Title: Kounterterms in anti-de Sitter gravity

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Abstract: <p>As an alternative to the Holographic Renormalization procedure, we</p>

<p>introduce a regularization scheme for AdS gravity based on the addition</p>

<p>boundary terms which are a given polynomial of the extrinsic and </p>

<p>intrinsic curvatures (Kounterterms).</p>

<p> </p>

<p>Since these terms are closely related to either topological invariants </p>

<p>or Chern-Simons densities in the corresponding dimension, they can be </p>

<p>easily generalized to other gravity theories (Einstein-Gauss-Bonnet, </p>

<p>Lovelock, etc.).</p>

<p> </p>

<p>Finally, a general prescription on how to obtained the standard </p>

<p>counterterm series in AdS gravity is given.â€‹ </p>

Kounterterms in anti-de Sitter Gravity

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Outline

1. Variational problem in EH gravity

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2. Quasilocal (Brown-York) stress tensor
3. Holographic Renormalization and Local Counterterms

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2. Quasilocal (Brown-York) stress tensor
3. Holographic Renormalization and Local Counterterms
4. Holographic stress tensor
5. Alternative regularization scheme in AdS gravity: Kounterterms

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1. Variational problem in EH gravity
2. Quasilocal (Brown-York) stress tensor
3. Holographic Renormalization and Local Counterterms
4. Holographic stress tensor
5. Alternative regularization scheme in AdS gravity: Kounterterms
6. From extrinsic to intrinsic regularization
7. Quasilocal vs holographic stress tensor: two examples

Variational problem in EH gravity

- **EH gravity action in $D = d + 1$ dimensions**

$$I_{EH} = \frac{1}{16\pi G} \int_M d^{d+1}x \sqrt{-\mathcal{G}} \left(R - 2\Lambda \right)$$

- **Variation of the action**:

$$\delta I_{EH} = \frac{1}{16\pi G} \int_M d^{d+1}x \sqrt{-\mathcal{G}} \, \mathcal{E}_\mu^\nu \left(\mathcal{G}^{-1} \delta \mathcal{G} \right)_\nu^\mu + \int_{\partial M} d^d x \, \Theta$$

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- **Surface term:**

$$\Theta = \frac{1}{16\pi G} \sqrt{-h} \, n_\mu \, \delta_{[\alpha\beta]}^{[\mu\nu]} \mathcal{G}^{\beta\varepsilon} \delta\Gamma_{\nu\varepsilon}^\alpha$$

Variational problem in EH gravity

- **In Gauss-normal coordinates**

$$ds^2 = \quad N^2(\rho)d\rho^2 + h_{ij}(\rho, x)dx^i dx^j$$

Variational problem in EH gravity

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$$ds^2 = N^2(\rho)d\rho^2 + h_{ij}(\rho,x)dx^i dx^j$$

Extrinsic curvature $K_{ij} = -\frac{1}{2N}\partial_\rho h_{ij}$

$$\Gamma^\rho_{ij} = \frac{1}{N}K_{ij}, \qquad \Gamma^i_{\rho j} = -NK^i_j$$

- **On-shell variation of the action** $(n_\mu = (N, \vec{0}))$:

$$\delta I_{EH} = \frac{1}{16\pi G}\int_{\partial M} d^d x\sqrt{-h}\left(2\delta K + K^i_j(h^{-1}\delta h)^j_i\right)$$

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- **Gibbons-Hawking term:**

$$I_{Dirichlet} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^d x\sqrt{-h}K$$

- **Dirichlet problem for the metric:**

$$\delta I_{Dirichlet} = \frac{1}{16\pi G}\int\limits_{\partial M} d^d x\sqrt{-h}\left(K^i_j - K\delta^i_j\right)(h^{-1}\delta h)^j_i$$

Quasilocal stress tensor

- **Variation of the Dirichlet action respect to** h_{ij}

$$\delta I_{Dir} = \int_{\partial M} d^d x \, \frac{1}{2} \sqrt{-h} \, T^{ij}[h] \, \delta h_{ij}$$

- **Brown-York tensor is conserved, i.e.,** $\nabla_i T^{ij} = 0$.

