Title: Lie algebraic formulation of tt* equations and topological strings

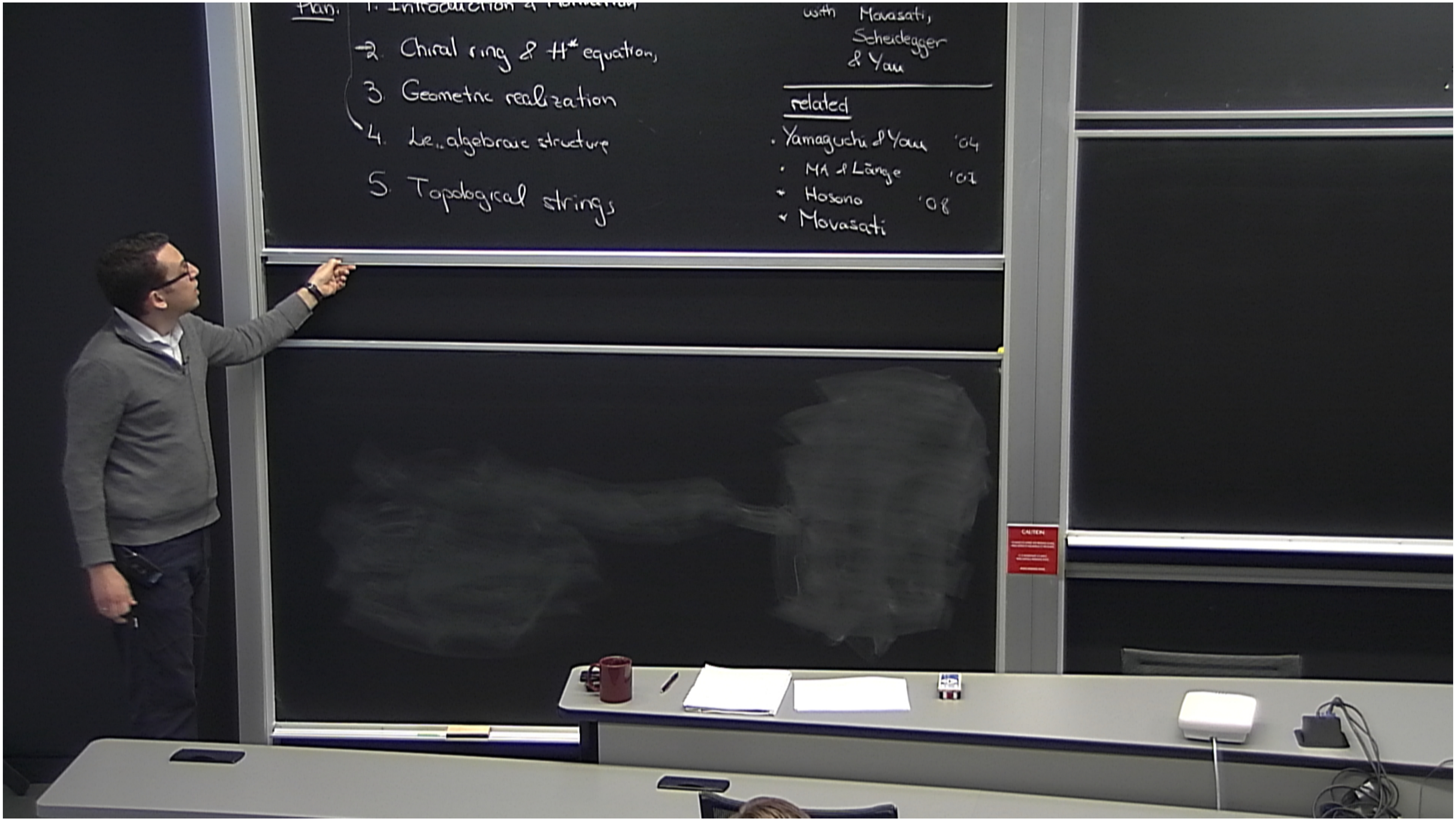
Date: Jan 22, 2015 11:00 AM

URL: http://pirsa.org/15010132

Abstract: <p>The tt* equations define a flat connection on the moduli spaces of 2d, N=2 quantum field theories. For conformal theories with c=3d, which can be realized as nonlinear sigma models into Calabi-Yau d-folds, this flat connection is equivalent to special geometry for threefolds and to its analogs in other dimensions. I will show that the non-holomorphic content of the tt* equations in the cases d=1,2,3 is captured in terms of finitely many generators of special functions, which close under derivatives. The generators are understood as coordinates on a larger moduli space. This space parameterizes a freedom in choosing representatives of the chiral ring while preserving a constant topological metric. Geometrically, the freedom corresponds to a choice of forms on the target space respecting the Hodge filtration and having a constant pairing. Linear combinations of vector fields on that space are identified with generators of a Lie algebra. This Lie algebra replaces the anti-holomorphic derivatives of tt* and provides these with a finer and algebraic meaning. The generators of the differential rings of special functions are given by quasi-modular forms for d=1 and their generalizations in d=2,3. For d=3, this can be used to provide a purely Lie algebraic formulation of the higher genus topological string theory amplitudes and of the BCOV holomorphic anomaly equations which govern them.</p>


based on
Plan
3 Geometric realization
5 Topological strings
related


tt* equations & topological strings
Plan. 1. Introduction & Motivation
2. Chiral ring & tt* equation,
3 Geometric realization
4. Lie algebraic structure
also
1410.1889
with Movasati,
Scheidegger
& Yau
related
Yamaguchi & Yau '04
MA & Länge '07
Hosono '08
Movasati



Plan.
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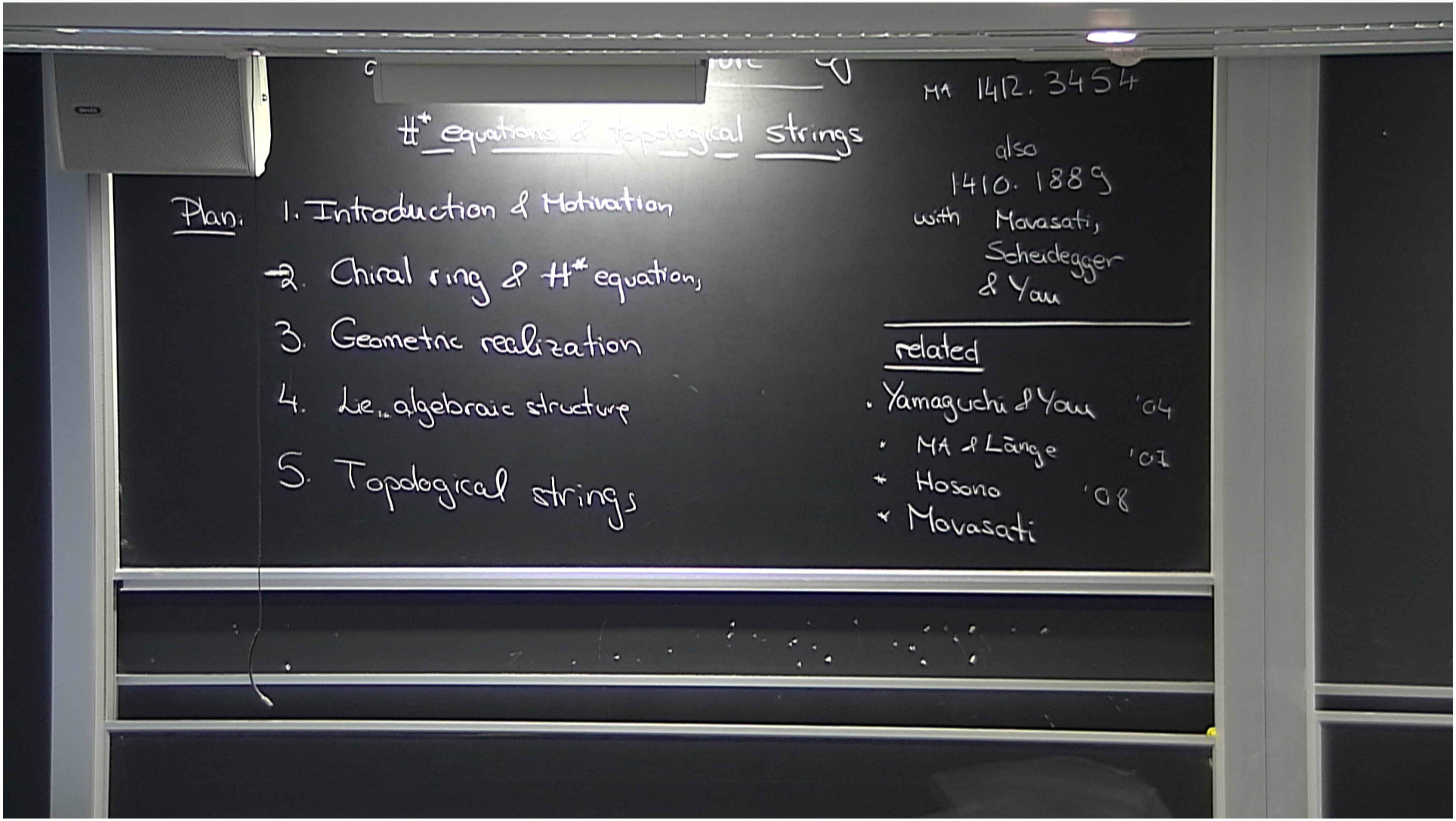
tt* equations & topological strings
MA 1412.3454
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with Movasati,
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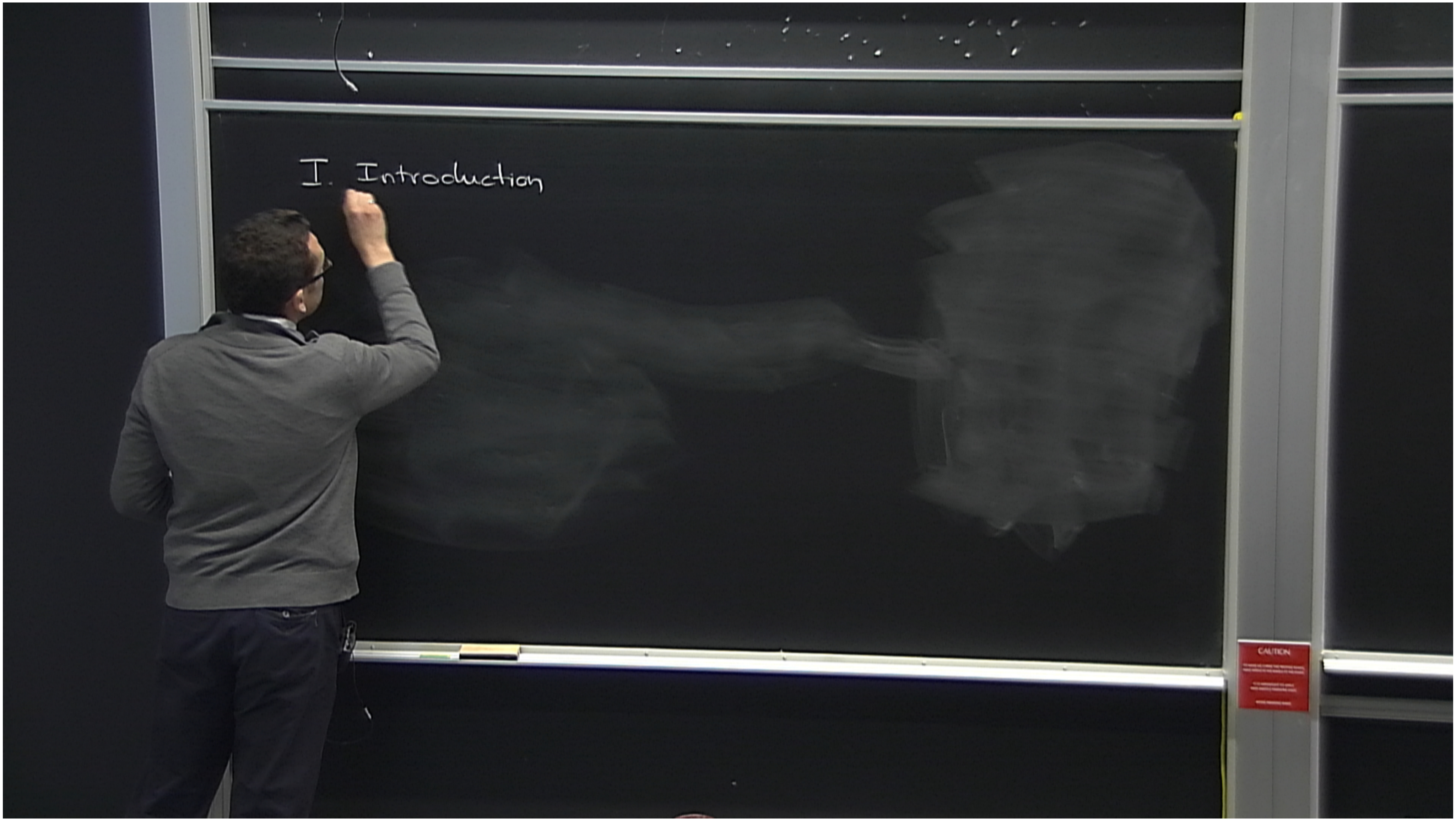
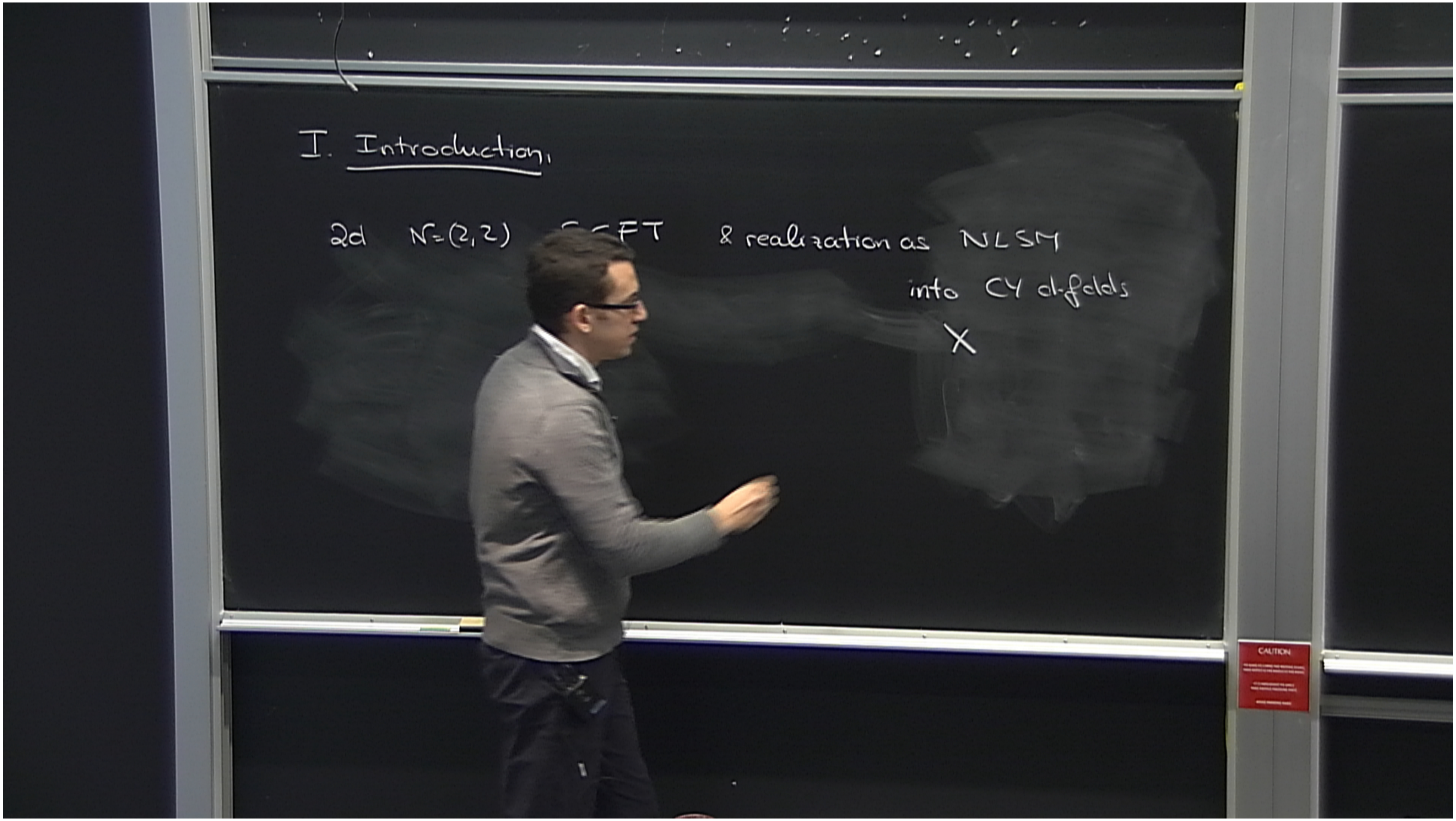
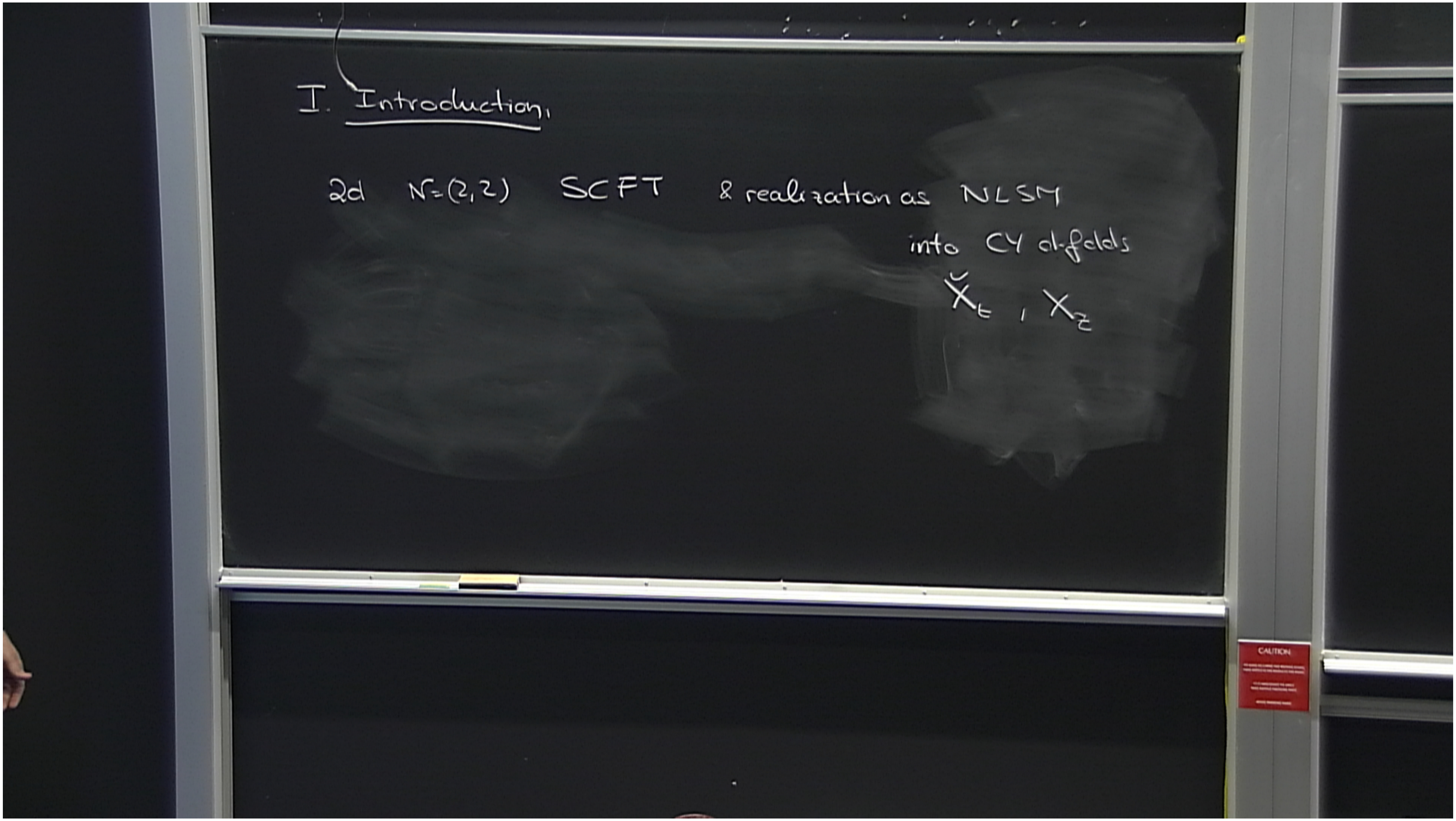
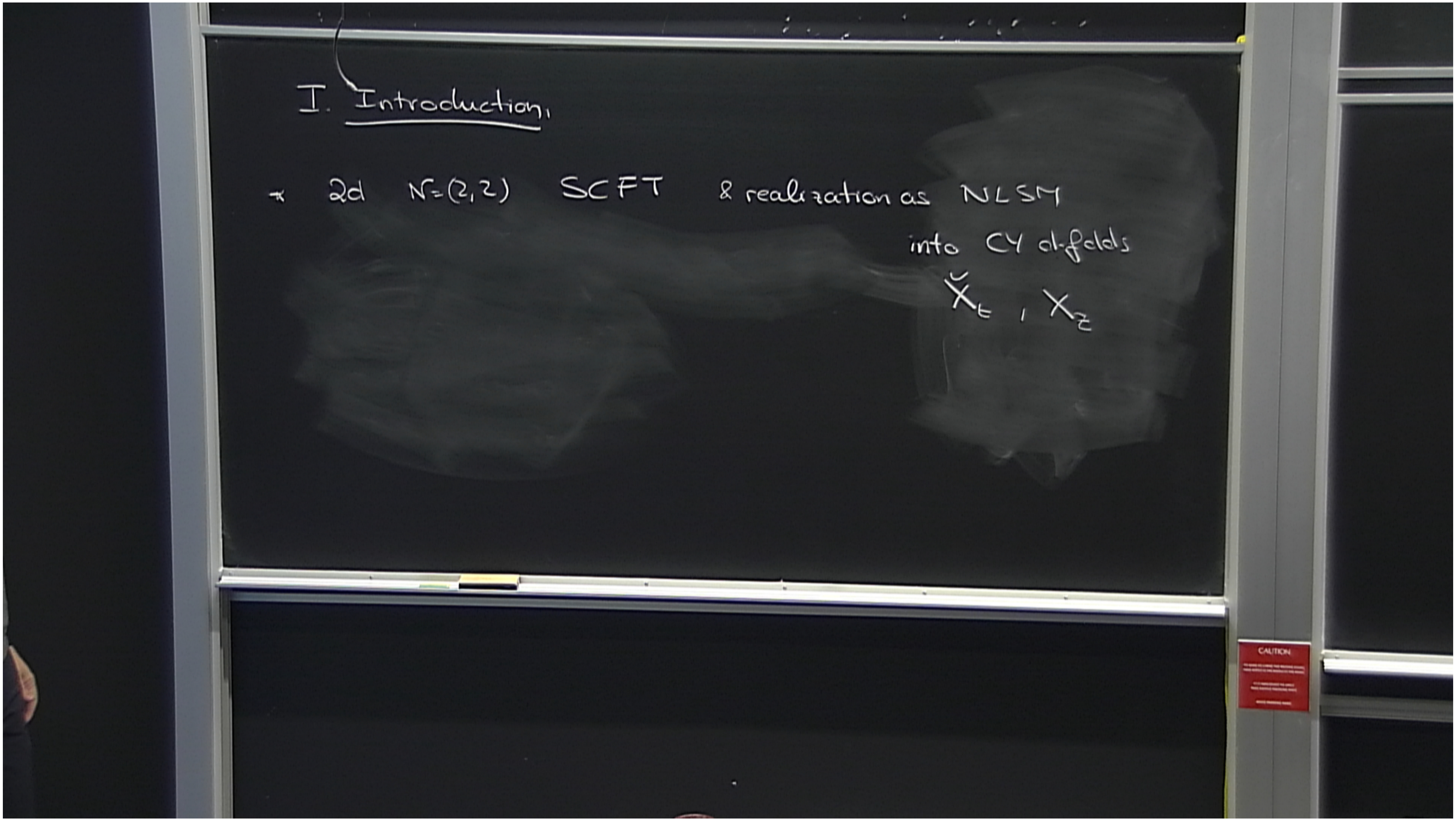

I. Introduction


I. Introduction
2d N=(2,2)
& realization as NLSM
into CY d-folds
X


I. Introduction.
2d N=(2,2) SCFT & realization as NLSM
into CY d-folds
X̌_t , X_z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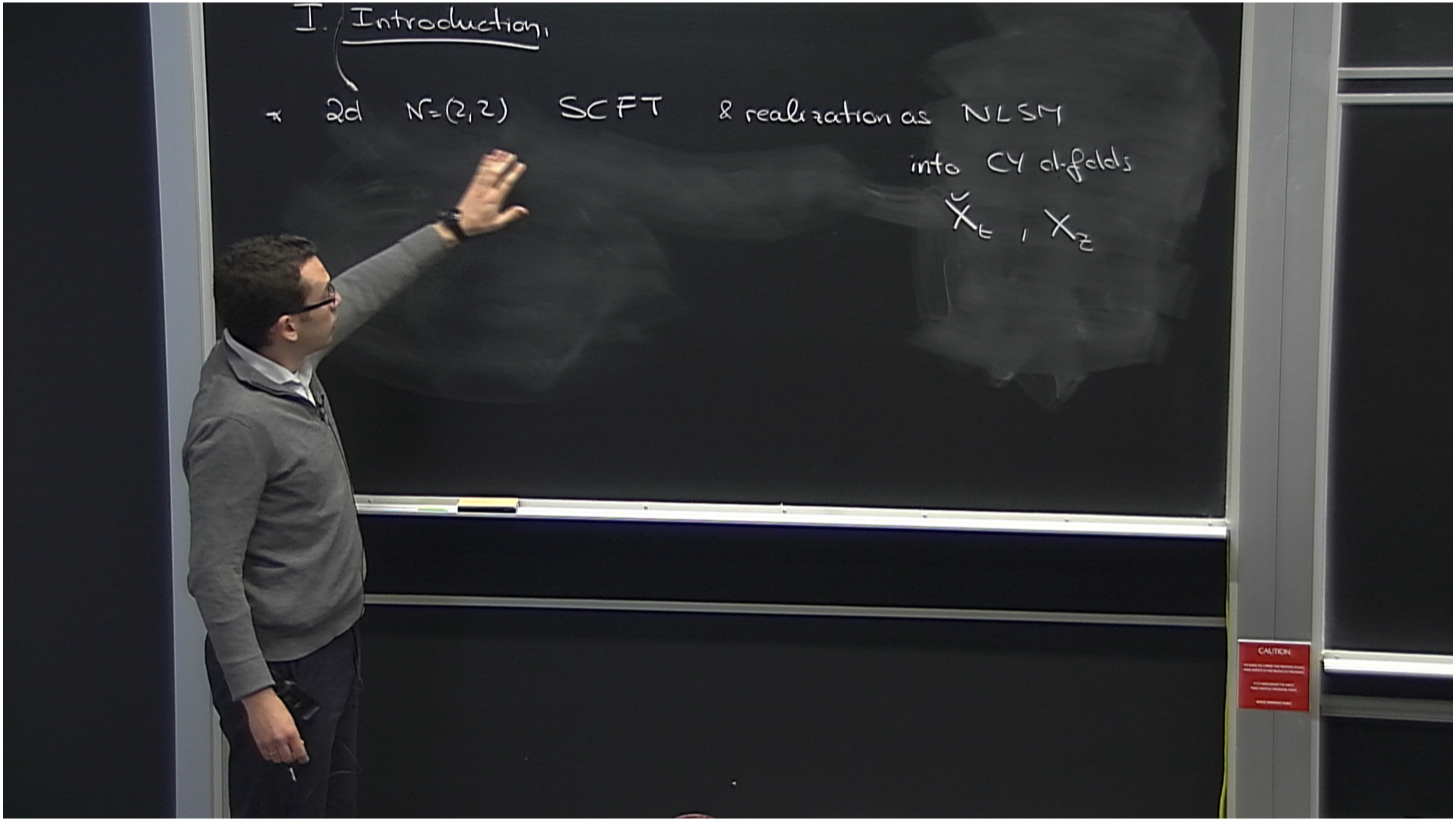
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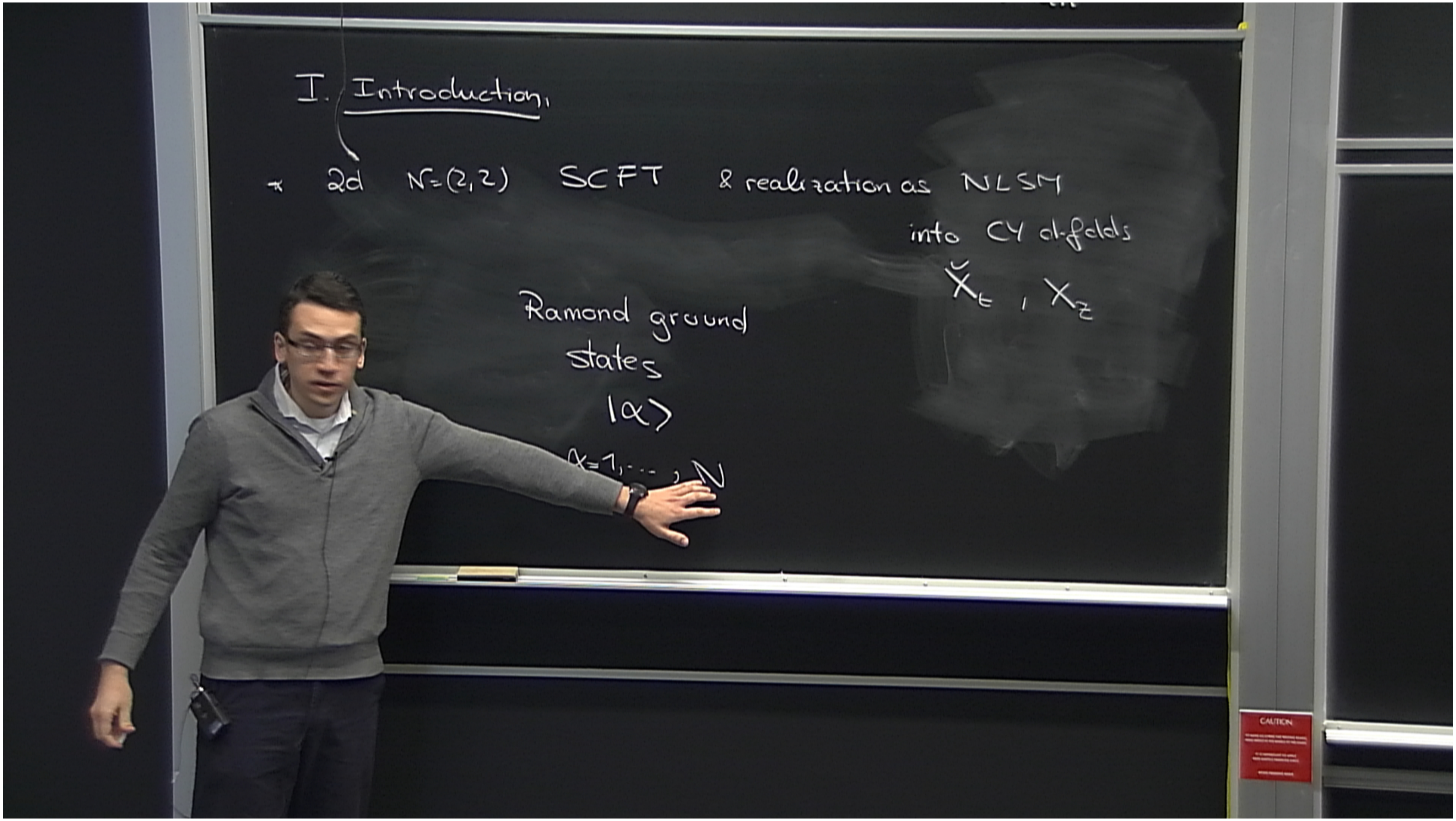
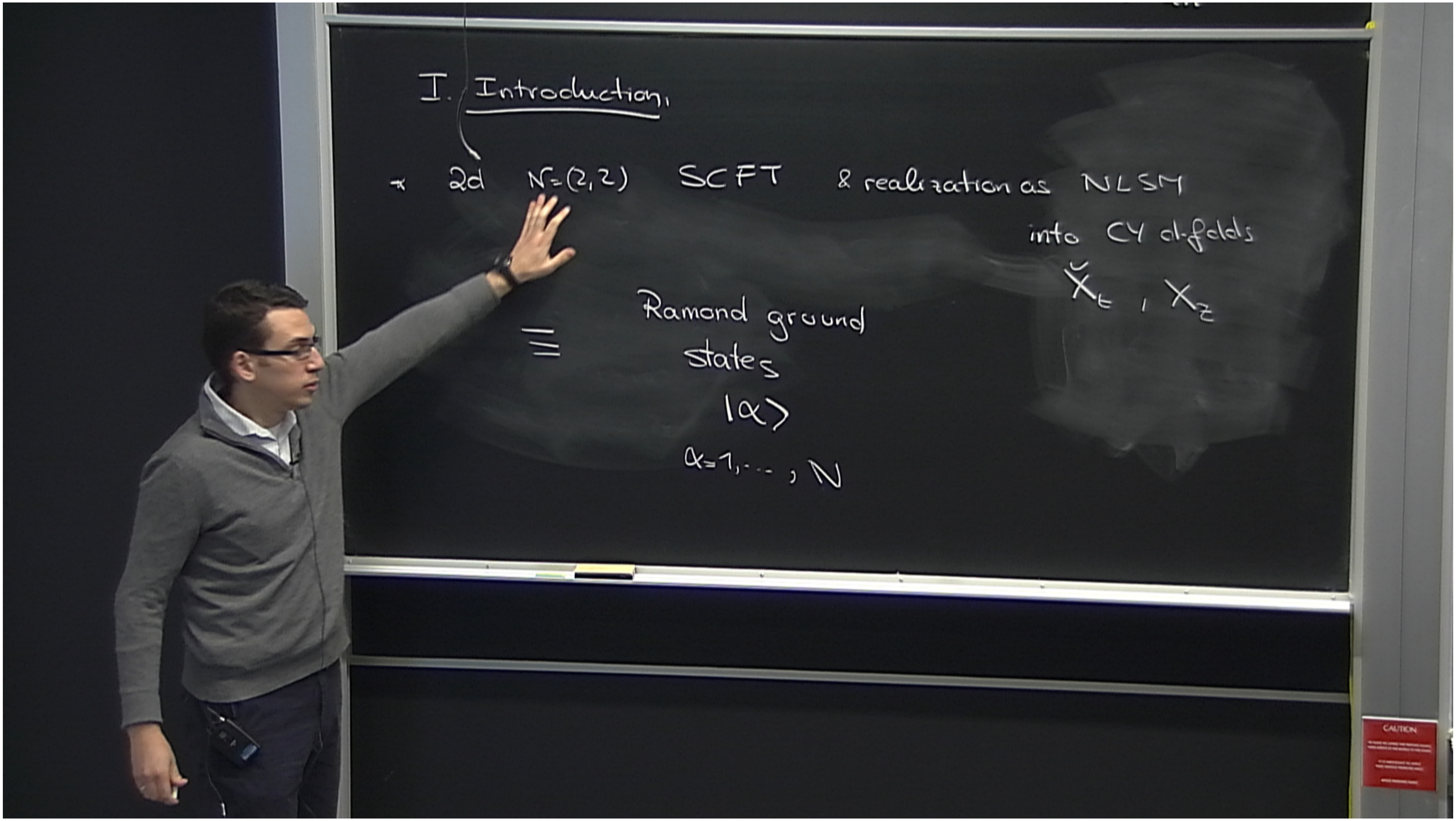
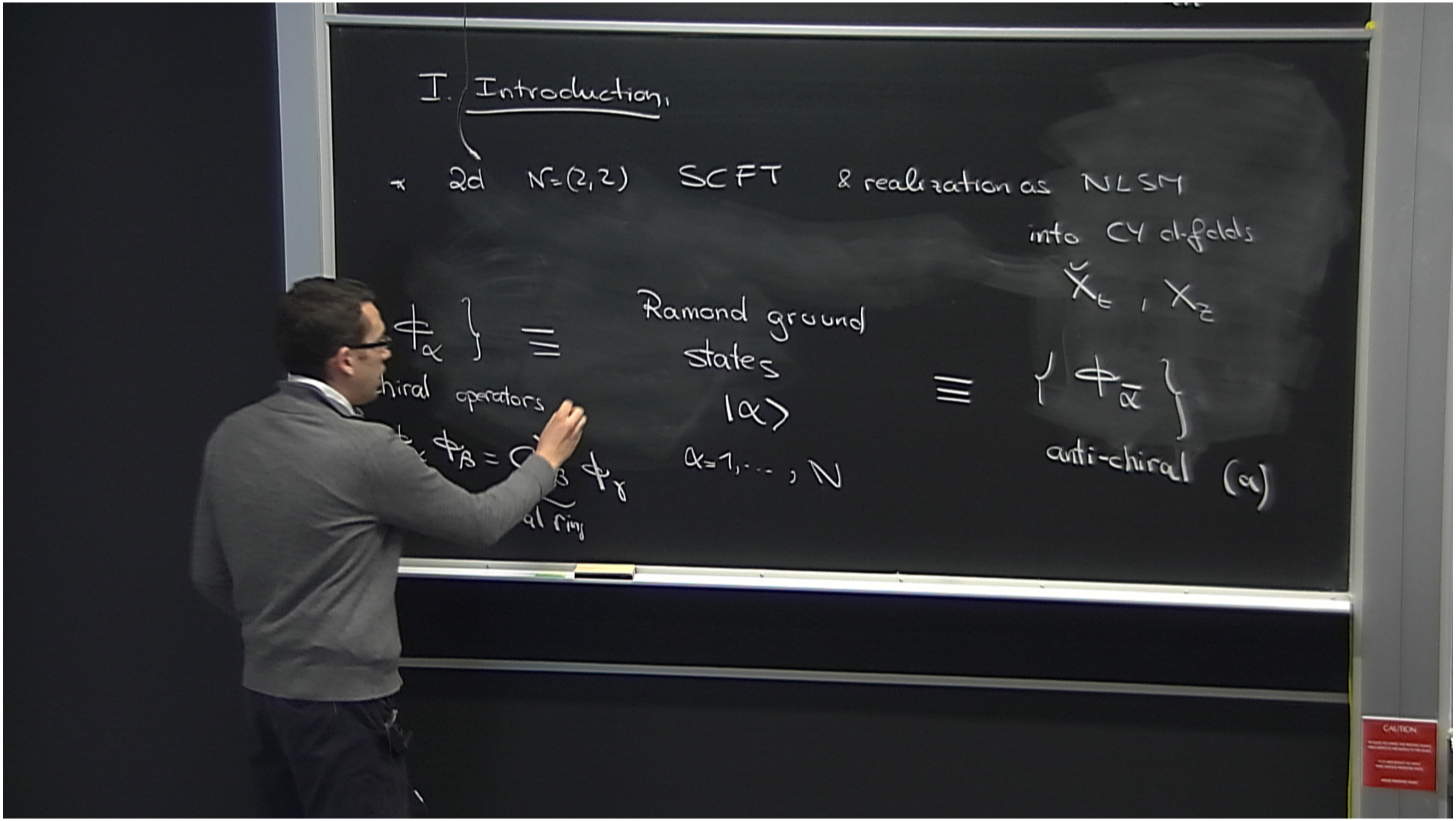

I. Introduction
* 2d N=(2,2) SCFT & realization as NLSM
into CY d-folds
X̌t , Xz
Ramond ground states
|α⟩
α=1,...,N


I. Introduction
* 2d $\mathcal{N}=(2,2)$ SCFT & realization as NLSM
into CY 3-folds
$\check{X}_t$, X_z
≡ Ramond ground states
$|\alpha\rangle$
$\alpha=1,\dots,N$
CAUTION


I. Introduction
2d N=(2,2) SCFT & realization as NLSM
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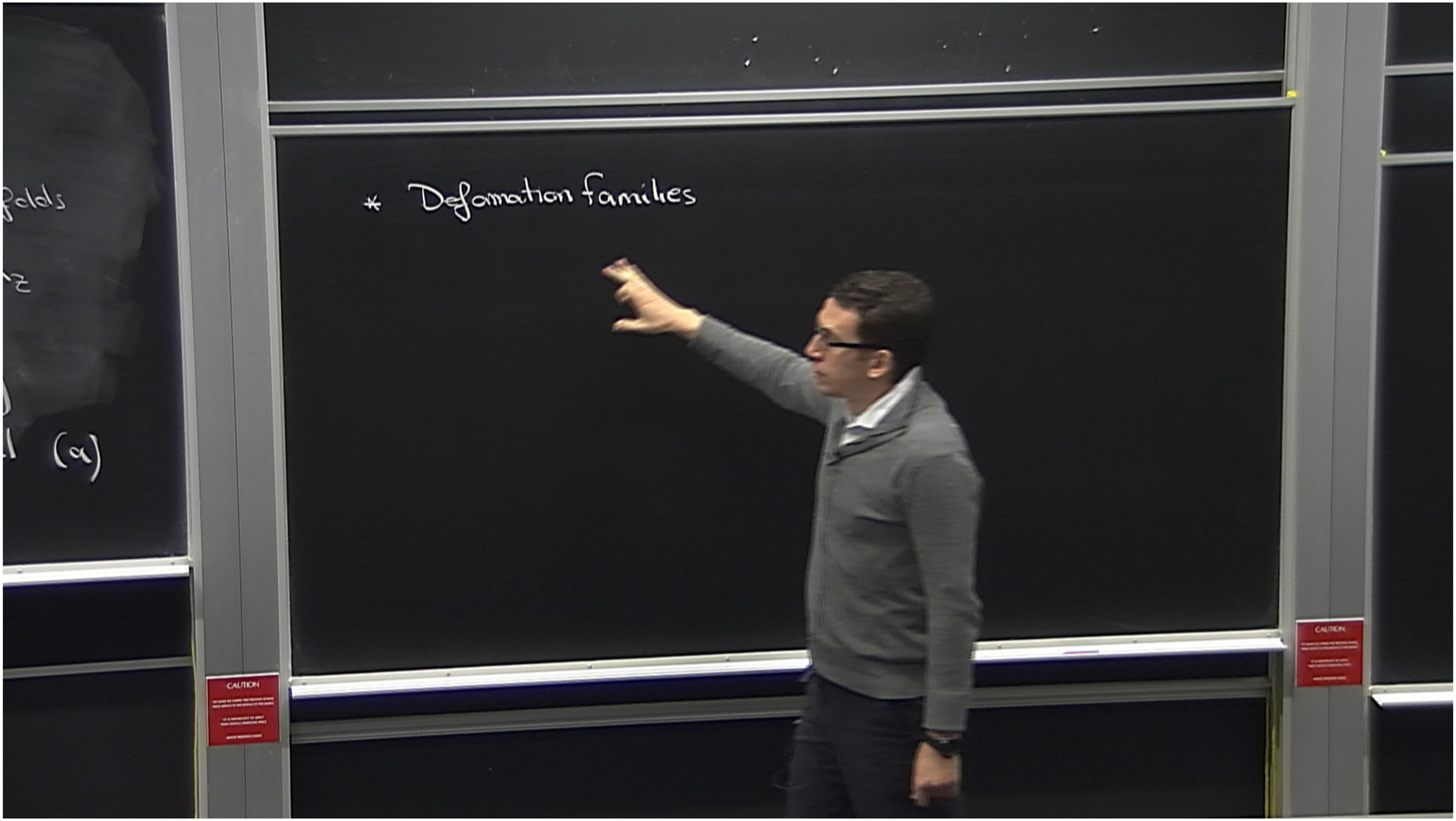
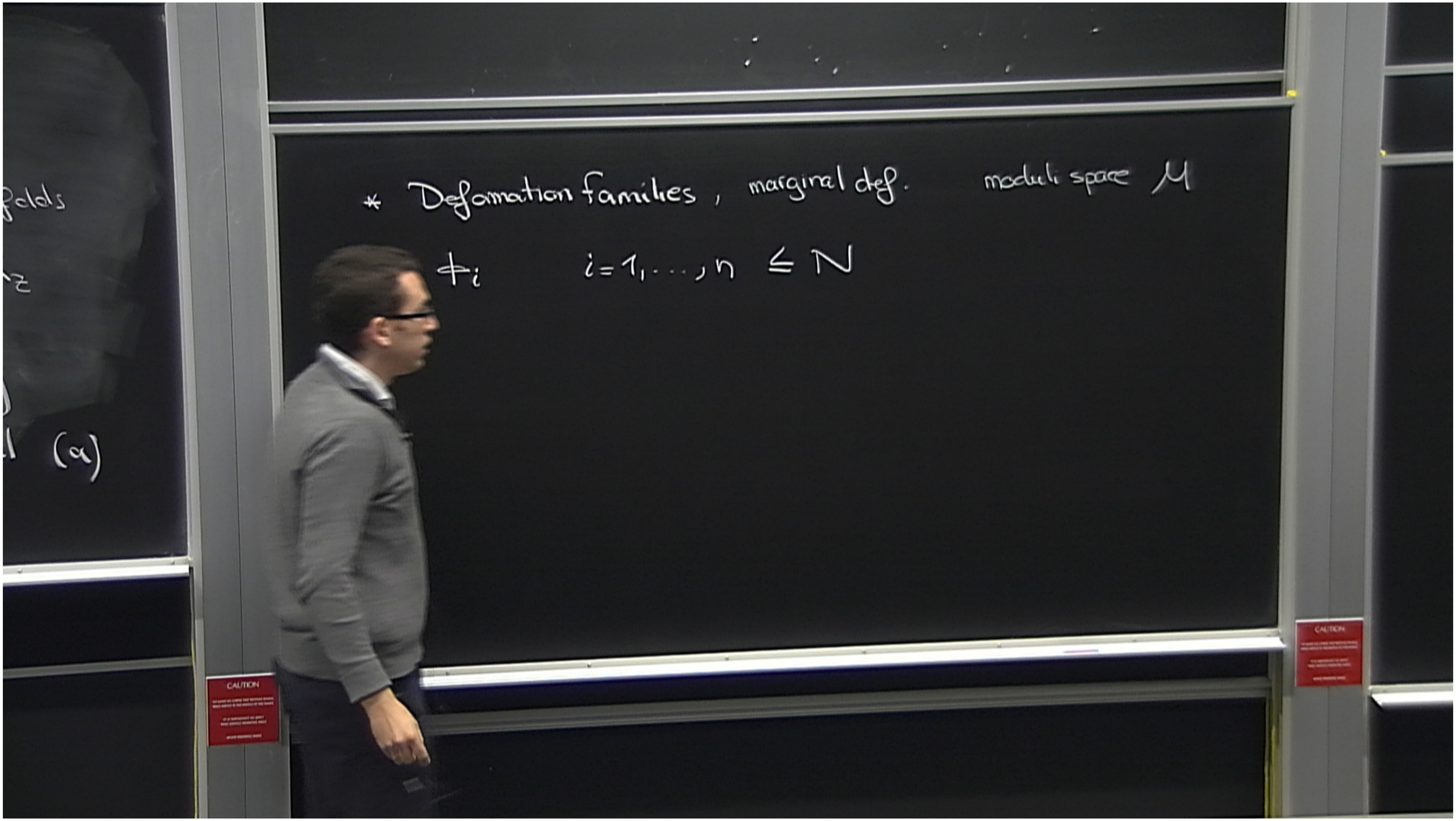
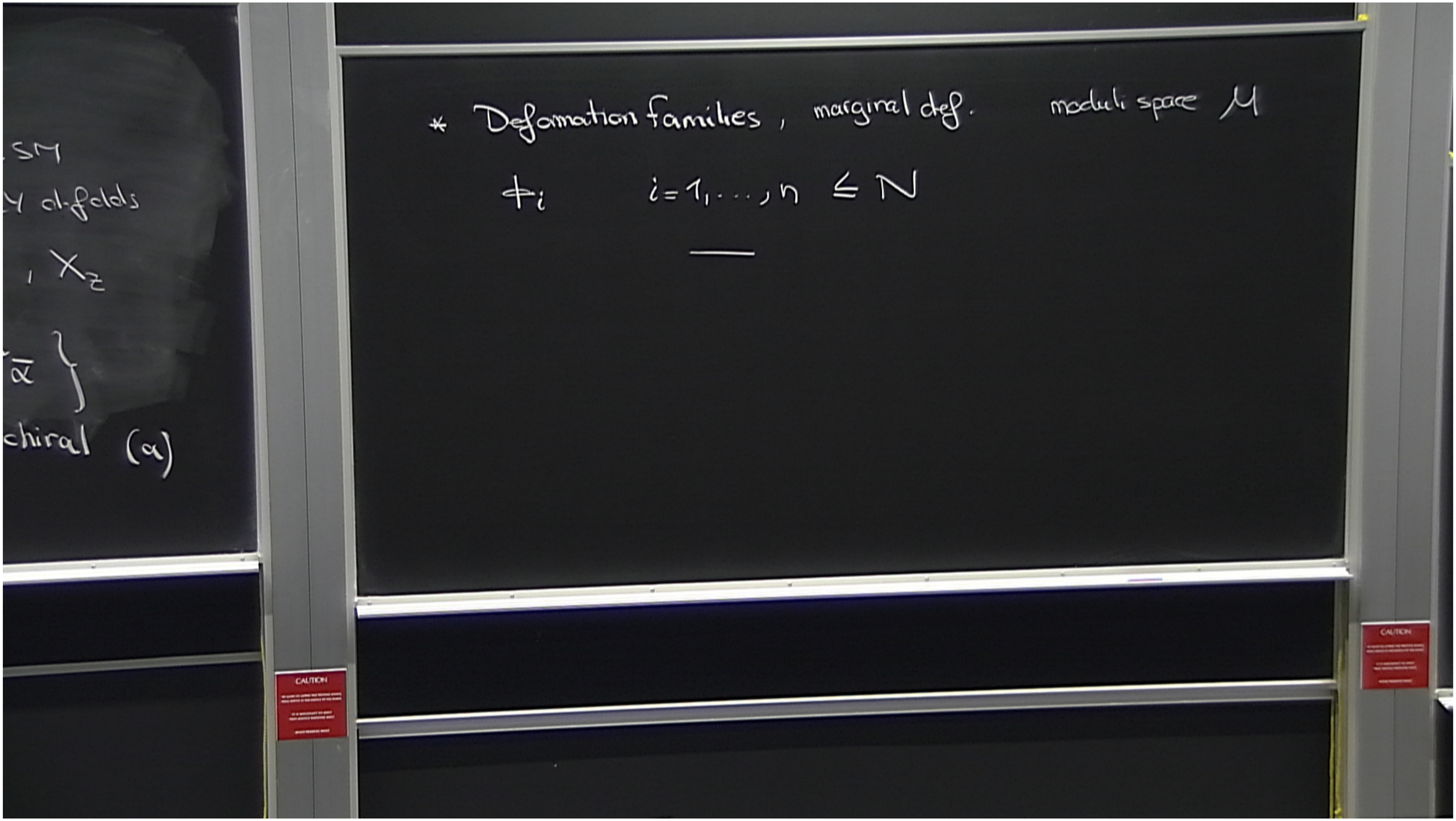
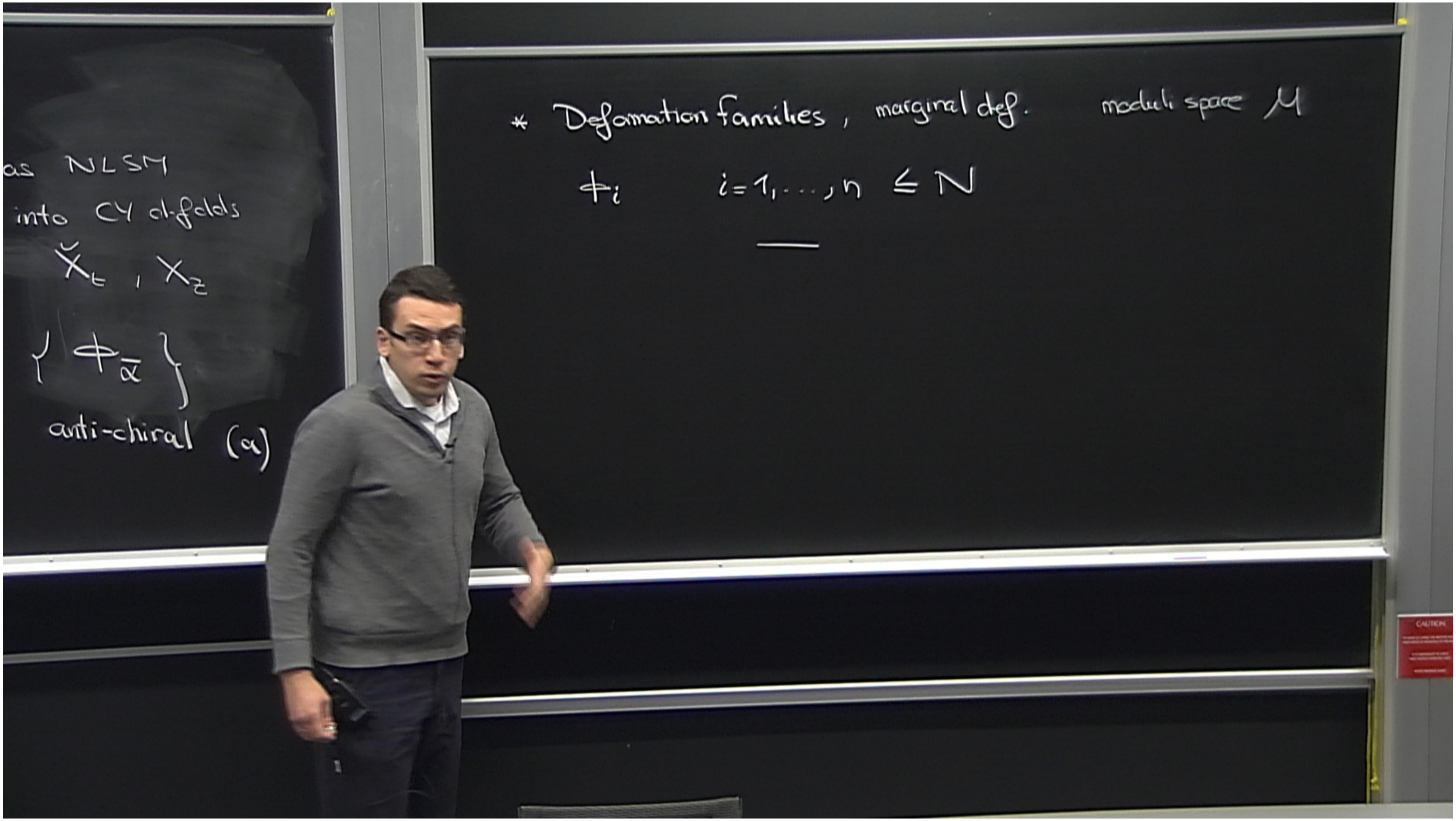
Ramond ground
states
|α>
α=1, ..., N
anti-chiral
operators


