Title: Mirror Theorem between Landau-Ginzburg models

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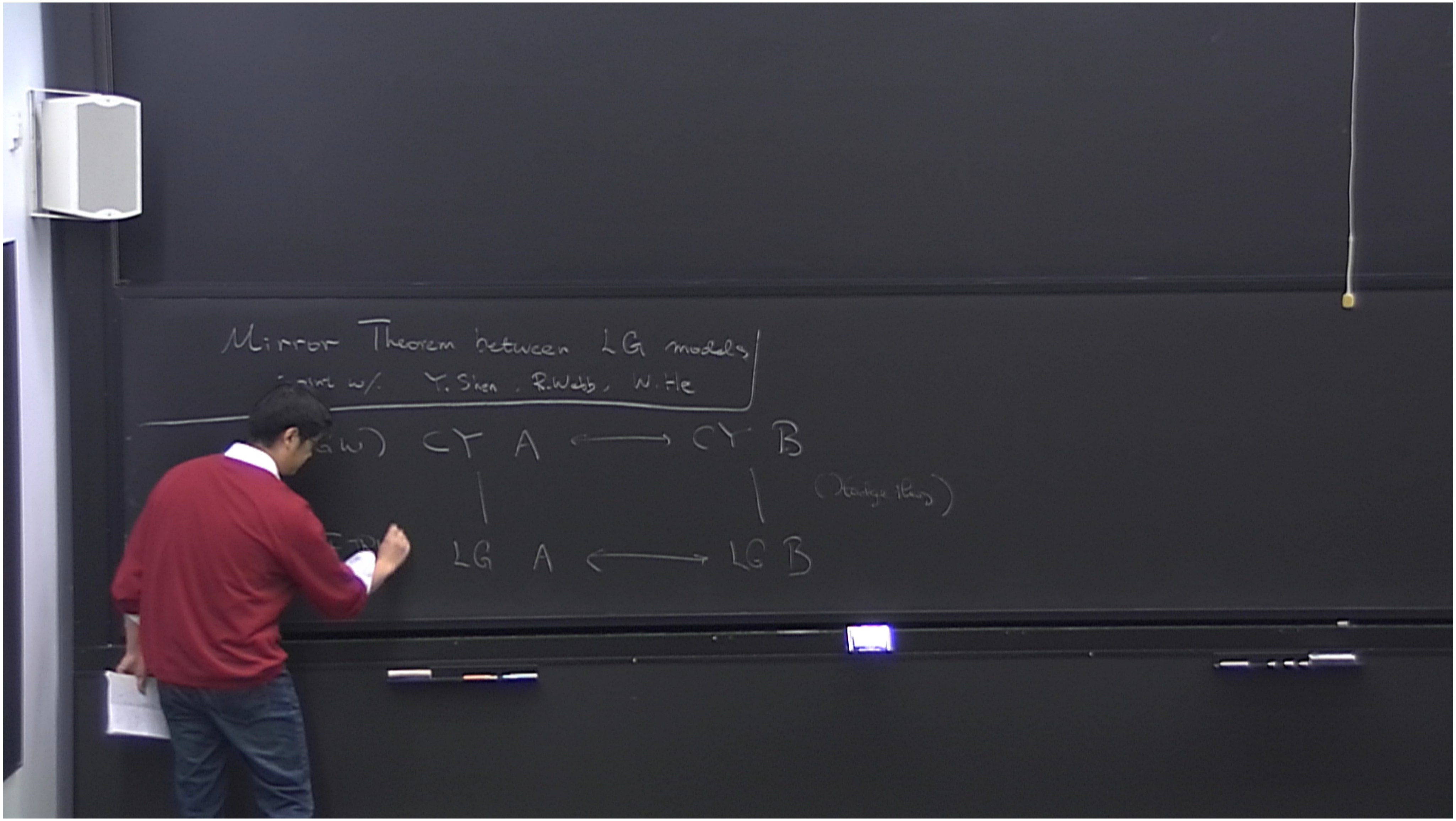
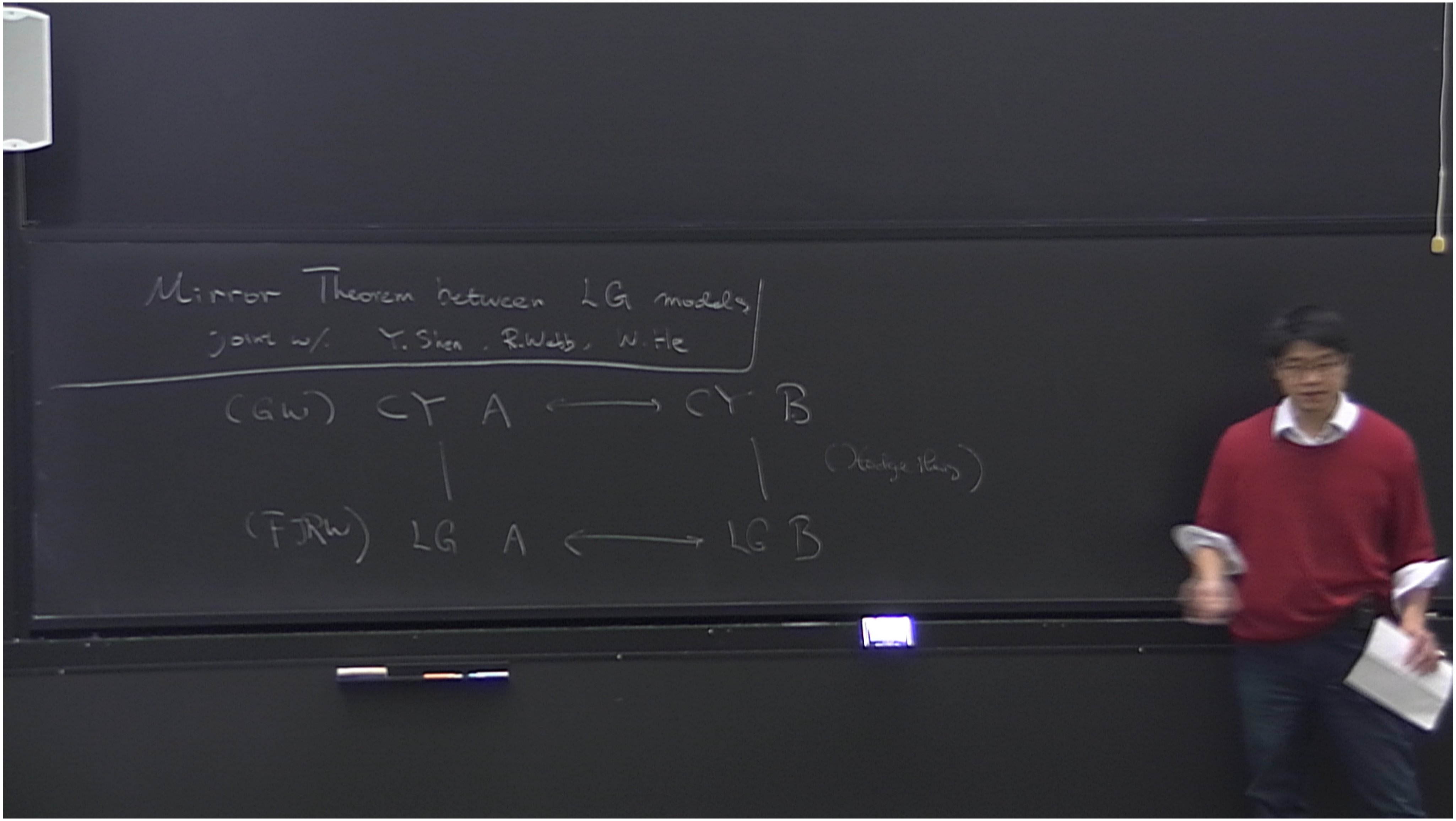
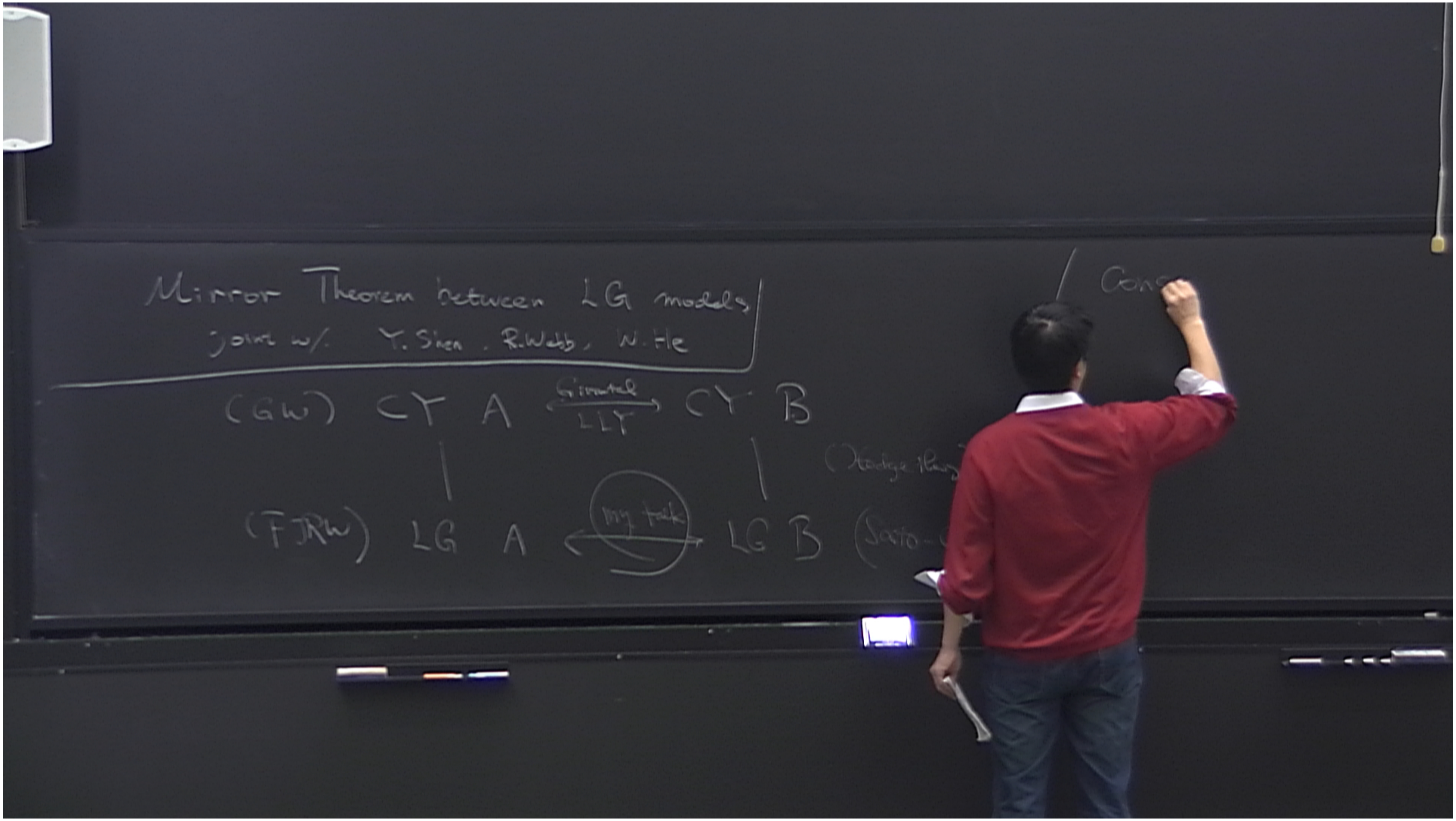
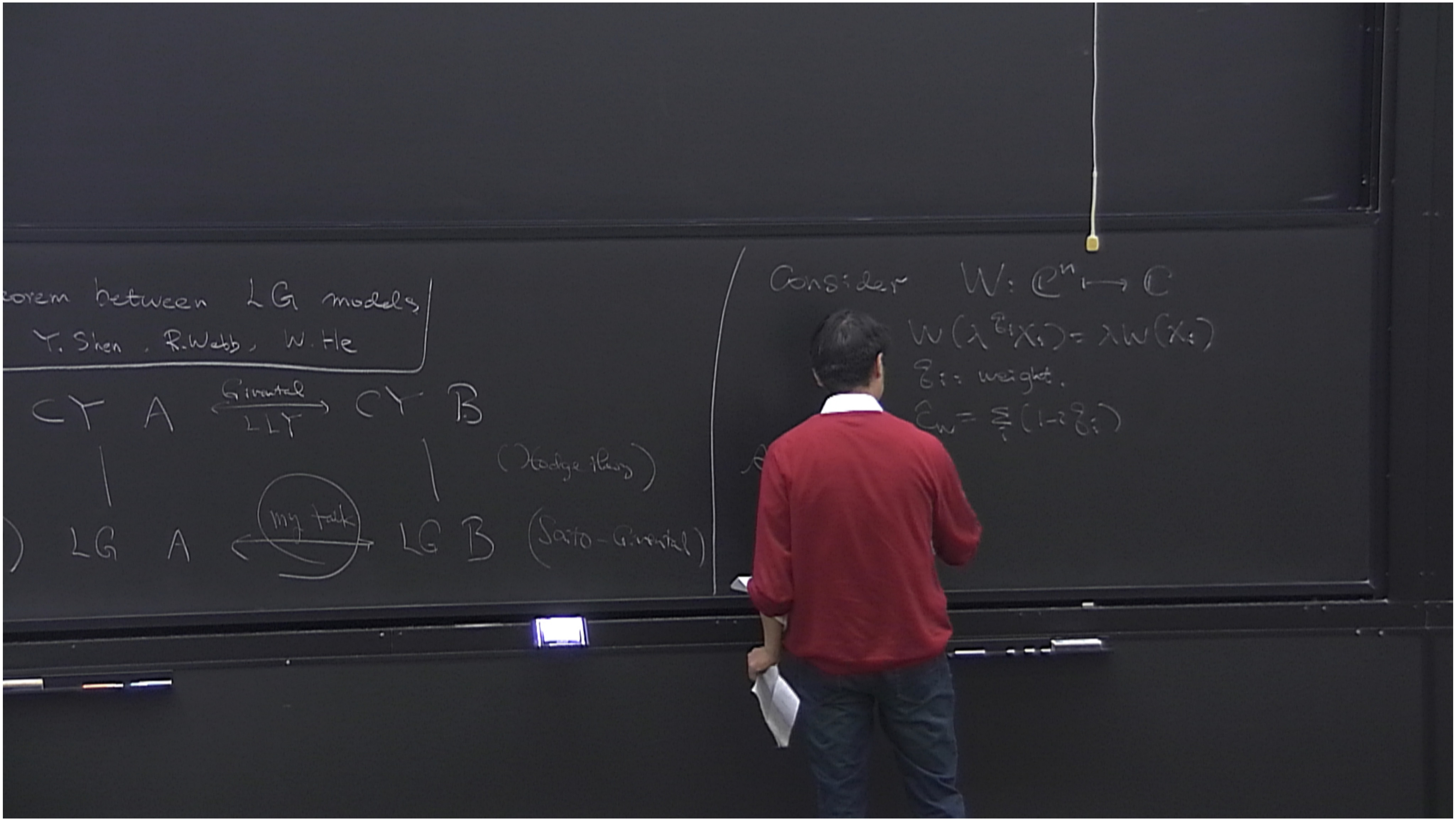
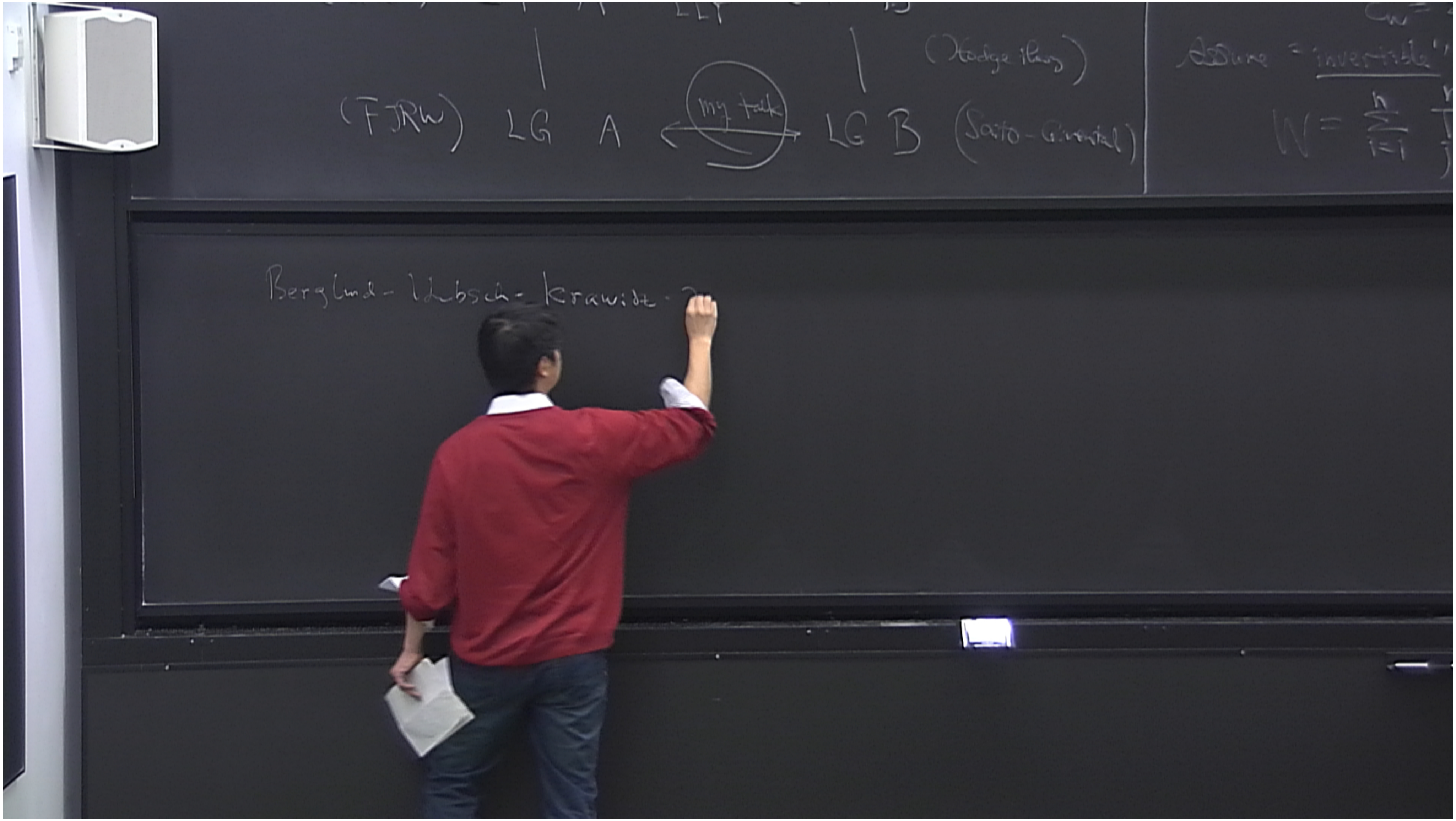
Abstract: <p>In this talk, I will prove the Landau-Ginzburg mirror symmetry conjecture for general quasi-homogenous singularities, i.e., the FJRW theory (LG A-model) of such polynomials is equivalent to the Saito-Givental theory (LG B-model) of the mirror polynomial. This is joint work with Weiqiang He, Rachel Webb and Yefeng Shen. </p>

