Title: Standard Model-10

Date: Jan 16, 2015 10:15 AM

URL: http://pirsa.org/15010086

Abstract:

Part 2 (Renormalization & QED)

in free theory for $\phi^{(0)}(x)$ "Bare" field. in lagrangian.

$$\int d^4x e^{ipx} \langle 0|T\{\phi^{(0)}(x)\phi^{(0)}(0)\}|0\rangle = \frac{i}{p^2 - m^2 + i\varepsilon} \Rightarrow \text{Pole Has Residue "}i\text{"}$$

Problem: w/ interactions:

$$\int d^4x e^{ipx} \langle \Omega|T\{\phi^{(0)}(x)\phi^{(0)}(0)\}|\Omega\rangle = \frac{i(Z)}{p^2 - m^2 + i\varepsilon}$$

Z is "wave function" Renormaliz. Constant. (field strength Renormaliz.)

By Hand, define $\phi^{(r)}(x) = \frac{1}{\sqrt{Z_\phi}}\phi^{(0)}(x)$

$$\int d^4x\, e^{ipix} \langle\Omega|T\left(\frac{\phi^{(0)}(x)}{\sqrt{Z_\phi}}\frac{\phi^{(0)}(0)}{\sqrt{Z_\phi}}\right)|\Omega\rangle = \frac{i}{p^2-m^2+i\varepsilon} + \cdots$$

dont mess up Pole

What Happens to Lagrangian?

$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu\phi^{(0)})\partial_\mu\phi^{(0)} + \cdots$ write in Renormalized fields.

$= \frac{Z_\phi}{2}\partial_\mu\phi^{(r)}\partial_\mu\phi^{(r)} + \cdots$ Kinetic term is Non-Canonical

ction"

th Renormalize)

Renormalized objects obey usual feynman Rules.

$$\int d^4x\, e^{ipx} \langle \Omega | T\{\underbrace{\phi^{(r)}(x)\phi^{(r)}(0)}\}|\Omega\rangle = \frac{i}{p^2 - m^2 + i\varepsilon} + (\cdots)$$

line in feyn. diagrams.

$\mathcal{L}_e$ becomes:

$$= \underbrace{\frac{1}{2}\partial_\mu\phi^{(r)}\partial^\mu\phi^{(r)}}_{\text{proper kinetic term.}} + \underbrace{\frac{1}{2}(Z_\varphi - 1)\partial_\mu\phi^{(r)}\partial^\mu\phi^{(r)}}_{\text{New interaction (!)}}$$

define New Interaction

$$\frac{1}{2}\delta_\varphi\,\partial_\mu\phi^{(r)}\partial^\mu\phi^{(r)}$$

term in Potential!

New feynman Rule: $\quad -\!-\!\otimes\!-\!- = i\delta_3(p^2)$

define δ_3 inside the theory to be consistent with our desired pole in propagator @ $p^2 = m^2$)

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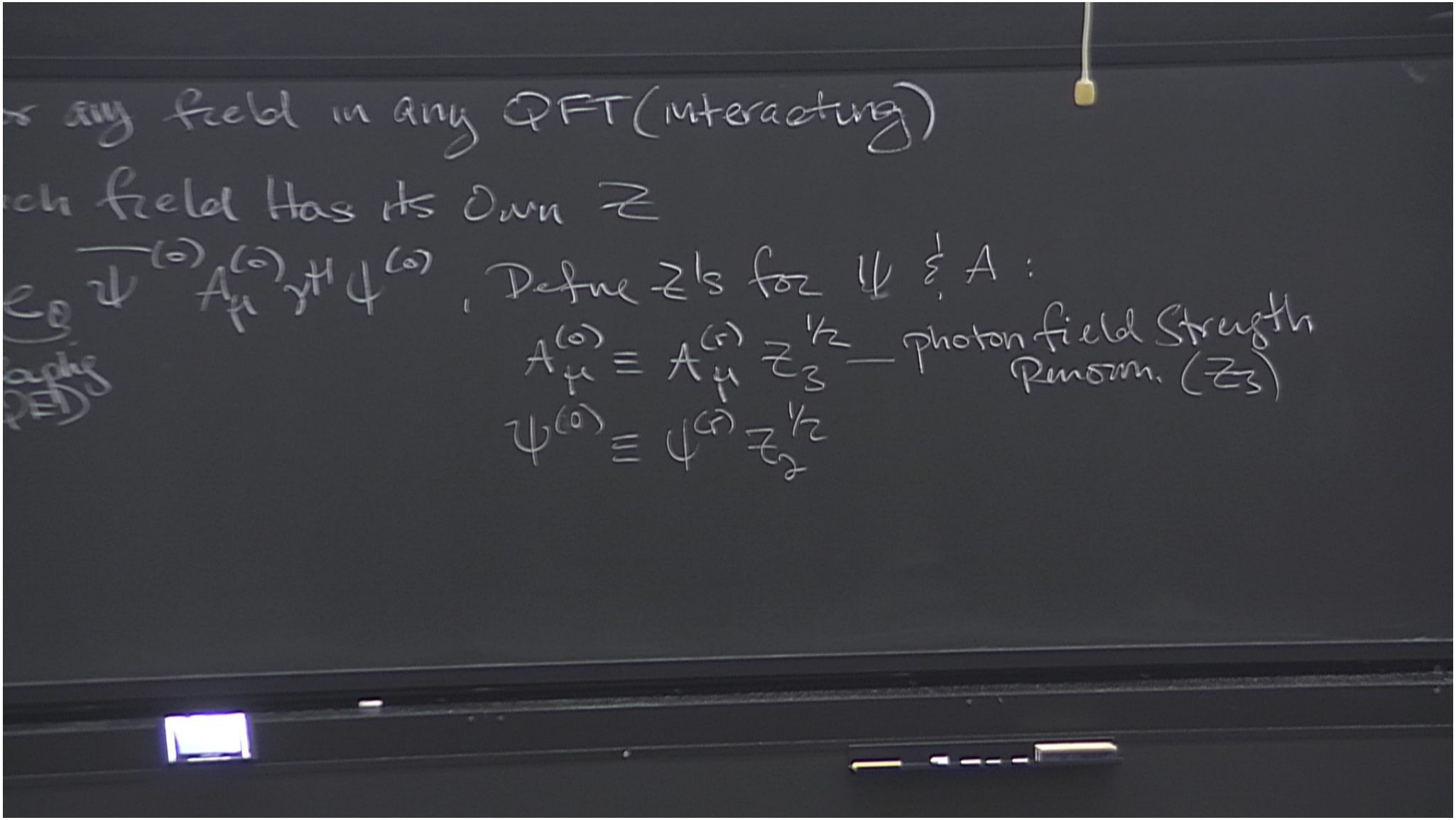
Must be included, wherever there is possibility of pole being shifted!

Nothing Special about scalar ϕ → true for any field in any

Consider QED w/ fermion (Dirac) ψ, Each field has its

$$\mathcal{L}_{QED} = -\frac{1}{4}\left(\partial_\mu A_\nu^{(0)} - \partial_\nu A_\mu^{(0)}\right)^2 + i\bar{\psi}^{(0)}\not{\partial}\psi^{(0)} + \underbrace{e_0\,\bar{\psi}^{(0)} A_\mu^{(0)} \gamma^\mu \psi^{(0)}}_{\text{Coupling QED}}$$

If this were whole story,
No ψ, A interactions
All z's are $= 1$


ny field in any QFT (interacting)
ch field Has its Own Z
$\bar{\psi}^{(0)} A_\mu^{(0)} \gamma^\mu \psi^{(0)}$, Define Z's for ψ & A:
$A_\mu^{(0)} \equiv A_\mu^{(r)} Z_3^{1/2}$ — photon field Strength Renorm. (Z_3)
$\psi^{(0)} \equiv \psi^{(r)} Z_2^{1/2}$

$$\mathcal{L}_{QED} = -\frac{1}{4} Z_3 (F_{\mu\nu}^{(r)})^2 + Z_2 i \bar{\psi}^{(r)} \underbrace{\not{\partial}}_{\gamma^\mu \partial_\mu} \psi^{(r)} + e_0 Z_2 Z_3^{1/2} A_\mu^{(r)} \bar{\psi}^{(r)} \gamma^\mu \psi^{(r)}$$

By Convention: $e_0 Z_2 Z_3^{1/2} \equiv Z_1 e_{phys.}$ — Renormalized electric chg.

Gauge Invariance: Requires

$$Z_2 \bar{\psi}^{(r)} \left(i\not{\partial} + e_{phys.} \underbrace{A^{(r)}}_{A_\mu^{(r)} \gamma^\mu} \right) \psi^{(r)} \Rightarrow$$

$\equiv \gamma^\mu D_\mu$

only if $Z_1 = Z_2$ in QED (does, See QFT Matthew Schwartz textbook)

Note. If $Z_1 = Z_2$

$$e_0 \equiv Z_3^{-1/2} e_{phys}$$

Physical Coupling determined Only b

$$\mathcal{L}_{QED} = -\frac{1}{4} Z_3 (F_{\mu\nu}^{(r)})^2 + Z_2 i \bar{\psi}^{(r)} \underbrace{\partial\!\!\!/}_{\gamma^\mu \partial_\mu} \psi^{(r)} + e_0 Z_2 Z_3^{1/2} A_\mu^{(r)} \bar{\psi}^{(r)} \gamma^\mu \psi^{(r)}$$

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Gauge Invariance: $A_\mu^{(r)} \gamma^\mu$

Requires:

$$Z_2 \bar{\psi}^{(r)} \left(\underbrace{i\partial\!\!\!/ + e_{phys.} A\!\!\!/^{(r)}}_{\equiv \gamma^\mu D_\mu} \right) \psi^{(r)} \Rightarrow$$

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Physical Coupling determined Only b

$\mathcal{L}^{(0)} = \bar{\psi}^{(0)} \not{\partial} \psi^{(0)} + e_0 Z_2 Z_3^{1/2} A_\mu^{(r)} \bar{\psi}^{(r)} \gamma^\mu \psi^{(r)}$

$\not{\partial} = \gamma^\mu \partial_\mu$

$Z_1 e_{phys}$ — Renormalized electric chg.

only if $Z_1 = Z_2$ in QED

$\Rightarrow$ (does; See QFT Matthew Schwartz textbook)

Note. If $Z_1 = Z_2$

$$e_0 \equiv Z_3^{-1/2} e_{phys}$$

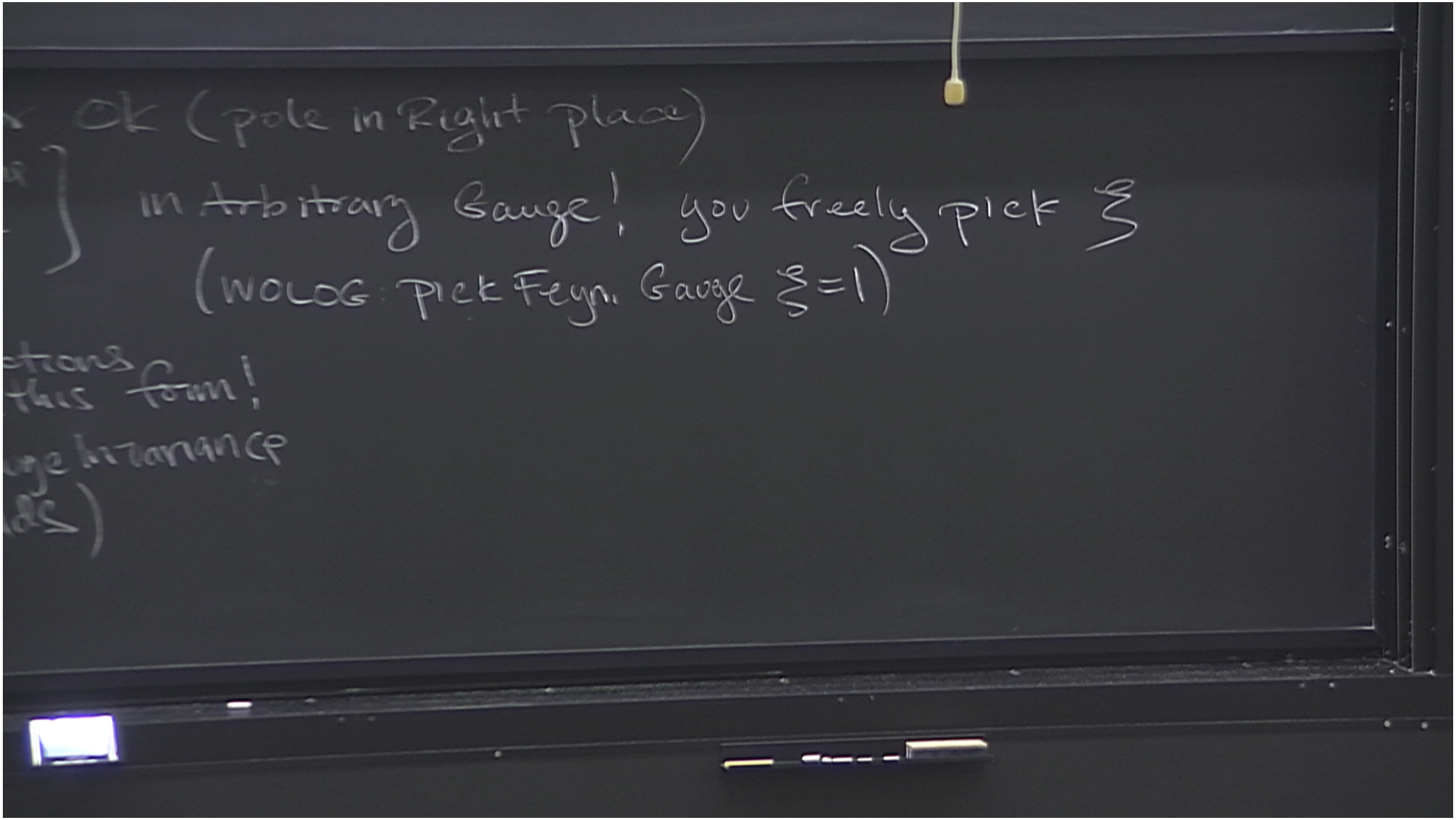
Physical Coupling determined Only by Z_3