* Deformation families


* Deformation families , marginal def. moduli space M
ϕi i=1,...,n ≤ N


* Deformation families , marginal def. moduli space $\mathcal{M}$
ϕ_i $i = 1, \dots, n \leq N$
chiral (a)


as NLSM
into CY cl-folds
anti-chiral (a)
* Deformation families , marginal def.
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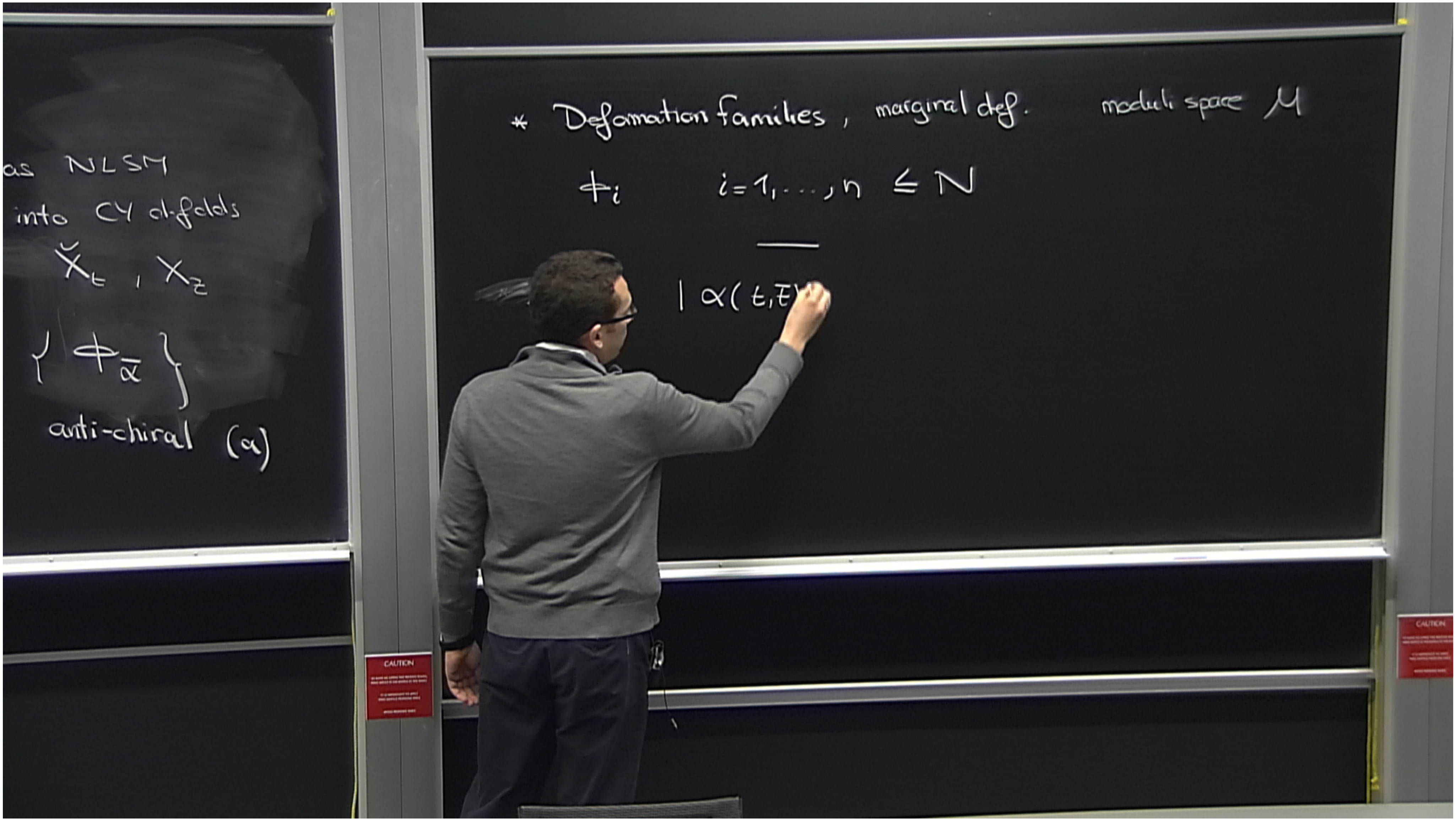
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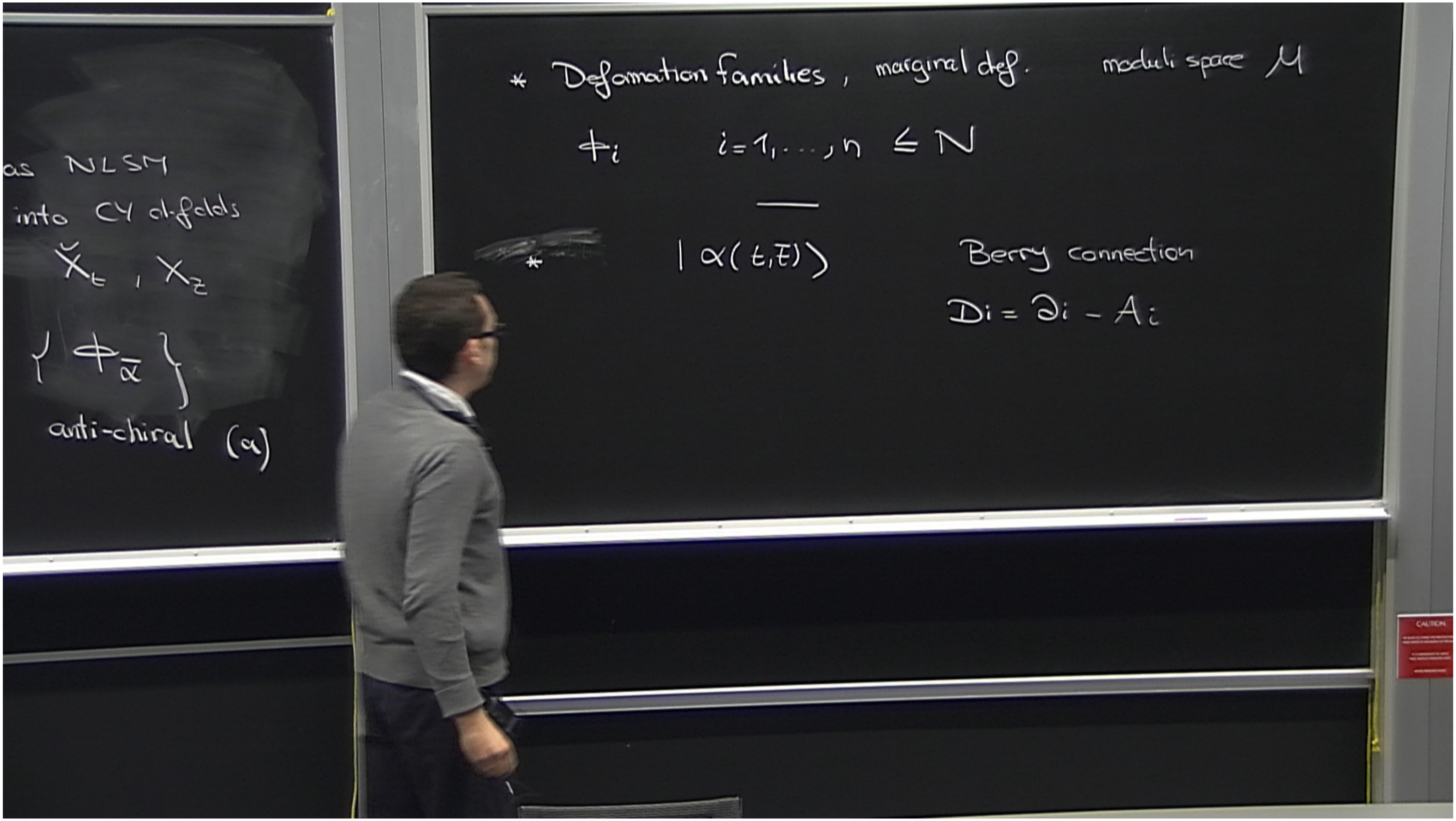
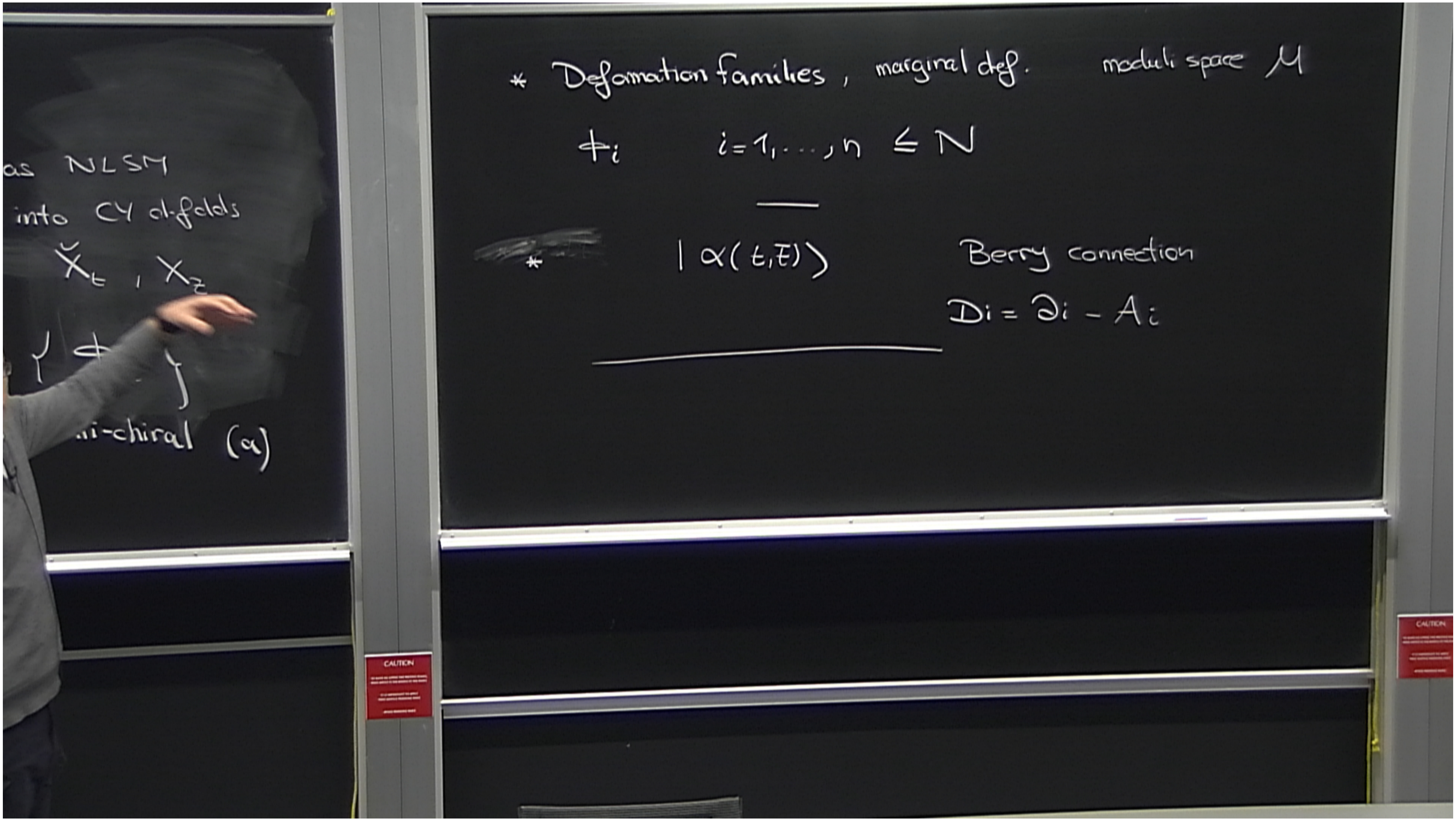

as NLSM
into CY d-folds
X̌_t , X_z
{ φ_ᾱ }
anti-chiral (a)
* Deformation families , marginal def.
moduli space M
φ_i
i = 1, ... , n ≤ N
* |α(t, t̄)⟩
Berry connection
D_i = ∂_i − A_i


as NLSM
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$\check{X}_t$, X_z
-chiral (α)
* Deformation families , marginal def.
moduli space $\mathcal{M}$
t_i $i=1,\dots,n \leq N$
$|\alpha(t,\bar{t})\rangle$
Berry connection
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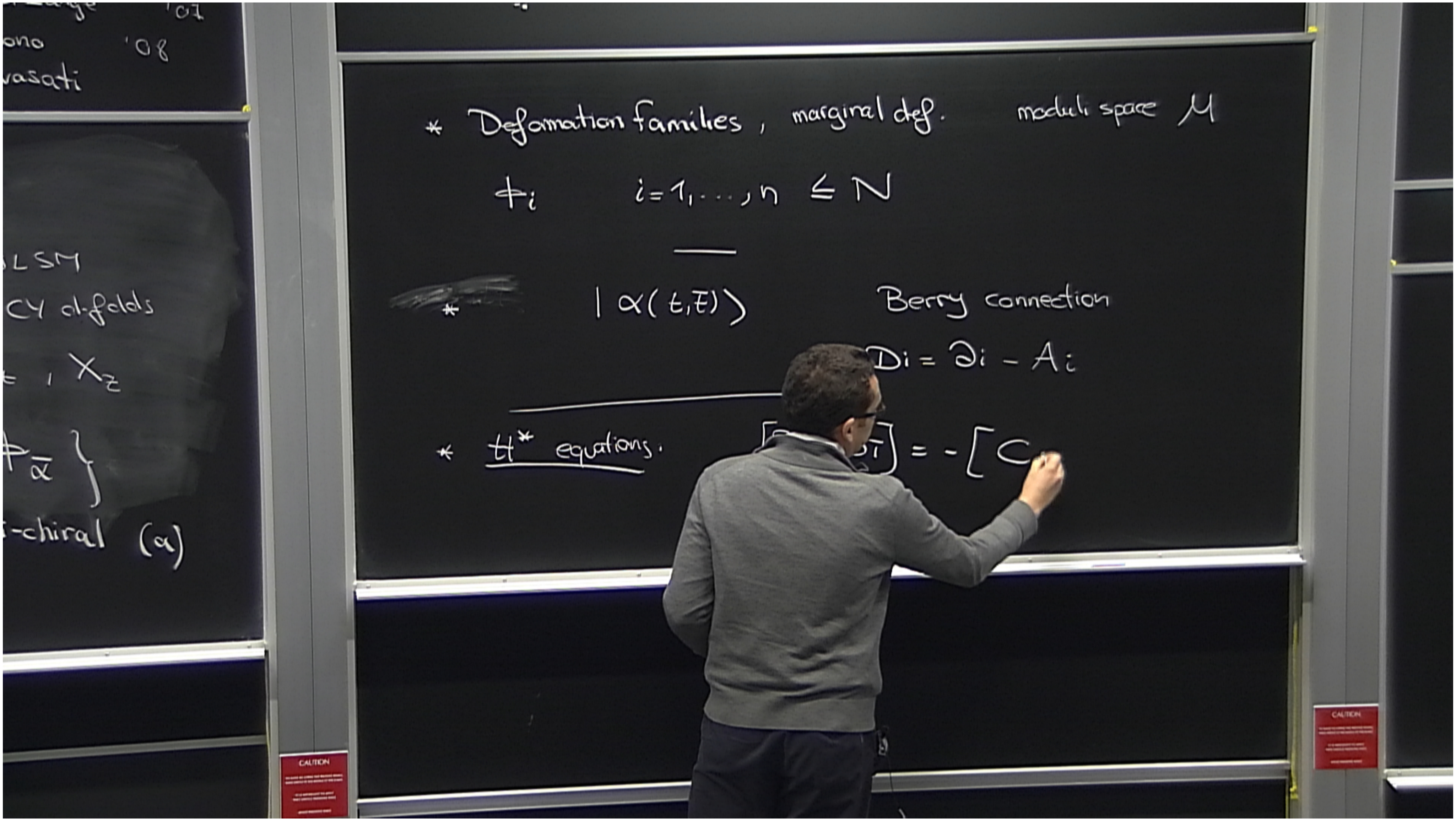
* Deformation families , marginal def. moduli space M
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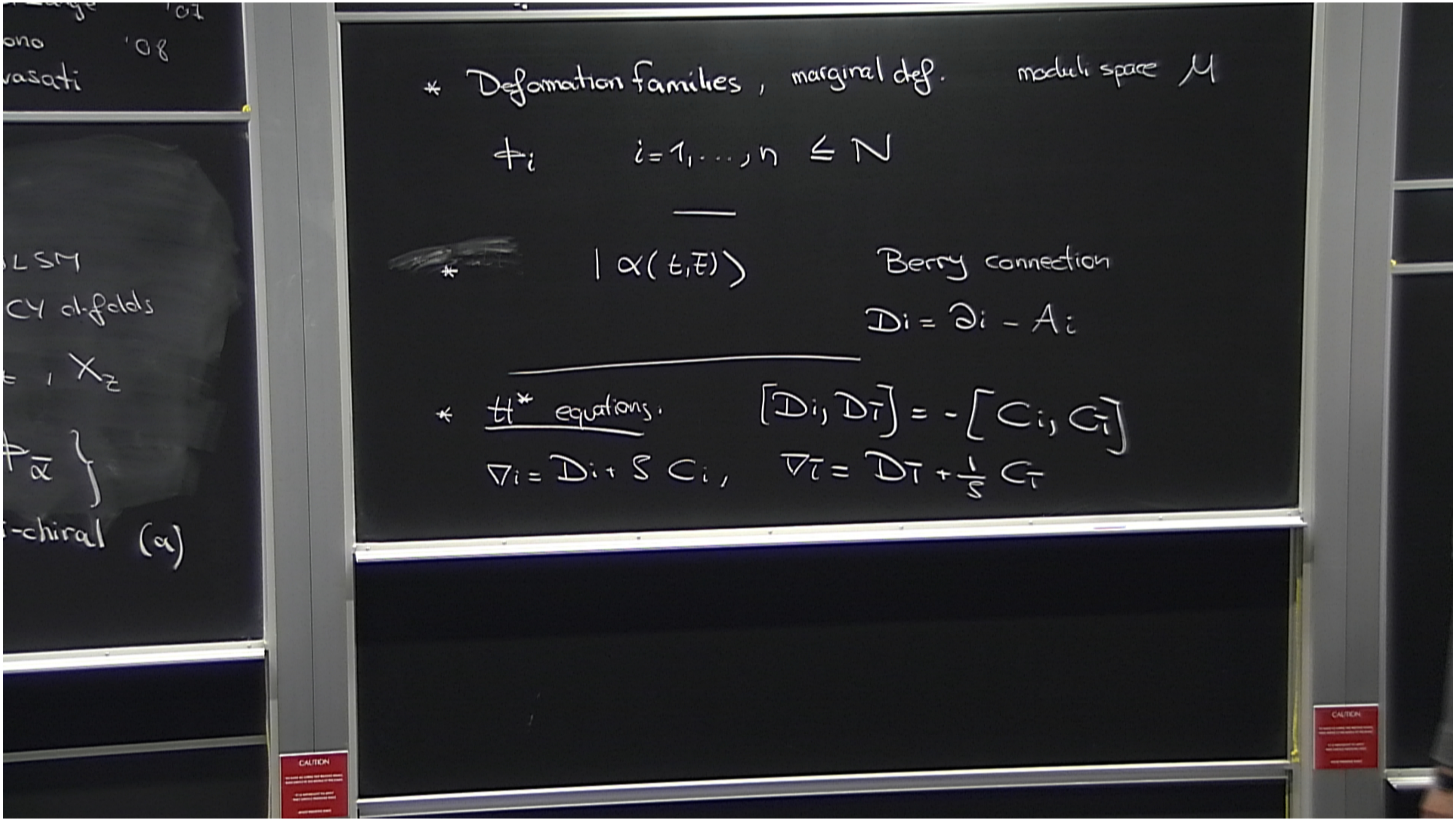
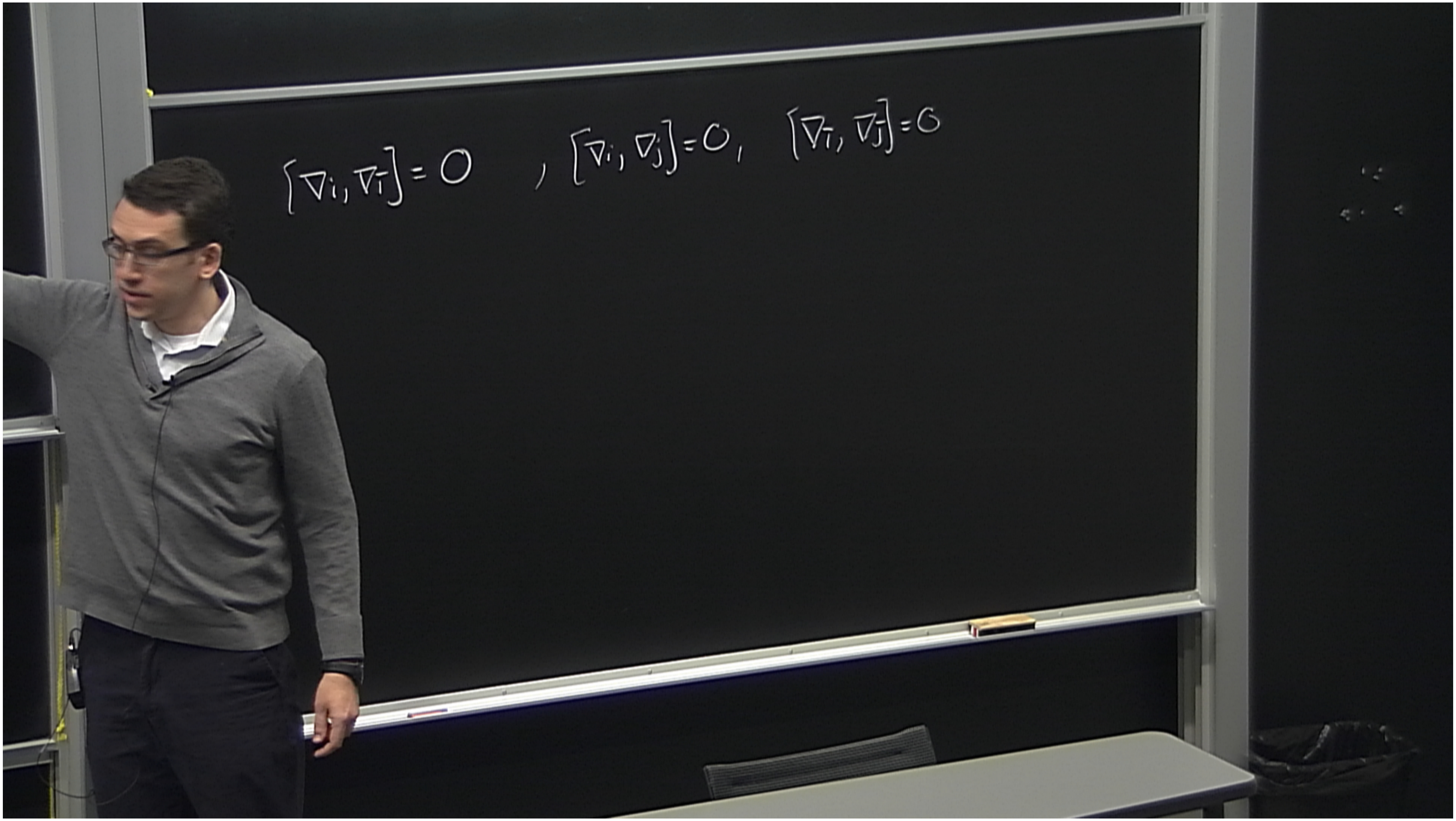
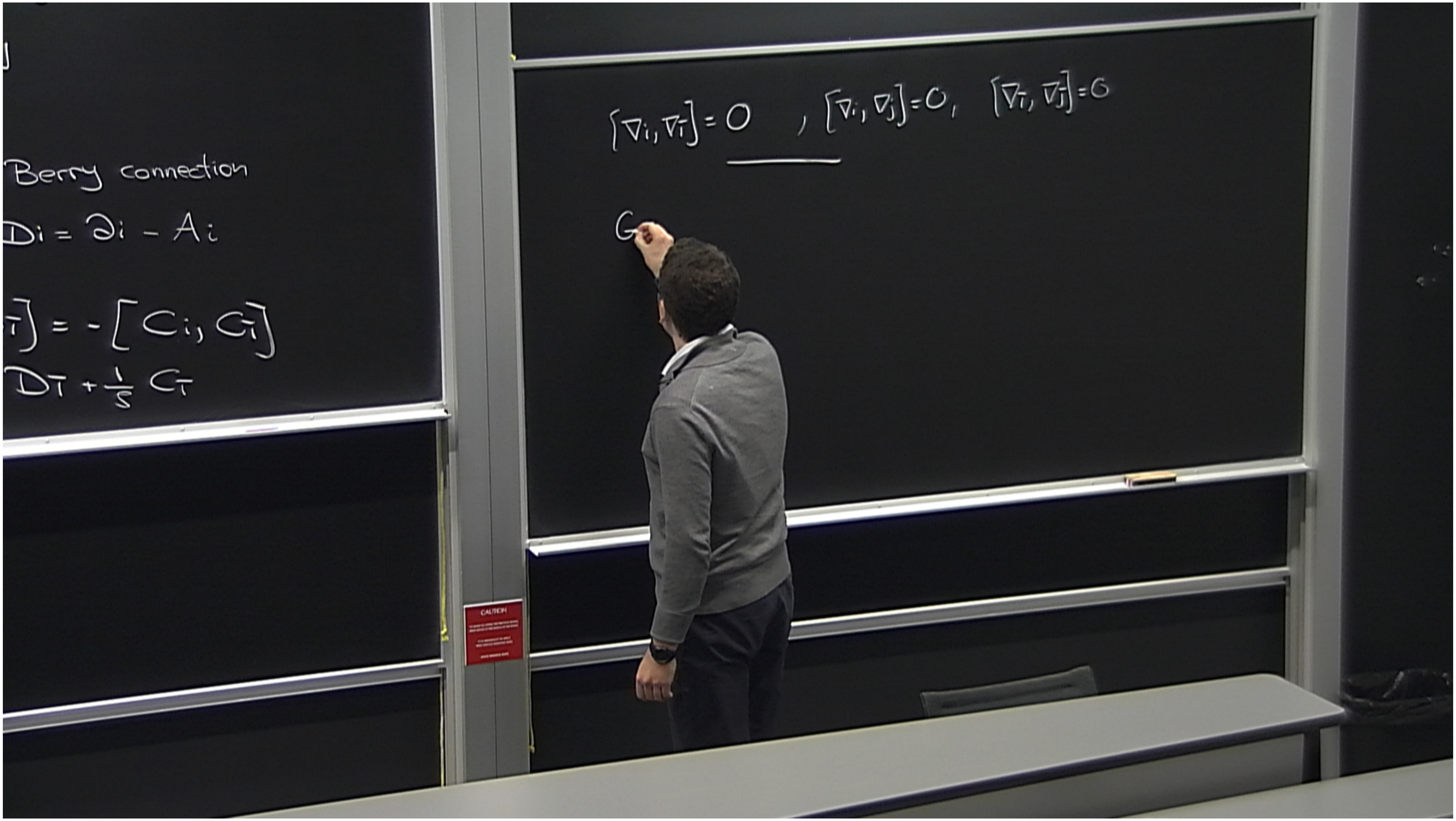
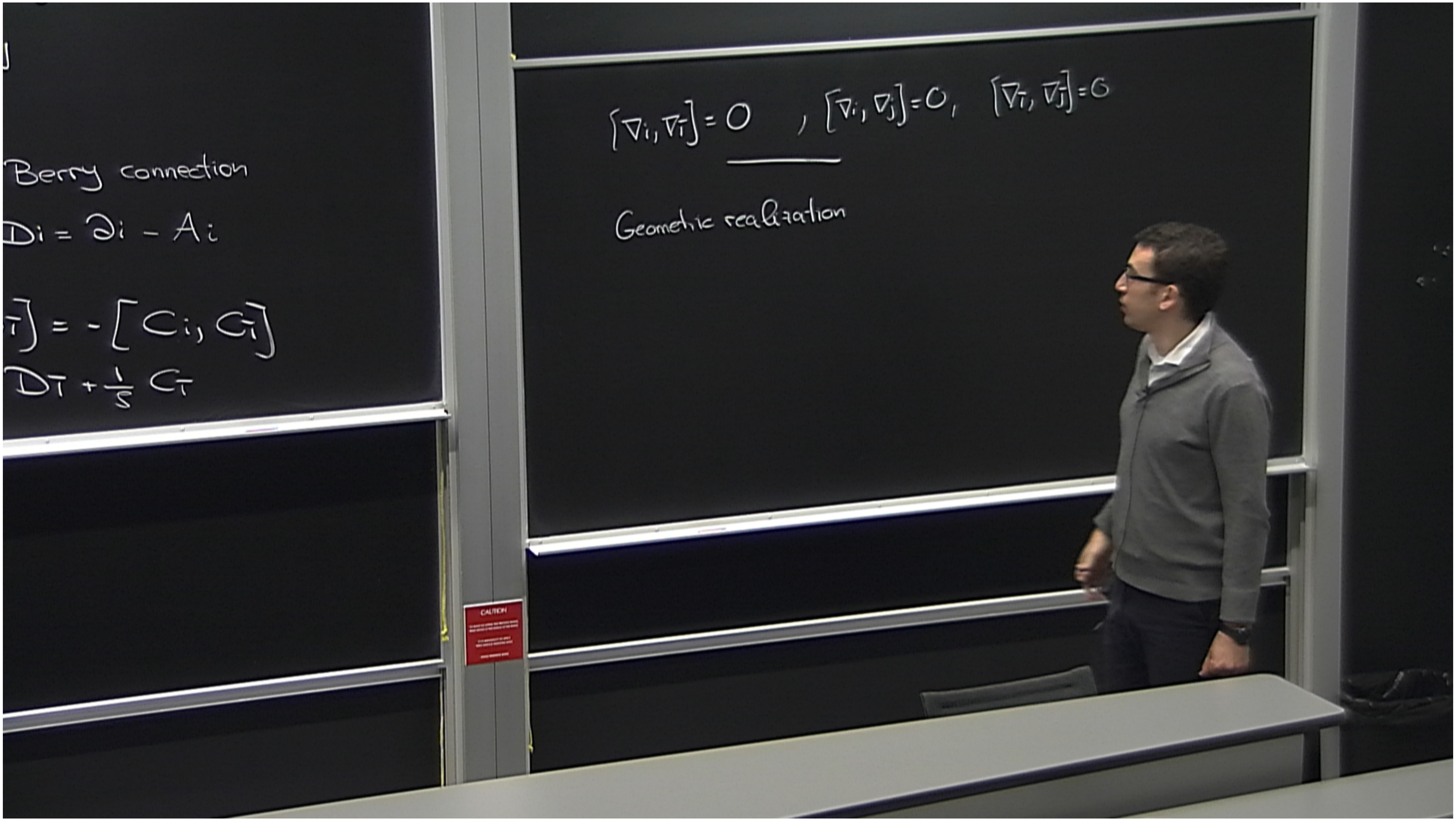
* |α(t,t̄)⟩ Berry connection
Di = ∂i − Ai
* tt* equations.
-chiral (α)

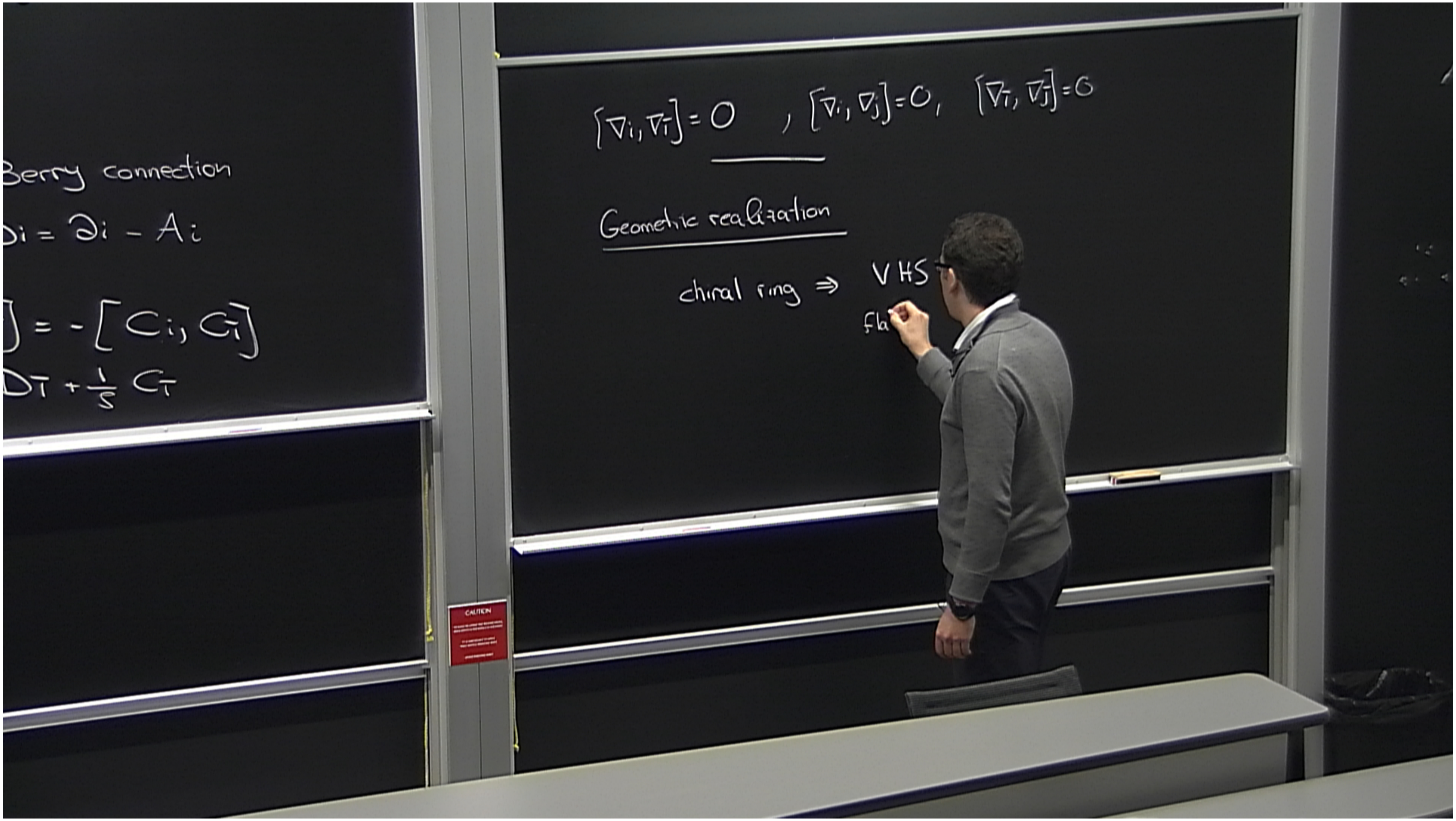


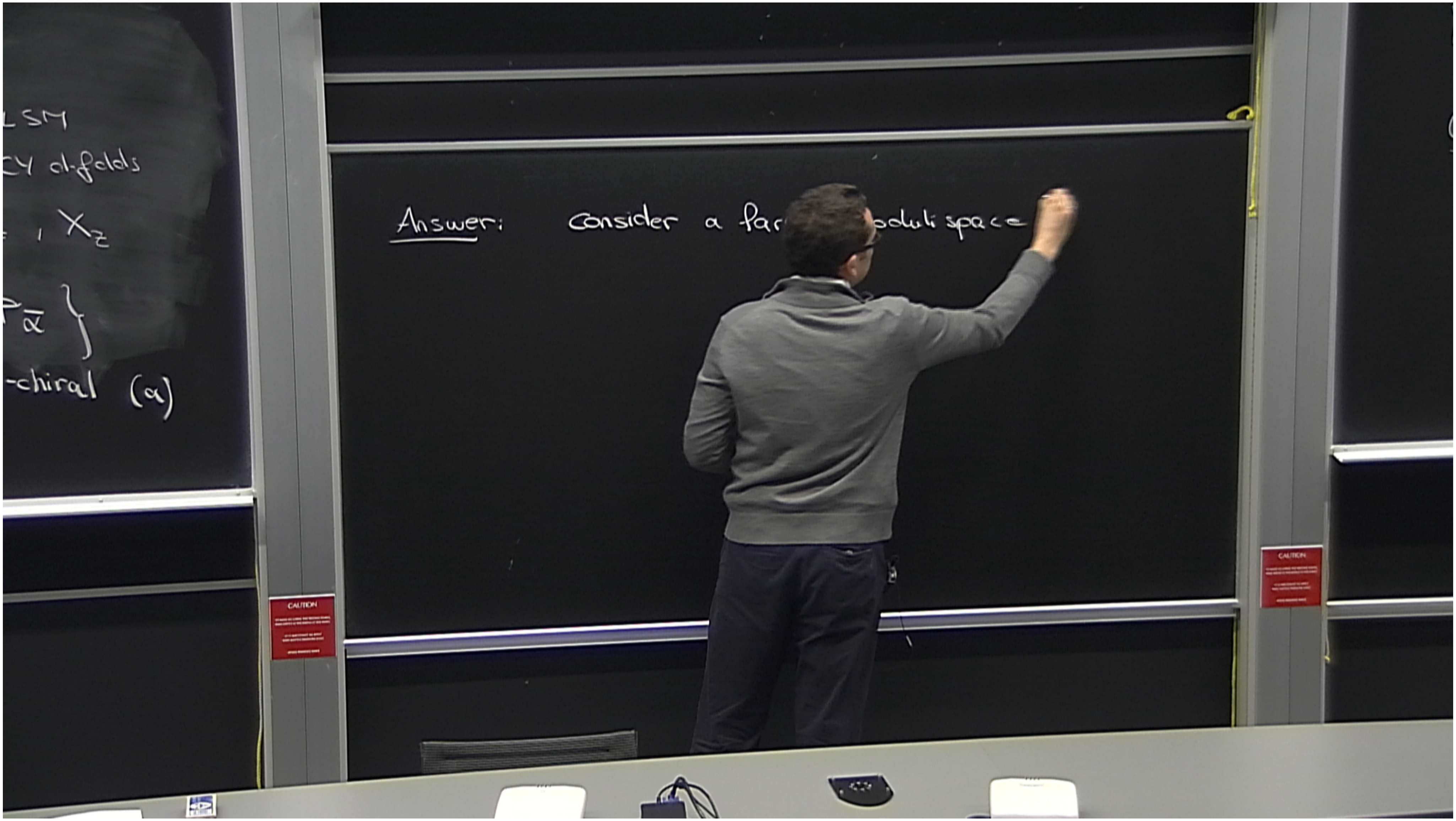
* Deformation families , marginal def. moduli space M
t_i i = 1, . . . , n ≤ N
* |α(t, t̄)⟩ Berry connection
D_i = ∂_i − A_i
* tt* equations. [D_i, D_ī] = −[C_i, C_ī]
∇_i = D_i + ζ C_i , ∇_ī = D_ī + 1/ζ C_ī

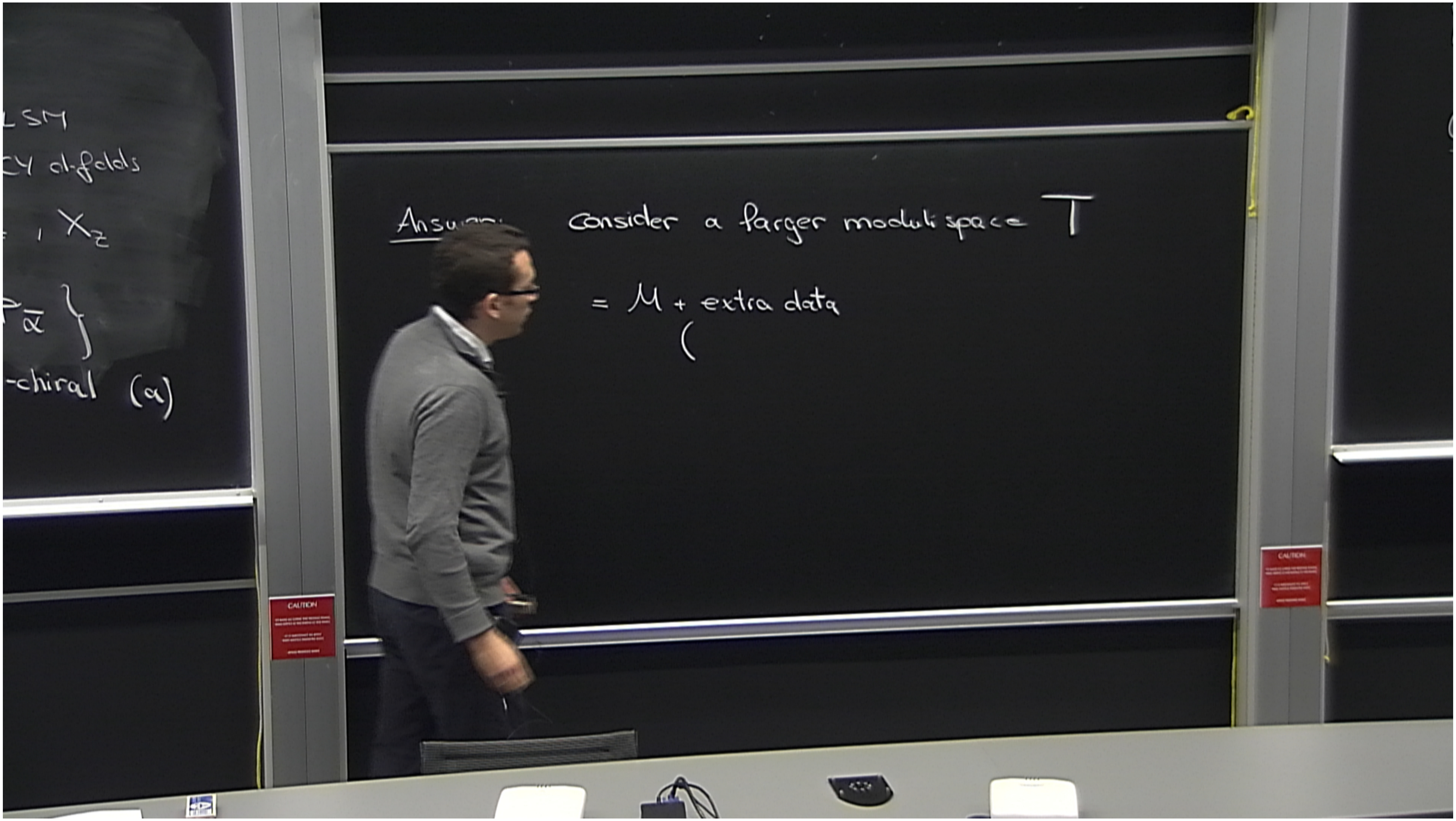

[∇i, ∇ī] = 0 , [∇i, ∇j] = 0, [∇ī, ∇j̄] = 0


Berry connection
Di = ∂i − Ai
] = −[Ci, Cī]
Dī + 1/S Cī
[∇i, ∇ī] = 0 , [∇i, ∇j] = 0, [∇ī, ∇j̄] = 0
CAUTION

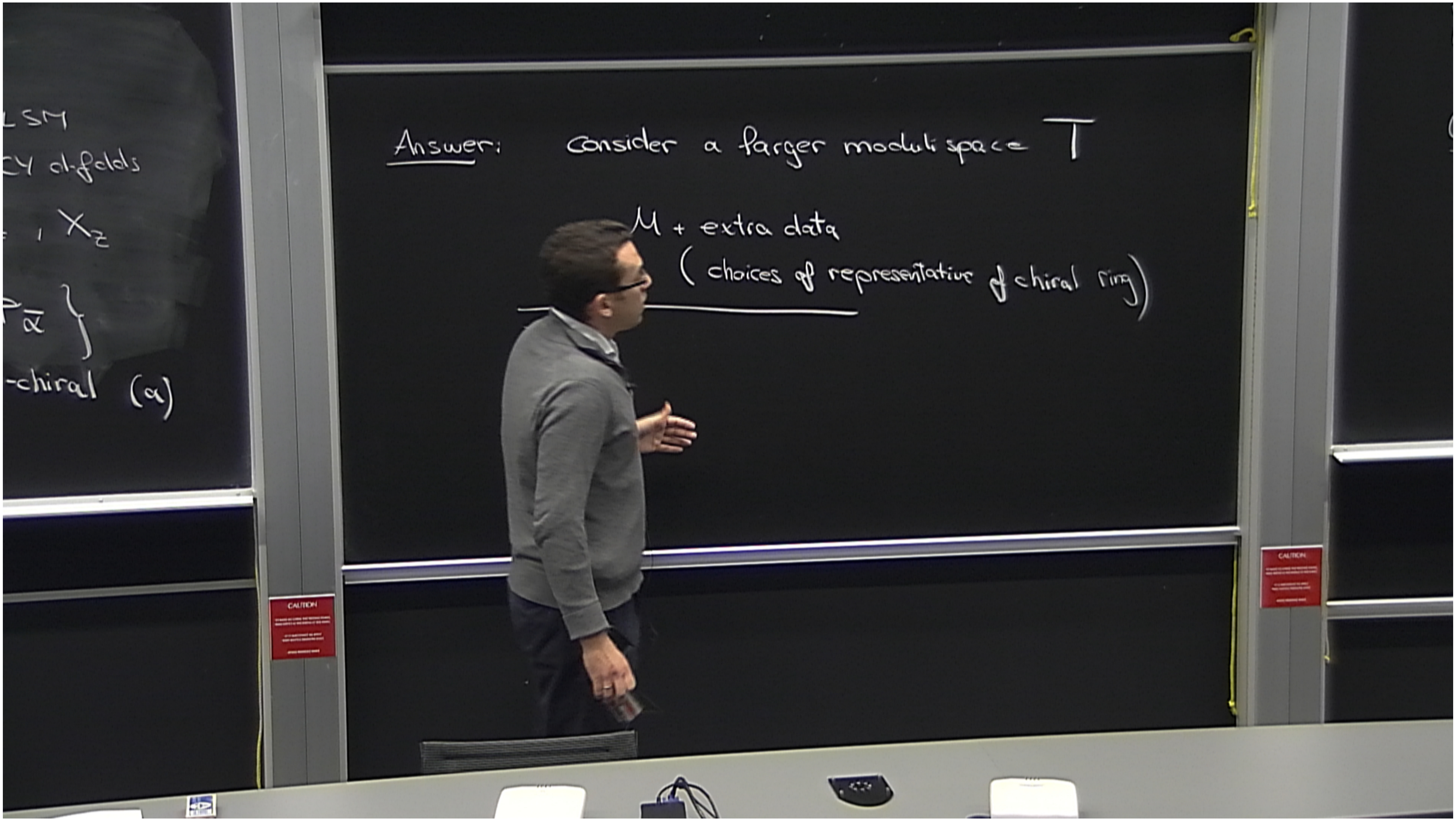

Berry connection
Geometric realization

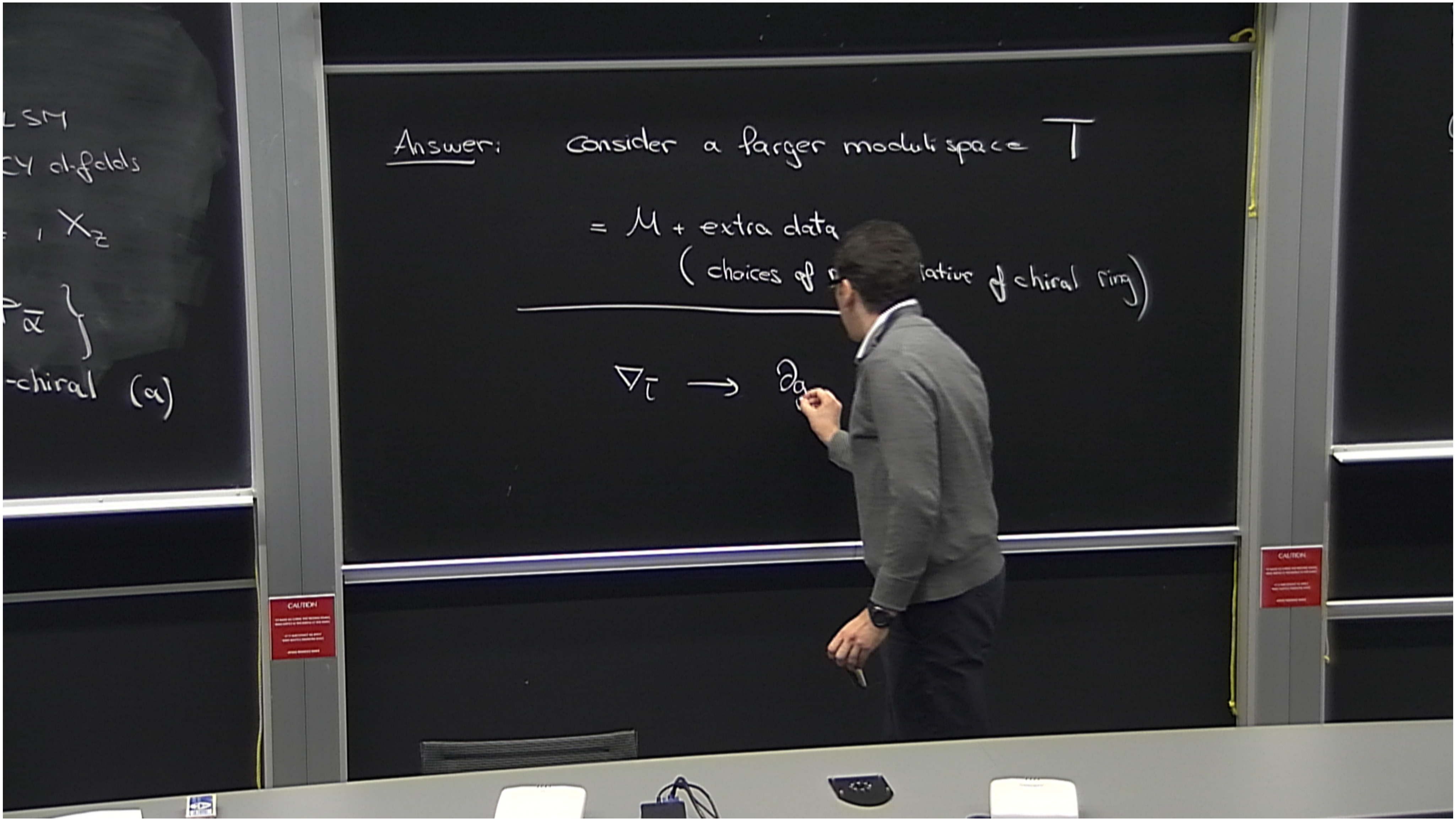

erry connection
i = ∂i − Ai
] = − [Ci, Cī]
[∇i, ∇ī] = 0 , [∇i, ∇j] = 0, [∇ī, ∇j̄] = 0
Geometric realization
chiral ring ⇒ VHS
CAUTION

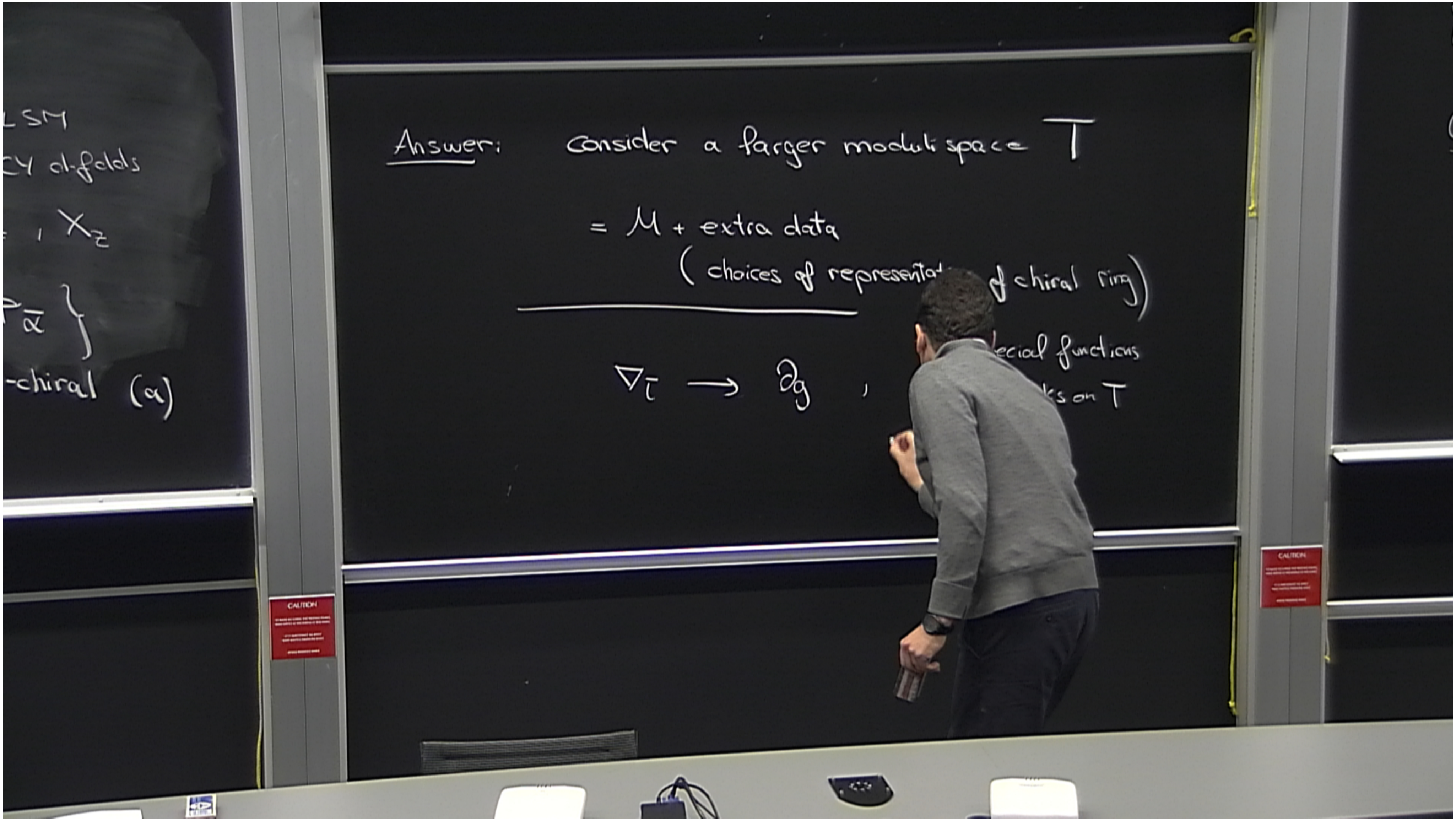

LSM
CY d-folds
-chiral (a)
Answer: Consider a
space



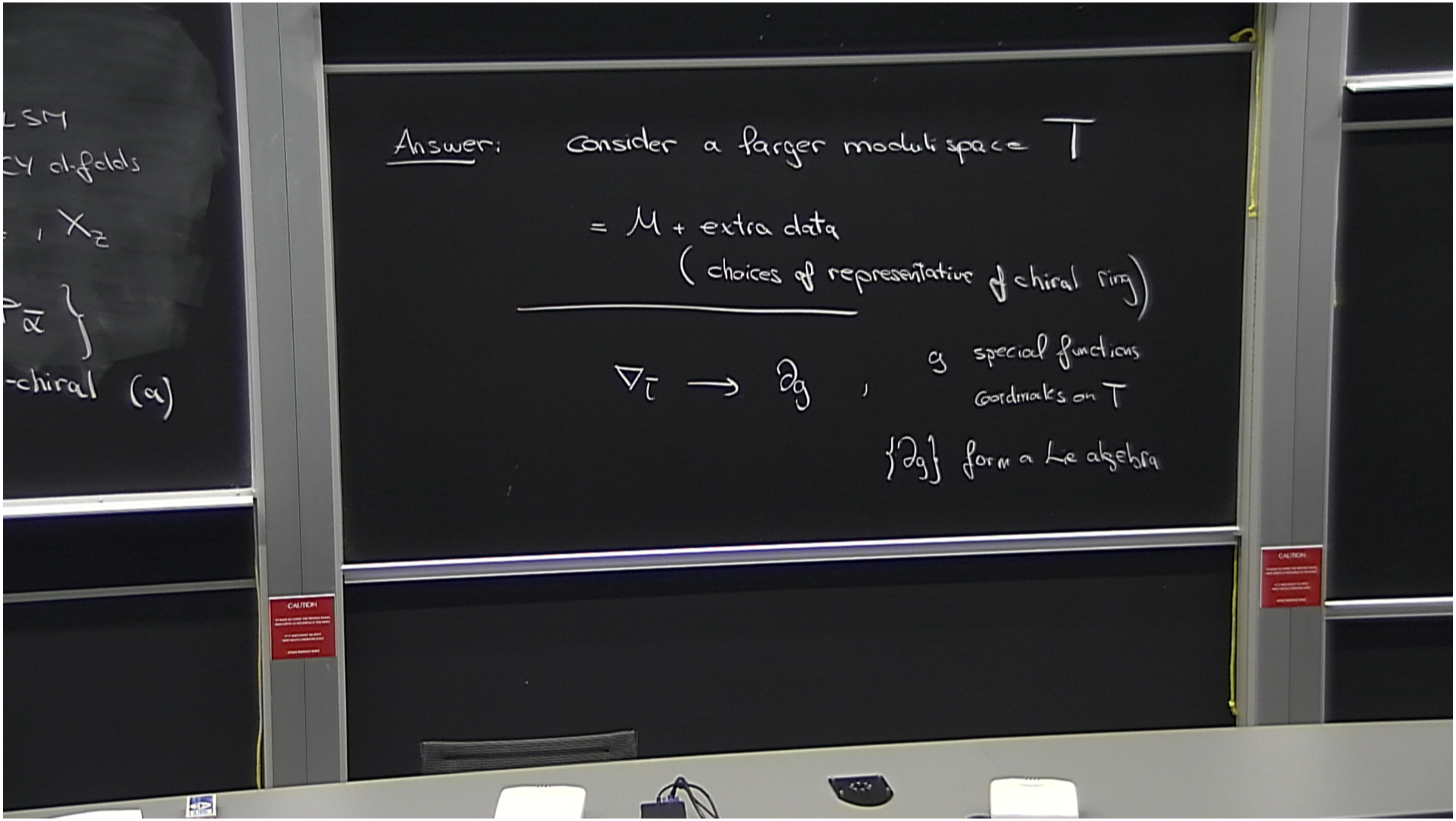
Answer: consider a larger moduli space T
= M + extra data
CY d-folds
-chiral (a)
CAUTION

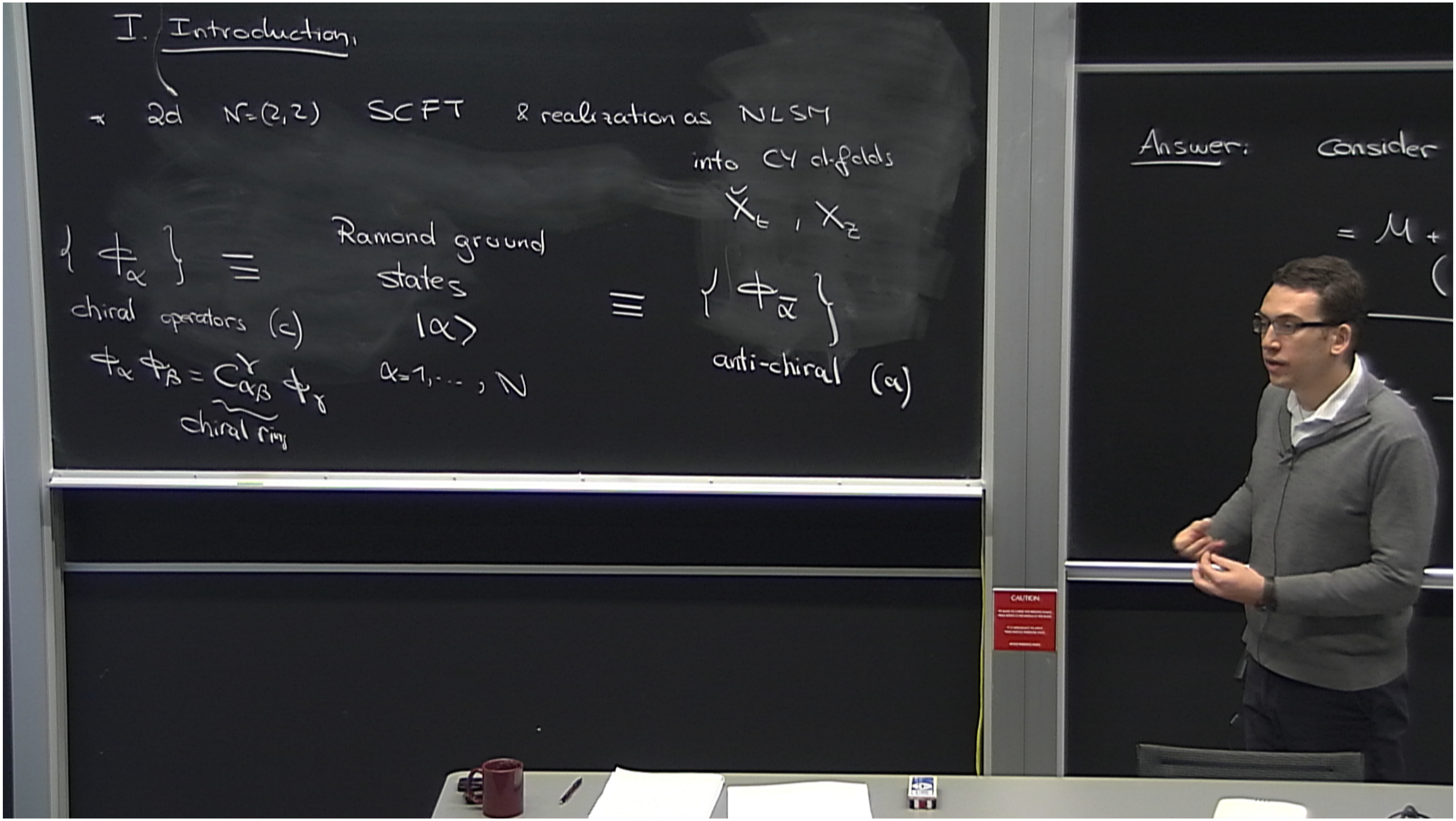

Answer: Consider a larger moduli space T
M + extra data
(choices of representative of chiral ring)


Answer: Consider a larger moduli space T
= M + extra data
(choices of
tative of chiral ring)
∇τ → ∂a
LSM
CY d-folds
-chiral (α)
CAUTION



LSM
CY d-folds
-chiral (α)
Answer: Consider a larger moduli space T
= M + extra data
(choices of representat
of chiral ring)
ecial functions
∇τ → ∂g ,
ks on T
CAUTION


Answer: Consider a larger moduli space T
= M + extra data
(choices of representative of chiral ring)
∇τ → ∂g , g special functions
coordinates on T
{∂g} form a Lie algebra
LSM
CY d-folds
, X2
-chiral (a)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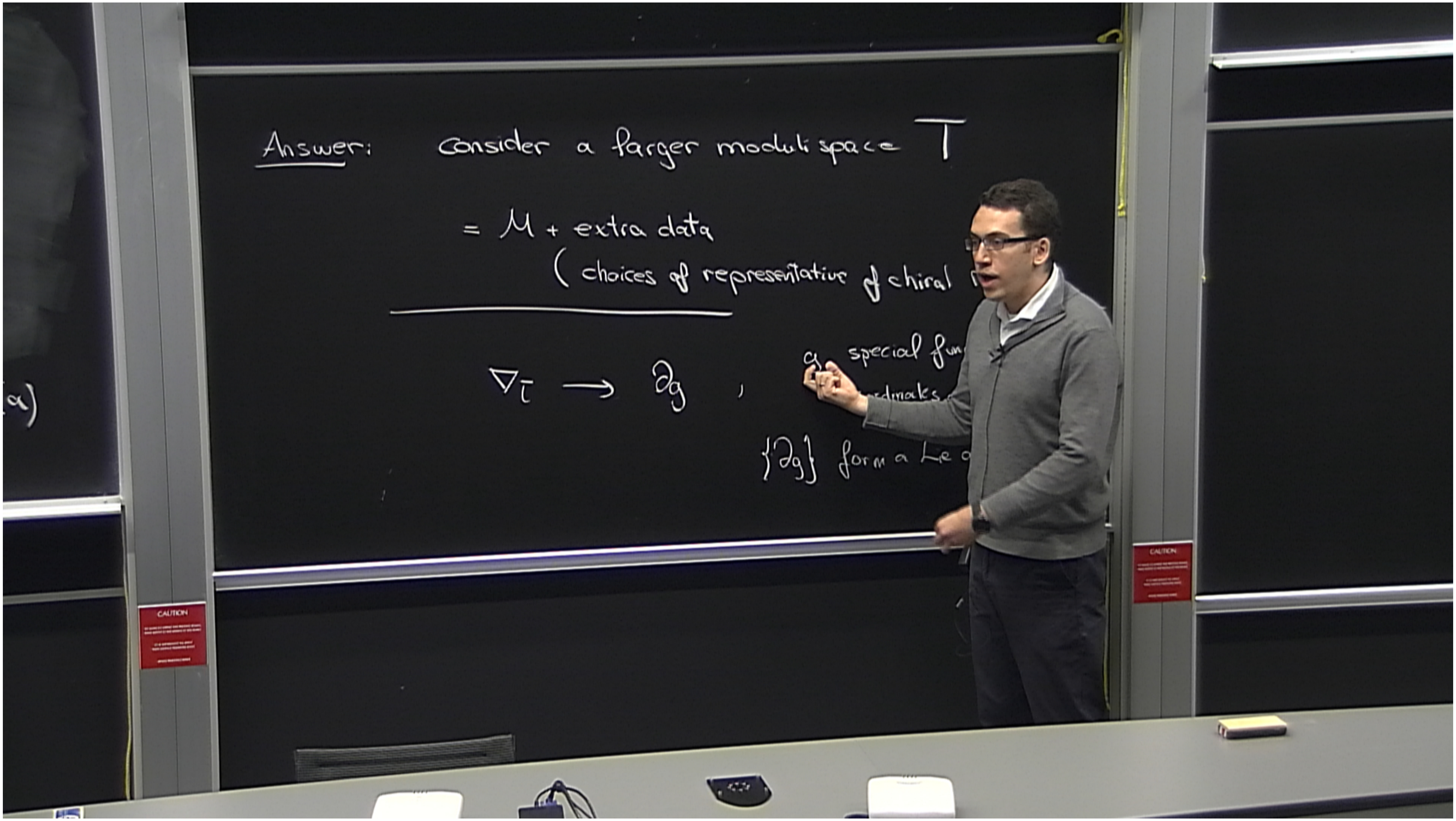
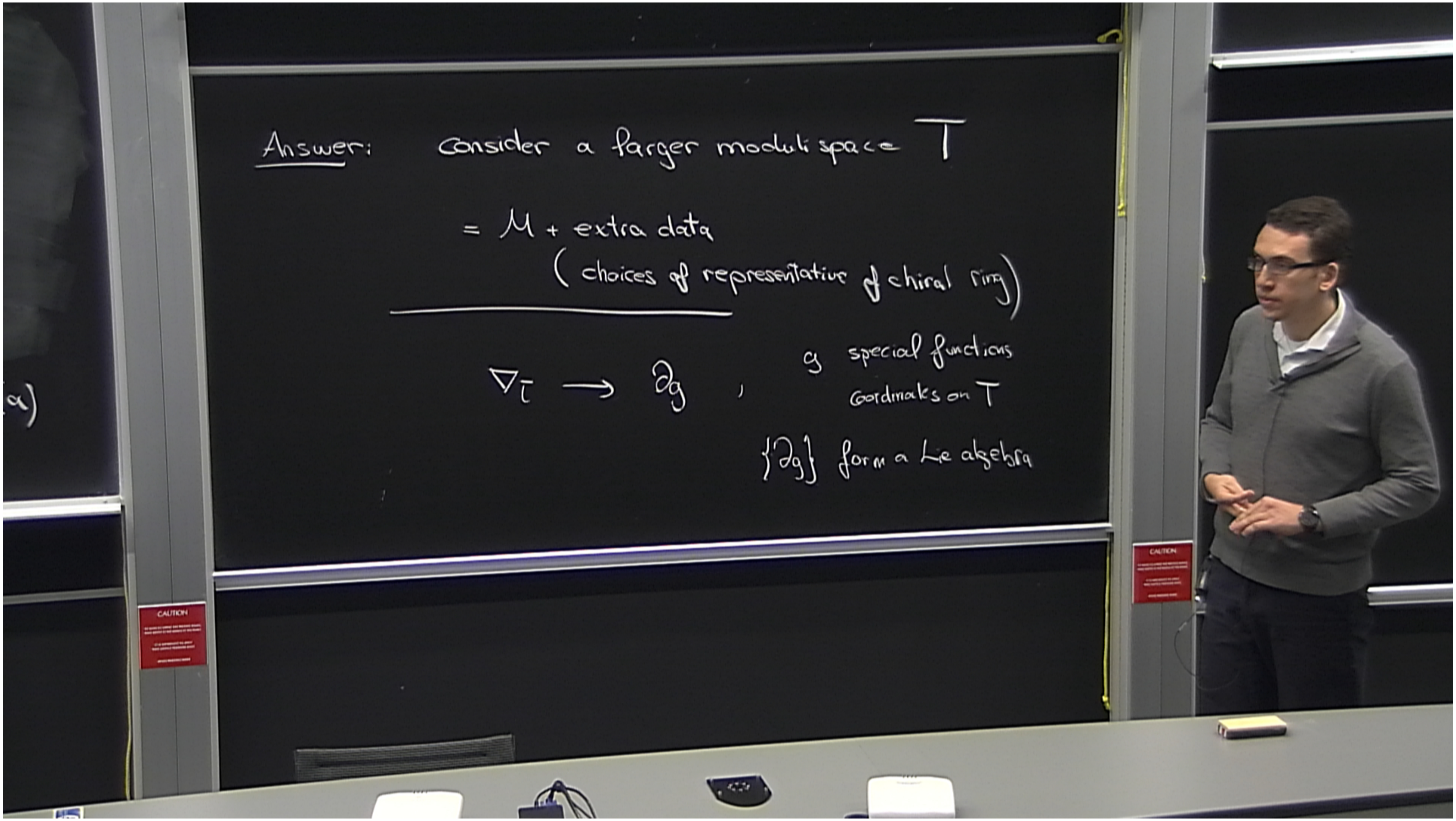
X̌_t , X_z
{ φ_α } ≡ Ramond ground states ≡ { φ_ᾱ }
chiral operators (c)
|α>
anti-chiral (a)
φ_α φ_β = C_αβ^γ φ_γ
α=1, ... , N
chiral ring
Answer: Consider
CAUTION


