Title: Gravitational Physics-1

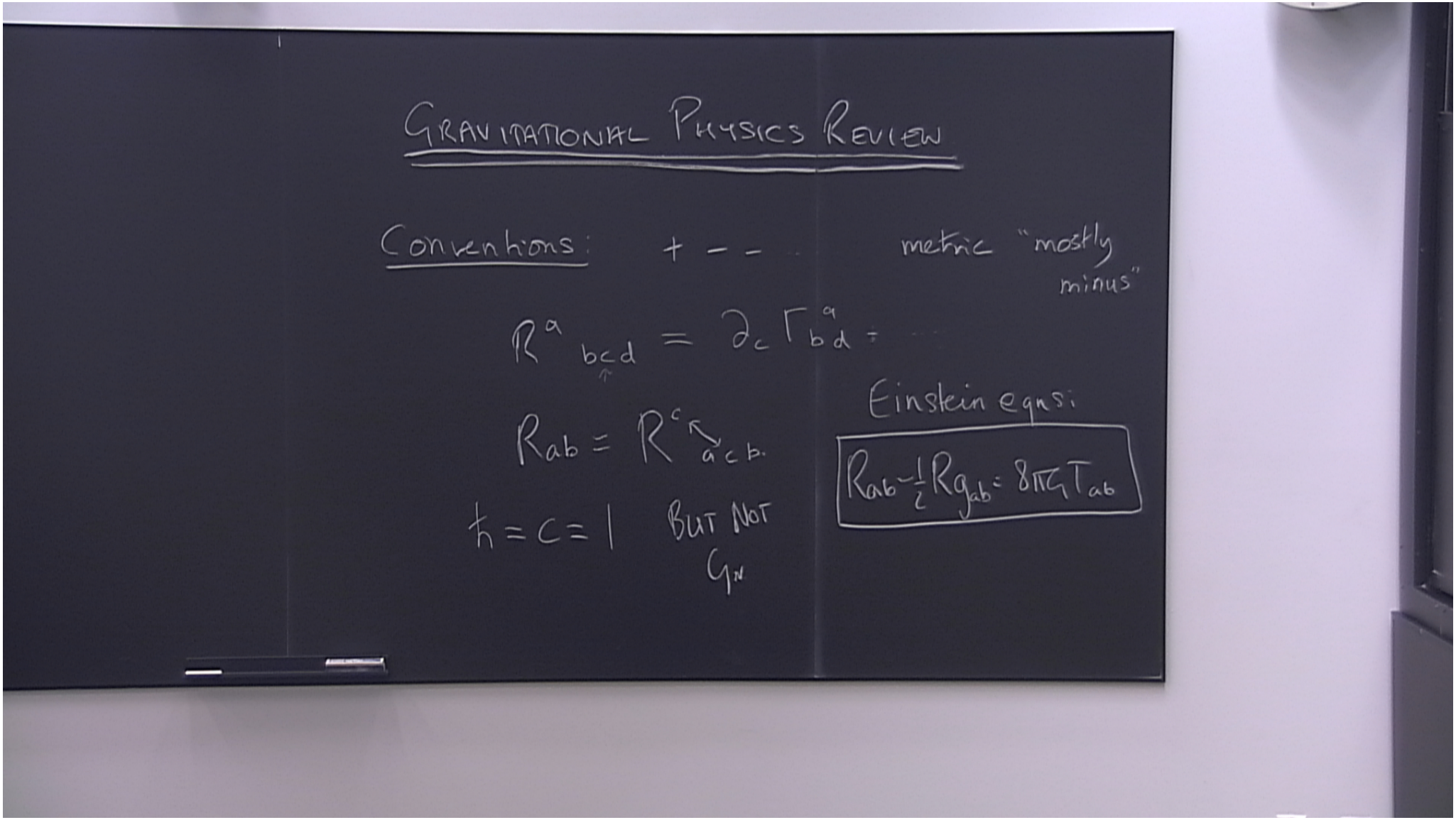
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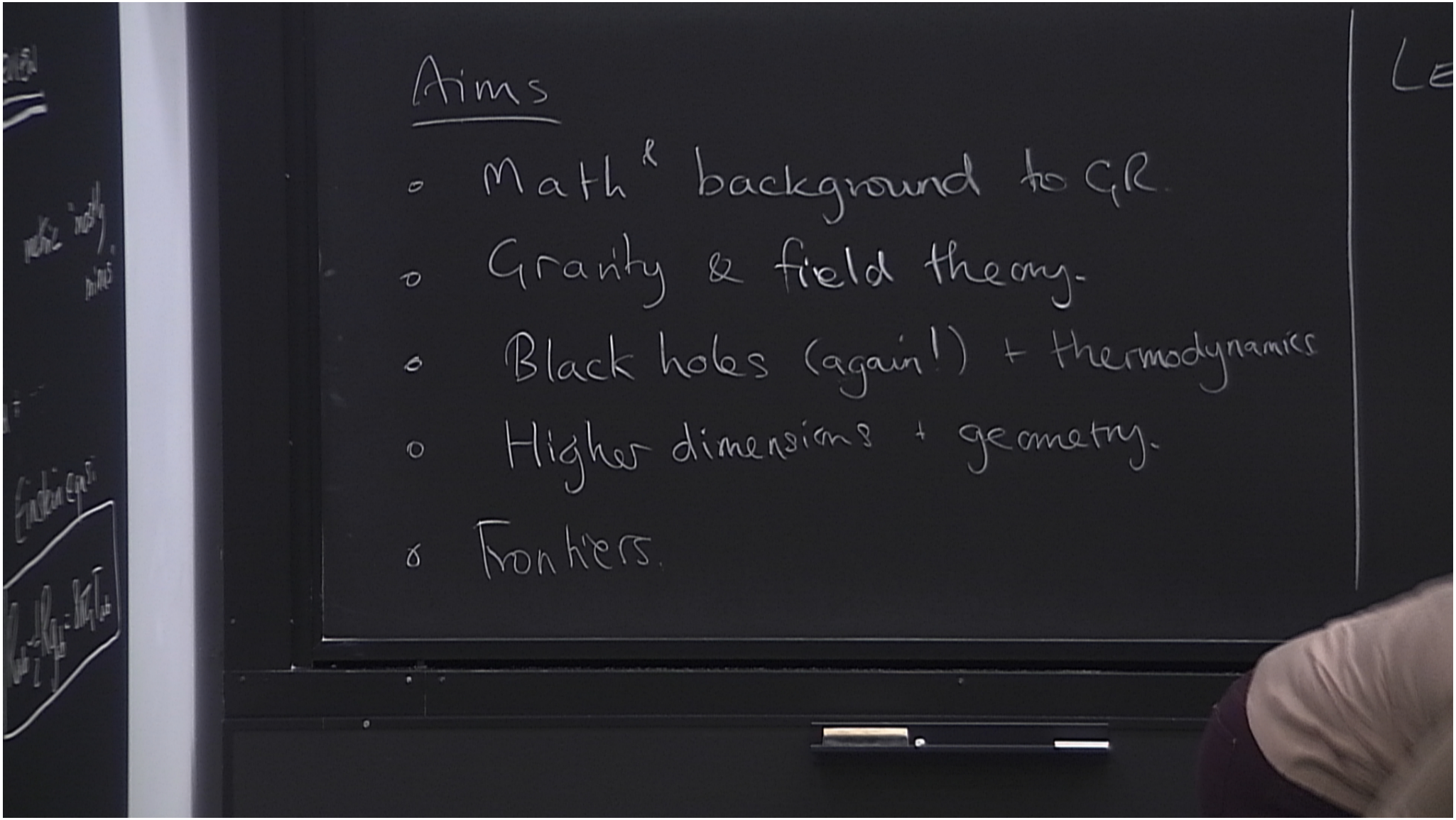
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Abstract:


GRAVITATIONAL PHYSICS REVIEW
Conventions + - -
metric "mostly


GRAVITATIONAL PHYSICS REVIEW
Conventions + - -
metric "mostly


GRAVITATIONAL PHYSICS REVIEW
Conventions: + − −
metric "mostly minus"
$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \ldots$
$R_{ab} = R^c{}_{acb}$
$\hbar = c = 1$ BUT NOT G_N
Einstein eqns:
$R_{ab} - \frac{1}{2} R g_{ab} = 8\pi G T_{ab}$


Aims
Math & background to GR
Gravity & field theory.
Black holes (again!) + thermodynamics
Higher dimensions + geometry.
Frontiers.