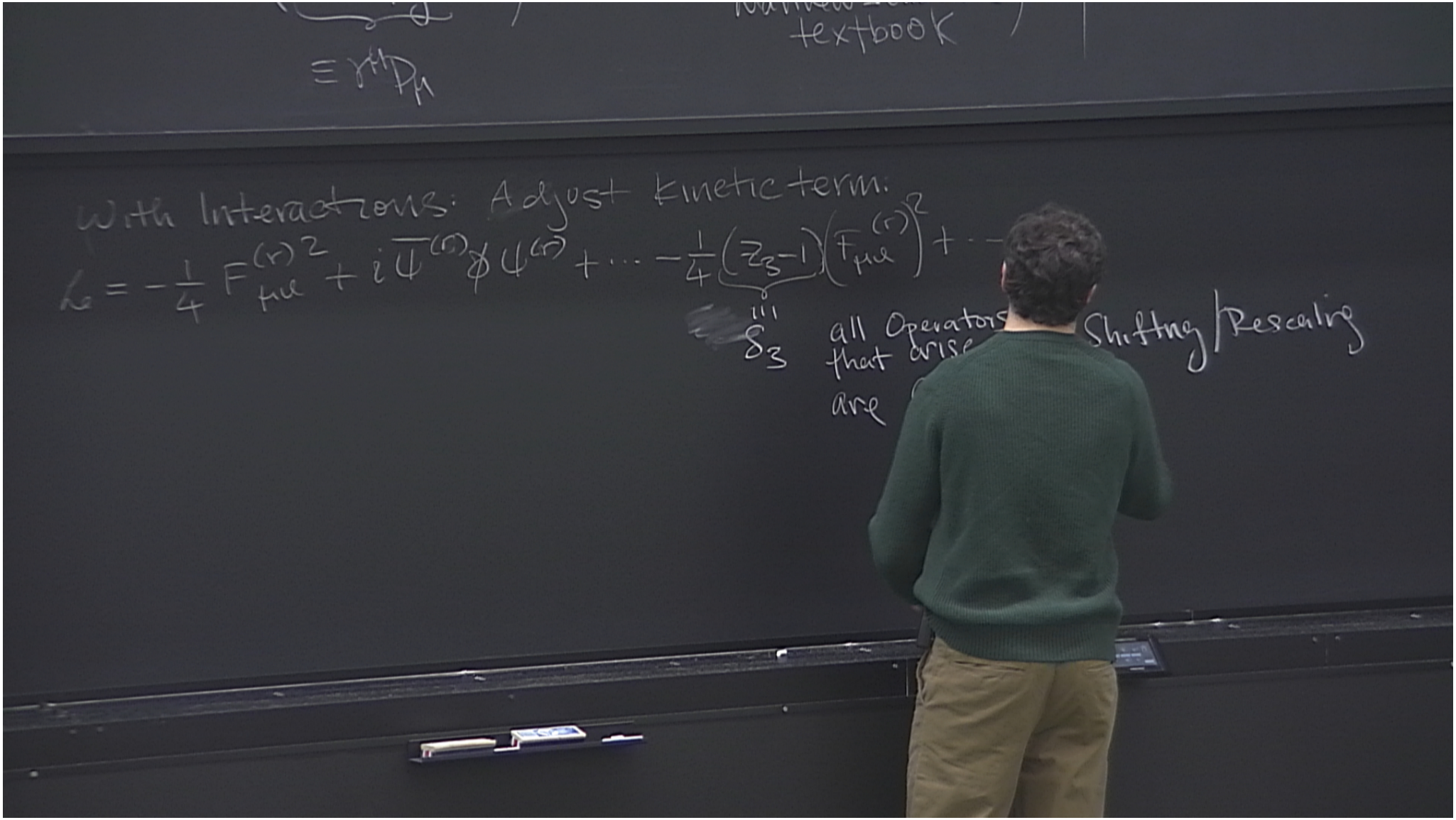
Goal: find Z_3, chosen to Make A_μ propagator Ok (pole

Propagator (free theory) $iG_{\mu\nu}(p^2) \equiv \frac{-i}{p^2}\left(g^{\mu\nu} - (1-\xi)\frac{p^\mu p^\nu}{p^2}\right)$ in Arbit

(wolo

photon prop $\longrightarrow \frac{-i}{p^2}(g^{\mu\nu})$ all Corrections will Have this form! (if Gauge Invariance Holds)


ck (pole in Right place)
in ~~Arb~~itrary Gauge! you freely pick ξ
(WOLOG: Pick Feyn. Gauge ξ=1)
etrans
this ~~form~~!
ge Invariance
ds)


textbook
With Interactions: Adjust kinetic term:
δ_3
Shifting/Rescaling

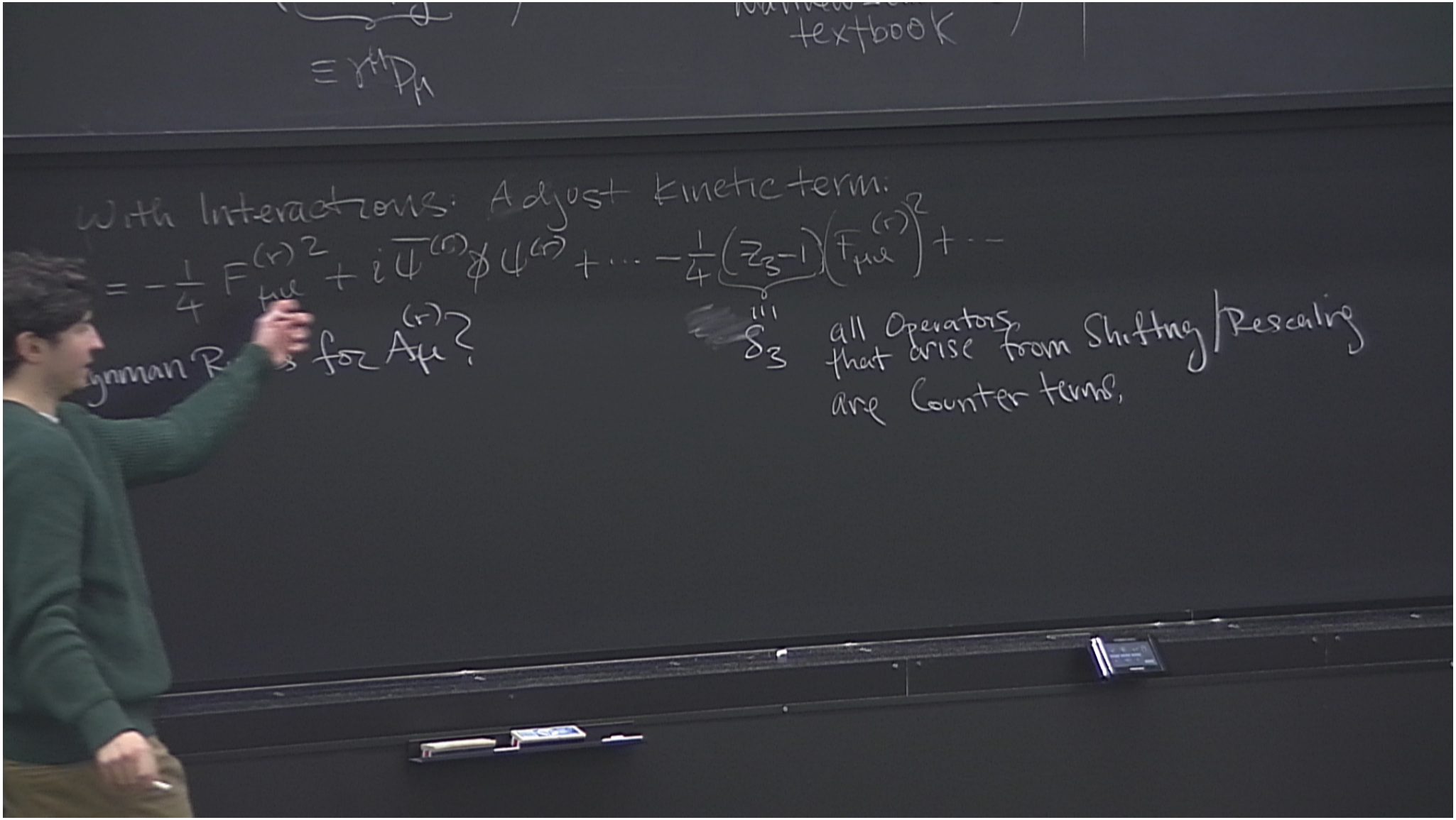
$\equiv \gamma^\mu D_\mu$

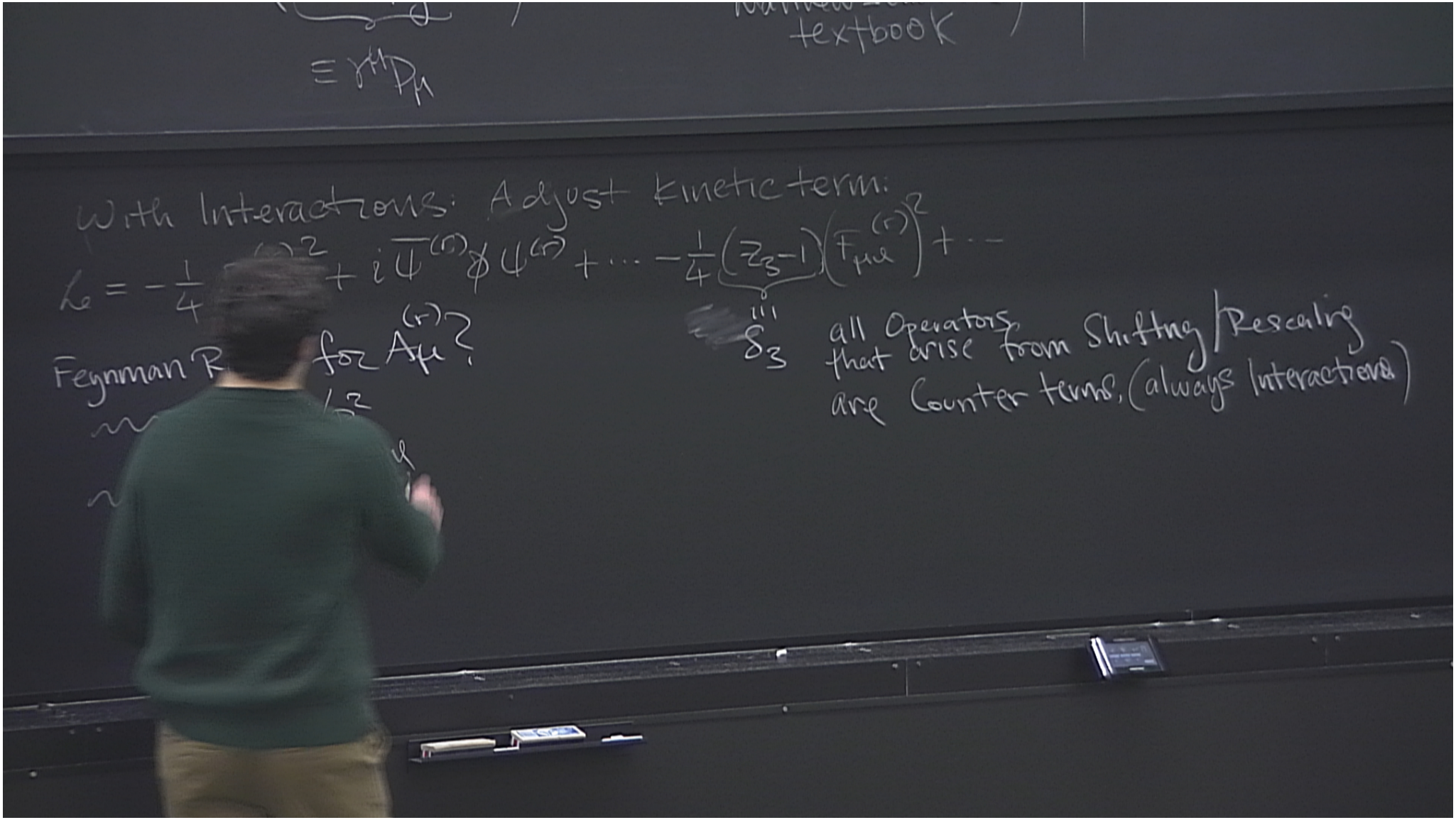
textbook

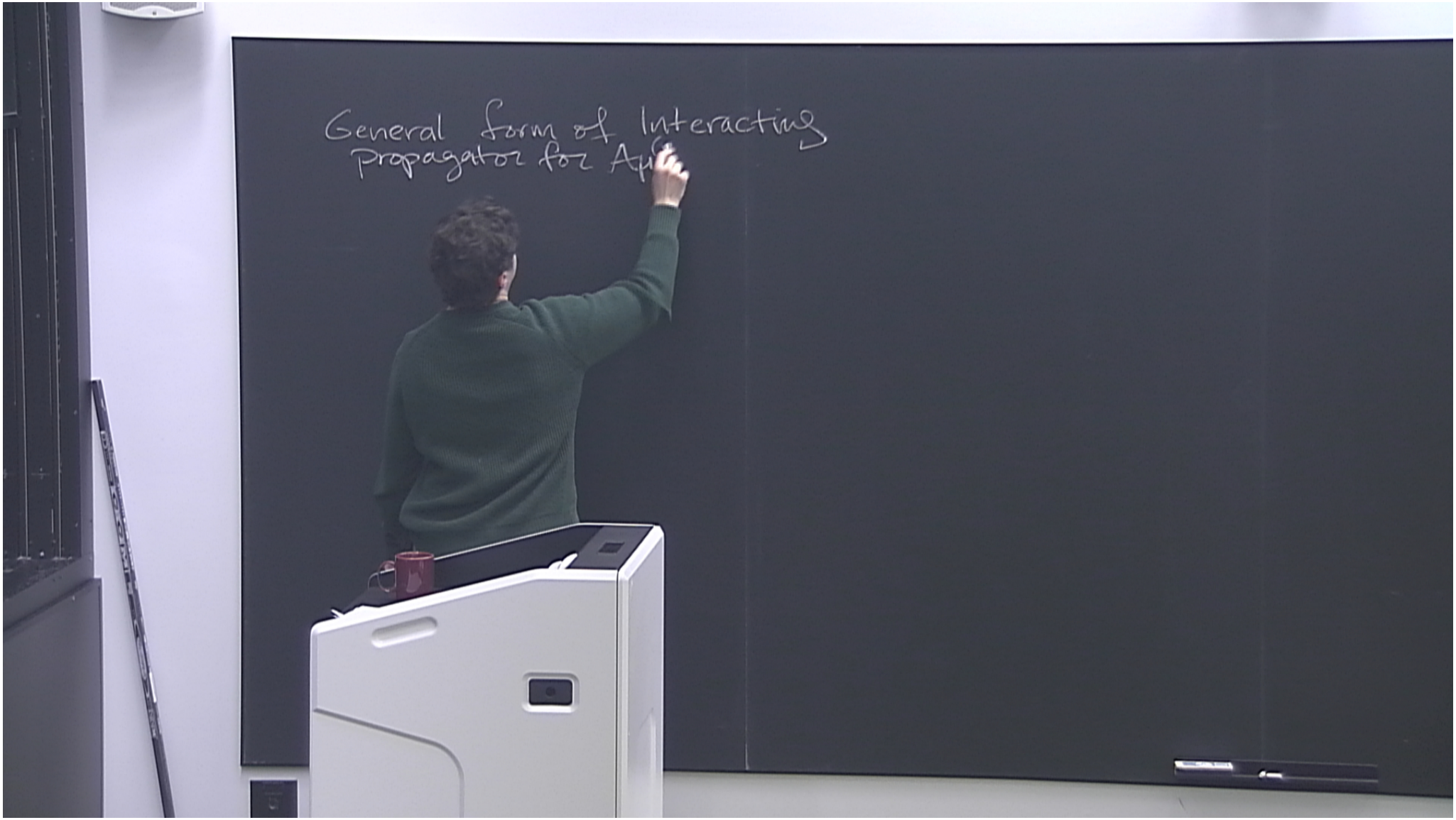
With interactions: Adjust kinetic term:

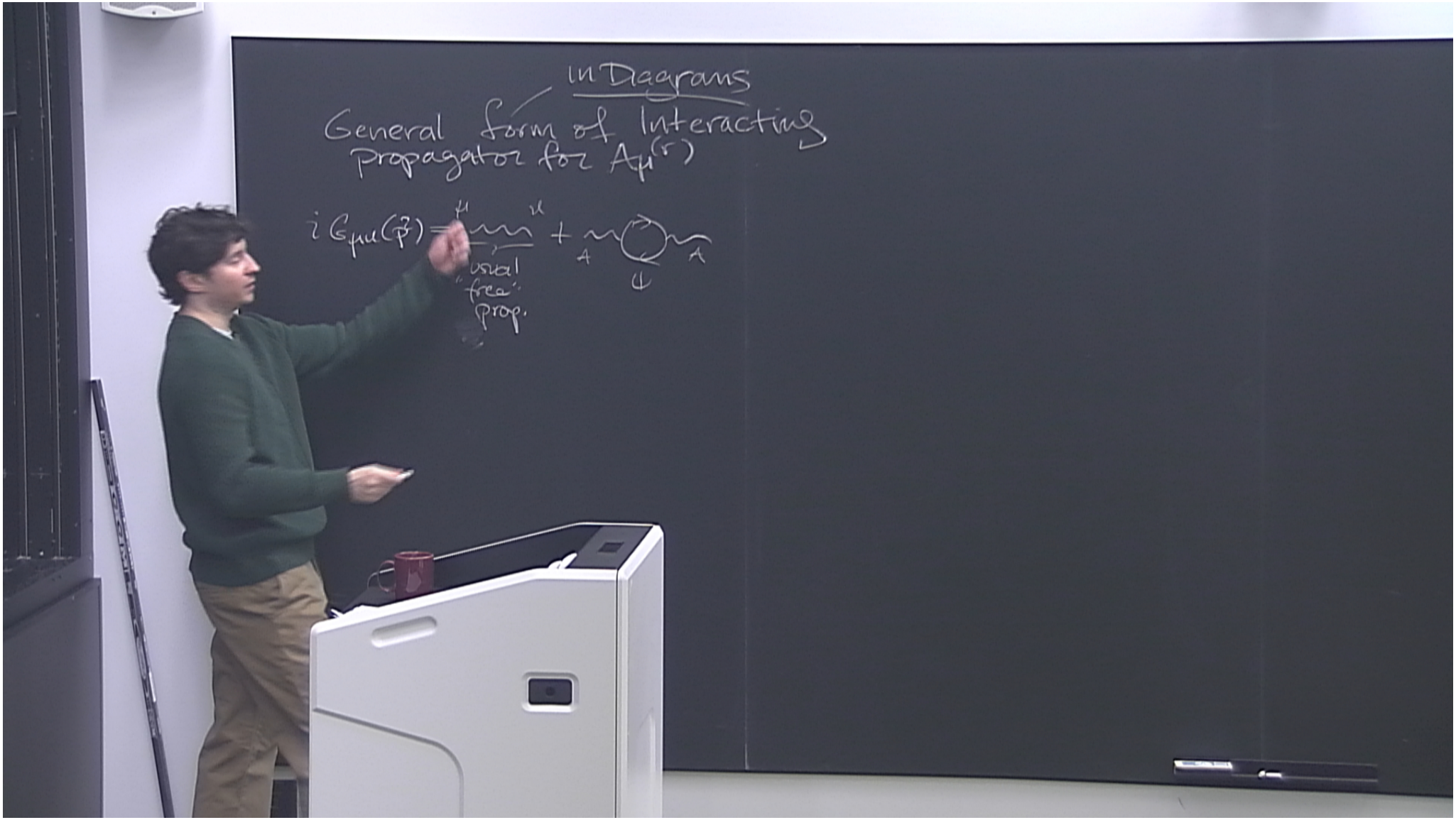
$$\mathcal{L} = -\frac{1}{4} F^{(r)2}_{\mu\nu} + i\bar{\psi}^{(0)} \not{\partial} \psi^{(0)} + \cdots - \frac{1}{4}\underbrace{(Z_3 - 1)}_{\delta_3}\left(F^{(r)}_{\mu\nu}\right)^2 + \cdots$$

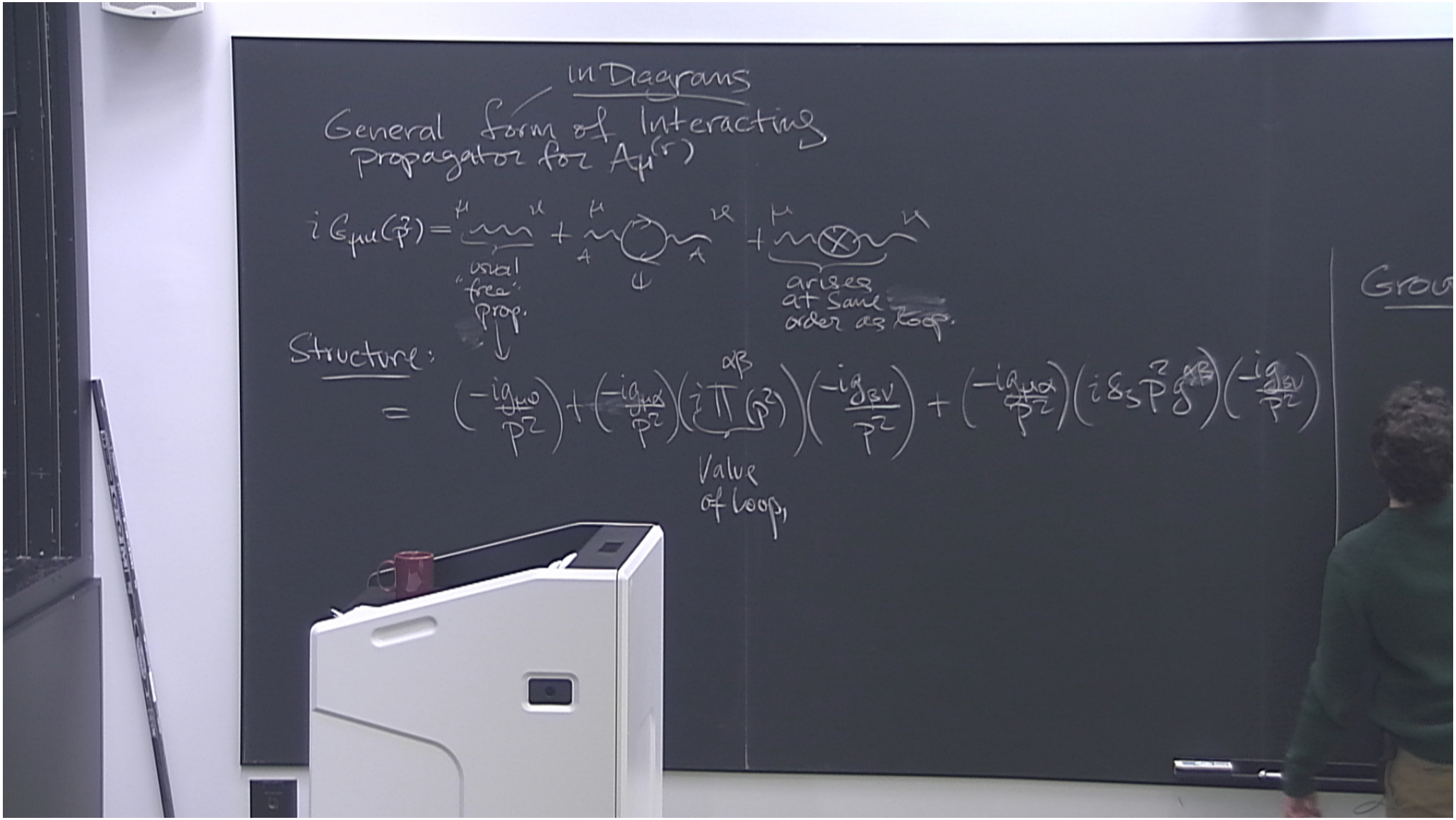
all Operators that arise from Shifting/Rescaling are Counter terms,

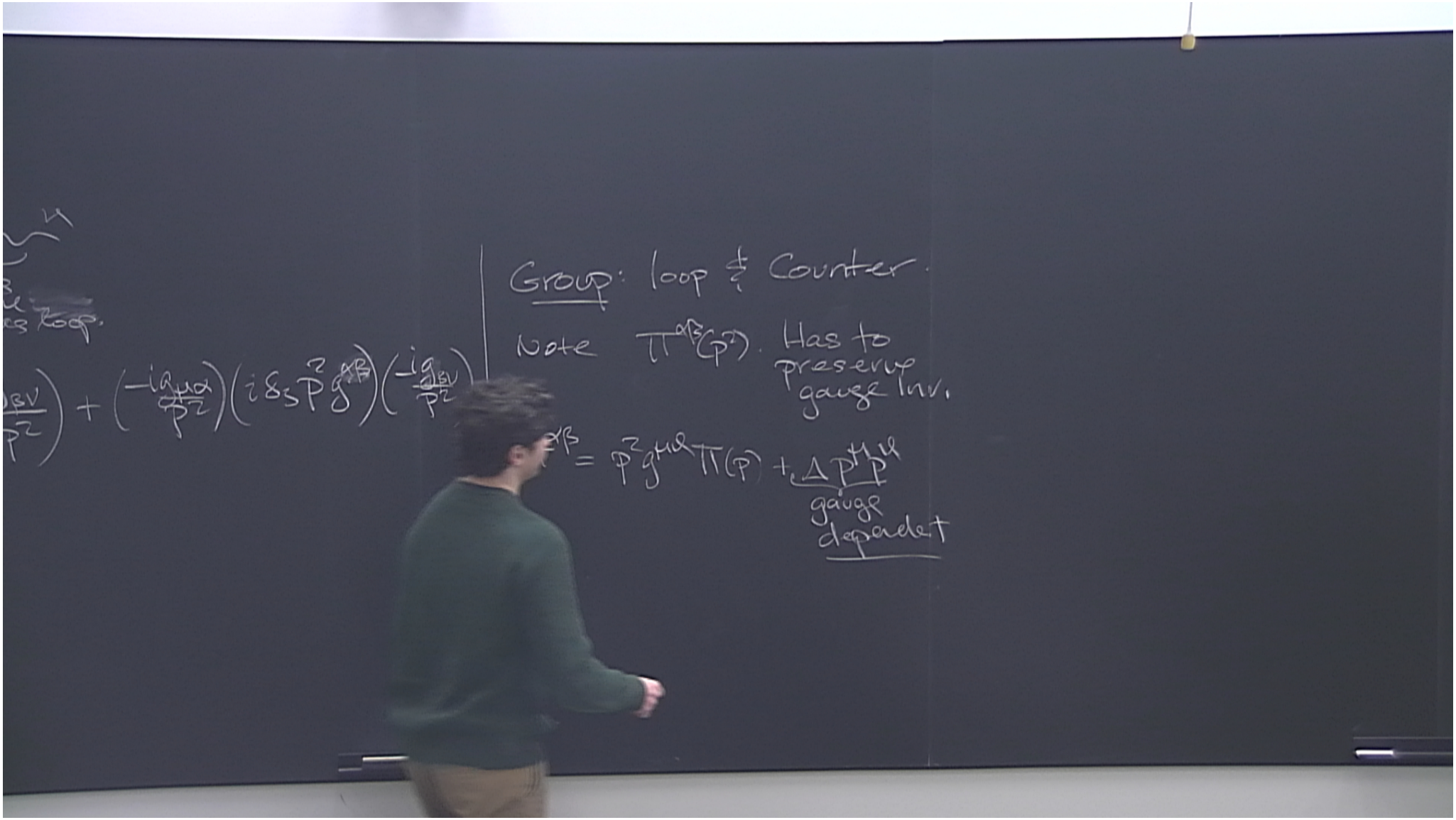

textbook
With interactions: Adjust kinetic term:
for $A_\mu^{(r)}$?
δ_3
all Operators that arise from Shifting/Rescaling are Counter terms,

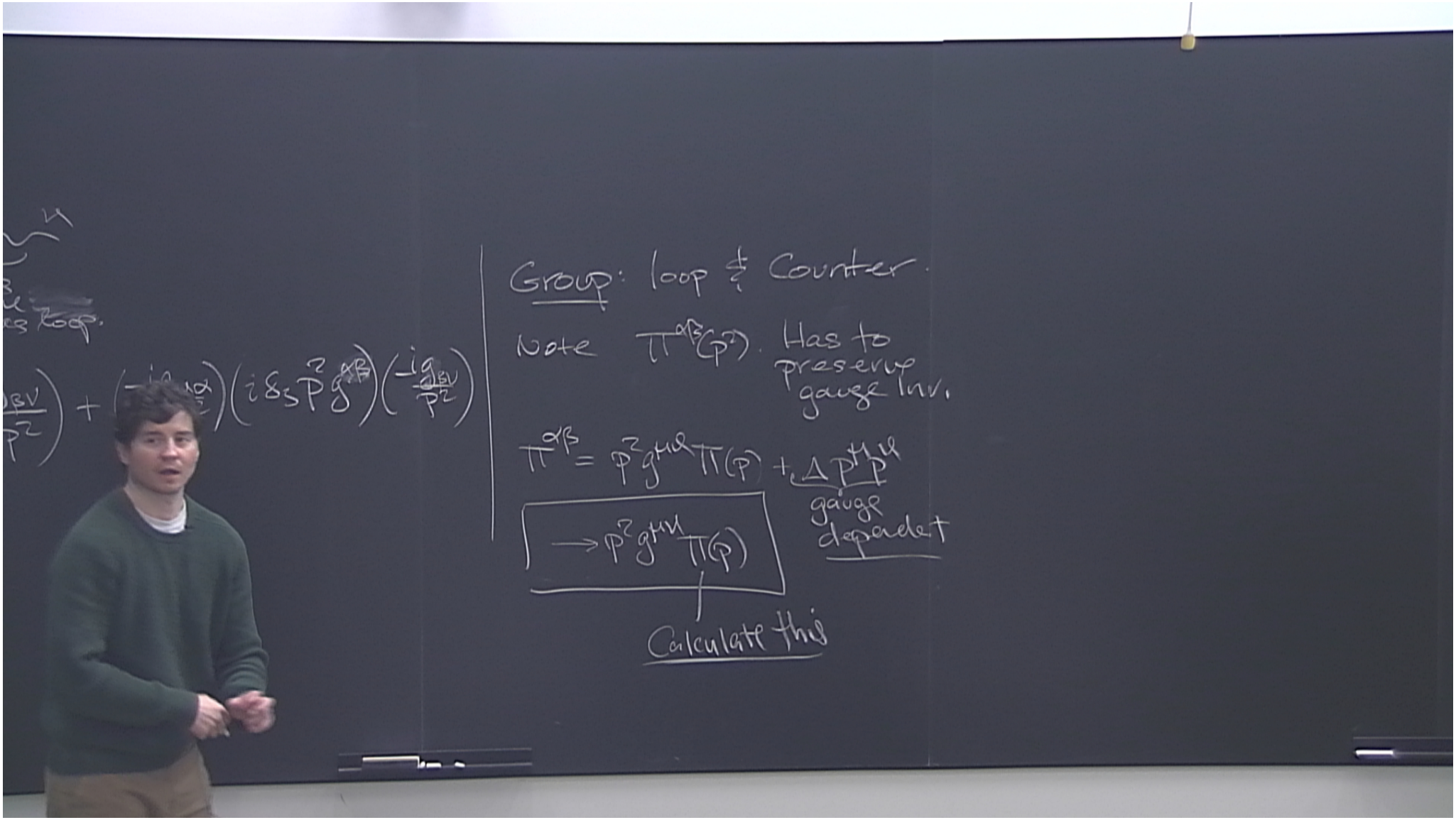

textbook
With Interactions: Adjust kinetic term:
all Operators that arise from Shifting/Rescaling are Counter terms, (always Interactions)


General form of Interacting
Propagator for


in Diagrams
General form of Interacting Propagator for $A_\mu(x)$
$iG_{\mu\nu}(p) =$
usual "free" prop.
+
A
A


in Diagrams
General form of Interacting Propagator for $A_\mu^{(r)}$
$iG_{\mu\nu}(p^2) =$
usual "free" prop.
arises at same order as loop.
Structure:
$= \left(\frac{-ig_{\mu\nu}}{p^2}\right) + \left(\frac{-ig_{\mu\alpha}}{p^2}\right)\left(i\Pi^{\alpha\beta}(p^2)\right)\left(\frac{-ig_{\beta\nu}}{p^2}\right) + \left(\frac{-ig_{\mu\alpha}}{p^2}\right)\left(i\delta_5 p^2 g^{\alpha\beta}\right)\left(\frac{-ig_{\beta\nu}}{p^2}\right)$
Value of loop,
Grou