Answer: Consider a larger moduli space T
= M + extra data
(choices of representative of chiral
∇τ → ∂g ,
g special
{∂g} form a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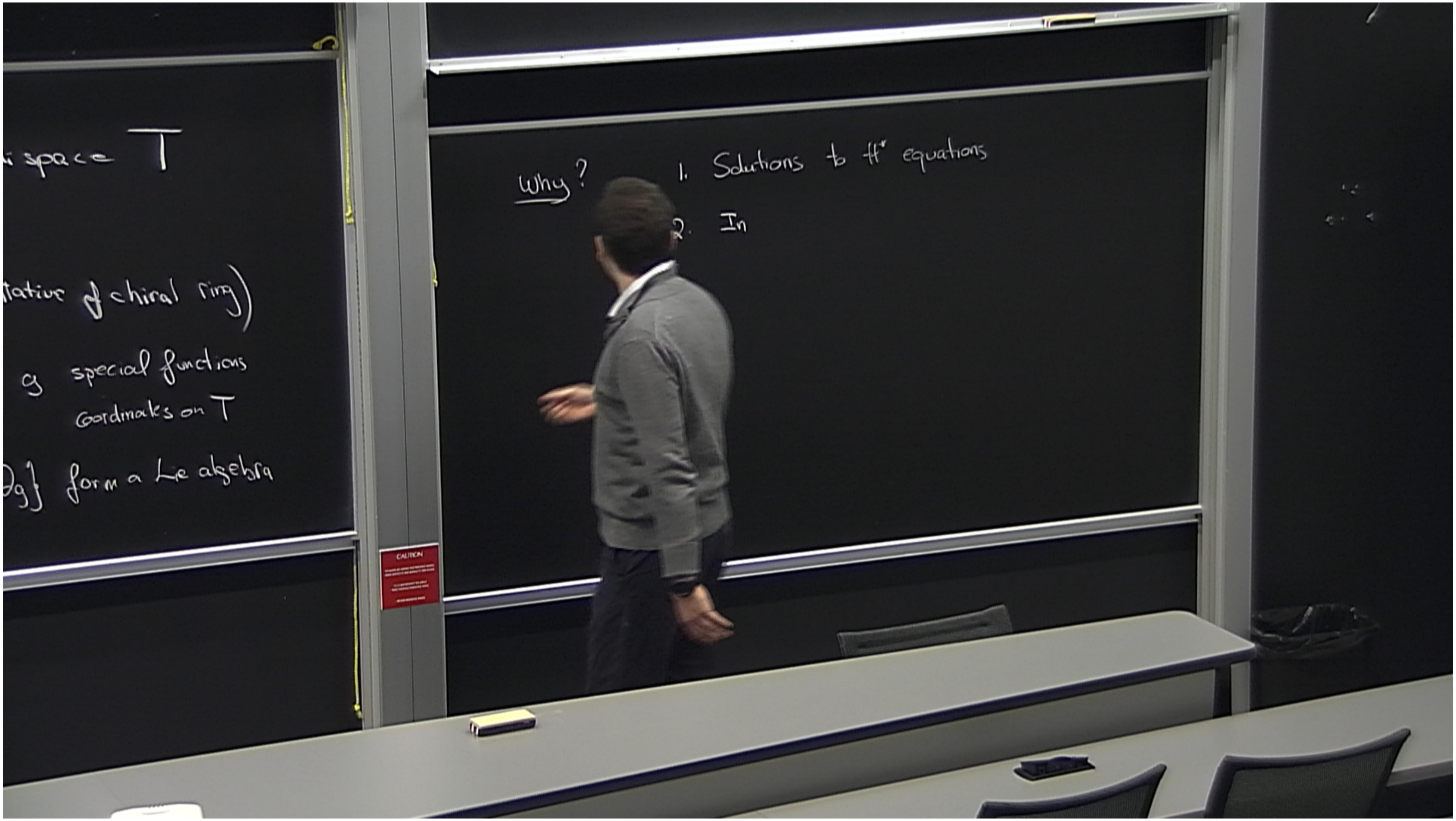
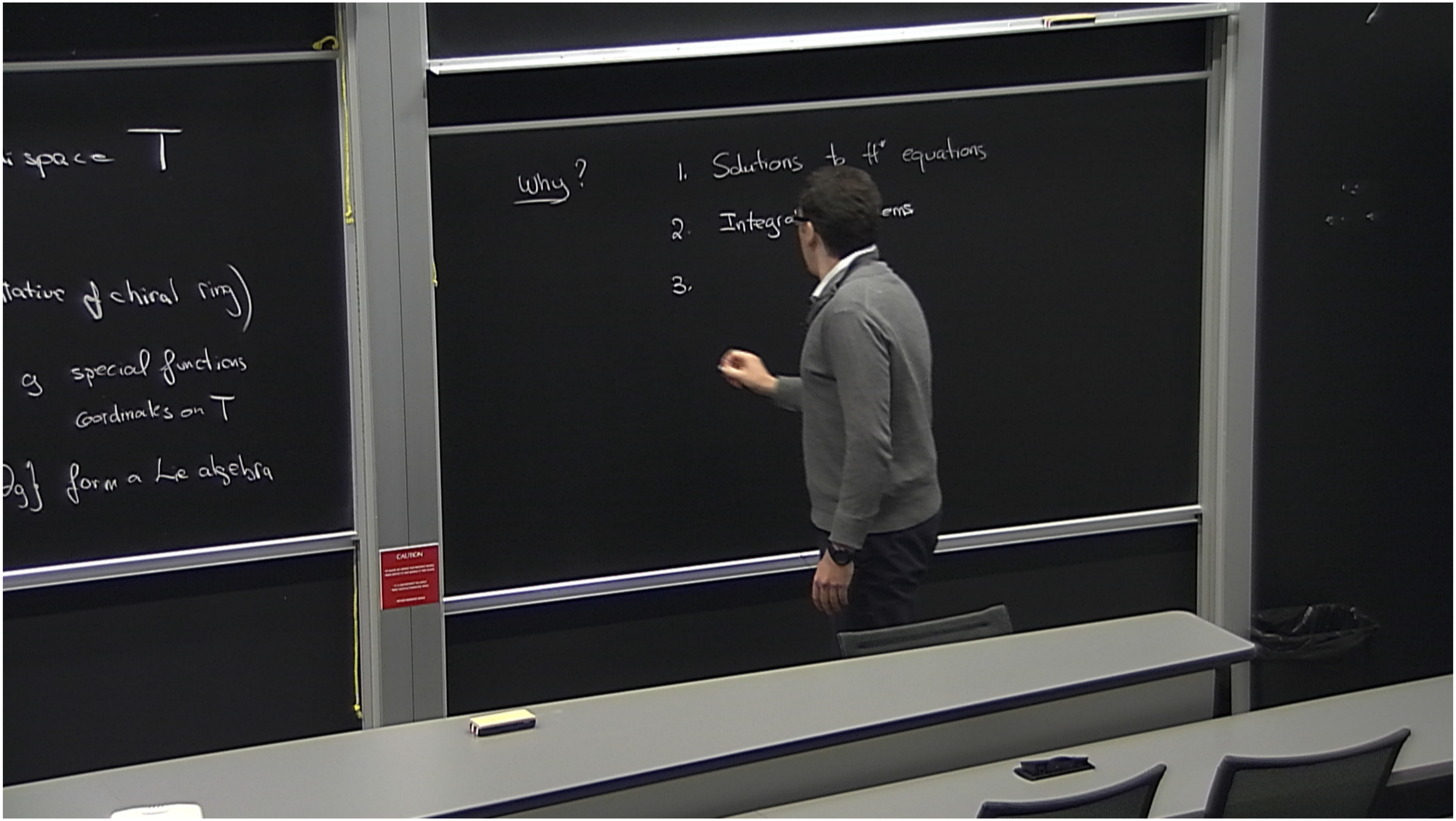

space T
tative of chiral ring)
g special functions
Coordinates on T
form a Lie algebra
Why?
1. Solutions to tt* equations
2. In
CAUTION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Why?
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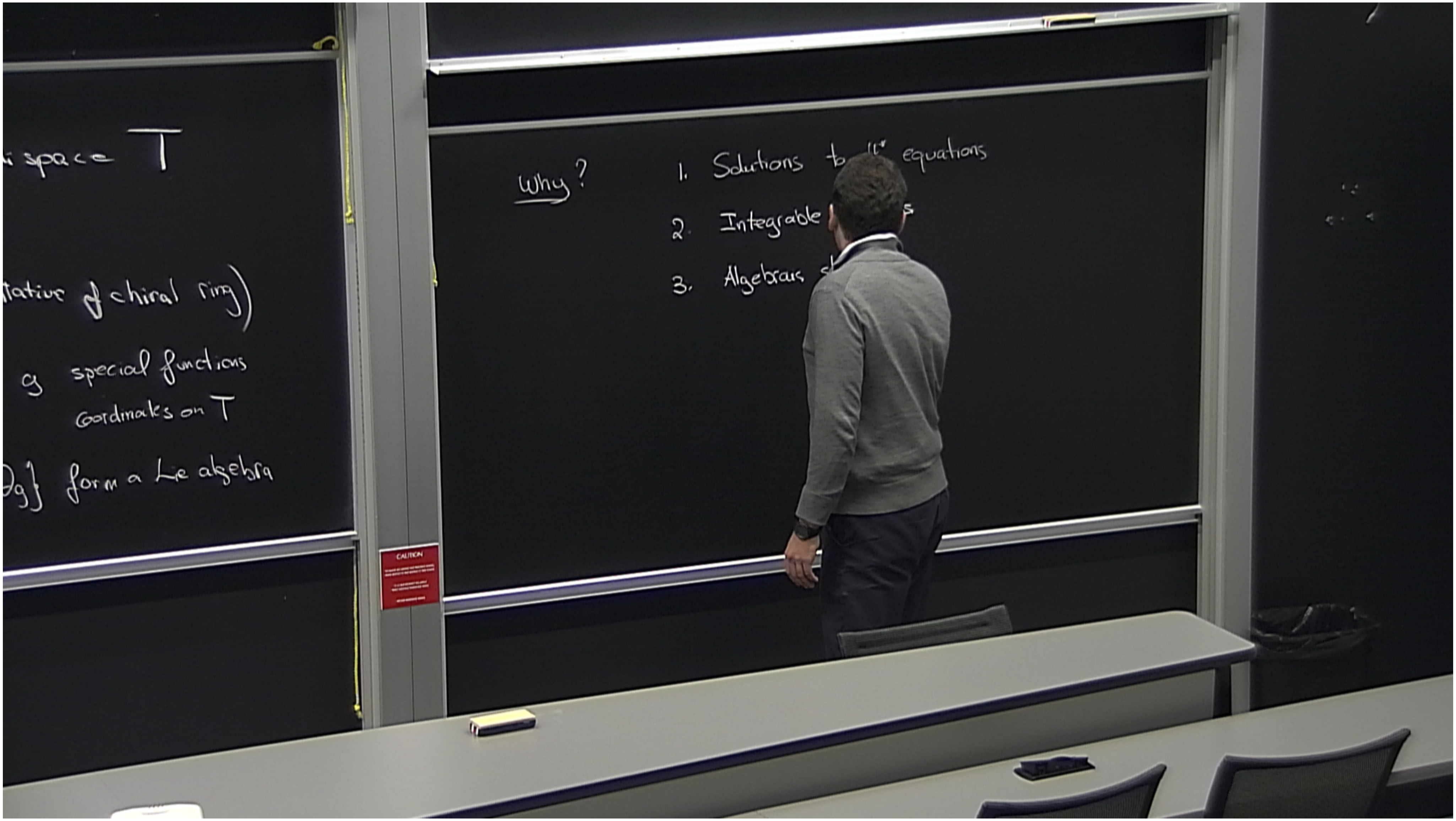
2.
3.
CAUTION


space T
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Coordinates on T
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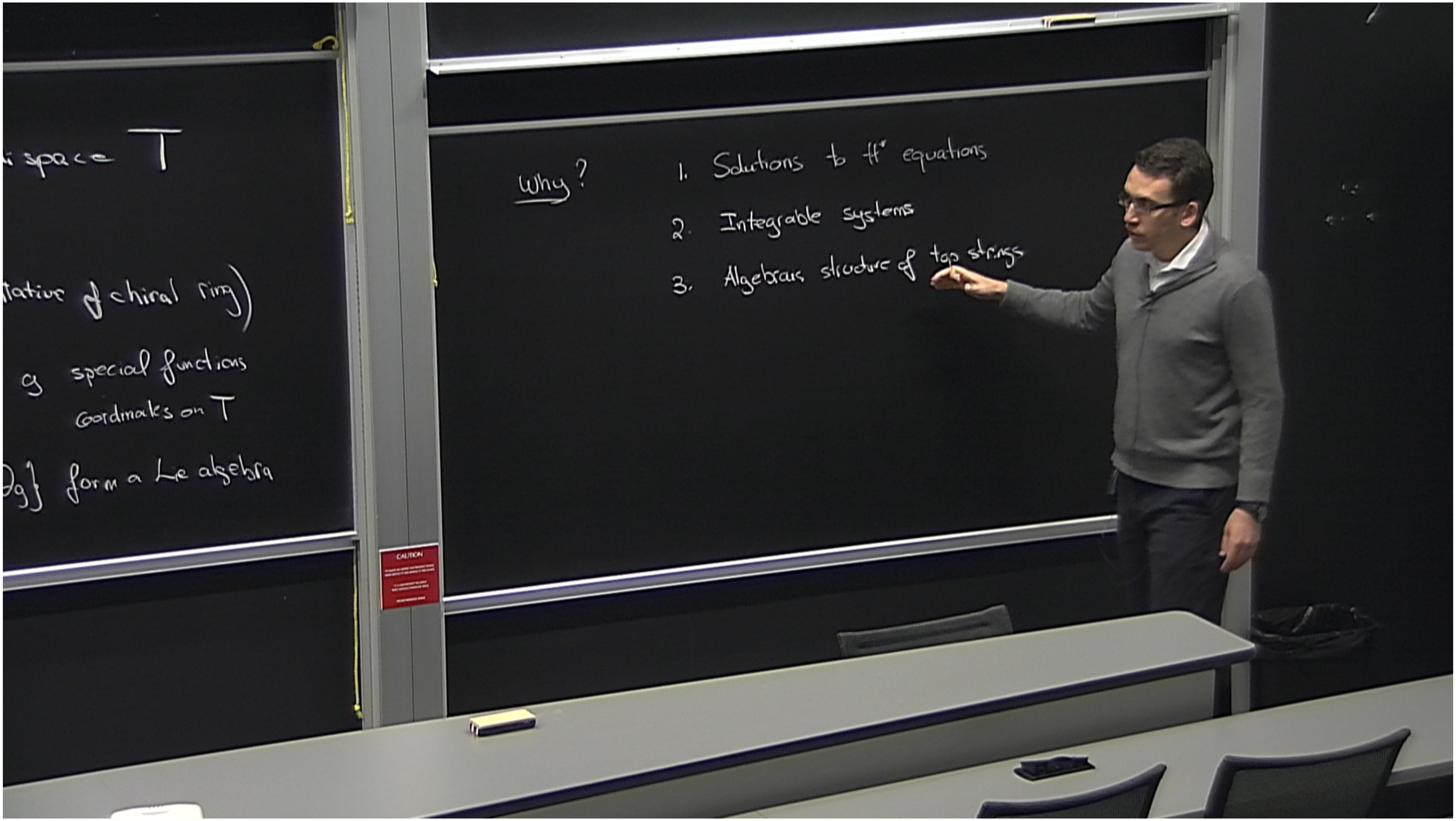
Why?
1. Solutions to equations
2. Integrable
3. Algebras



space T
(tative of chiral ring)
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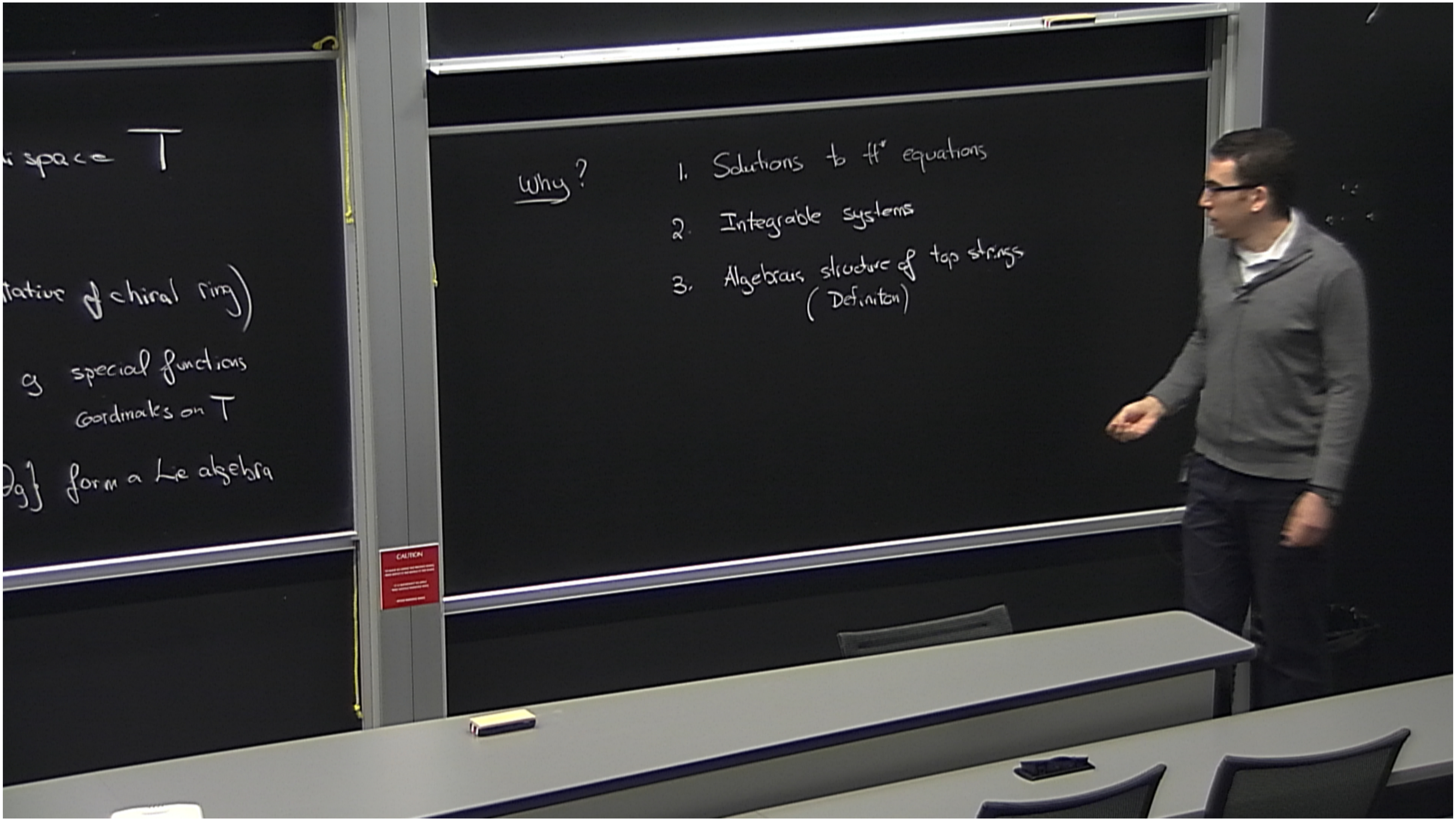
g} form a Lie algebra
Why?
1. Solutions to tt* equations
2. Integrable systems
3. Algebraic structure of top strings


space T
of chiral ring)
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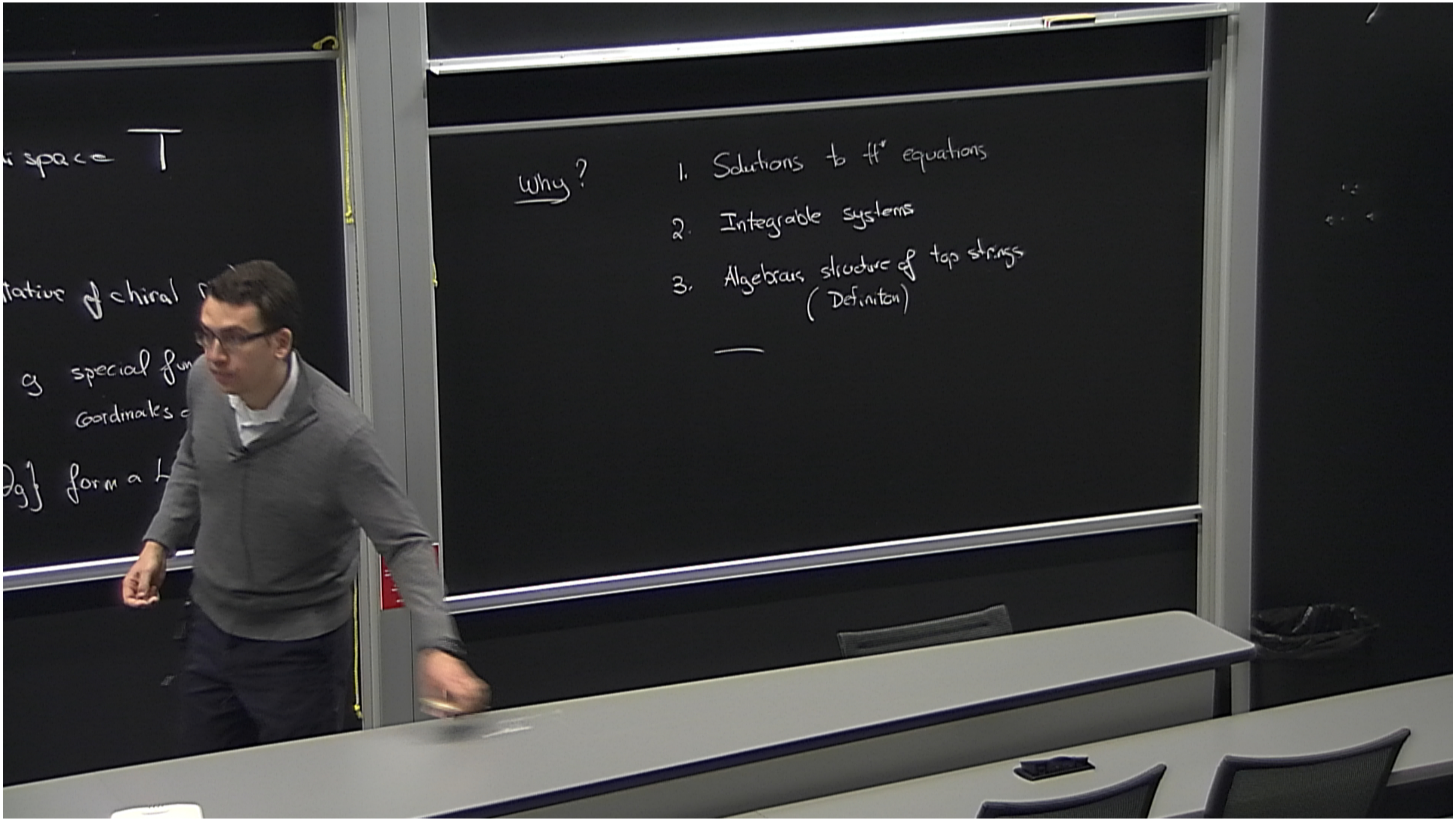
Why?
1. Solutions to tt* equations
2. Integrable systems
3. Algebraic structure of top strings
(Definition)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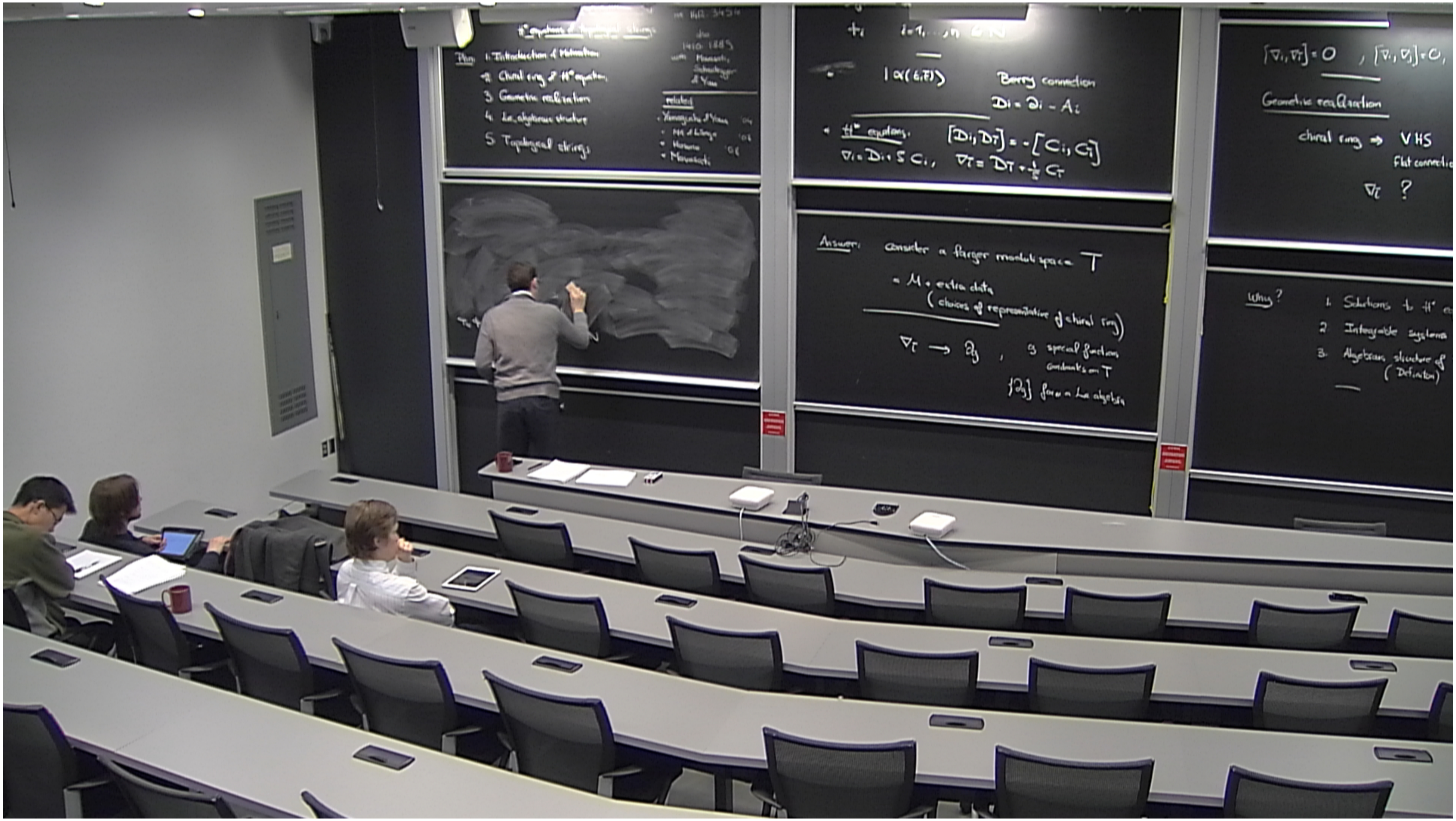
(Definition)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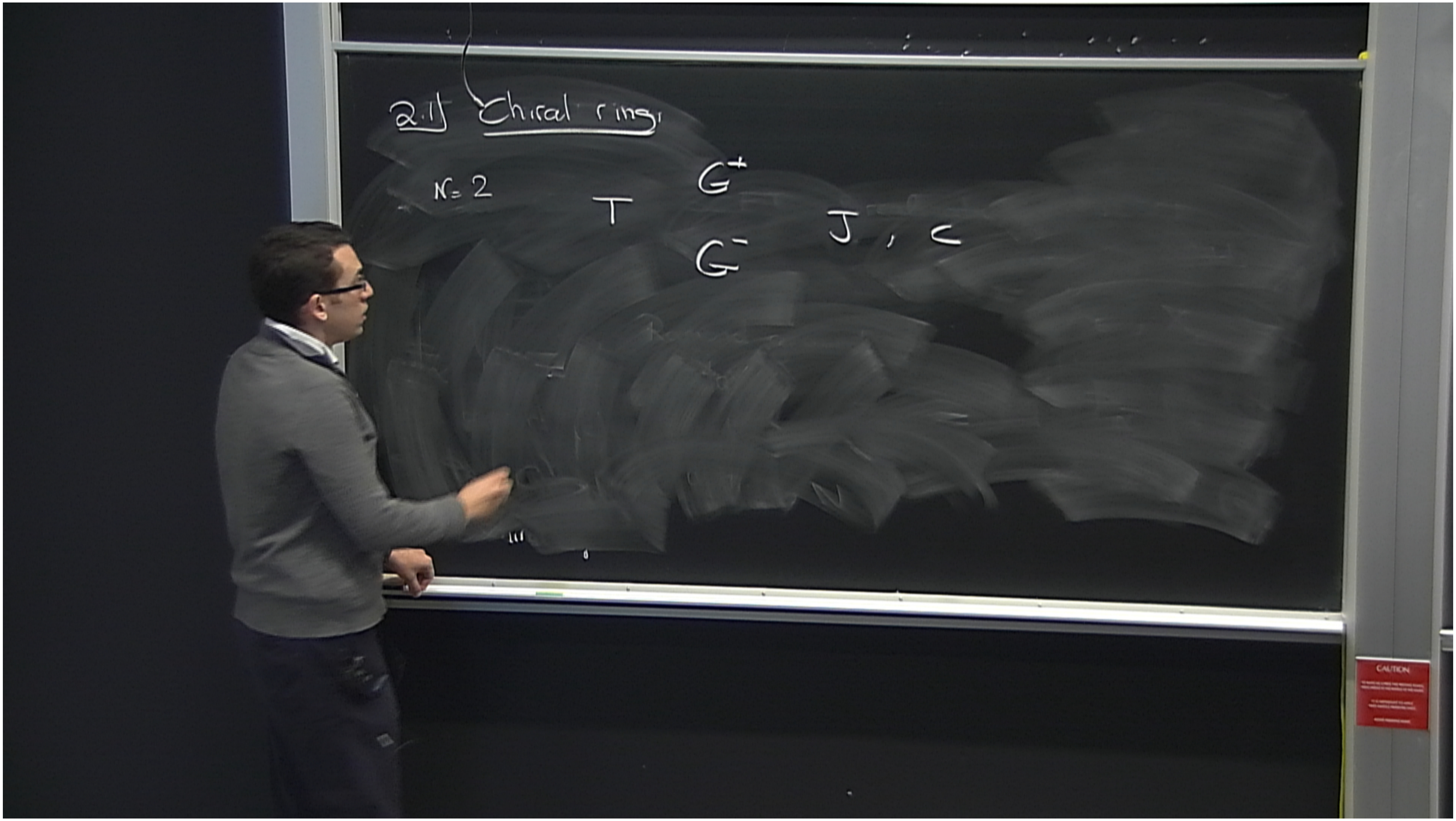
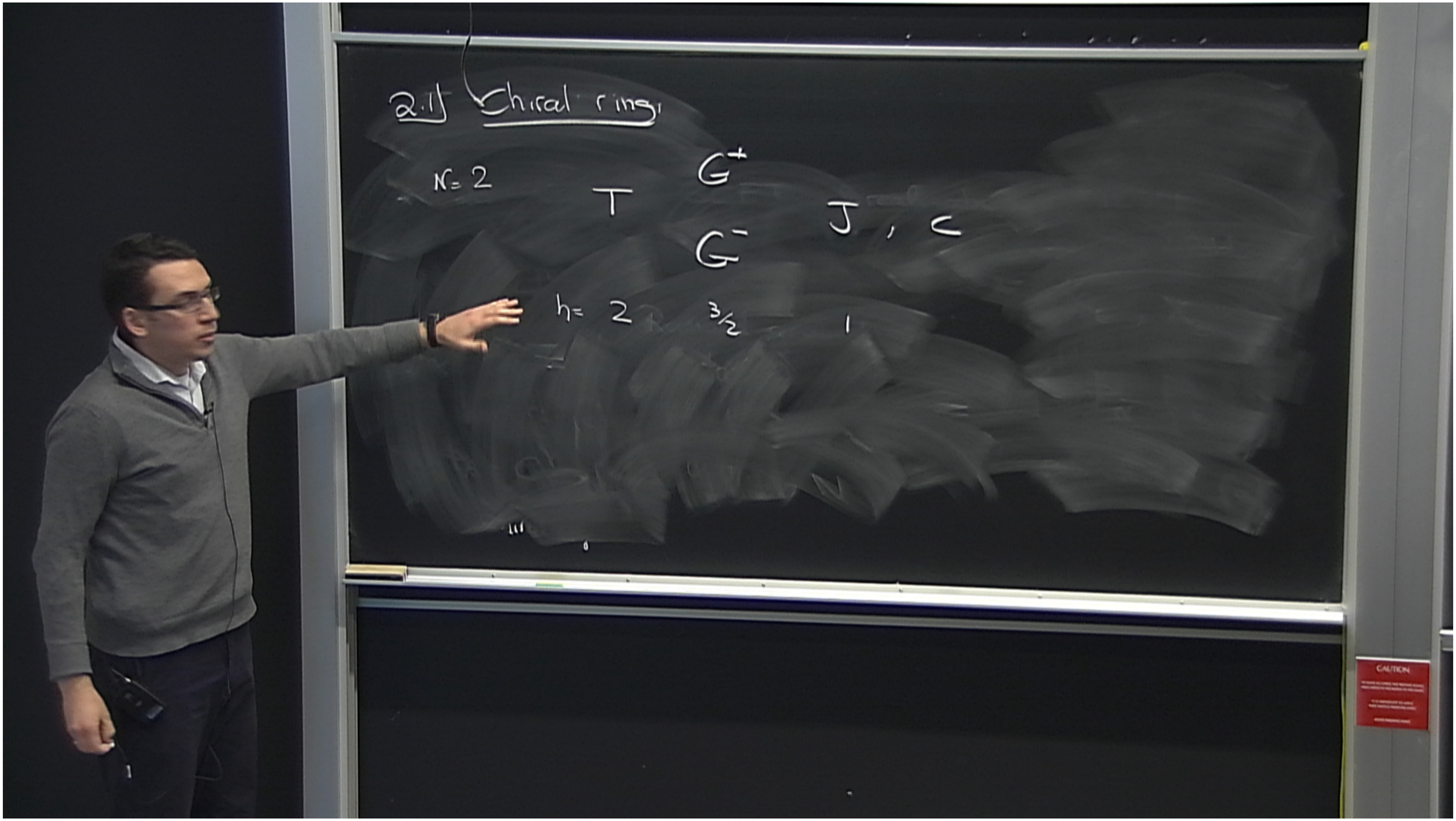
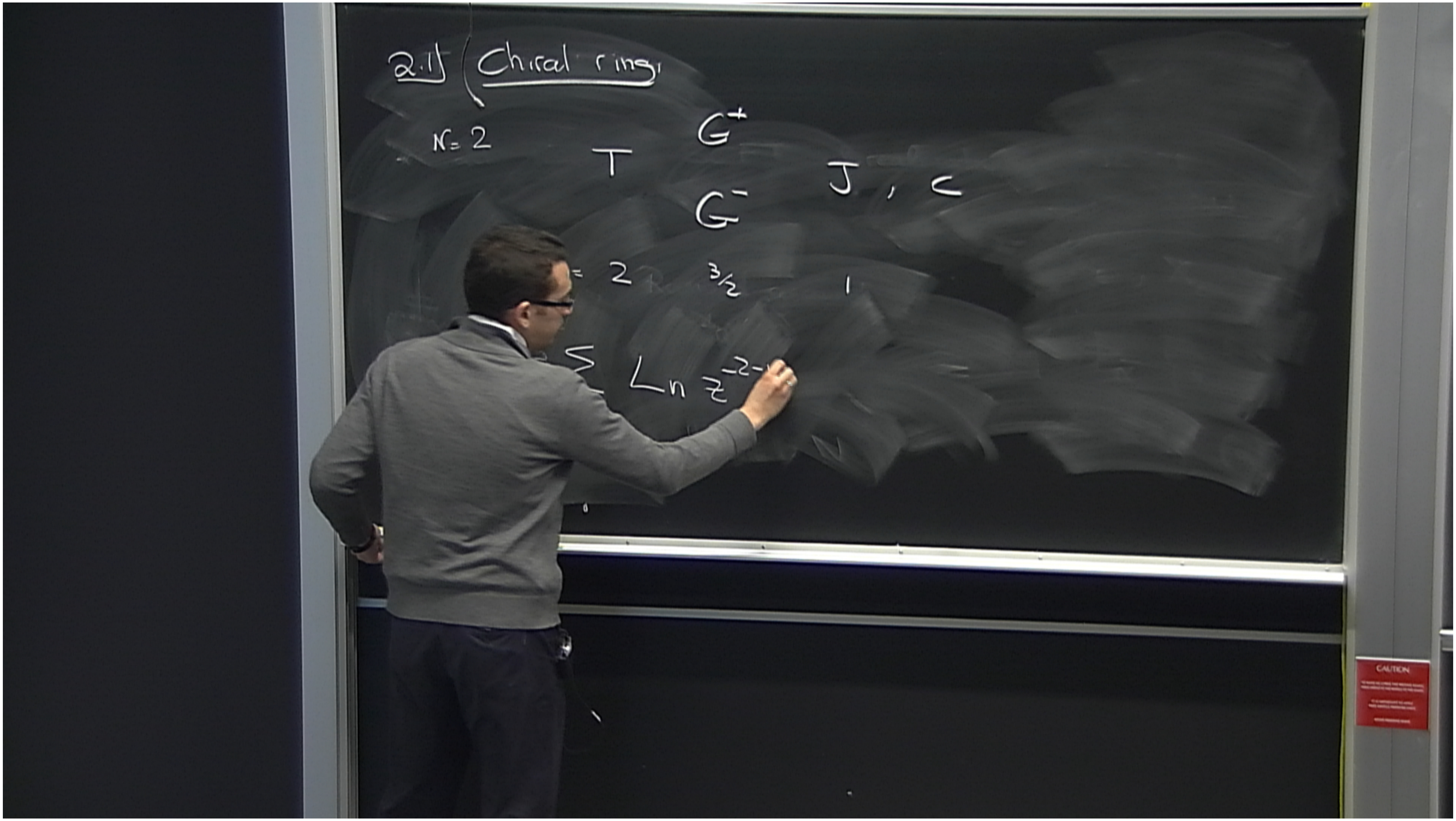
5. Topological strings
Berry connection
Answer: consider a larger moduli space T
= M + extra data
Geometric realization
chiral ring ⇒ VHS
Why?
2. Integrable systems


2.1) Chiral ring:
N=2
T
G+
G−
J , c



2.1) Chiral rings
N= 2
T
G+
G−
J , c
h= 2
3/2
1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G+
G−
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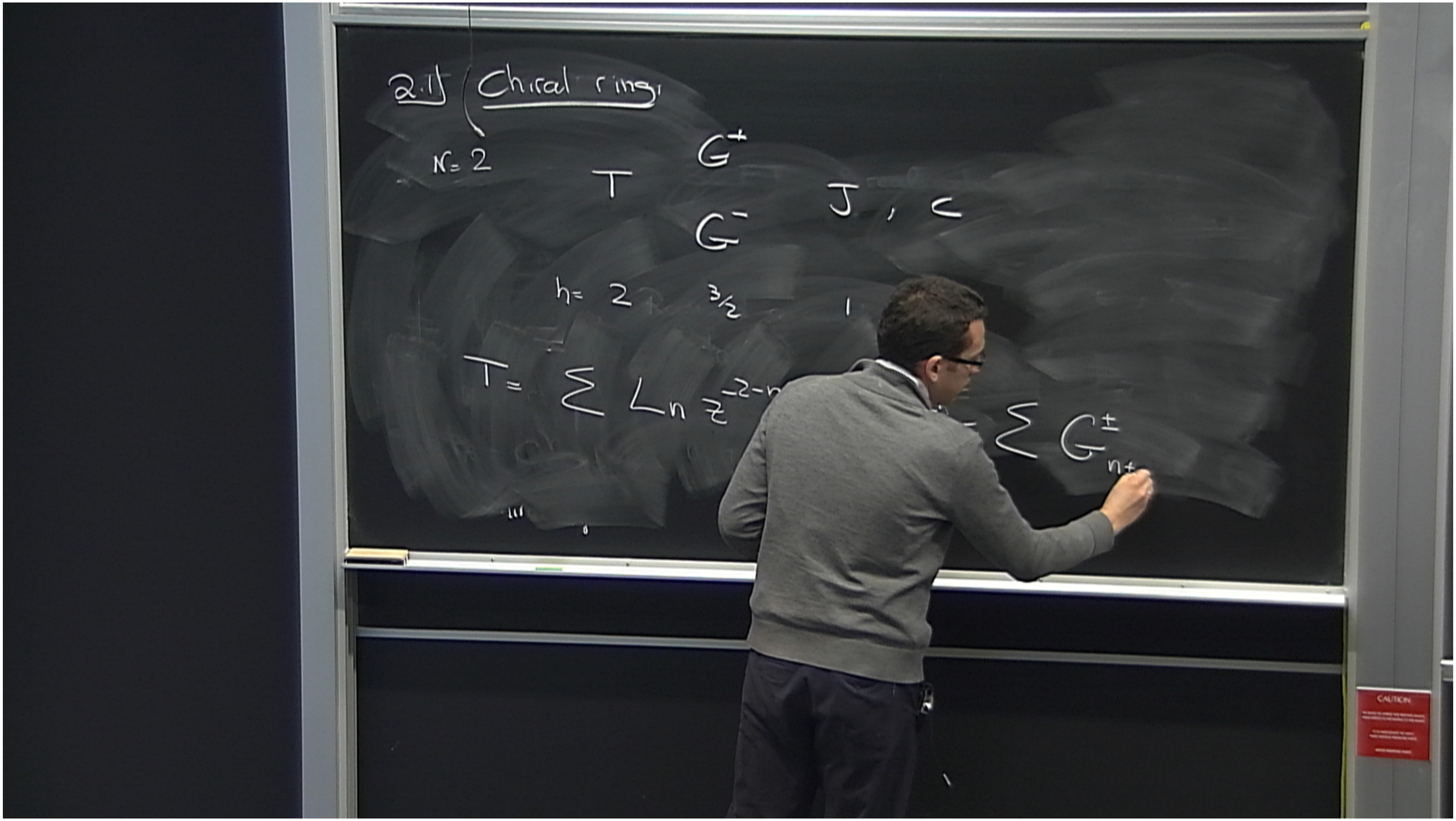
2
3/2
1
Σ Ln


2.1) Chiral rings
N= 2
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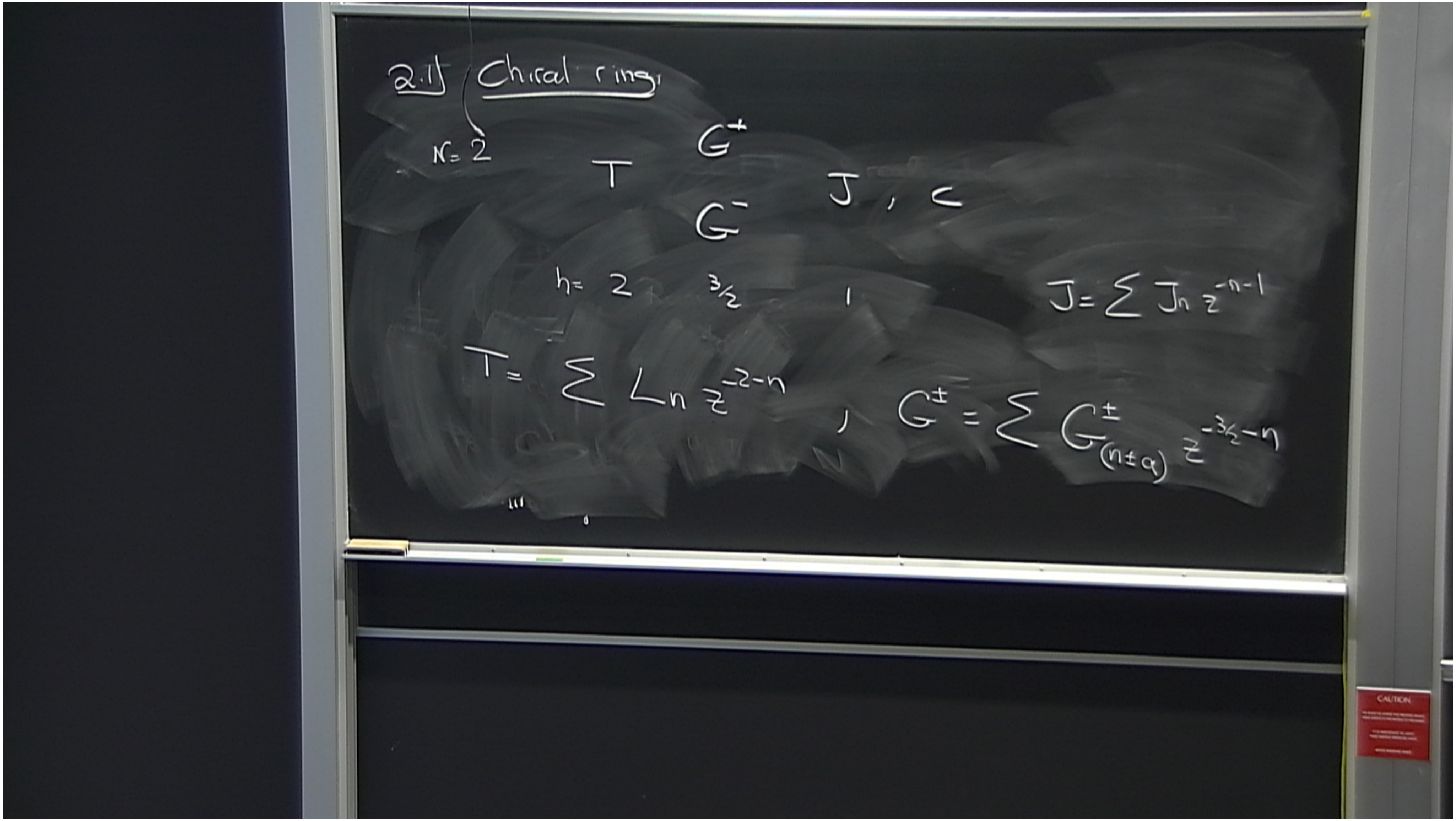
G+
G−
J , c
h= 2
3/2
1
T= Σ Ln z−2−n
Σ G±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N = 2
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G−
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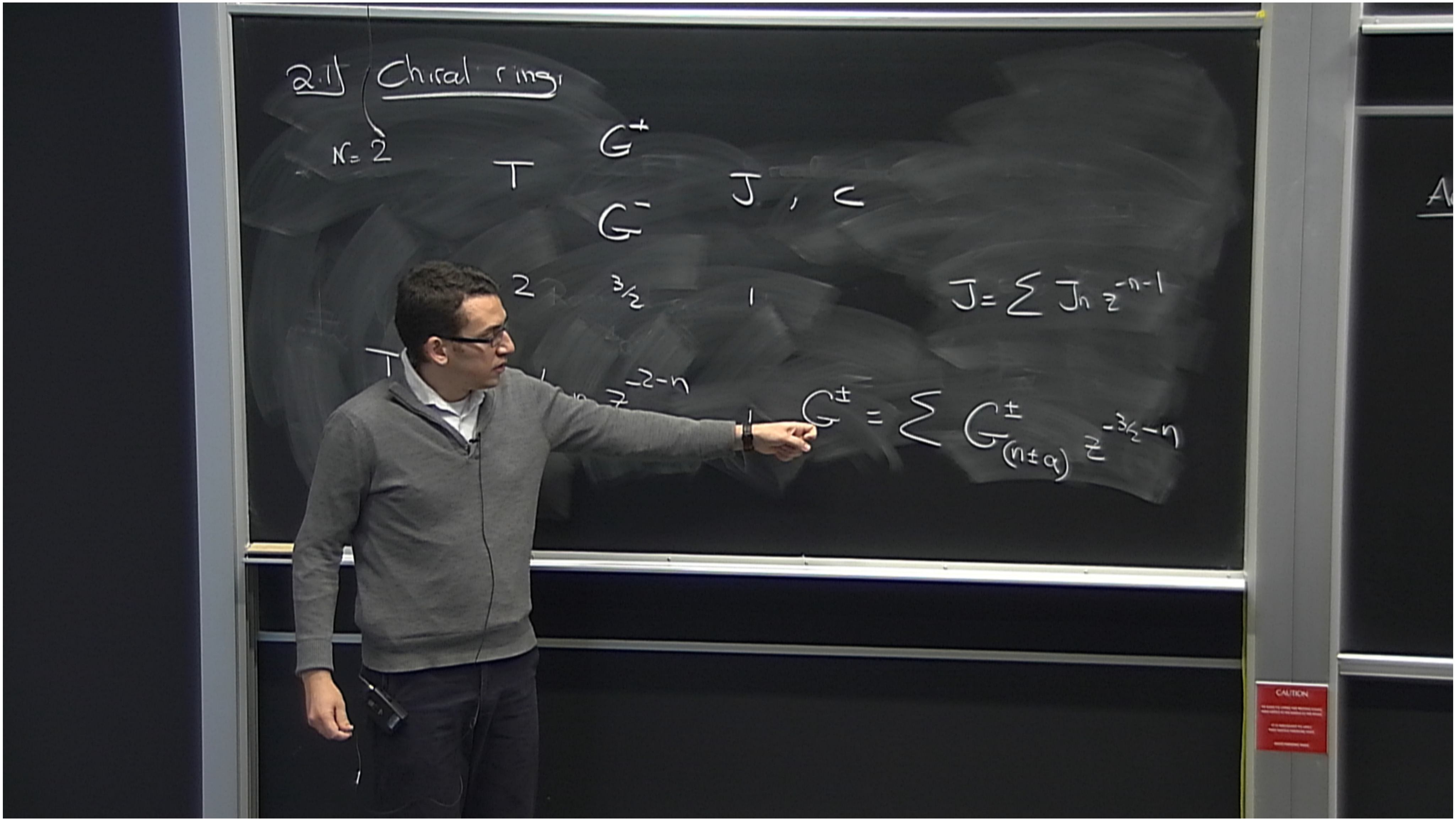
h = 2 3/2 1
J = ∑ Jn z^(−n−1)
T = ∑ Ln z^(−2−n)
G± = ∑ G±(n±a) z^(−3/2−n)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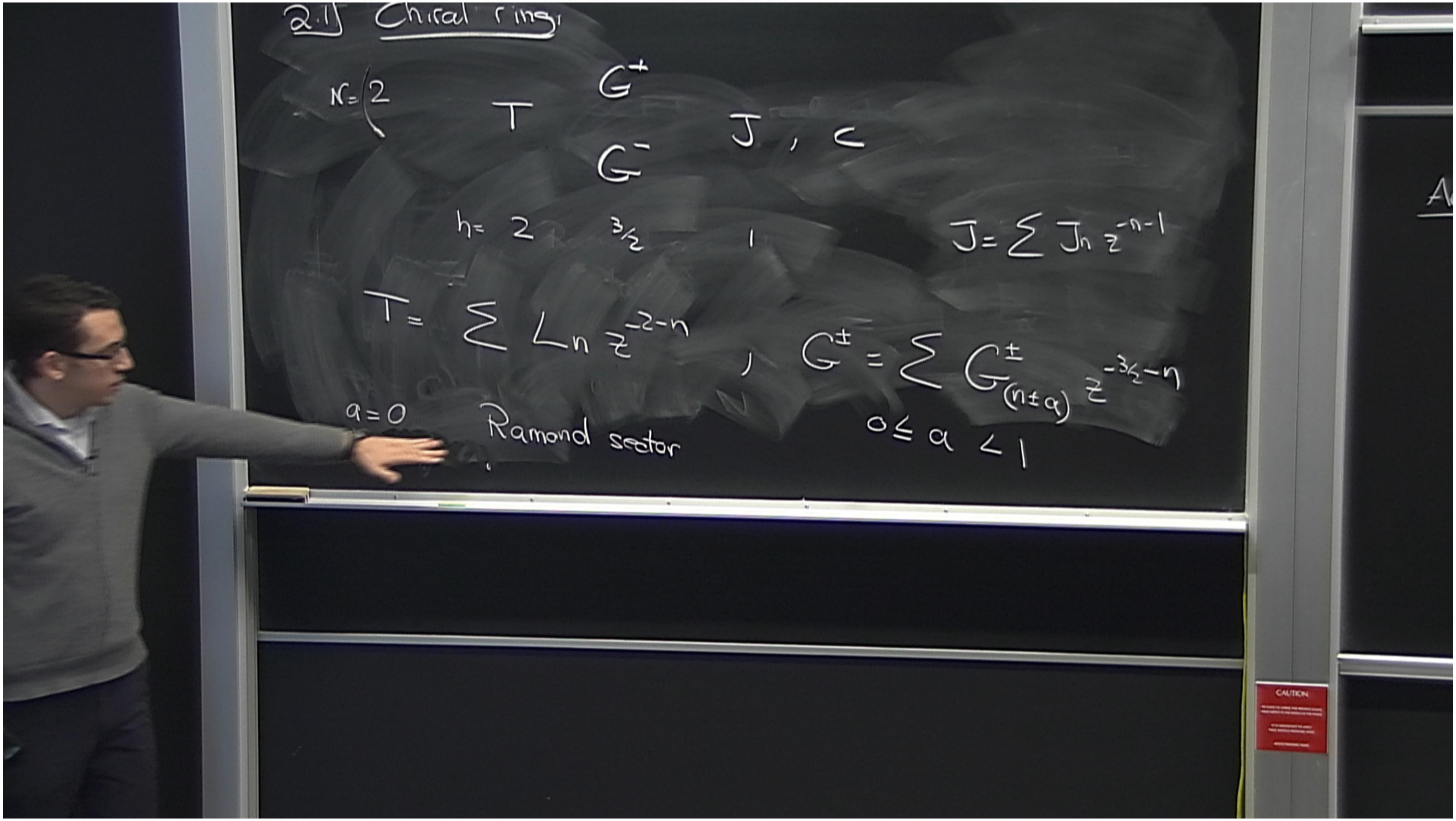
2.1 Chiral ring:
N=2
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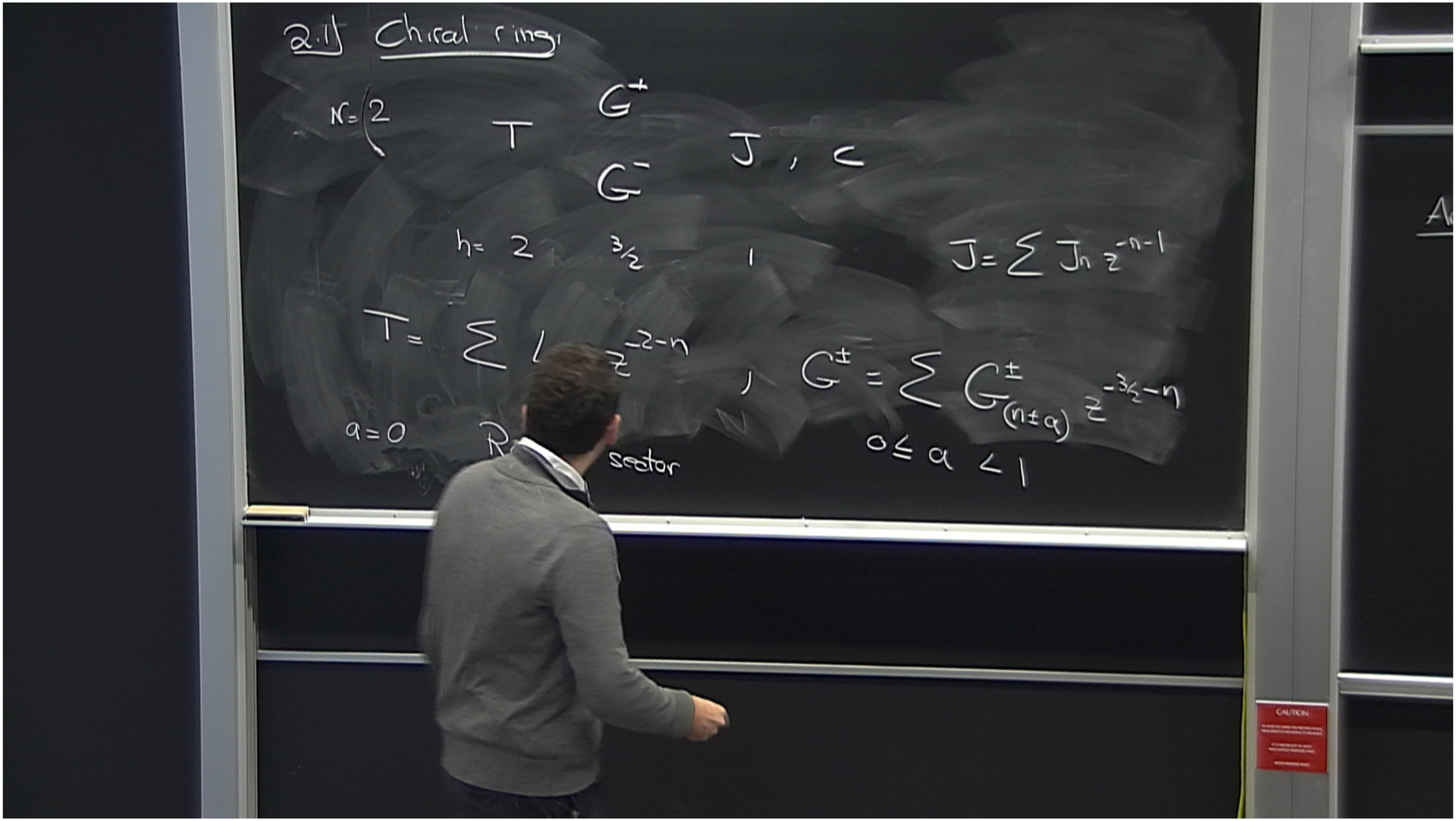
G−
J , c
h= 2
3/2
1
J= ∑ Jn z-n-1
T= ∑ Ln z-2-n
G± = ∑ G±(n±a) z-3/2-n
a=0
Ramond sector
0≤ a < 1
CAUTION


