Title: Getting rid off the Barbero-Immirzi parameter in LQG

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Abstract: <p>TBA</p>

Getting Rid Of Barbero-Immirzi Parameter in LQG

Introduction

(1) What is γ?

$$S_H[e,\omega] = \int \frac{1}{2}\varepsilon_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega) + \frac{1}{\gamma}\delta_{IJKL}\, e^I \wedge e^J \wedge F^{KL}(\omega)$$

$\rightarrow$ No effect on class. solut°

(2) Why γ?

'86 : Ashtekar $\rightarrow \gamma = \pm i$

Gauge theory $G = SL(2,\mathbb{C})$

'96 : $\gamma \in \mathbb{R}$

(3) Problems! $a_j = 8\pi\gamma\, l_p^2 \sqrt{j(j+1)}$

(4) Observations : $\boxed{\gamma = \pm i}$ $j \in \frac{\mathbb{N}}{2}$

Chern-Simons Theo... BH

a) ...eme: How SO(2) CS ?

b) Few words on CS

2. <u>Counting BH microstates</u>

a) Quantizat° of CS

b) Verlinde Formulae

c) Back to BH; semi-classical

3. <u>Analytic continuat°</u> : $\gamma = \pm i$

a) Naive continuat°

b) Rig. continuat° : $S \simeq \frac{a_H}{4 \ell_P^2}$

4. <u>Discussion</u> : 3D gravity

...

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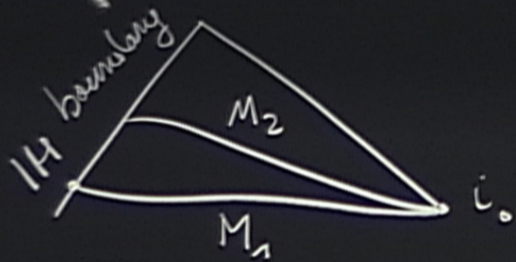
1. SU(2) CS and BH in LQG

a) **Frame**: (Non-Rotat°) Spherical BH



$\left\{\begin{array}{l}\text{Null-boundary} \\ \text{Topology: } S^2 \times \mathbb{R} \\ \text{IH condit° : } \ldots\end{array}\right.$

→ **Symplectic structure**: $A = A_a^i J_i$

$$\{A_a^i(x), A_b^j(y)\} = \frac{4\pi}{k}\,\varepsilon_{ab}\,\delta^{ij}\,\delta(x-y) \;;\quad k = \frac{a_H}{2\pi G\gamma(\cdots)}$$

→ **Coupling between bulk and bound**

$$\underbrace{F(A)}_{\text{boundary}} \propto \underbrace{E}_{\text{bulk}}$$

$\Rightarrow$ $\boxed{\text{CS Theory with } \left\{\begin{array}{l} G = SU(2) \\ \text{level } k \propto a_H \\ \text{Excitat°} \rightarrow \text{punctures}\end{array}\right.}$

$$\mathcal{P} = \{ A \mid F(A) = 0 \text{ but } \mathrm{Hol}_\ell(A) \in \mathcal{C}_\ell \} / \text{Gauge transformat}^\circ$$

$$\{ A_a^i(x) ; A_b^j(y) \} = \frac{4\pi}{k} \varepsilon_{ab} \delta^{ij} \delta(x-y)$$

2. Counting BH microstates

a) Quantizat° of CS theory

- Path integral quantization

$$Z_{CS}(M;k) = \int [\mathcal{D}A]\, e^{i S_{CS}(A)}$$

$$\begin{cases} [\mathcal{D}A^g] = [\mathcal{D}A] \text{ with } A^g = g^{-1} A g + g^{-1} dg \\ S_{CS}(A^g) = S_{CS}(A) + \frac{k}{4\pi} \underbrace{n_g(M)}_{\in \mathbb{N}} \end{cases}$$

$$\Rightarrow e^{i \frac{k}{4\pi} n_g(M)} = 1 \Rightarrow \underline{k \in \mathbb{Z}}$$

$$\{ A_a^i(x), A_b^j(y) \} = \frac{4\pi}{k} \varepsilon_{ab} \delta^{ij} \delta(x-y)$$

2. Counting BH microstates

a) Quantizat° of CS Theory

. Path integral quantization:

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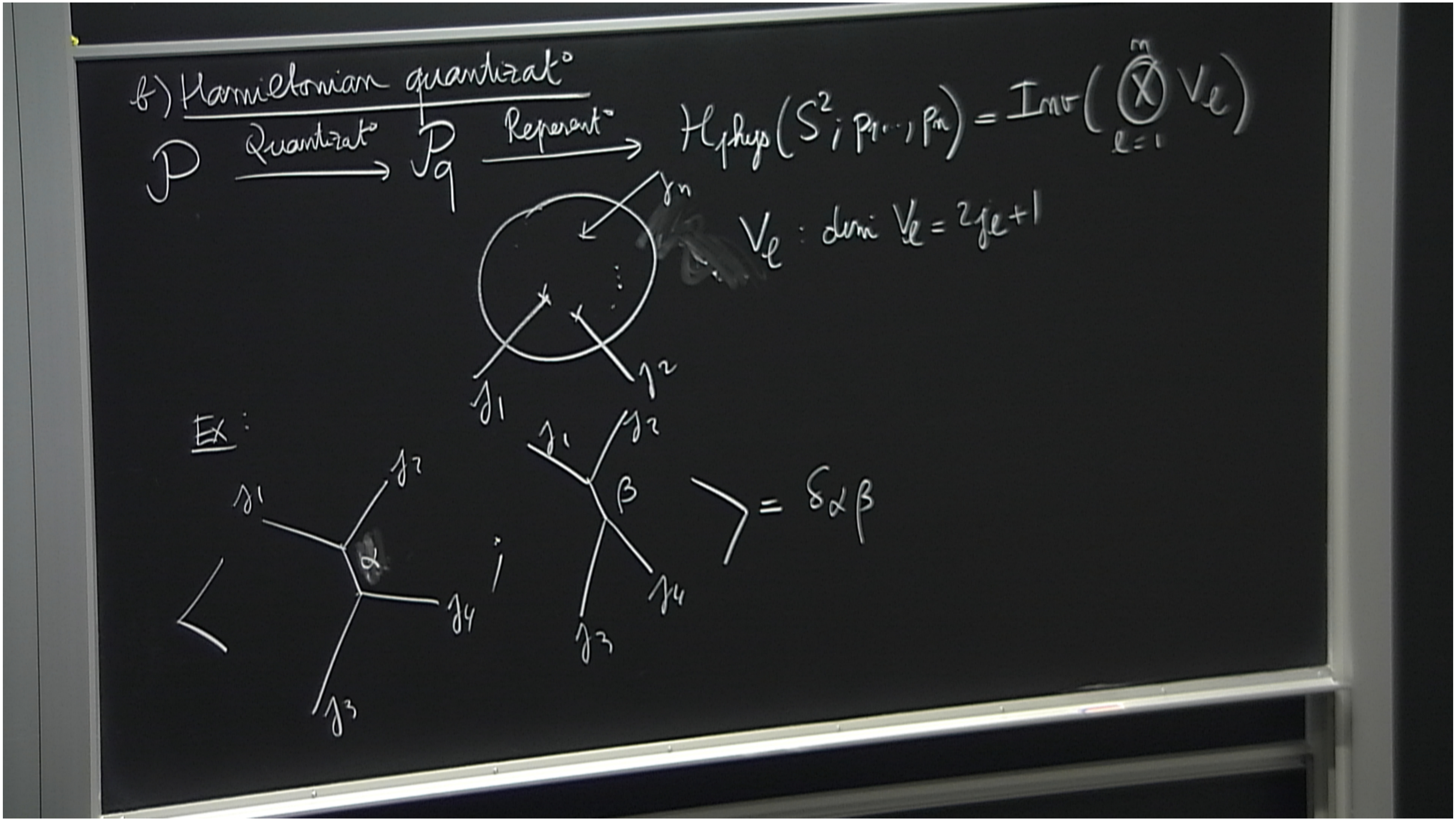