Group: loop & Counter.
Note $\Pi^{\alpha\beta}(p^2)$. Has to preserve gauge inv.
$= p^2 g^{\mu\nu}\Pi(p) + \Delta p^\mu p^\nu$
gauge dependent


Group: loop & Counter.
Note $\Pi^{\alpha\beta}(p^2)$. Has to preserve gauge inv.
$\Pi^{\alpha\beta} = p^2 g^{\mu\nu}\Pi(p) + \Delta p^\mu p^\nu$
gauge dependent
$\rightarrow p^2 g^{\mu\nu}\Pi(p)$
Calculate this

$\left(\frac{-ig_{\beta\nu}}{p^2}\right)$

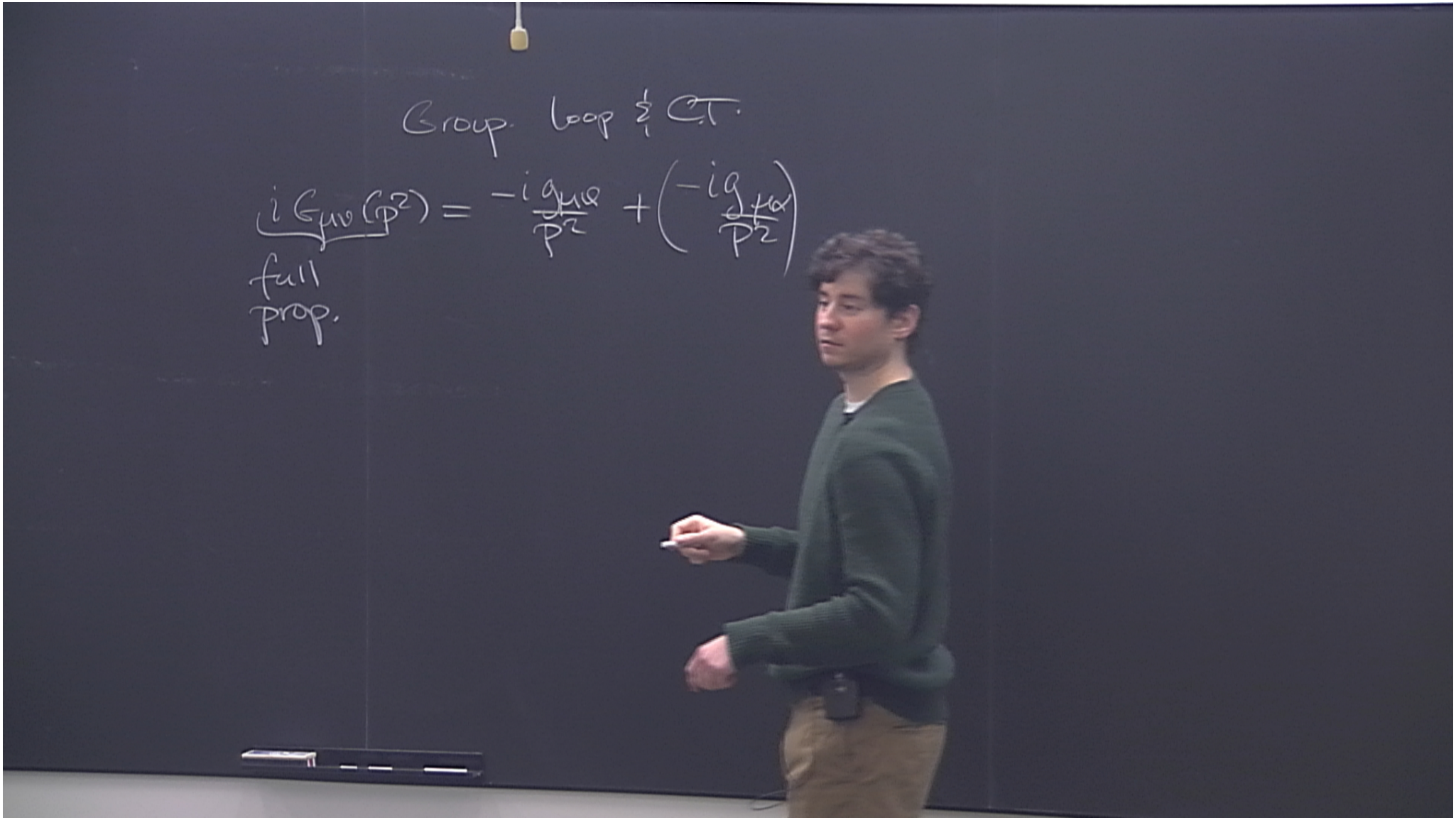
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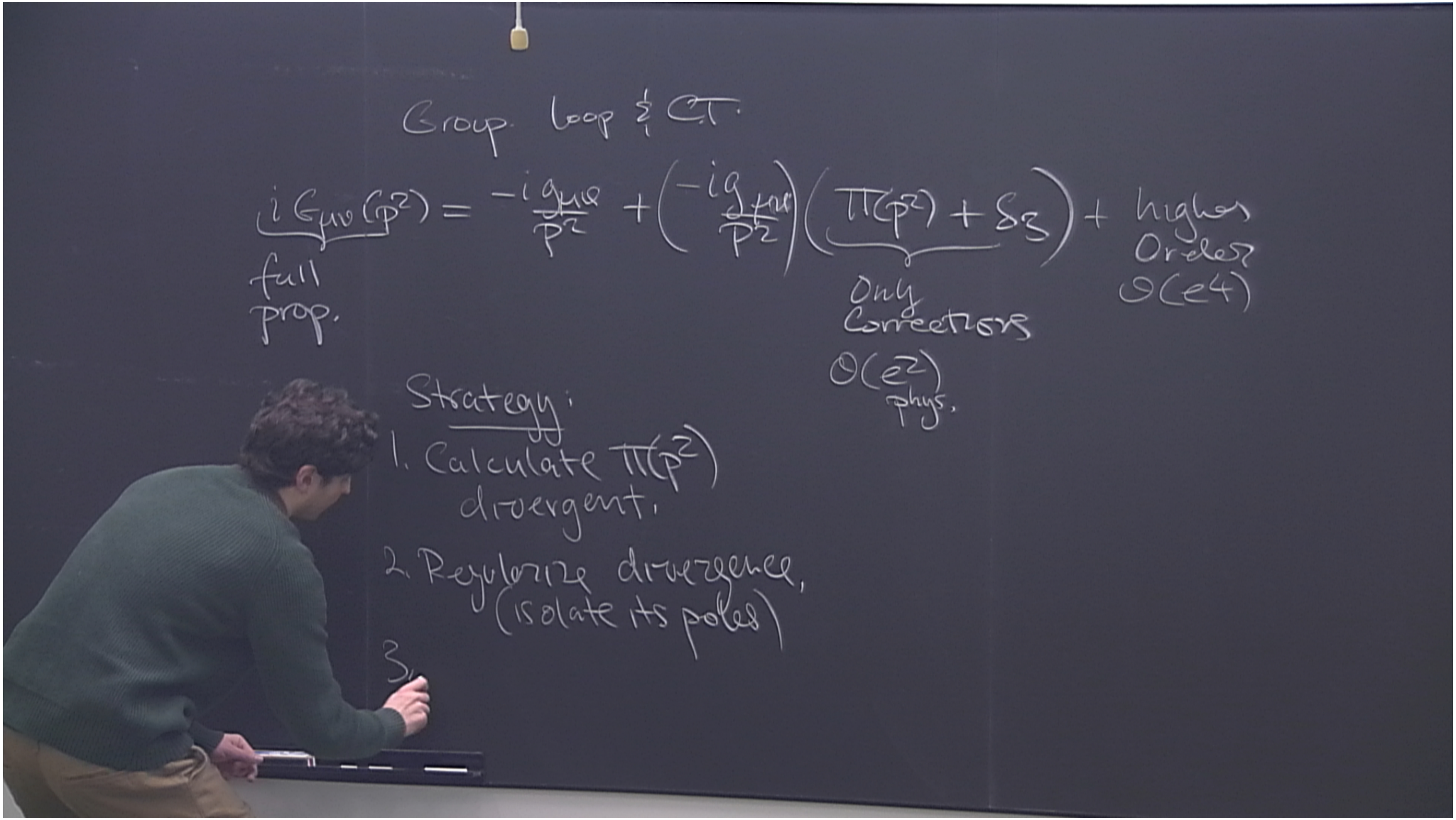
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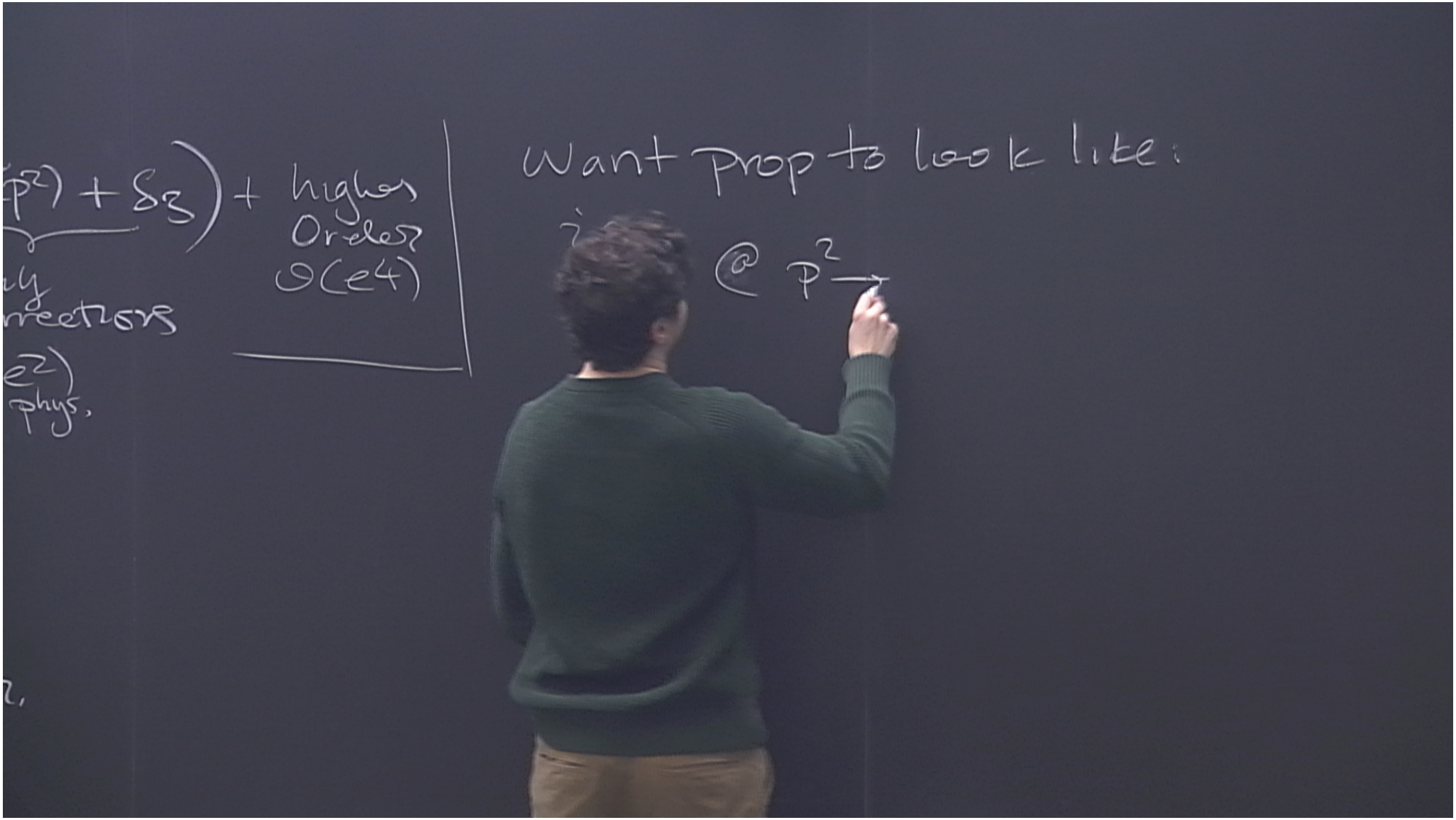
$$\Pi^{\alpha\beta} = p^2 g^{\mu\nu}\Pi(p) + \underbrace{\Delta p^\mu p^\nu}_{\text{gauge dependent}}$$

$$\rightarrow p^2 g^{\mu\nu}\Pi(p)$$

Calculate this


Group loop & C.T.
$iG_{\mu\nu}(p^2) = -i\frac{g_{\mu\nu}}{p^2} + \left(-ig\frac{}{p^2}\right)$
full
prop.


Group loop & C.T.
$iG_{\mu\nu}(p^2) = -i\frac{g_{\mu\nu}}{p^2} + \left(-i\frac{g_{\mu\nu}}{p^2}\right)\left(\Pi(p^2) + \delta_3\right) +$ higher Orders $O(e^4)$
full prop.
Only Corrections
$O(e^2)$ phys.
Strategy:
1. Calculate $\Pi(p^2)$ divergent.
2. Regularize divergence, (isolate its poles)
3.


Want prop to look like:
higher Orders O(e4)

S_3) + higher Orders $O(e^4)$

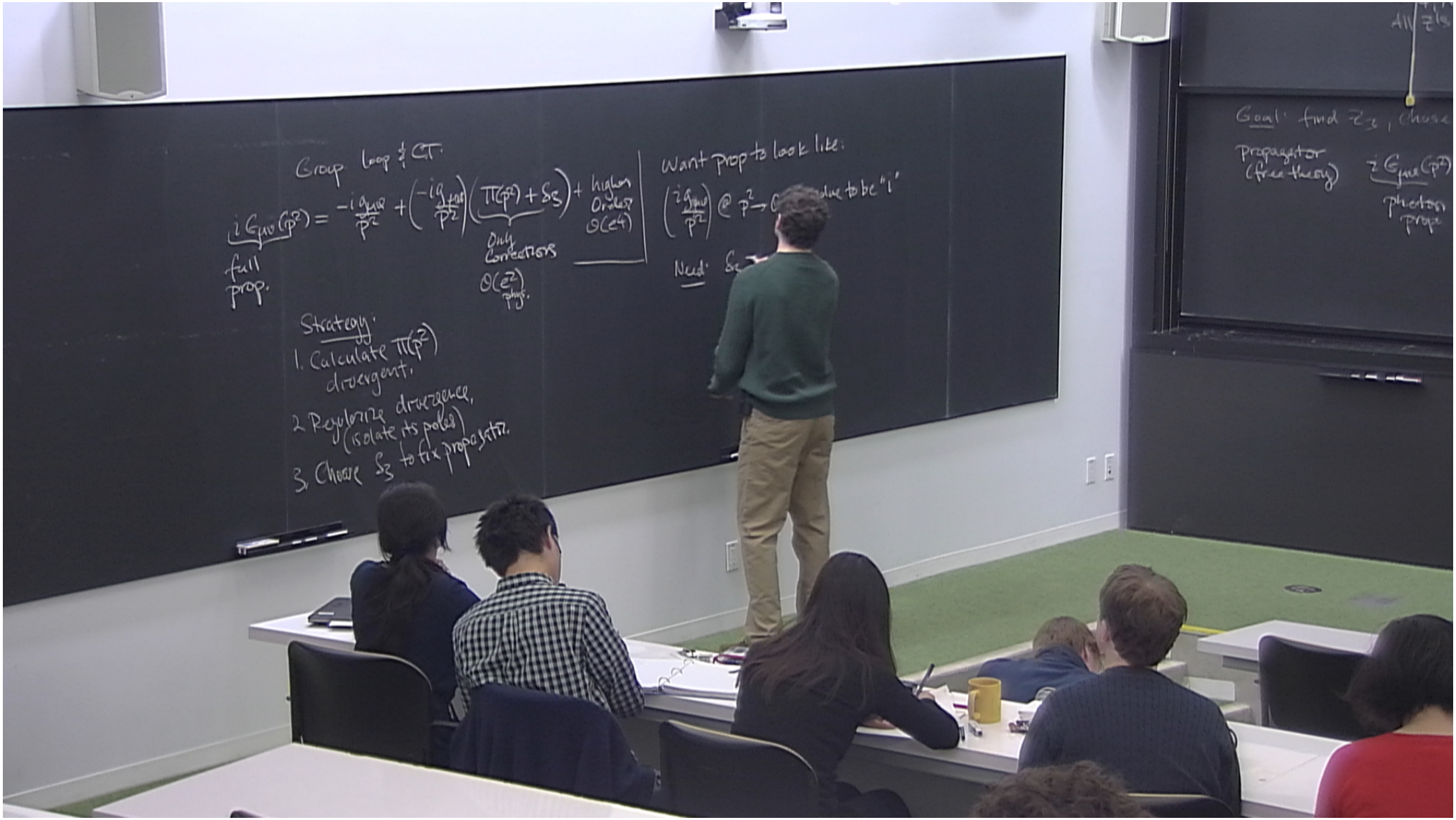
Want Prop to look like:

$\frac{i g_{\mu\nu}}{p^2}$ @ $p^2 \to 0$ Residue to be "i"

δ_3) + higher Orders $O(e^4)$

Want Prop to look like:

$\left(\frac{i g_{\mu\nu}}{p^2}\right)$ @ $p^2 \to 0$ Residue to be "i"

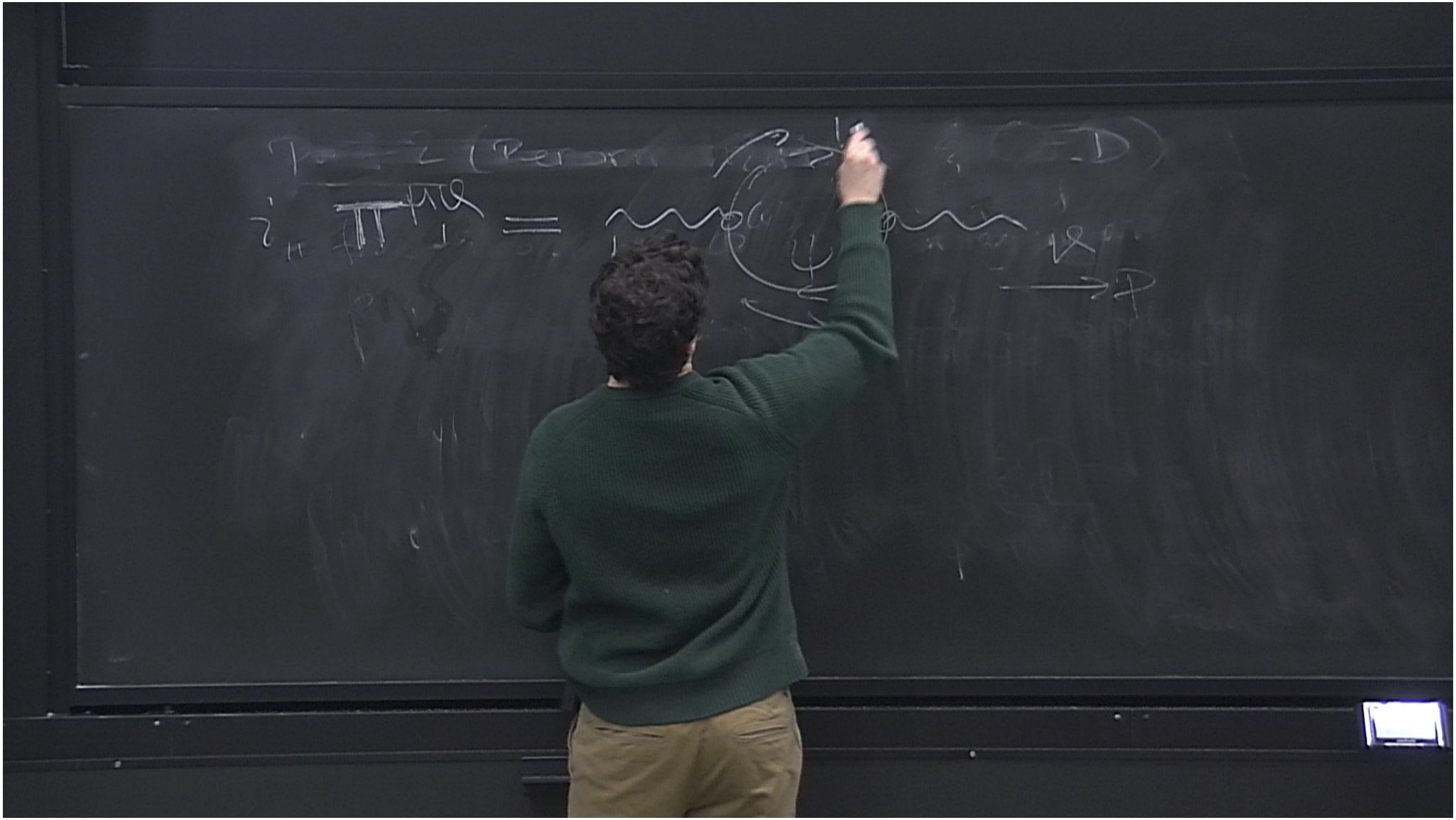

Group loop & CT.
full prop.
Only Corrections
Strategy:
1. Calculate Π(p²) divergent.
2. Regularize divergence, (isolate its poles)
3. Choose δ3 to fix propagator.
Want prop to look like.
Need:
Goal: find Z3, Choose
Propagator (free theory)
photon prop

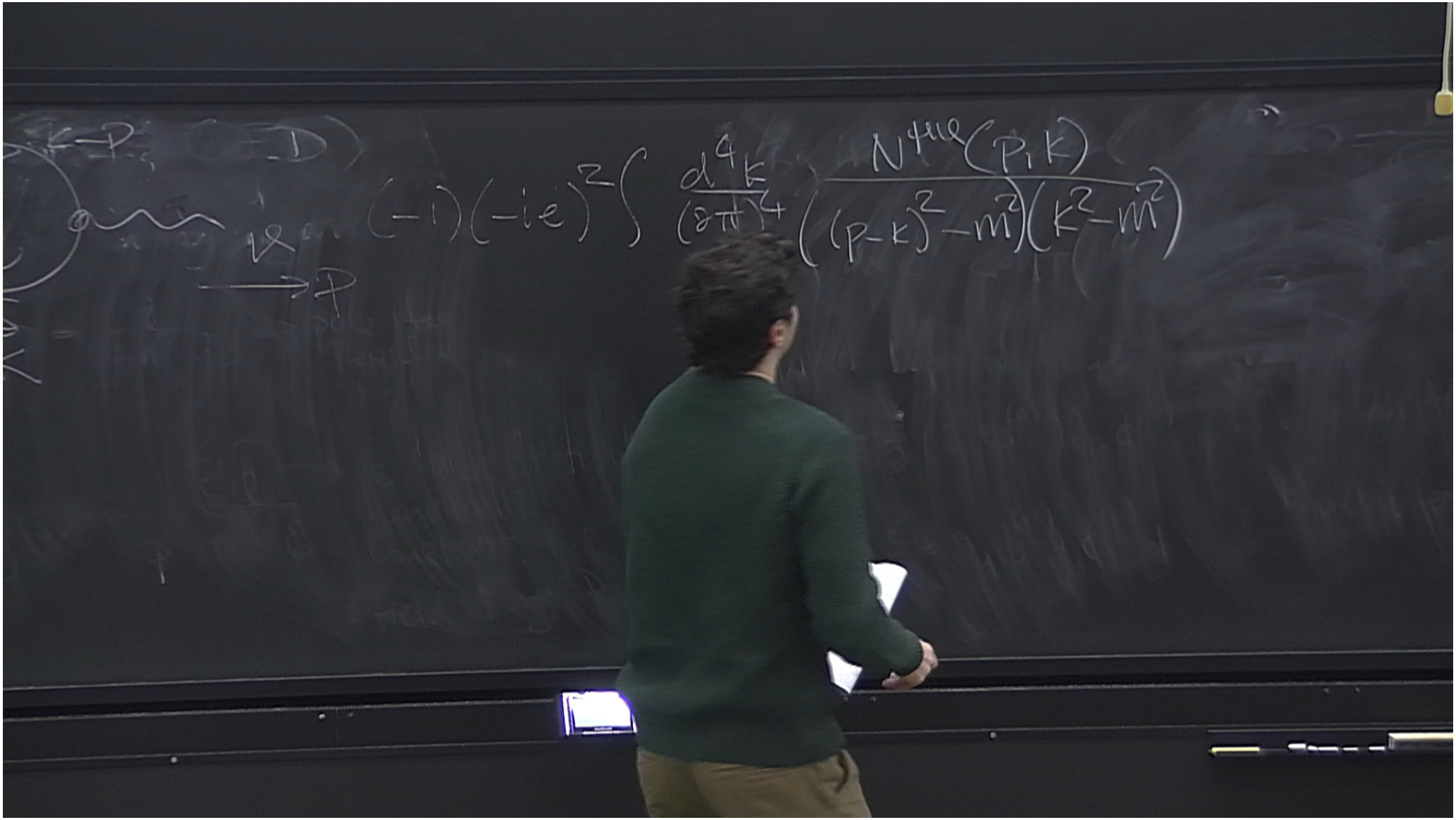
δ_3) + higher Orders $O(e^4)$

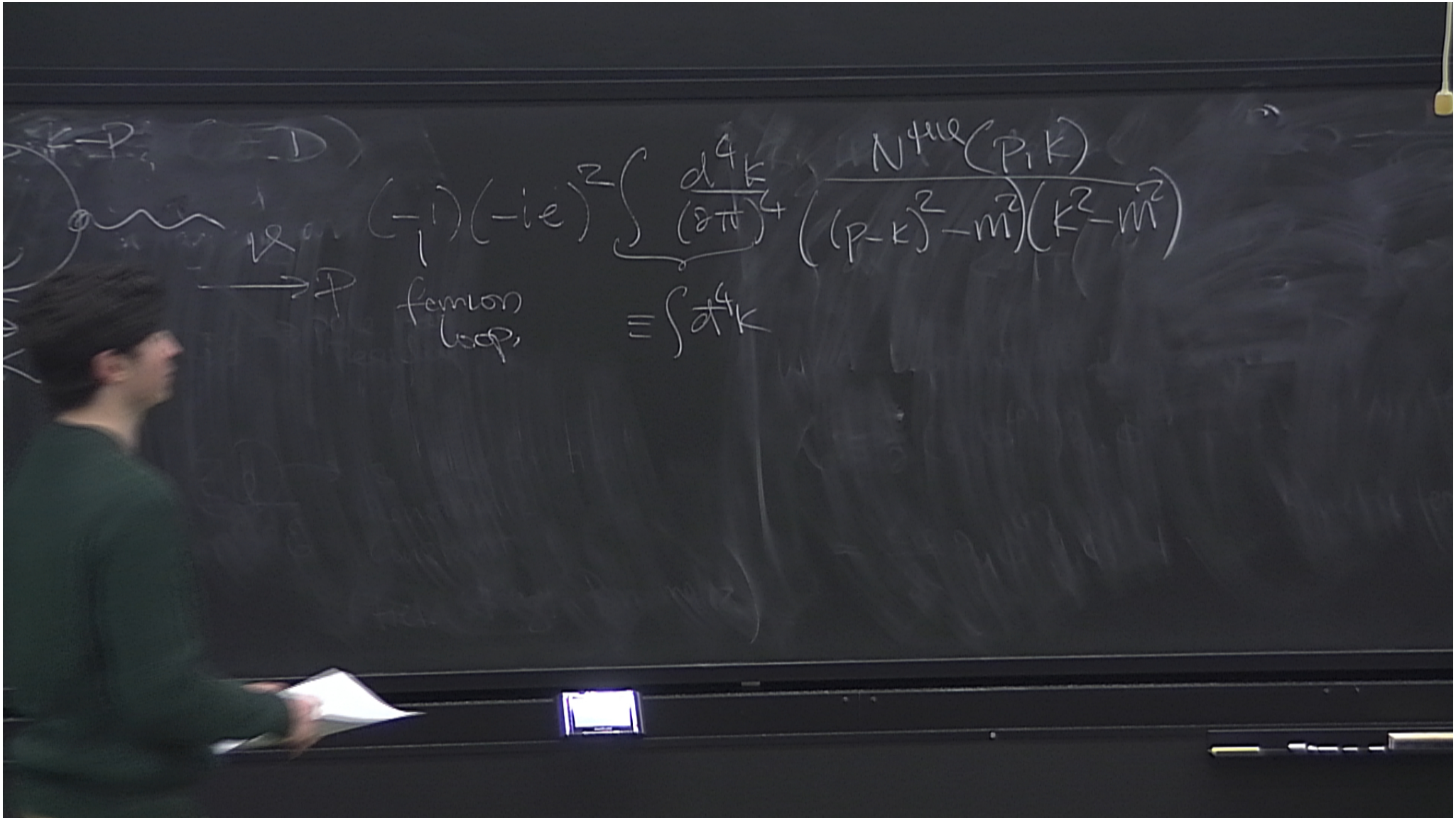
Want Prop to look like:

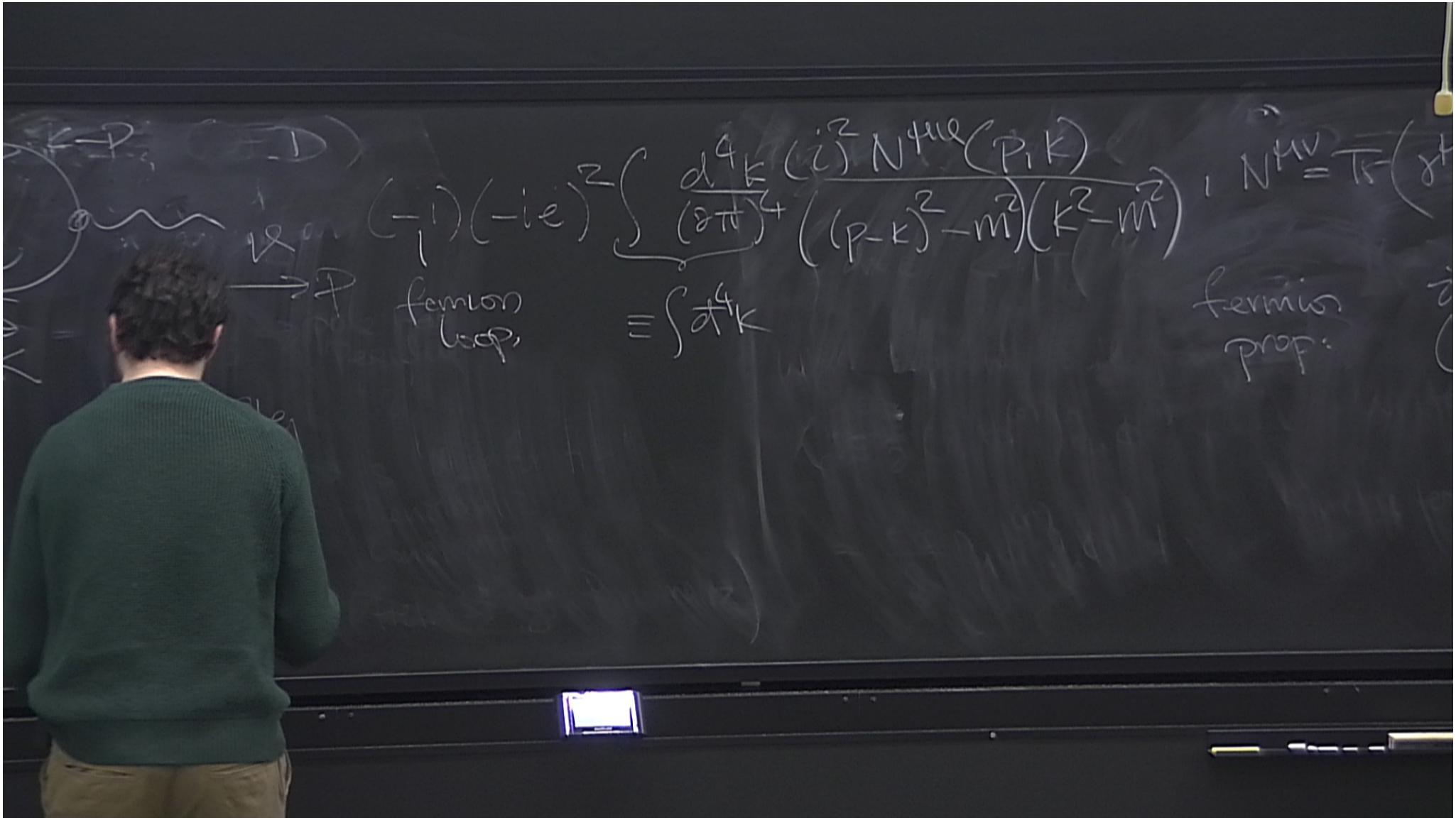
$\left(\frac{i g_{\mu\nu}}{p^2}\right)$ @ $p^2 \to 0$ Residue to be "i"