2.1) Chiral rings
N=(2
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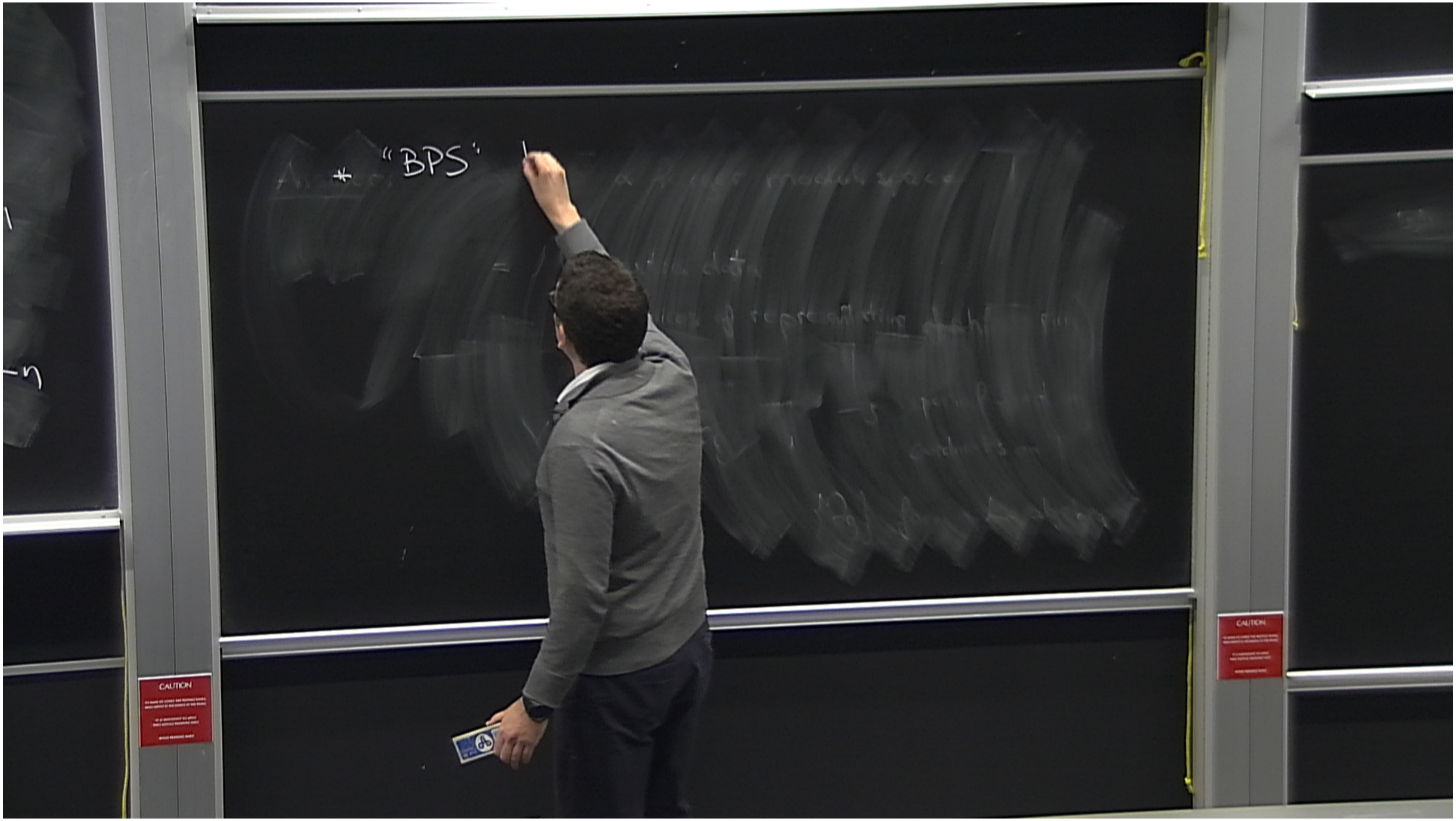
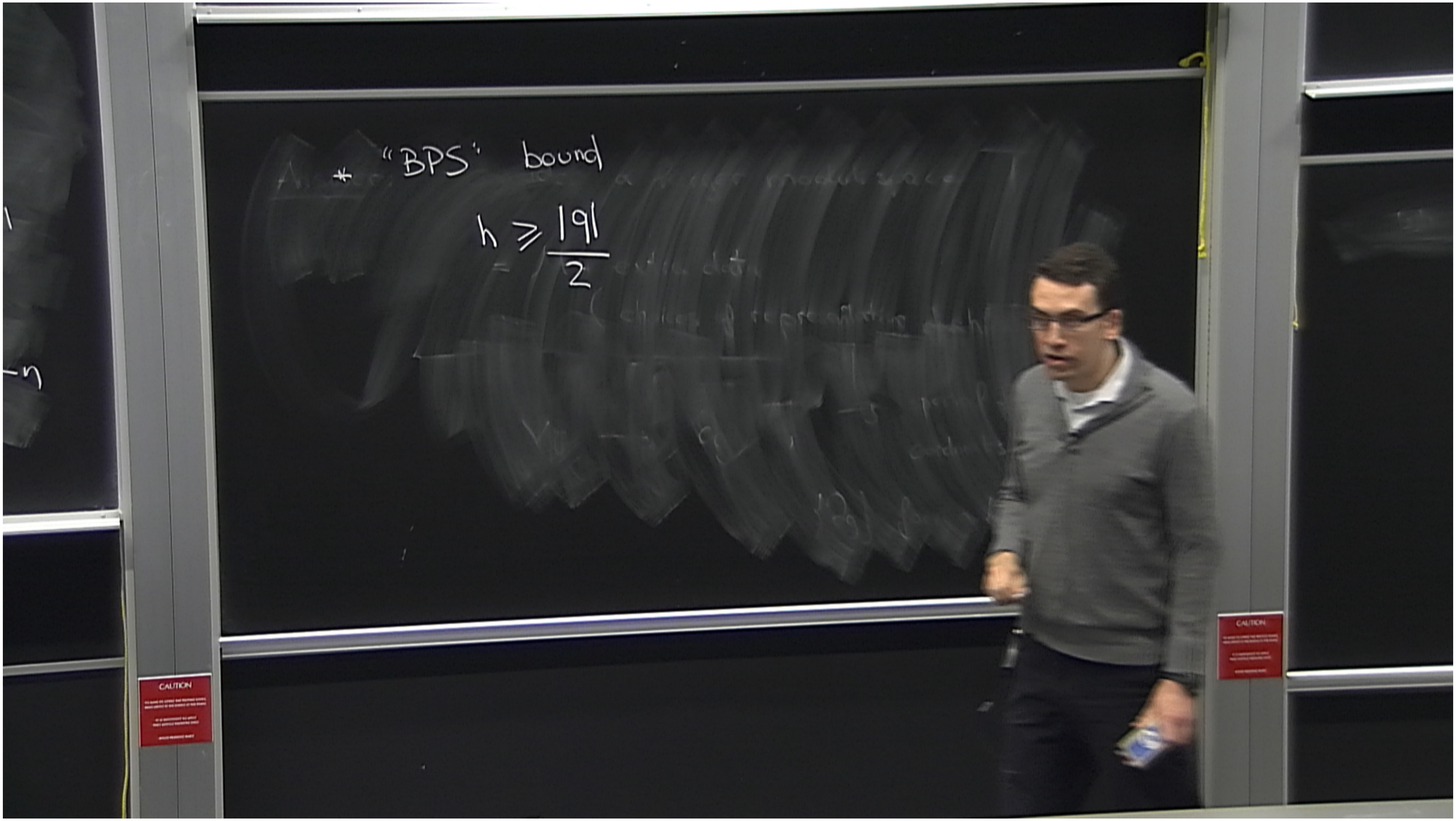
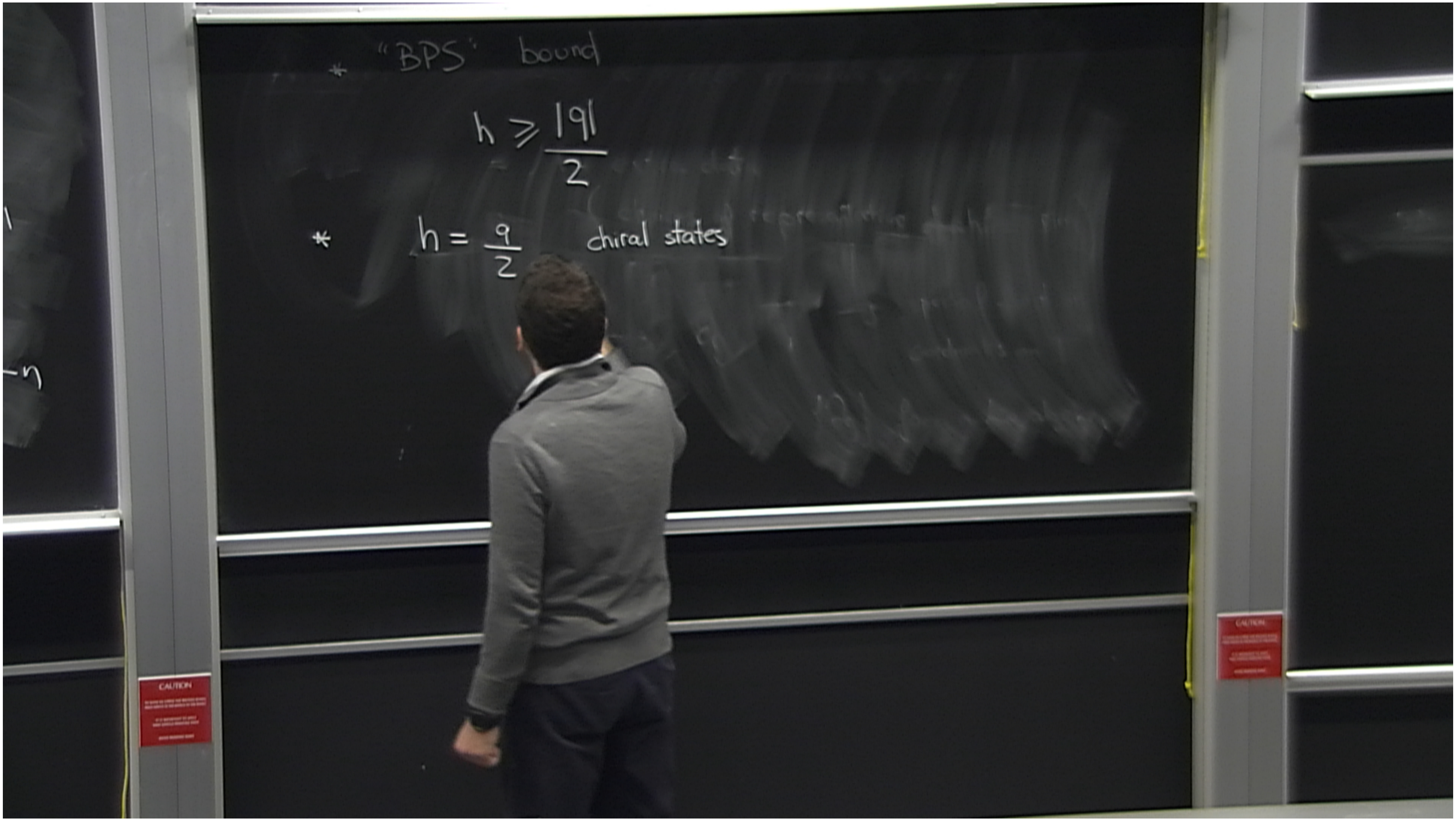
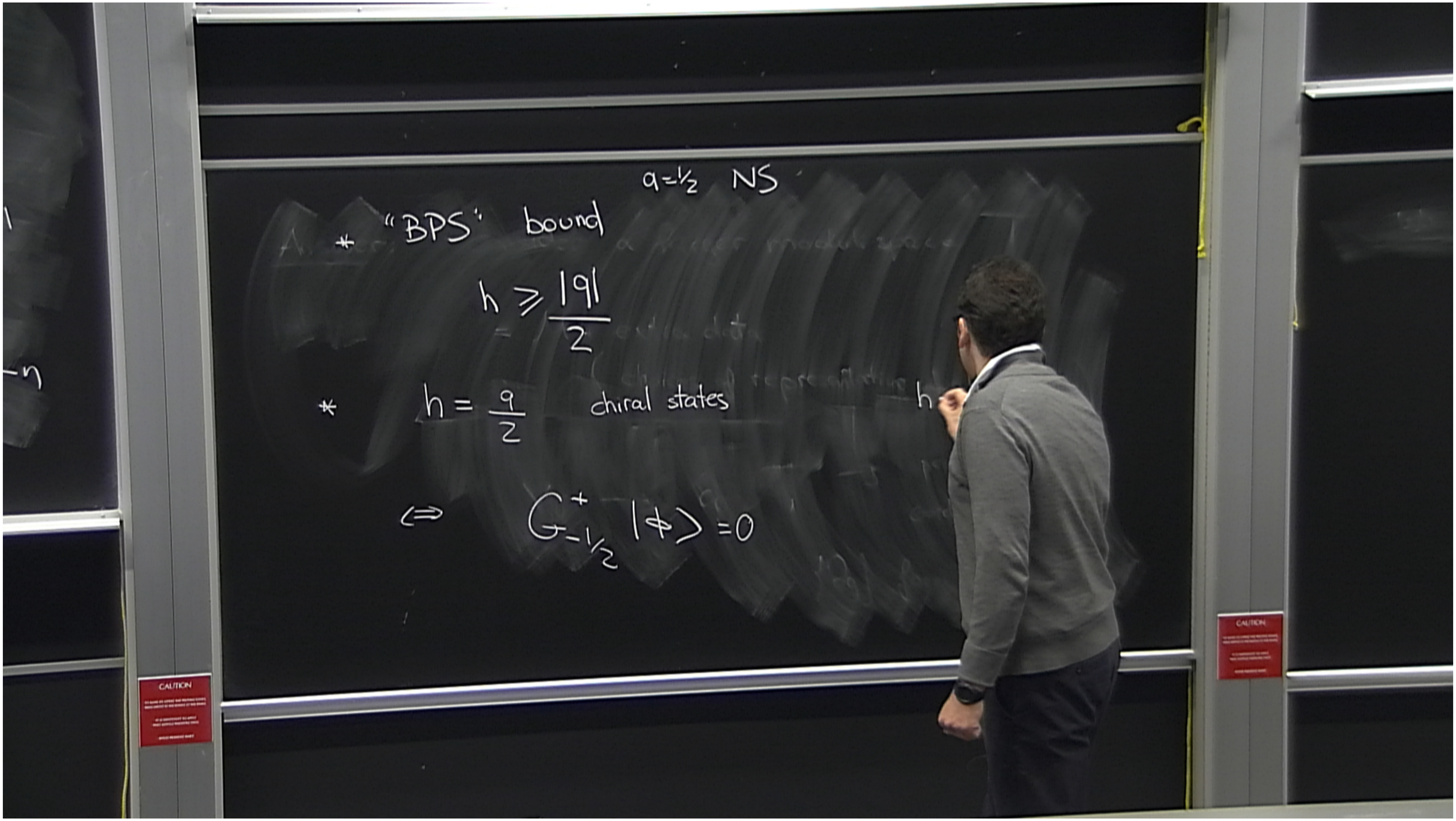
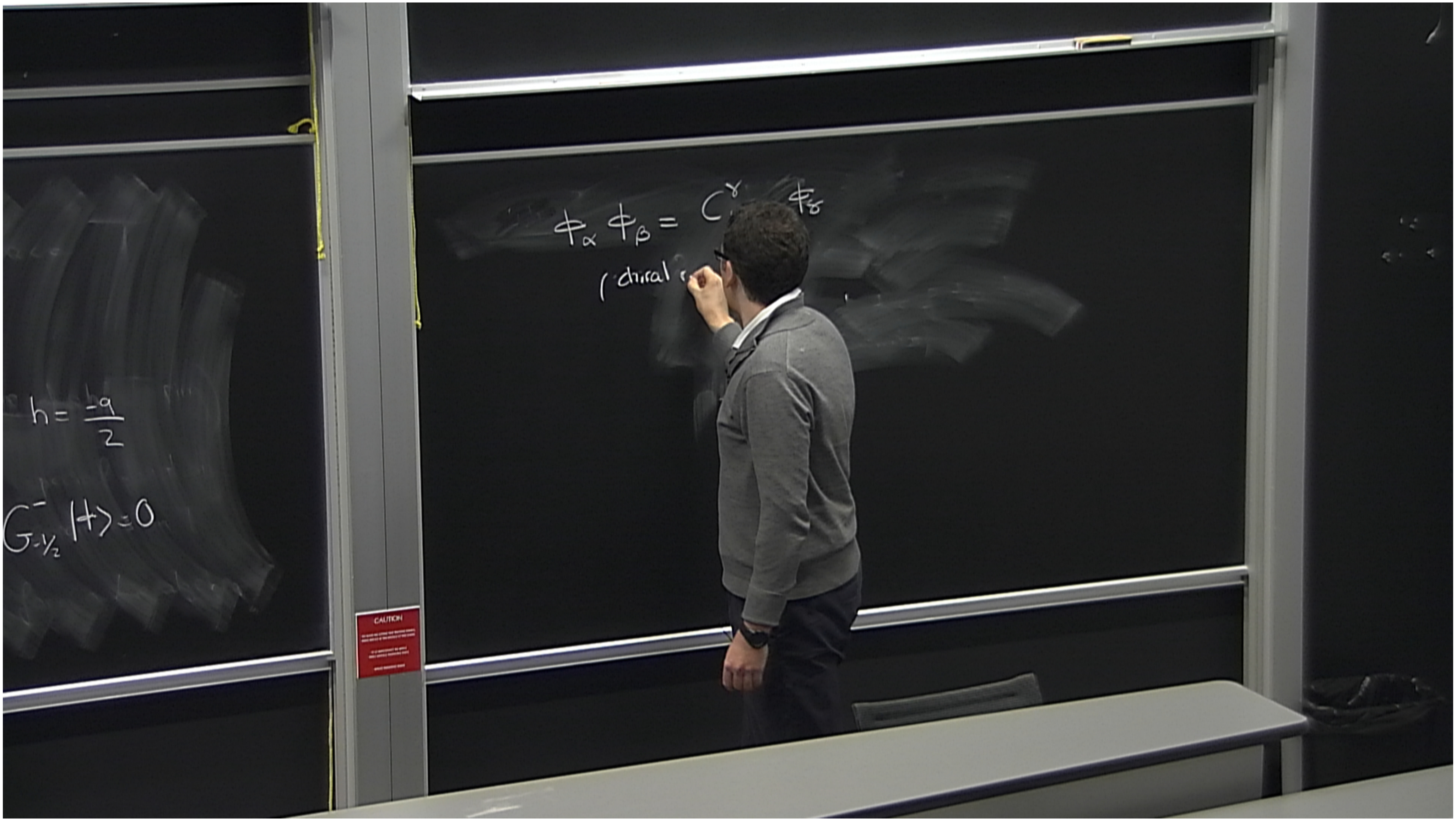
J= ∑ Jn z-n-1
T= ∑
z-2-n
G± = ∑ G±(n±a) z-3/2-n
a=0
sector
0 ≤ a < 1

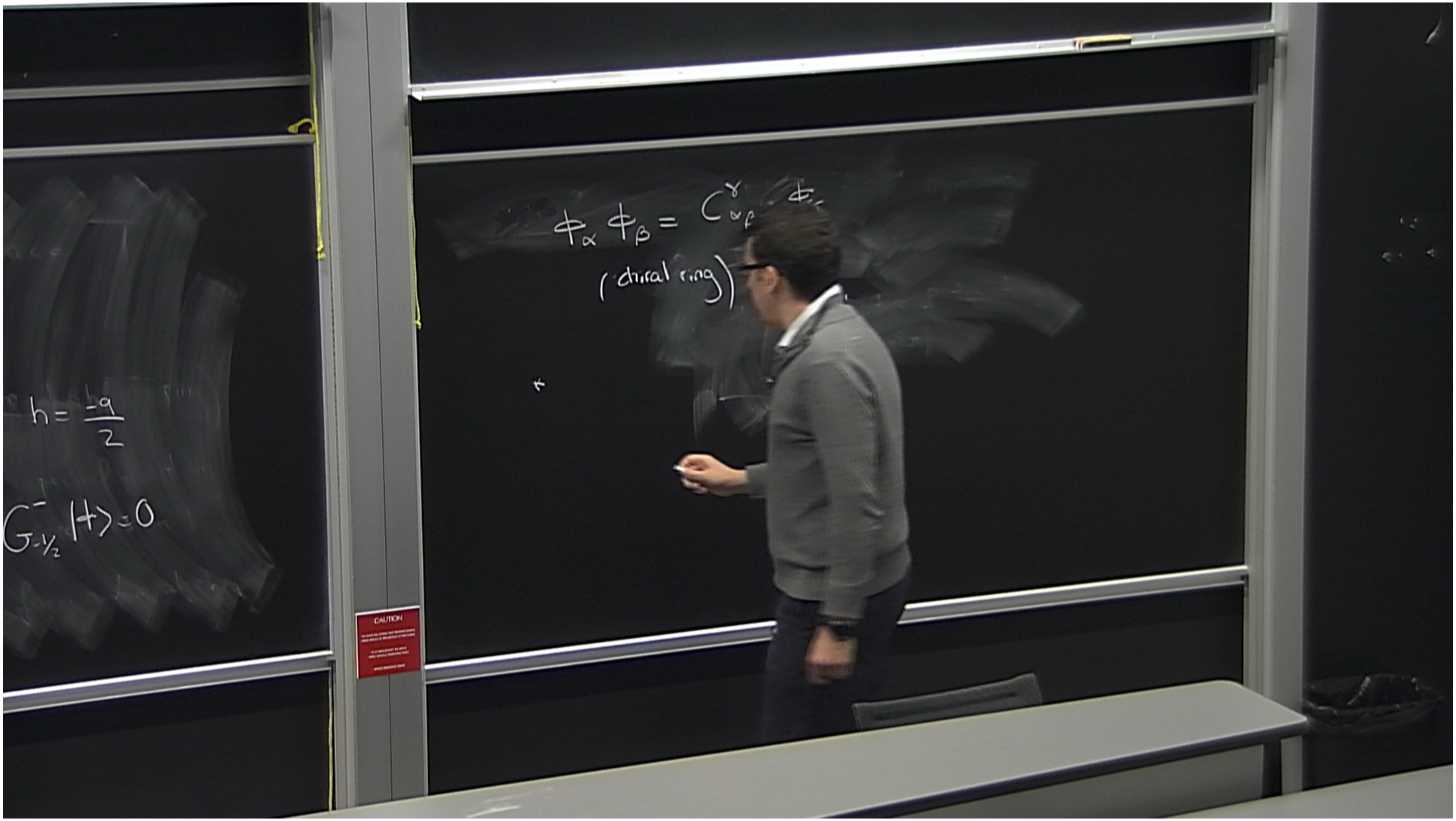

* "BPS"


* "BPS" bound
$h \geq \frac{|q|}{2}$

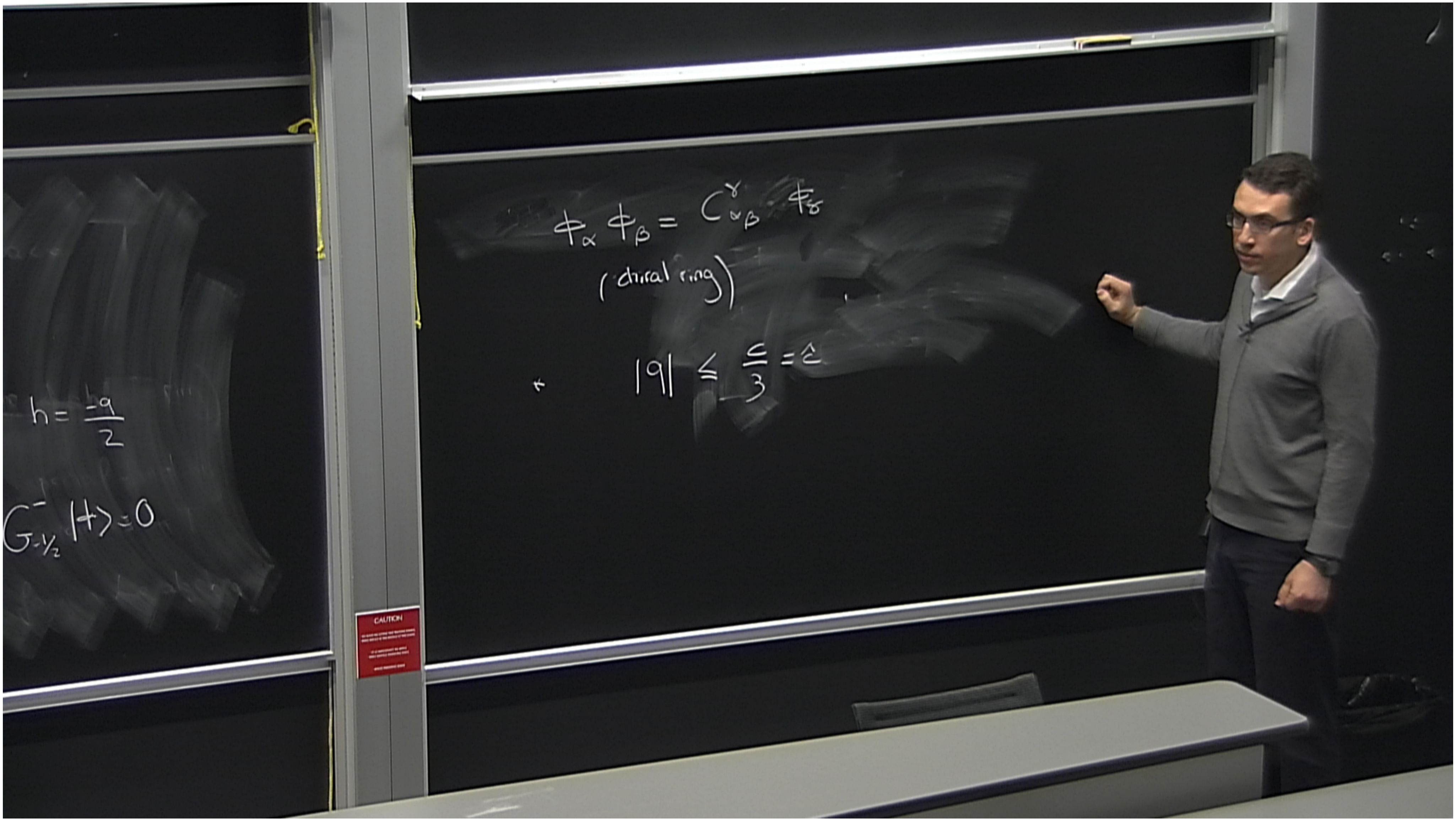

* "BPS" bound
h ≥ |q|/2
* h = q/2 chiral states


q=½ NS
* "BPS" bound
h ≥ |q|/2
* h = q/2 chiral states
h
⟺ G+−½ |φ⟩ =0

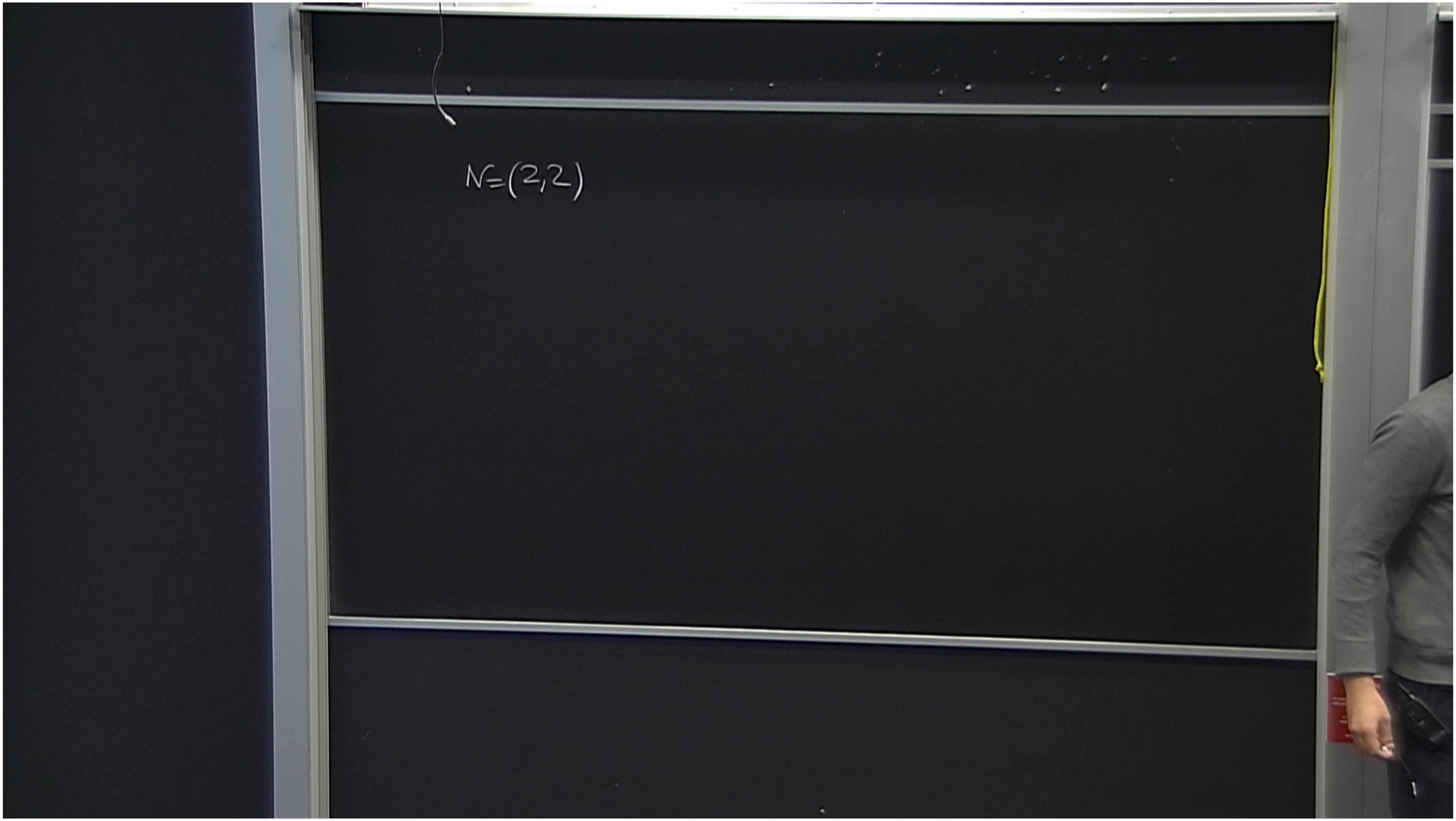

$\phi_\alpha \phi_\beta = C^\gamma \phi_\gamma$
(chiral
$h = \frac{-q}{2}$
$G^-_{-1/2} |+\rangle = 0$

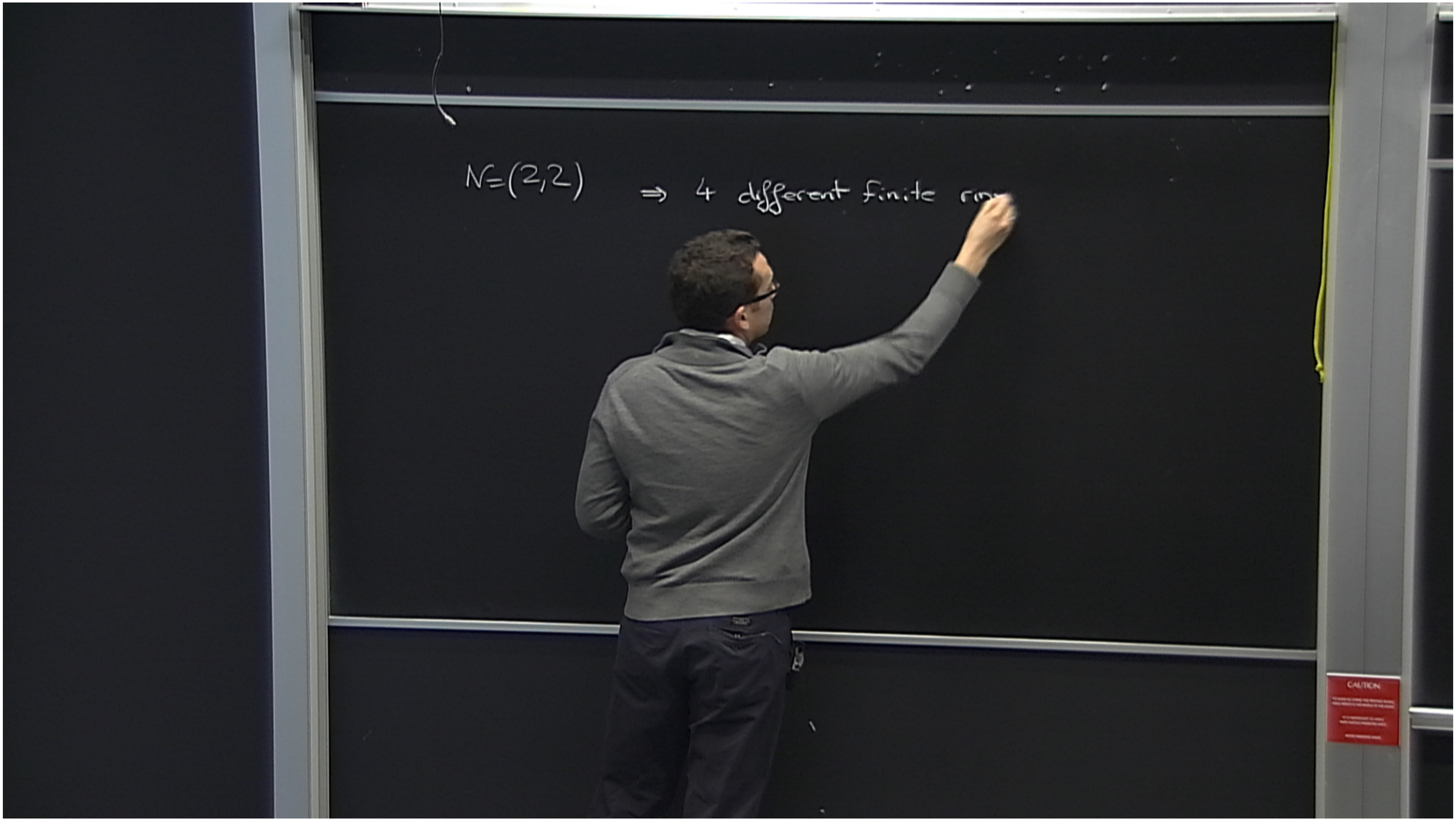


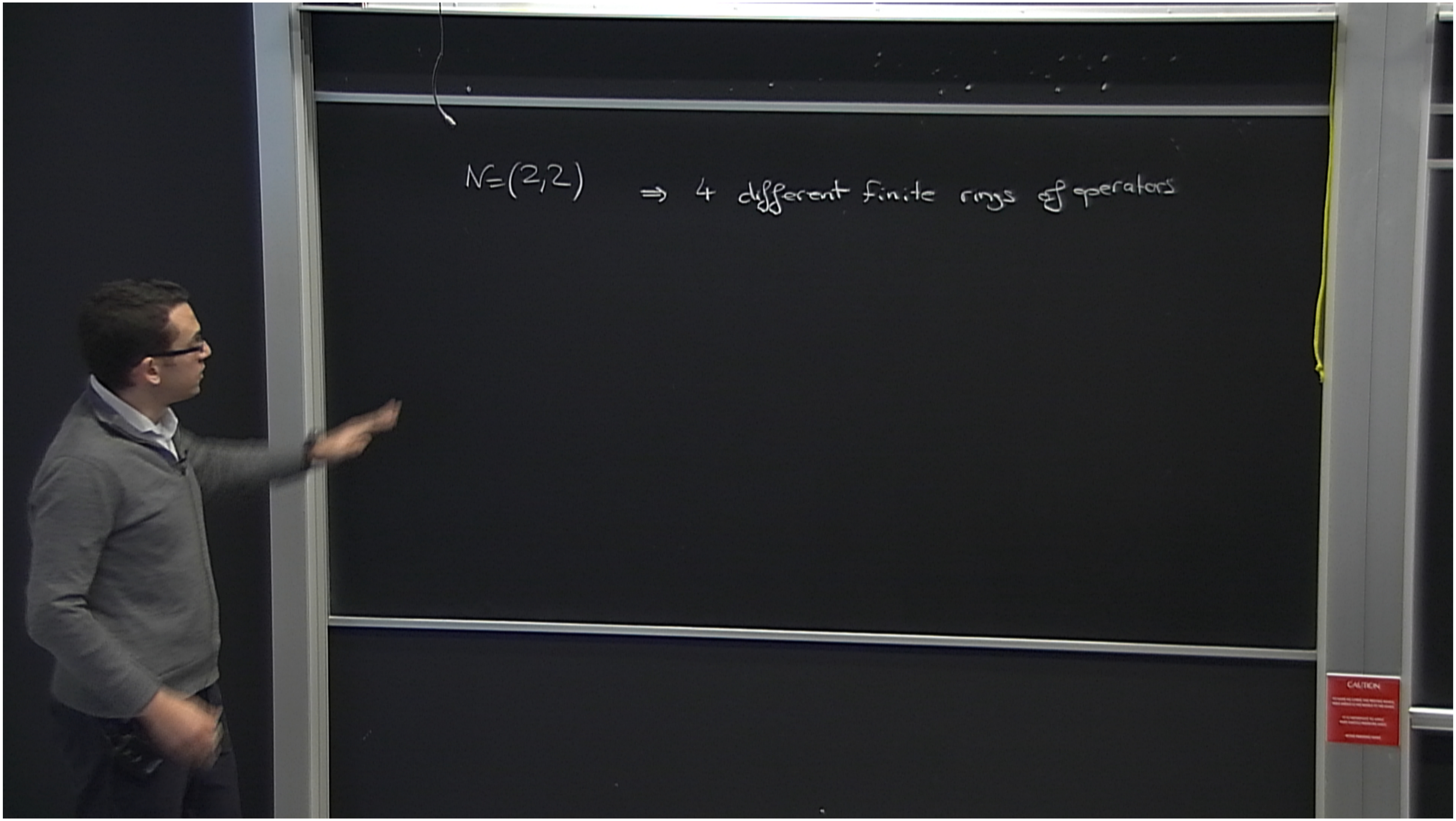
$\phi_\alpha \phi_\beta = C^\gamma_{\alpha\beta}$
(chiral ring)
$h = \frac{-q}{2}$
$G^-_{-1/2} |+\rangle = 0$
CAUTION

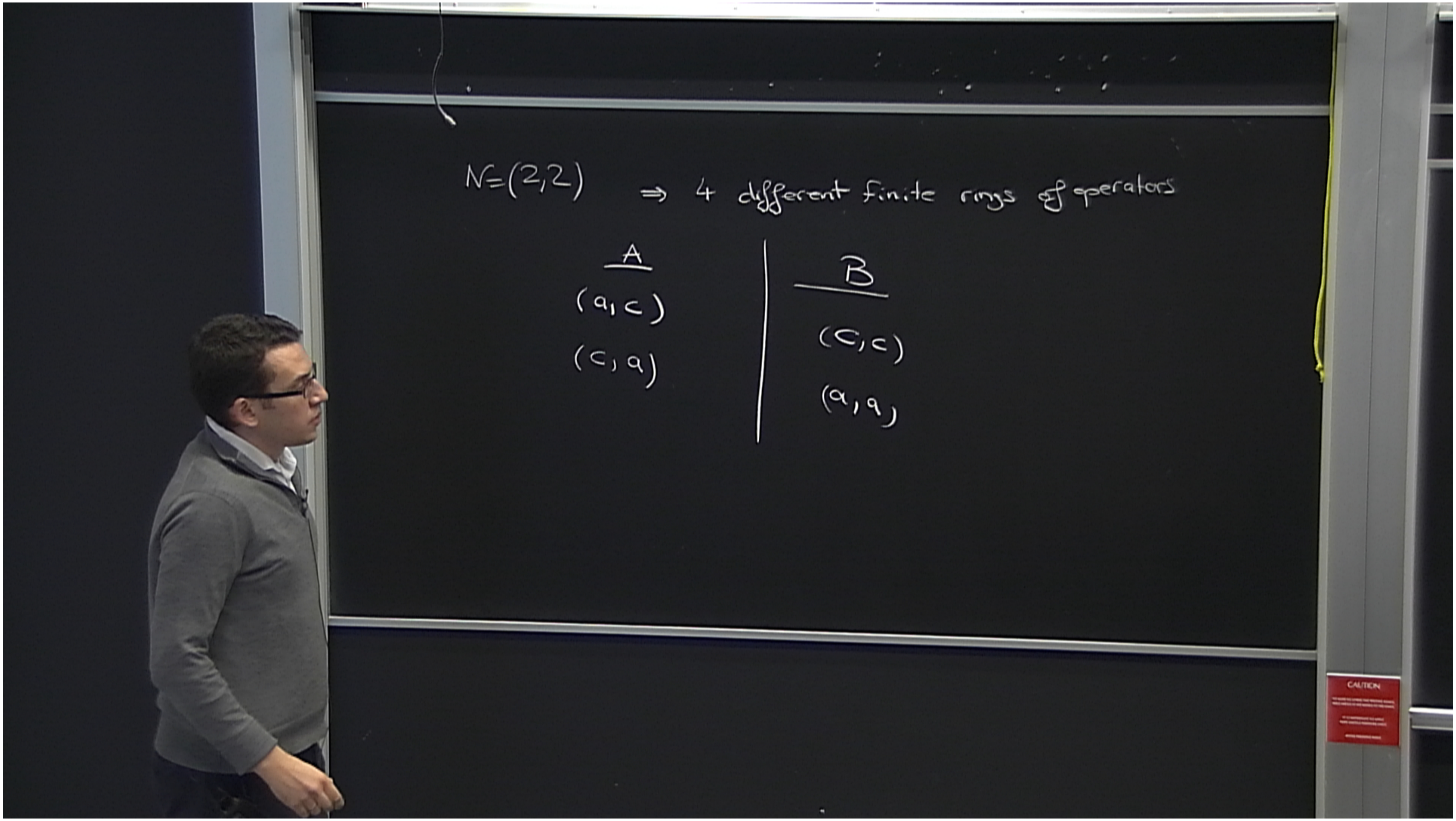


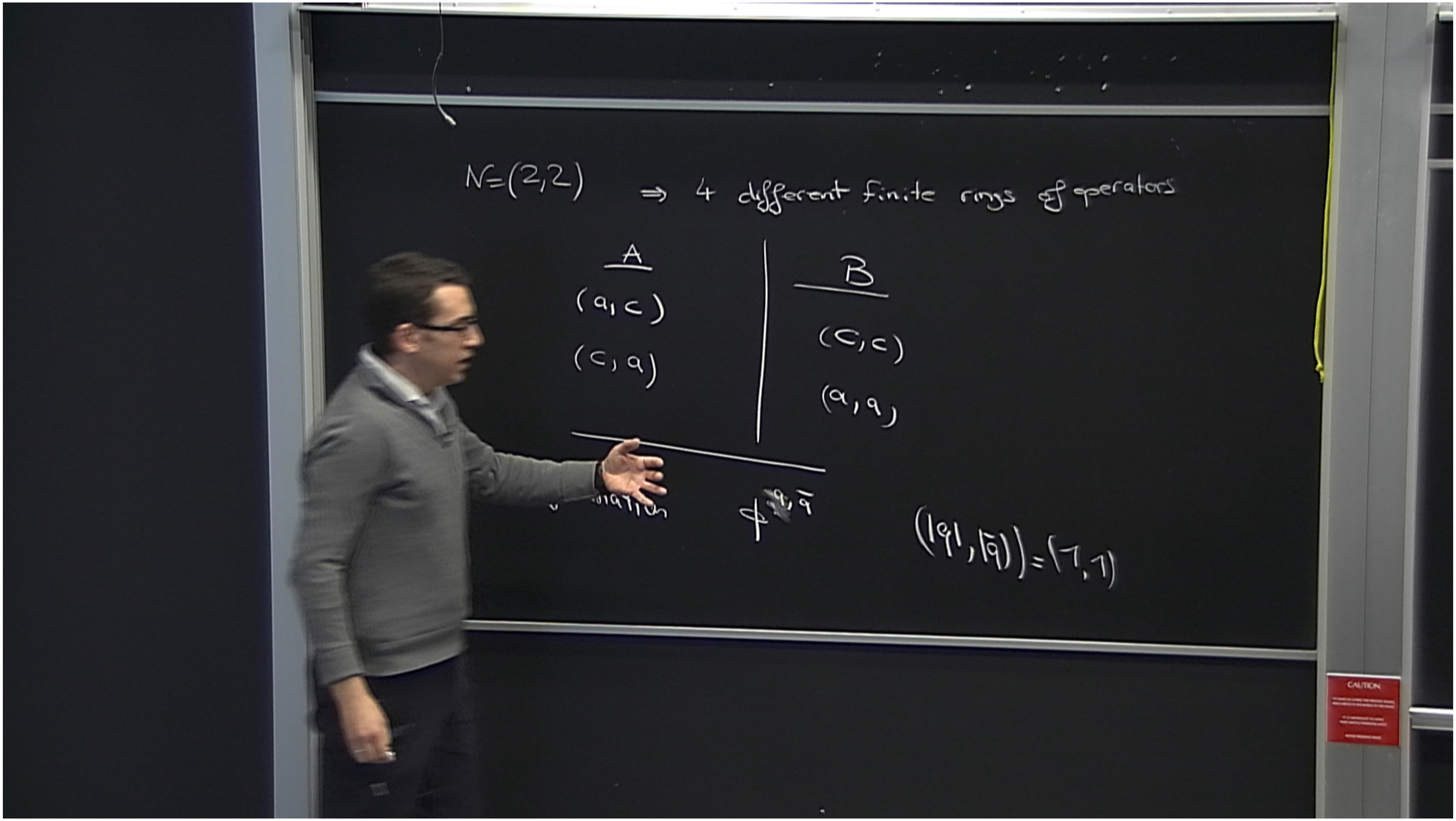
$\phi_\alpha \phi_\beta = C^\gamma_{\alpha\beta} \phi_\gamma$
(chiral ring)
$|q| \le \frac{c}{3} = \hat{c}$
$h = \frac{\pm q}{2}$
$G^-_{-1/2} |+\rangle = 0$
CAUTION

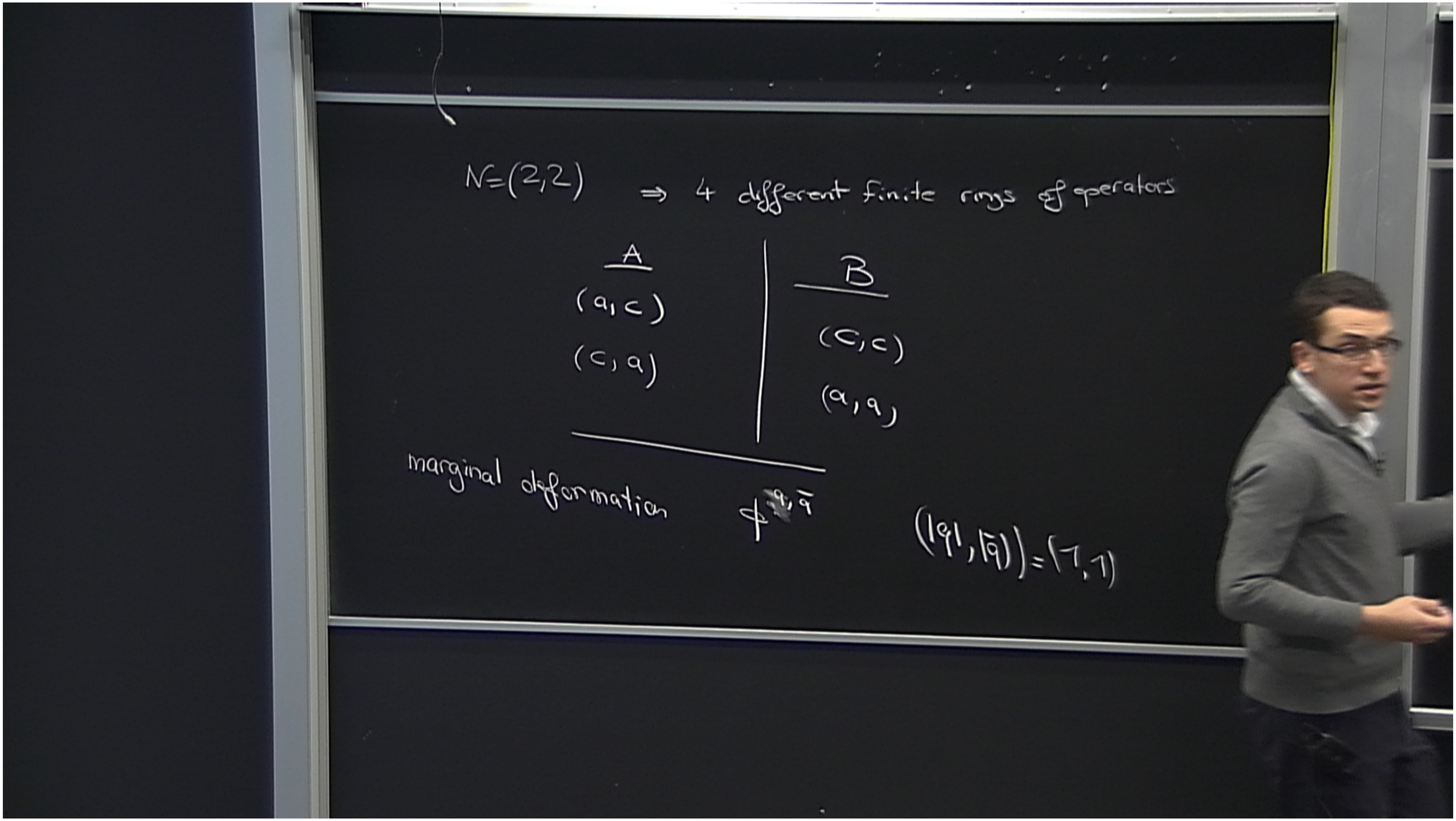

N=(2,2)

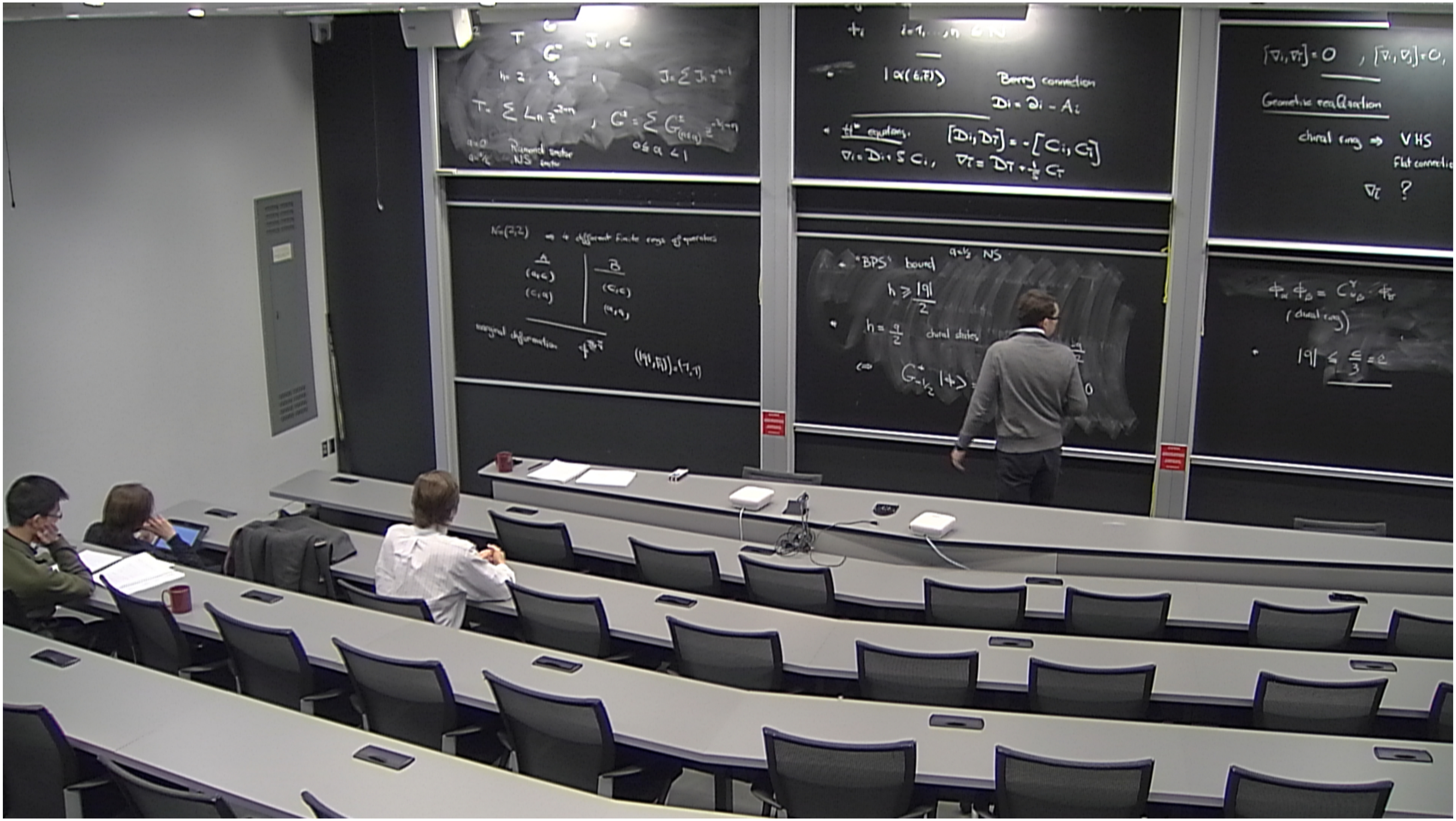

N=(2,2) ⇒ 4 different finite

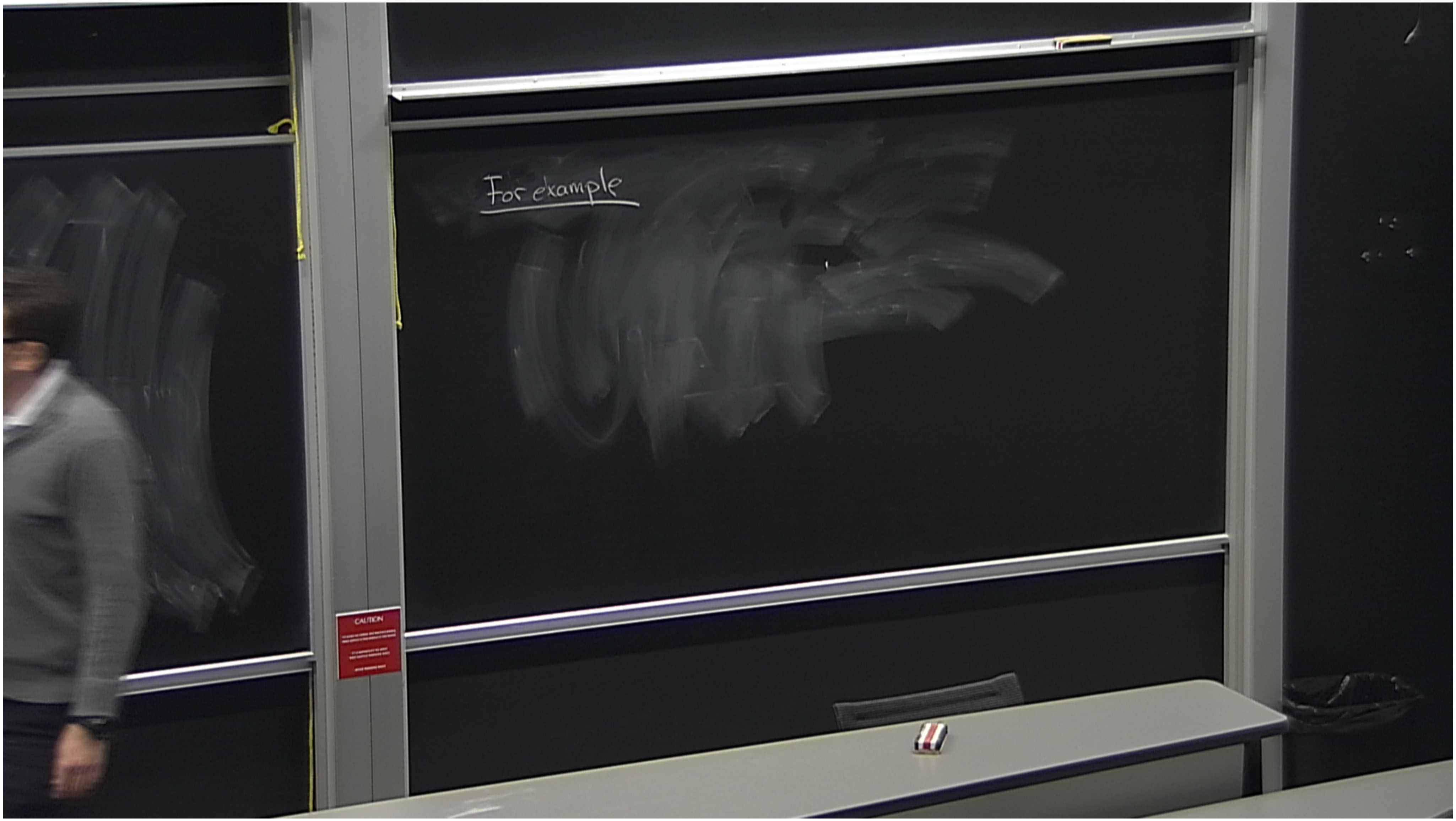

N=(2,2) ⇒ 4 different finite rings of operators

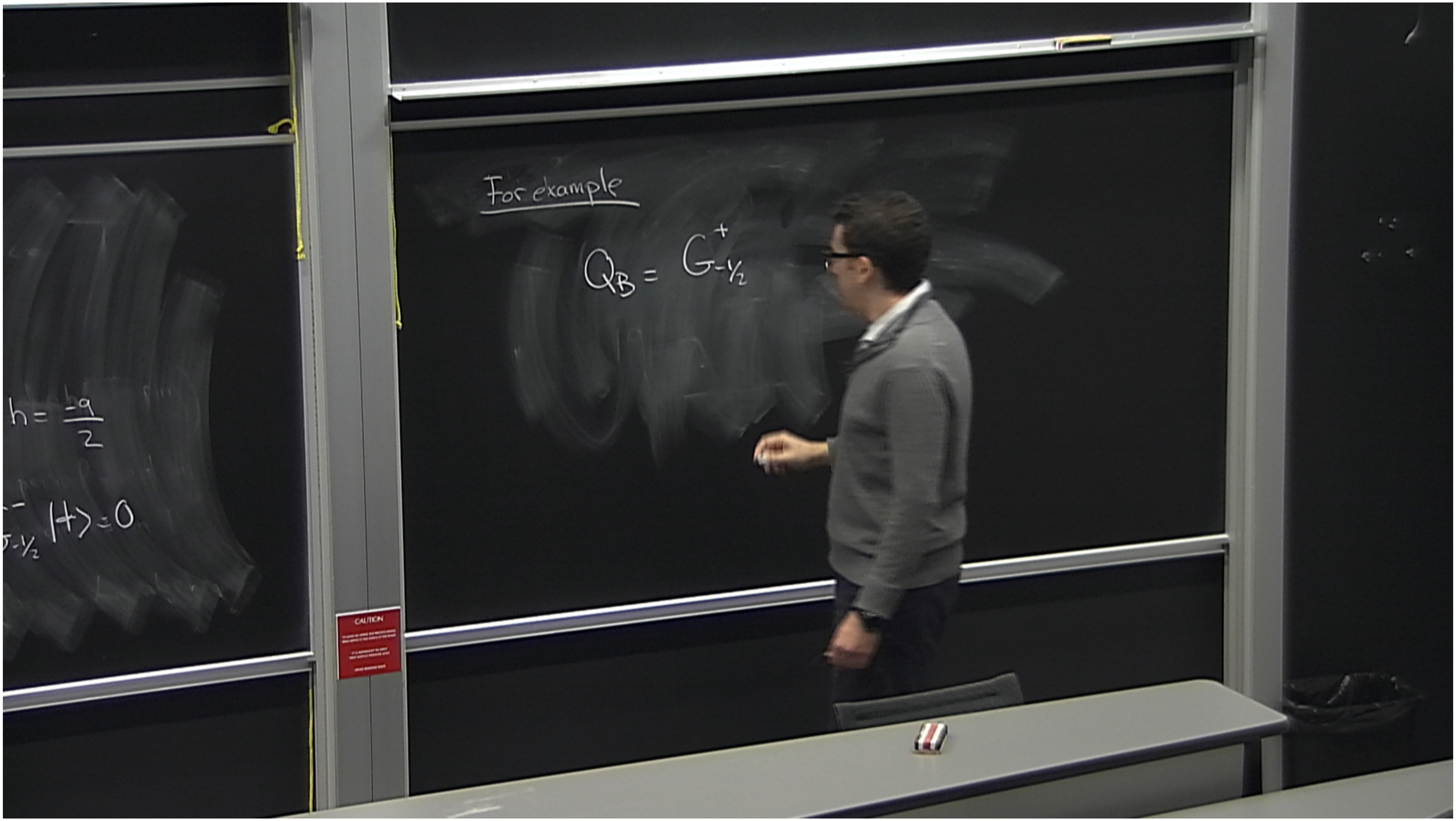

N=(2,2) ⇒ 4 different finite rings of operators
A
(a, c)
(c, a)
B
(c, c)
(a, a,

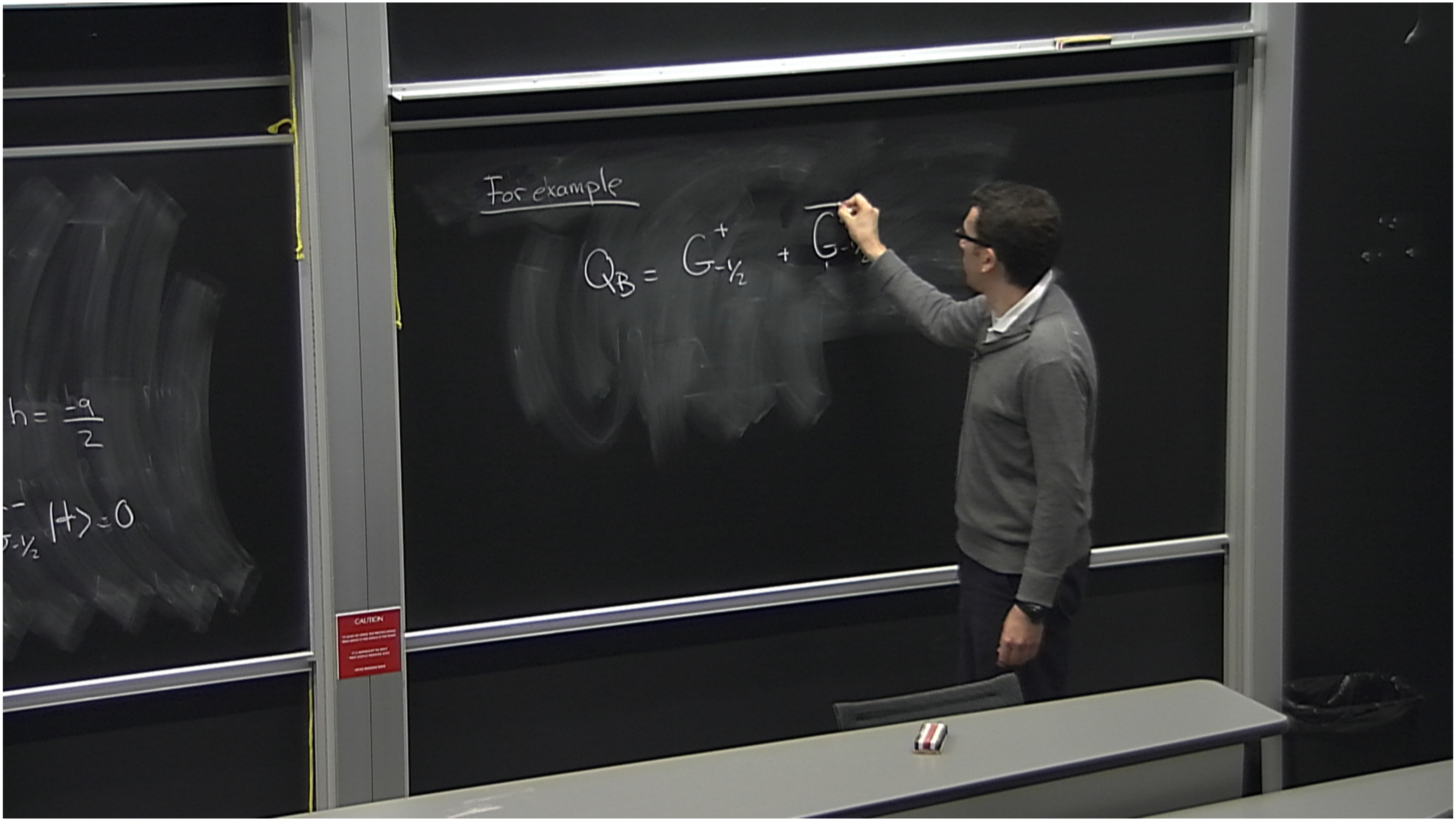

N=(2,2) ⇒ 4 different finite rings of operators
A
(a, c)
(c, a)
B
(c, c)
(a, a)
(|q|, |q̄|) = (7, 1)