LECTURE 1 Revision: Differential Geometry

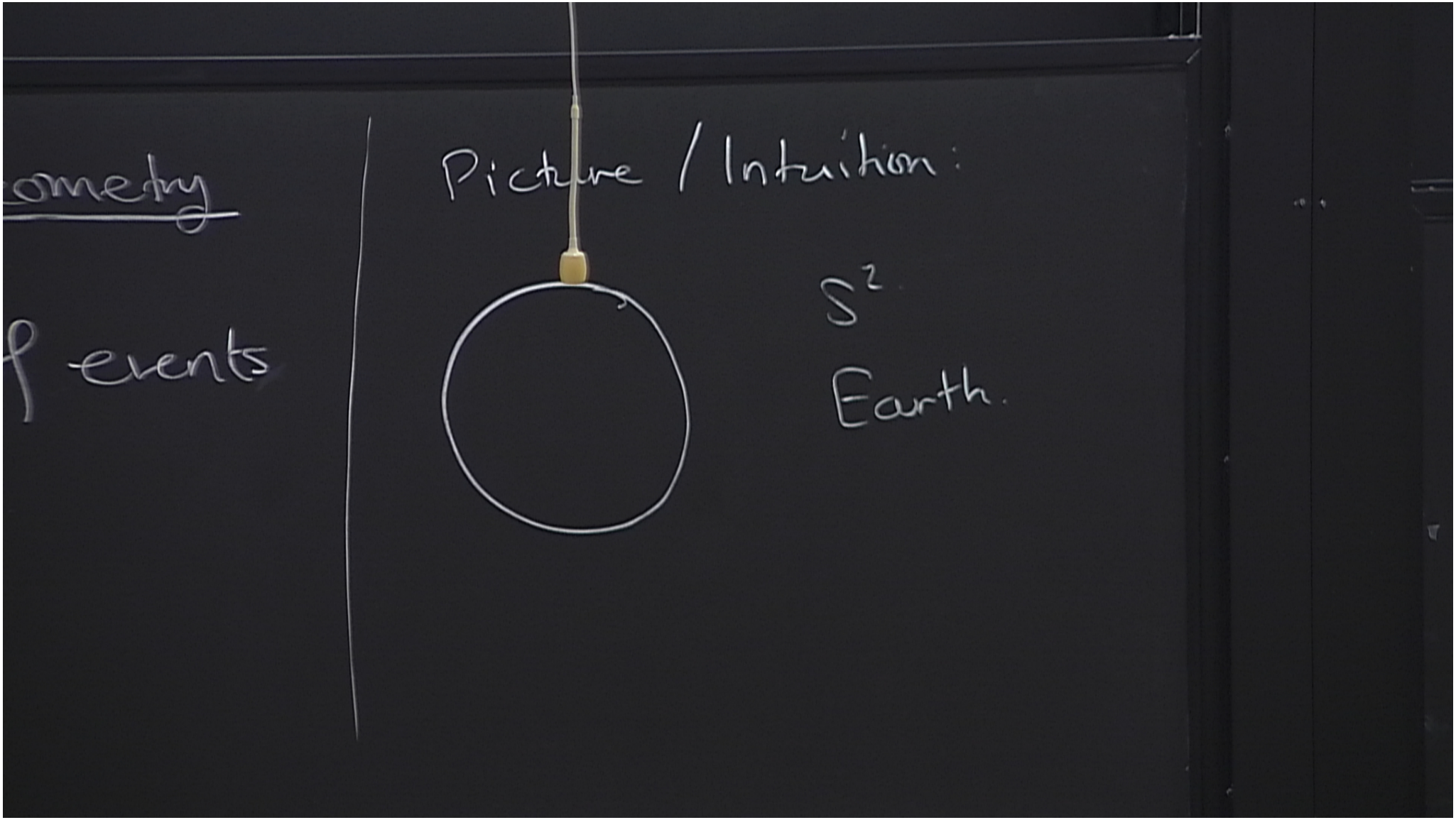
Defn: A manifold is a set of events that look locally like $\mathbb{R}^n$

Picture / Intuition



S^2

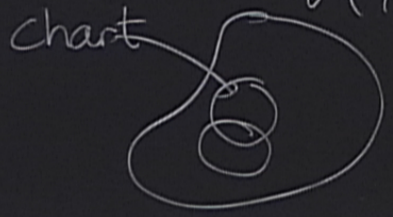
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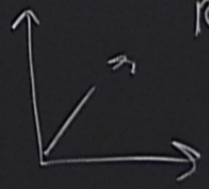

ometry
f events
Picture / Intuition:
S^2
Earth.