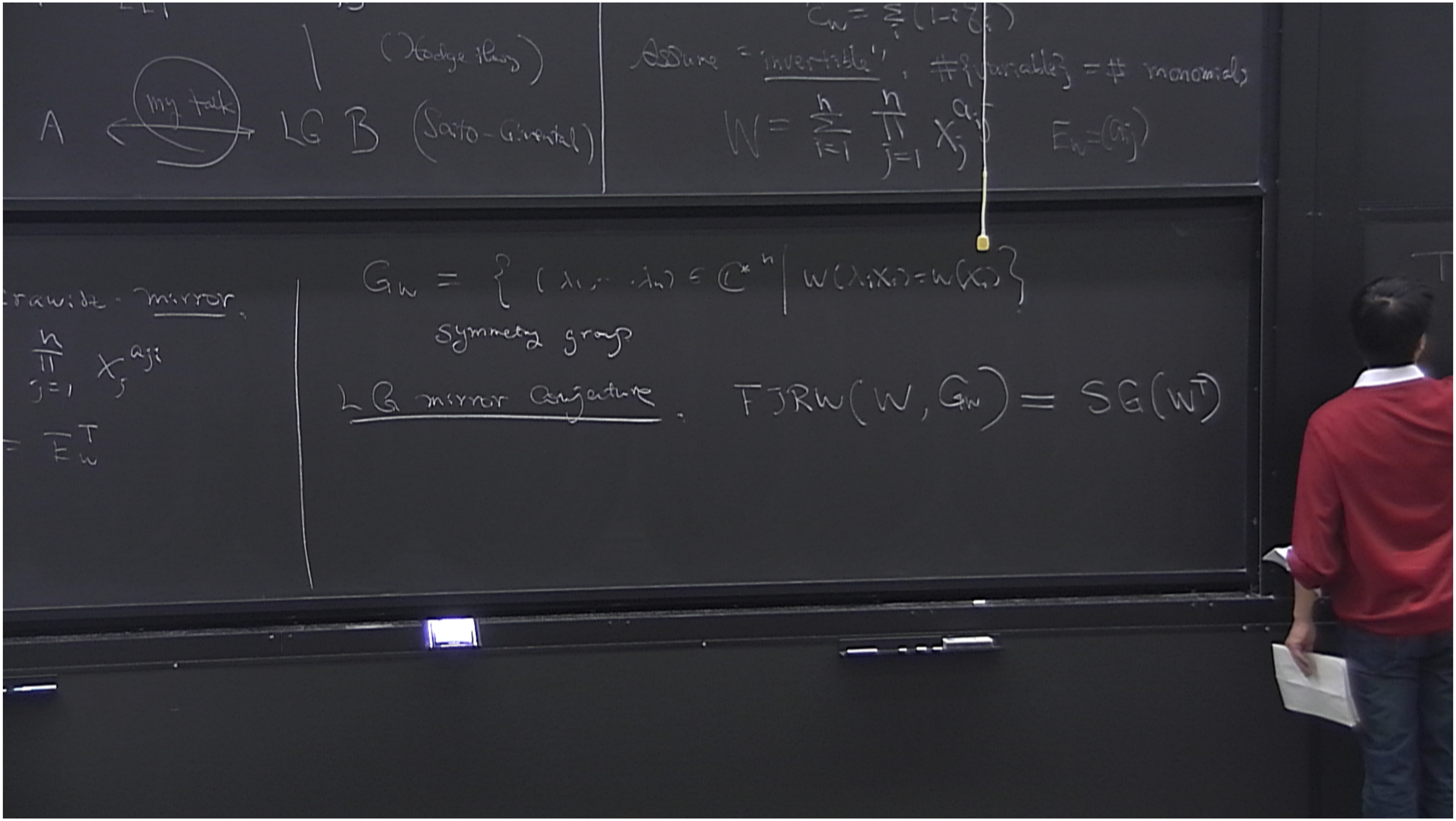

Mirror Theorem between LG models
w/. Y. Shen, R. Webb, W. He
CY A ⟷ CY B
(Hodge theory)
LG A ⟷ LG B