- **Conserved current:**

$$J^i = T^{ij} \xi_j \implies Q[\xi] = \int_{\Sigma} \sqrt{\sigma} \, u_i T^{ij} \xi_j$$

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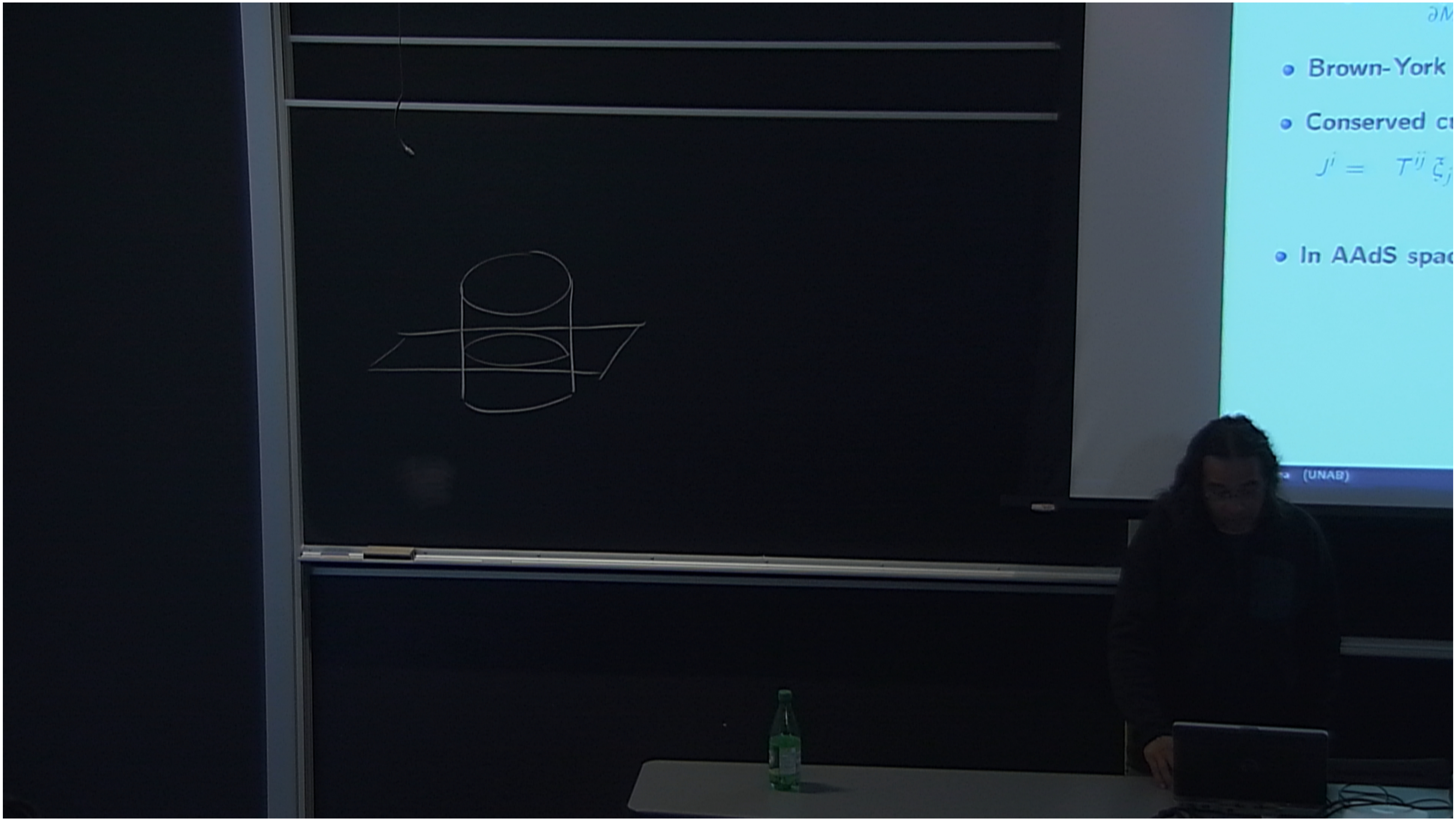
Brown-York
Conserved
In AAdS
(UNAB)

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- **In AAdS spacetimes** $Q[\xi]$ **is divergent, even in** $D = 3$.