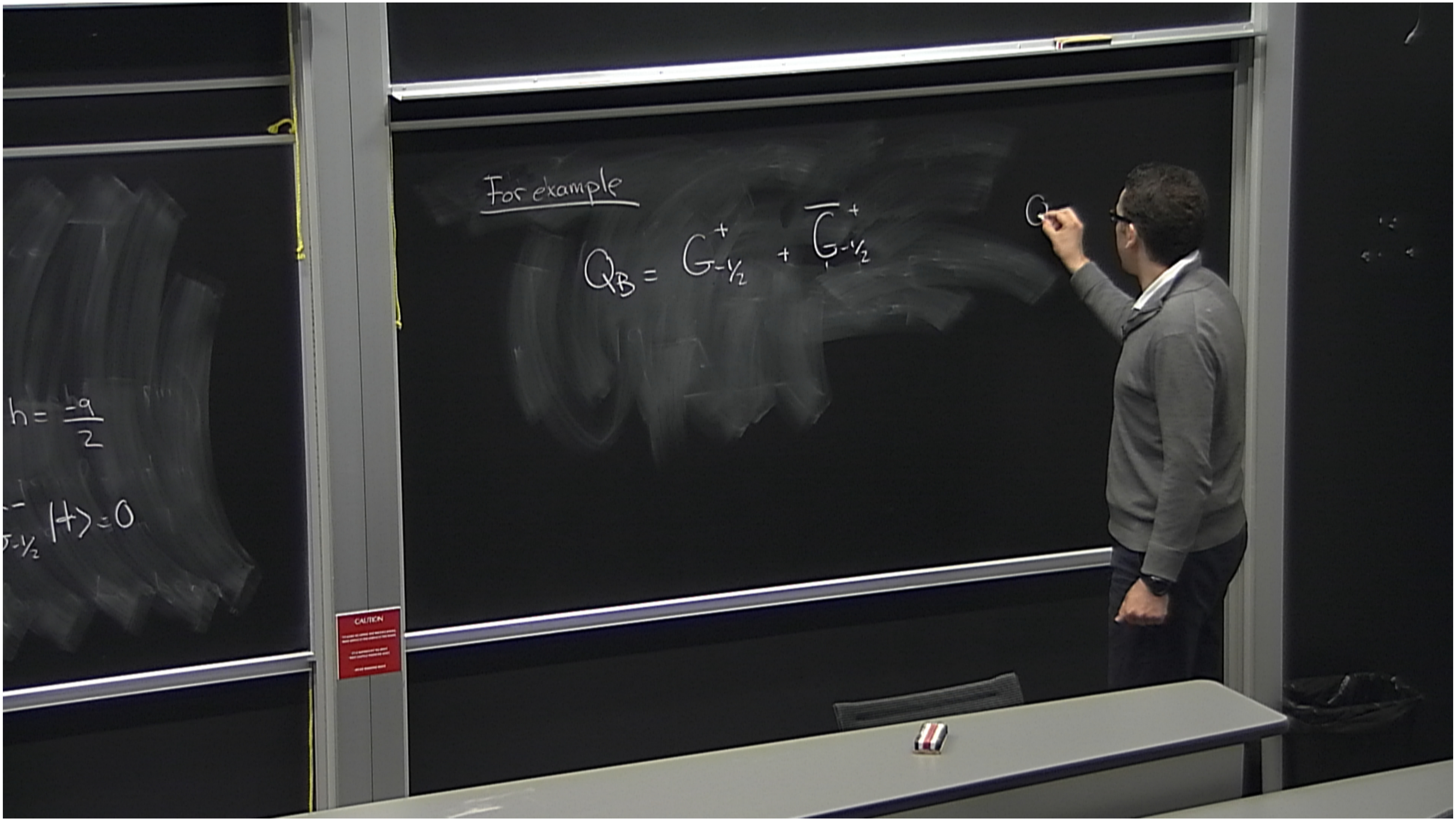

N=(2,2) ⇒ 4 different finite rings of operators
A
(a,c)
(c,a)
B
(c,c)
(a,a)
marginal deformation
(|q|,|q̄|)=(1,1)

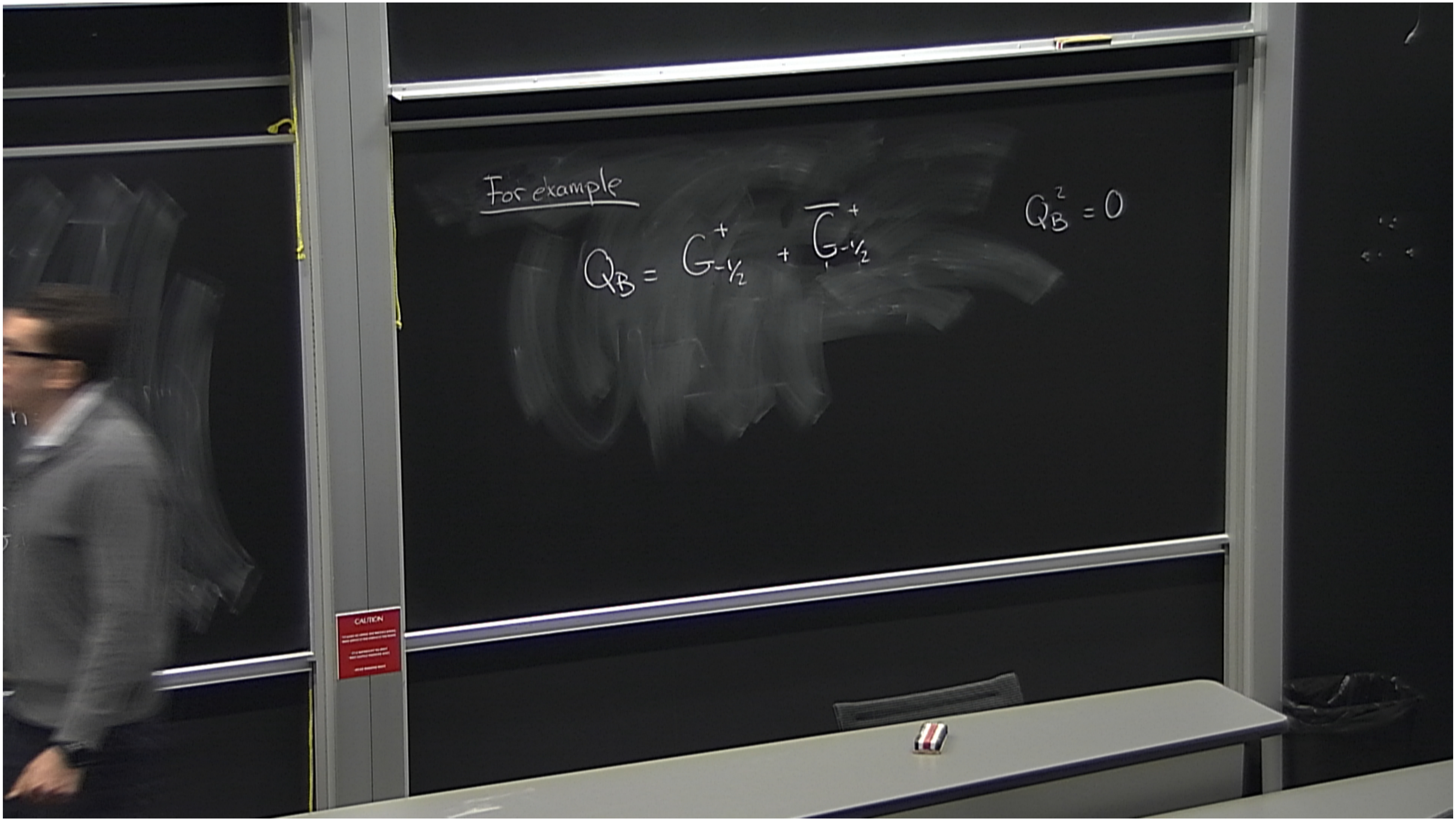


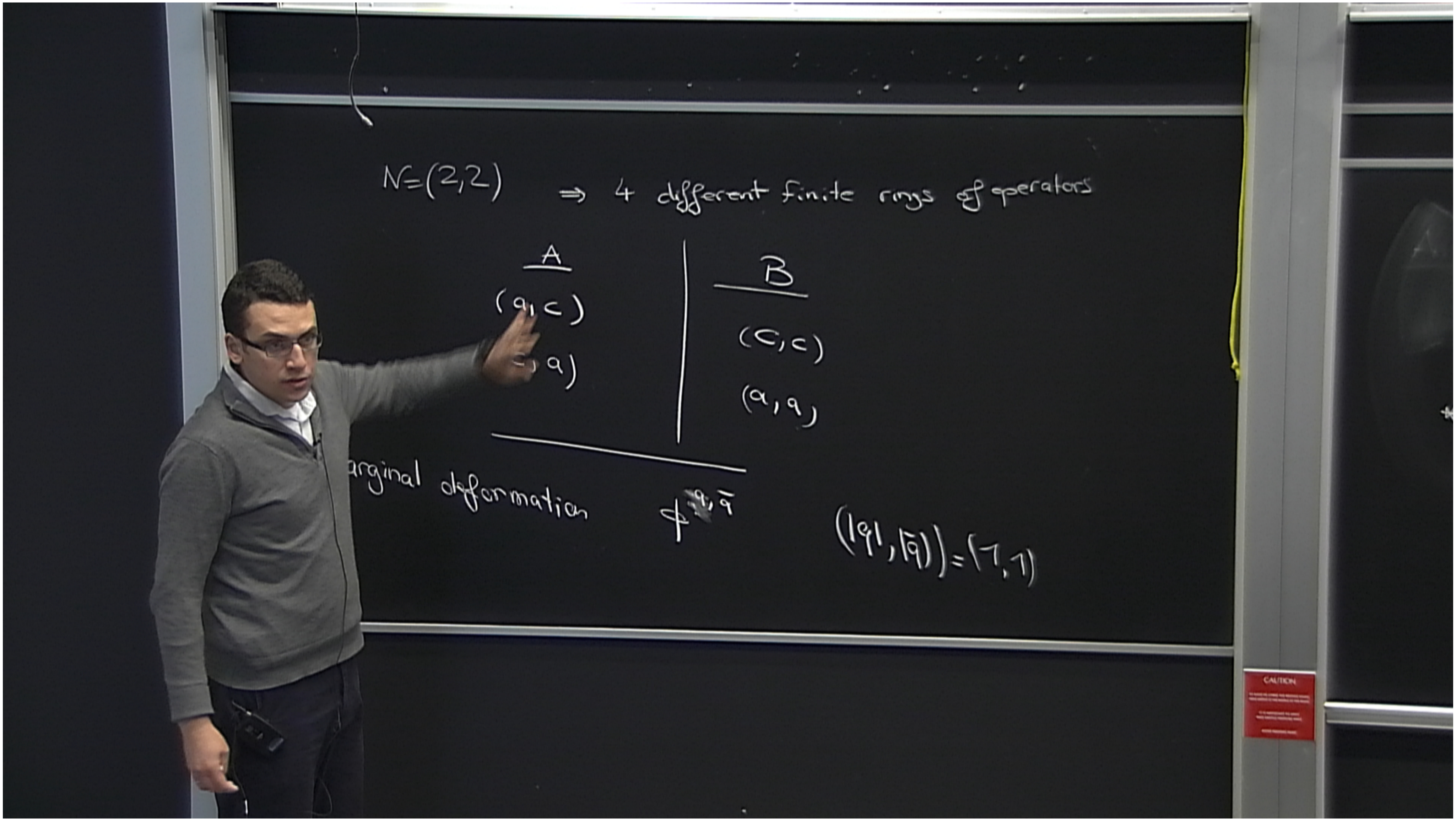

For example
CAUTION

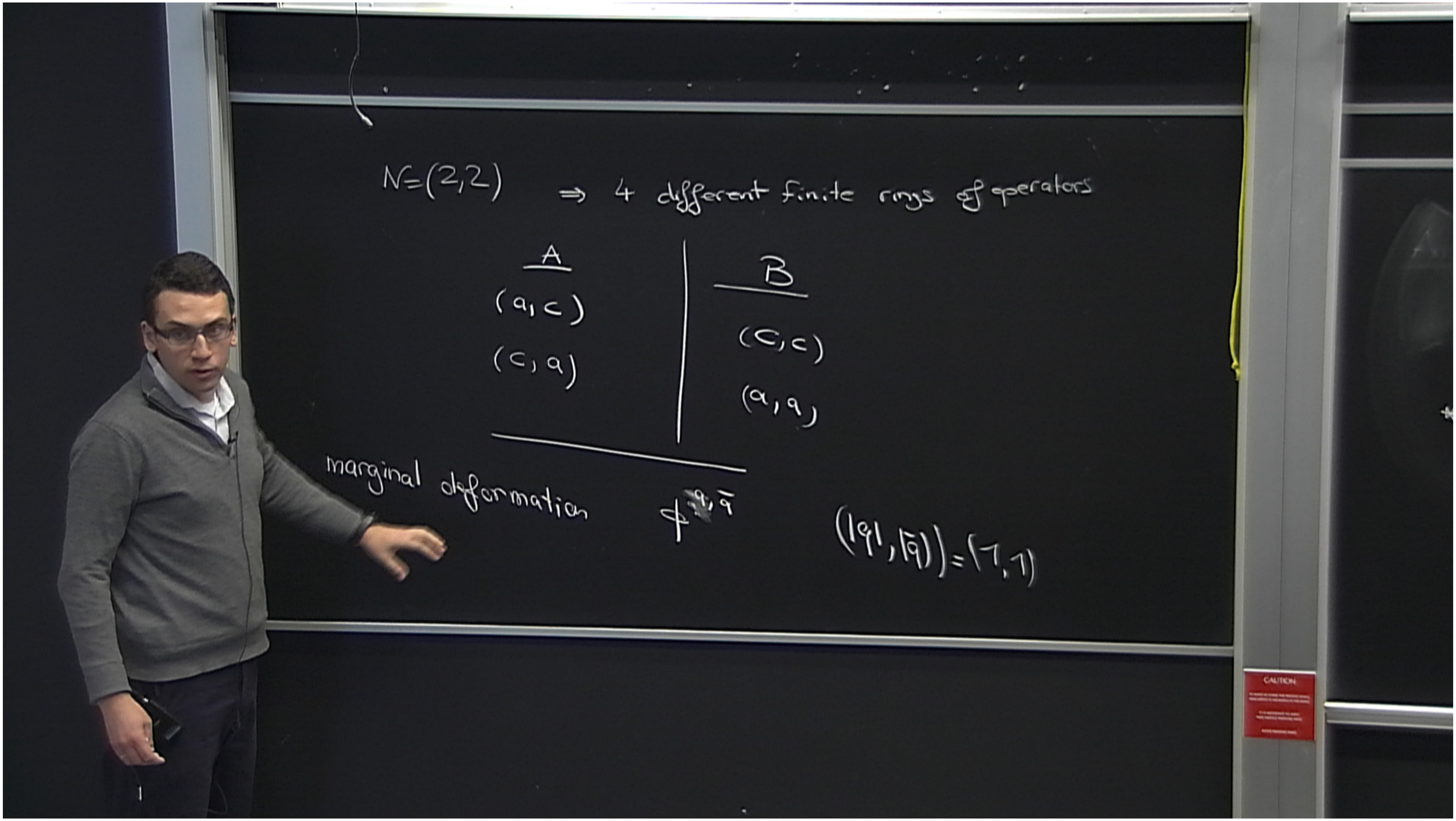

For example
$Q_B = G^+_{-1/2}$
$h = \frac{-q}{2}$
$|+\rangle = 0$

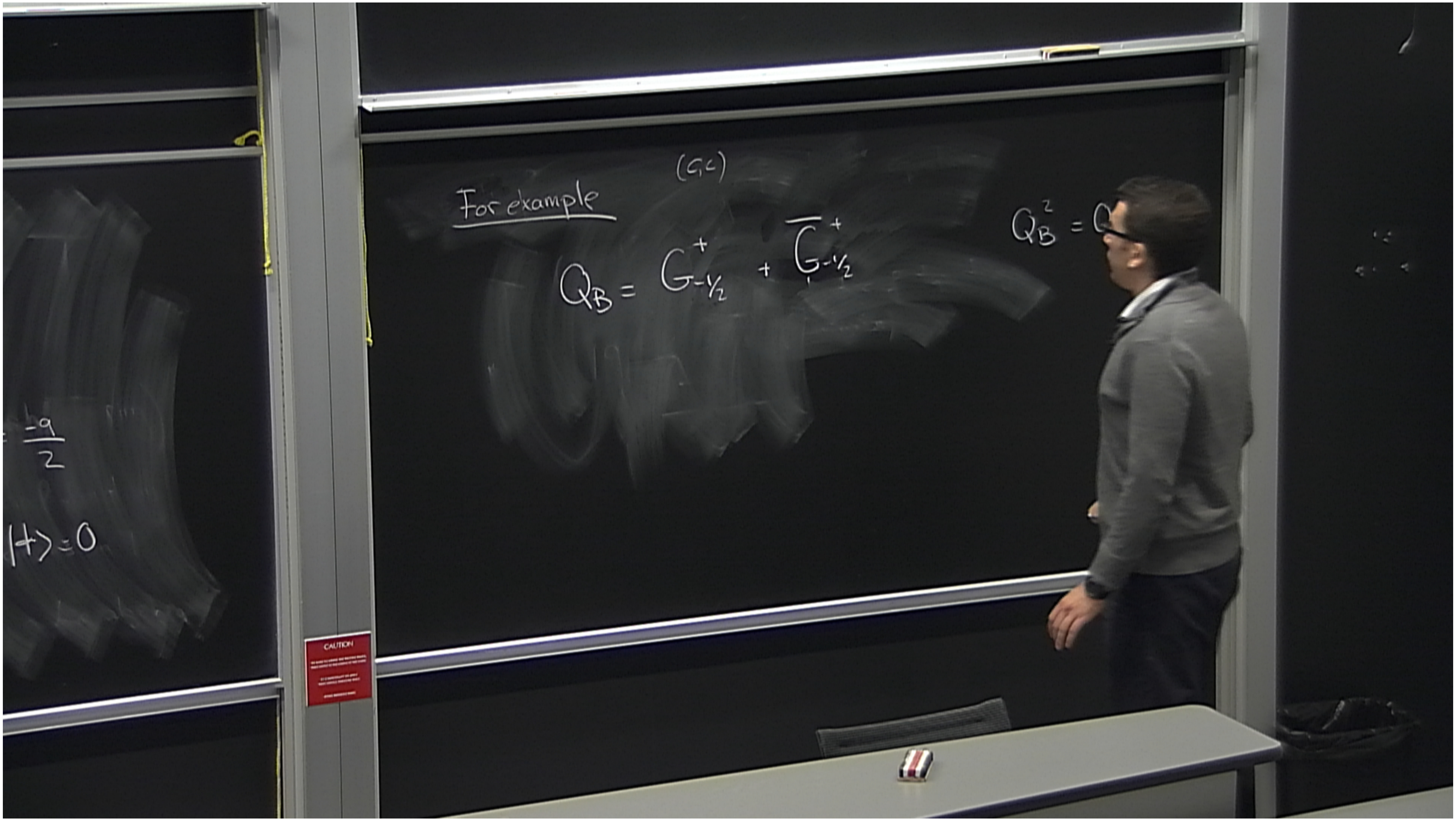

For example
QB = G+ -1/2 +

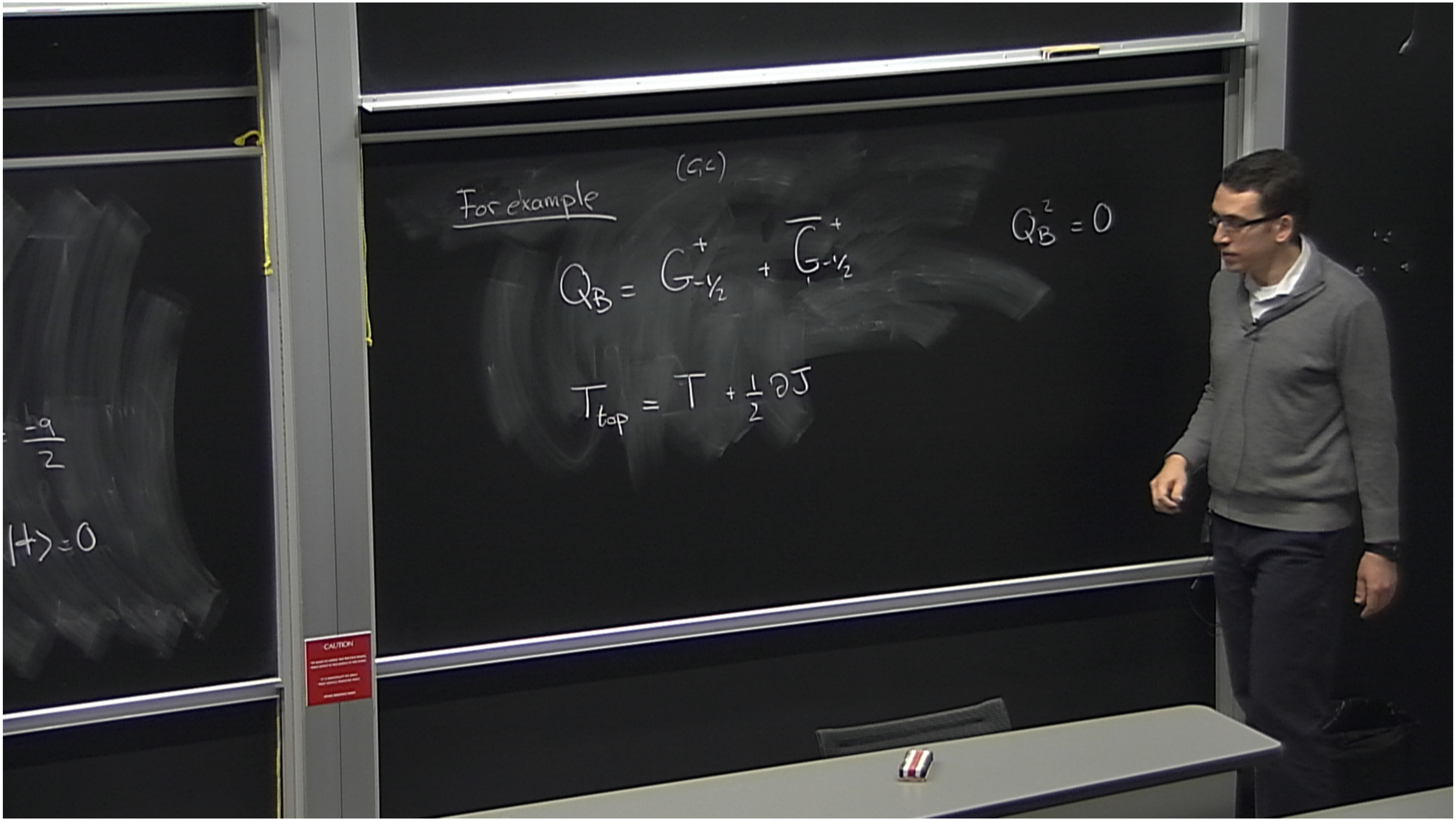

For example
Q_B = G^+_{-1/2} + \bar{G}^+_{-1/2}
h = -9/2
|+> = 0

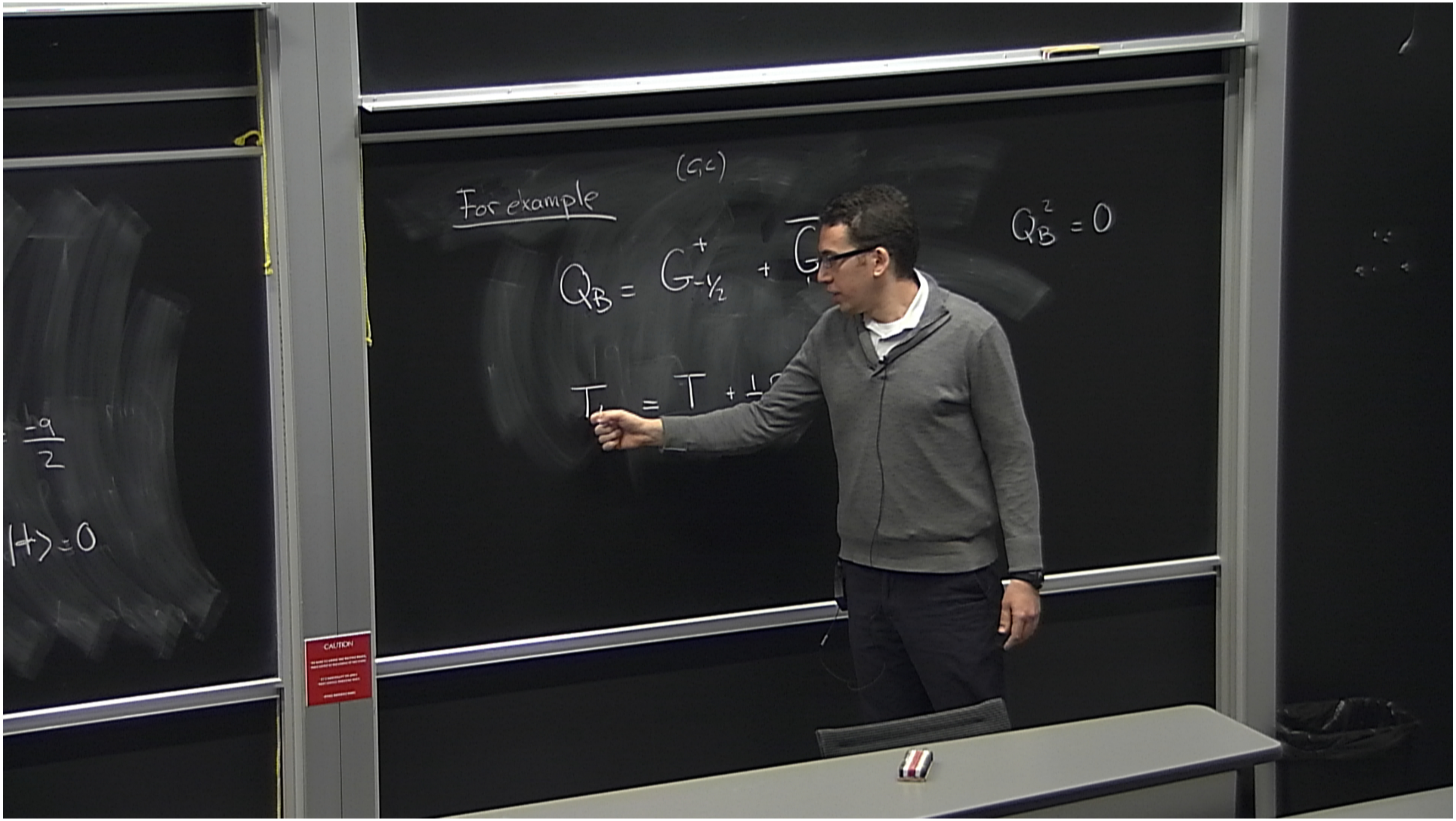

For example
$Q_B = G^+_{-1/2} + \bar{G}^+_{-1/2}$
$Q_B^2 = 0$
CAUTION

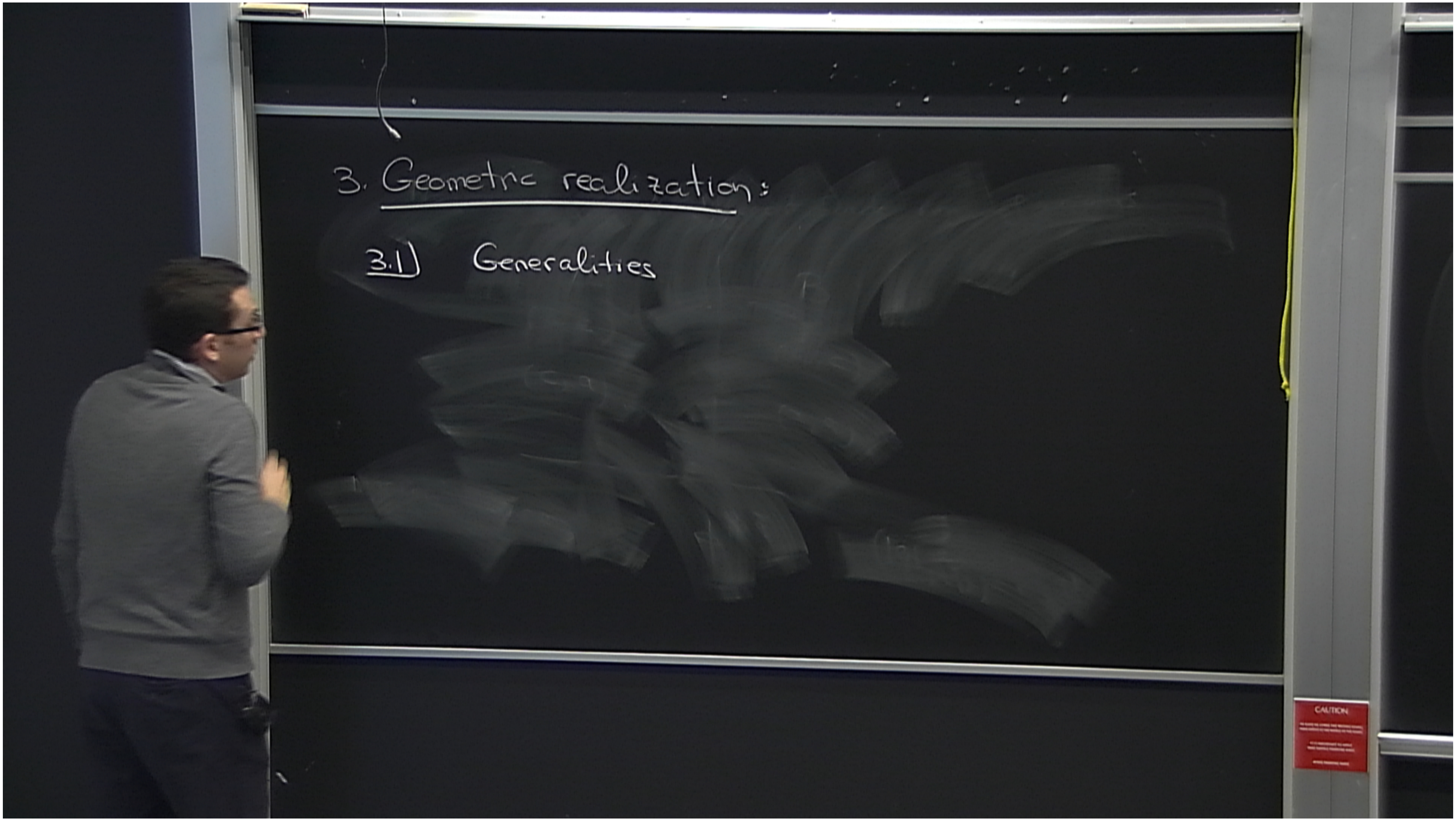

N=(2,2) ⇒ 4 different finite rings of operators
A
(a,c)
B
(c,c)
(a,a)
deformation
(|q|,|q̄|)=(1,1)
CAUTION

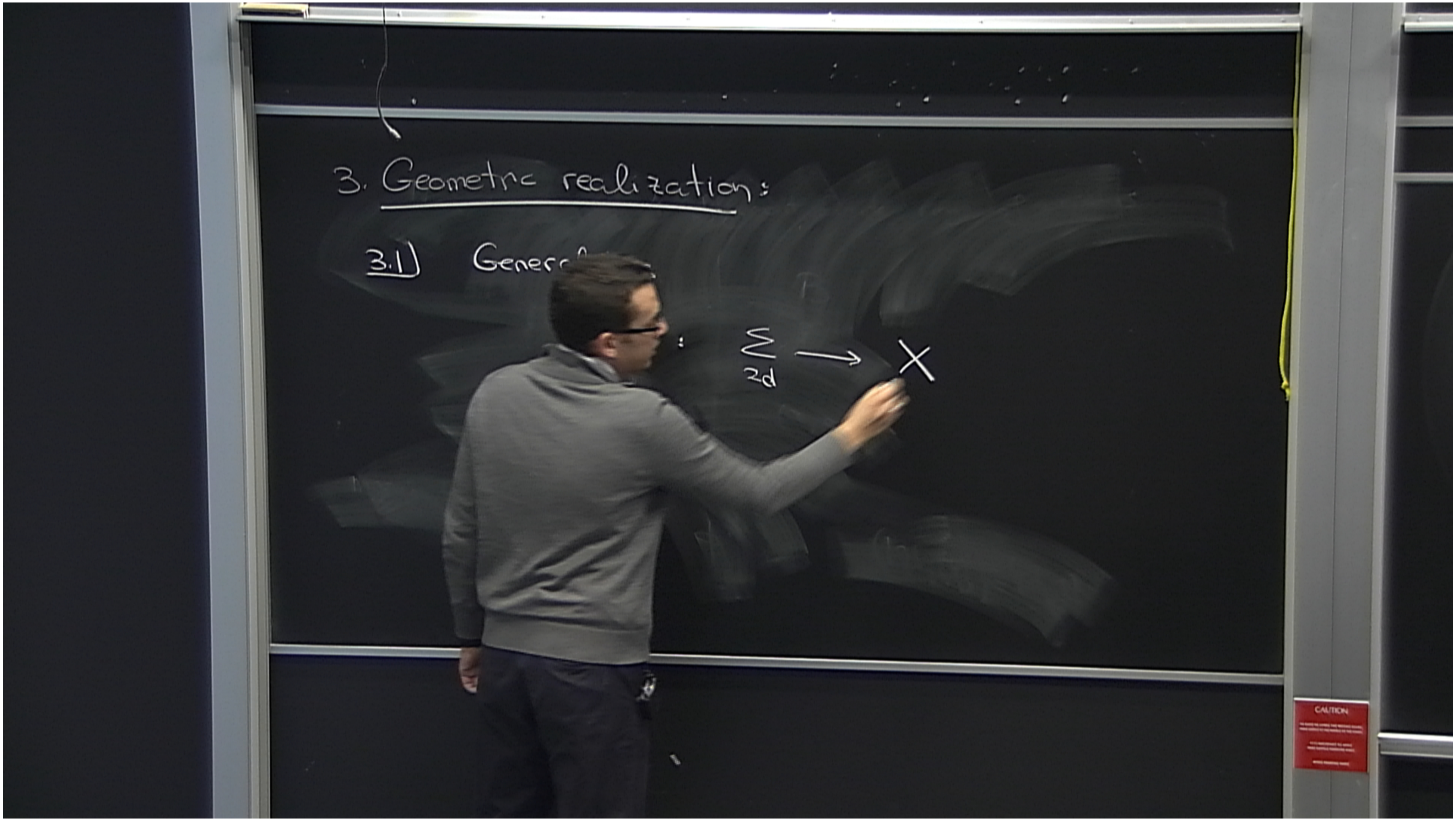

N=(2,2) ⇒ 4 different finite rings of operators
A
(a,c)
(c,a)
B
(c,c)
(a,a)
marginal deformation
(|q|,|q̄|)=(1,1)
CAUTION

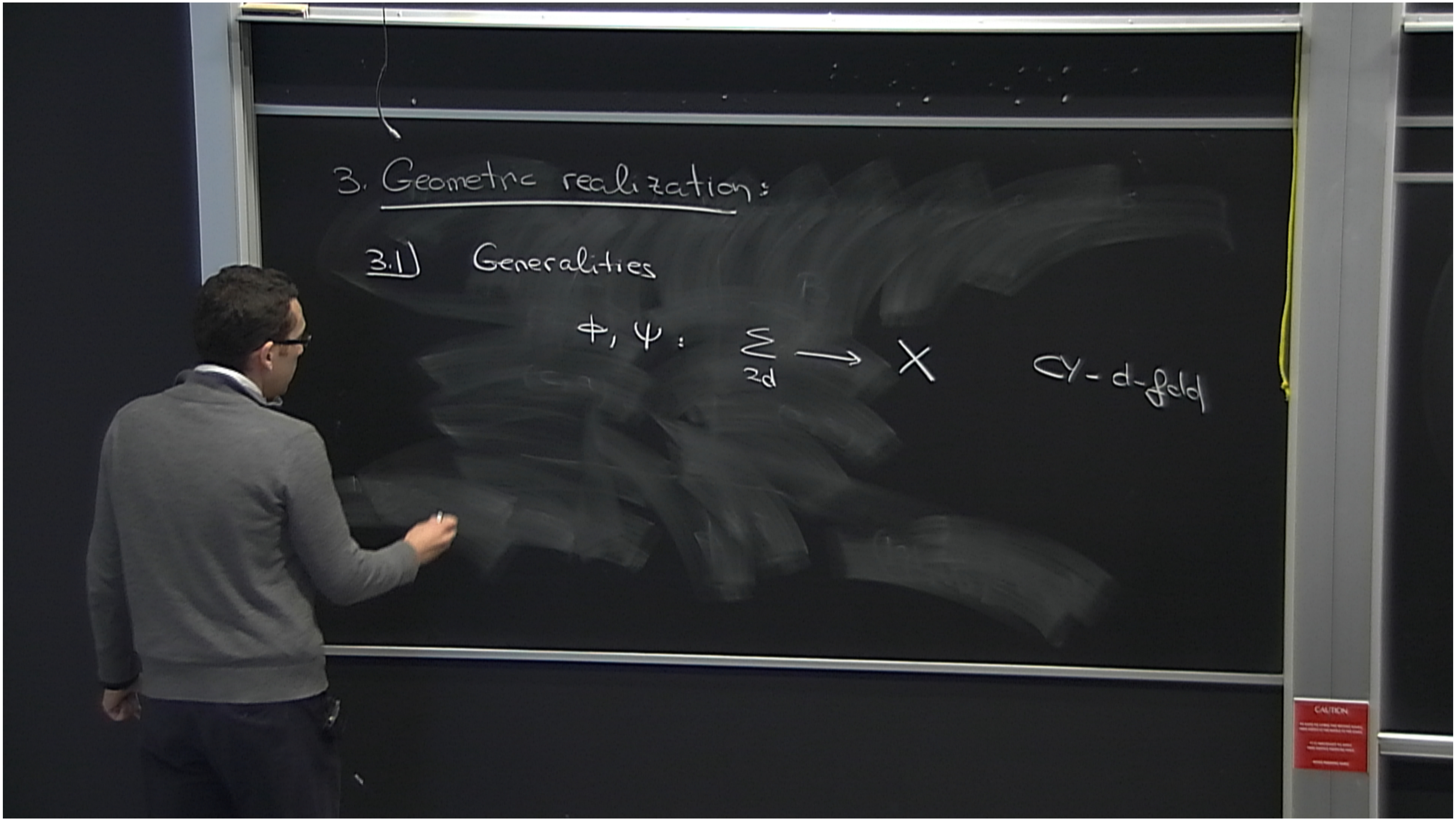

(GC)
For example
$Q_B = G^+_{-1/2} + \bar{G}^+_{-1/2}$
$Q_B^2 =$
$|+\rangle = 0$

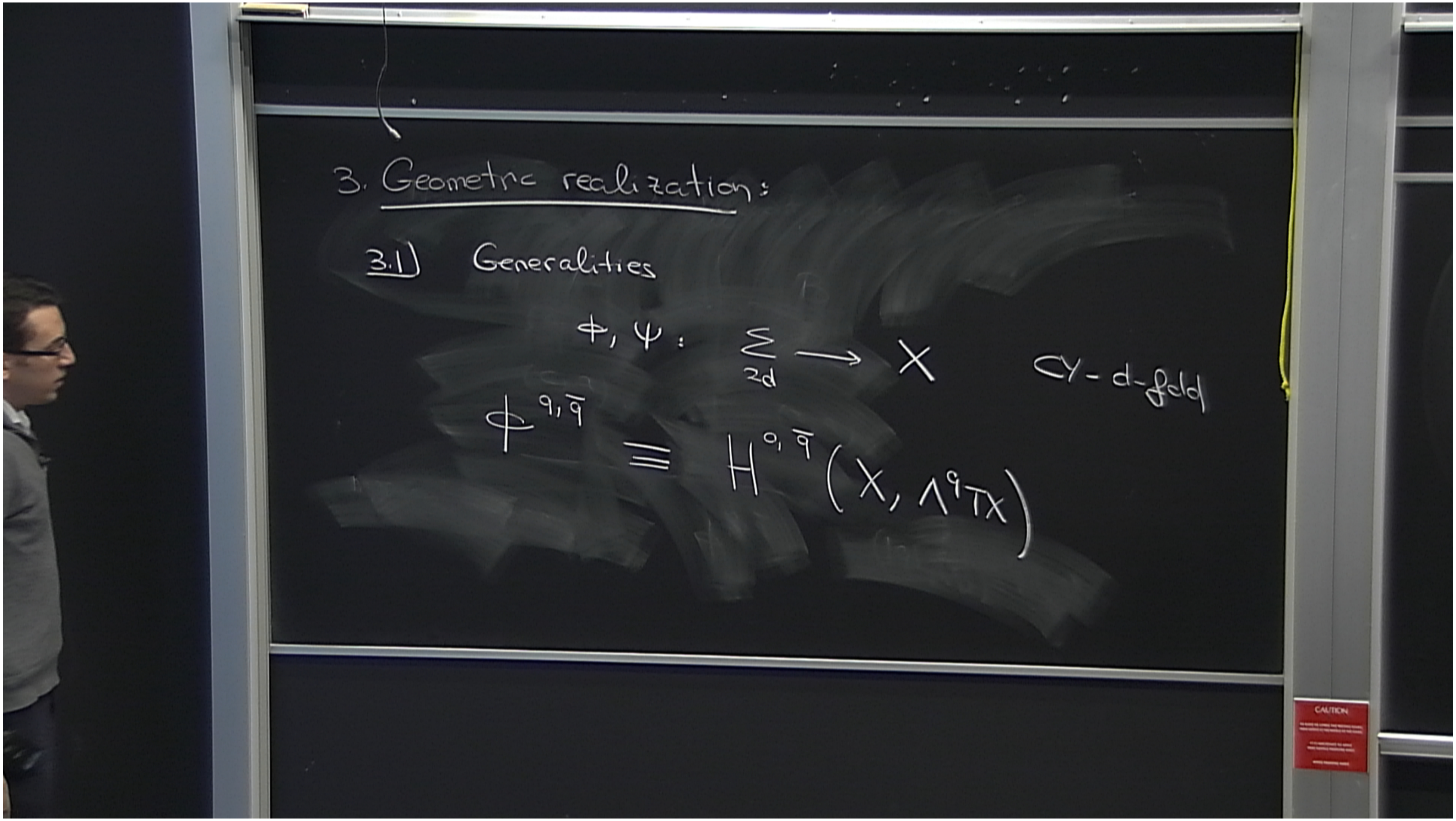

(GC)
For example
Q_B = G^+_{-1/2} + \bar{G}^+_{-1/2}
Q_B^2 = 0
T_top = T + 1/2 ∂J
CAUTION