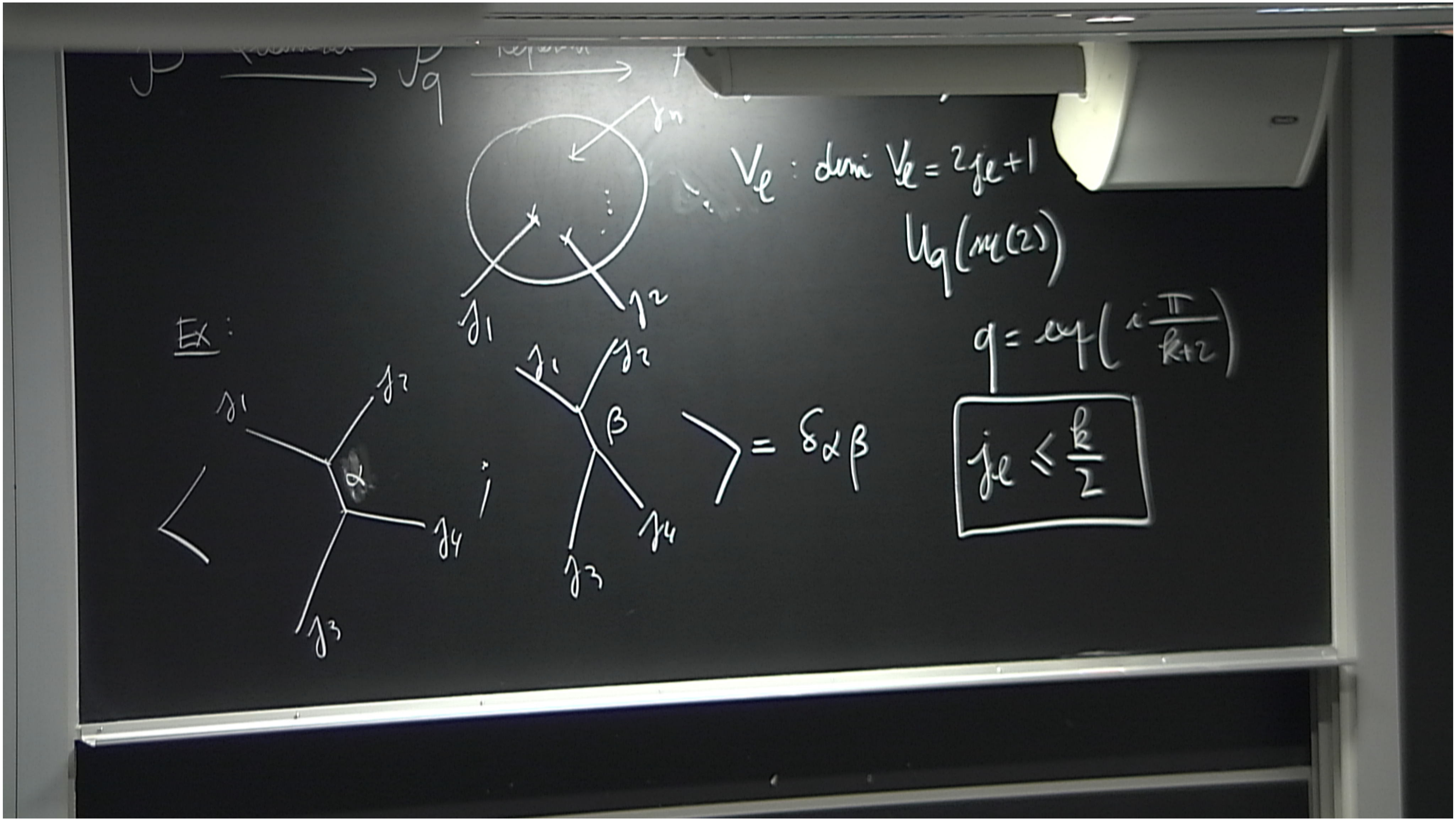
Rmqs: $|Z_{CS}|^2 = Z_{TV}$