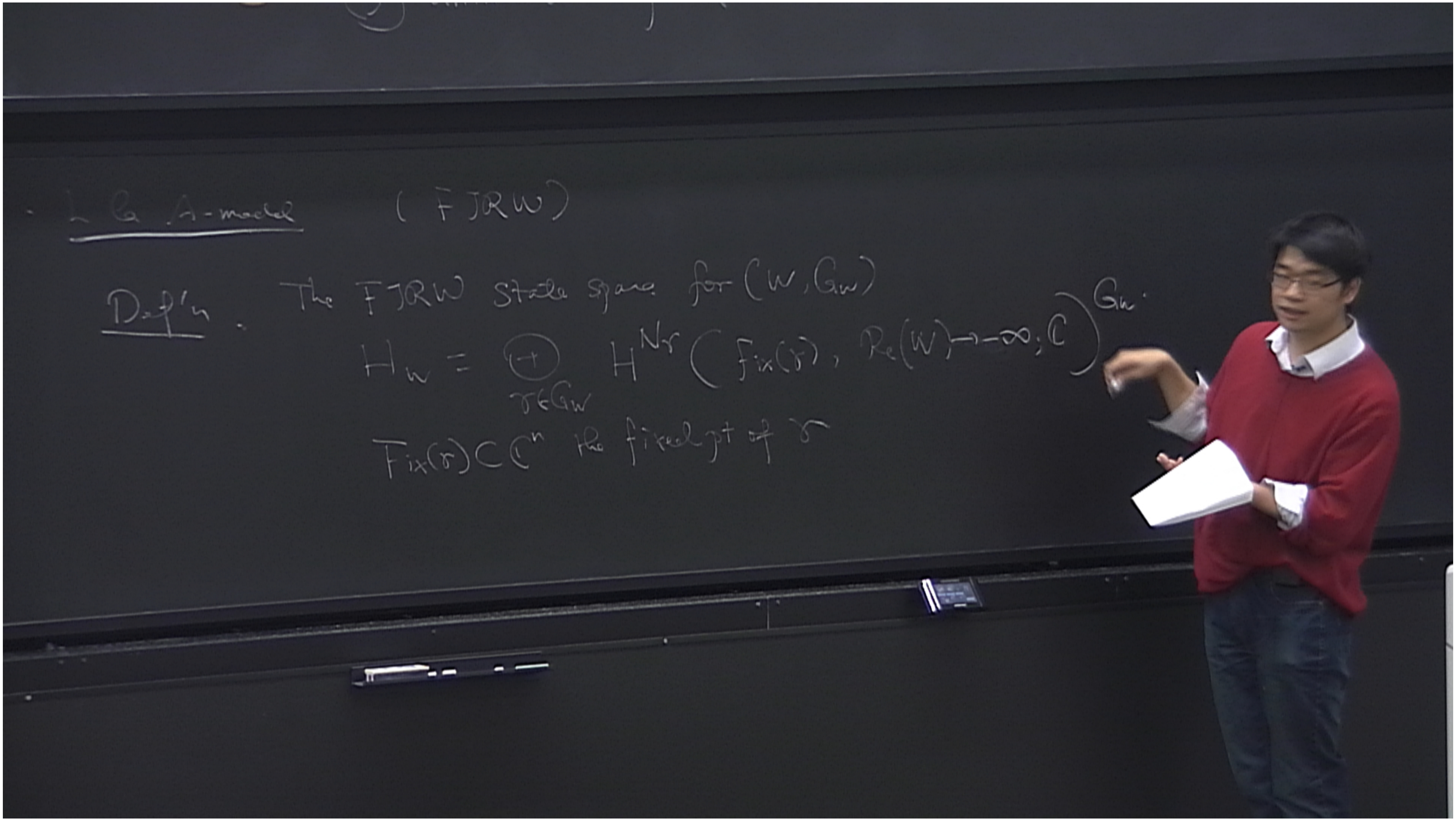

Mirror Theorem between LG models
joint w/ Y. Shen, R. Webb, N. He
(GW) CY A ⟷ CY B
(Hodge theory)
(FJRW) LG A ⟷ LG B

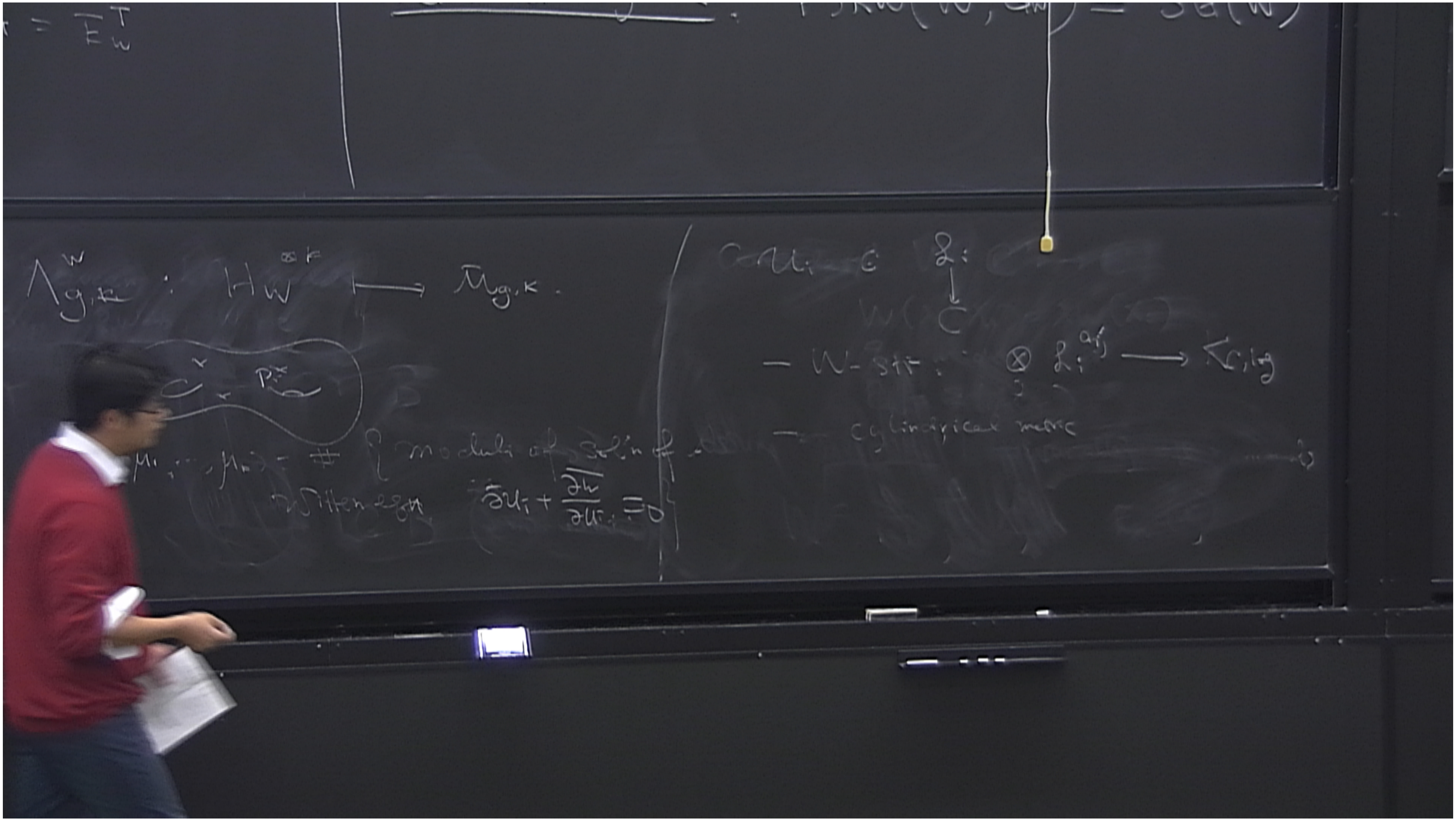

Mirror Theorem between LG models
joint w/. Y. Shen, R. Webb, W. He
(GW) CY A
Givental
LLY
CY B
(FJRW) LG A
my talk
LG B