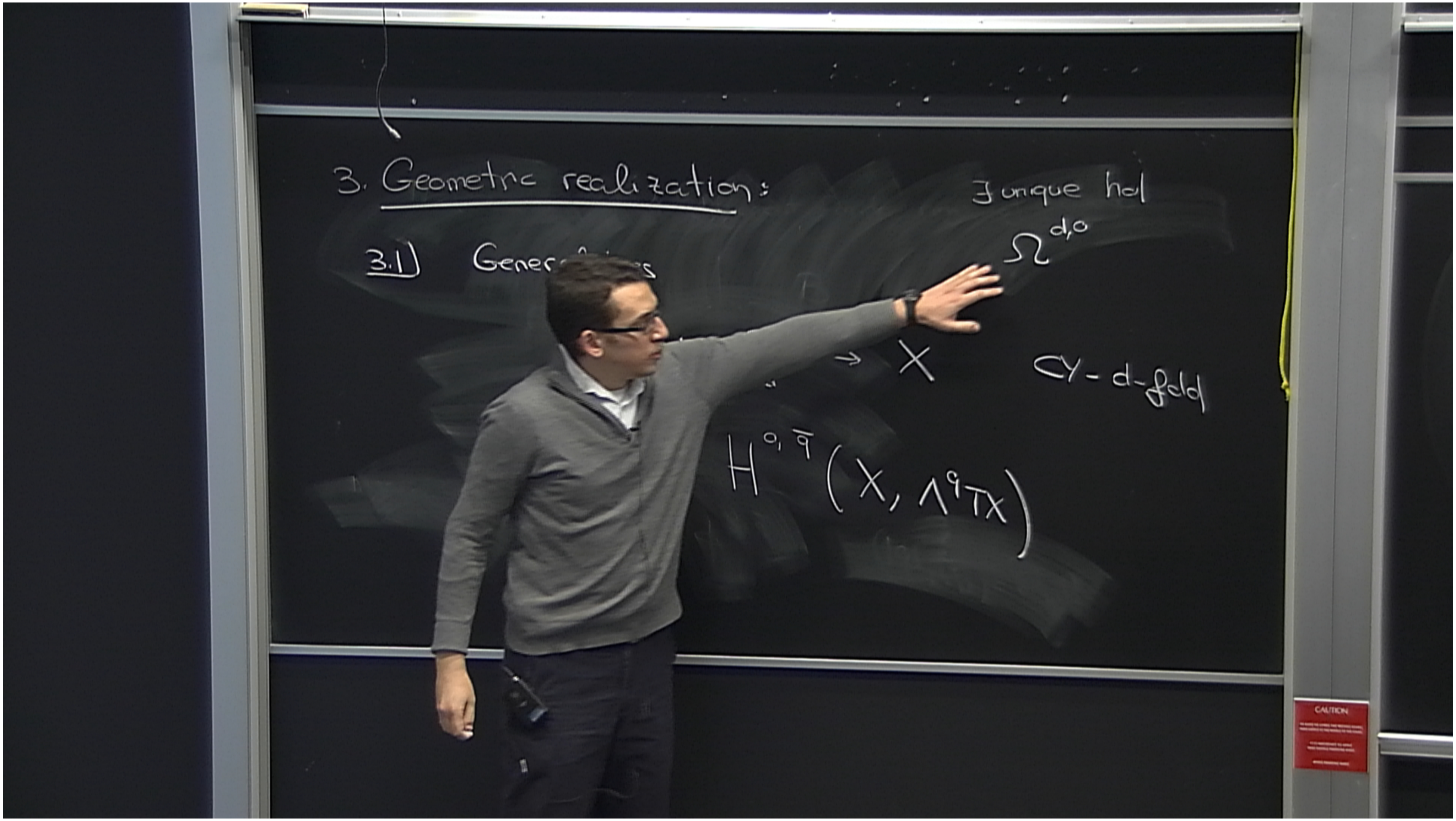

For example
(GC)
$Q_B^2 = 0$
CAUTION

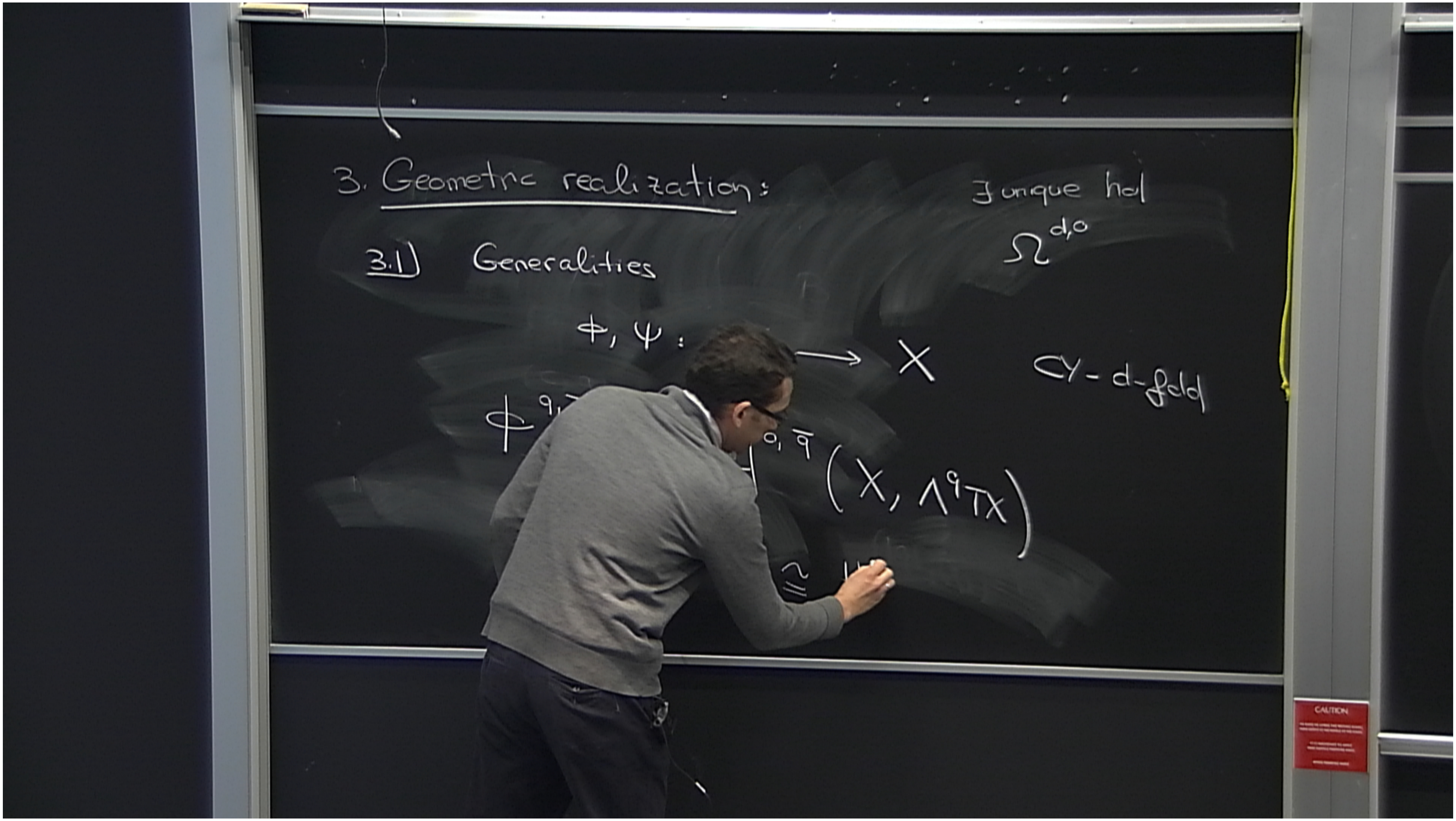

3. Geometric realization:
3.1) Generalities

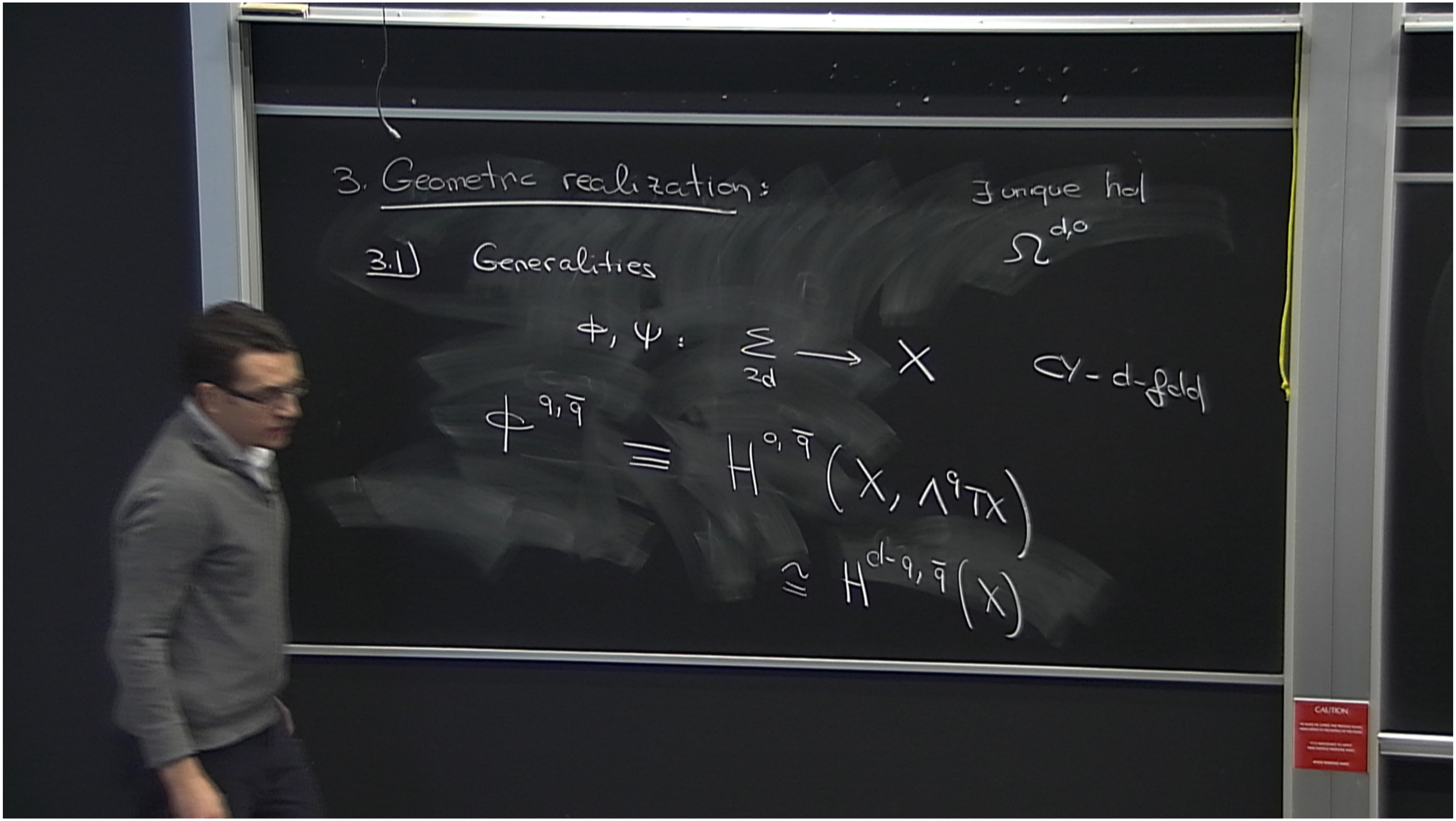

3. Geometric realization:
3.1) Genera
Σ
2d
→ X

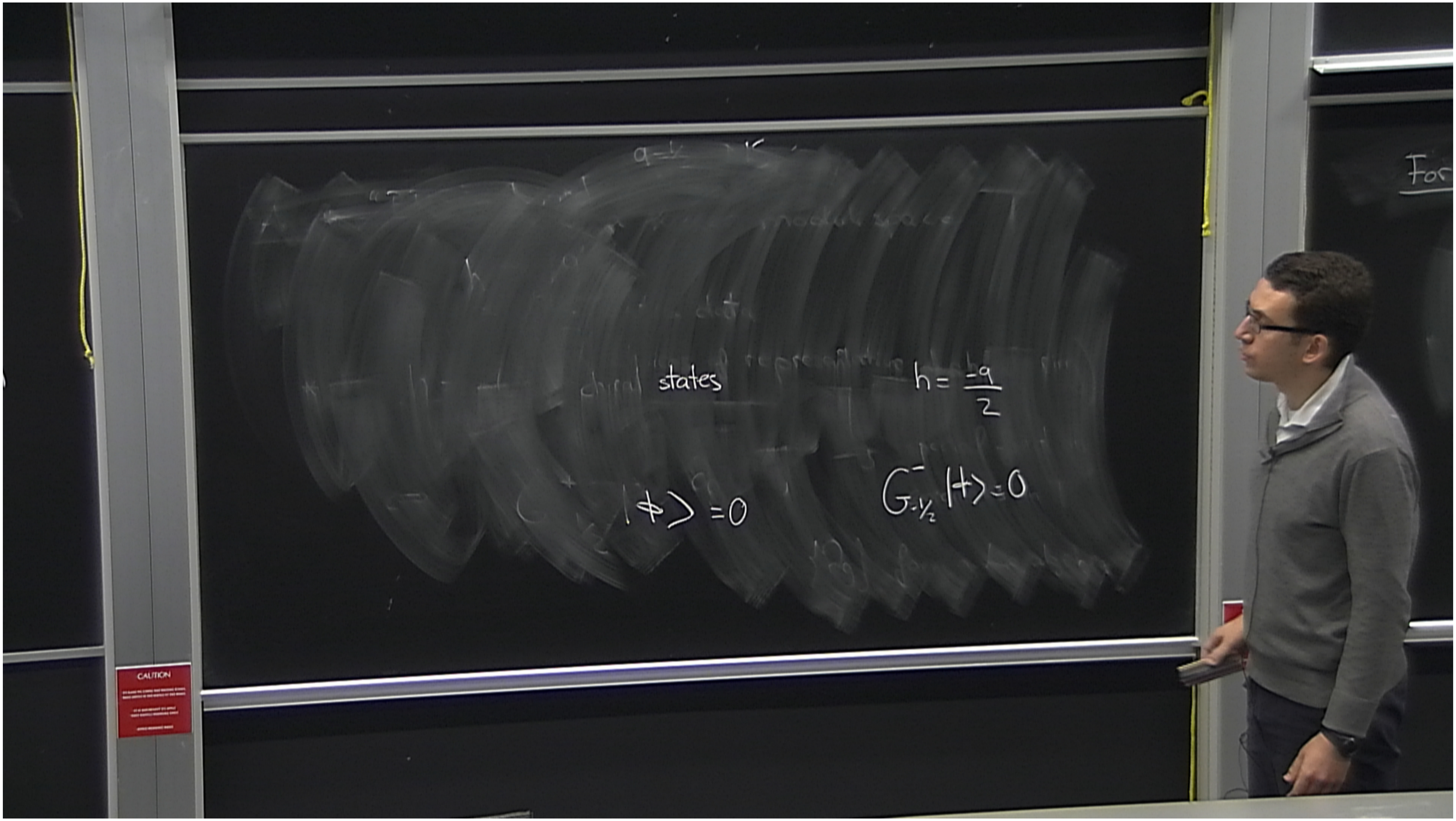

3. Geometric realization:
3.1) Generalities
Φ, Ψ : Σ → X
2d
CY - d-fold

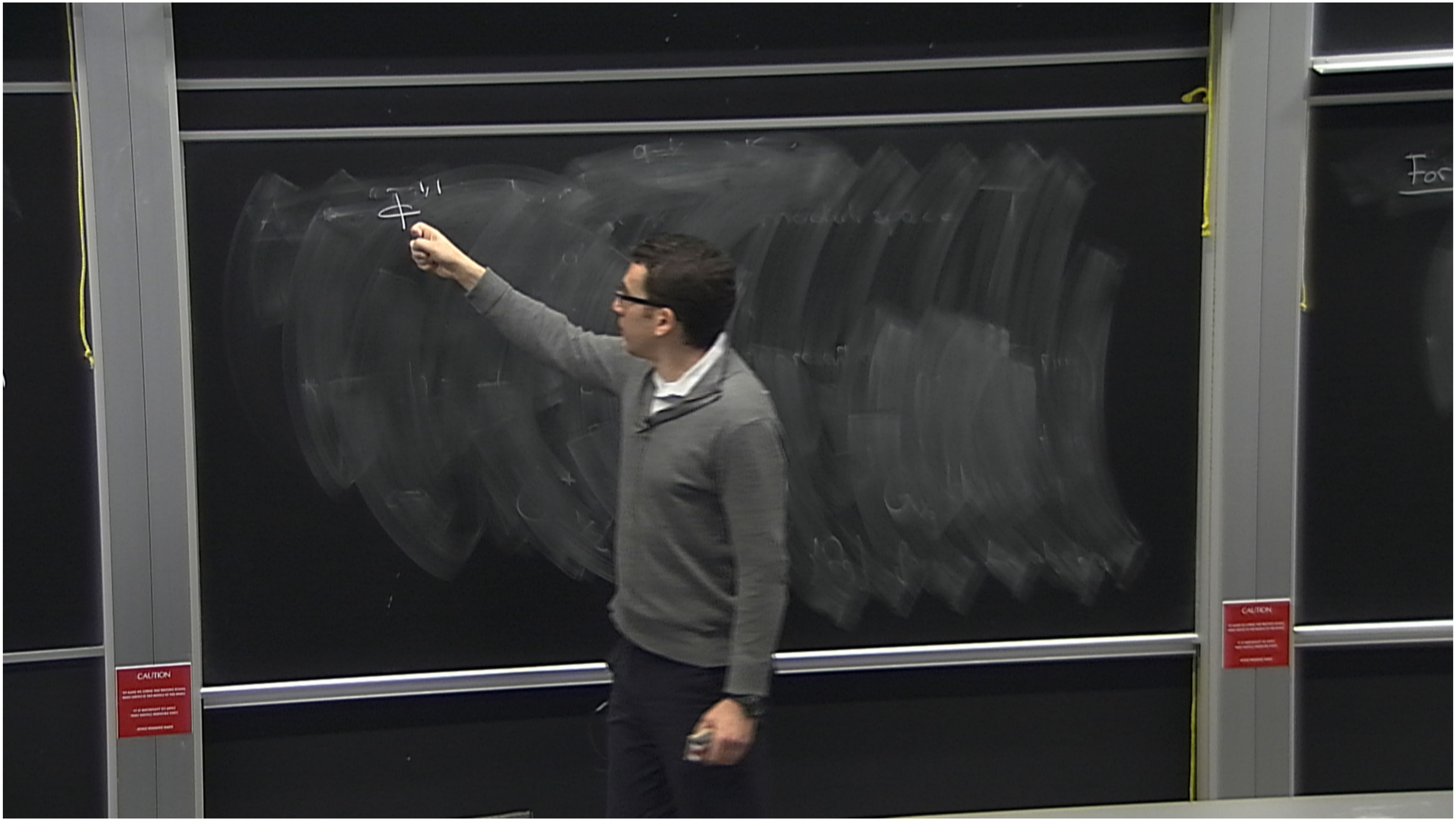

3. Geometric realization:
3.1) Generalities
φ, ψ : Σ 2d ⟶ X
CY-d-fold
φ^{q,q̄} ≡ H^{0,q̄}(X, Λ^q TX)

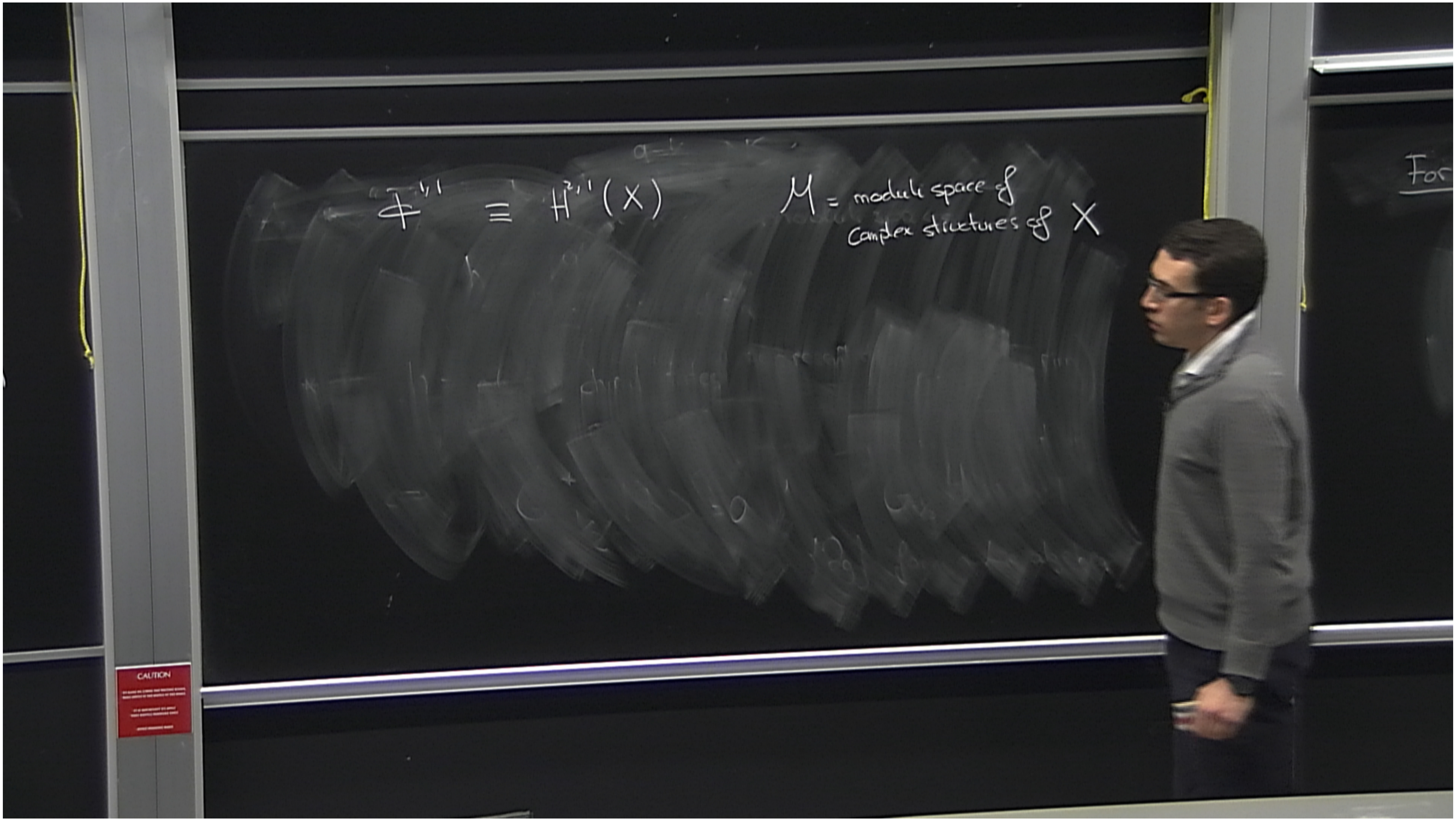

3. Geometric realization:
3.1)
∃ unique hol
$\Omega^{d,0}$
$\to X$
CY - d-fold
$H^{0,\bar{q}}(X, \Lambda^q TX)$
CAUTION

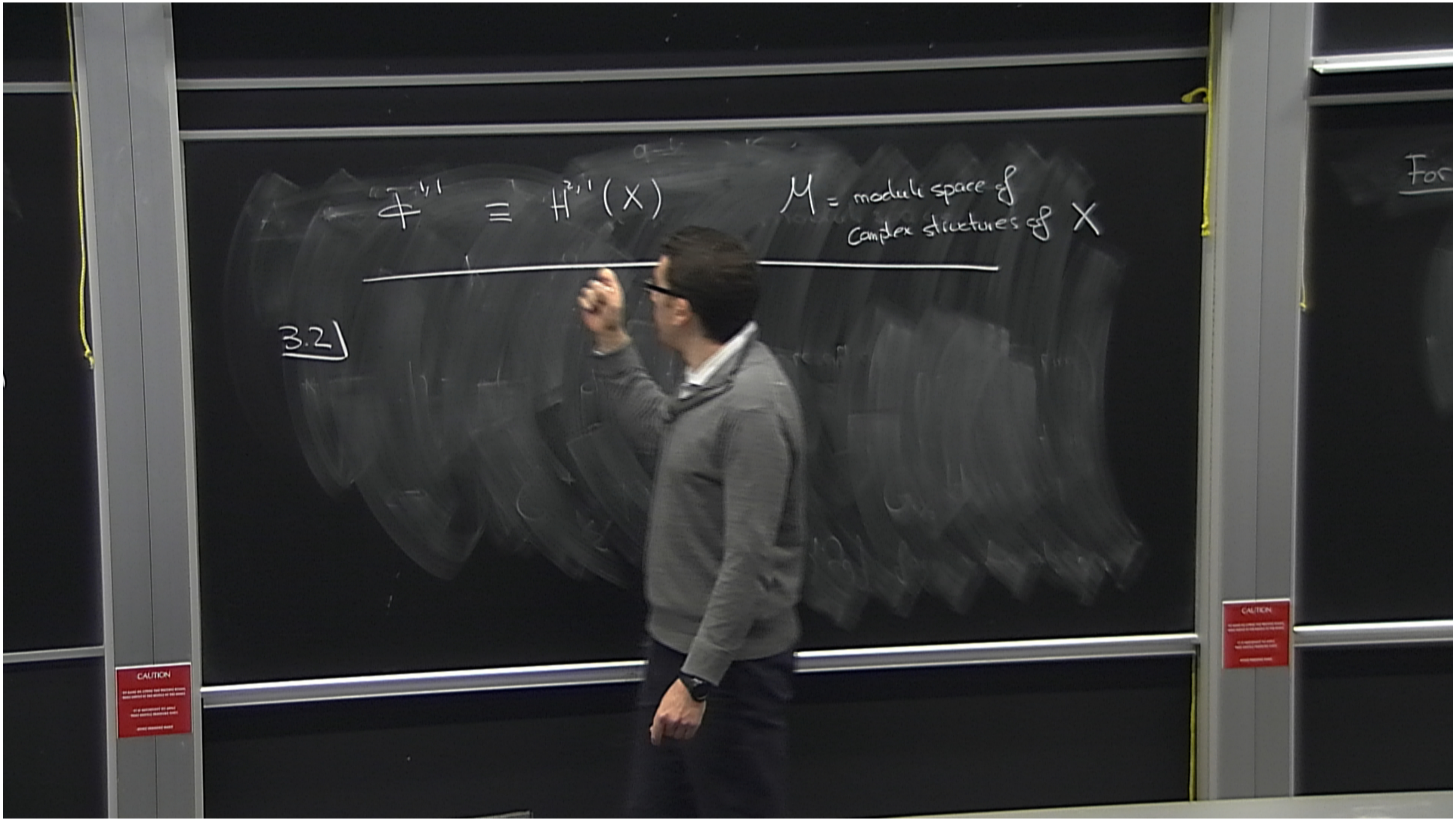

3. Geometric realization:
3.1) Generalities
∃ unique hol
Ω d,0
φ, ψ :
→ X
CY - d-fold
(X, Λ q TX)

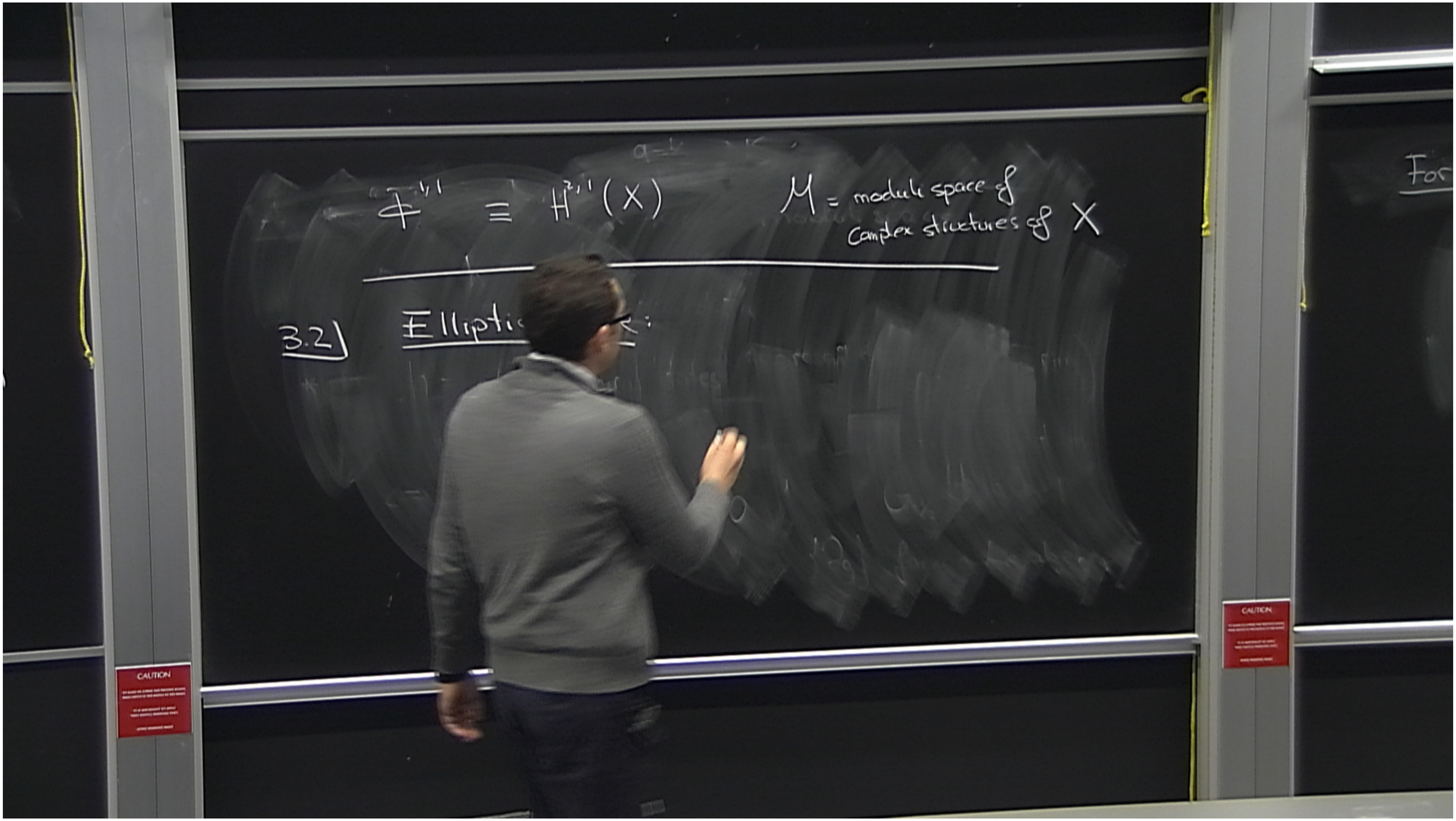

3. Geometric realization:
3.1) Generalities
$\phi, \psi : \Sigma_{2d} \longrightarrow X$
CY - d-fold
∃ unique hol
$\Omega^{d,0}$
$\phi^{q,\bar{q}} \equiv H^{0,\bar{q}}(X, \Lambda^q TX) \cong H^{d-q,\bar{q}}(X)$

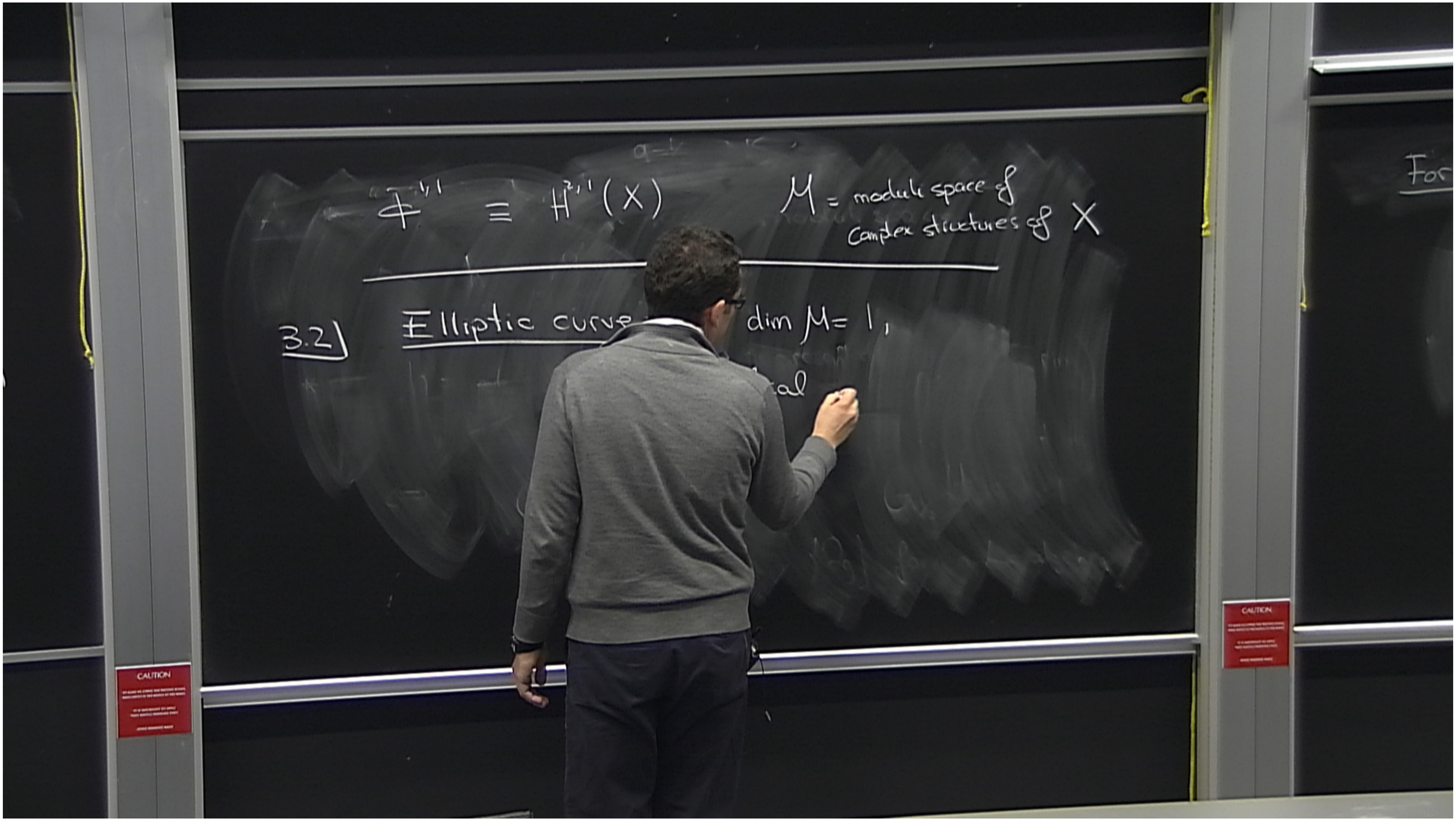

states
h = -q/2
|ϕ⟩ = 0
G⁻-½ |+⟩ = 0
CAUTION


CAUTION
CAUTION

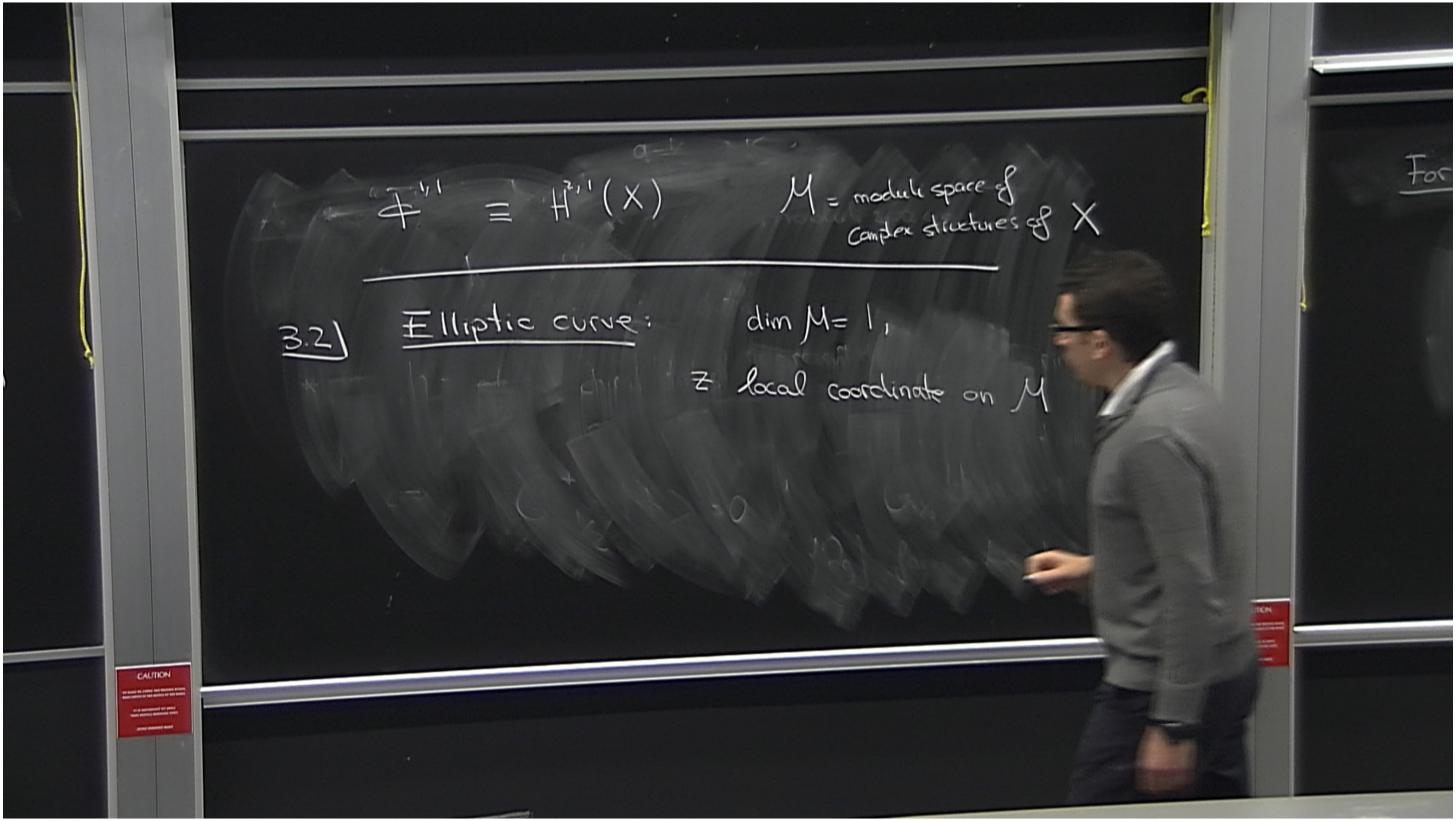

$\Phi^{1,1} = H^{2,1}(X)$
$\mathcal{M}$ = moduli space of complex structures of X
For

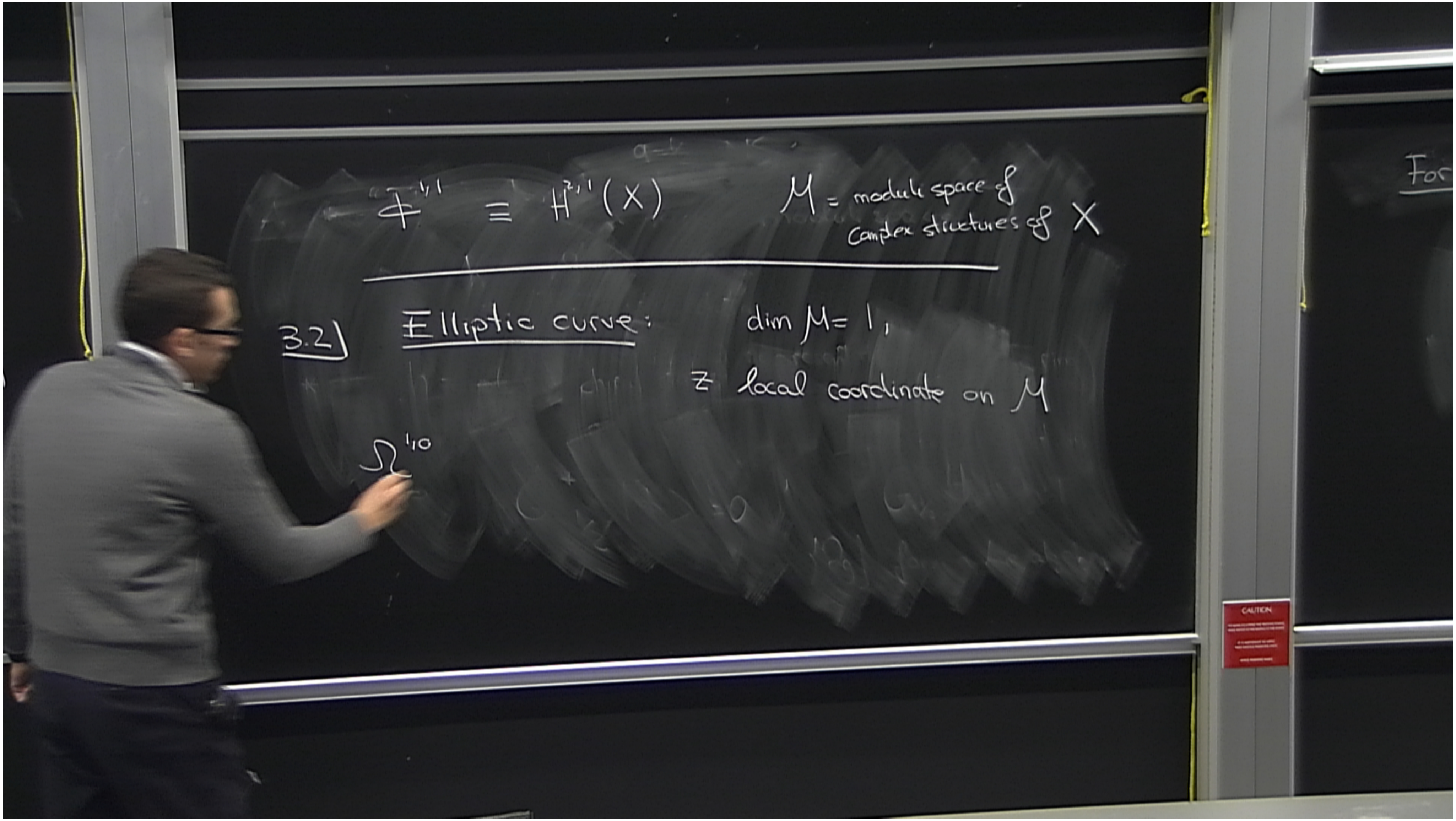

Φ^{1,1} = H^{2,1}(X)
M = moduli space of complex structures of X
3.2)
For
CAUTION

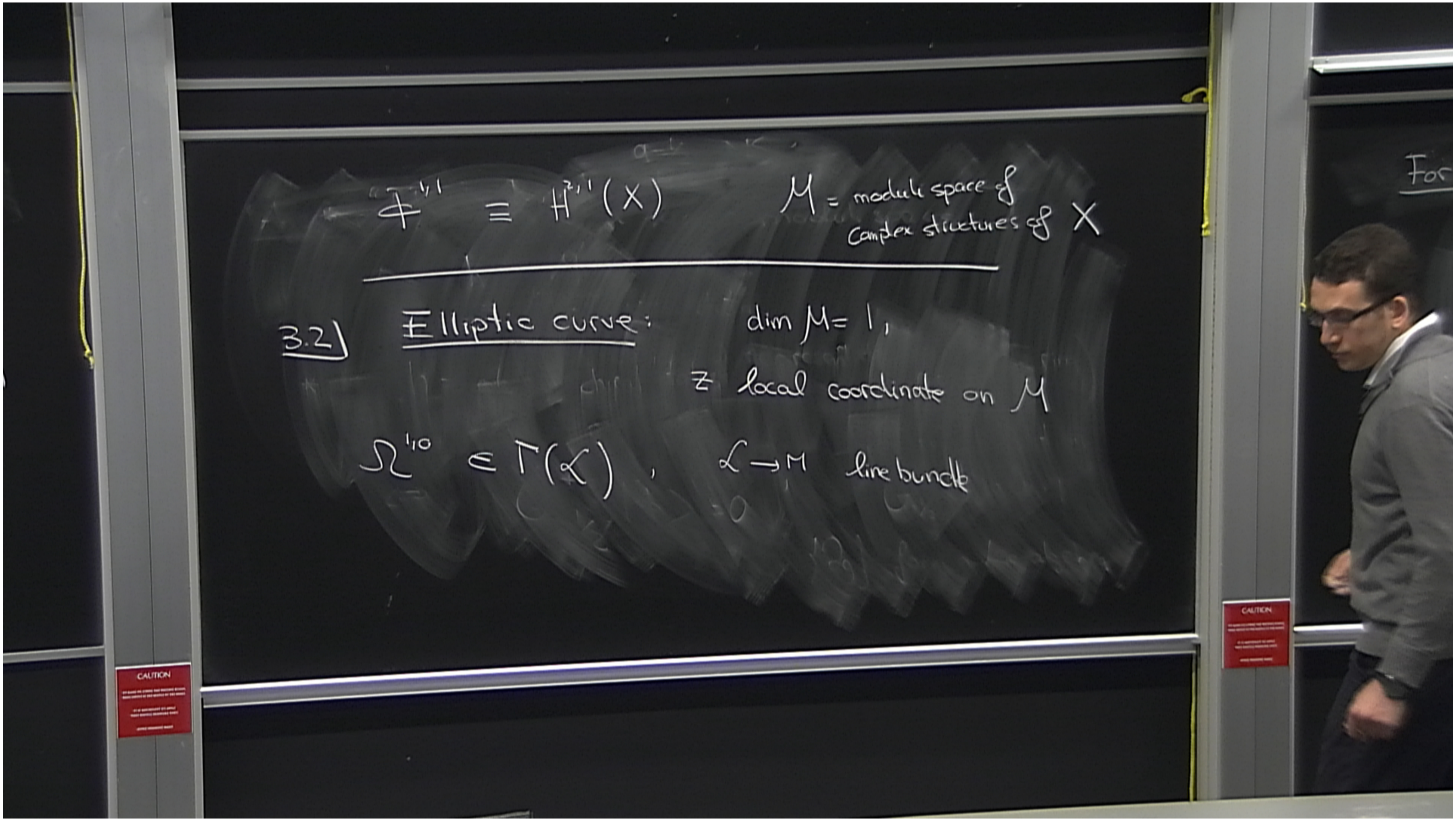

Φ^{1,1} = H^{2,1}(X)
M = moduli space of
complex structures of X
3.2) Ellipti

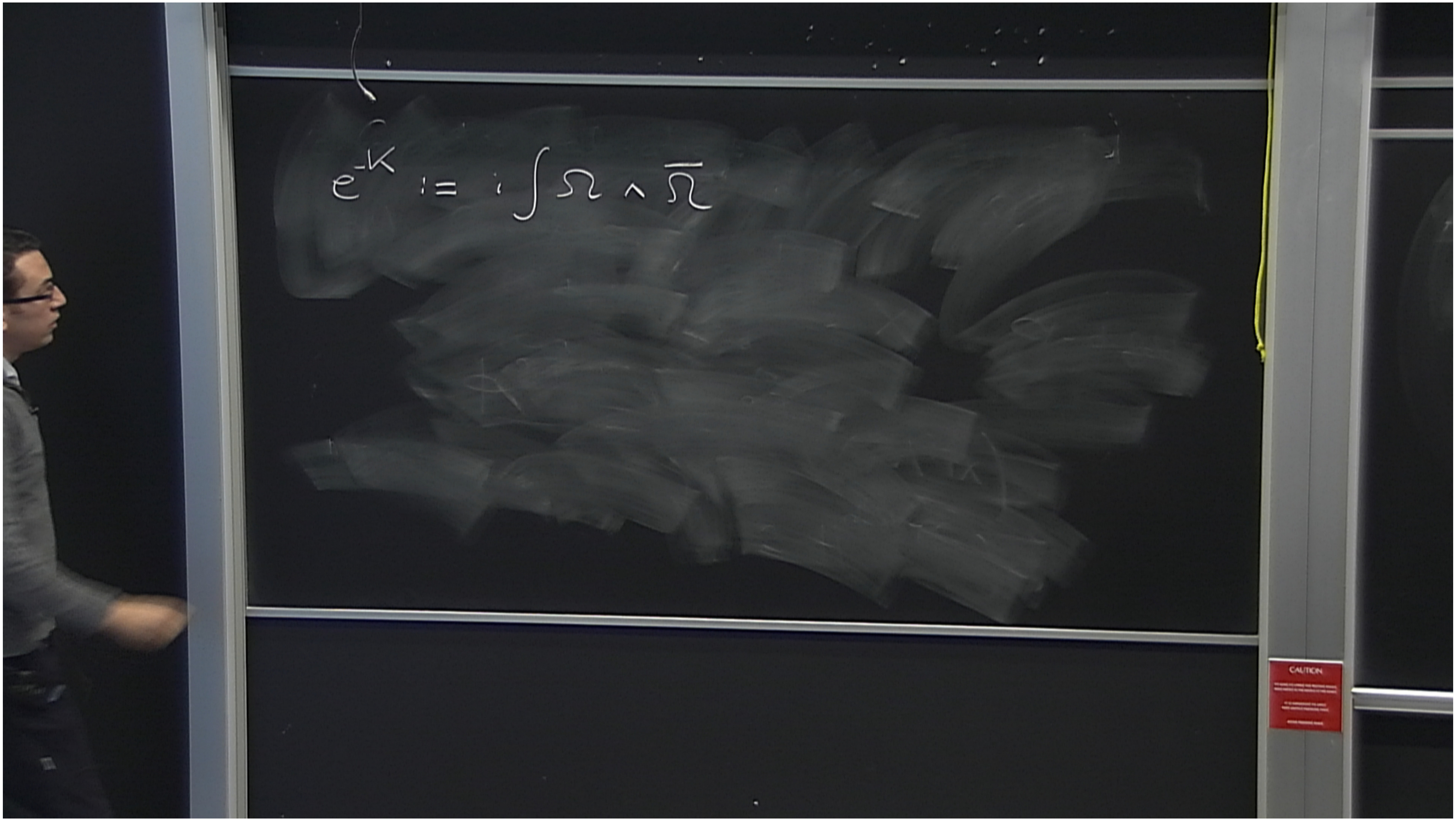


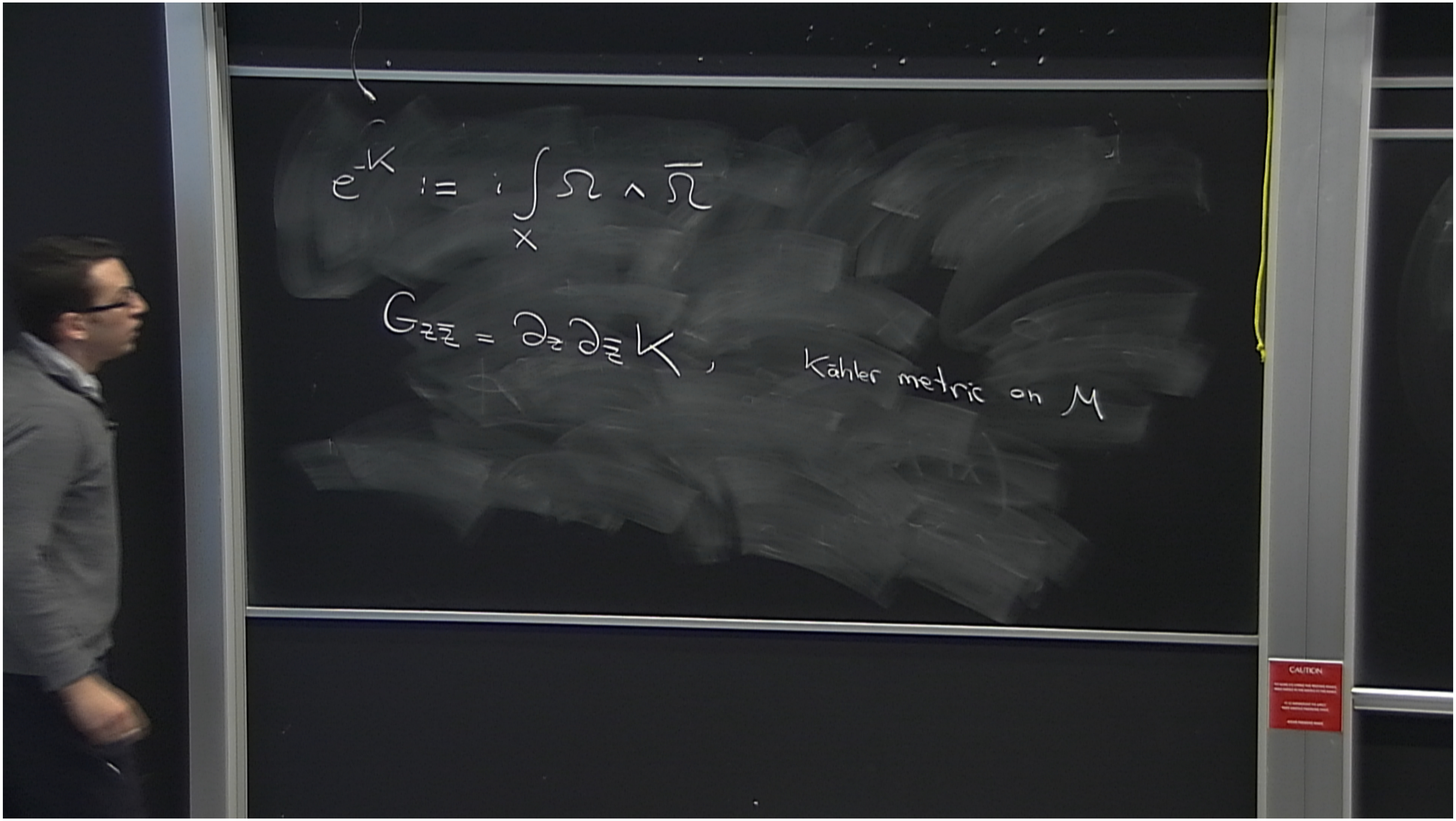
Φ^{1,1} = H^{2,1}(X)
M = moduli space of complex structures of X
3.2) Elliptic curve
dim M = 1,
CAUTION
CAUTION

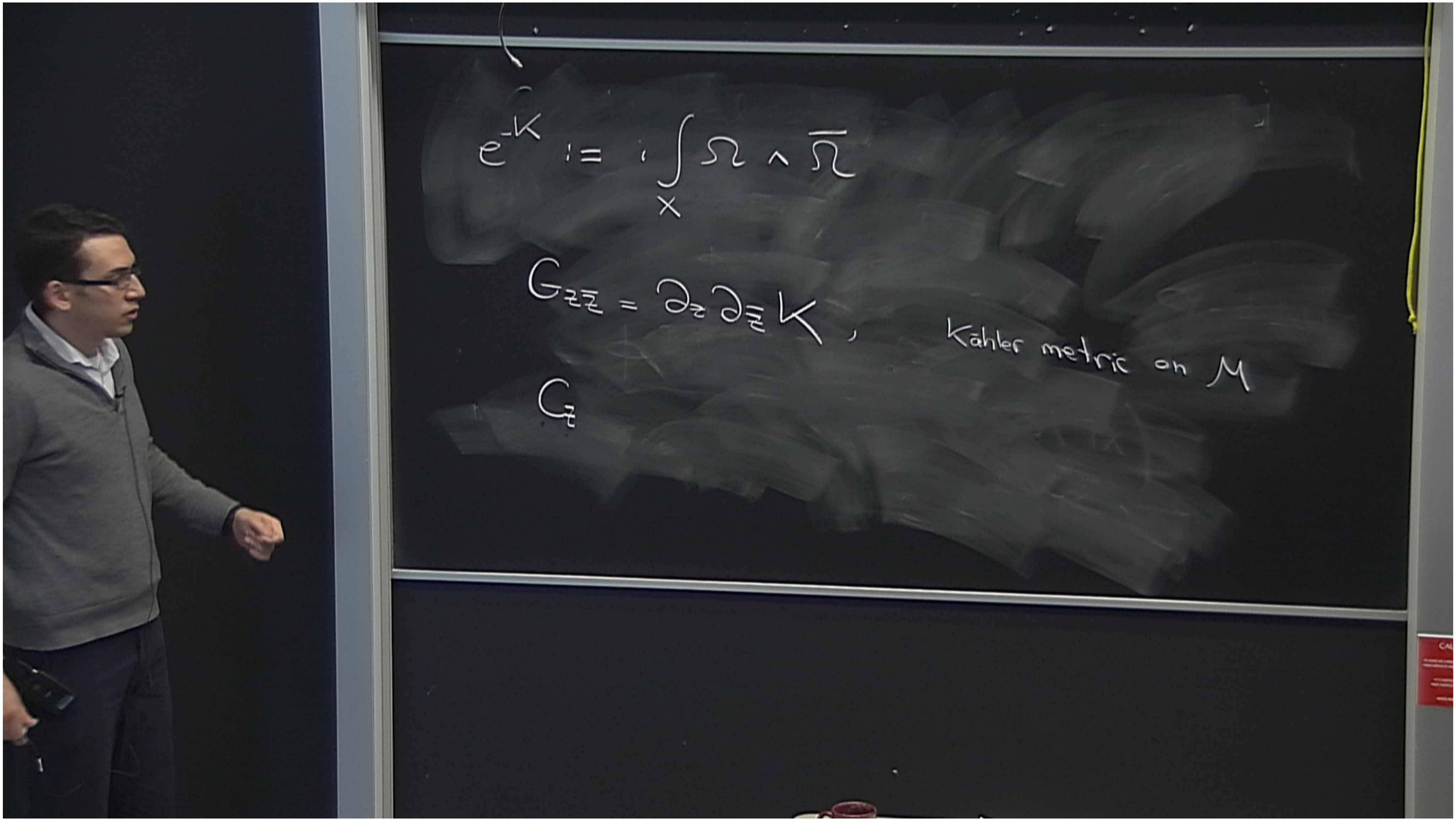

Φ^{1,1} = H^{2,1}(X)
M = moduli space of complex structures of X
3.2) Elliptic curve:
dim M = 1,
z local coordinate on M
CAUTION
For

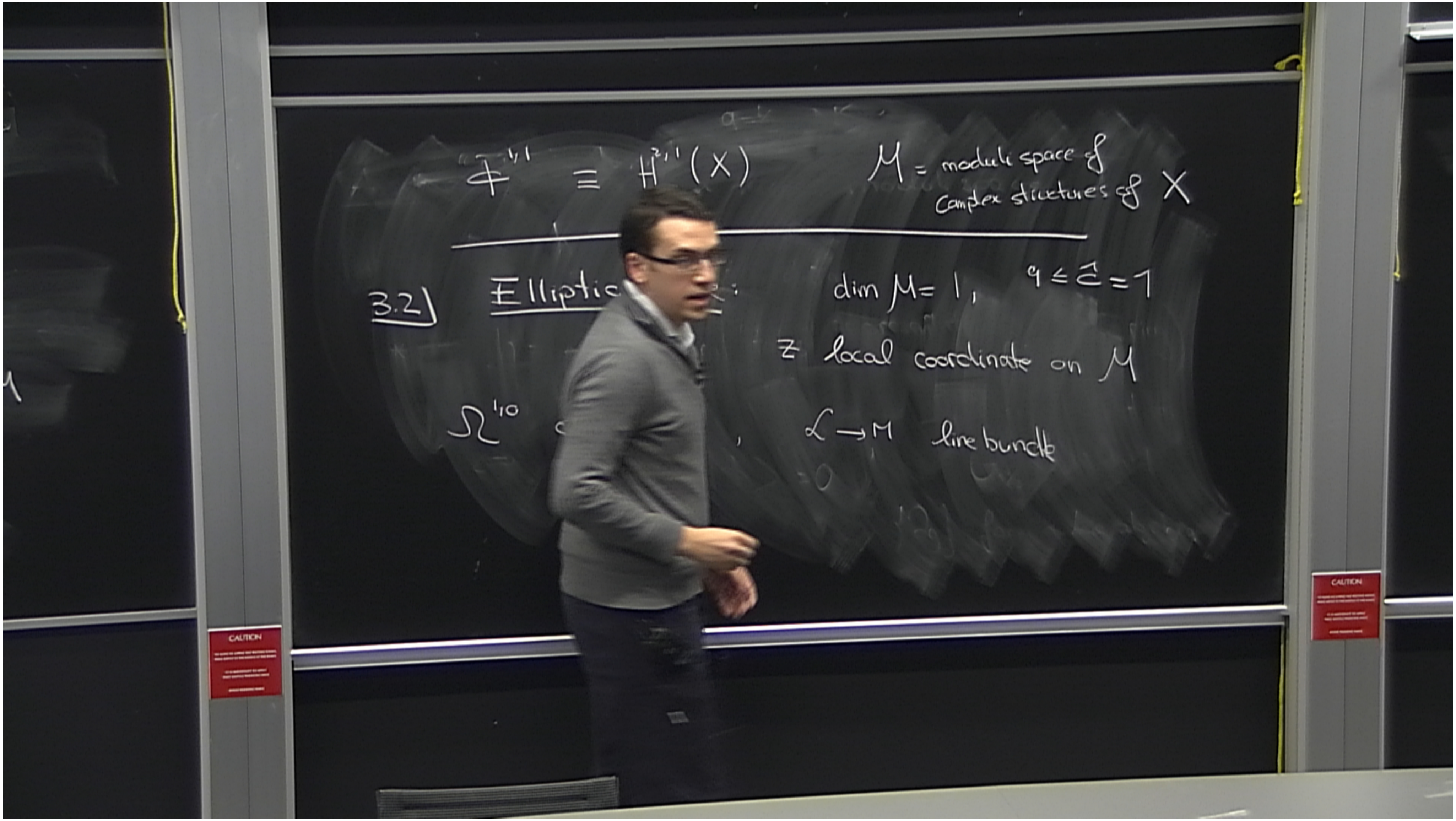

Φ^{2,1} = H^{2,1}(X)
M = moduli space of complex structures of X
3.2) Elliptic curve:
dim M = 1,
z local coordinate on M
Ω^{1,0}
For

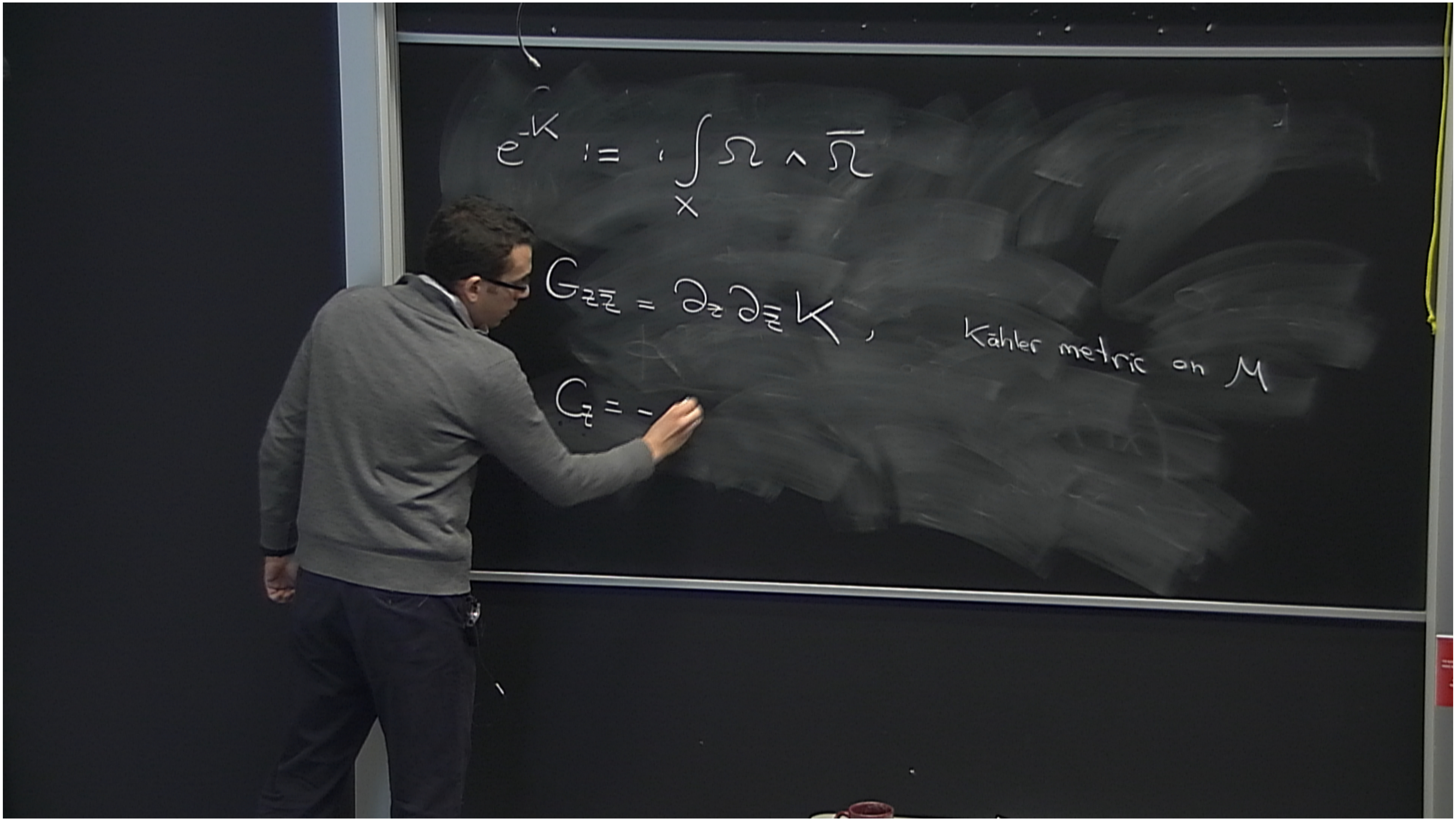

$\Phi^{1,1} \cong H^{2,1}(X)$
$\mathcal{M}$ = moduli space of complex structures of X
3.2) Elliptic curve:
dim $\mathcal{M}$ = 1,
z local coordinate on $\mathcal{M}$
$\Omega^{1,0} \in \Gamma(\mathcal{L})$,
$\mathcal{L} \to M$ line bundle
For

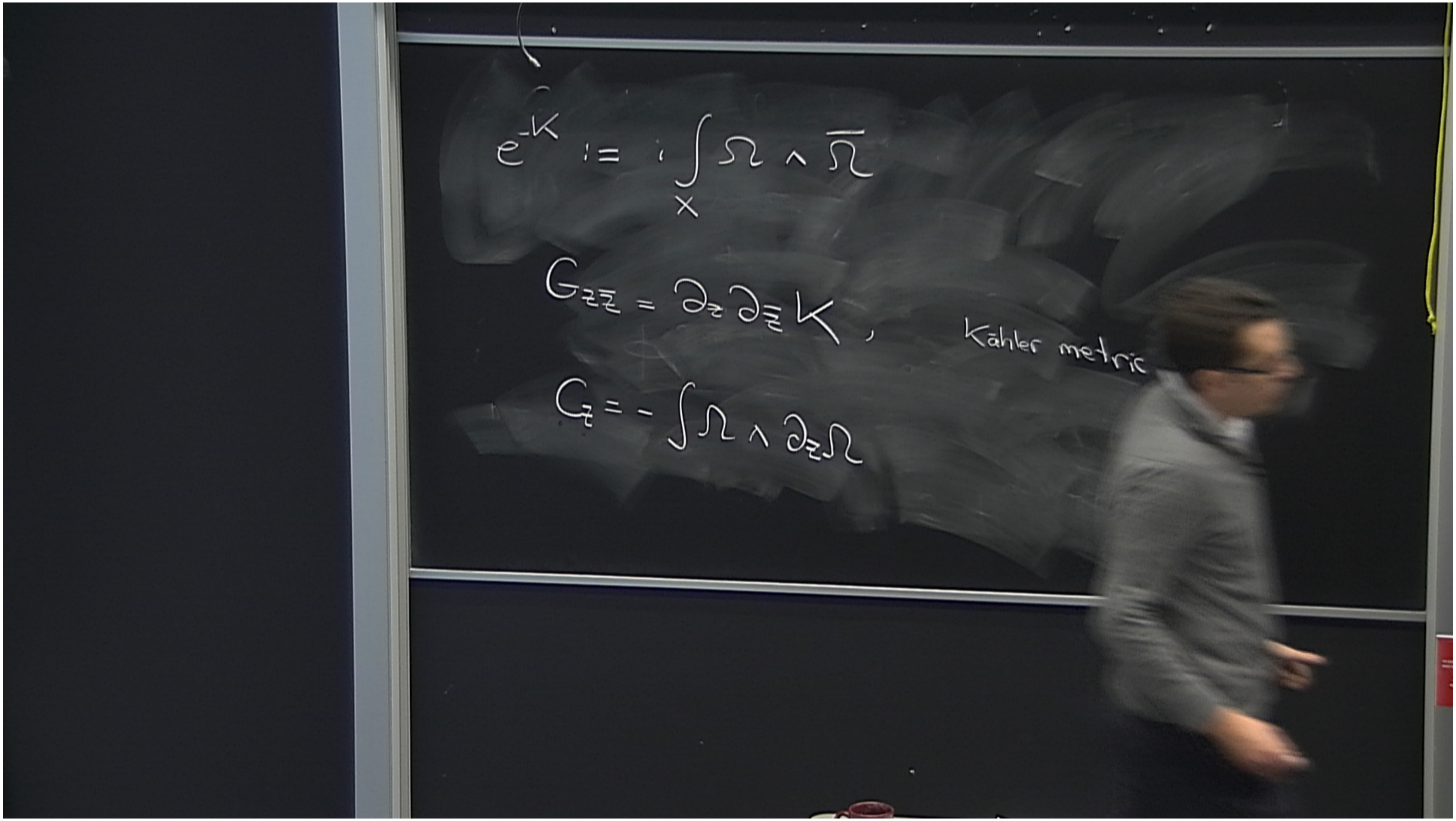

$e^{-K} := i \int \Omega \wedge \bar{\Omega}$

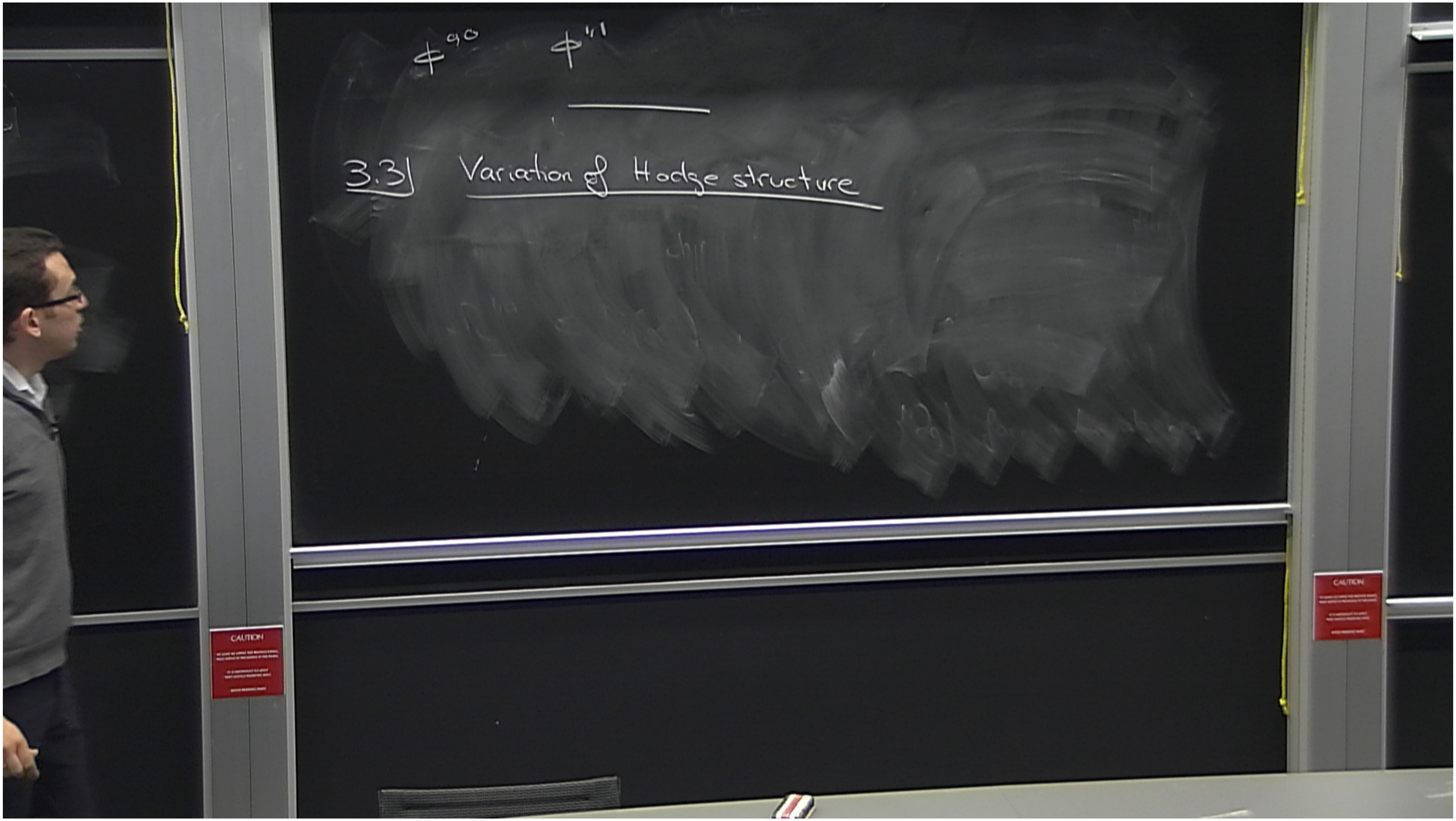

$e^{-K} := i \int_X \Omega \wedge \bar{\Omega}$
$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K$, Kähler metric on $\mathcal{M}$

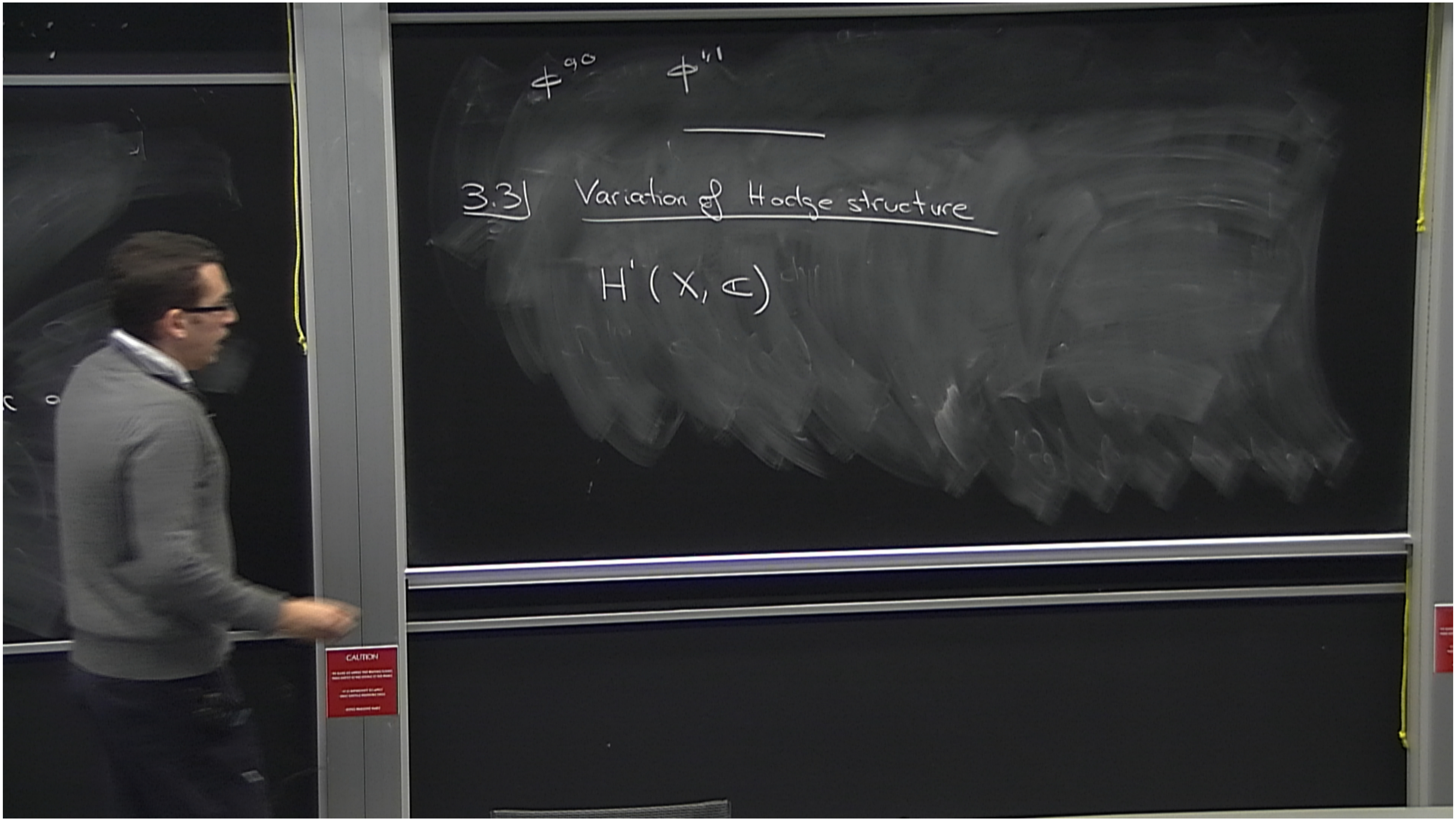

$e^{-K} := i \int_X \Omega \wedge \overline{\Omega}$
$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K$,
Kähler metric on $\mathcal{M}$
G_z