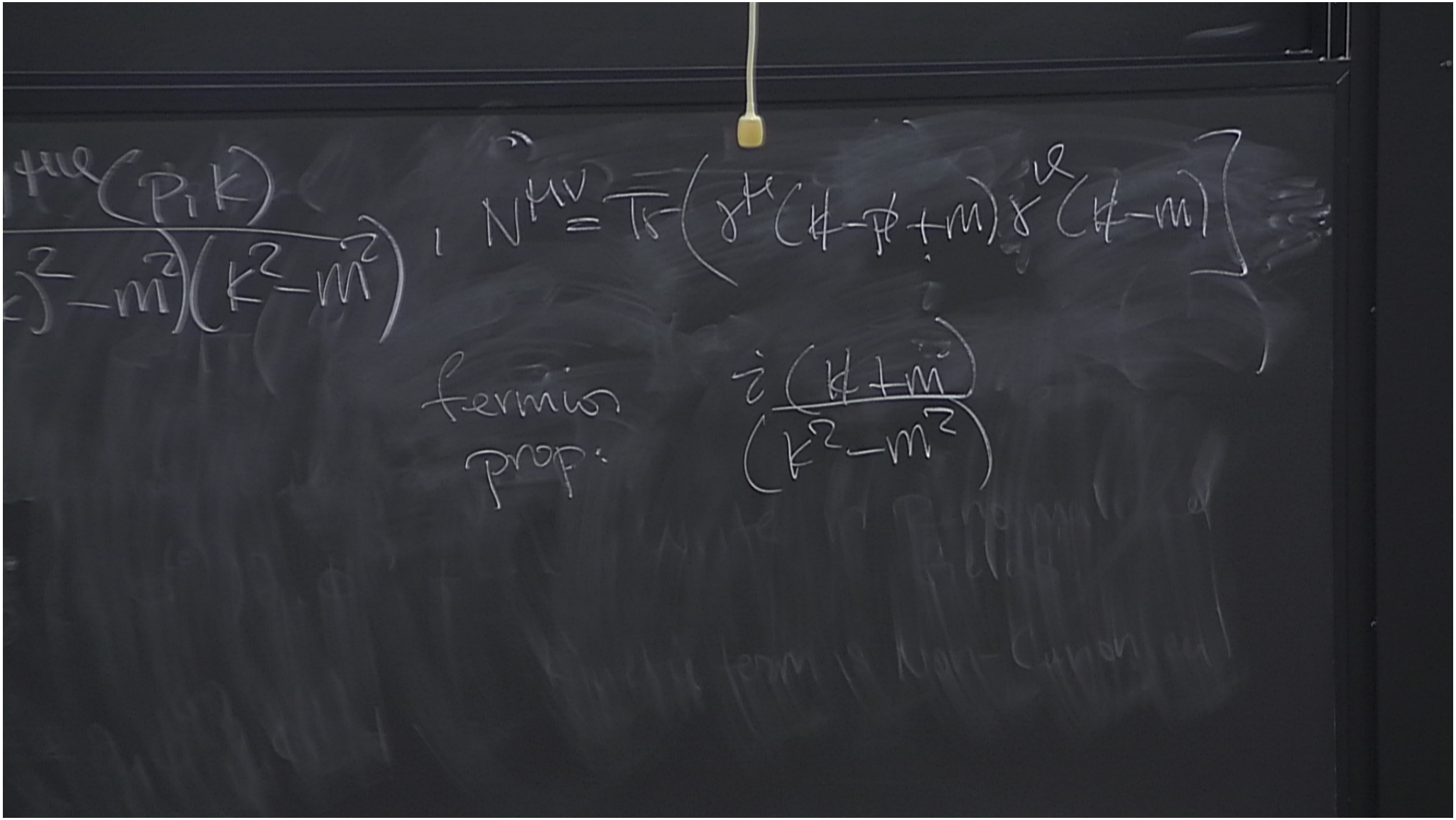
Need: $\boxed{\delta_3 = -\Pi(0)}$ Renormalization Condition

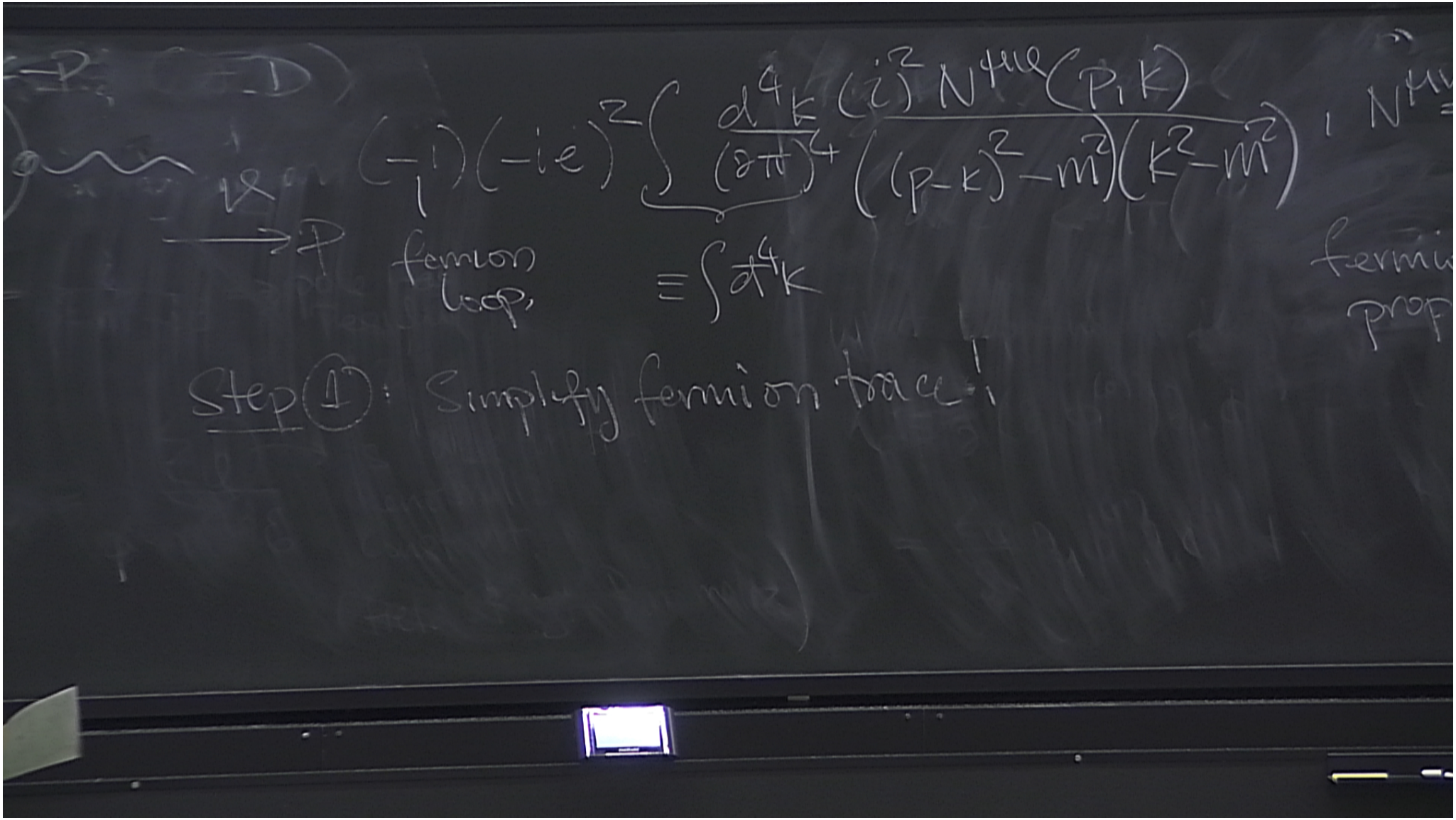


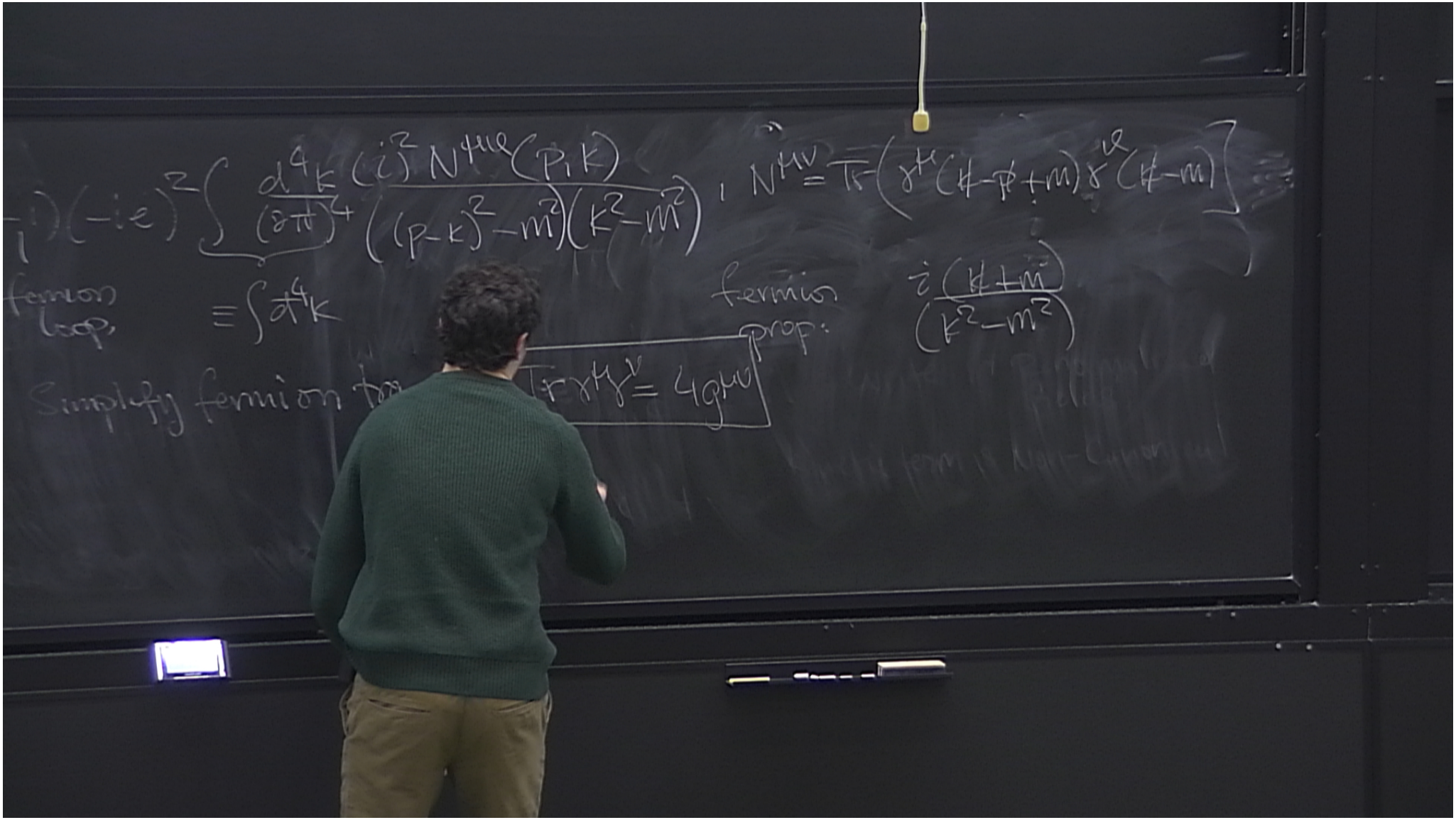

p-k
k
p
$(-1)(-ie)^2\int \frac{d^4k}{(2\pi)^4} \frac{N^{\mu\nu}(p,k)}{((p-k)^2-m^2)(k^2-m^2)}$

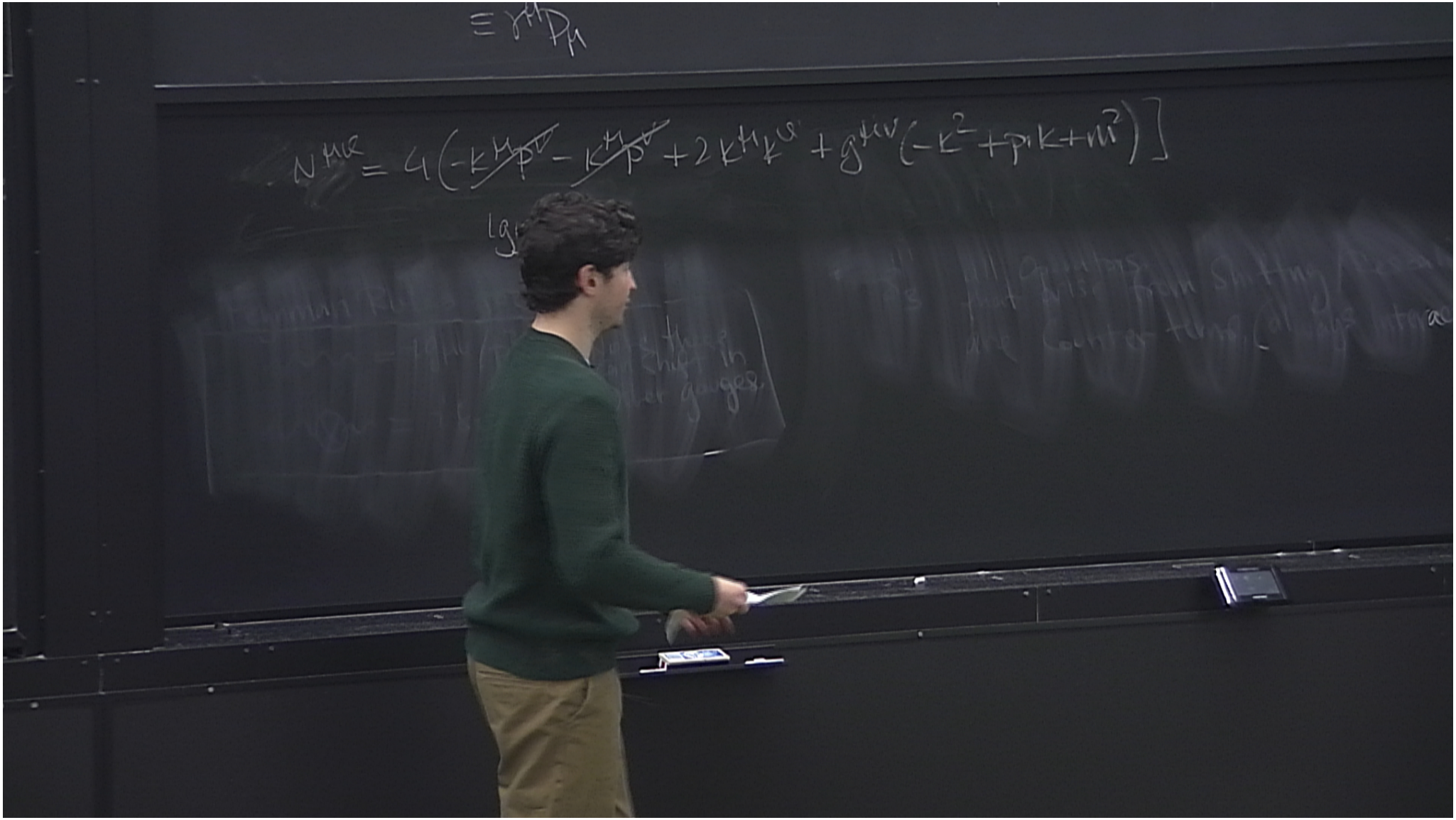

$(-1)(-ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{N^{\mu\nu}(p,k)}{((p-k)^2-m^2)(k^2-m^2)}$
fermion loop
$\equiv \int d^4k$