LECTURE 1 Revision: Differential Geometry

Defn: A manifold is a set of events that look locally like $\mathbb{R}^n$

$\mathcal{M}$ / spacetime



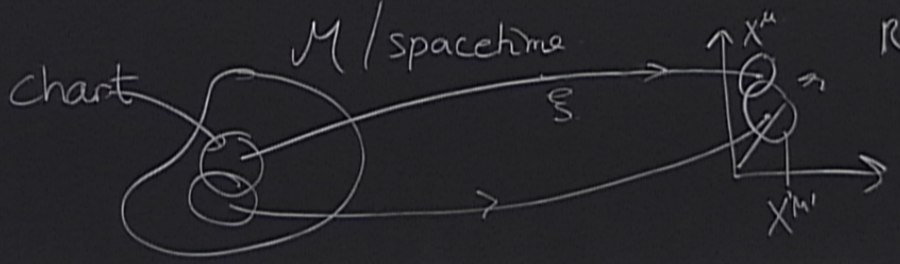


Picture / Int



LECTURE 1 Revision: Differential Geometry

Defn: A manifold is a set of events that look locally like $\mathbb{R}^n$



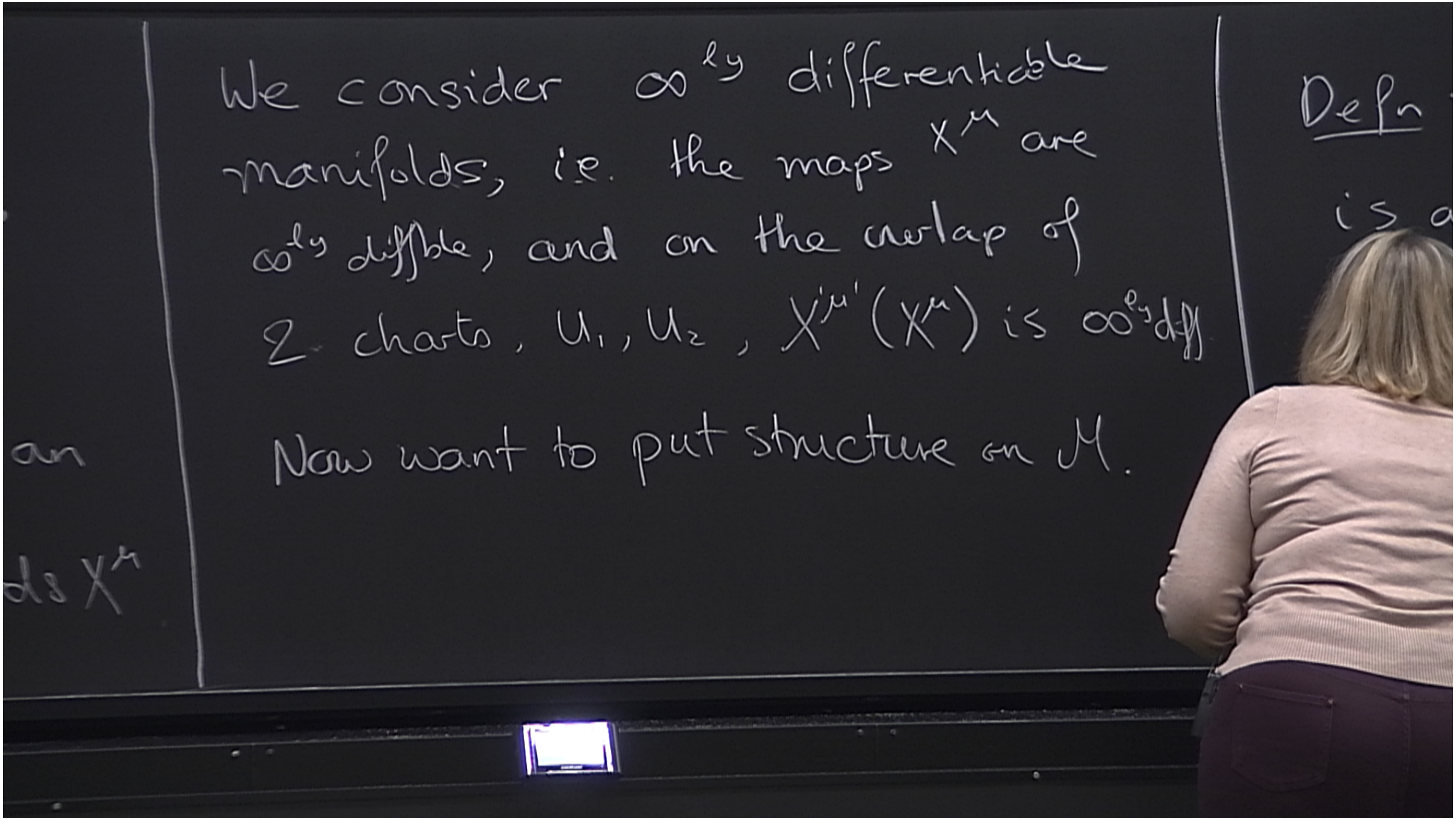
Picture / Intui

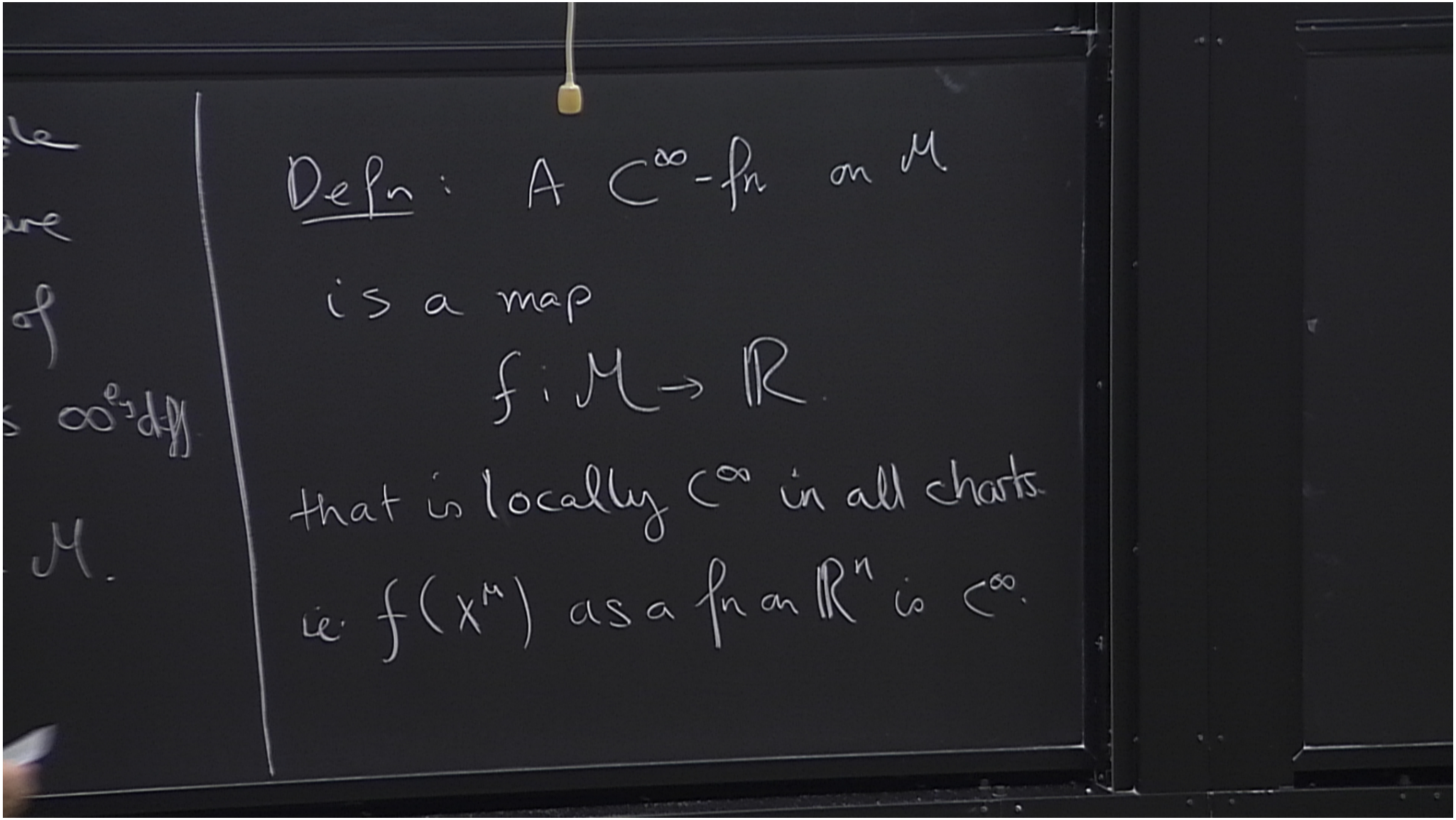
The manifold is covered by an atlas of open sets (charts) U_i,

$$\mathcal{M} = \bigcup_i U_i$$

each of which has a map U_i into an open subset of $\mathbb{R}^n$, giving co-ords x^μ

We c
mani
∞^{ly} d
2 c
Now


We consider ∞^{ly} differentiable manifolds; i.e. the maps x^μ are ∞^{ly} diffble, and on the overlap of 2 charts, U_1, U_2, $x'^{\mu'}(x^\mu)$ is ∞^{ly} diff
Now want to put structure on M.
Defn
is a
an
ds x^μ