$Z_{CS}(M, K, k) \rightarrow$ Jones Polyn.


b) Hamiltonian quantizat°
$\mathcal{P} \xrightarrow{\text{Quantizat°}} \mathcal{P}_q \xrightarrow{\text{Represent°}} \mathcal{H}_{phys}(S^2; p_1, \dots, p_n) = \mathrm{Inv}\left(\bigotimes_{\ell=1}^{n} V_\ell\right)$
V_ℓ : dim $V_\ell = 2j_\ell + 1$
j_n
j_1
j_2
Ex:
j_1
j_2
j_3
j_4
α
β
$= \delta_{\alpha\beta}$


V_ℓ : dim $V_\ell = 2j_\ell + 1$
$U_q(su(2))$
$q = \exp\left(i\frac{\pi}{k+2}\right)$
$\boxed{j_\ell \leq \frac{k}{2}}$
Ex:
j_1 j_2 j_3 j_4 α
j_1 j_2 j_3 j_4 β
$= \delta_{\alpha\beta}$
j_1 j_2 j_n

c) Verlinde Formula

$d_\ell = 2j_\ell + 1$

$$N_k(j_1,\dots,j_n) = \dim \mathcal{H}_{phys}(S^2, j_1,\dots,j_n)$$

$$= \frac{2}{k+2}\sum_{p=1}^{k+1} \sin^2\left(\frac{\pi}{k+2}p\right) \prod_{\ell=1}^{n} \frac{\sin\left(\frac{\pi}{k+2} p\, d_\ell\right)}{\sin\left(\frac{\pi}{k+2}p\right)}$$

d) Back to BH: semi-classical entropy

$k \gg 1$

$$N_\infty(j_1,\dots,j_n) = \frac{2}{\pi}\int_0^\pi d\theta \,\sin^2\theta \prod_{\ell=1}^{n} \frac{\sin(d_\ell\theta)}{\sin\theta}$$

Ex: $j_1 = \cdots = j_n = j \Rightarrow a_H = 4\pi \ell_p^2 \gamma\, n \sqrt{d^2-1}$

$$N_\infty(a_H) = \frac{2}{\pi}\int_0^\pi \sin^2\theta \left(\frac{\sin d\theta}{\sin\theta}\right)^n \propto d^n \Rightarrow S = \ln N_\infty(a_H) = n \ln d$$

$$S = \frac{a_H}{4\ell_p^2} \times \left(\frac{\gamma_0}{\gamma}\right) \quad \text{with } \gamma_0 = \frac{\ln d}{\pi\sqrt{d^2-1}}$$

3. Analytic continuat° : $\gamma = \pm i$

a) Naive continuat°

LQG : $a_j = 8\pi\gamma \ell_p^2 \sqrt{j(j+1)} = 8\pi i \ell_p^2 \sqrt{j(j+1)} \in i\mathbb{R}$

$$j = -\frac{1}{2} + is \Rightarrow a_s = 8\pi \ell_p^2 \sqrt{s^2 + \frac{1}{4}} \in \mathbb{R}$$

SU(1,1)

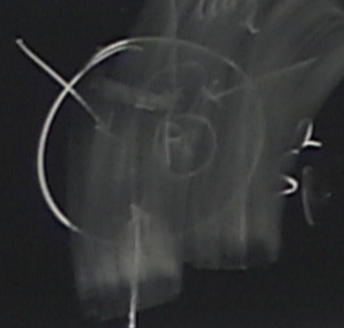
b) <u>Few words of CS theory</u> . $M = 3D$ Mfd

3D Gauge theory: $G = SU(2)$ $\quad A = A_\mu^i J_i\,,\ J_i \in su(2)\ ;\ tr(J_i J_j) = \delta_{ij}$

$$S_{CS}(A) = \frac{k}{4\pi}\int_M tr\left(A \wedge dA + \frac{2}{3} A \wedge [A \wedge A]\right)$$