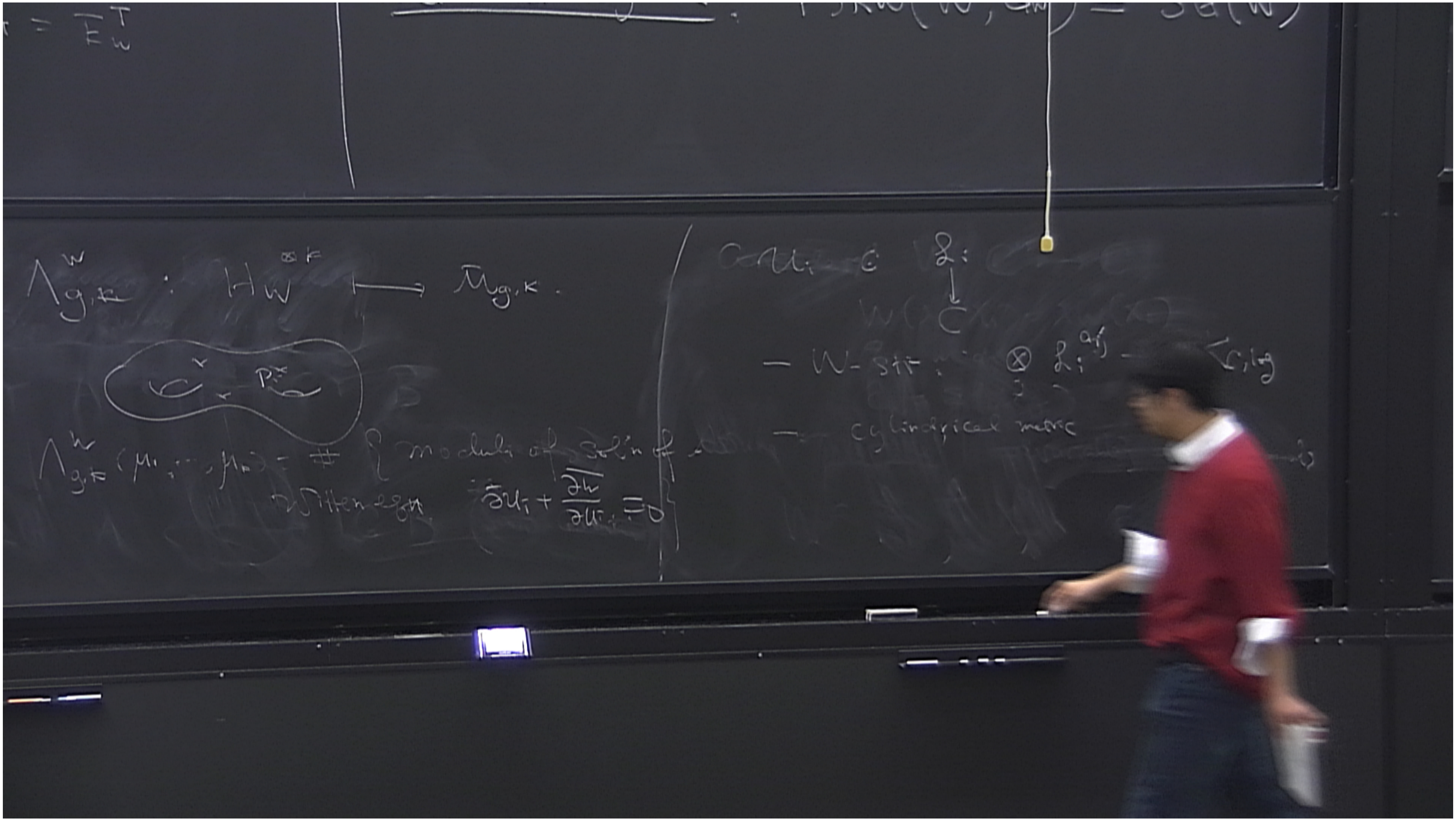

between LG models
Y. Shen, R. Webb, W. He
CY A
Givental
LLY
CY B
(Hodge theory)
LG A
my talk
LG B
(Saito-Givental)
Consider W: ℂⁿ ⟼ ℂ
W(λ^{q_i} x_i) = λW(x_i)
q_i: weight.
ĉ_W = Σ_i (1-2q_i)

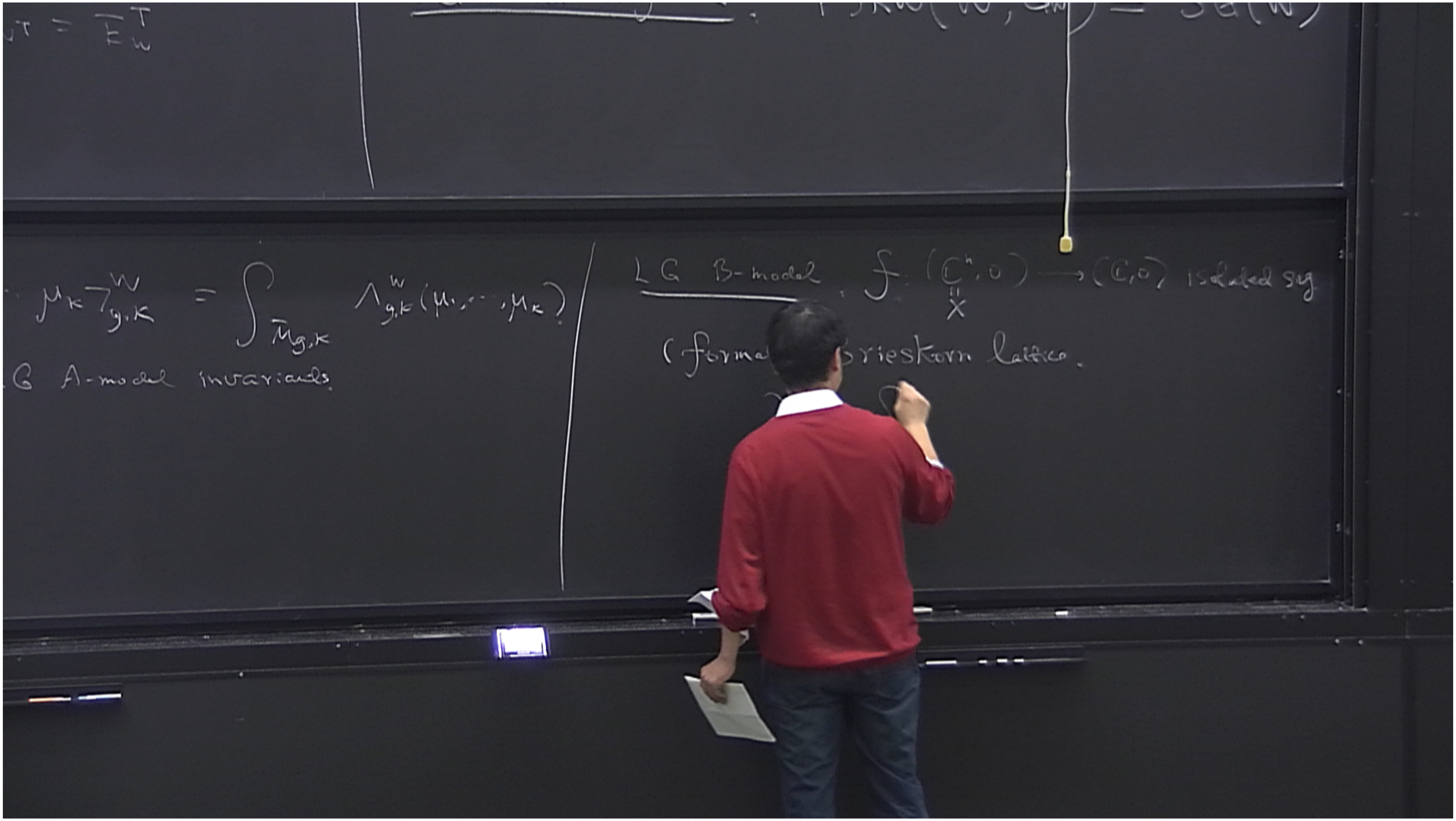

(Hodge theory)
(FJRW) LG A
my talk
LG B (Saito-Givental)
Berglund-Hübsch-Krawitz


(Hodge theory)
A ← my talk → LG B (Saito-Givental)
Assume "invertible", #{variables} = # monomials
W = Σ_{i=1}^{n} Π_{j=1}^{n} X_j^{a_ij}   E_W = (a_ij)
rawitz - mirror
E_W^T
G_W = { (λ_1, ..., λ_n) ∈ C*^n | W(λ_i X_i) = W(X_i) }
symmetry group
LG mirror Conjecture. FJRW(W, G_W) = SG(W^T)


LG A-model (FJRW)
Def'n. The FJRW state space for (W, Gw)
Hw = ⊕ H^Nγ (Fix(γ), Re(W)→-∞; C)^Gw.
γ∈Gw
Fix(γ) ⊂ C^n the fixed pt of γ


moduli of sol'n of
W-str
cylindrical metric


modului of sol'n of
Witten eqn
cylindrical metric
W-str


LG B-model
isolated sing
rieskorn lattice.
LG A-model invariants

$\langle \ \rangle^W_{g,k} = \int_{\overline{\mathcal{M}}_{g,k}} \Lambda^W_{g,k}(\mu_1,\cdots,\mu_k)$

-model invariants

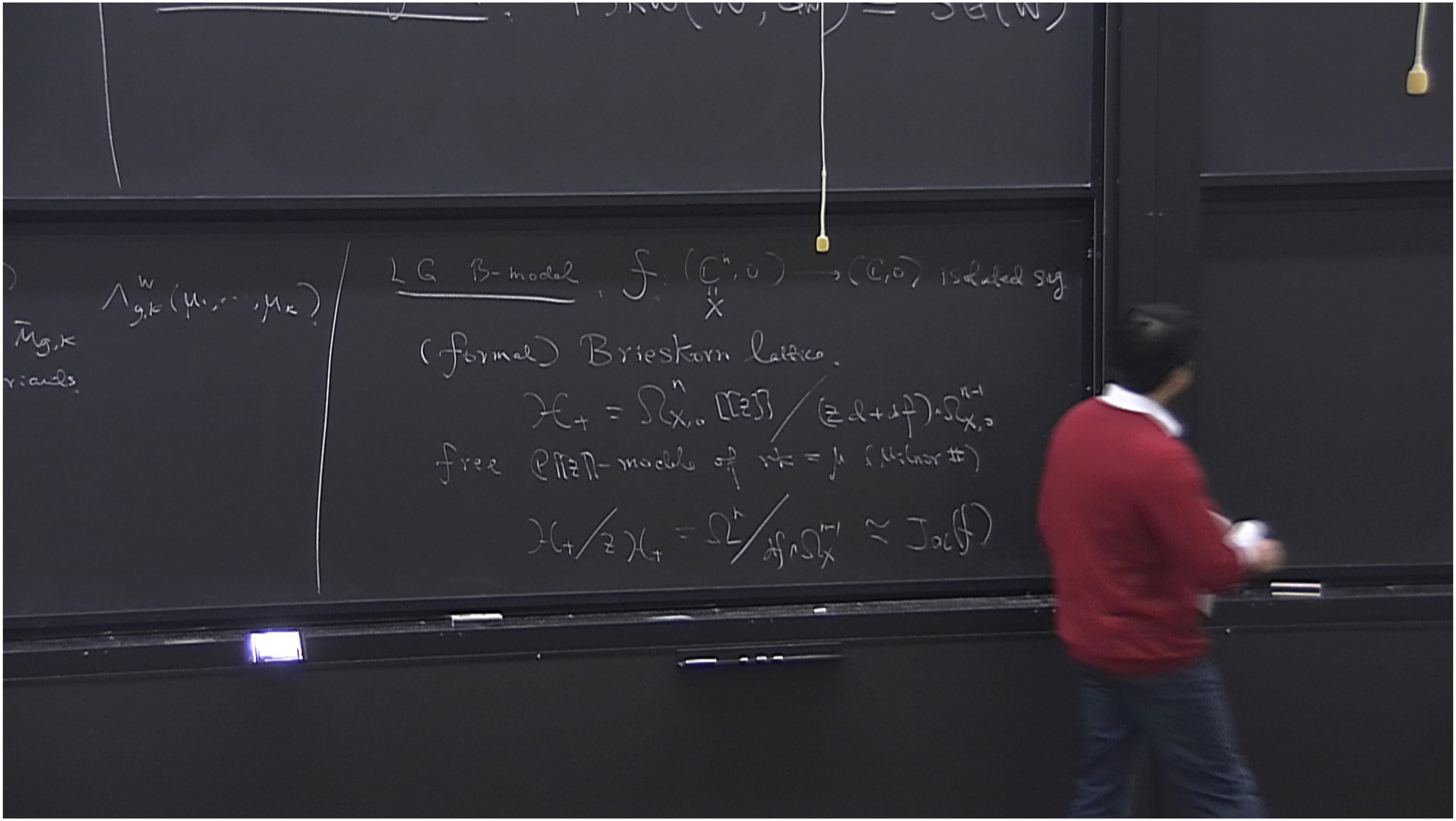
LG B-model: $f : (\mathbb{C}^n, 0) \to (\mathbb{C}, 0)$ isolated sing, $\mathbb{C}^n = X$