- **We can always add local counterterms such that** $T^{ij}[h]$ **is regular**

$$I_{ren} = I_{EH} - \frac{1}{8\pi G} \int_{\partial M} d^d x \sqrt{-h} K + \int_{\partial M} d^d x \, \mathcal{L}_{ct}(h, \mathcal{R}, \nabla \mathcal{R})$$

Counterterm method

- **Regularized AdS gravity action** (holographic renormalization) [Henningson, Skenderis JHEP 9807:023(1998)]

$$I_{ren} = \frac{1}{16\pi G}\int_M d^{d+1}x\,\sqrt{-\mathcal{G}}\,(R-2\Lambda) - \frac{1}{8\pi G}\int_{\partial M} d^d x\,\sqrt{-h}\,K + \\ + \int_{\partial M} d^d x\,\mathcal{L}_{ct}(h,\mathcal{R},\nabla\mathcal{R})$$

$$\Lambda = -\frac{d\,(d-1)}{2\ell^2}$$

- **Renormalized quasi-local stress tensor**: $T^{ij}_{ren}[h] = \frac{2}{\sqrt{-h}}\frac{\delta I_{ren}}{\delta h_{ij}}$.

- **Background-independent charges:** Vacuum energy for global AdS (Casimir energy for boundary CFT) [Balasubramanian, Kraus CMP 208: 413 (1999)]

Holographic Renormalization

- For Asymptotically AdS (AAdS) spacetimes, Fefferman-Graham (FG) form of the metric

$$ds^2 = \frac{\ell^2}{4\rho^2}\,d\rho^2 + \frac{1}{\rho}\,g_{ij}(x,\rho)\,dx^i dx^j \qquad (1)$$

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- the boundary of the spacetime is at $\rho = 0$
- $g_{ij}(x,\rho)$ accepts a regular expansion in powers of ρ

$$g_{ij}(x,\rho) = g_{(0)ij}(x) + \rho g_{(1)ij}(x) + \rho^2 g_{(2)ij}(x) + \cdots .$$

- $g_{(0)ij}$ is the boundary data for the *holographic reconstruction of the spacetime*, i.e., solving $g_{(k)}$ as a covariant funtional of $g_{(0)}$

Holographic Renormalization

- For instance

$$g_{(1)ij} = \frac{1}{d-2}\left(\mathcal{R}_{(0)ij} - \frac{1}{2(d-1)}g_{(0)ij}\mathcal{R}_{(0)}\right)$$

$$\begin{aligned} g_{(2)ij} = \frac{1}{d-4}\Big(&-\frac{1}{8(d-1)}\nabla_i\nabla_j\mathcal{R}_{(0)} + \frac{1}{4(d-2)}\nabla_k\nabla^k\mathcal{R}_{(0)ij} \\ &-\frac{1}{8(d-1)(d-2)}g_{(0)ij}\nabla_k\nabla^k\mathcal{R}_{(0)} - \frac{1}{2(d-2)}\mathcal{R}^{kl}_{(0)}\mathcal{R}_{(0)ikjl} \\ &+\frac{(d-4)}{2(d-2)^2}\mathcal{R}_{(0)ik}\mathcal{R}^{k}_{(0)j} + \frac{1}{(d-1)(d-2)^2}\mathcal{R}_{(0)}\mathcal{R}_{(0)ij} \\ &+\frac{1}{4(d-2)^2}\mathcal{R}_{(0)kl}\mathcal{R}^{kl}_{(0)}g_{(0)ij} - \frac{3d}{16(d-1)^2(d-2)^2}\mathcal{R}^2_{(0)}g_{(0)ij}\Big) \end{aligned}$$

Counterterm method in AdS gravity

- $$\mathcal{L}_{ct} = \frac{d-1}{\ell}\sqrt{-h} + \frac{\ell\sqrt{-h}}{2(d-2)}\mathcal{R} + \frac{\ell^3\sqrt{-h}}{2(d-2)^2(d-4)}\left(\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d}{4(d-1)}\mathcal{R}^2\right) + \frac{\ell^5\sqrt{-h}}{(d-2)^3(d-4)(d-6)}\left(\frac{3d-2}{4(d-1)}\mathcal{R}\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{d(d+2)}{16(d-1)^2}\mathcal{R}^3 - 2\mathcal{R}^{ij}\mathcal{R}^{kl}\mathcal{R}_{ijkl} - \frac{d}{4(d-1)}\nabla_i\mathcal{R}\nabla^i\mathcal{R} + \nabla^k\mathcal{R}^{ij}\nabla_k\mathcal{R}_{ij}\right) + \ldots$$