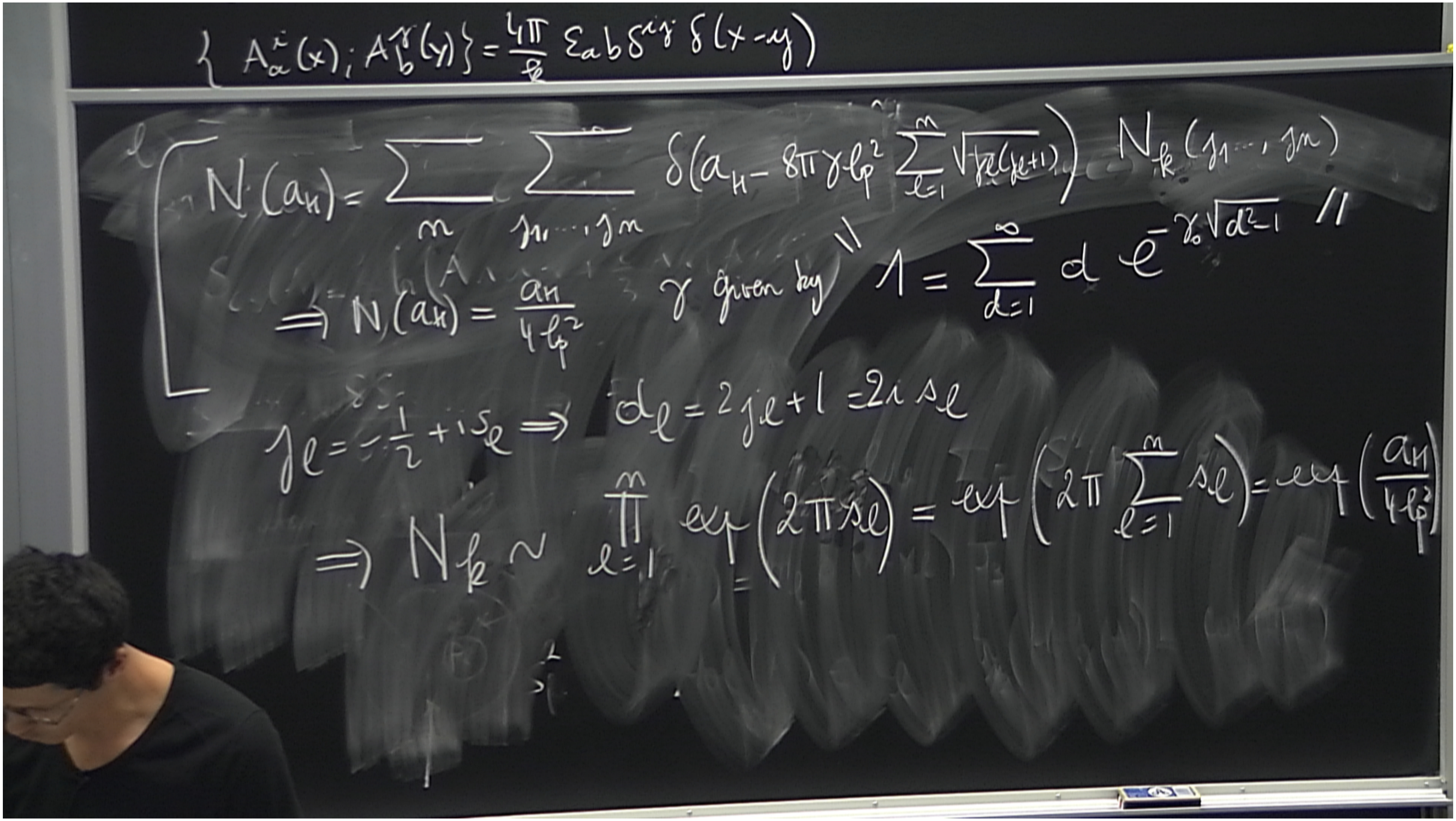
<u>e.o.m.</u> : $\dfrac{\delta S_{CS}}{\delta A} = 0 \Rightarrow F(A) = 0$

<u>Hamiltonian formulation</u> : $M = \Sigma \times \mathbb{R} \rightarrow$ here $\Sigma = (S^2; p_1, \dots, p_m)$



$$F(A) = m_\ell\, \delta(x - x_\ell)\, J_3$$

$$Hol_\ell(A) \in C_\ell \qquad tr_{1/2}\left(Hol_\ell(A)\right) = \cos(m_\ell)$$


$\{A_a^i(x), A_b^j(y)\} = \frac{4\pi}{k} \varepsilon_{ab} \delta^{ij} \delta(x-y)$
$N(a_H) = \sum_{n} \sum_{j_1 \cdots j_n} \delta\left(a_H - 8\pi\gamma l_p^2 \sum_{l=1}^{n} \sqrt{j_l(j_l+1)}\right) N_k(j_1 \cdots, j_n)$
$\Rightarrow N(a_H) = \frac{a_H}{4 l_p^2}$ γ given by $\quad 1 = \sum_{d=1}^{\infty} d\, e^{-\gamma_0 \sqrt{d^2-1}}$
$j_l = -\frac{1}{2} + i s_l \Rightarrow d_l = 2j_l + 1 = 2i s_l$
$\Rightarrow N_k \sim \prod_{l=1}^{n} \exp(2\pi s_l) = \exp\left(2\pi \sum_{l=1}^{n} s_l\right) = \exp\left(\frac{a_H}{4 l_p^2}\right)$