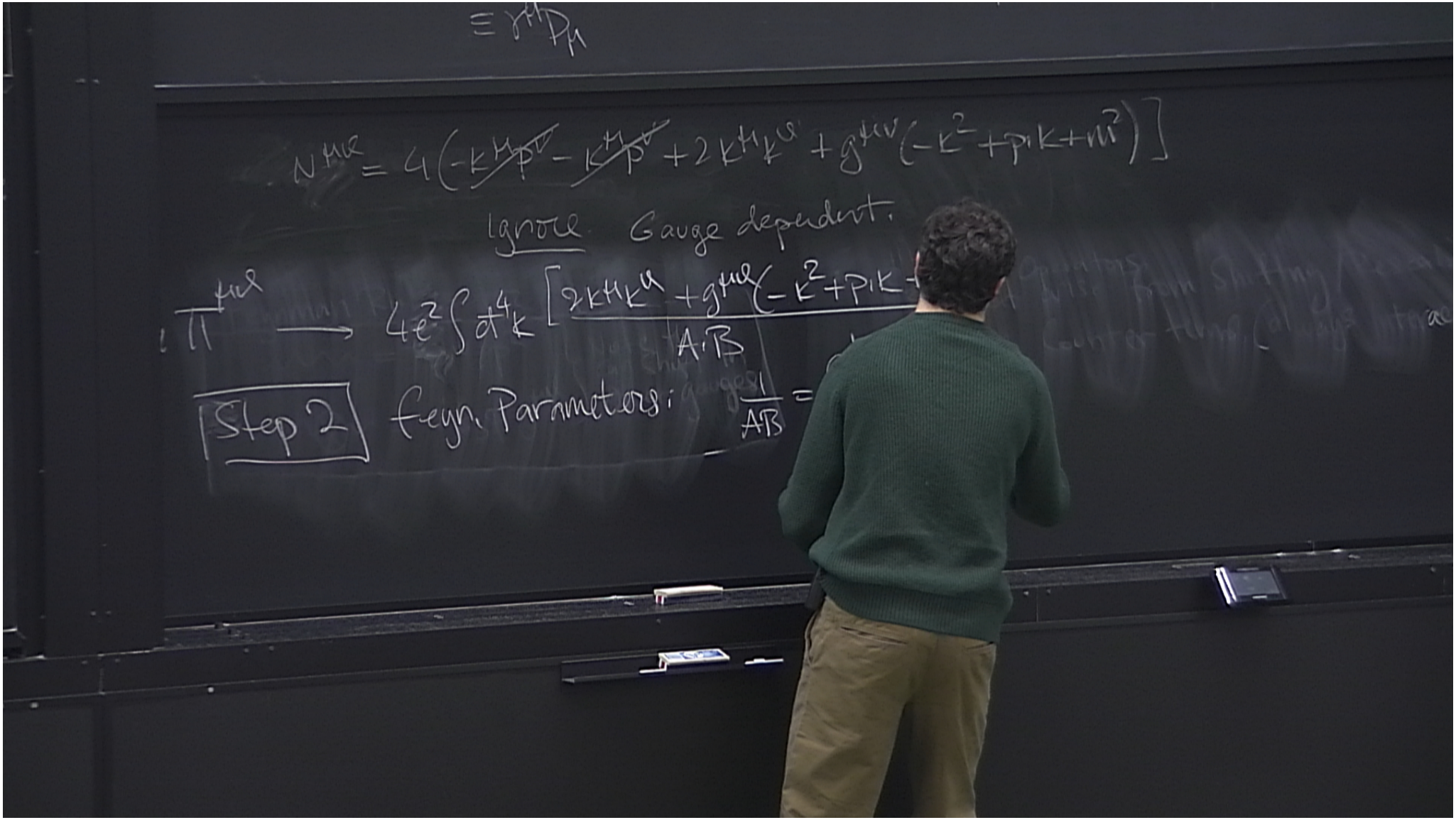

k−P
P
k
(−1)(−ie)²∫ d⁴k/(2π)⁴ (i)³ N^μν(P,k) / ((P−k)²−m²)(k²−m²)
N^μν = Tr(γ
fermion loop
≡ ∫d⁴k
fermion prop:


$\frac{(p_i k)}{(k^2-m^2)}$
$N^{\mu\nu} = \mathrm{Tr}\left[\gamma^\mu(\not{k}-\not{p}+m)\gamma^\nu(\not{k}-m)\right]$
fermion prop:
$\frac{i(\not{k}+m)}{(k^2-m^2)}$


$(-1)(-ie)^2 \int \frac{d^4k}{(2\pi)^4} \frac{(i)^2 N^{\mu\nu}(p,k)}{((p-k)^2-m^2)(k^2-m^2)}$
fermion loop
$\equiv \int d^4k$
Step (1): Simplify fermion trace!


fermion loop
Simplify fermion
fermion prop:
Tr γμγν = 4gμν


$\equiv \gamma^\mu p_\mu$
$N^{\mu\nu} = 4\left(-k^\mu p^\nu - k^\mu p^\nu + 2k^\mu k^\nu + g^{\mu\nu}(-k^2 + p\cdot k + m^2)\right]$


$\equiv \gamma^\mu p_\mu$
$N^{\mu\nu} = 4\left(-k^\mu p^\nu - k^\mu p^\nu + 2k^\mu k^\nu + g^{\mu\nu}(-k^2 + p\cdot k + m^2)\right)$
Ignore Gauge dependent.
$\Pi^{\mu\nu} \longrightarrow 4e^2 \int d^4k \left[\frac{2k^\mu k^\nu + g^{\mu\nu}(-k^2 + p\cdot k}{A\cdot B}\right.$
Step 2 Feyn. Parameters: $\frac{1}{AB} =$

$\equiv \gamma^\mu p_\mu$

$N^{\mu\nu} = 4\left(-k^\mu p^\nu - k^\mu p^\nu + 2k^\mu k^\nu + g^{\mu\nu}(-k^2 + p\cdot k + m^2)\right]$

Redefine k_μ

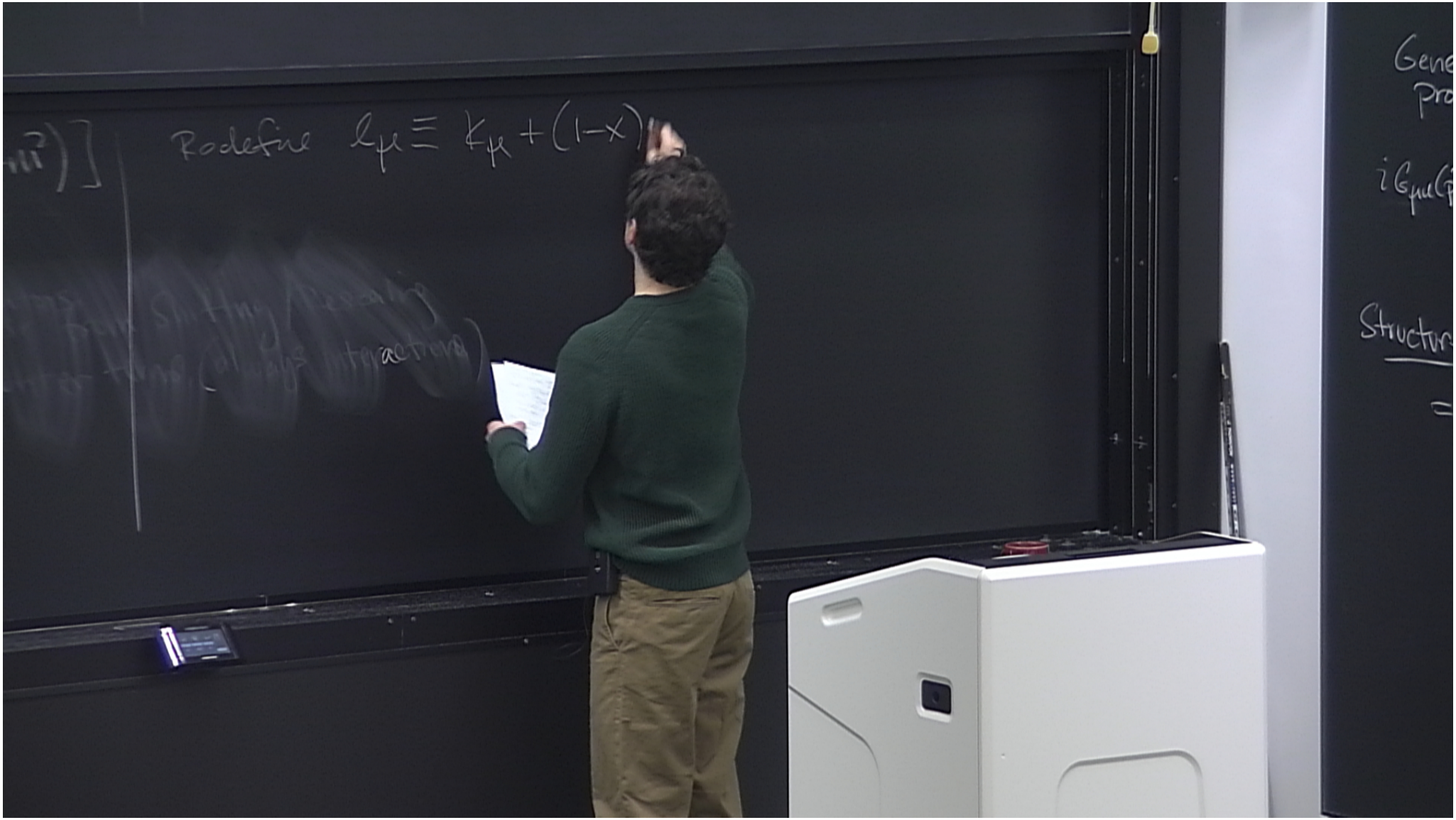
Ignore Gauge dependent.

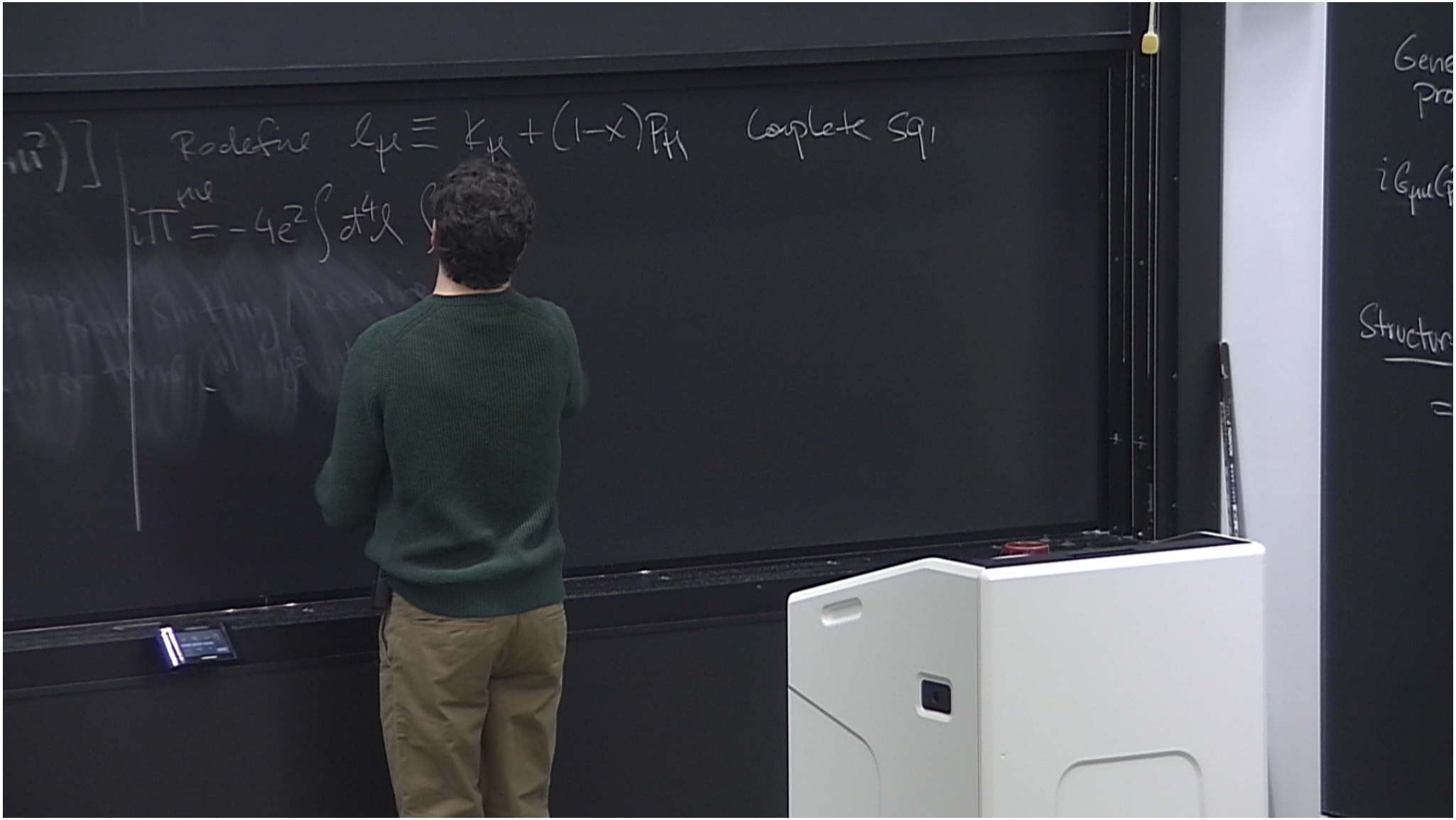
$$i\Pi^{\mu\nu} \longrightarrow 4e^2 \int d^4k \left[\frac{2k^\mu k^\nu + g^{\mu\nu}(-k^2 + p\cdot k + m^2)}{A\cdot B}\right]$$