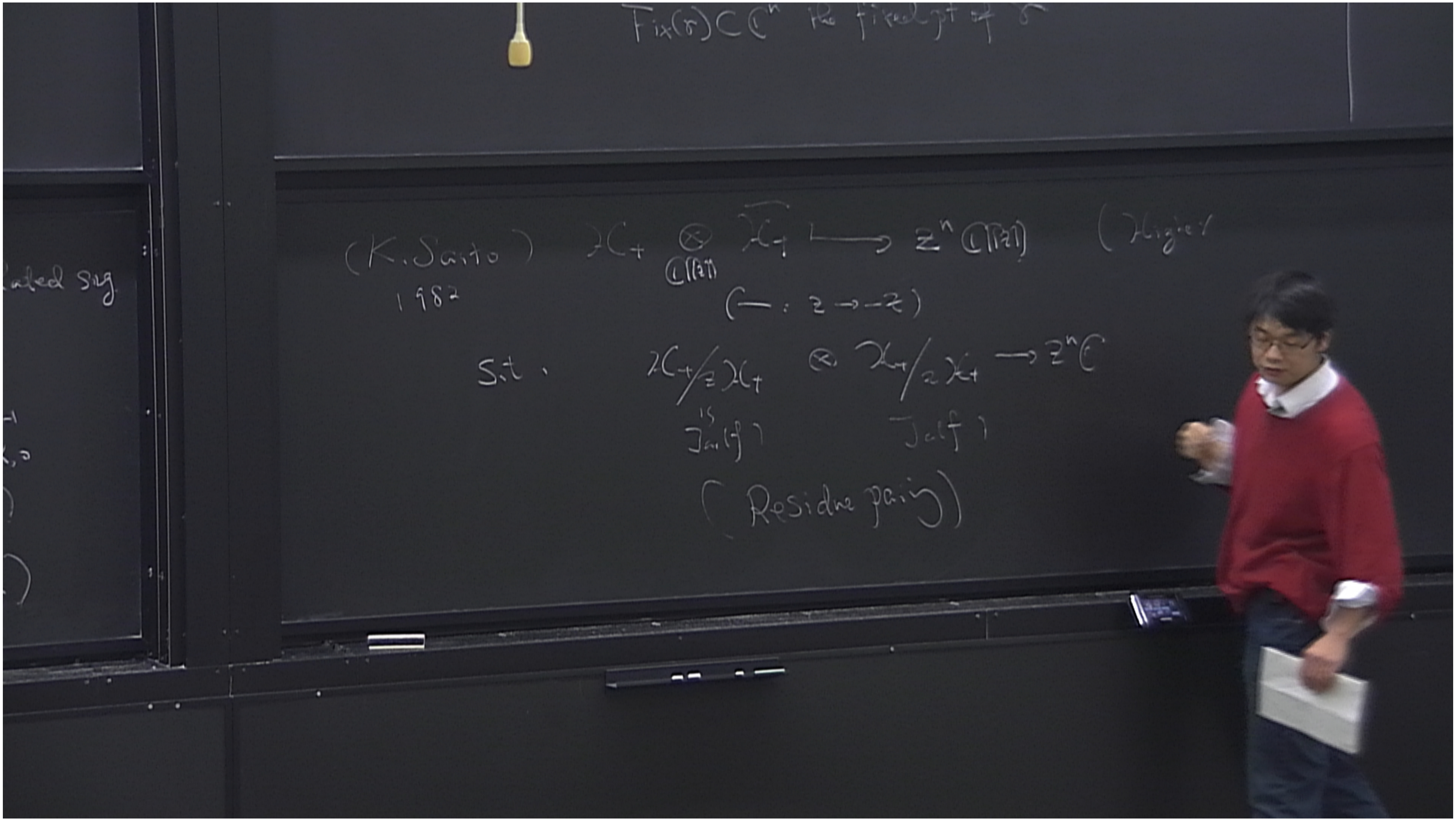
(formal) Brieskorn lattice.

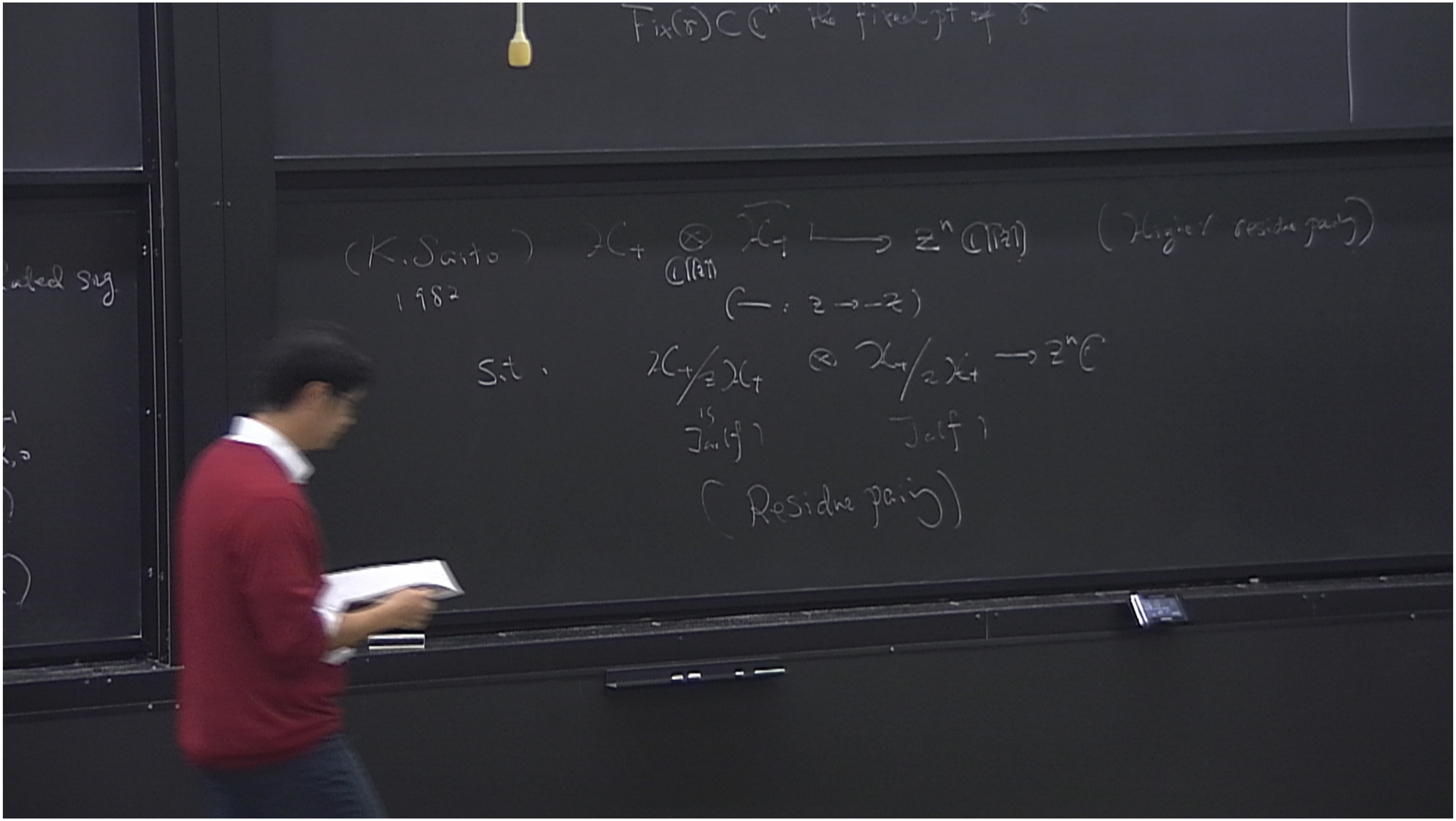
$$\mathcal{H}_+ = \Omega^n_{X,0}[[z]] \big/ (z\,d + df\wedge)\,\Omega^{n-1}_{X,0}$$

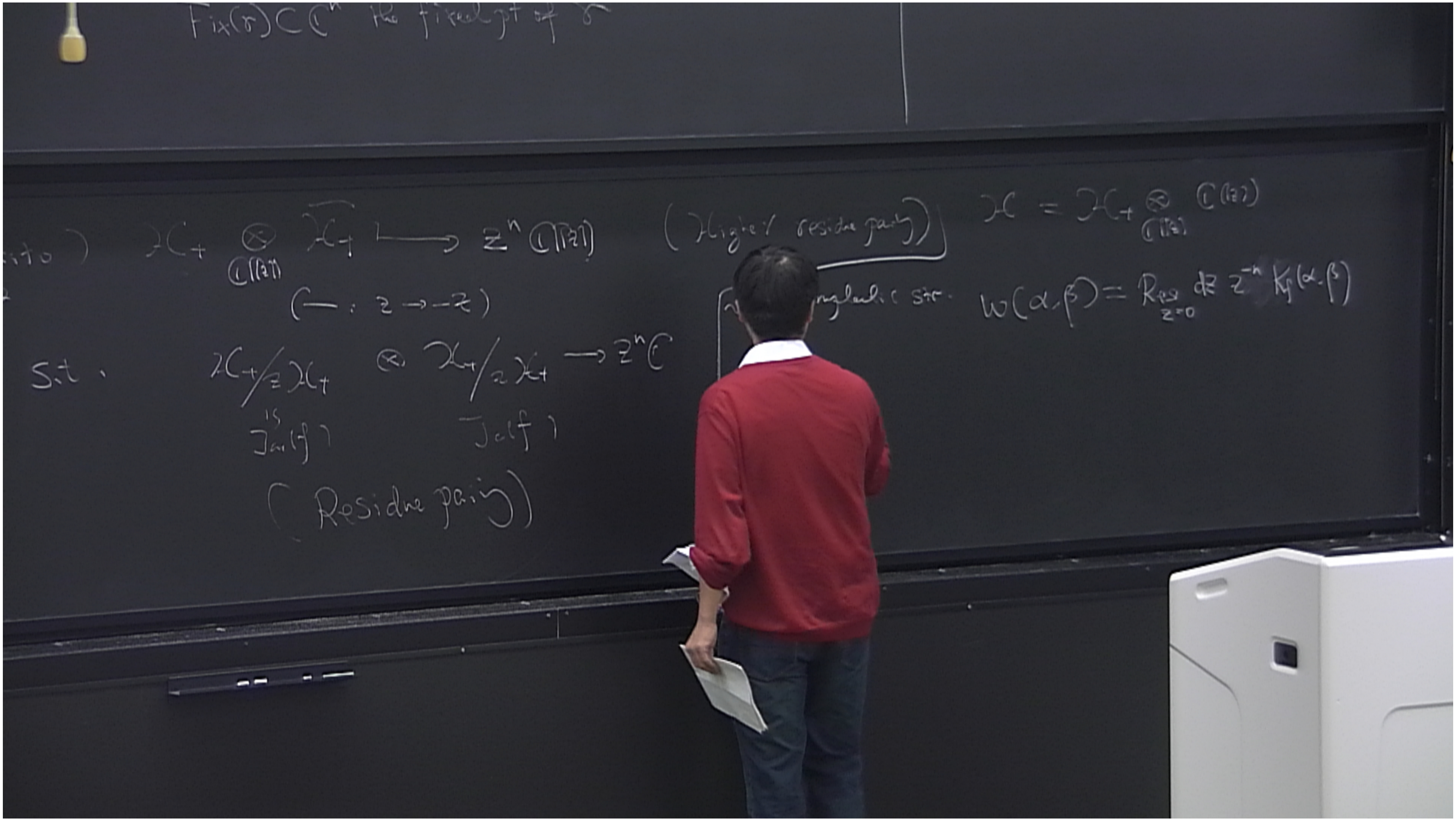
free $\mathbb{C}[[z]]$-module of rk $= \mu$ (Milnor #)