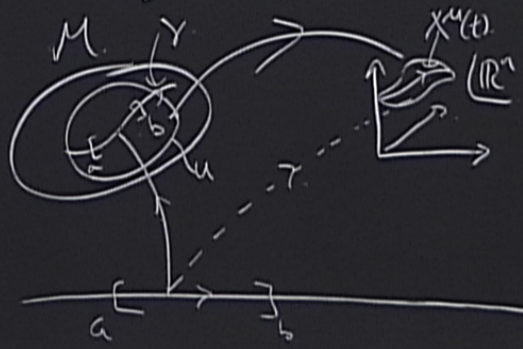

Defn: A C^∞-fn on $\mathcal{M}$
is a map
$f : \mathcal{M} \to \mathbb{R}$
that is locally C^∞ in all charts.
ie. $f(x^\mu)$ as a fn on $\mathbb{R}^n$ is C^∞.

each of which has a map U_i into an open subset of $\mathbb{R}^n$, giving co-ords x^μ

Now want

Defn: A C^∞ curve in a map

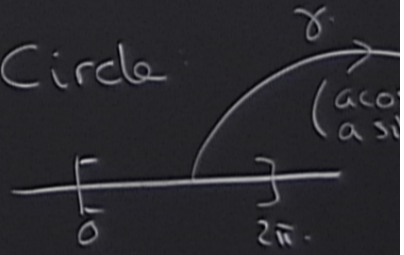
$\gamma : \mathbb{R} \to M$ s.t. image in $\mathbb{R}^n$ in C^∞

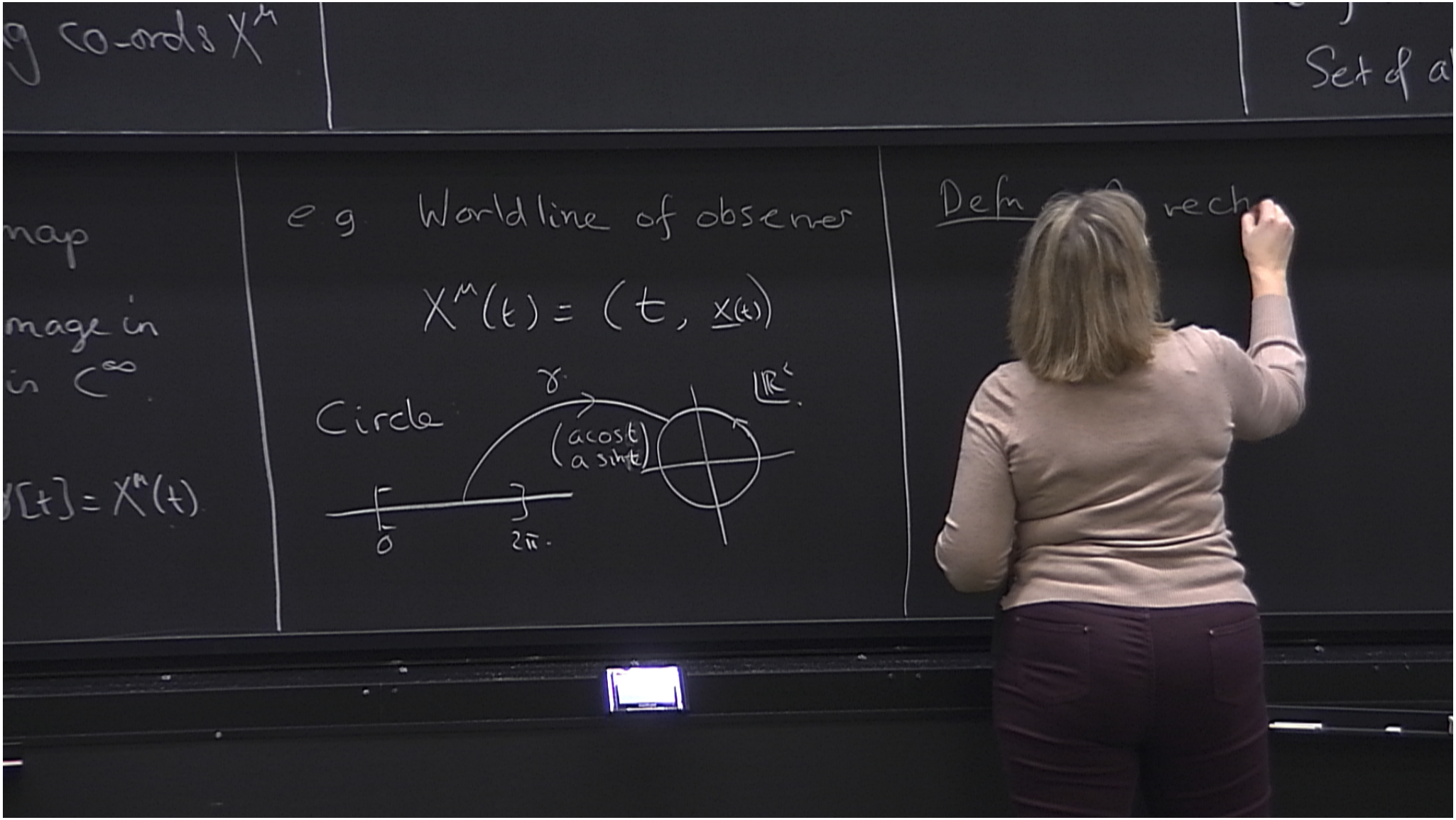


e.g. World line

$x^\mu(t) =$

Circle




g co-ords X^μ
Set of a
map
mage in
in C^∞
$[t] = X^\mu(t)$
e g World line of observer
$X^\mu(t) = (t, \underline{x}(t))$
Circle
γ
$\mathbb{R}^2$
(a cost)
a sint
0
2π
Defn
rech

i.e. $f(x^\mu)$ as a fn on $\mathbb{R}^4$ is C^∞.
Set of all such fns is $C^\infty(M)$

of observer

$x(t)$

$\mathbb{R}^4$



Defn. A vector is defined as a tangent to a curve - a linear operator

i.e. $f(x^\mu)$ as a fn on $\mathbb{R}^4$ is C^∞.

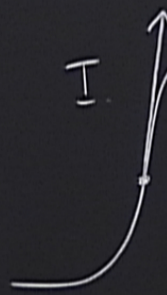
Set of all such fns is $C^\infty(M)$

f observer

$x(t)$

$\mathbb{R}^4$

Defn. A <u>vector</u> is defined as a tangent to a curve - a linear operator



$I : C^\infty(M) \to C^\infty(M)$

$f \longmapsto \frac{df}{dt}$ at P

For $f \in C^\infty(M)$, $f(t) = f \circ \gamma(t)$

is a $\mathbb{R}$-fn hence diffble.

Since $If = \frac{df}{dt} \quad \forall f \in C^\infty(M)$

we may write (in a local chart)

$$I = T^\mu \frac{\partial}{\partial x^\mu}$$

Note I is de

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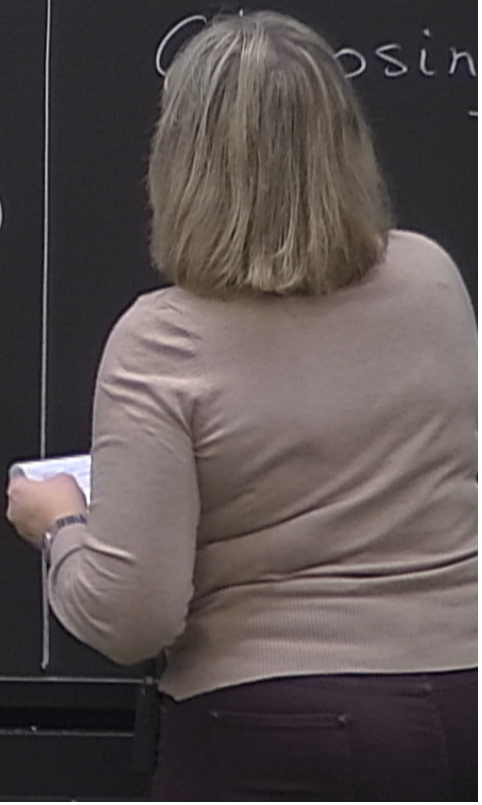
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Note I

