Title: Spinodal Instabilities and Super-Planckian Excursions in Natural Inflation

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Abstract: <p>Models such as Natural Inflation that use Pseudo-Nambu-Goldstone bosons (PNGB's) as the inflaton are attractive for many reasons. However, they typically require trans-Planckian field excursions $\Delta \Phi>M_{\rm Pl}$, due to the need for an axion decay constant $f>M_{\rm Pl}$ to have both a sufficient number of e-folds {\em and} values of $n_s,\ r$ consistent with data. Such excursions would in general require the addition of all other higher dimension operators consistent with symmetries, thus disrupting the required flatness of the potential and rendering the theory non-predictive. We show that in the case of Natural Inflation, the existence of spinodal instabilities (modes with tachyonic masses) can modify the inflaton equations of motion to the point that versions of the model with $f<M_{\rm Pl}$ can still inflate for the required number of e-folds. The instabilities naturally give rise to two separate phases of inflation with different values of the Hubble parameter H one driven by the zero mode, the other by the unstable fluctuation modes. The values of n_s and r typically depend on the initial conditions for the zero mode, and, at least for those examined here, the values of r tend to be unobservably small.</p>

Spinodal Natural Inflation

Rich Holman, PI Cosmology Seminar
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based on arXiv:1412.6879 w/ Albrecht and Ben Richard

Why so flat? The eta problem

To get enough e-folds of inflation we need the slow-roll parameters to be small

$$\epsilon = \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1$$

$$\eta = M_P^2\left(\frac{V''(\phi)}{V(\phi)}\right) \ll 1$$

The problem arises if we consider the inflaton as described by an effective field theory valid only at scales below some cutoff, such as the Planck scale.

If this theory is obtained by integrating out "Planck Slop" fields, the slow roll parameters can get corrected:

$$\Delta V = cV(\phi)\frac{\phi^2}{\Lambda^2} \Rightarrow \Delta\eta \sim 2c\left(\frac{M_P}{\Lambda}\right)^2$$

Generically, flatness is not protected against quantum corrections!

Calling Mr. Natural! PNGB's to the Rescue?

How can we protect the flatness of the potential?

Shift symmetry!

$$\Phi \to \Phi + c$$

Only the kinetic term (and higher derivatives) is invariant under shifts; that's a little too much symmetry!

Nambu-Goldstone bosons exhibit this symmetry. But if the underlying symmetry is explicitly broken as well we can generate a potential.

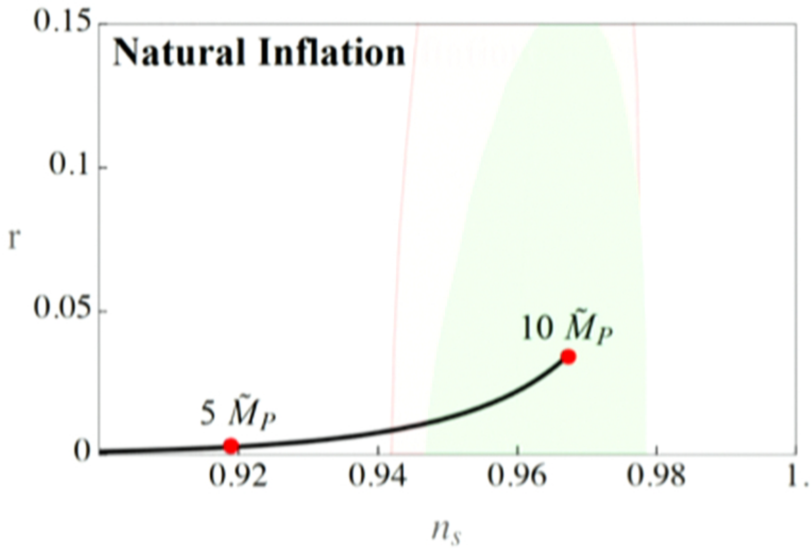
Following QCD axion with an anomalous U(1) symmetry, try the potential

$$V(\Phi) = \Lambda^4 \left(1 + \cos\frac{\Phi}{f}\right)$$

Natural Inflation

What does it take to get a good inflationary scenario out of natural inflation?

$$\Phi_{NI} = 2f \sin^{-1}\left[\exp\left(\frac{2f^2 \log(\sin\left(\frac{\Phi_E}{2f}\right) - N}{2f^2}\right)\right]$$



From Sanz and Croon, arXiv:1411.7809

Need f greater than the Planck mass!

This is a problem from the EFT point of view!

$$V_{NR} = \Phi^4 \left(\frac{\Phi}{\tilde{M}_P} \right)^n$$

Higher order terms of this form would dominate over the original potential as the field travels a distance f.