Kounterterms

- *Extrinsic* **regularization** $\tilde{I}_{ren} = \quad I_{EH} + c_d \int\limits_{\partial M} d^d x \, B_d(h, K, \mathcal{R})$

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- **Inspired by a simple observation (EH+GB in** $D = 4$**)**

$$I = \frac{1}{16\pi G} \int\limits_M d^4x \sqrt{-\mathcal{G}} \left[(R - 2\Lambda) + \alpha \left(R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} - 4R_{\mu\nu} R^{\mu\nu} + R^2 \right) \right]$$

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- **Euclidean action for Sch-AdS black hole:**

$$G = \beta^{-1} I^E = \frac{M}{2} \left(1 + \frac{4}{\ell^2} \alpha \right) - TS' + \frac{\pi r^3}{4G\ell^2} \left(1 - \frac{4}{\ell^2} \alpha \right)$$

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- **Correct black hole thermo $\Rightarrow$ GB coupling** $\alpha = \frac{\ell^2}{4}$

- **Finite Noether current also implies** $\alpha = \frac{\ell^2}{4}$
  [Aros, Contreras, Olea, Troncoso, Zanelli, PRL 84, 1647 (2000)]

Kounterterms

- Euler Theorem in $D = 4$ dimensions

$$\int_M d^4x \, GB = 32\pi^2 \chi(M) + \int_{\partial M} d^3x \, B_3$$

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$$\int_M d^4x\, GB = 32\pi^2 \chi(M) + \int_{\partial M} d^3x\, B_3$$

- Kounterterms = given polynomial in the extrinsic and intrinsic curvatures (K_{ij} and $\mathcal{R}^{kl}_{ij}(h)$)

$$\begin{aligned} B_3 &= 4\sqrt{-h}\,\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2}\,\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}\,K^{j_2}_{i_2} K^{j_3}_{i_3} \right) \\ &= 4\sqrt{-h}\left[-2(\mathcal{R}^i_j - \frac{1}{2}\delta^i_j \mathcal{R}) K^j_i - \frac{2}{3} K^i_j K^j_k K^k_i + K(K^i_j K^j_i - \frac{1}{3}K^2) \right] \end{aligned}$$

$$c_3 = \ell^2 / 64\pi G$$

Kounterterms

- $D = 2n$ dimensions [Olea, JHEP 0506: 023 (2005)]

$$B_{2n-1} = 2n\sqrt{-h}\int_0^1 dt\ \delta^{[i_1\cdots i_{2n-1}]}_{[j_1\cdots j_{2n-1}]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3} - t^2 K^{j_2}_{i_2} K^{j_3}_{i_3}\right)\times\cdots$$

$$\cdots\times\left(\frac{1}{2}\mathcal{R}^{j_{2n-2}j_{2n-1}}_{i_{2n-2}i_{2n-1}} - t^2 K^{j_{2n-2}}_{i_{2n-2}} K^{j_{2n-1}}_{i_{2n-1}}\right)$$

$$c_{2n-1} = (-\ell^2)^{n-1}/(16\pi G n(2n-2)!)$$

Kounterterms

- Kounterterms in $D = 2n + 1$ [Olea, JHEP 0704: 073 (2007)]

$$B_{2n} = 2n \int_0^1 dt \int_0^t ds\, \delta^{[j_1 \cdots j_{2n}]}_{[i_1 \cdots i_{2n}]} K^{i_1}_{j_1} \delta^{i_2}_{j_2} \left(\frac{1}{2} \mathcal{R}^{i_3 i_4}_{j_3 j_4} - t^2 K^{i_3}_{j_3} K^{i_4}_{j_4} + \frac{s^2}{\ell^2} \delta^{i_3}_{j_3} \delta^{i_4}_{j_4} \right) \times \cdots$$

$$\cdots \times \left(\frac{1}{2} \mathcal{R}^{i_{2n-1} i_{2n}}_{j_{2n-1} j_{2n}} - t^2 K^{i_{2n-1}}_{j_{2n-1}} K^{i_{2n}}_{j_{2n}} + \frac{s^2}{\ell^2} \delta^{i_{2n-1}}_{j_{2n-1}} \delta^{i_{2n}}_{j_{2n}} \right) .$$