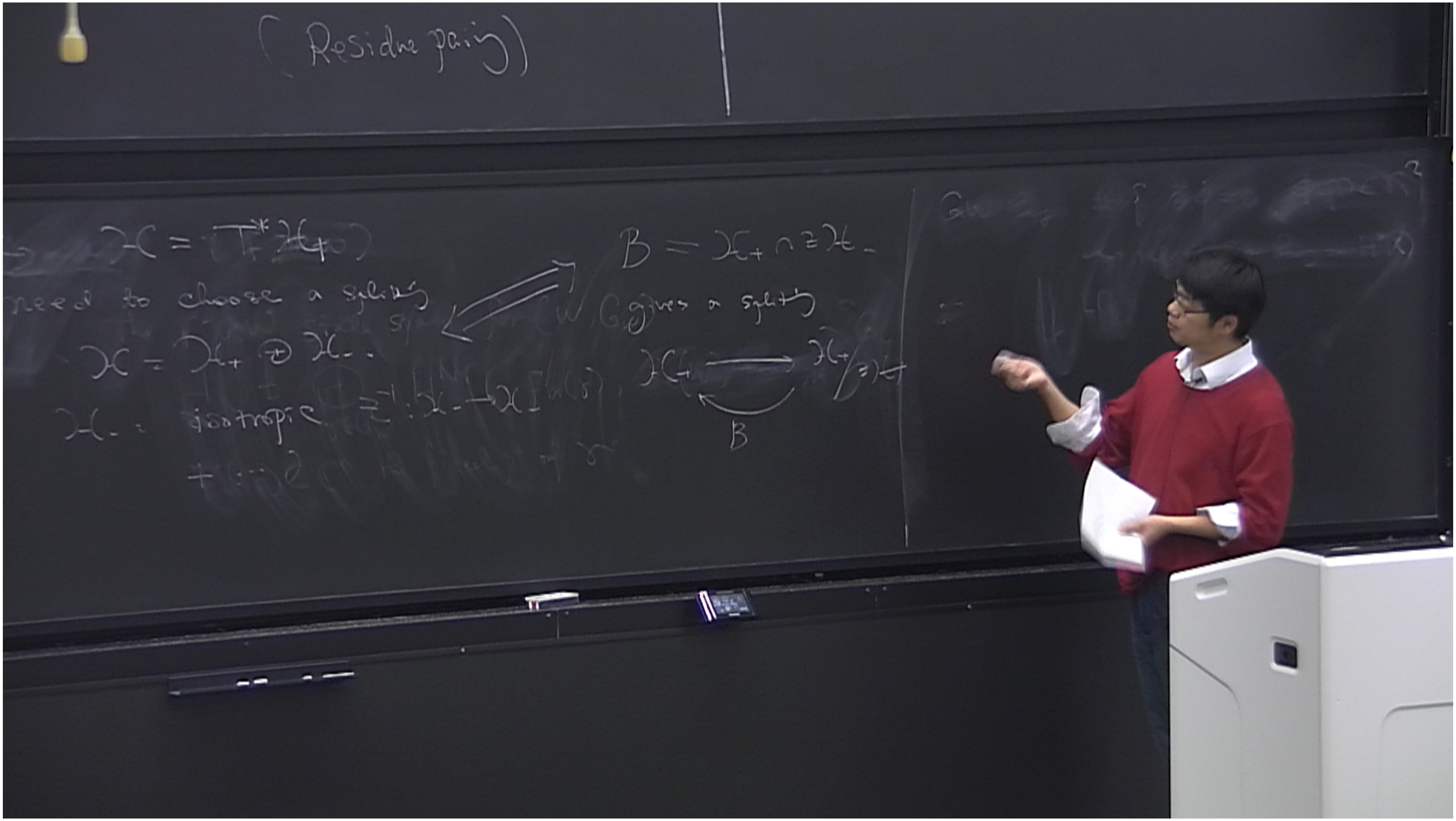
$$\mathcal{H}_+ / z\mathcal{H}_+ = \Omega^n \big/ df\wedge\Omega^{n-1}_X \simeq \mathrm{Jac}(f)$$

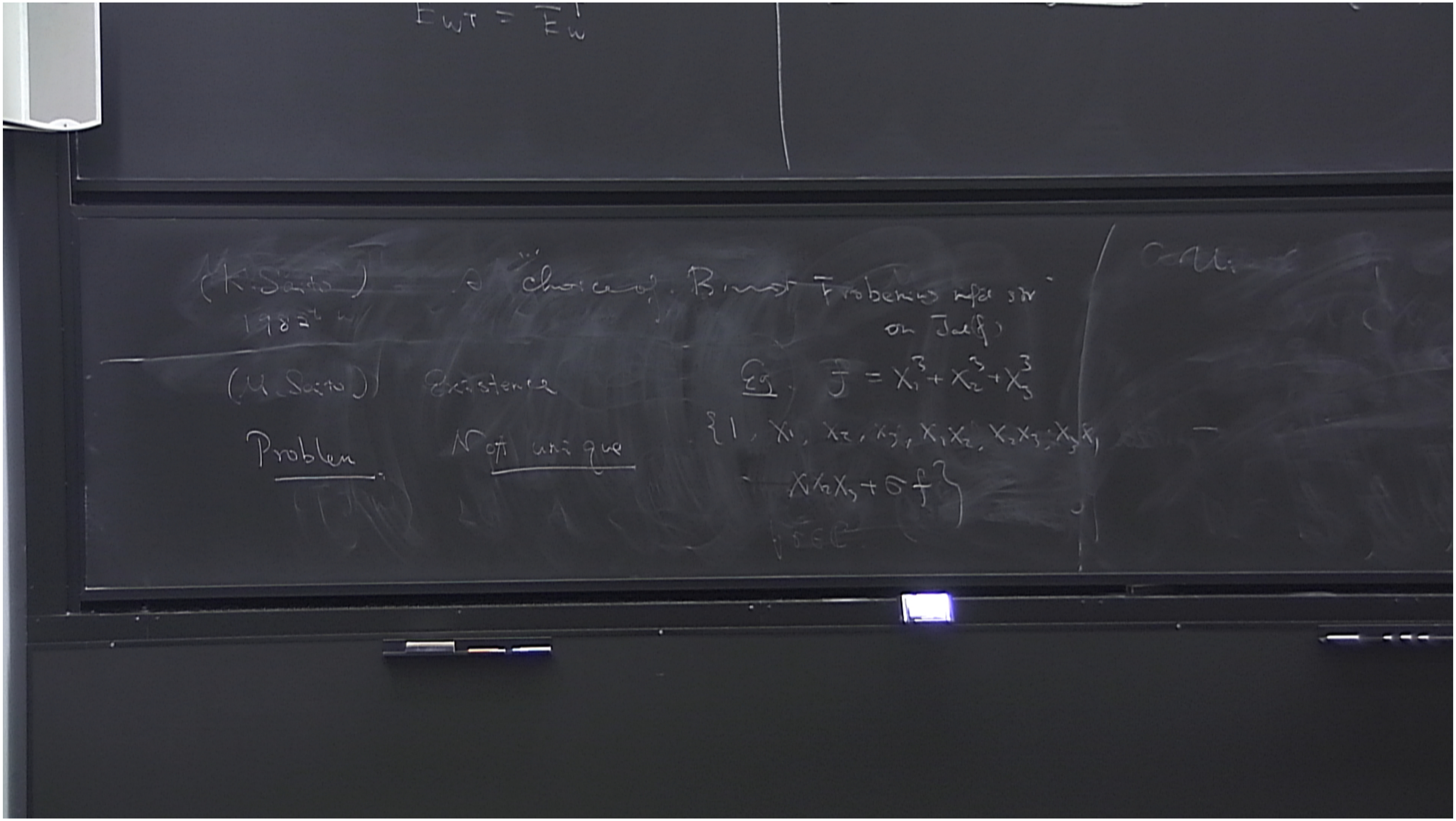

LG B-model
(formal) Brieskorn lattice
free

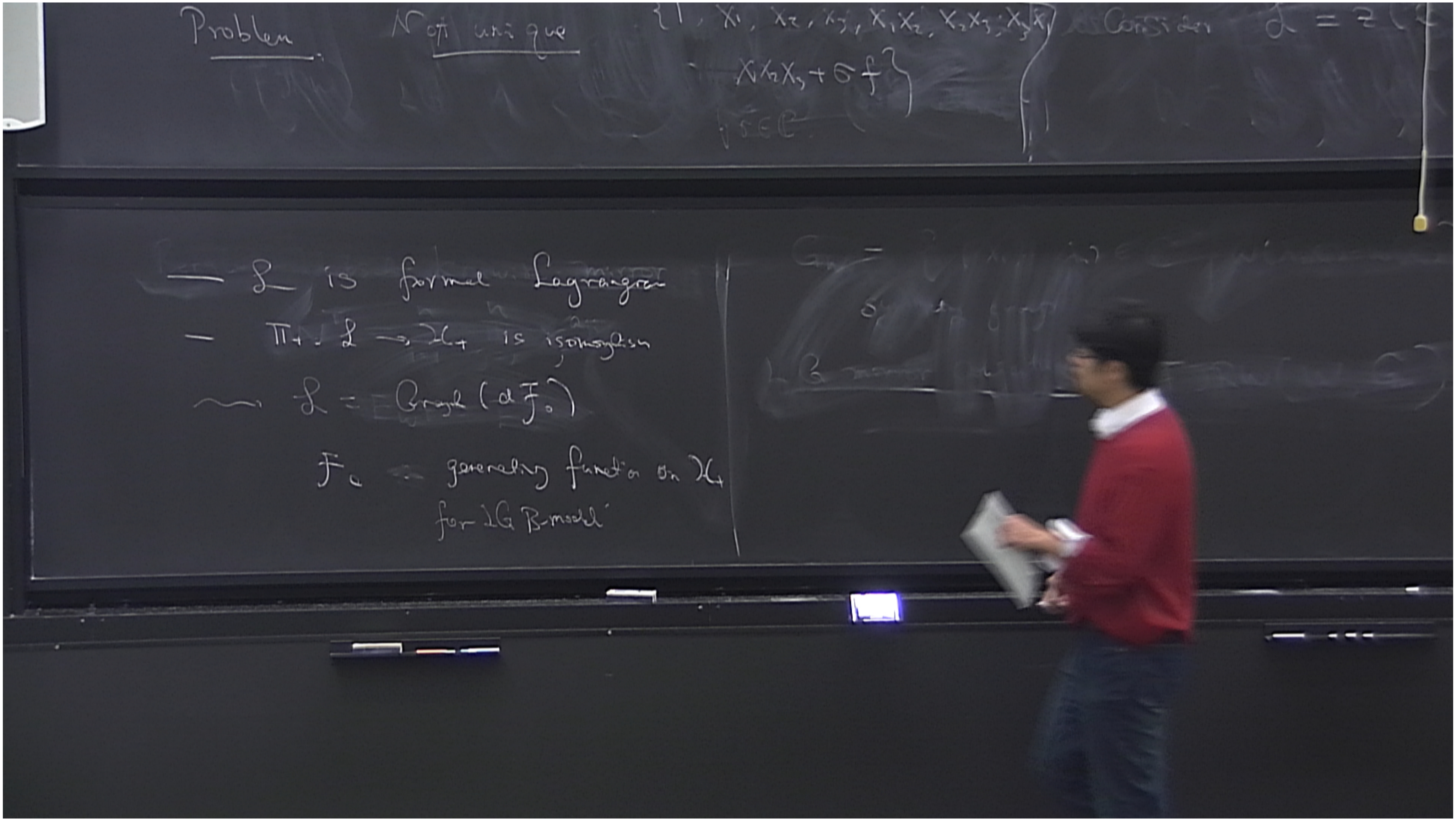

(K. Saito)
1982
s.t.
(Residue pairing)