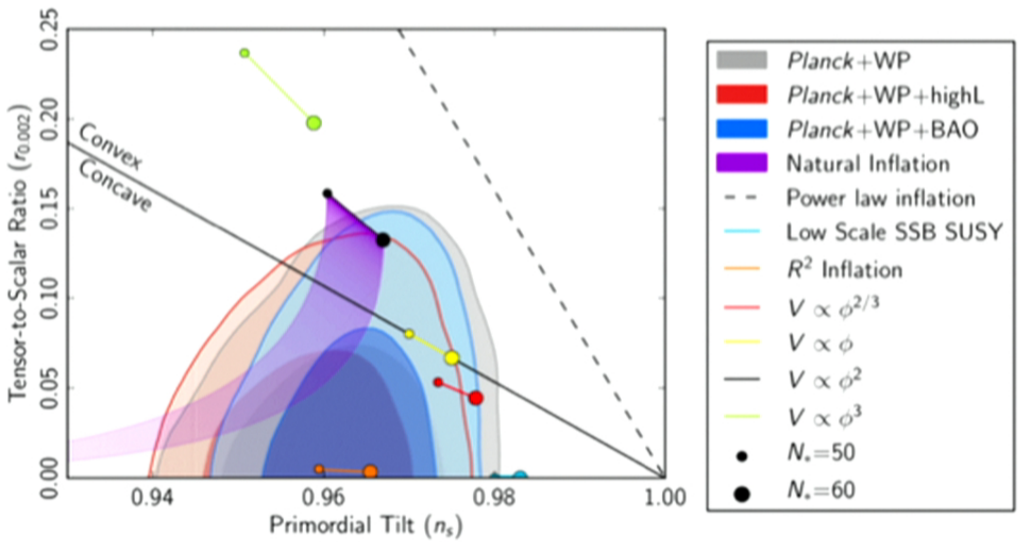
It's Mr. Natural's Excellent Super-Planckian adventure!

There are many possible solutions to this problem

- ★ Axion Monodromy
- ★ N-flation
- ★ Aligned Inflation

We'd like to try something a little different.

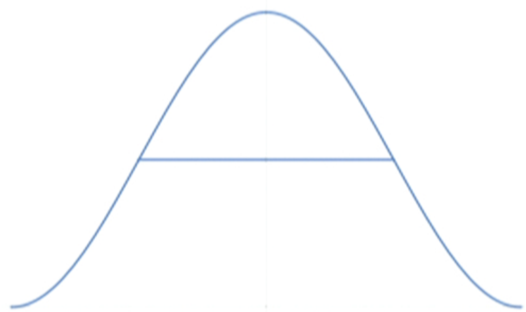
Spinodalling for e-folds



First note that the Planck data prefers small-field inflation, with a concave potential during inflation.

In all the approaches to inflation in these potentials, the dynamics of the inflaton is treated as if fixed JUST by the potential

But for these concave potentials, this is NOT CORRECT!



If zero mode is above the spinodal line where the effective mass vanishes, low wavenumber modes will be TACHYONIC

One resummation technique uses the Hartree approximation, which is the N=1 version of the large-N approximation

(RH and Dan Cormier PRD62 (2000), 023520)

$$\psi^{2n} \to \frac{(2n)!}{2^n(n-1)!}\langle\psi^2\rangle^{n-1}\psi^2 - \frac{(2n)!(n-1)}{2^n n!}\langle\psi^2\rangle^n$$

$$\psi^{2n+1} \to \frac{(2n+1)!}{2^n n!}\langle\psi^2\rangle^n\psi$$

This makes the interacting theory a Gaussian one, but with memory of the interactions encoded in the self-consistent calculation of the two point function

Diagrammatically, this resums “cactus” diagrams.

Now, take the NI potential, insert the split into zero mode and fluctuations, then apply the Hartree-ization process to this

$$\cos\left(\frac{\psi}{f}\right) \rightarrow \left(1 - \frac{(\psi^2 - \langle\psi^2\rangle)}{2f^2}\right)\exp\left(-\frac{\langle\psi^2\rangle}{2f^2}\right),$$

$$\sin\left(\frac{\psi}{f}\right) \rightarrow \frac{\psi}{f}\exp\left(-\frac{\langle\psi^2\rangle}{2f^2}\right)$$

$$\ddot{\phi} + 3H(t)\dot{\phi} - \frac{\Lambda^4}{f}\exp\left(-\frac{\langle\psi^2\rangle}{2f^2}\right)\sin\left(\frac{\phi}{f}\right) = 0,$$

$$\ddot{f}_k + 3H(t)\dot{f}_k + \left[\frac{k^2}{a^2(t)} - \frac{\Lambda^4}{f^2}\exp\left(-\frac{\langle\psi^2\rangle}{2f^2}\right)\cos\left(\frac{\phi}{f}\right)\right]f_k = 0$$

$$H^2(t) = \frac{1}{3\tilde{M}_{\rm Pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\langle\dot{\psi}^2\rangle + \frac{1}{2a^2}\left\langle(\vec{\nabla}\psi)^2\right\rangle + \Lambda^4\left(1 + \cos\left(\frac{\phi}{f}\right)\exp\left(-\frac{\langle\psi^2\rangle}{2f^2}\right)\right)\right]$$