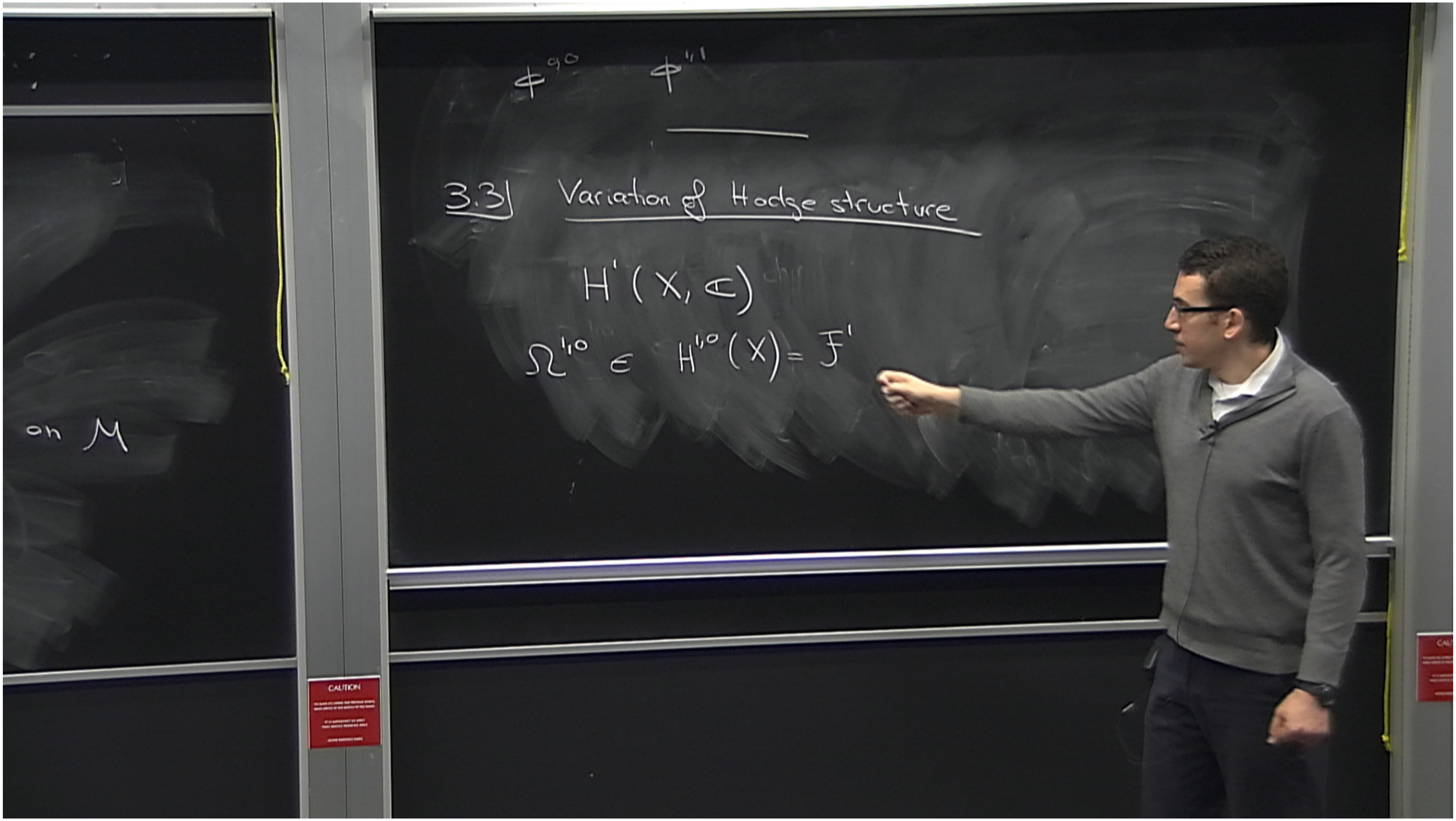

M = moduli space of complex structures of X
3.2) Ellipti
dim M = 1,
z local coordinate on M
L → M line bundle
CAUTION

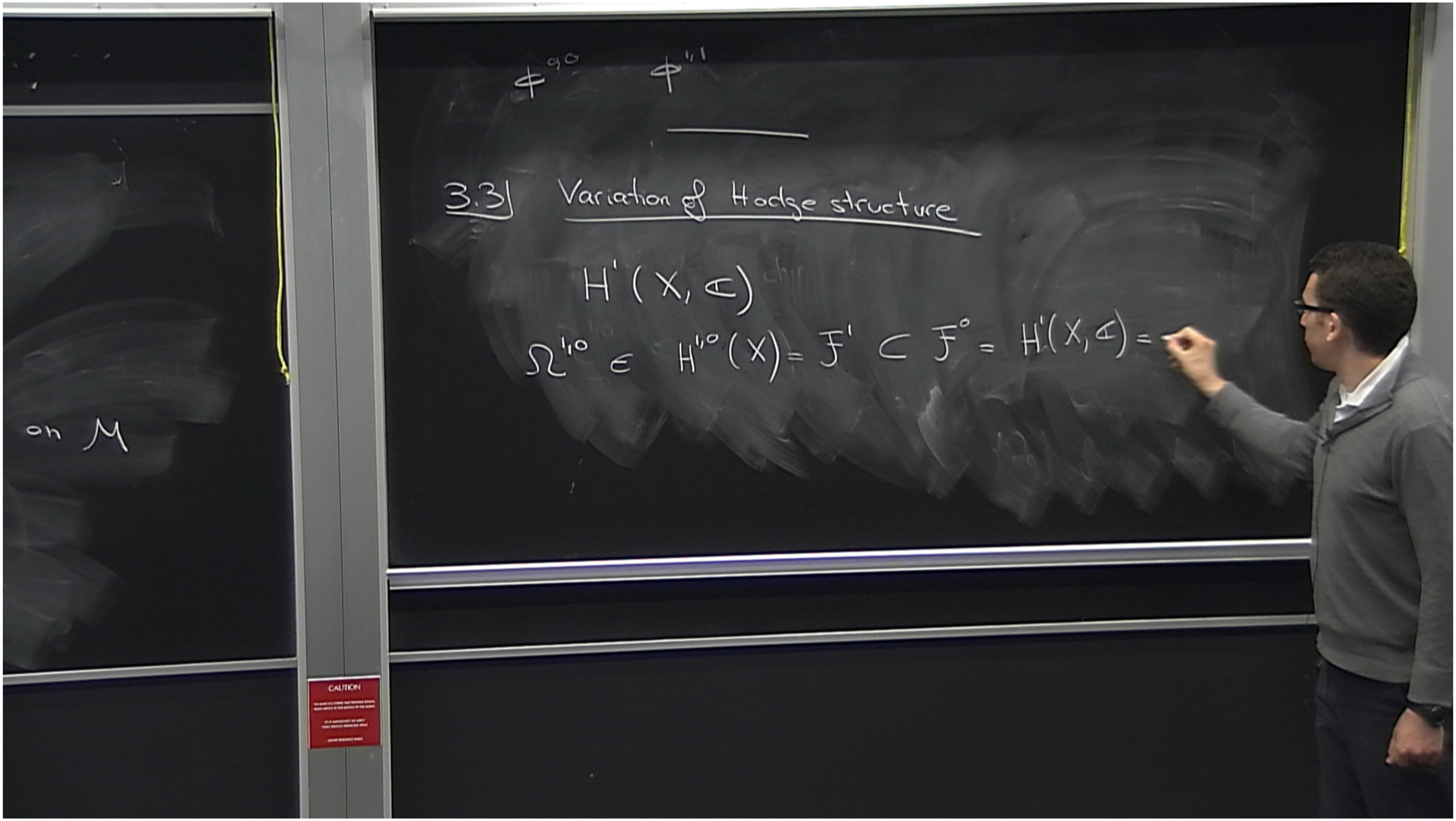

$e^{-K} := i \int_X \Omega \wedge \bar{\Omega}$
$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K$,
Kähler metric on $\mathcal{M}$
$G_z = -$

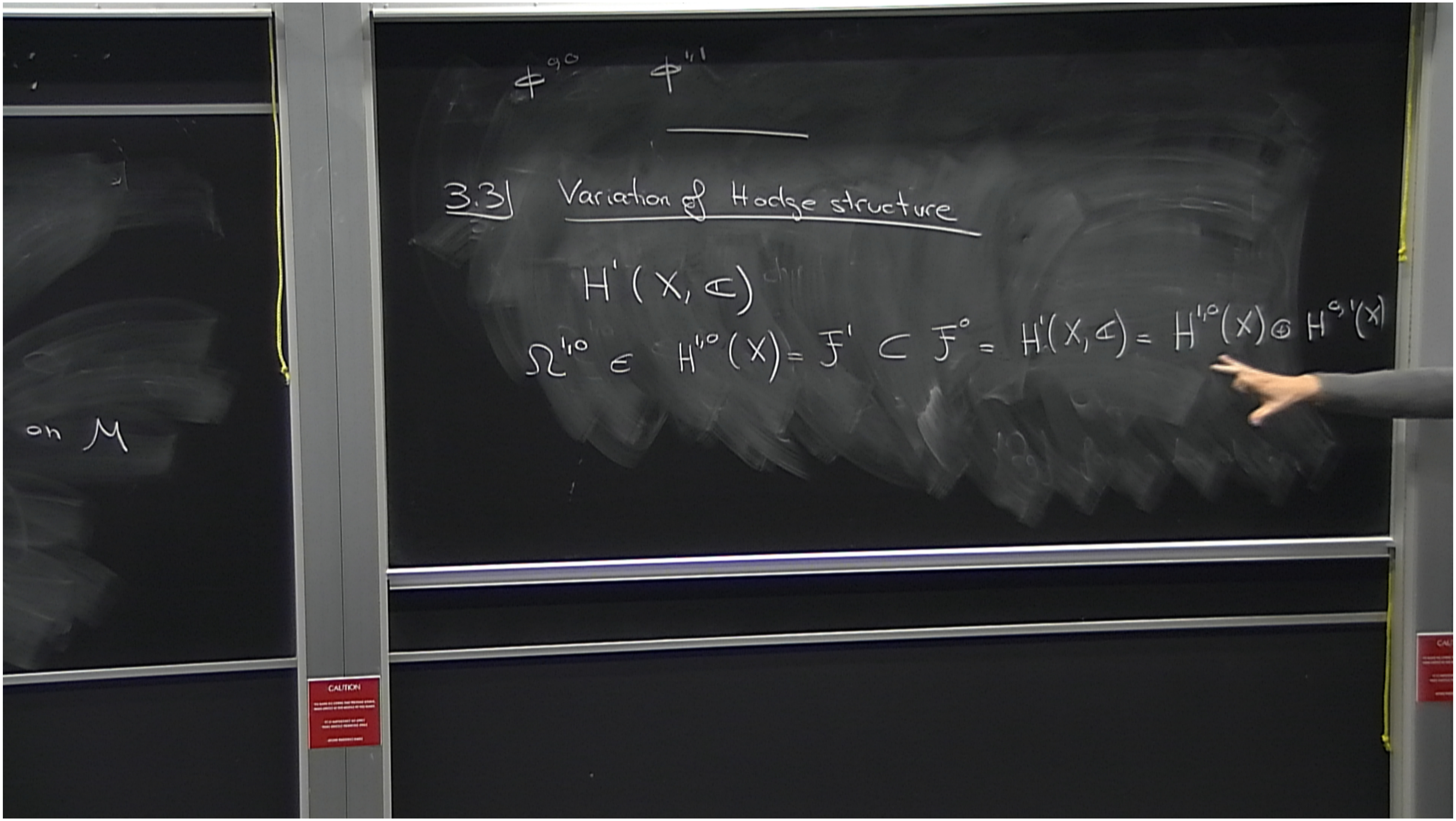

$e^{-K} := i\int_X \Omega \wedge \bar{\Omega}$
$G_{z\bar{z}} = \partial_z \partial_{\bar{z}} K$, Kähler metric
$G_z = -\int \Omega \wedge \partial_z \Omega$

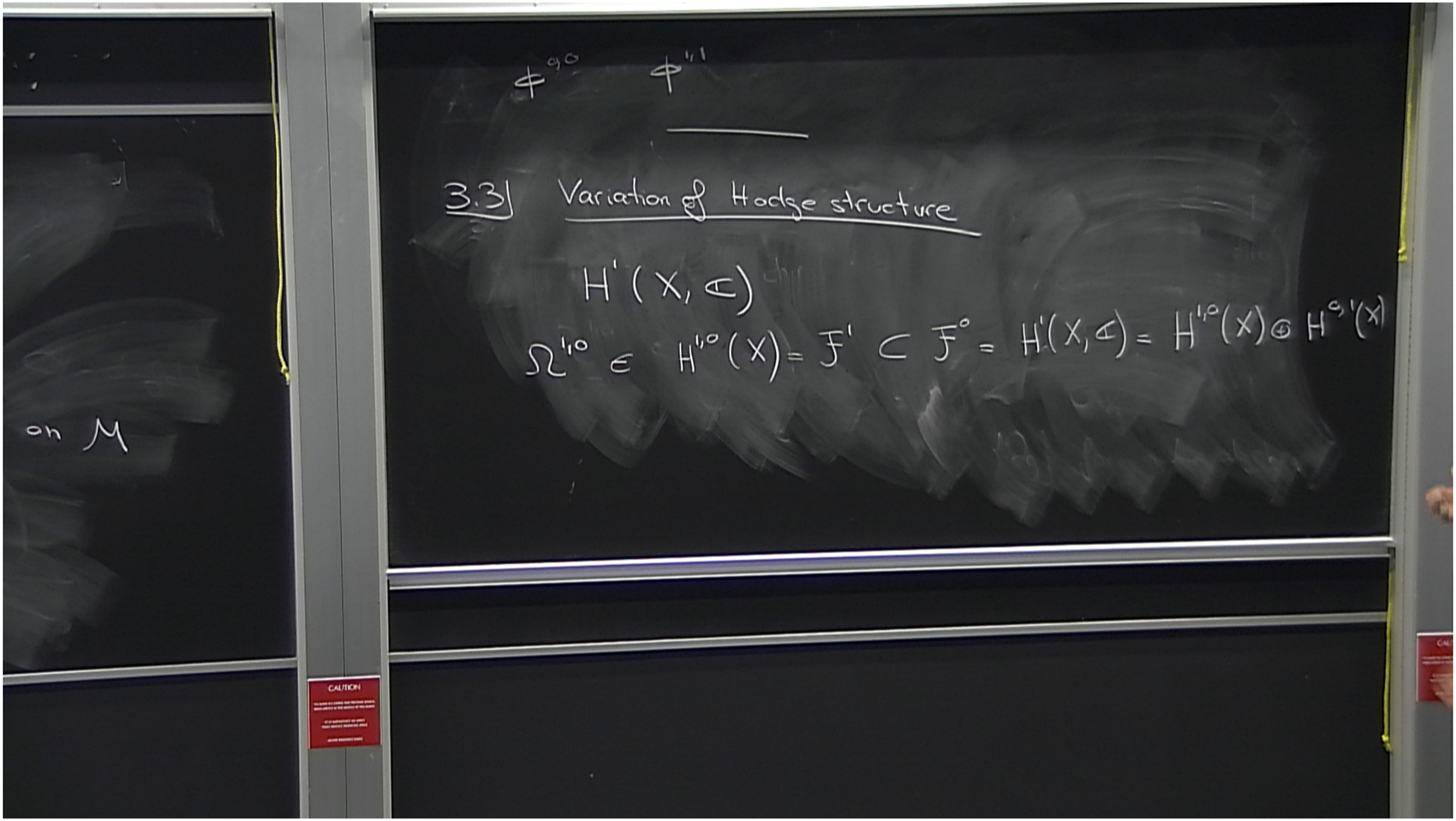

$\Phi^{2,0}$ $\Phi^{1,1}$
3.3| Variation of Hodge structure
CAUTION
CAUTION

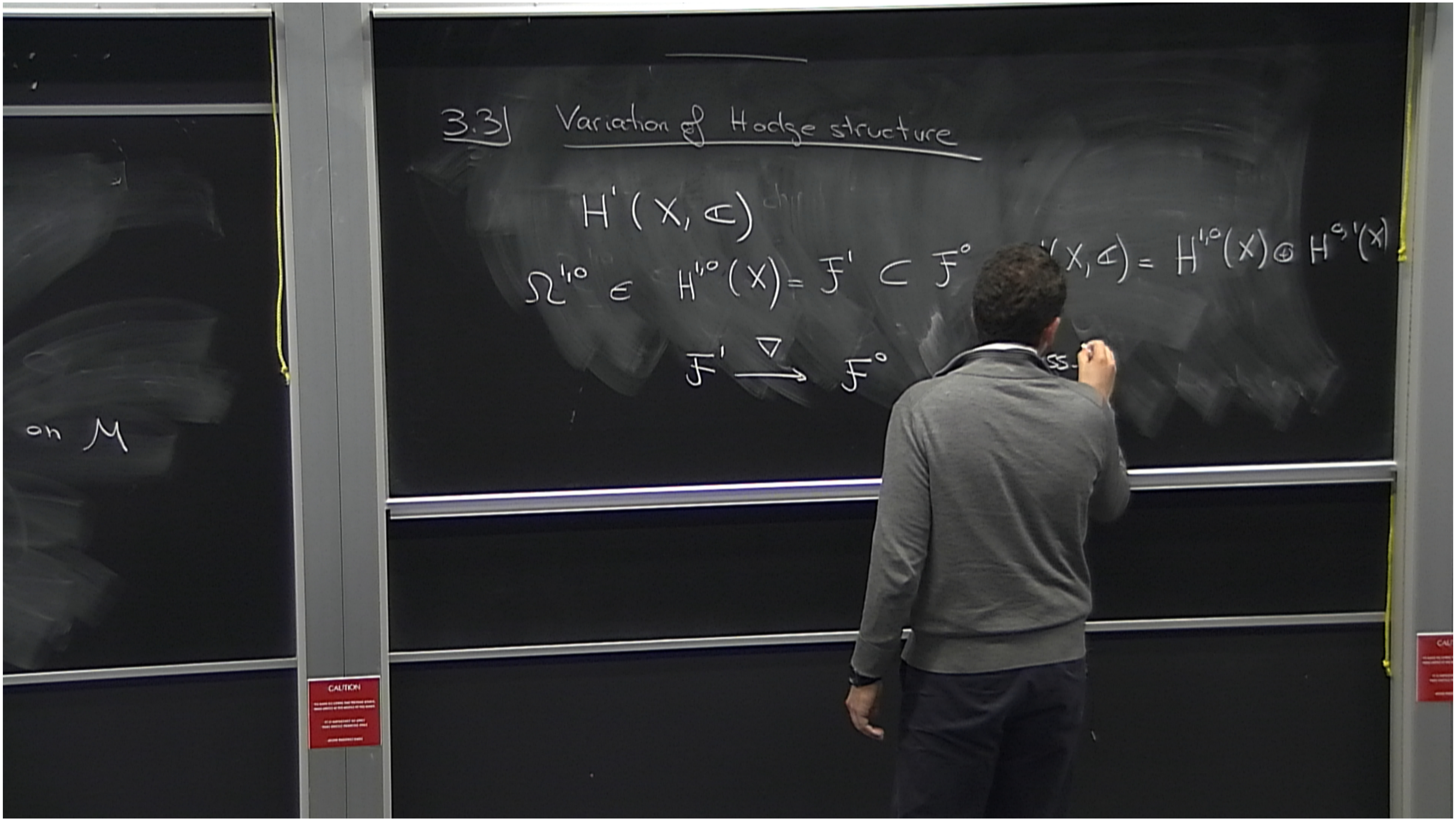

3.3) Variation of Hodge structure
H¹(X, ℂ)

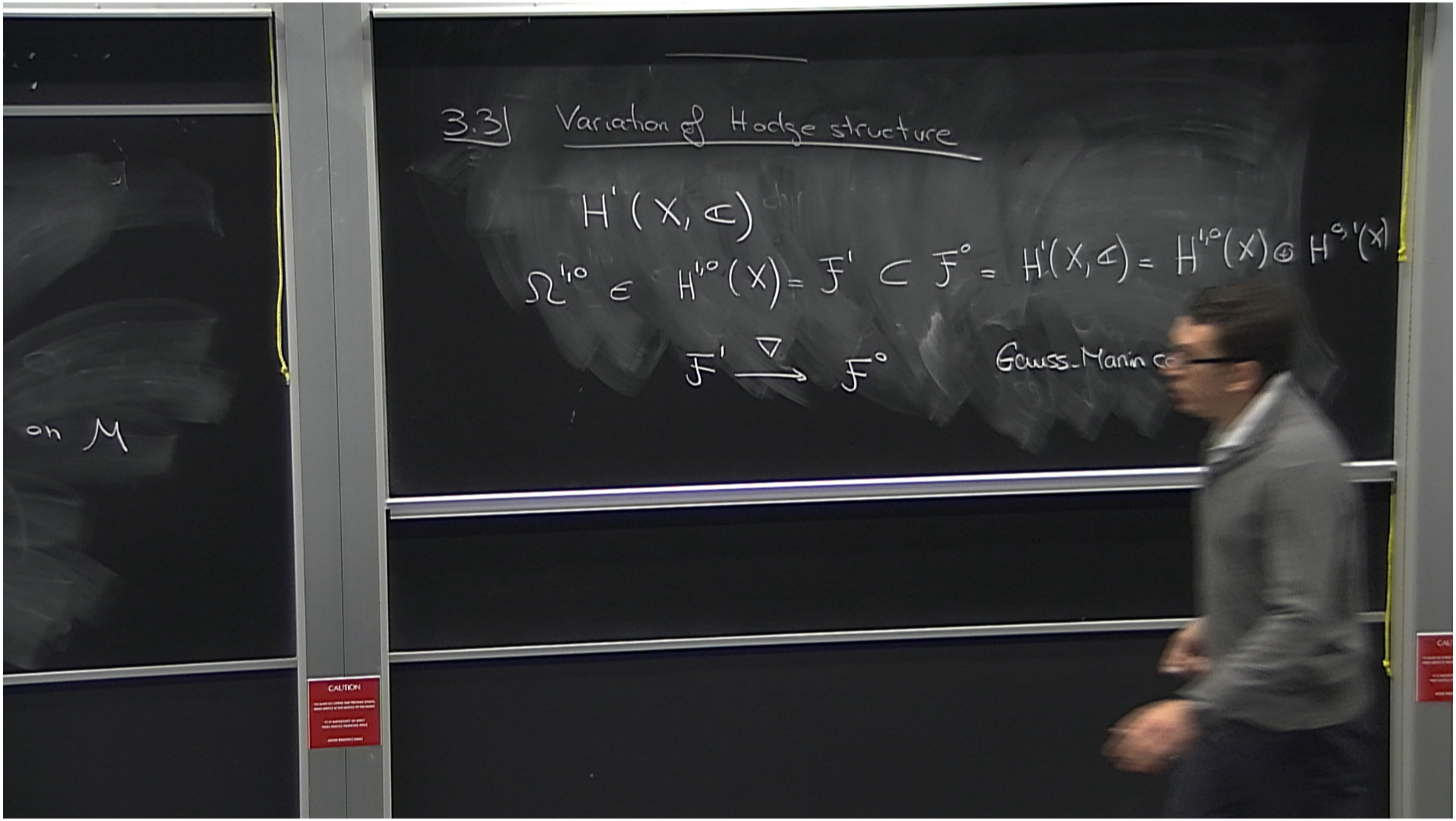

3.3| Variation of Hodge structure
H'(X, ℂ)
Ω^{1,0} ∈ H^{1,0}(X) = F'
on M

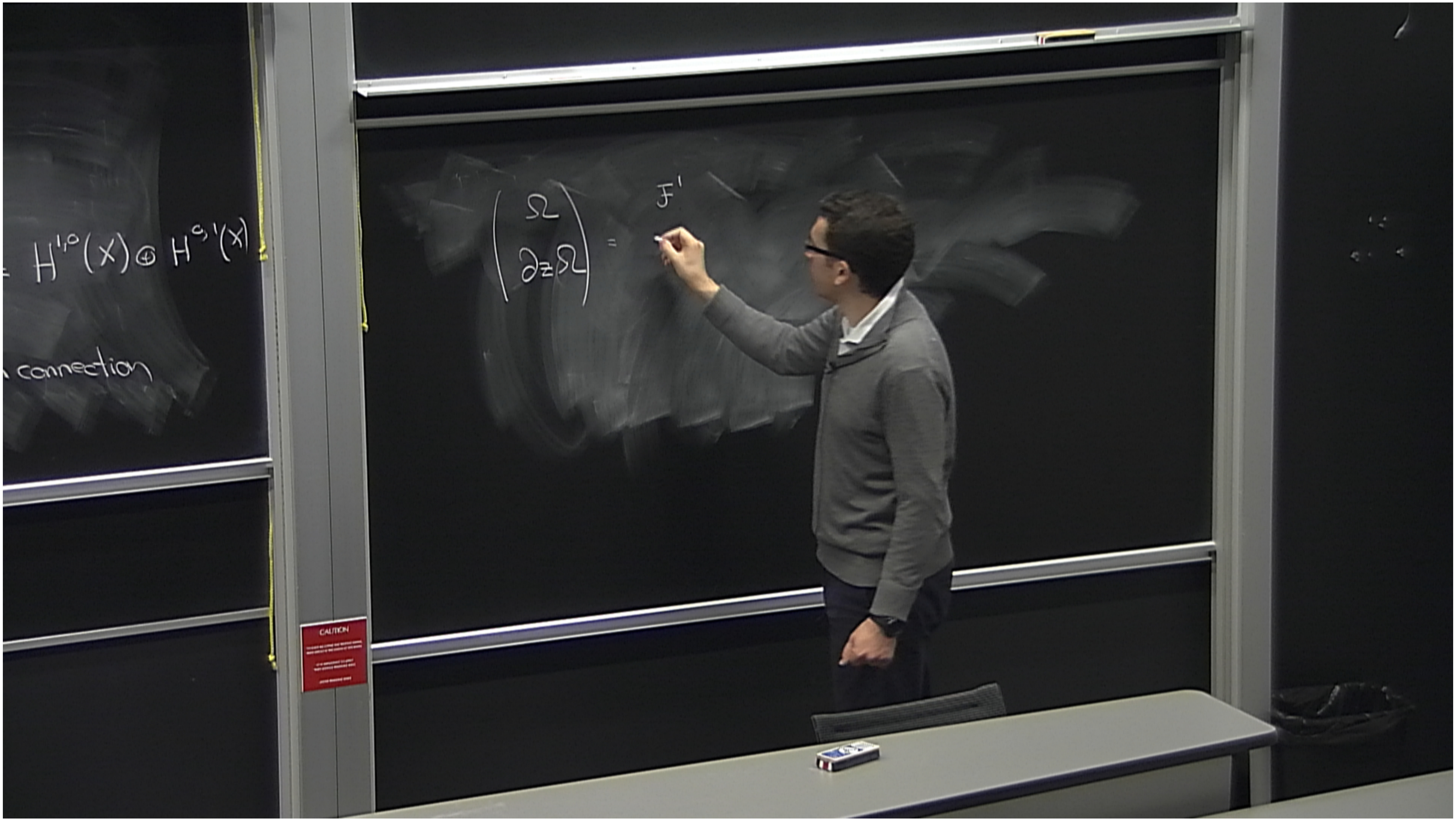

3.3 Variation of Hodge structure
on M
CAUTION

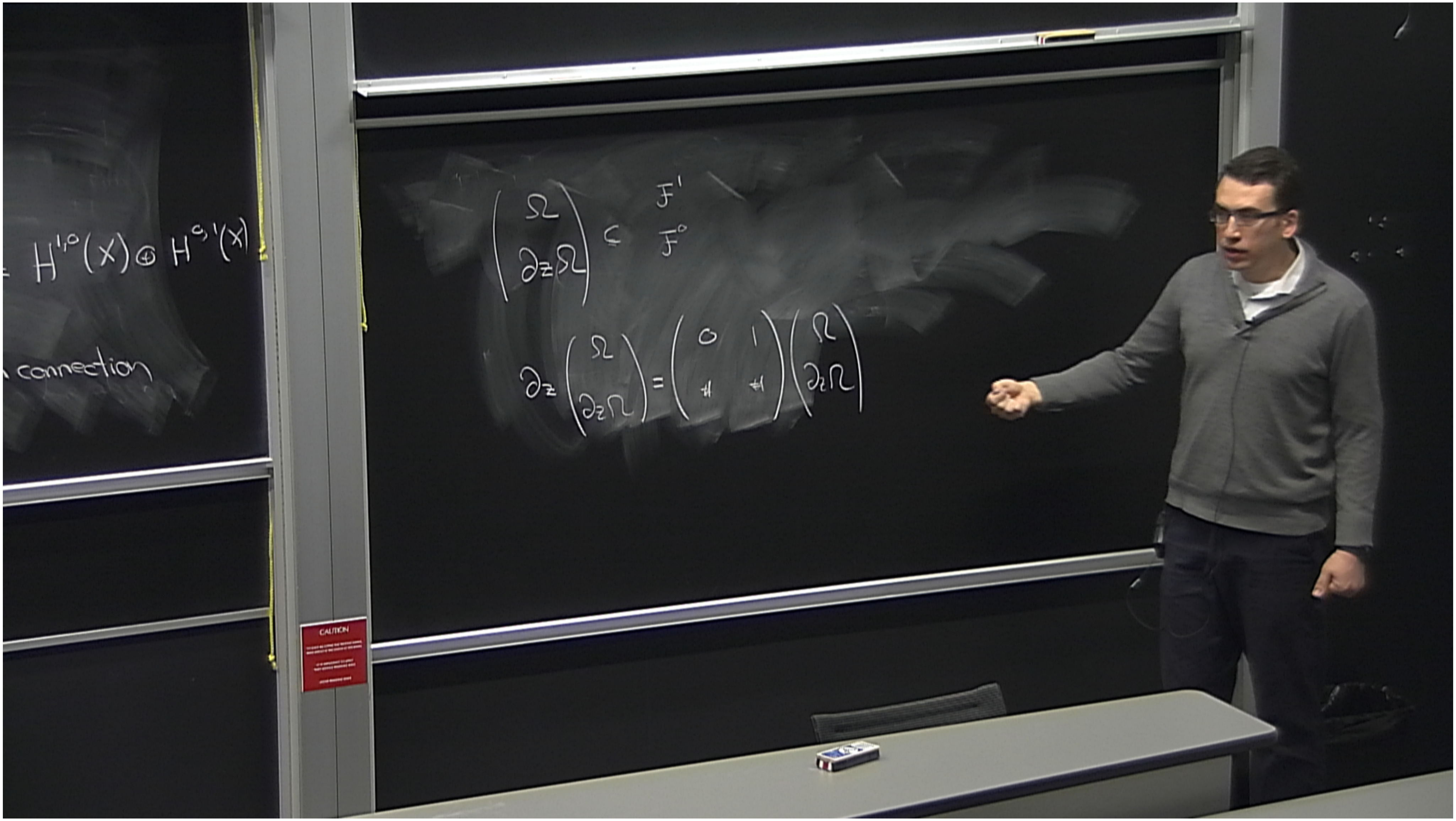

φ^{0,0} φ^{1,1}
3.3 Variation of Hodge structure
H^1(X, ℂ)
Ω^{1,0} ∈ H^{1,0}(X) = F^1 ⊂ F^0 = H^1(X, ℂ) = H^{1,0}(X) ⊕ H^{0,1}(X)
on M
CAUTION


on M
3.3| Variation of Hodge structure
H'(X, C)
Ω^{1,0} ∈ H^{1,0}(X) = F' ⊂ F° = H'(X, C) = H^{1,0}(X) ⊕ H^{0,1}(X)
CAUTION

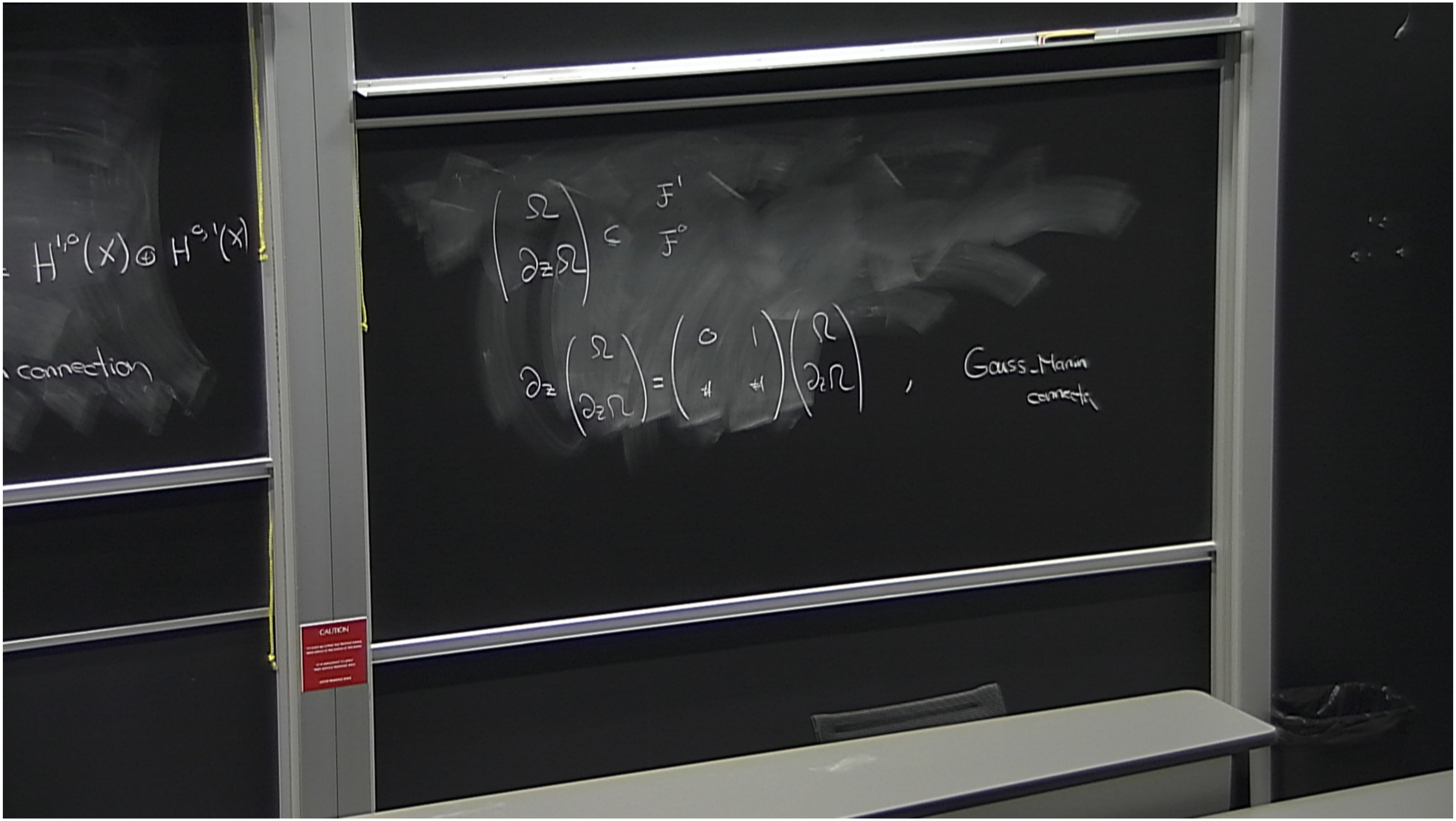

3.3| Variation of Hodge structure
H'(X, C)
Ω^1,0 ∈ H^1,0(X) = F' ⊂ F°
(X, C) = H^1,0(X) ⊕ H^0,1(X)
F' ∇ F°
on M

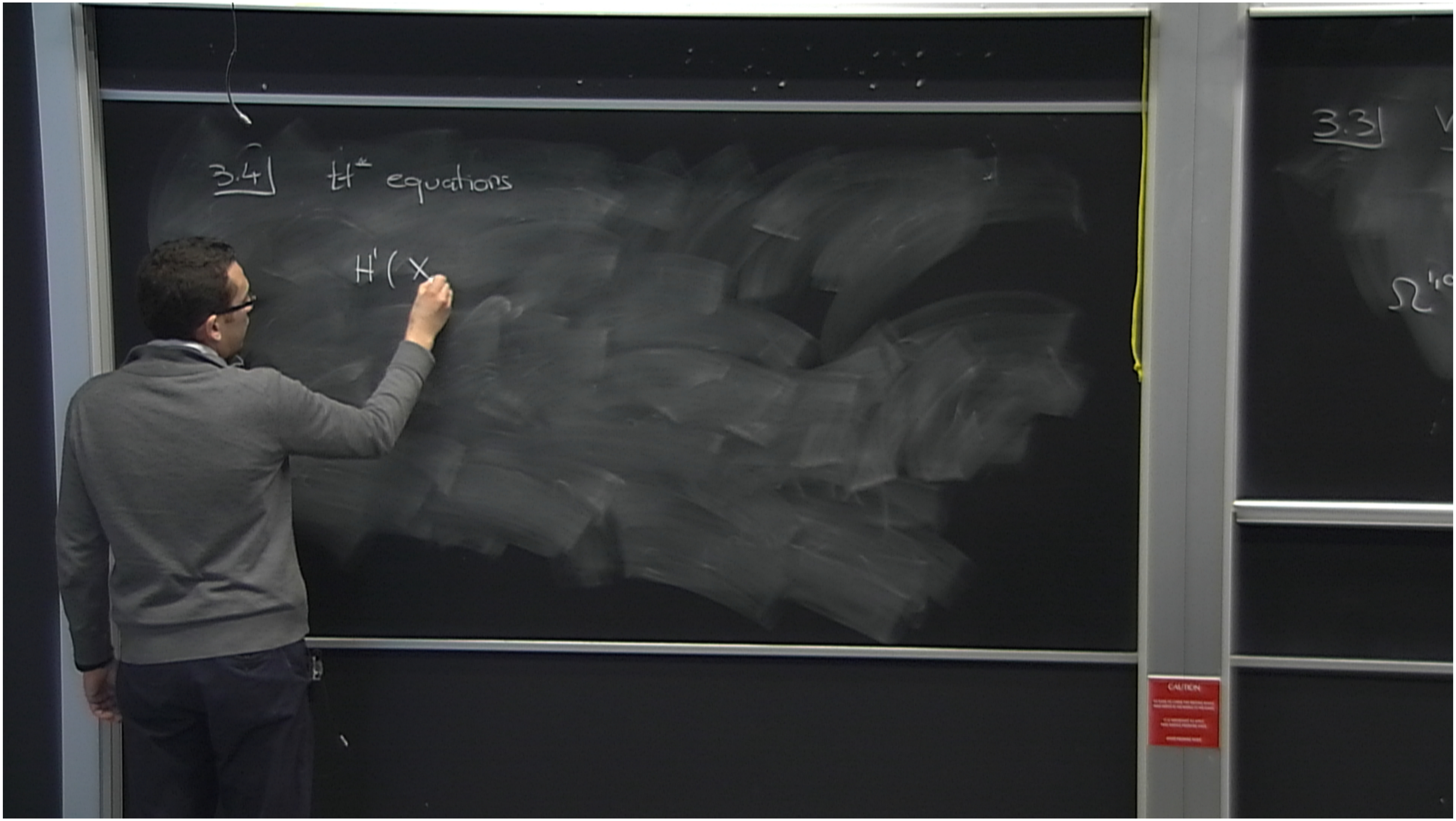

on M
3.3) Variation of Hodge structure
H'(X, ℂ)
Ω^{1,0} ∈ H^{1,0}(X) = F' ⊂ F° = H'(X, ℂ) = H^{1,0}(X) ⊕ H^{0,1}(X)
F' ∇→ F°
Gauss-Marin
CAUTION

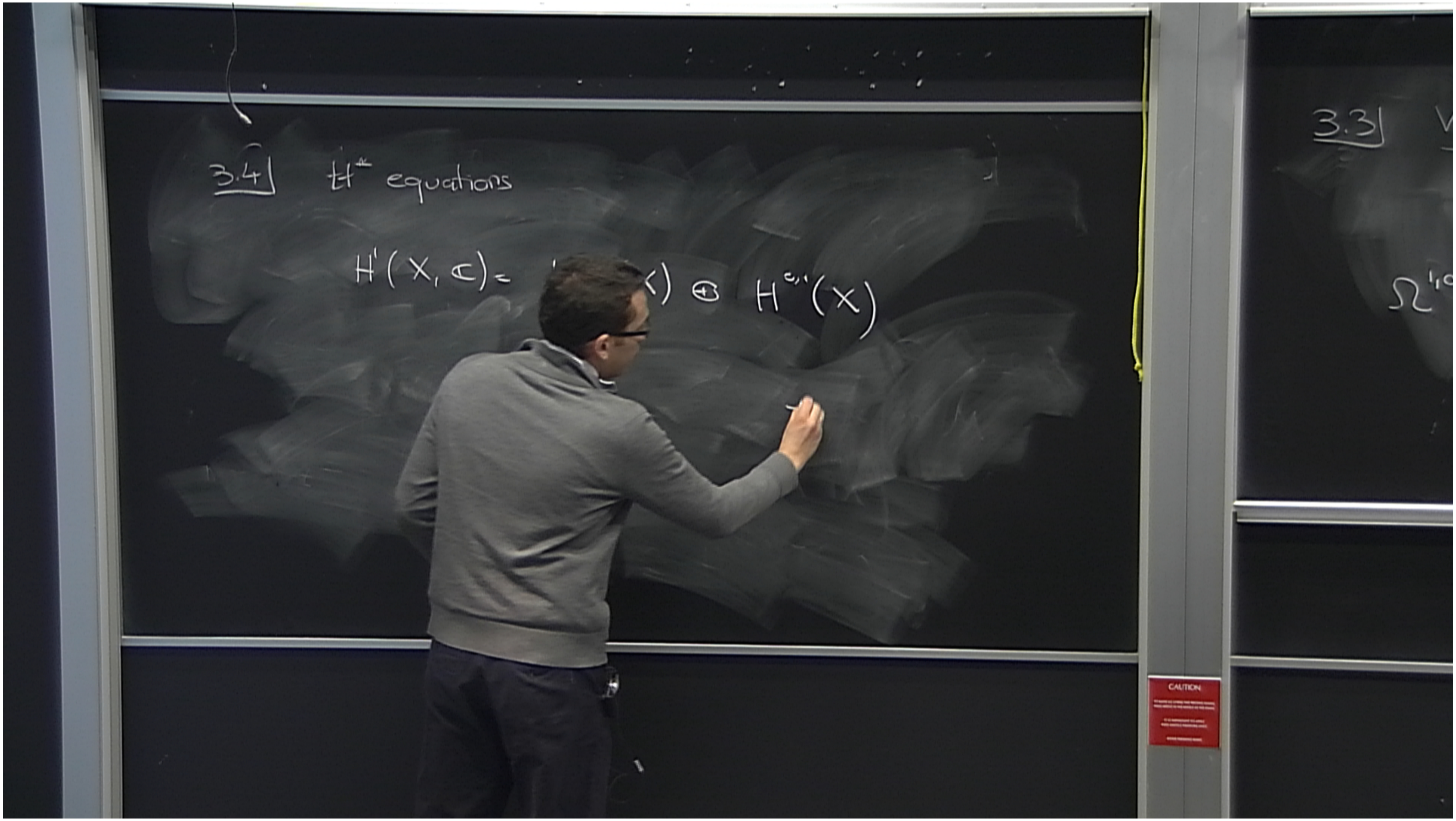

$H^{1,0}(X) \oplus H^{0,1}(X)$
connection
$\begin{pmatrix} \Omega \\ \partial_z \Omega \end{pmatrix} =$
CAUTION

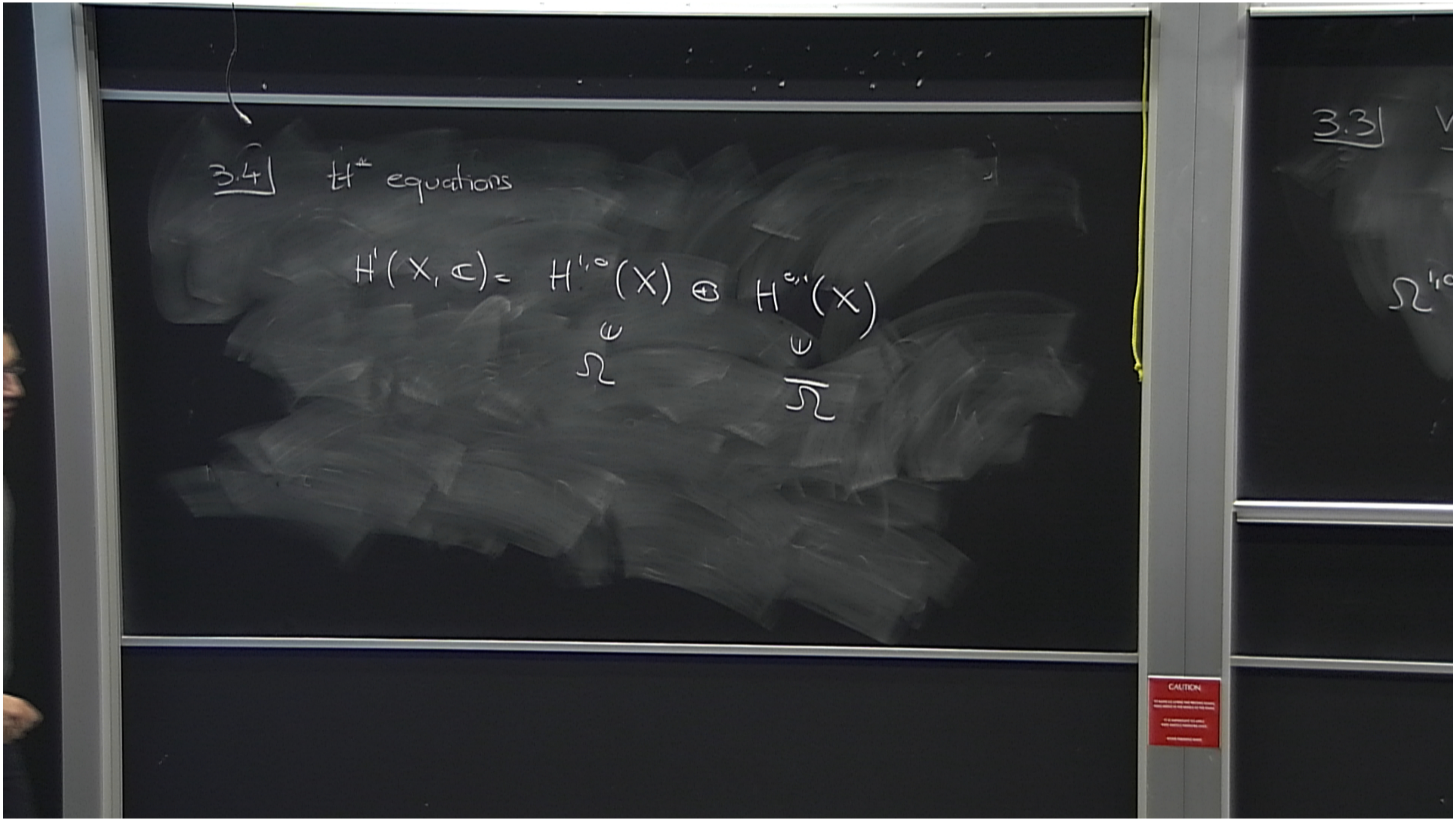


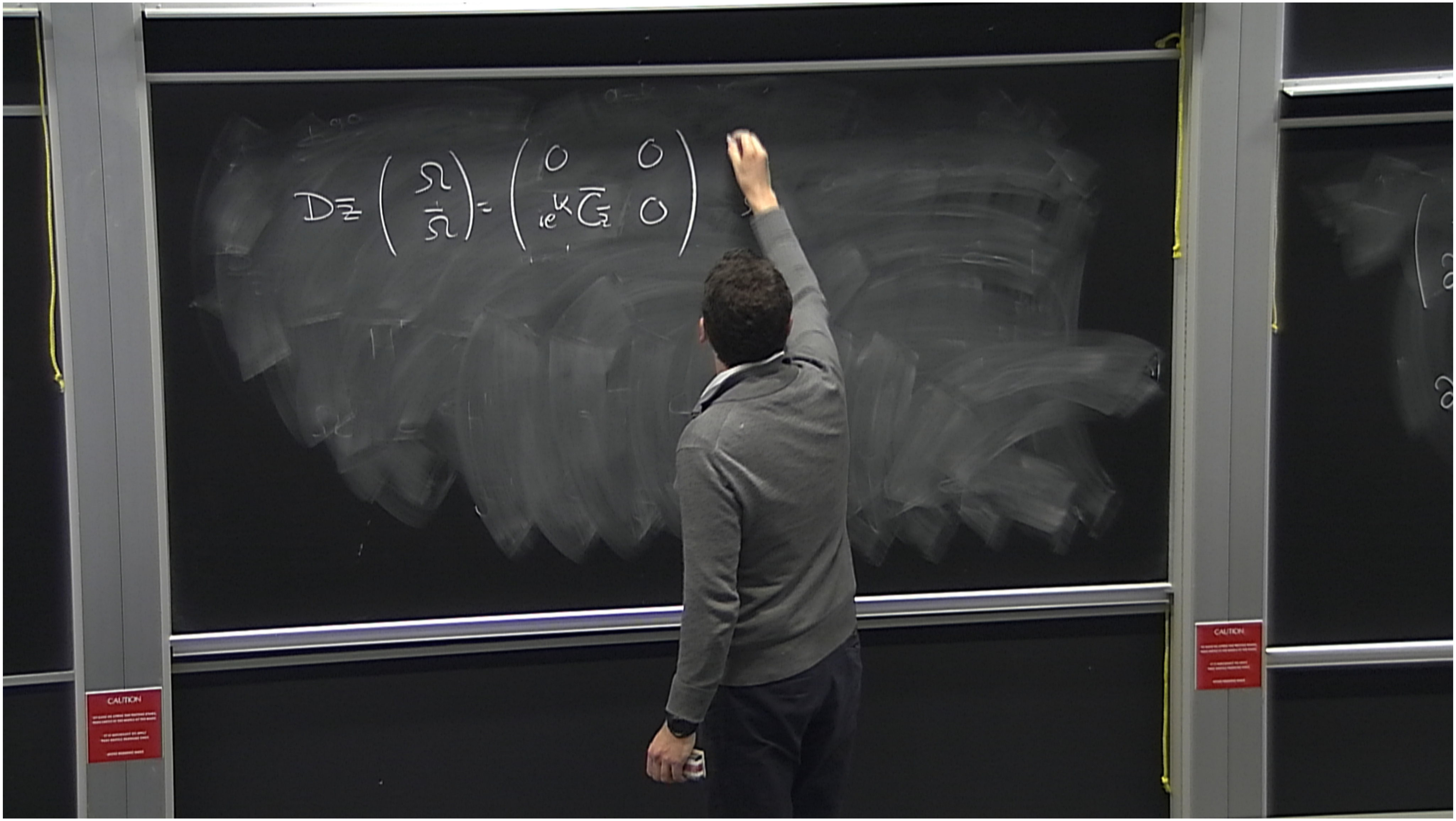
H^{1,0}(X) ⊕ H^{0,1}(X)
connection
CAUTION


H^{1,0}(X) ⊕ H^{0,1}(X)
connection
(Ω, ∂zΩ) ⊂ F^1, F^0
∂z (Ω, ∂zΩ) = (0 1; * *)(Ω, ∂zΩ) ,
Gauss-Manin connection

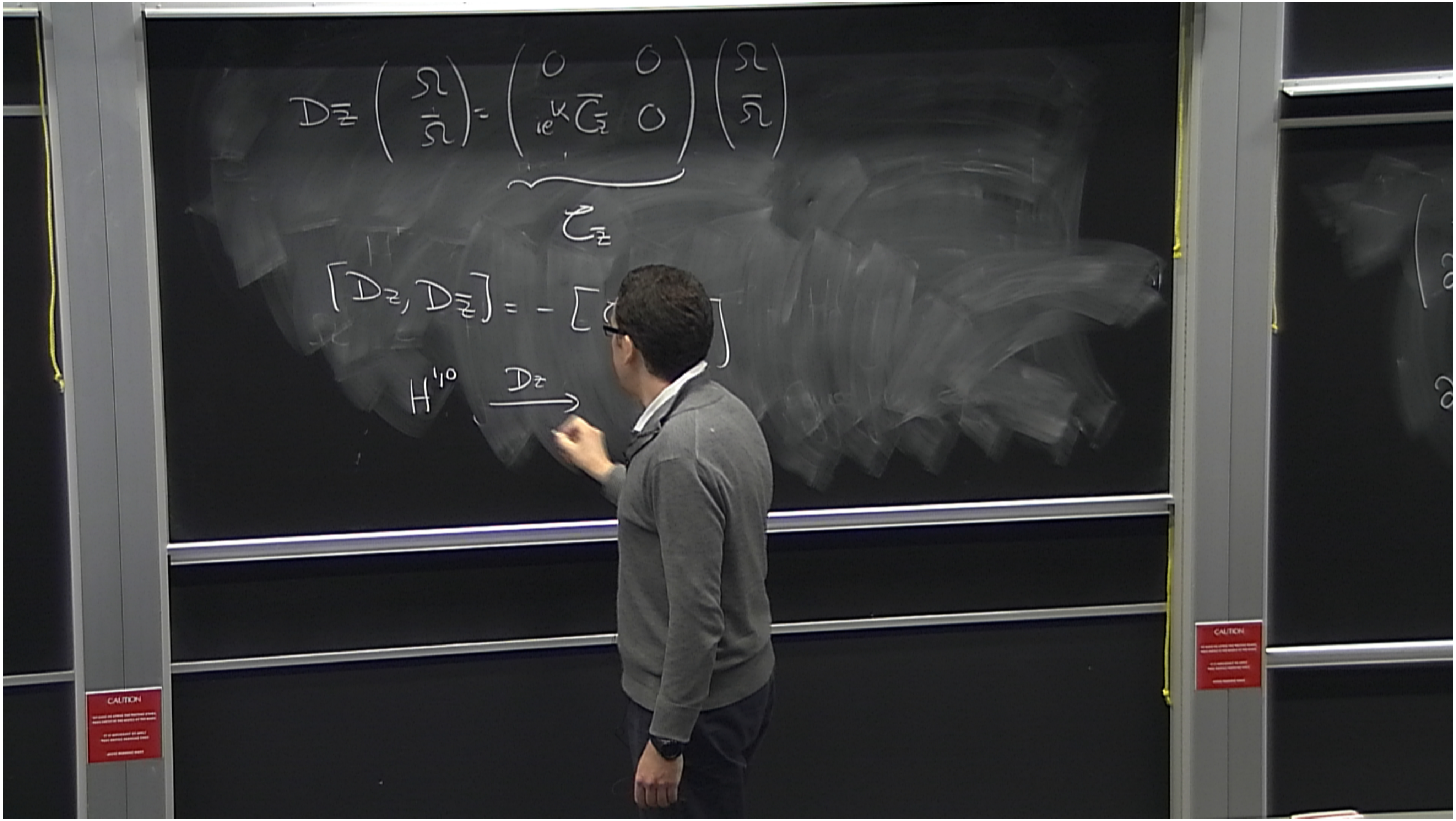

3.4 tt* equations
H'(X
3.3

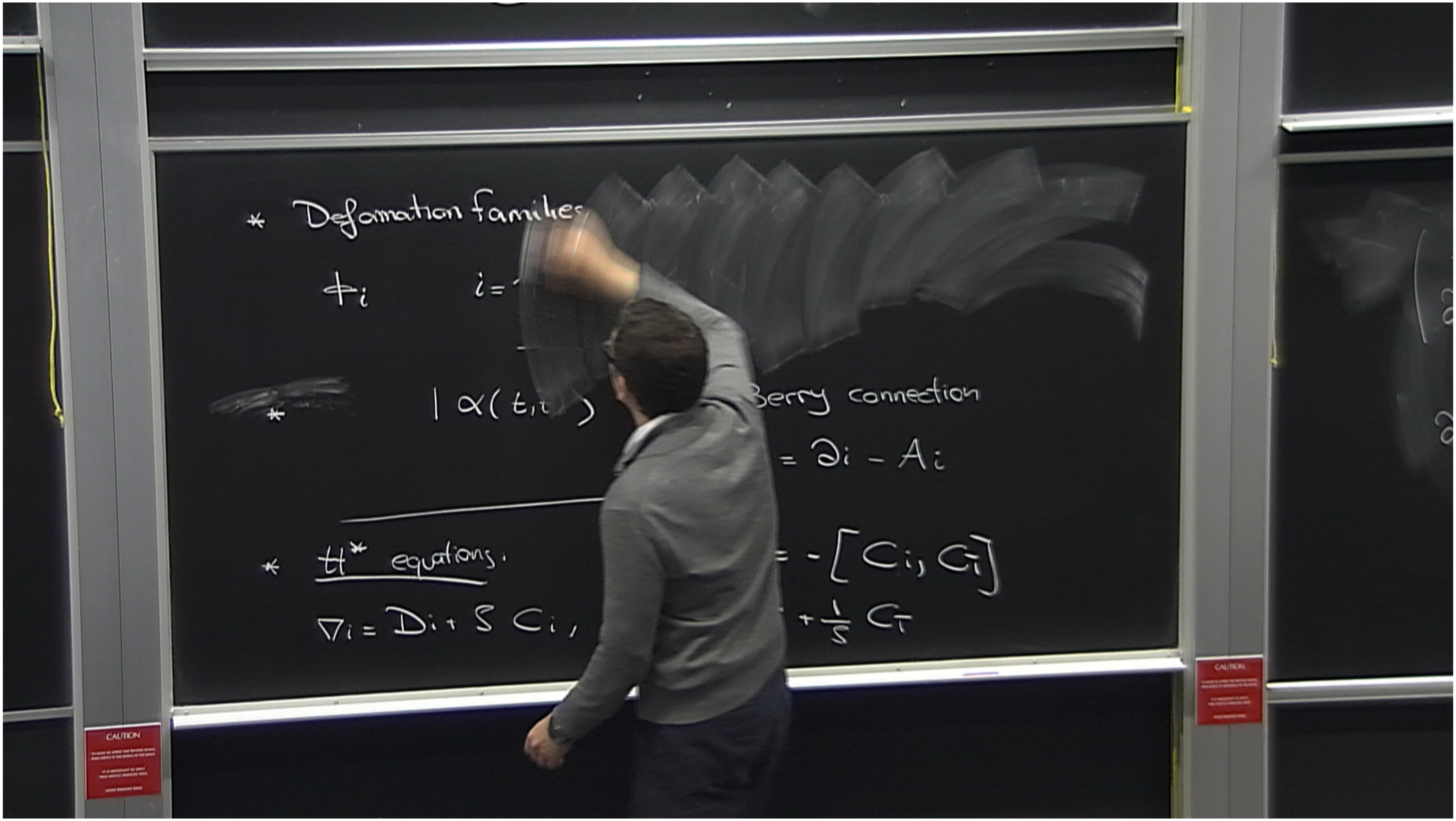

3.4
tt* equations
3.3

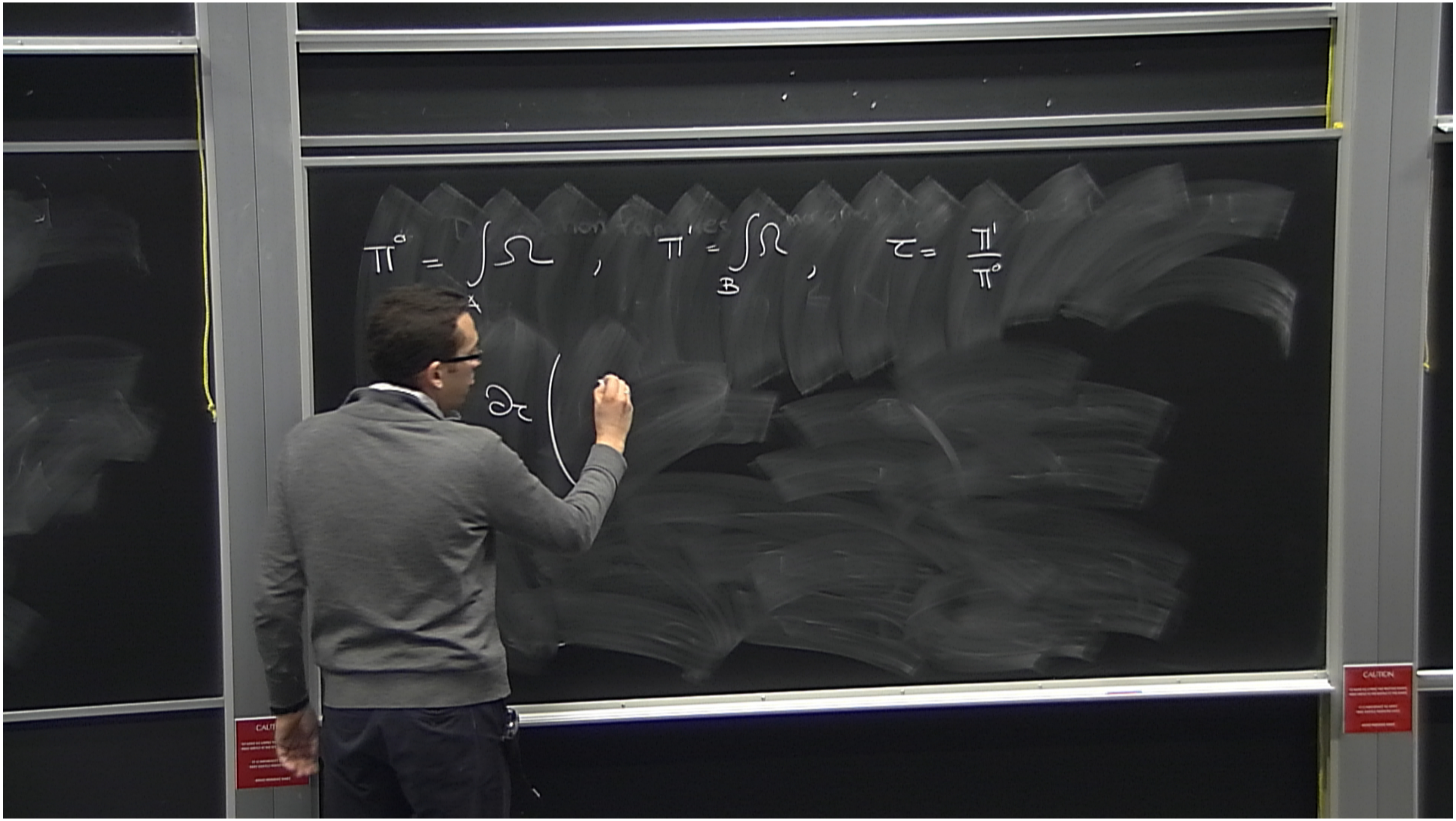

3.4 tt* equations
$H^1(X,\mathbb{C}) = H^{1,0}(X) \oplus H^{0,1}(X)$
Ω
$\overline{\Omega}$