C osin



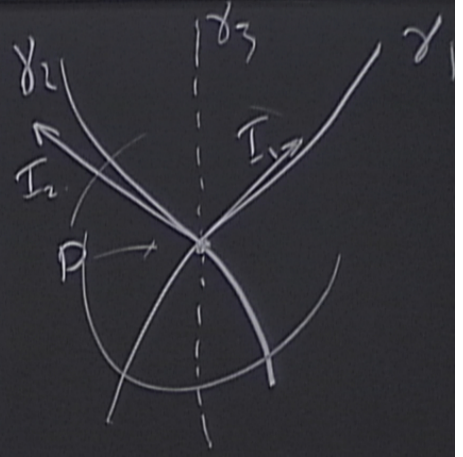
$f \circ \gamma(t)$

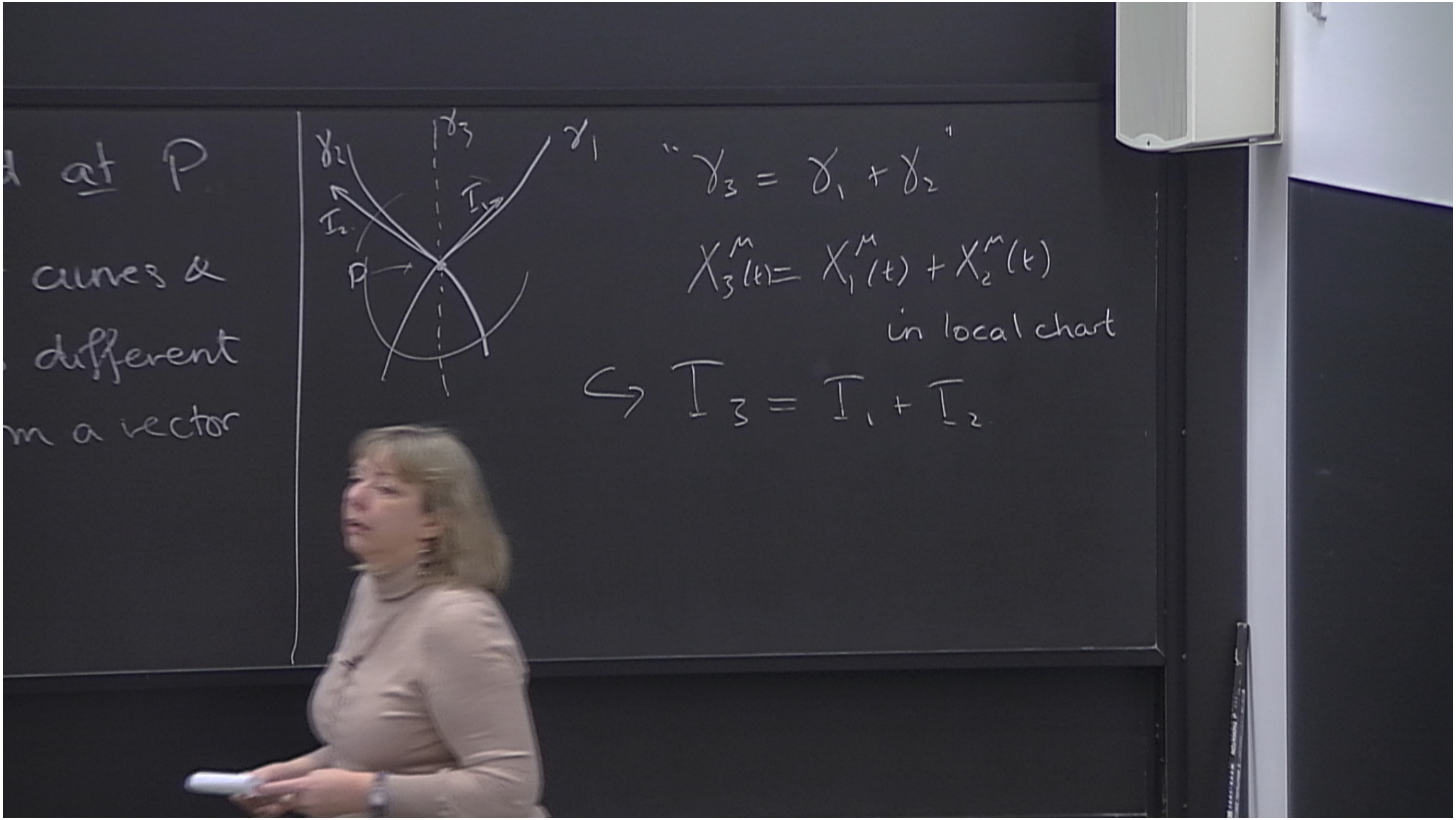
$\in C^\infty(M)$

$t)$

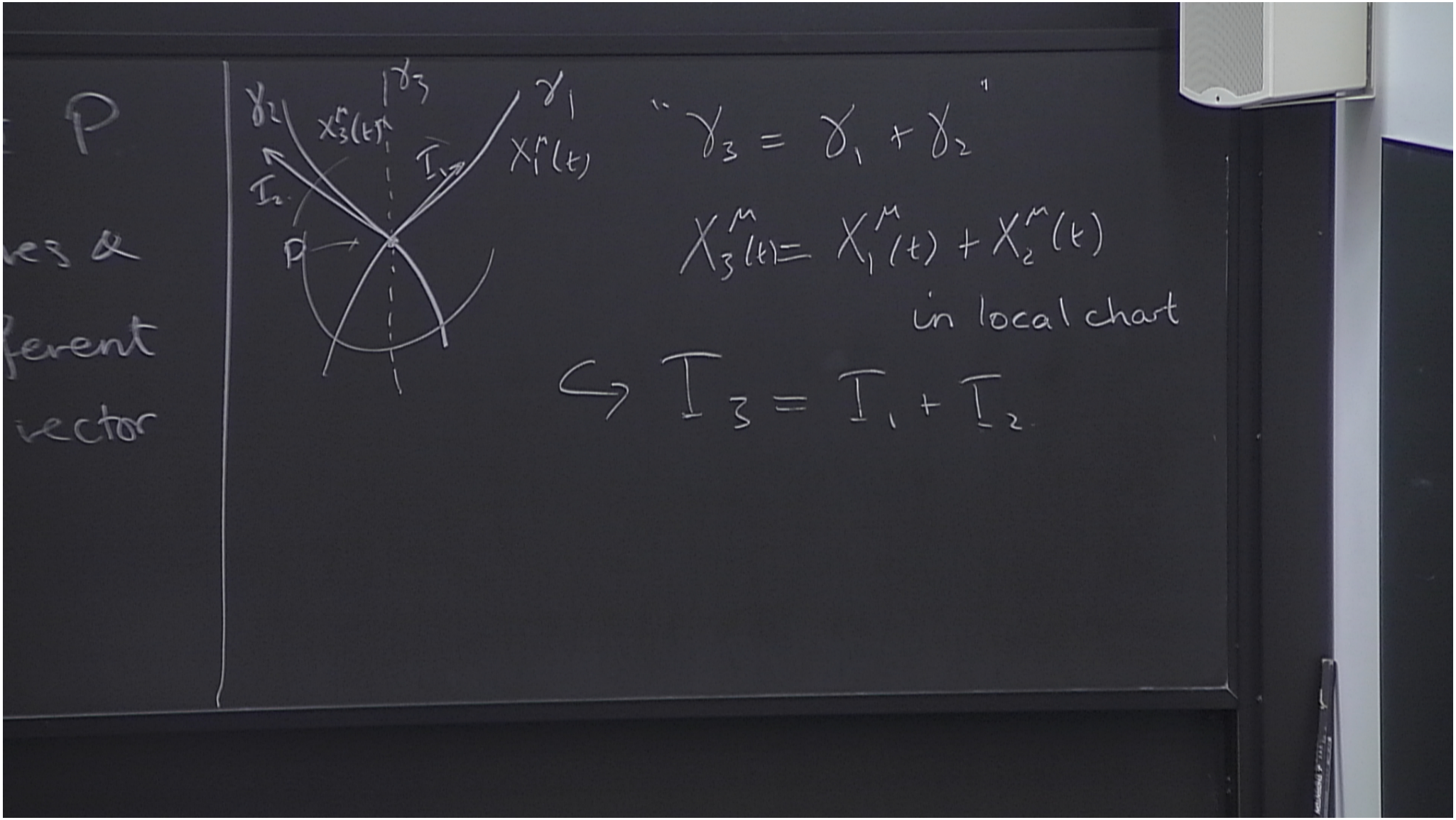
Note I is defined at P

Choosing different curves & parametrization gives different operators which form a vector space at P.




at P
curves &
different
m a vector
γ_2
γ_3
γ_1
I_1
I_2
P
$\because \gamma_3 = \gamma_1 + \gamma_2$
$X_3^\mu(t) = X_1^\mu(t) + X_2^\mu(t)$
in local chart
$\hookrightarrow I_3 = I_1 + I_2$




γ3
γ2
γ1
$X_3^n(t)$
$X_1^n(t)$
I_3
I_2
I_1
p
$\gamma_3 = \gamma_1 + \gamma_2$
$X_3^M(t) = X_1^M(t) + X_2^M(t)$
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on $\mathbb{R}^n$ is C^∞.

is $C^\infty(M)$

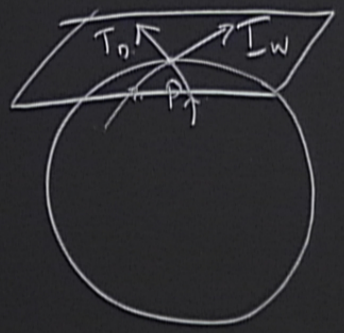
a tangent

ator

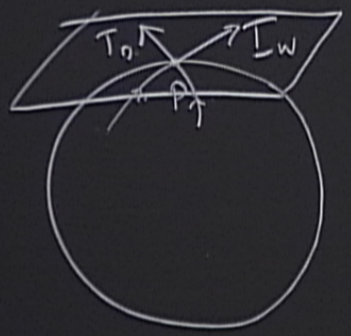
$\to C^\infty(M)$

$\to \frac{df}{dt}$ at P.

For S^2:



For S^2:



Bases $T_p(M)$ is a vector space

so we can choose a basis

e.g. $\frac{\partial}{\partial x^\mu}$ $\Leftrightarrow$ coord basis

also we will use an orthonormal basis

Gravitational Physics Review

Conventions: + − − metric "mostly minus"

$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \ldots$

$R_{ab} = R^c{}_{acb}$

