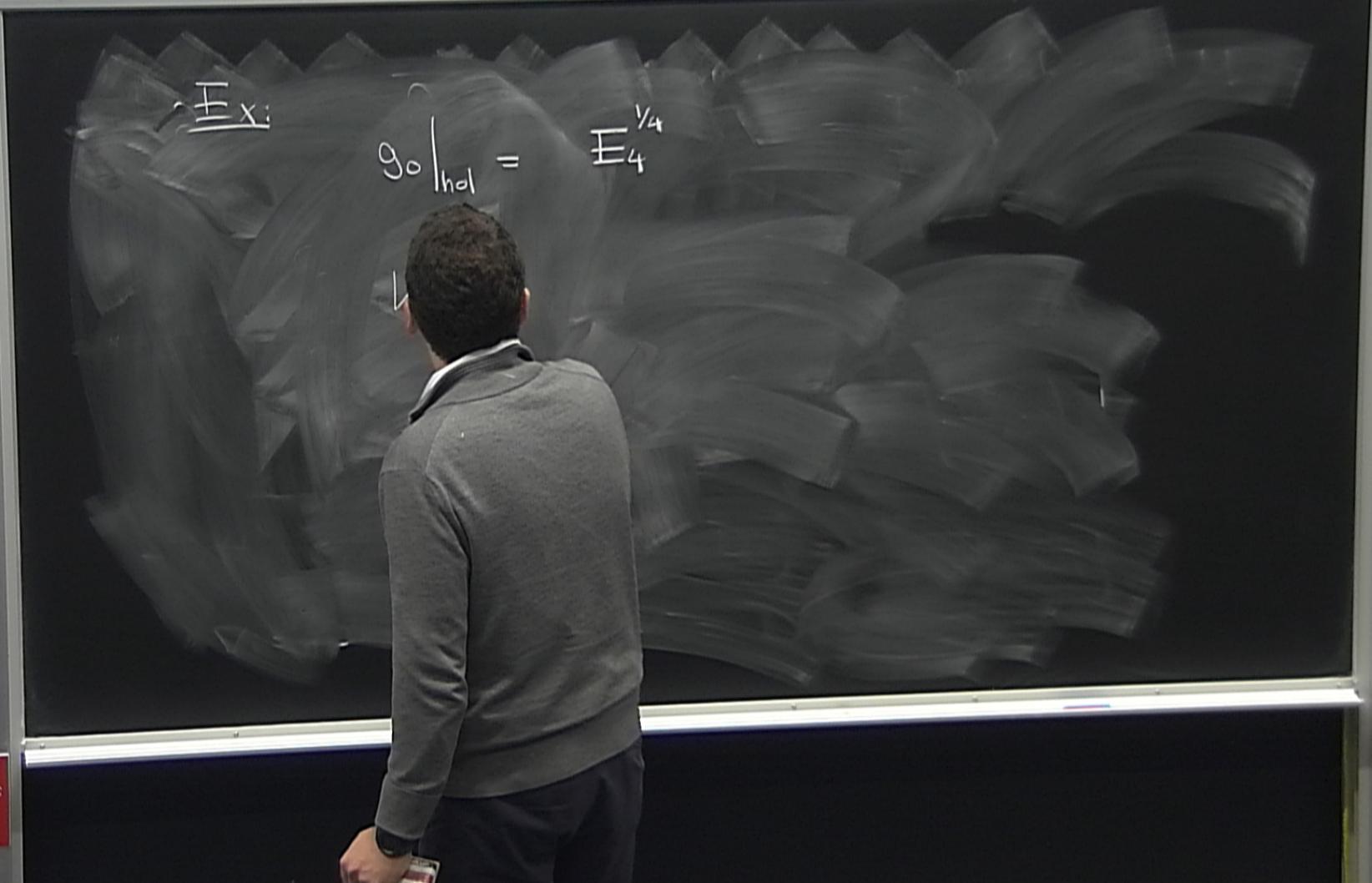
(z, \mathbb{C})

L



$$\partial_t \vec{N}_t = \underbrace{\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}}_{J_+} \vec{N}_t$$

tt^* equations \leftrightarrow $sl(2, \mathbb{C})$ Lie algebra on T

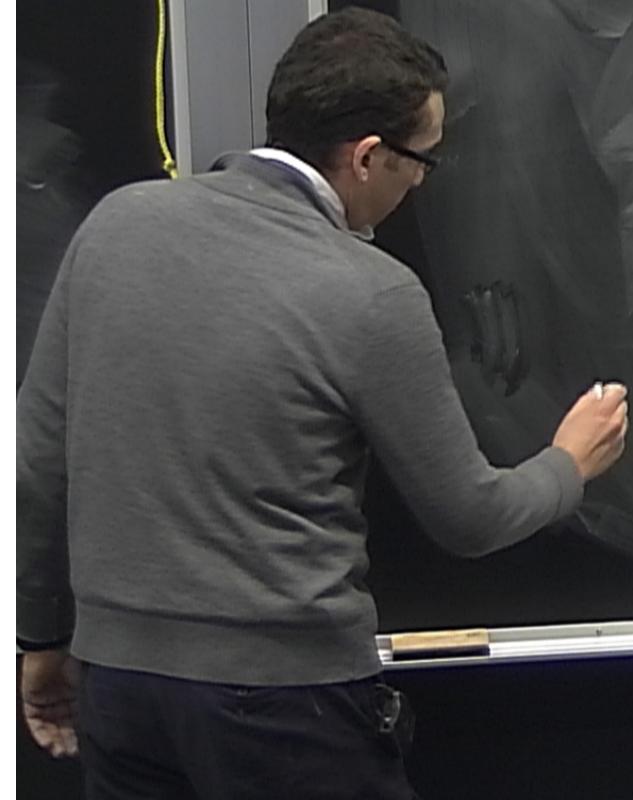


Σ_x

$$g_0|_{hd} = E_4^{\gamma_4}$$

$$K_t|_{hd} = -\frac{E_z}{12}$$

Σ



CAUTION