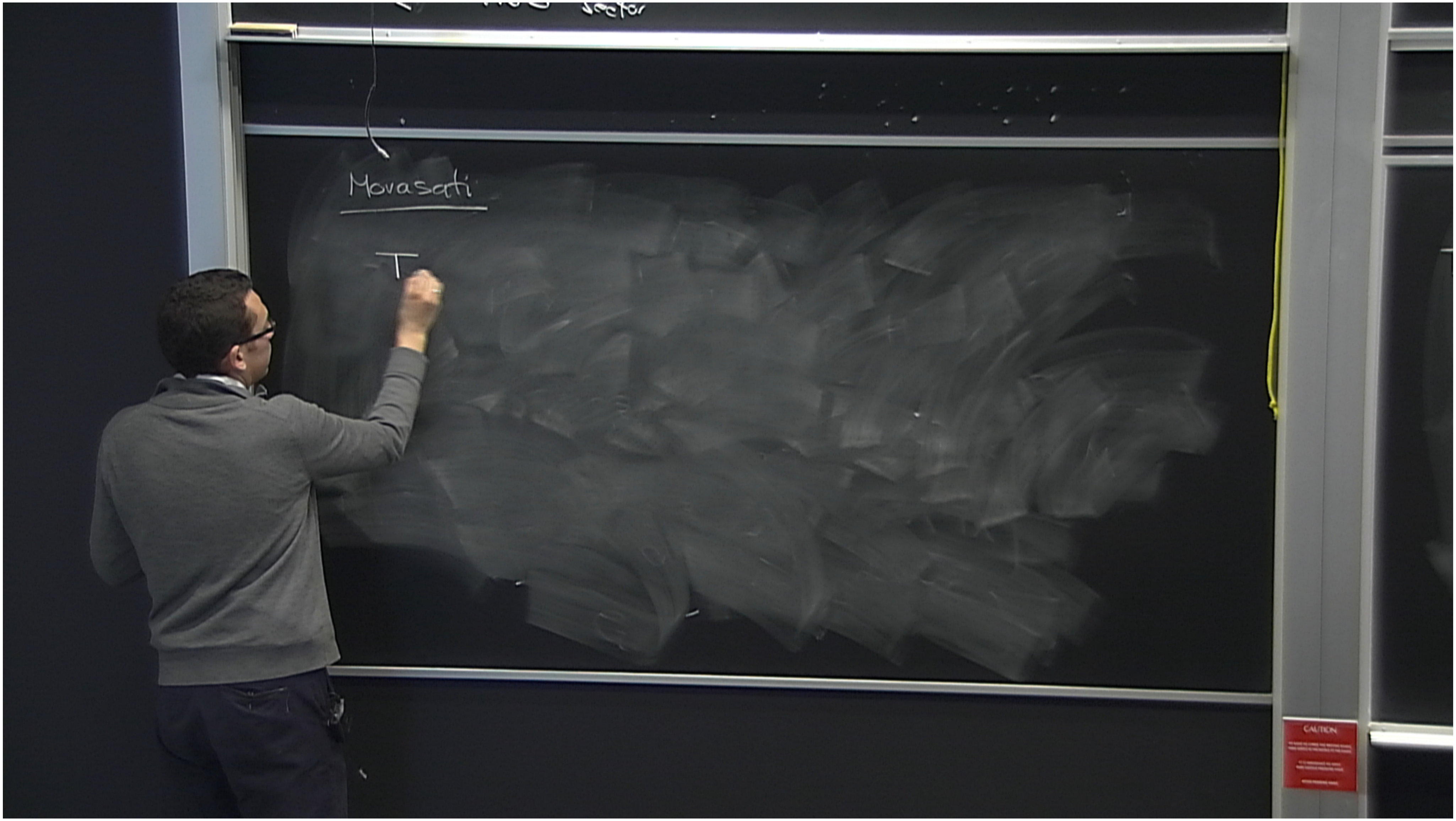


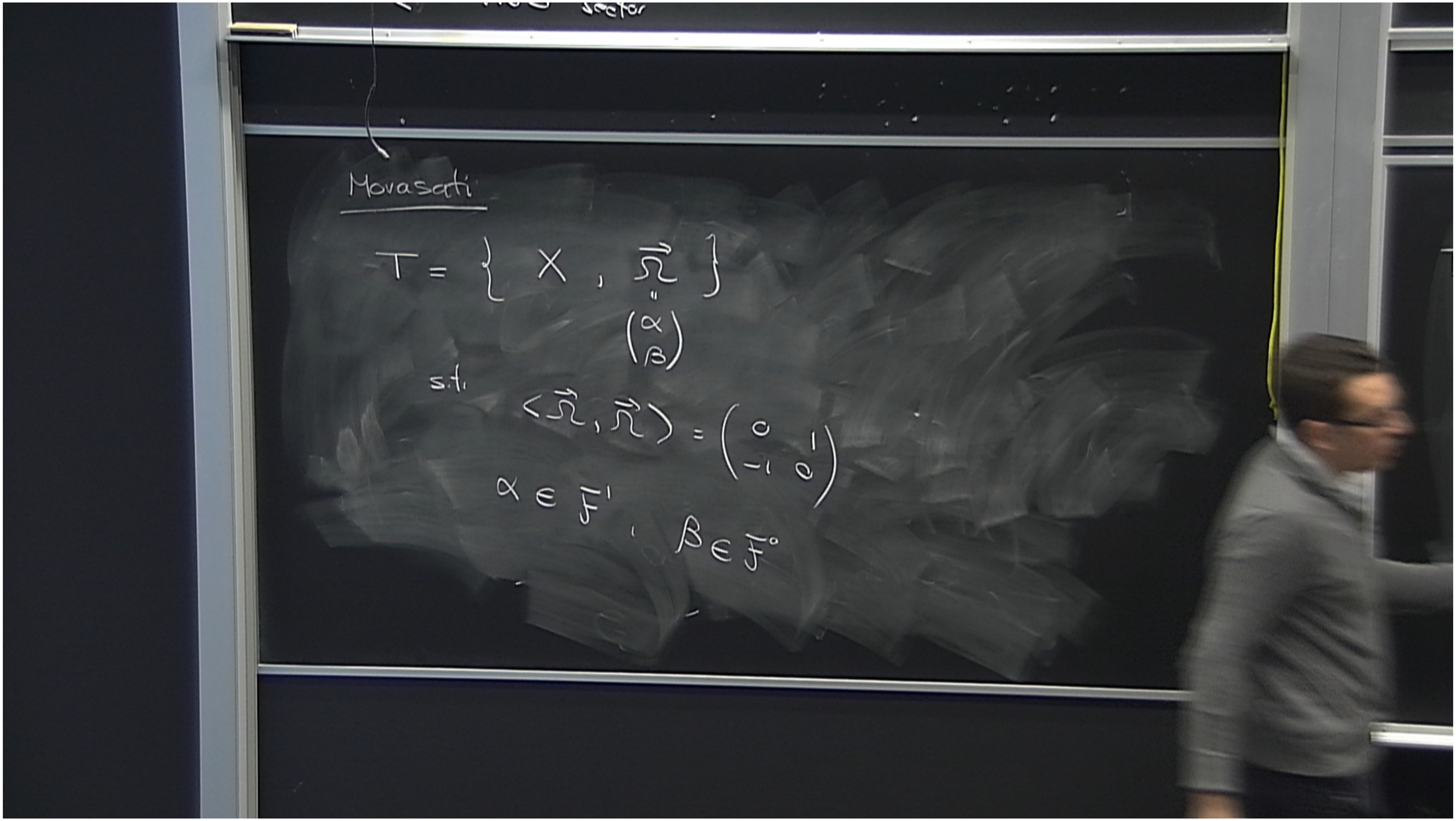
$D\bar{z}\begin{pmatrix}\Omega\\ \dot{\Omega}\end{pmatrix} = \begin{pmatrix}0 & 0\\ ie^{K}\bar{G}_{\bar{z}} & 0\end{pmatrix}$

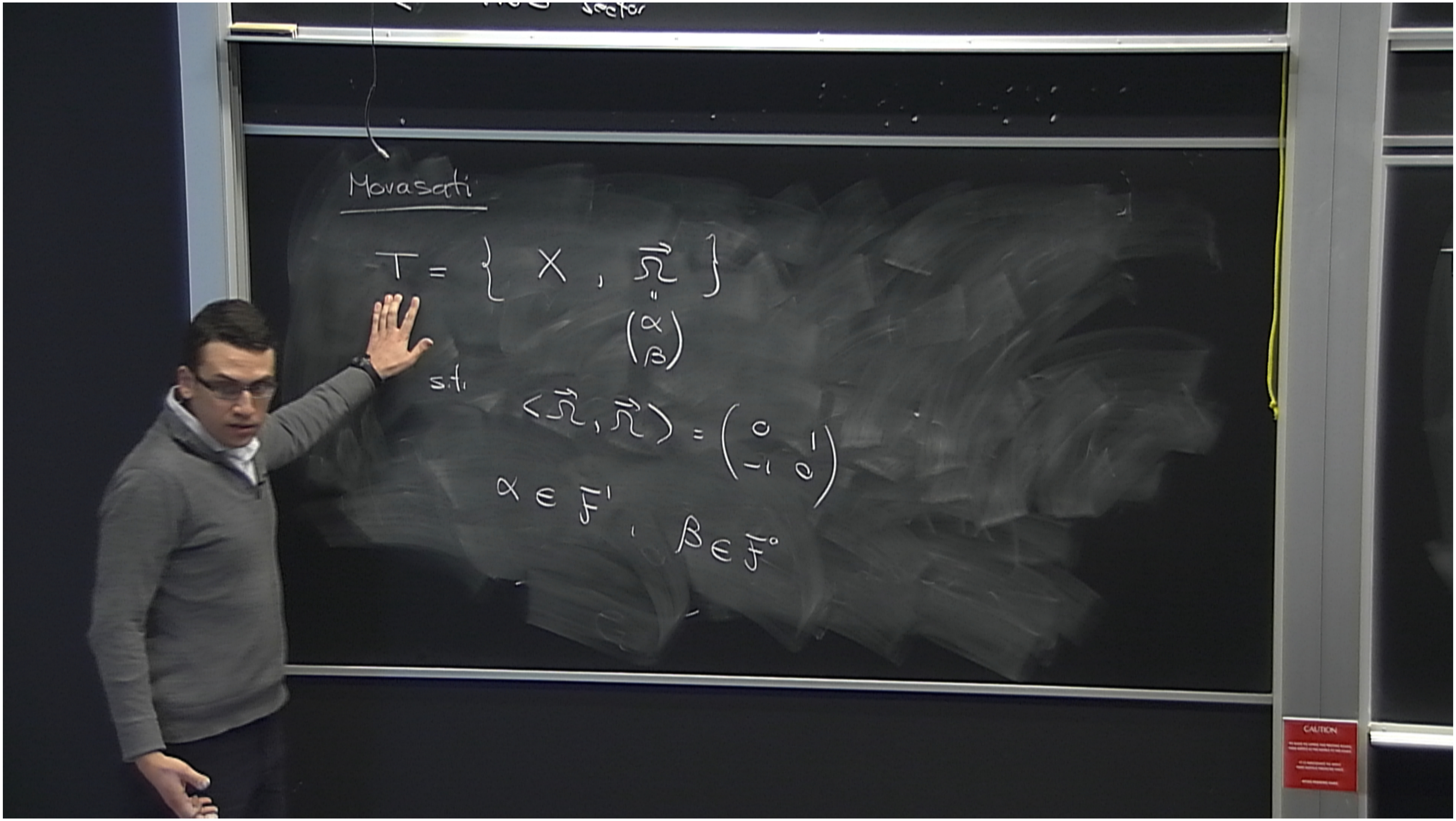

$D_{\bar{z}} \begin{pmatrix} \Omega \\ \bar{\Omega} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ ie^{K} \bar{C}_{\bar{z}} & 0 \end{pmatrix} \begin{pmatrix} \Omega \\ \bar{\Omega} \end{pmatrix}$
$C_{\bar{z}}$
$[D_z, D_{\bar{z}}] = - [$
$H^{1,0} \xrightarrow{D_z}$


* Deformation families
Berry connection
= ∂i − Ai
* tt* equations.
∇i = Di + S Ci ,
= − [Ci , Cī]
+ 1/S Cī

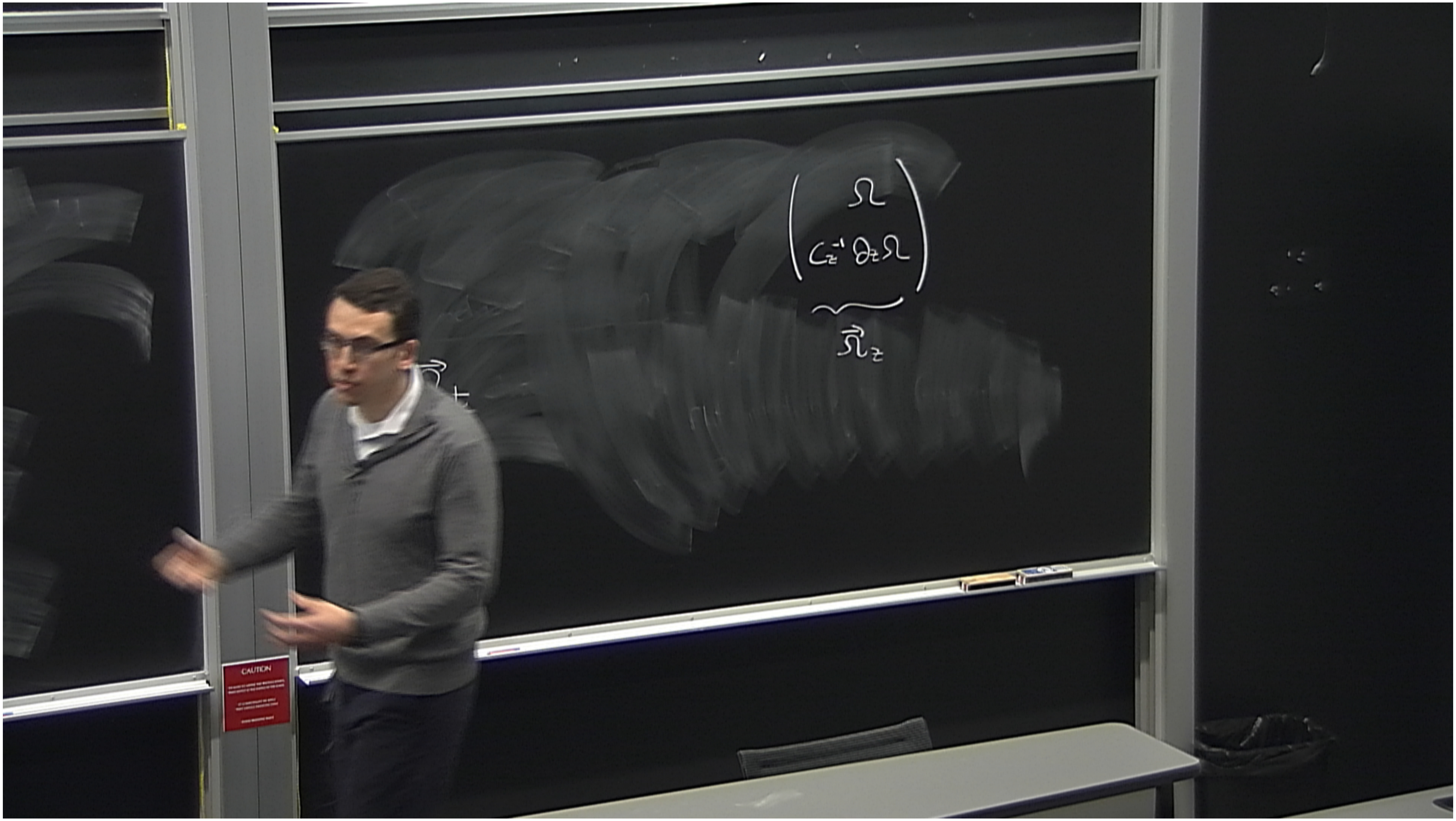

Π° = ∫_A Ω , Π' = ∫_B Ω , τ = Π'/Π°
∂τ (


Movasati

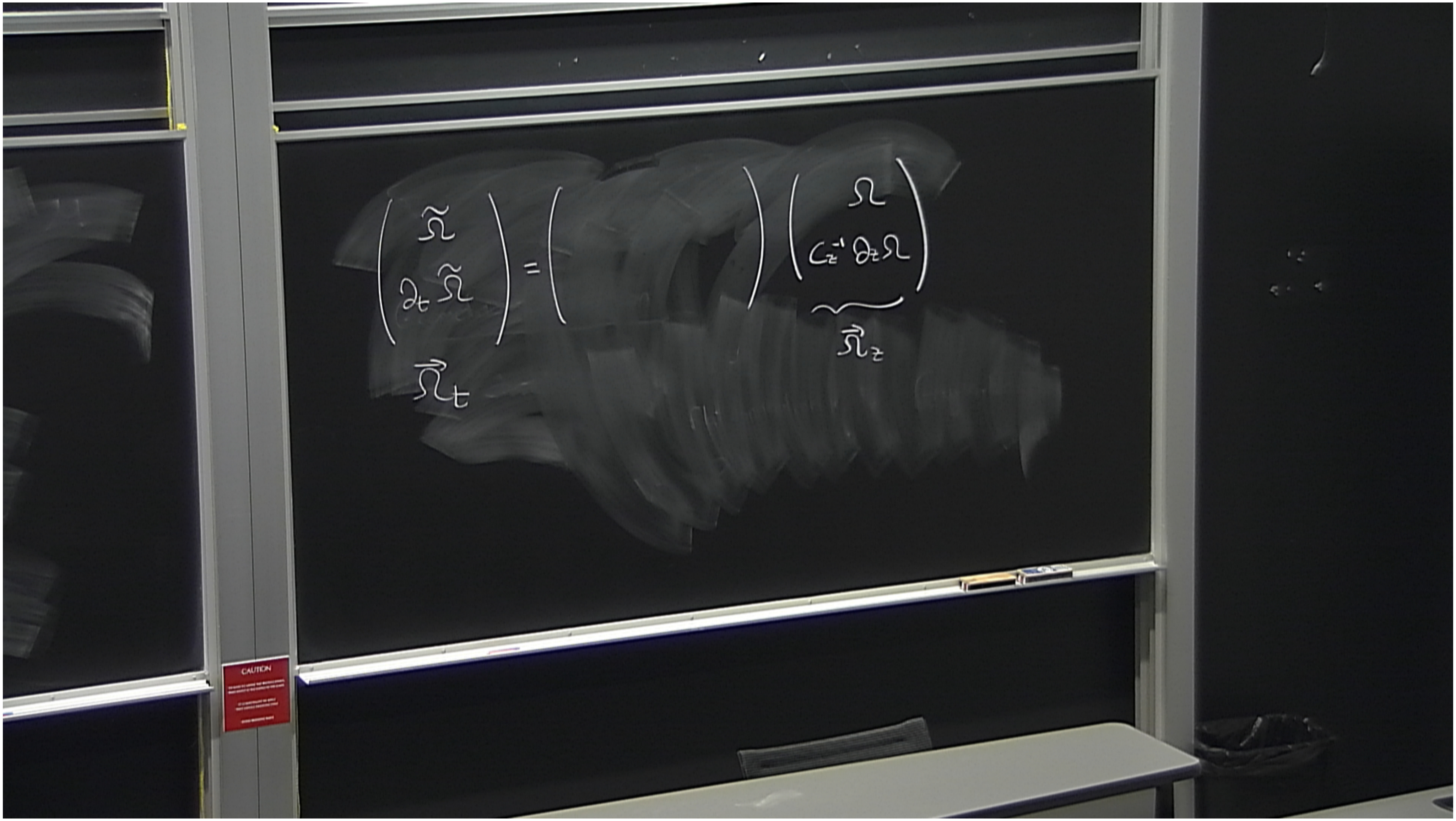

Movasati
T = { X , Ω⃗ }
(α β)
s.t. ⟨Ω⃗, Ω⃗⟩ = (0 1 / -1 e)
α ∈ F′ , β ∈ F°

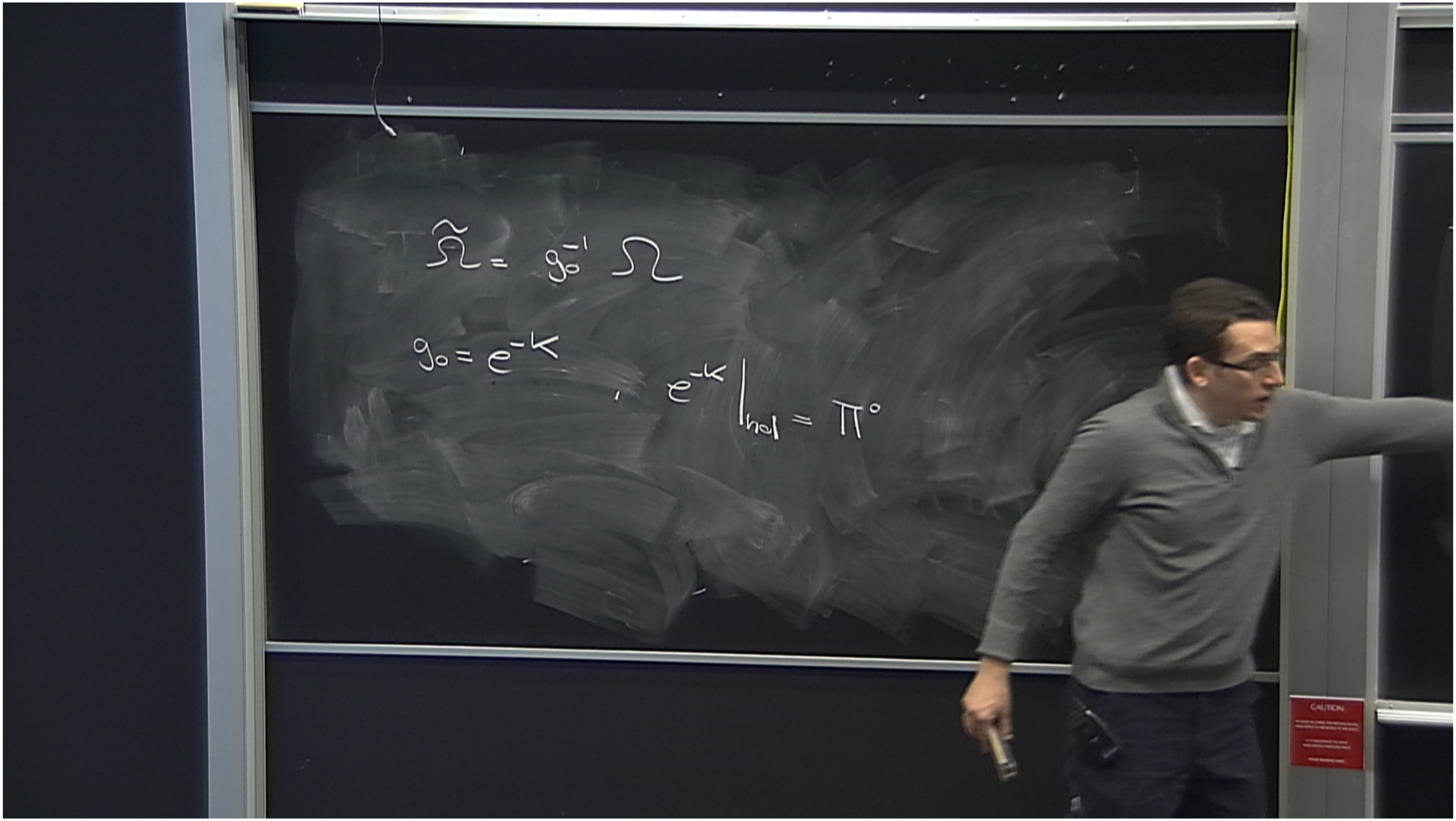

Movasati
T = { X , Ω⃗ }
(α β)
s.t.
⟨Ω⃗, Ω⃗⟩ = (0 1; −1 0)
α ∈ F¹ , β ∈ F⁰

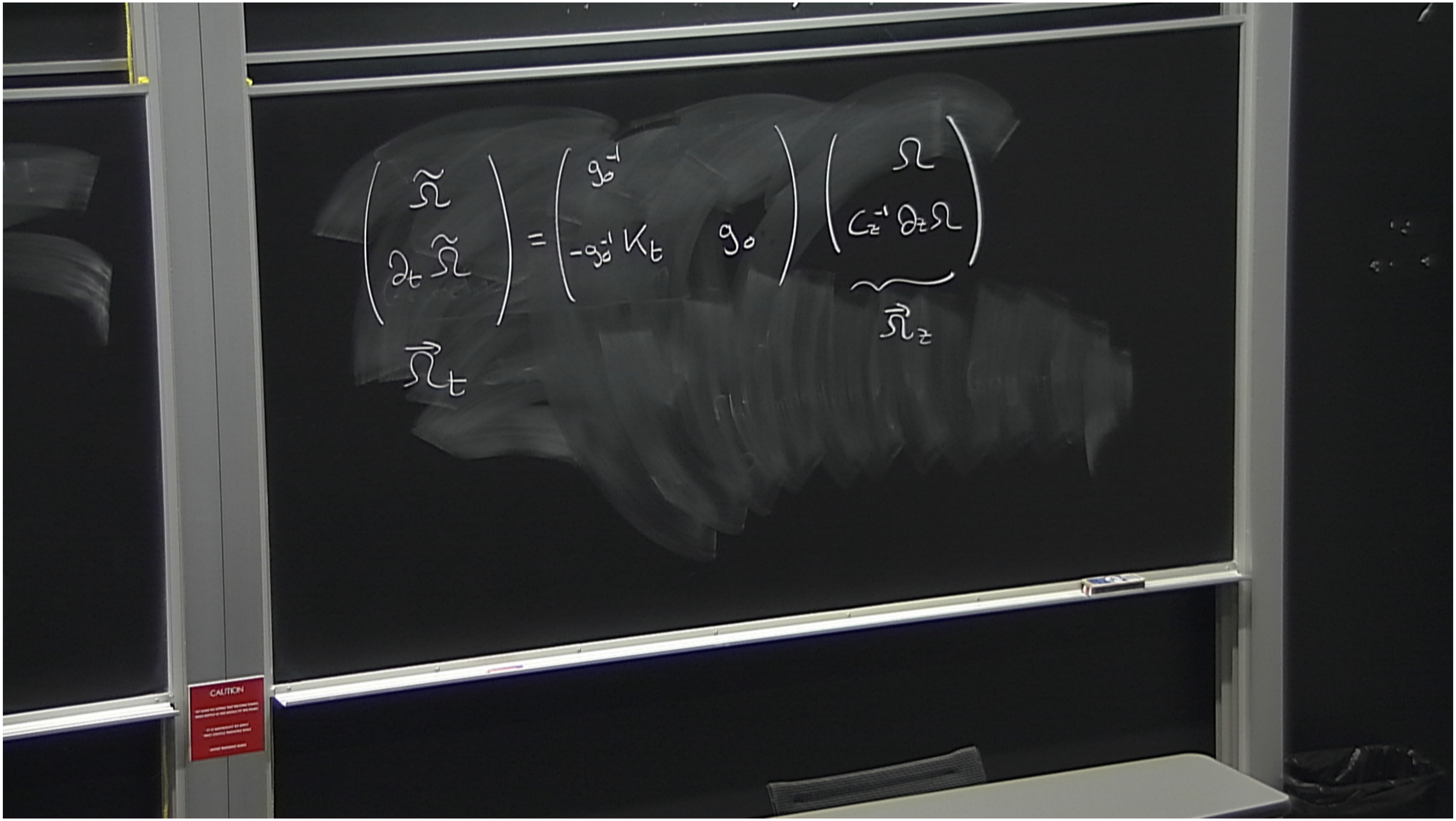

CAUTION

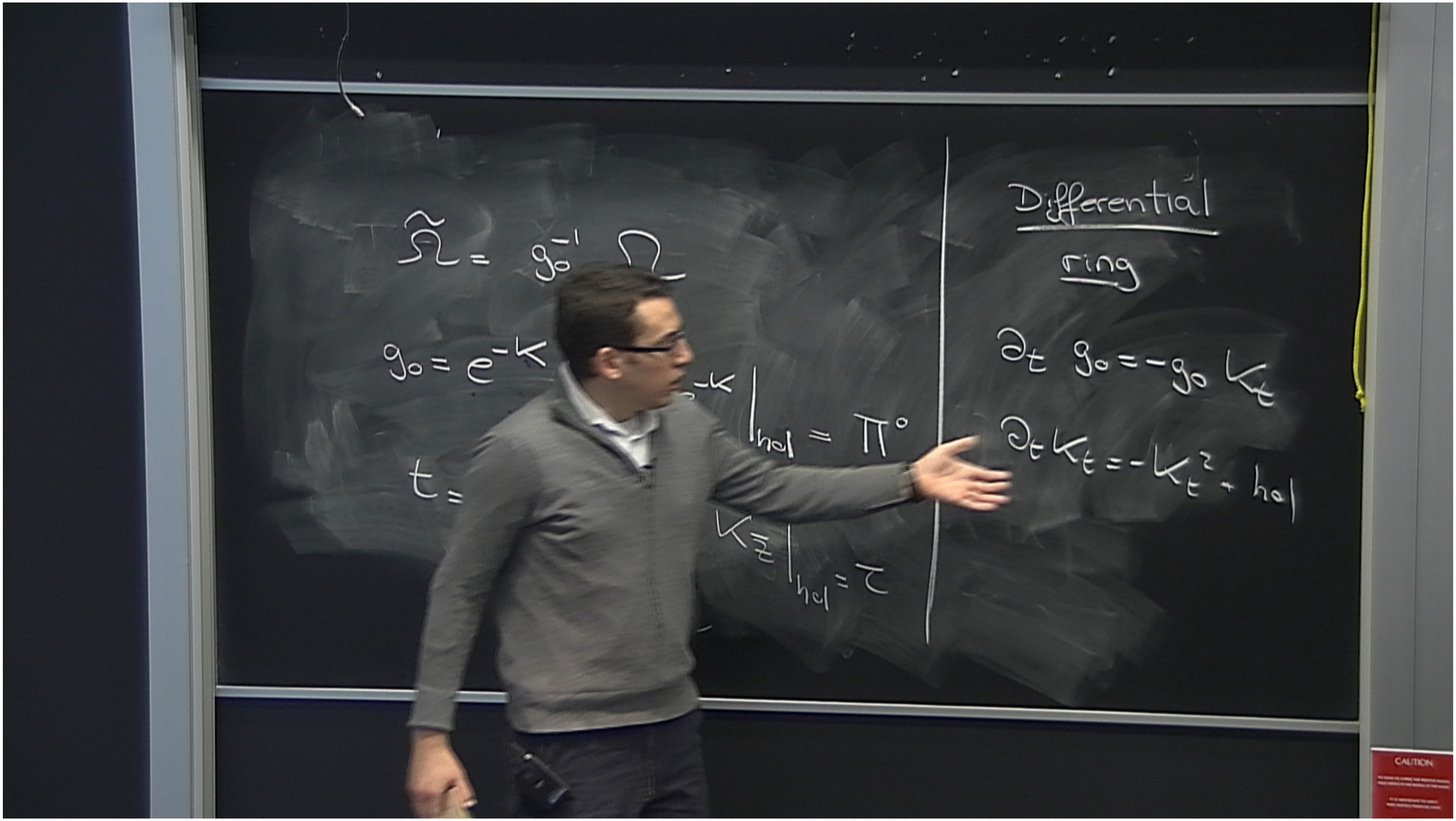


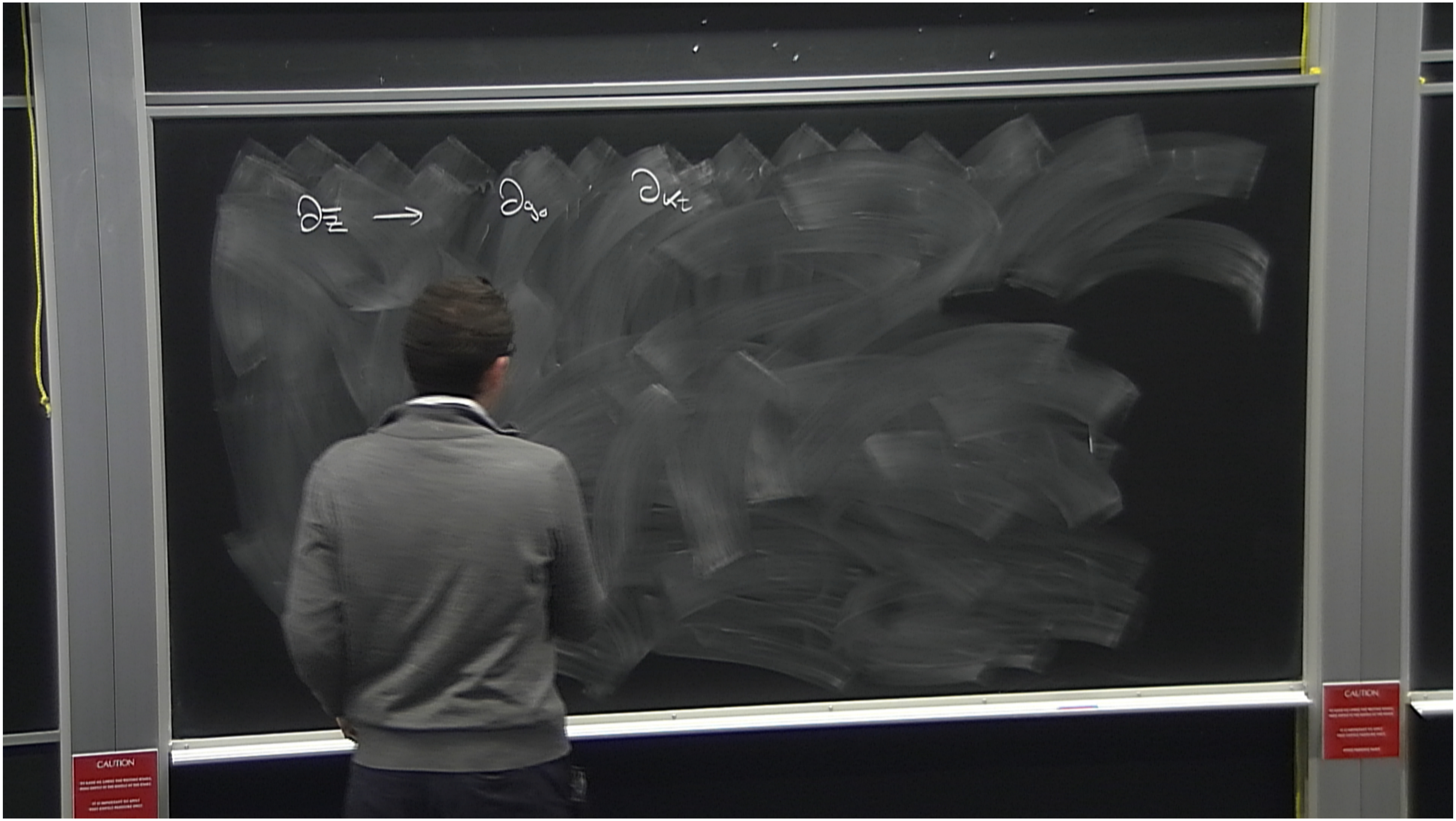
$\begin{pmatrix} \Omega \\ C_z^{-1} \partial_z \Omega \end{pmatrix}$
$\vec{\Omega}_z$


$\begin{pmatrix} \tilde{\Omega} \\ \partial_t \tilde{\Omega} \end{pmatrix} = \begin{pmatrix} \ \end{pmatrix} \begin{pmatrix} \Omega \\ C_z^{-1} \partial_z \Omega \end{pmatrix}$
$\vec{\Omega}_t$
$\vec{\Omega}_z$

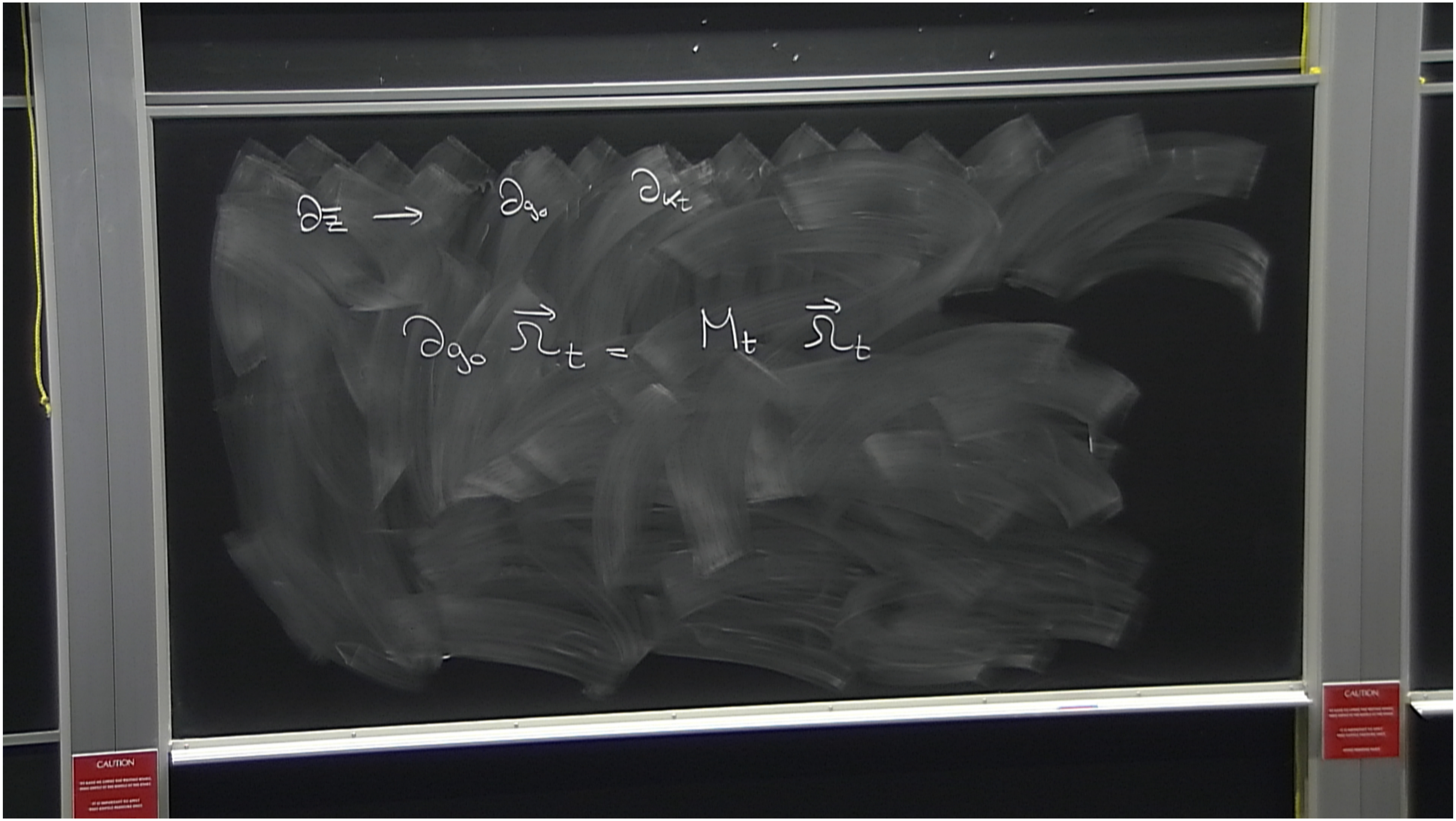

$\tilde{\Omega} = g_0^{-1}\,\Omega$
$g_0 = e^{-K}$
$e^{-K}\big|_{hol} = \Pi^0$

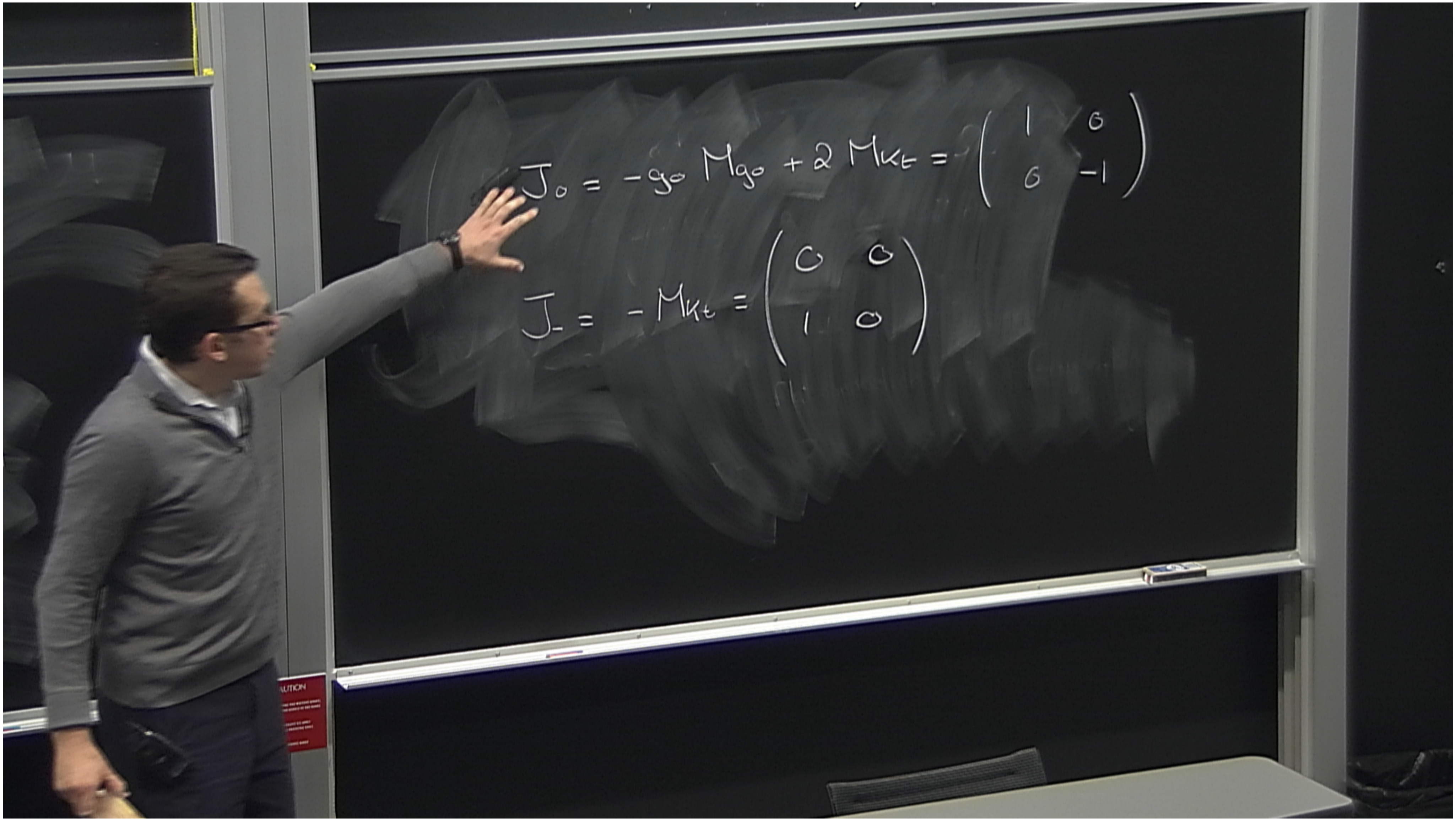

$\begin{pmatrix} \tilde{\Omega} \\ \partial_t \tilde{\Omega} \end{pmatrix} = \begin{pmatrix} g_0^{-1} & \\ -g_0^{-1} K_t & g_0 \end{pmatrix} \begin{pmatrix} \Omega \\ C_z^{-1} \partial_z \Omega \end{pmatrix}$
$\vec{\Omega}_t$
$\vec{\Omega}_z$
CAUTION

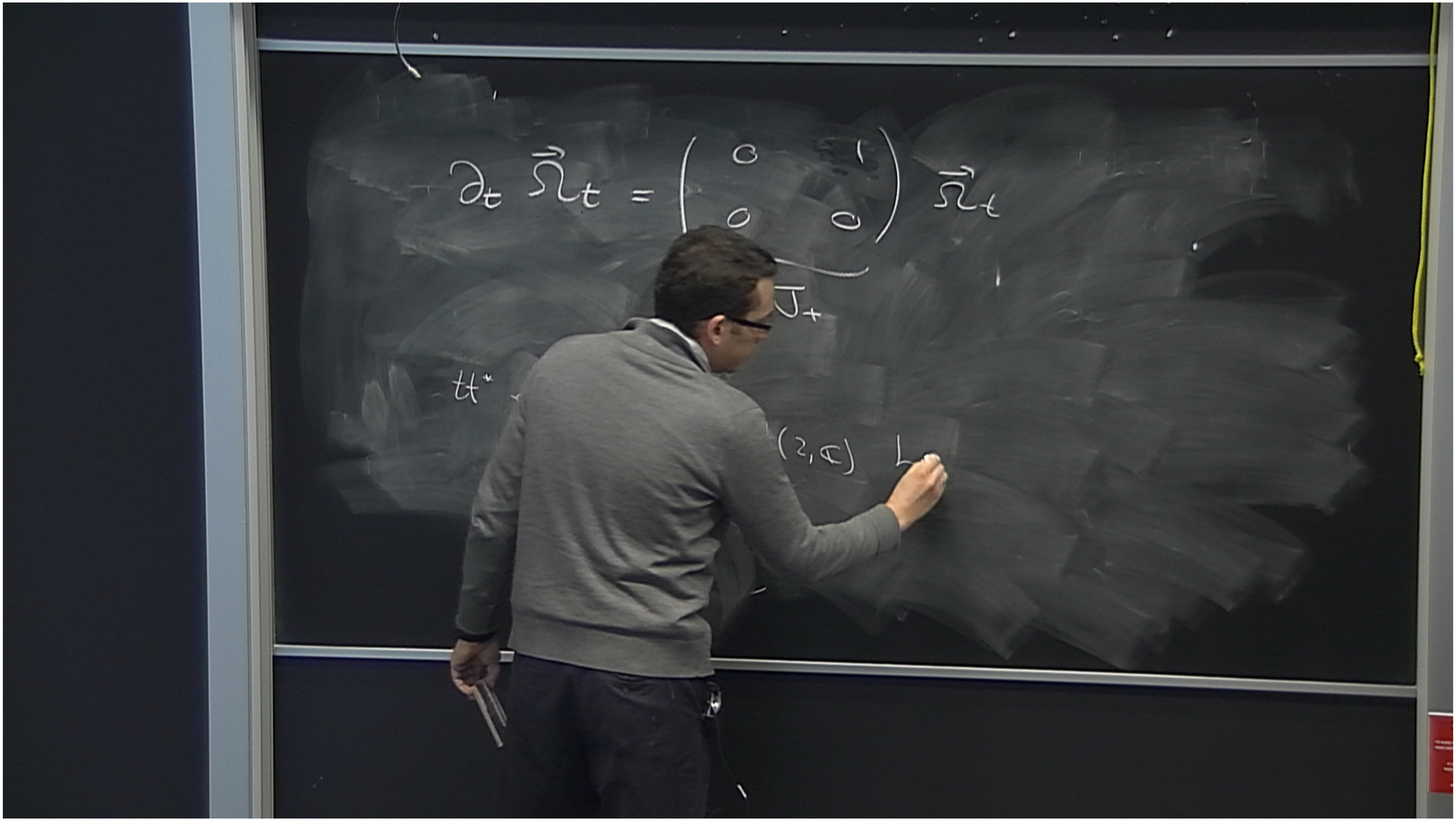

$\tilde{\Omega} = g_0^{-1}\,\Omega$
$g_0 = e^{-K}$
$|_{hol} = \Pi^0$
$t =$
$K_{\bar{z}}|_{hol} = \tau$
Differential ring
$\partial_t\, g_0 = -g_0\, K_t$
$\partial_t K_t = -K_t^2 + hol$

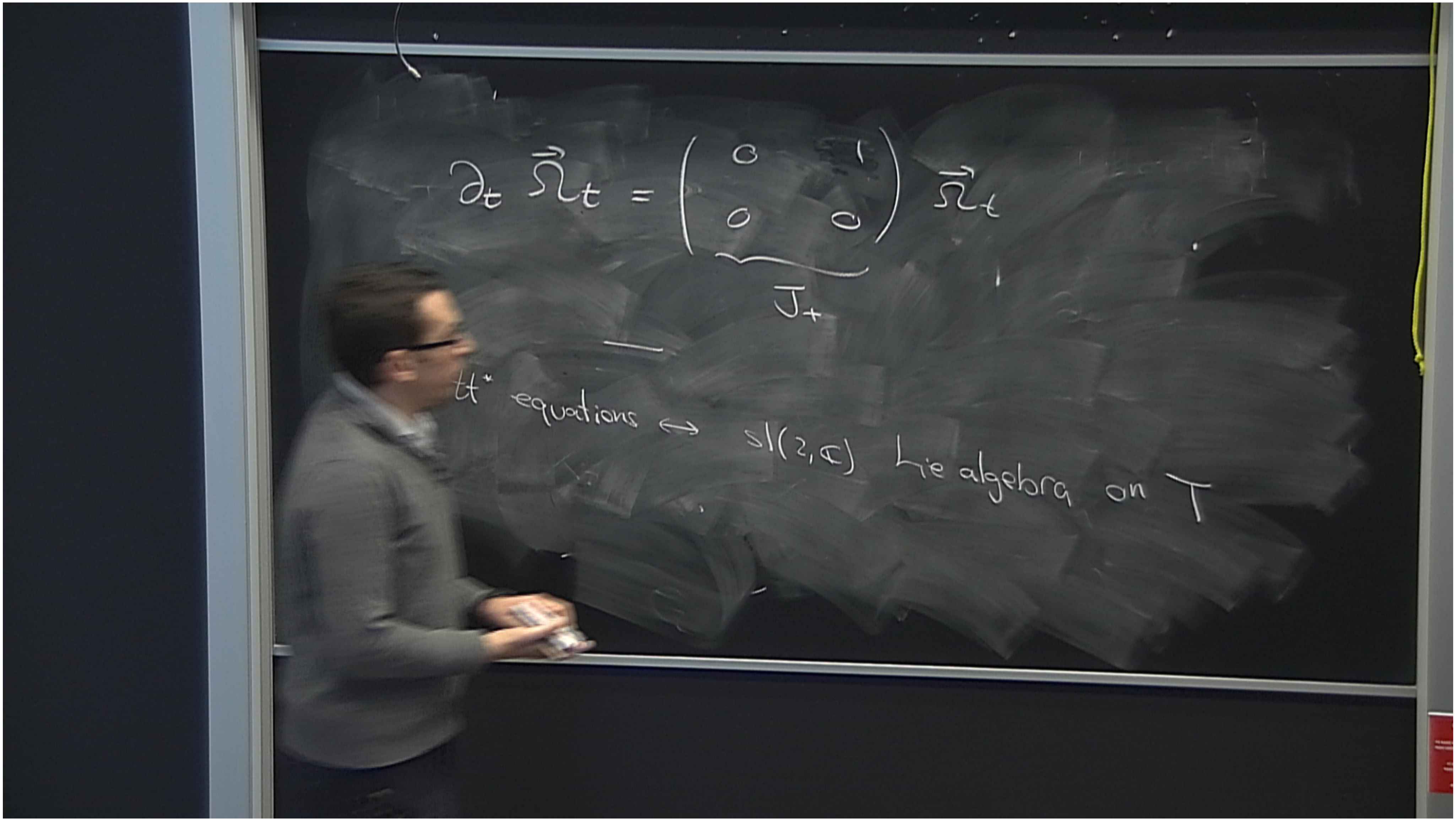


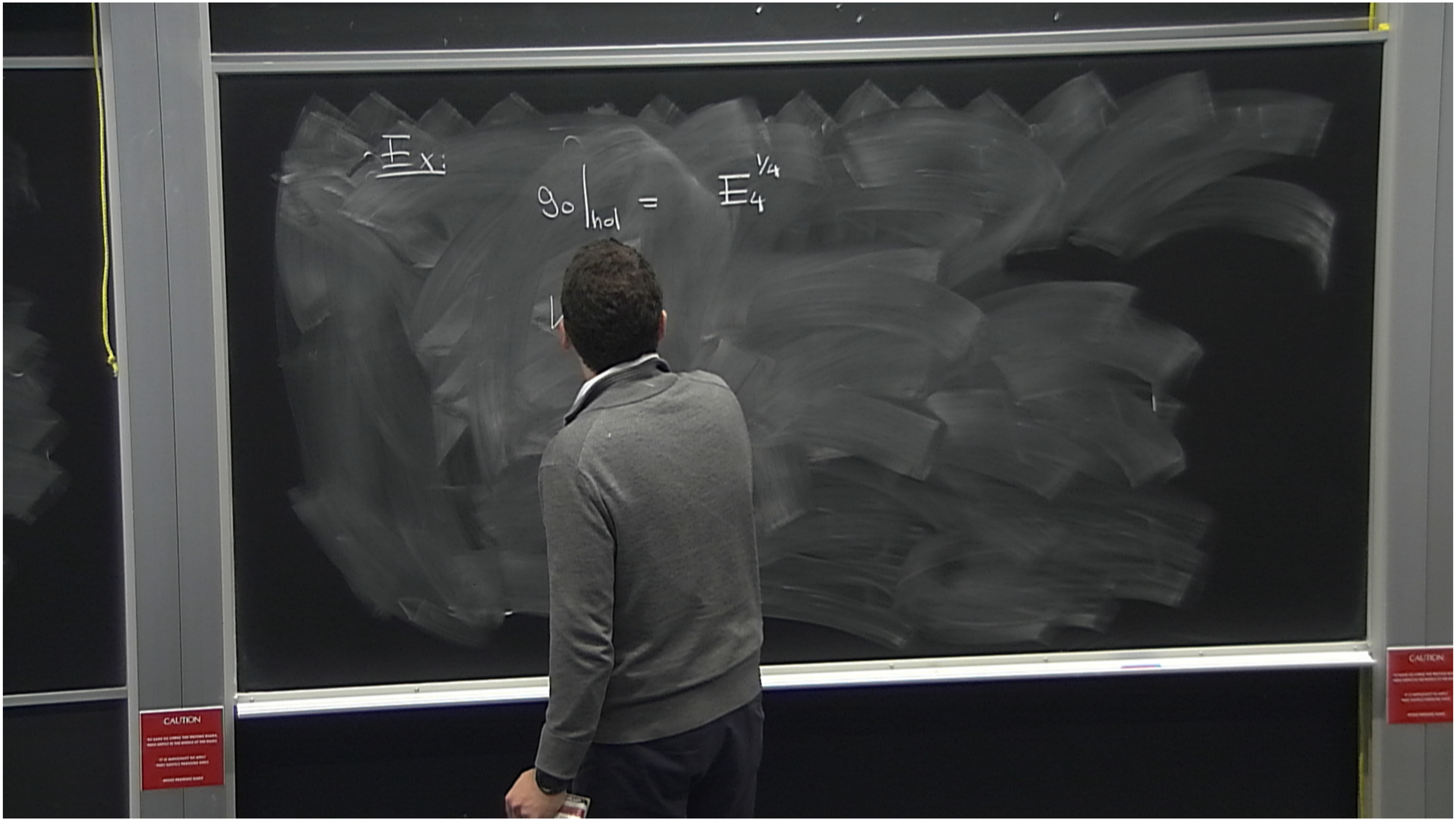
$\partial\Sigma \longrightarrow \partial g_0$
∂_{kt}

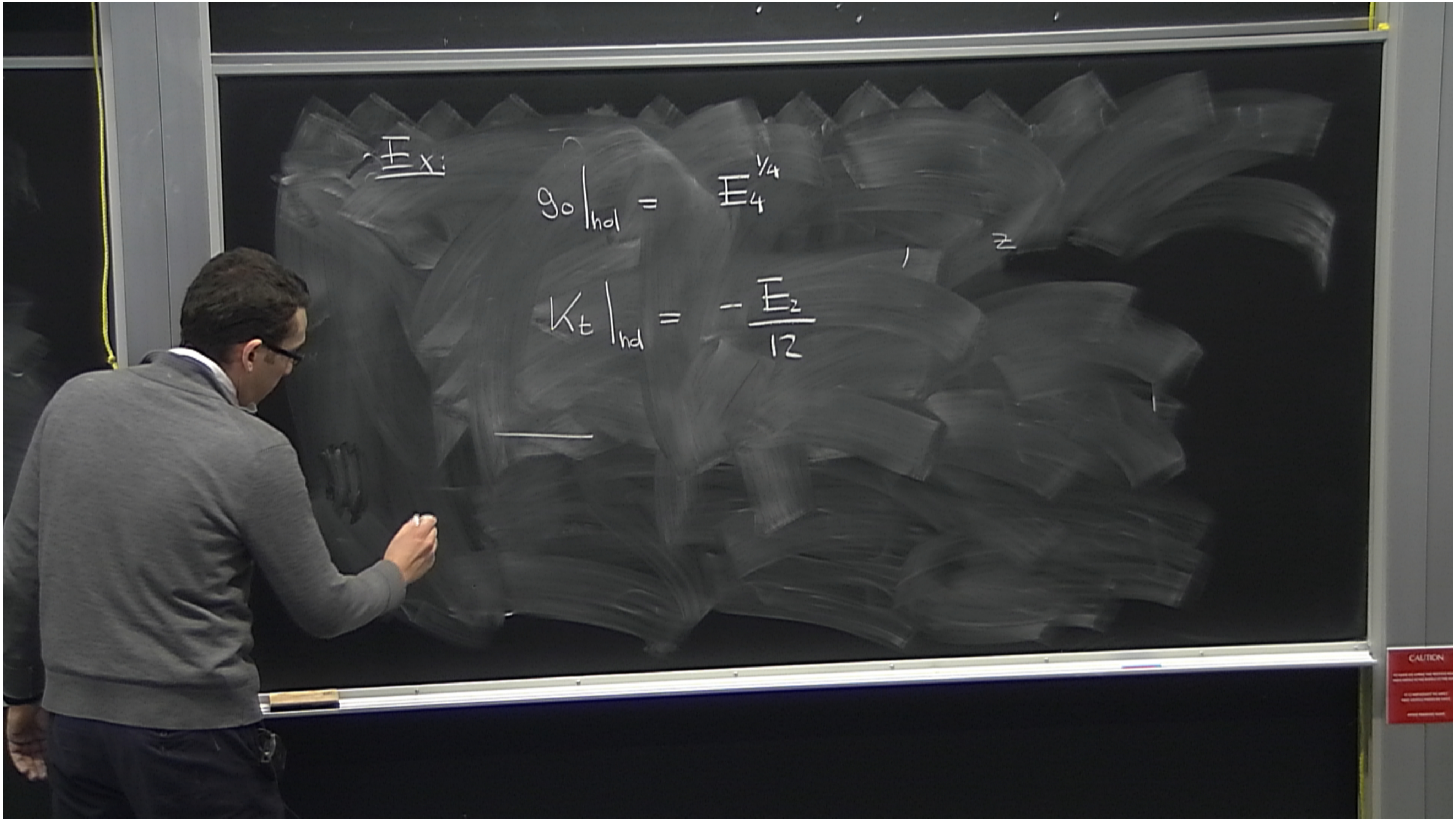

∂Z → ∂g₀ ∂Kt
∂g₀ Ω⃗t = Mt Ω⃗t


$J_0 = -g_0 M_{g_0} + 2 M_{Kt} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
$J_- = -M_{Kt} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$


∂t Ω t = (0 1 / 0 0) Ω t
J+
tt*
(2,C)


$\partial_t \vec{\Omega}_t = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \vec{\Omega}_t$
J_+
tt^* equations ↔ $sl(2,\mathbb{C})$ Lie algebra on T


Ex:
$g_0|_{hol} = E_4^{1/4}$


Ex:
$g_0|_{hol} = E_4^{1/4}$
$K_t|_{hol} = -\frac{E_2}{12}$