$\hbar = c = 1$ But Not G_N

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each of which has a map U_i into an open subset of $\mathbb{R}^n$, giving coords x^μ

Now want to put structure

Defn: A C^∞ curve in a map

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$\gamma(t) = x^\mu(t)$

e.g. Worldline of observer

$x^\mu(t) = (t, \underline{x}(t))$

Circle

$T(M)$

$e_a{}^\mu \frac{\partial}{\partial x^\mu}$

metric

$g_{ab} = \eta_{ab}$

Abstract Index Notation

Commonly we write T^μ to mean the vector $\bar{T}$ e.g. $\nabla_\mu T^\nu = \partial_\mu T^\nu + \Gamma^\nu_{\mu\lambda} T^\lambda$

but strictly, T^μ are the components of the vector $\bar{T}$ in coord basis $\frac{\partial}{\partial x^\mu}$

and are scalars!



Defn A covector is a map

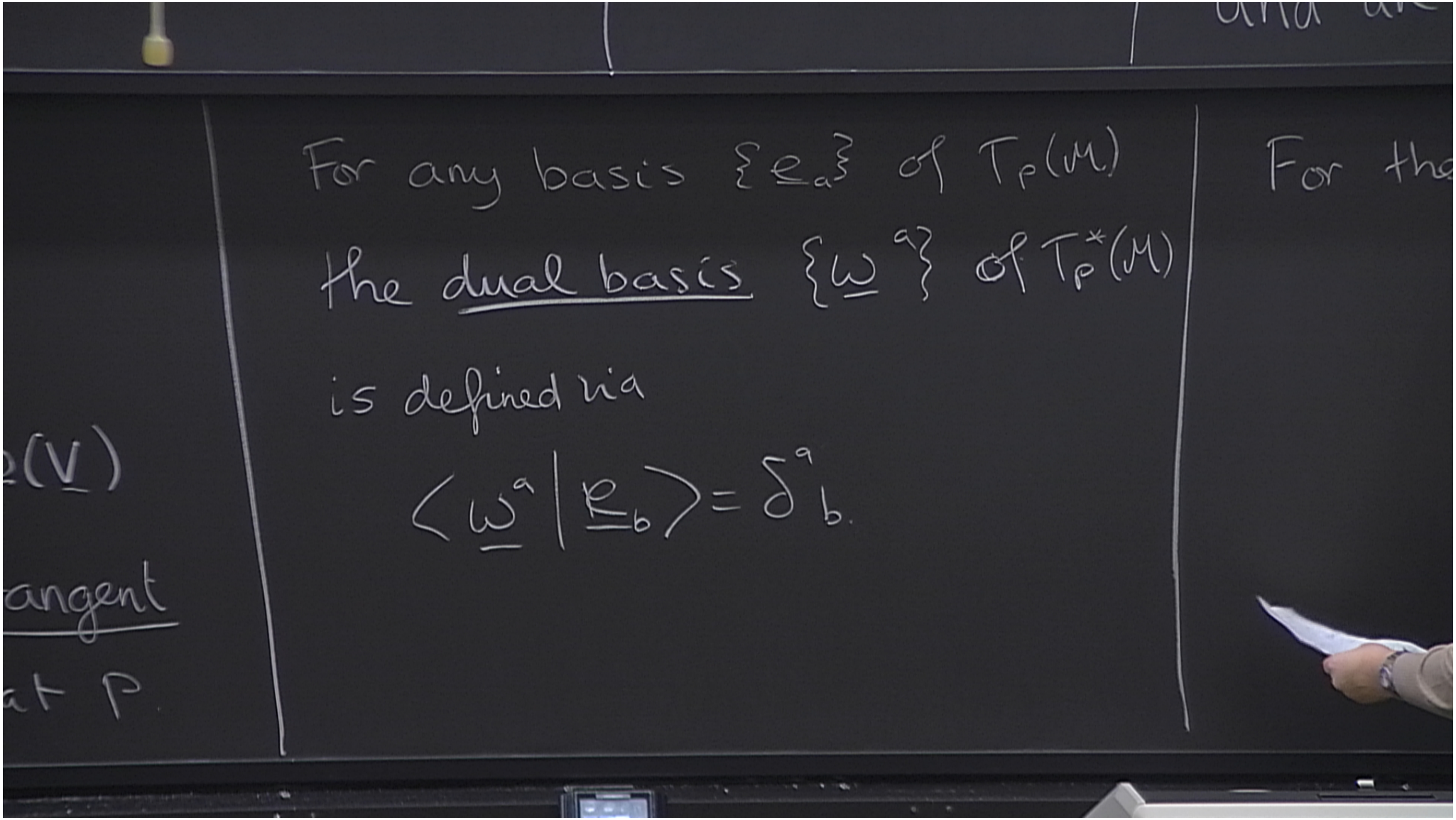
$$T_p(M) \longrightarrow \mathbb{R}$$

denoted as $\langle \underline{\omega} | \underline{V} \rangle$ or $\underline{\omega}(\underline{V})$

$\underline{\omega} \in T_p^*(M)$ - the cotangent space at p

For any

the du


For any basis $\{\underline{e}_a\}$ of $T_p(M)$
the dual basis $\{\underline{\omega}^a\}$ of $T_p^*(M)$
is defined via
$\langle \underline{\omega}^a | \underline{e}_b \rangle = \delta^a_b$.