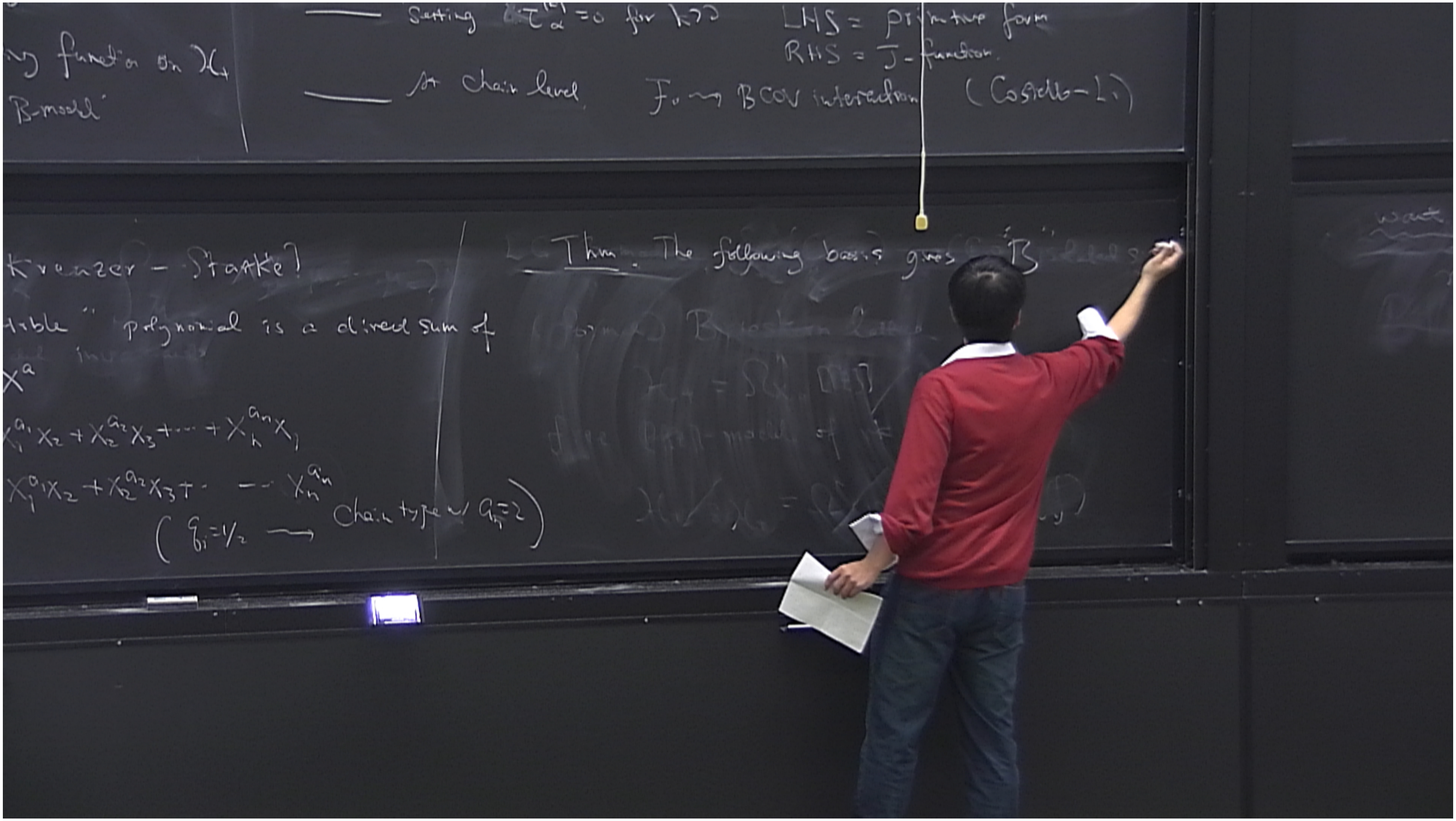

Fix(σ) ⊂ ℂ^n the fixed pt of σ
(K. Saito)
1982
(Higher residue pairing)
S.t.
(Residue pairing)

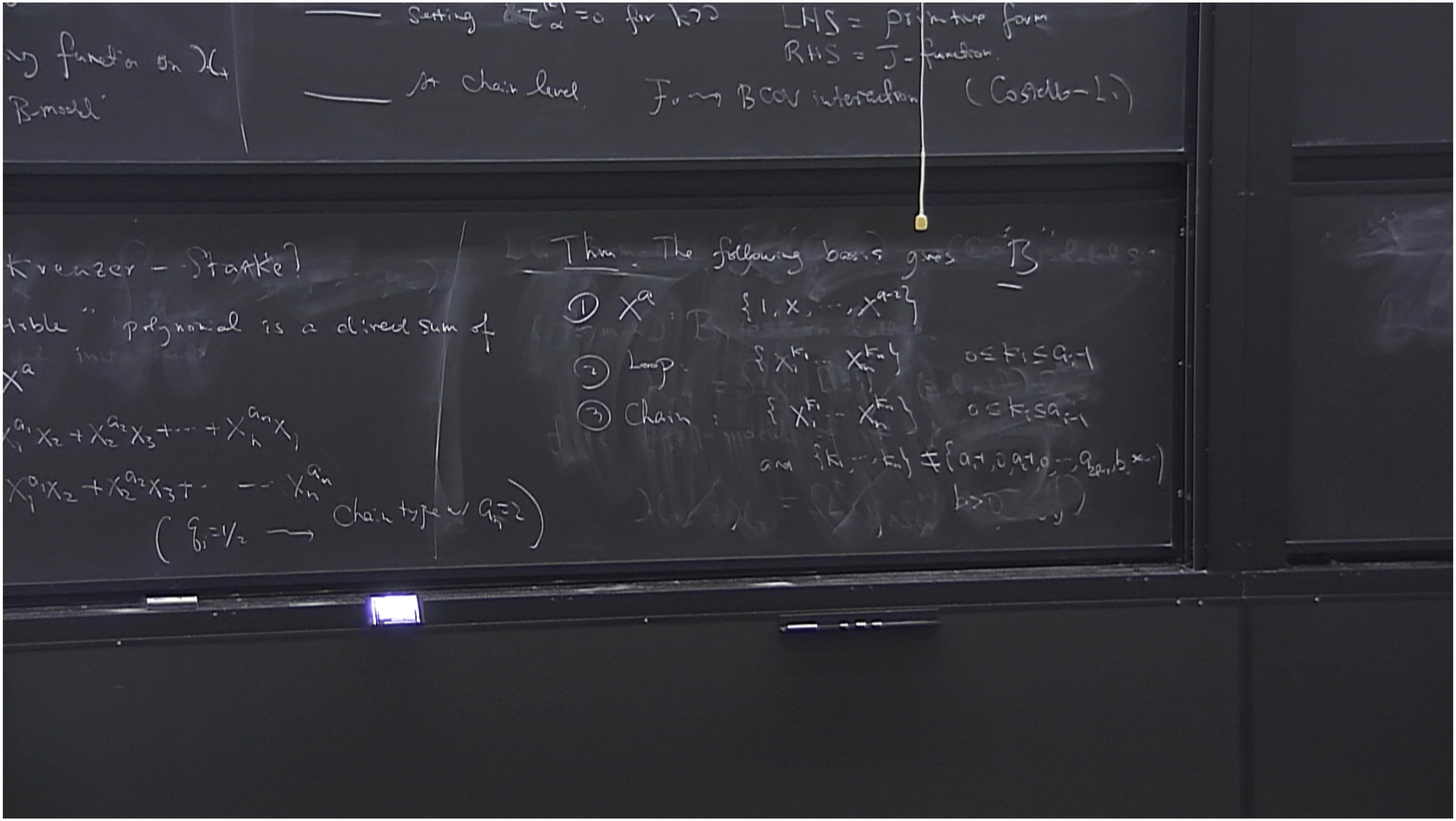

(Higher residue pairing)
(Residue pairing)