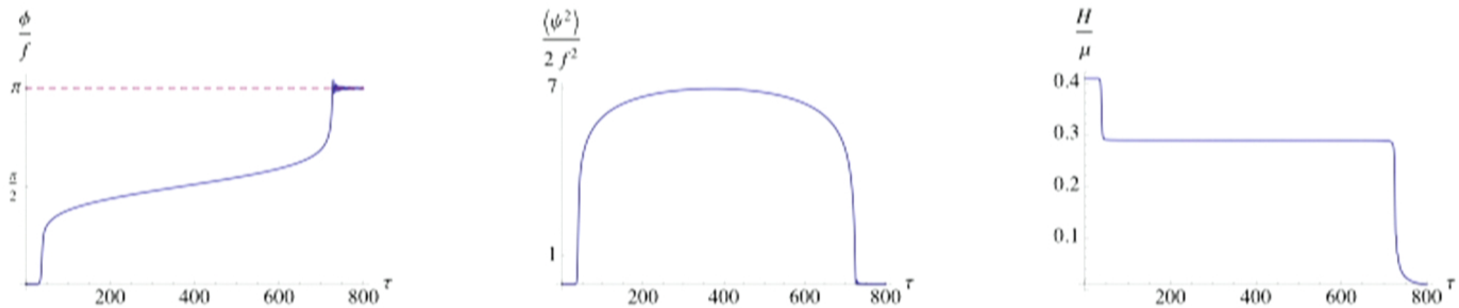
We see that the mode equations depend on the two point function which in turn depends on the modes; this is the self-consistency of the system.

What happens?

Depending on where the zero mode is placed, the dynamics starts off as usual, but then becomes fluctuation dominated.

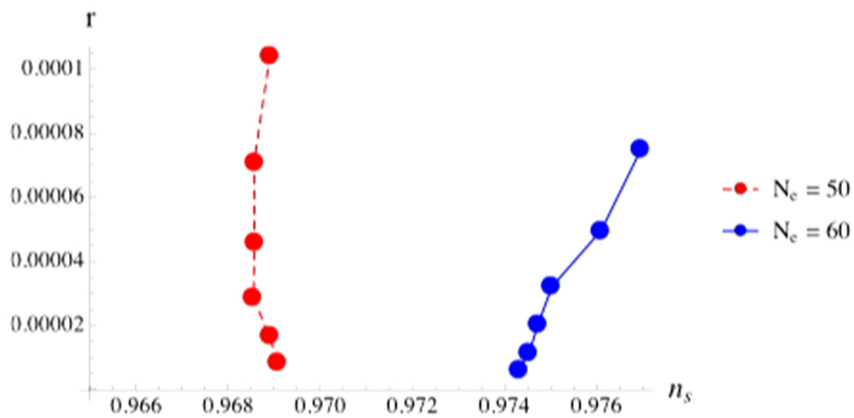
The hope would be that even for parameters that would NOT allow for enough inflation just from the potential, the fluctuation dominated phase would kick in some more e-folds

Let's pick a situation where f is smaller than the Planck scale.



The usual dynamics would give about 5 e-folds or so. But once the zero mode gets near the spinodal region, fluctuations grow non-perturbatively and a SECOND phase of inflation starts! This gives rise to the rest of the e-folds needed.

What about r and ns?



This is a collection of models with about the same number (~200) of e-folds but with f less than the Planck mass. r tends to be small (but we all know BICEP didn't see primordial B modes, right?) but ns is within the error contours from PLANCK.

Interesting fact: The background dynamics can be reproduced by looking at the zero mode and a second zero mode constructed from the long-wavelength modes

$$\sigma(t) = \langle \psi^2 \rangle_{k<aH}^{\frac{1}{2}}$$

$$\ddot{\phi} + 3H\dot{\phi} + \mathcal{V}'(\sigma, \phi) = 0$$

$$\ddot{\sigma} + 3H\dot{\sigma} + \mathcal{V}''(\sigma, \phi) = 0$$

$$H^2 = \frac{1}{3\tilde{M}_P^2}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\sigma}^2 + \mathcal{V}(\sigma, \phi)\right]$$

$$\mathcal{V}(\sigma, \phi) \equiv \sum_{n=0}^{\infty} \frac{1}{2^n n!} \sigma^{2n} V^{(2n)}(\phi)$$

$$\mathcal{V}(\sigma, \phi)_{NI} = \Lambda^4 \left(1 + \exp\left(-\frac{\sigma^2}{2f^2}\right)\cos\frac{\phi}{f}\right)$$

Take-Home Lessons, Questions and Stuff to do

Take-Home Lesson 1: You HAVE to include quantum dynamics to get the full evolution of the system! Just using the effective potential is NOT enough.

This is especially important in light of PLANCK wanting us to be in the concave part of the potential during inflation. There will always be spinodal regions in this case.

Take-Home Lesson 2: Spinodal effects can save NI from itself! No need for super-Planckian excursions (unless r stays large!)

Question: How do we know that the Hartree approximation captures the right physics?

It's true that the HA is uncontrolled. On the other hand, the fluctuations have to be tamed somehow; neglecting this is NOT an option!

AND.. the same effect shows up in large-N. However, there the spinodal line lies at the bottom of the well, where the Goldstone modes appear. If the potential is set to be zero there, no second phase of inflation will appear.

To do:

1. Better grip on HA
2. Delineate parameter space of spinodal NI
3. Other models, maybe with more than one axion