Kounterterms

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Kounterterms in higher-curvature gravity

- Einstein-Gauss-Bonnet AdS [Kofinas, Olea, PRD D74:084035 (2006)]

$$I_{EGB} = \frac{1}{16\pi G}\int\limits_{M} d^D x\sqrt{-\mathcal{G}}\left[(R-2\Lambda)+\alpha(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}-4R_{\mu\nu}R^{\mu\nu}+R^2)\right]$$

- Effective AdS radius

$$\ell_{\text{eff}}^{-2} = \frac{1\pm\sqrt{1-\frac{4\alpha(D-3)(D-4)}{\ell^2}}}{2\alpha\,(D-3)\,(D-4)}\,,$$

- Kounterterm series B_d preserves its form if $\ell \longrightarrow \ell_{\text{eff}}$
  One only needs to change $c_d^{EH} \longrightarrow c_d^{EGB}$.

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- Effective AdS radius

$$\ell_{\text{eff}}^{-2} = \frac{1 \pm \sqrt{1 - \frac{4\alpha(D-3)(D-4)}{\ell^2}}}{2\alpha (D-3)(D-4)} ,$$

- Kounterterm series B_d preserves its form if $\ell \longrightarrow \ell_{\text{eff}}$
  One only needs to change $c_d^{EH} \longrightarrow c_d^{EGB}$.
- Lovelock-AdS gravity [Kofinas, Olea, JHEP 0711:069 (2007)]

From extrinsic to intrinsic regularization

- Regularized action

$$I_{reg} = I_{EH} + \frac{\ell^2}{16\pi G} \int\limits_{\partial M} d^3x \sqrt{-h}\, \delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1} \left(\frac{1}{2} \mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3} K^{j_2}_{i_2} K^{j_3}_{i_3} \right).$$

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- Adding zero...

$$I_{reg} = I_{EH} - \frac{1}{8\pi G}\int\limits_{\partial M} d^3x\sqrt{-h}\,K + \int\limits_{\partial M} d^3x\,\mathcal{L}_{ct}.$$

$$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G}\sqrt{-h}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]} K^{j_1}_{i_1}\left(\frac{1}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(h) - \frac{1}{3}K^{j_2}_{i_2}K^{j_3}_{i_3} + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right).$$

- And expanding...

$$K^i_j = \frac{1}{\ell}\delta^i_j - \rho\ell S^i_j(g) + \mathcal{O}(\rho^2)$$

$$S^i_j(g) = \frac{1}{D-3}\left(\mathcal{R}^i_j(g) - \frac{1}{2(D-2)}\delta^i_j\mathcal{R}(g)\right)$$

From extrinsic to intrinsic regularization

- $$\mathcal{L}_{ct} = \frac{\ell^2}{16\pi G}\frac{\sqrt{-g}}{\rho^{3/2}}\delta^{[i_1 i_2 i_3]}_{[j_1 j_2 j_3]}\left(\frac{\delta^{j_1}_{i_1}}{\ell} - \rho\ell S^{i_1}_{j_1}\right)\times$$
$$\times\left(\frac{\rho}{2}\mathcal{R}^{j_2 j_3}_{i_2 i_3}(g) - \frac{1}{3}\left(\frac{\delta^{j_2}_{i_2}}{\ell} - \rho\ell S^{i_2}_{j_2}\right)\left(\frac{\delta^{j_3}_{i_3}}{\ell} - \rho\ell S^{i_3}_{j_3}\right) + \frac{1}{\ell^2}\delta^{j_2}_{i_2}\delta^{j_3}_{i_3}\right) + \ldots$$