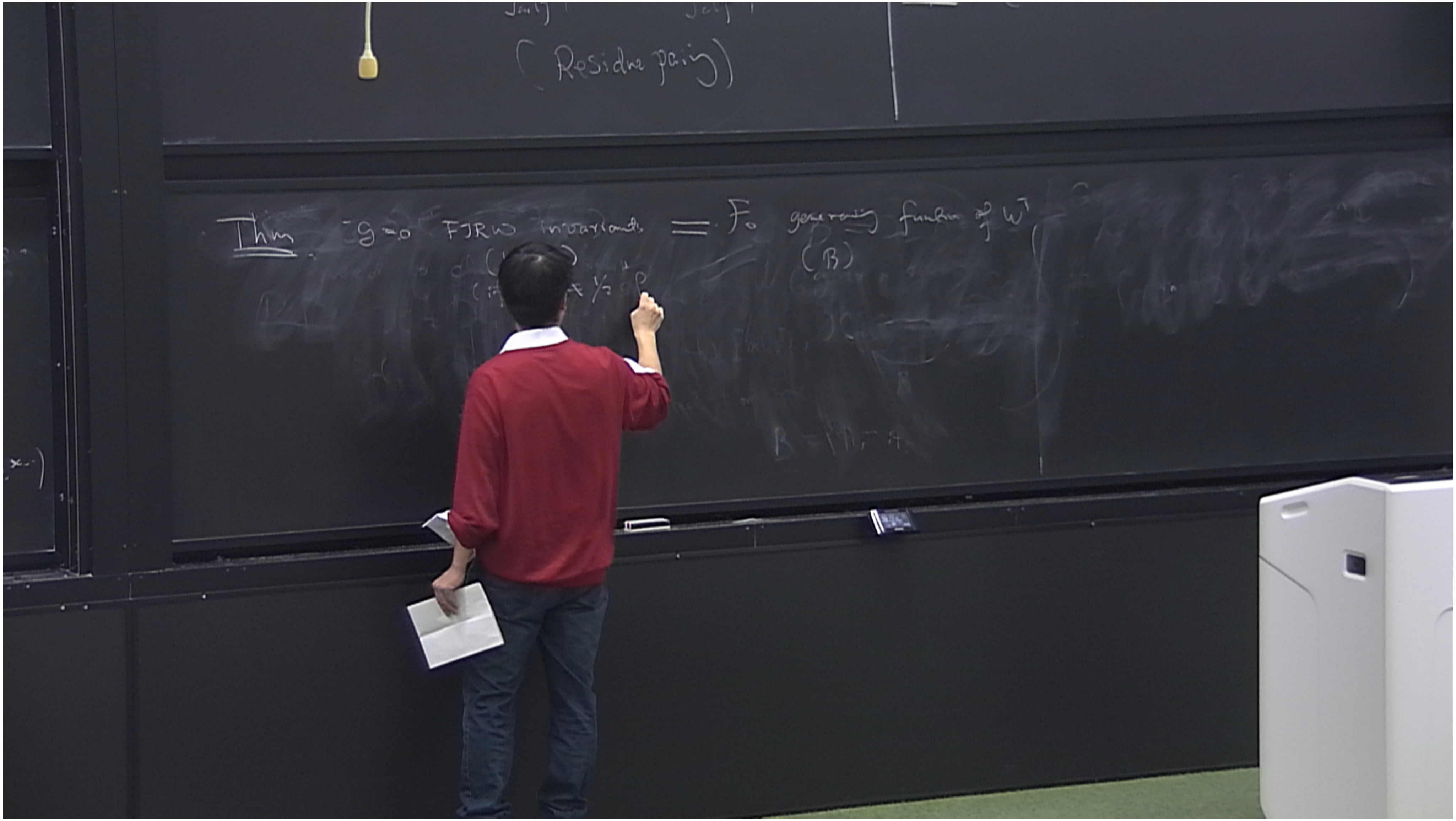

(Residue pairing)
need to choose a splitting
isotropic
gives a splitting
B

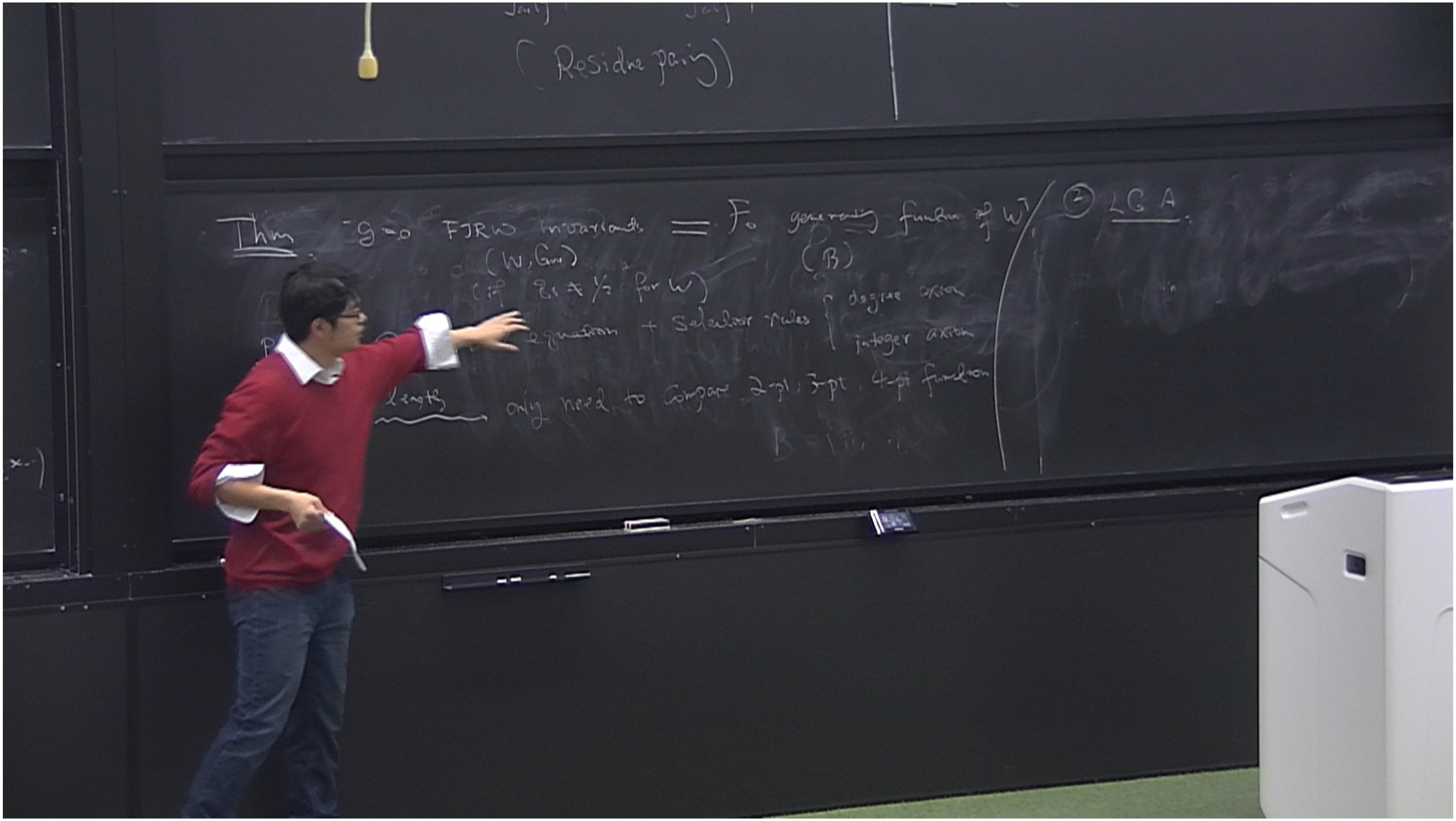

(K. Saito)
(M. Saito) Existence
Problem
Not unique
Eg. f = X1^3 + X2^3 + X3^3

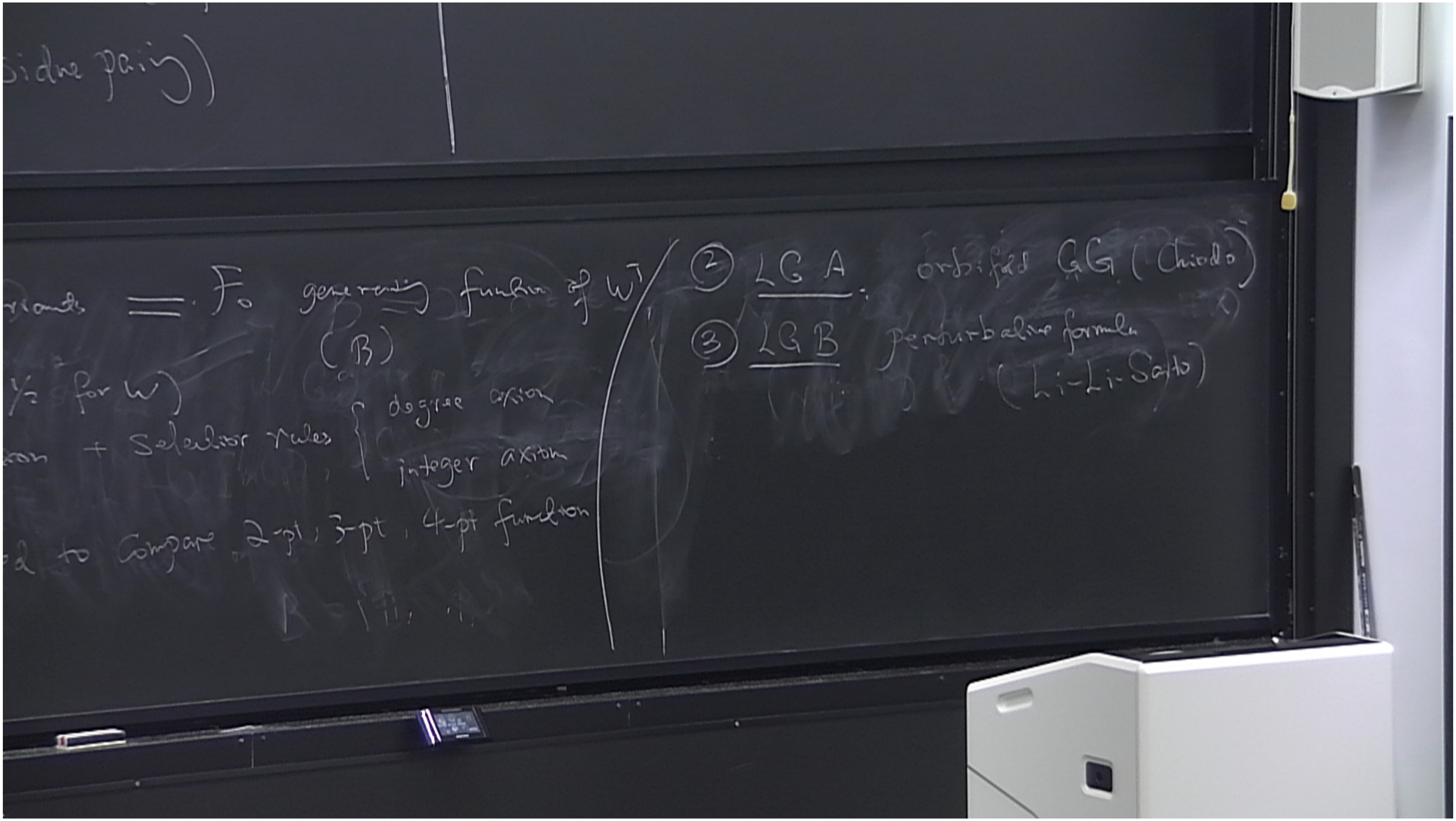

Problem
Not unique
Consider
is formal Lagrangian
is isomorphism
generating function on
for LG B-model


Setting
LHS = primitive form
RHS = J-function
B-model
At chain level
BCOV interaction
Kreuzer - Starke?
polynomial is a direct sum of
Chain type
Thm. The following basis gives


Setting
for
LHS =
RHS =
B-model
At chain level
BCOV interaction
Kreuzer - Starke?
polynomial is a direct sum of
Chain type
Thm. The following basis gives
① X^a $\{1, x, \cdots, x^{a-2}\}$
② Loop $\{X_1^{k_1} \cdots X_n^{k_n}\}$ $0 \le k_i \le a_i - 1$
③ Chain $\{X_1^{k_1} \cdots X_n^{k_n}\}$ $0 \le k_i \le a_i - 1$
and


(Residue pairing)
Thm g=0 FJRW invariants = F₀ generating function of W^T
(B)


(Residue pairing)
Thm
g=0 FJRW invariants = F₀ generating function of W^T
(W, G_max)
(B)
(if q_i ≠ 1/2 for W)
Selection rules
degree axiom
integer axiom
only need to compare 2-pt, 3-pt, 4-pt function
② LG A


= F0 generating function of W^T
(B)
for W)
+ Selection rules
degree axiom
integer axiom
to Compare 2-pt, 3-pt, 4-pt function
② LG A : orbifold GG (Chiodo)
③ LG B perturbative formula
(Li-Li-Saito)