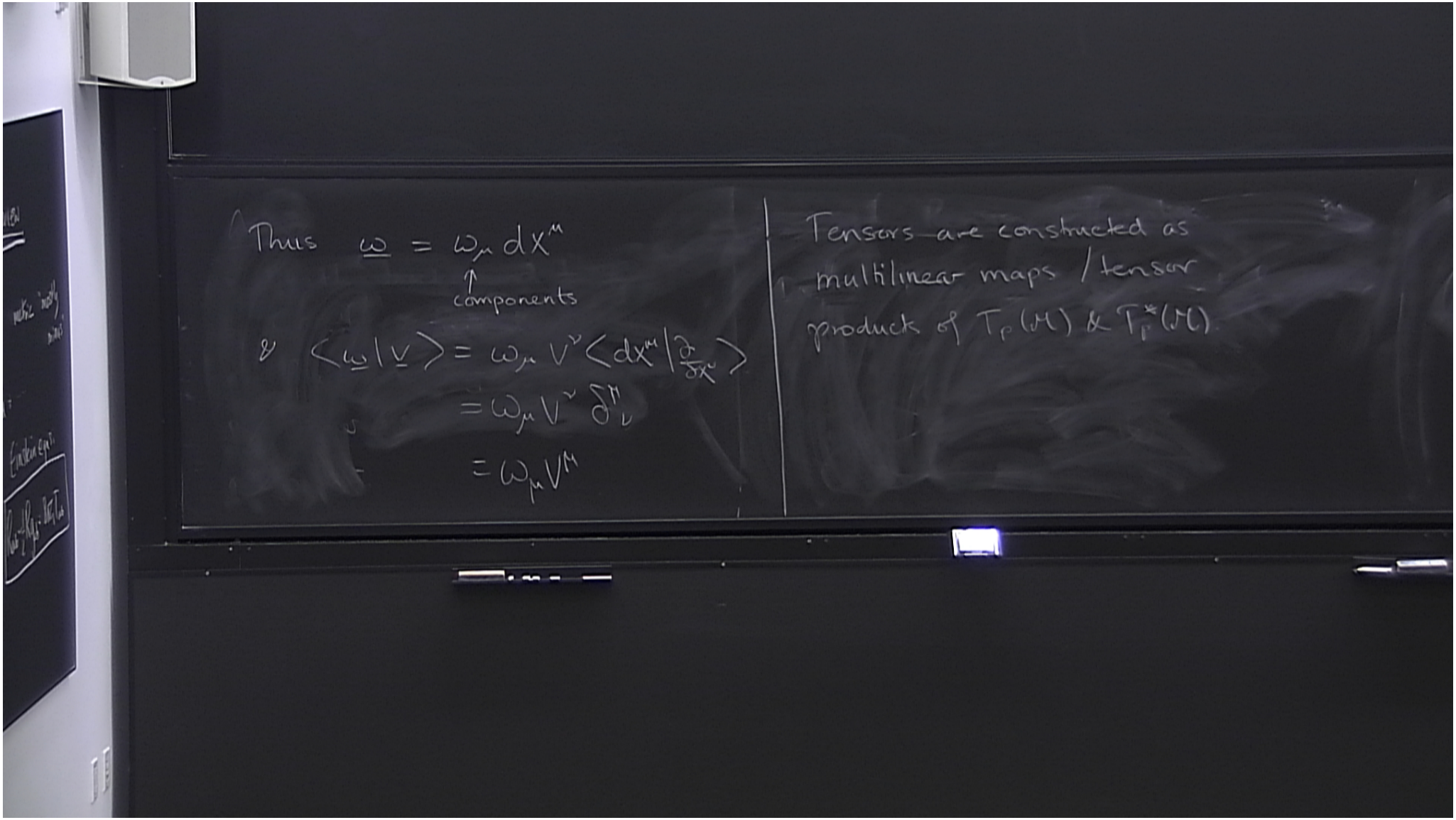
and are scalars!