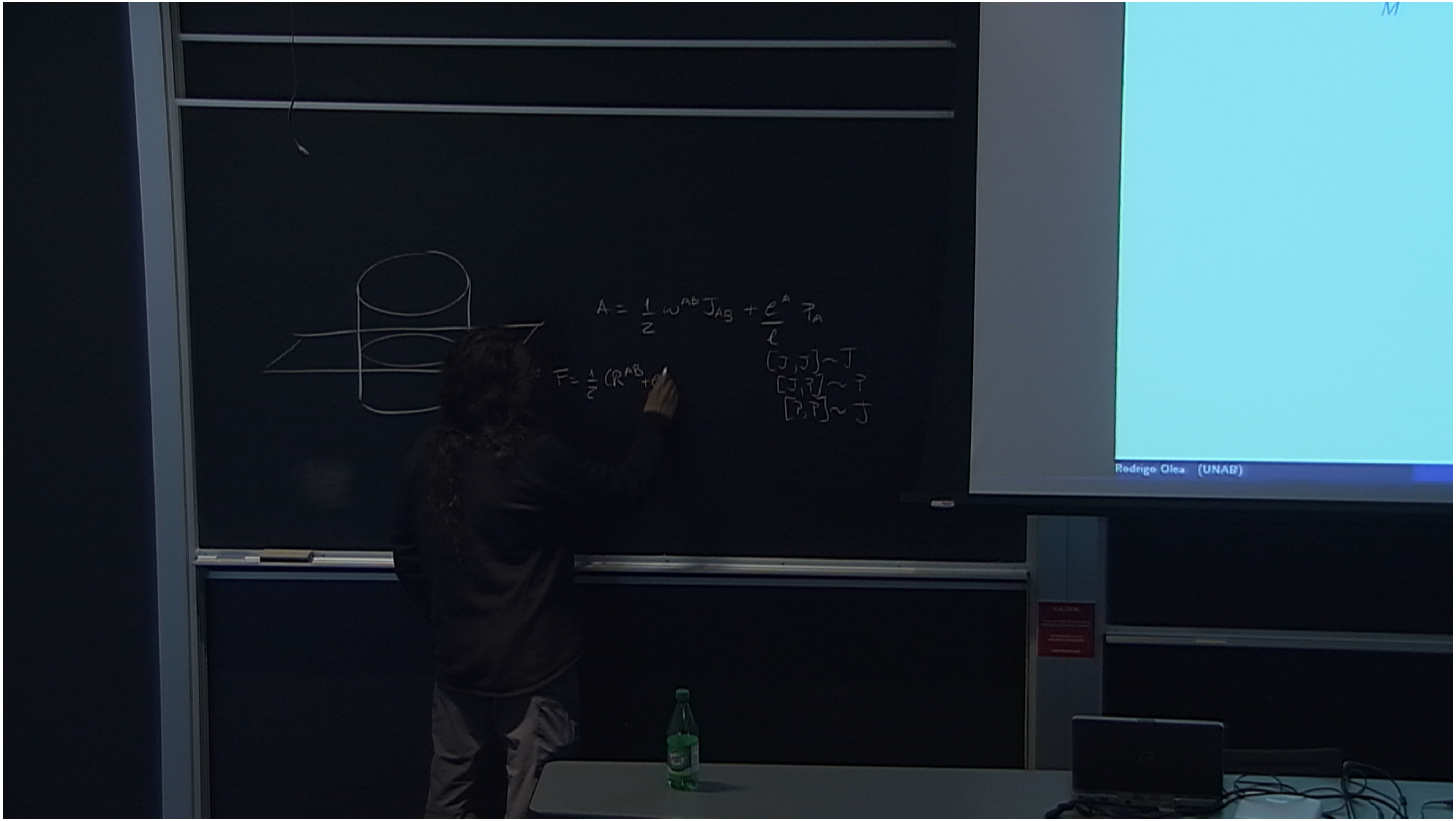
- *Extrinsic* counterterms turn into intrinsic (Balasubramanian-Kraus) ones

$$\begin{aligned}\mathcal{L}_{ct} &= \frac{1}{8\pi G}\frac{\sqrt{-g}}{\rho^{3/2}}\left(\frac{2}{\ell} + \frac{\ell}{2}\rho\mathcal{R}(g)\right) + \mathcal{O}(\rho^{1/2}) \\ &= \frac{1}{8\pi G}\sqrt{-h}\left(\frac{2}{\ell} + \frac{\ell}{2}\mathcal{R}(h)\right)\end{aligned}$$

Topological regularization

- And expanding...

$$\mathcal{L}_{ct} = \frac{\sqrt{-h}}{8\pi G}\left[\frac{(2n-2)}{\ell} + \frac{\ell}{2(2n-3)}\mathcal{R} + \right.$$
$$\left. + \frac{\ell^3}{2(2n-3)^2(2n-5)}\left(2\mathcal{R}^{ij}\mathcal{R}_{ij} - \frac{(2n+1)}{4(2n-2)}\mathcal{R}^2 - \frac{(2n-3)}{4}\mathcal{R}^{ijkl}\mathcal{R}_{ijkl}\right) + \cdots\right]$$


A = 1/2 ω^AB J_AB + e^A/ℓ P_A
[J,J] ~ J
[J,P] ~ P
[P,P] ~ J
Rodrigo Olea (UNAB)

Weyl tensor in the bulk...

- **MacDowell-Mansouri form of the action**

$$I_{ren} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-\mathcal{G}}\, \delta^{[\sigma\lambda\mu\nu]}_{[\gamma\delta\alpha\beta]} \left(R^{\gamma\delta}_{\sigma\lambda} + \frac{1}{\ell^2}\, \delta^{[\gamma\delta]}_{[\sigma\lambda]} \right) \left(R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\, \delta^{[\alpha\beta]}_{[\mu\nu]} \right).$$

- **The action is zero for global AdS spacetime**

$$I_{ren} = 0$$

- **Weyl tensor (on-shell)** $W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\, \delta^{[\alpha\beta]}_{[\mu\nu]}$

Weyl tensor in the bulk...

- **MacDowell-Mansouri form of the action**

$$I_{ren} = \frac{\ell^2}{256\pi G} \int\limits_M d^4x \sqrt{-\mathcal{G}}\, \delta^{[\sigma\lambda\mu\nu]}_{[\gamma\delta\alpha\beta]} \left(R^{\gamma\delta}_{\sigma\lambda} + \frac{1}{\ell^2}\, \delta^{[\gamma\delta]}_{[\sigma\lambda]} \right) \left(R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\, \delta^{[\alpha\beta]}_{[\mu\nu]} \right).$$

- **The action is zero for global AdS spacetime**

$$I_{ren} = 0$$

- **Weyl tensor (on-shell)** $W^{\alpha\beta}_{\mu\nu} = R^{\alpha\beta}_{\mu\nu} + \frac{1}{\ell^2}\, \delta^{[\alpha\beta]}_{[\mu\nu]}$
- **(On-shell) regularized action is equal to Conformal Gravity action**

$$I_{ren} = \frac{\ell^2}{64\pi G} \int\limits_M d^4x \sqrt{-\mathcal{G}}\, W_{\mu\nu\alpha\beta} W^{\mu\nu\alpha\beta}$$

O.Mišković and R.O., [arXiv:0902.2082];
J.Maldacena, [arXiv:1105.5632]

Critical Gravity

- 4D Critical Gravity

$$I_{CG} = \frac{1}{16\pi G} \int d^4x \sqrt{-\mathcal{G}} \left[\left(R + \frac{1}{\alpha} \right) + \alpha R^2 - 3\alpha R_{\mu\nu} R^{\mu\nu} \right]$$

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- $\alpha = \ell^2/6$

$$I_{CG} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-\mathcal{G}}\left[\left(R+\frac{6}{\ell^2}\right)-\frac{\ell^2}{2}\left(R_{\mu\nu}R^{\mu\nu}-\frac{1}{3}R^2\right)\right]$$

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- $I_{CG} = \frac{1}{16\pi G}\int d^4x\,\sqrt{-\mathcal{G}}\left[\left(R+\frac{6}{\ell^2}\right)-\frac{\ell^2}{4}\left(W^2-GB\right)\right]$
-

$$I = I_{ren} - \frac{\ell^2}{64\pi G}\int d^4x\,\sqrt{-\mathcal{G}}\,W^2$$

- $I_{CG} = 0$ for Einstein spaces [O.Mišković, R.O. and M. Tsoukalas, arXiv:1404.5993]