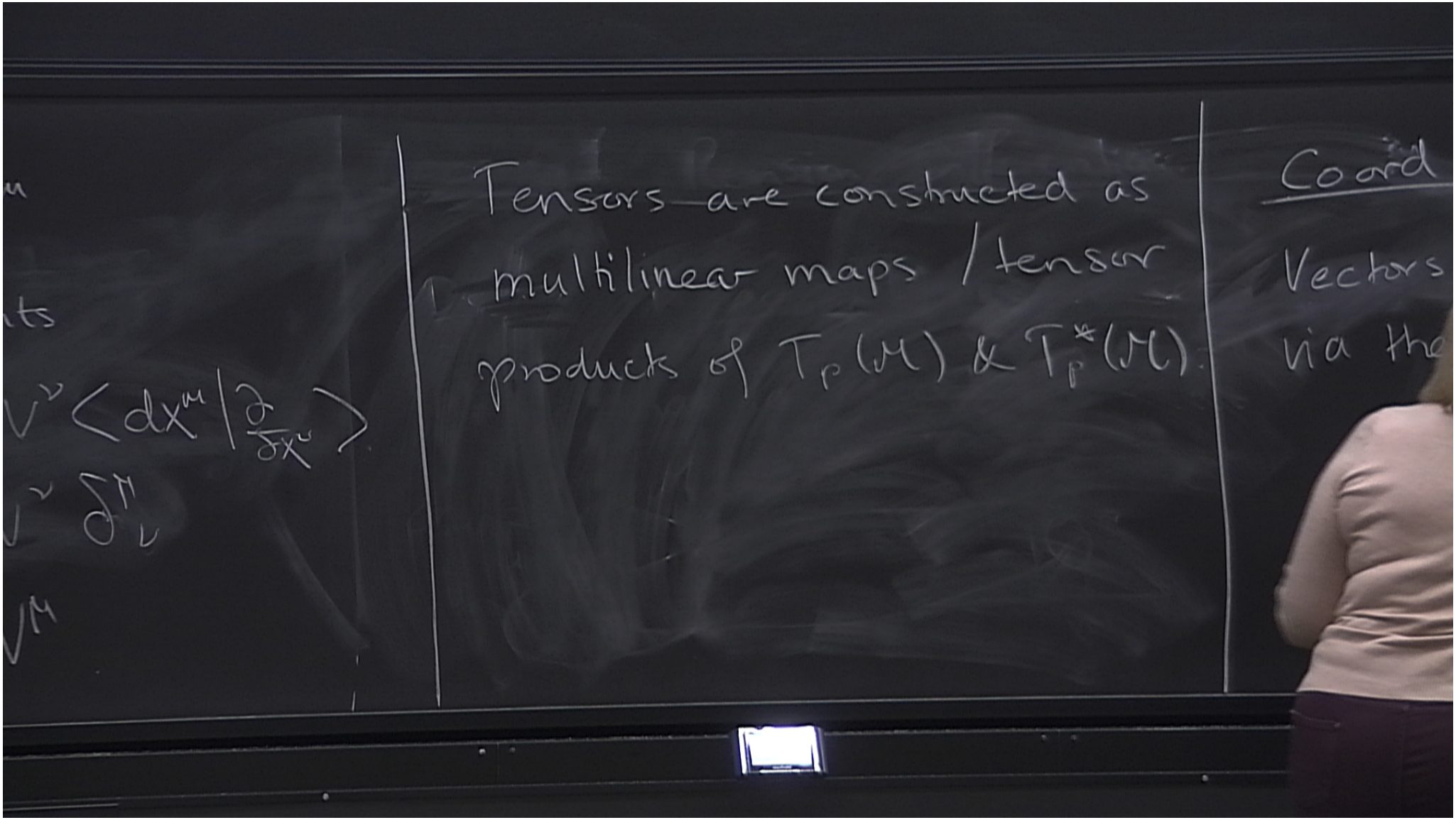
of $T_p(M)$

$\}$ of $T_p^*(M)$

For the coord basis, we write dx^μ, then

$$\langle dx^\mu | \frac{\partial}{\partial x^\nu} \rangle = \delta^\mu_\nu = \frac{\partial x^\mu}{\partial x^\nu}$$


Thus $\underline{\omega} = \omega_\mu dx^\mu$
components
& $\langle \underline{\omega} | \underline{v} \rangle = \omega_\mu V^\nu \langle dx^\mu | \frac{\partial}{\partial x^\nu} \rangle$
$= \omega_\mu V^\nu \delta^\mu_\nu$
$= \omega_\mu V^\mu$
Tensors are constructed as multilinear maps / tensor product of $T_p(M)$ & $T_p^*(M)$


Tensors are constructed as
multilinear maps / tensor
products of $T_p(M)$ & $T_p^*(M)$.
Coord
Vectors
via the

er ∞^{ly} differentiable

ie. the maps x^μ are

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to put structure on $\mathcal{M}$.

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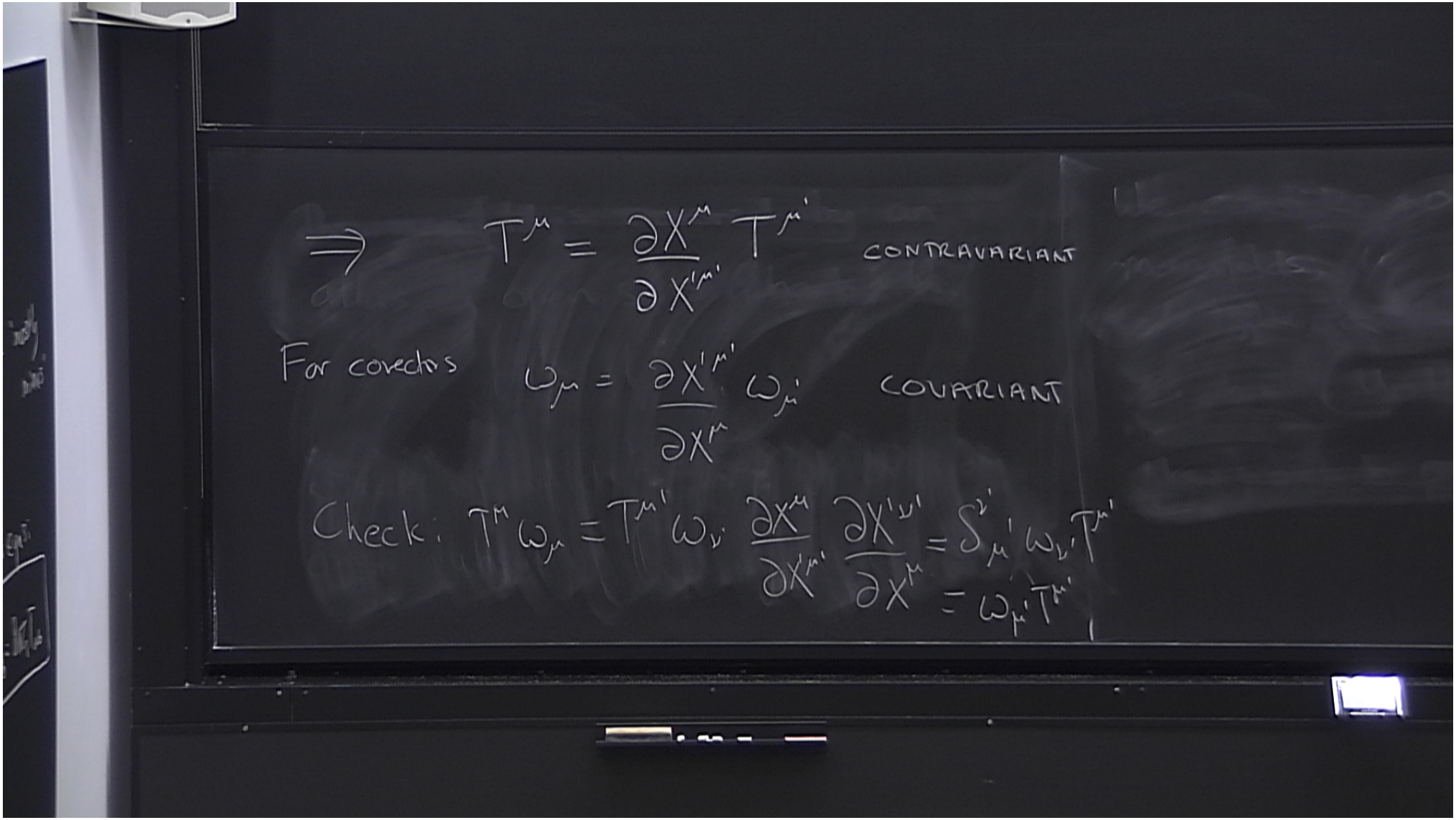
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Set of all such fns is $C^\infty(\mathcal{M})$


$\Rightarrow \quad T^{\mu} = \frac{\partial x^{\mu}}{\partial x'^{\mu'}} T^{\mu'}$ CONTRAVARIANT
For covectors $\omega_{\mu} = \frac{\partial x'^{\mu'}}{\partial x^{\mu}} \omega_{\mu'}$ COVARIANT
Check: $T^{\mu} \omega_{\mu} = T^{\mu'} \omega_{\nu'} \frac{\partial x^{\mu}}{\partial x'^{\mu'}} \frac{\partial x'^{\nu'}}{\partial x^{\mu}} = \delta^{\nu'}_{\mu'} \omega_{\nu'} T^{\mu'}$
$= \omega_{\mu'} T^{\mu'}$

CONTRAVARIANT

$$\frac{\partial x'^{\nu'}}{\partial x^{\mu}} = S^{\nu'}_{\mu'}\,\omega_{\nu'}\,T^{\mu'}$$

$$= \omega_{\mu'}\,T^{\mu'}$$

COVARIANT

- this is why differentiating vectors is problematic on a curved manifold

$$\frac{\partial T^{\mu}}{\partial x^{\nu}} = \frac{\partial x'^{\nu'}}{\partial x^{\nu}} \frac{\partial}{\partial x'^{\nu'}} \left(T^{\mu'} \frac{\partial x^{\mu}}{\partial x'^{\mu'}} \right)$$

$$= \frac{\partial x'^{\nu'}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x'^{\mu'}} \frac{\partial T^{\mu'}}{\partial x'^{\nu'}} + \frac{\partial x'^{\nu'}}{\partial x^{\nu}} \left(\frac{\partial^2 x^{\mu}}{\partial x'^{\nu'} \partial x'^{\mu'}} \right) T^{\mu'}$$