Quasilocal vs Holographic Stress Tensor

- Kounterterms: No clear identification of the boundary quasilocal stress tensor

$$\delta \tilde{I}_{ren} = \int\limits_{\partial M} d^{D-1}x \sqrt{-h} \left(\frac{1}{2} \tau_i^j \left(h^{-1} \delta h \right)_j^i + \Delta_i^j \delta K_j^i \right)$$

Holographic stress tensor not from a quasilocal one

- Example 1: AAdS sector in 3D Topologically Massive Gravity

$$I_{TMG} = I_{EH} + \frac{1}{32\pi G\mu} \int_M d^3x \left(\Gamma d\Gamma + \frac{2}{3}\Gamma^3 \right)$$

- But what about holographic stress tensor?

Variational problem in EGB gravity

- EH AdS+ GB term in 4D

$$I = \frac{1}{16\pi G}\int_M d^4x\sqrt{-\mathcal{G}}\left[(R-2\Lambda) + \frac{\ell^2}{4}(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2)\right]$$

- For any $D > 4$ (arbitrary GB coupling α)

$$\delta I_{GB} = \frac{\alpha}{4\pi G}\int_{\partial M} d^{D-1}x\sqrt{-h}\,\delta^{[jj_1j_2]}_{[ii_1i_2]}\left[\frac{1}{2}\left(h^{-1}\delta h\right)^i_k K^k_j + \delta K^i_j\right]\left(\frac{1}{2}\mathcal{R}^{i_1i_2}_{j_1j_2}(h) - K^{i_1}_{j_1}K^{i_2}_{j_2}\right)$$

- Gibbons-Hawking-Myers term for GB

$$\beta = -\frac{\alpha}{4\pi G}\sqrt{-h}\delta^{[j_1j_2j_3]}_{[i_1i_2i_3]}K^{i_1}_{j_1}\left(\frac{1}{2}\mathcal{R}^{i_2i_3}_{j_2j_3} - \frac{1}{3}K^{i_2}_{j_2}K^{i_3}_{j_3}\right),$$

Variational problem in GB gravity

- Dirichlet variation

$$\delta I_{GB} = \frac{\alpha}{8\pi G} \int_{\partial M} d^{D-1}x \sqrt{-h}\, \delta^{[j\, j_1 j_2 j_3]}_{[i\, i_1 i_2 i_3]} (h^{-1}\delta h)^i_j\, K^{i_1}_{j_1} \left(\frac{1}{2} \mathcal{R}^{i_2 i_3}_{j_2 j_3} - \frac{1}{3} K^{i_2}_{j_2} K^{i_3}_{j_3} \right)$$

- No Gibbons-Hawking term for $4D$ GB $\Rightarrow$ No quasilocal stress tensor

$$\delta I = \int_{\partial M} d^3x \sqrt{-h} \left(\frac{1}{2} \tau^j_i \left(h^{-1}\delta h \right)^i_j + \Delta^j_i\, \delta K^i_j \right)$$

where

$$\tau^j_i = \frac{1}{32\pi G} \delta^{[jkl]}_{[mnp]} K^m_i \left(R^{np}_{kl} + \frac{1}{\ell^2} \delta^{[np]}_{[kl]} \right), \quad \Delta^j_i = \frac{1}{32\pi G} \delta^{[jkl]}_{[inp]} \left(R^{np}_{kl} + \frac{1}{\ell^2} \delta^{[np]}_{[kl]} \right)$$

Conclusions and prospects

- The result of $D = 4$ is always true in $D = 2n$ (requires the asymptotic behavior of the Weyl tensor)

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- The result of $D = 4$ is always true in $D = 2n$ (requires the asymptotic behavior of the Weyl tensor)
- In $D = 2n + 1$ it is more subtle because counterterms ambiguity (finite counterterms that do not modify the Weyl anomaly).
- Comparison to Ashtekar-Magnon-Das charges [arXiv:1404.1411]
- Conformal mass in EGB [arXiv:1501.06861]
- Reading off the holographic (quasilocal) stress tensor from the variation of the action in other gravity theories (EGB, Lovelock, etc.) where Holographic Renormalization becomes extremely involved.