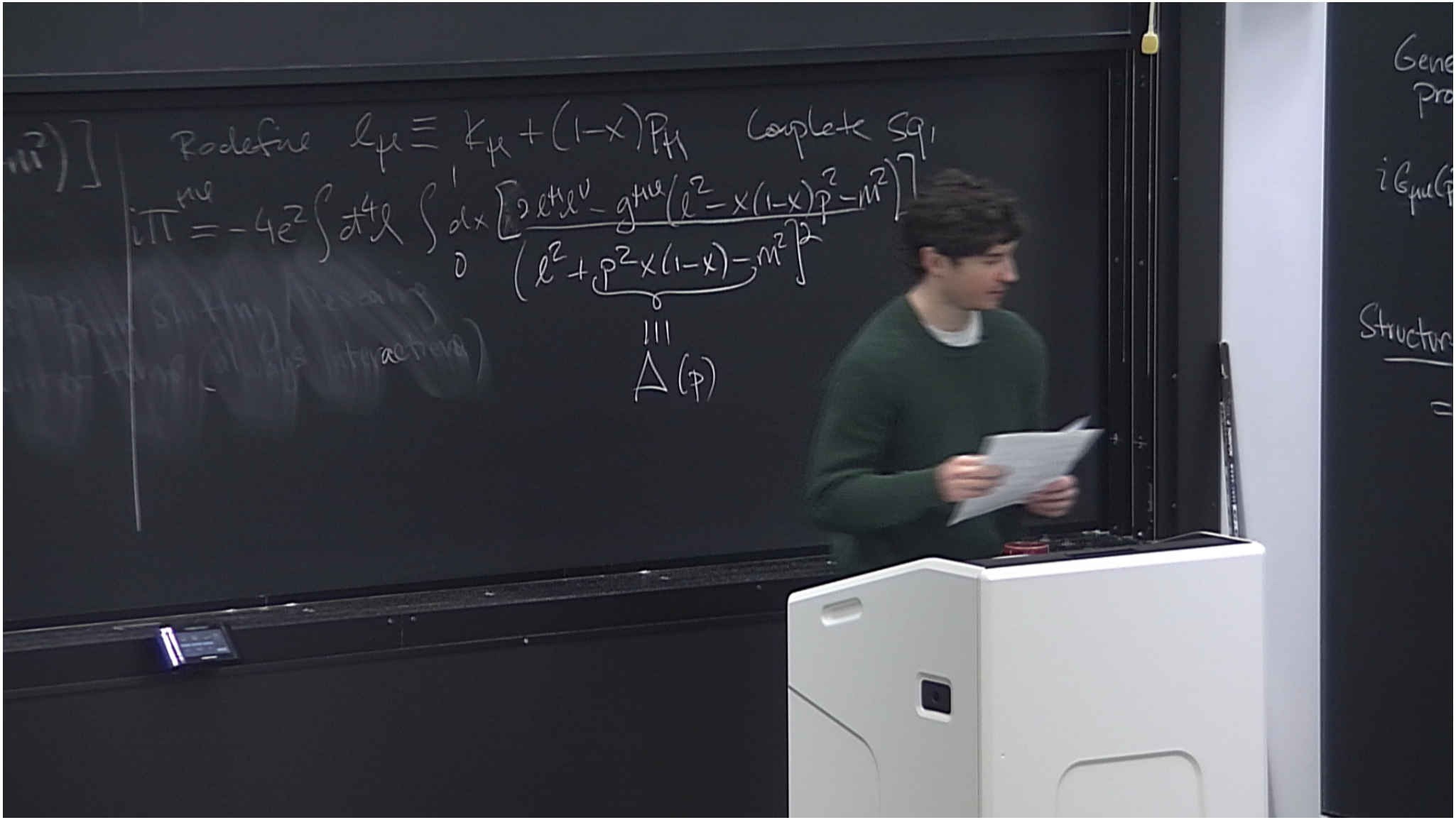
Step 2 Feyn. Parameters: $\frac{1}{AB} = \int_0^1 dx \frac{1}{(xA + (1-x)B)^2}$

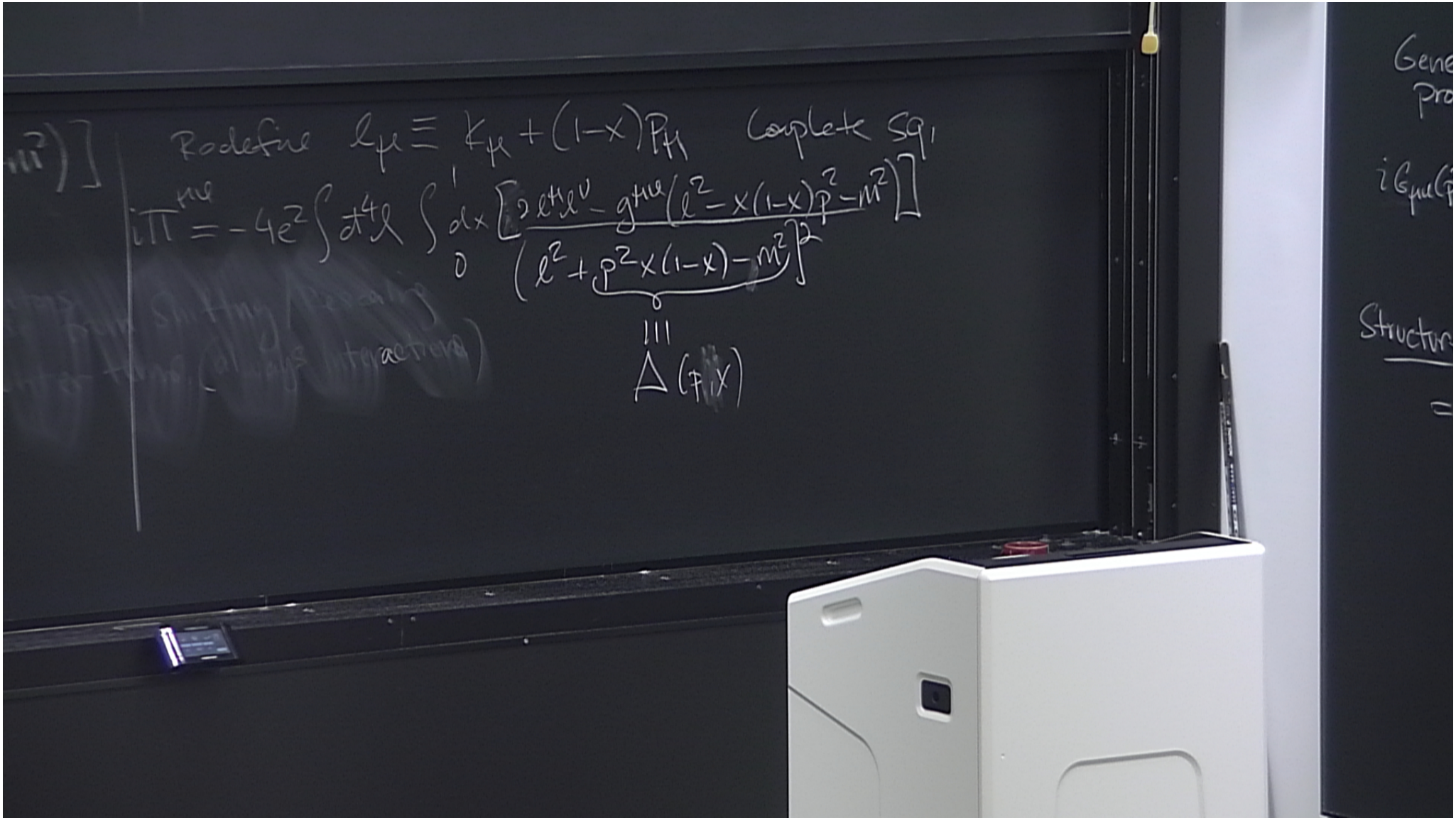
$A = k^2 - m^2$

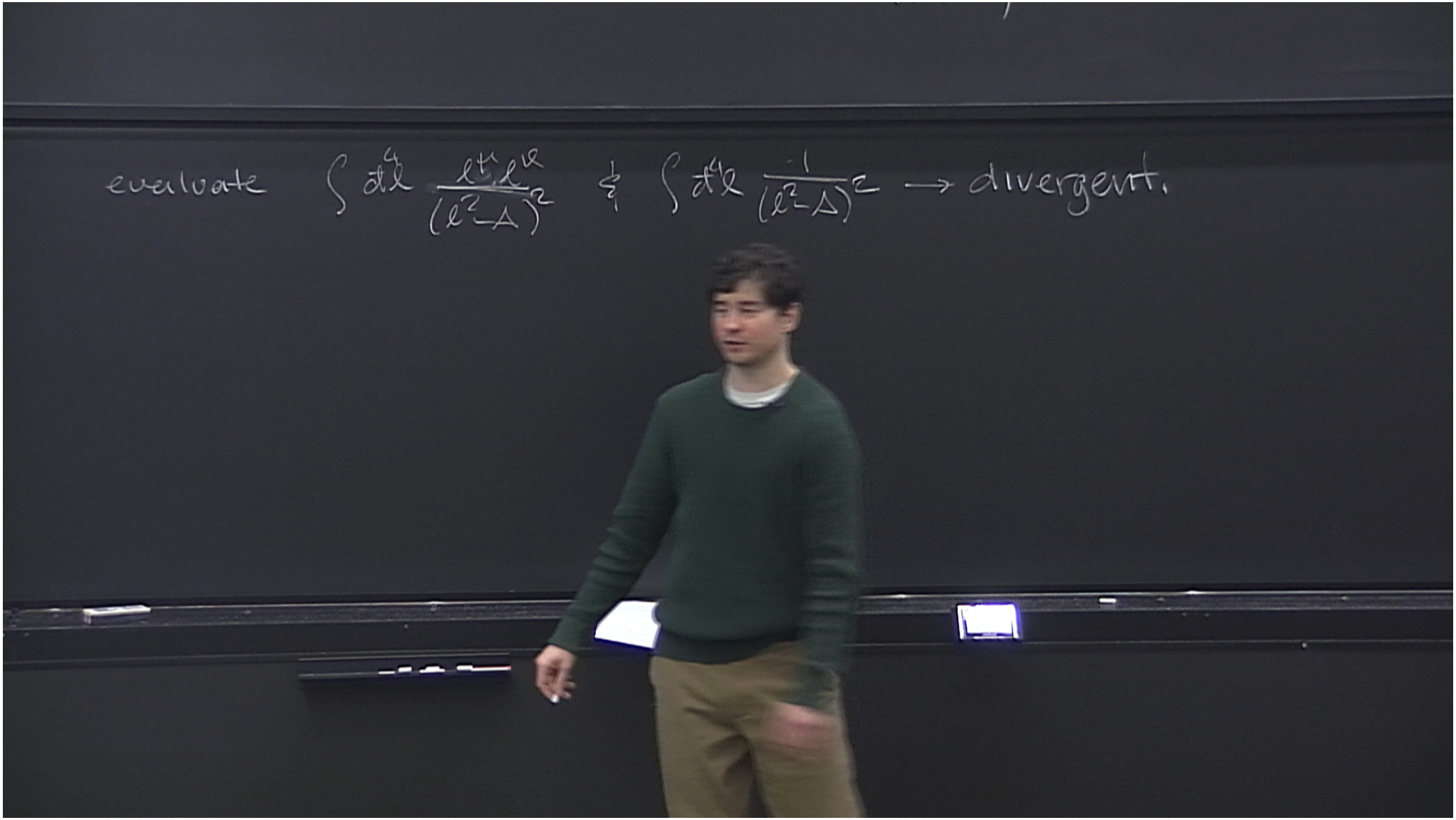
$B = (k-p)^2 - m^2$ → Den. Additive!


Redefine
Structur


Redefine $\ell_\mu \equiv k_\mu + (1-x)P_\mu$
Complete sq.
$i\Pi^{\mu\nu} = -4e^2 \int d^4\ell$
Structur


Redefine $\ell_\mu \equiv k_\mu + (1-x)p_\mu$ Complete Sq.
$i\Pi^{\mu\nu} = -4e^2 \int d^4\ell \int_0^1 dx \left[\frac{2\ell^\mu\ell^\nu - g^{\mu\nu}(\ell^2 - x(1-x)p^2 - m^2)}{(\ell^2 + p^2x(1-x) - m^2)^2} \right]$
$\Delta(p)$


Redefine $\ell_\mu \equiv k_\mu + (1-x)p_\mu$ Complete sq.
$i\Pi^{\mu\nu} = -4e^2 \int d^4\ell \int_0^1 dx \left[\frac{2\ell^\mu\ell^\nu - g^{\mu\nu}(\ell^2 - x(1-x)p^2 - m^2)}{(\ell^2 + p^2x(1-x) - m^2)^2} \right]$
$\Delta(p,x)$


evaluate $\int d^4k \frac{k^\mu k^\nu}{(k^2-\Delta)^2} \propto \int d^4k \frac{1}{(k^2-\Delta)^2} \rightarrow$ divergent.

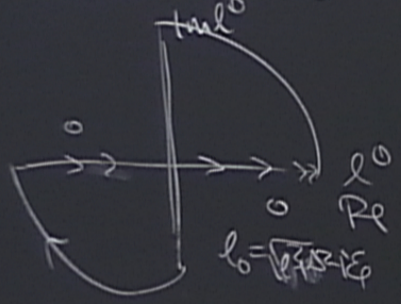
evaluate $\int d^4\ell \frac{\ell^\mu \ell^\nu}{(\ell^2-\Delta)^2} \neq \int d^4\ell \frac{1}{(\ell^2-\Delta)^2} \rightarrow$ divergent.

Work in d-dim

Wick Rotate:

Euclidean Space $\ell^0 = i\ell^0$

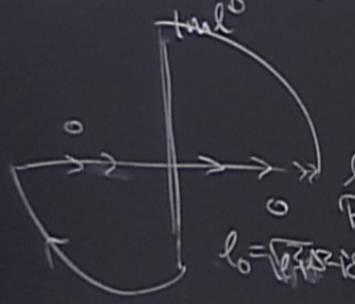
$\ell^2 = (\ell^0)^2 - \vec{\ell}^2 \rightarrow -\ell_E^2$



evaluate $\int d^4\ell \frac{\ell^\mu \ell^\nu}{(\ell^2-\Delta)^2}$ $\nleftrightarrow$ $\int d^4\ell \frac{1}{(\ell^2-\Delta)^2}$ → divergent.

Work in d-dim

Wick Rotate:



Euclidean Space $\ell_E^0 = i\ell^0$

$\ell^2 = (\ell^0)^2 - |\vec{\ell}|^2 \longrightarrow (\ell^0)^2 + (\vec{\ell})^2$

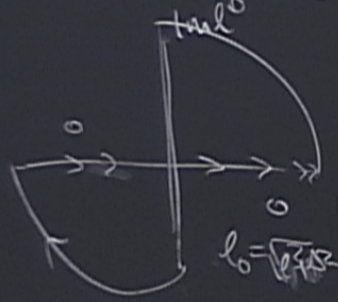
Spherical Coord. in d-dim.

$\frac{1}{(p^2-\Delta)+i\varepsilon}$ $\quad \ell_0 = +\sqrt{|\vec{\ell}|^2+\Delta}-i\varepsilon$

evaluate $\int d^4\ell \frac{\ell^\mu \ell^\nu}{(\ell^2-\Delta)^2}$ $\not\propto$ $\int d^4\ell \frac{1}{(\ell^2-\Delta)^2}$ → divergent.

Work in d-dim

Wick Rotate:



Euclidean Space $\ell_E^0 = i\ell^0$

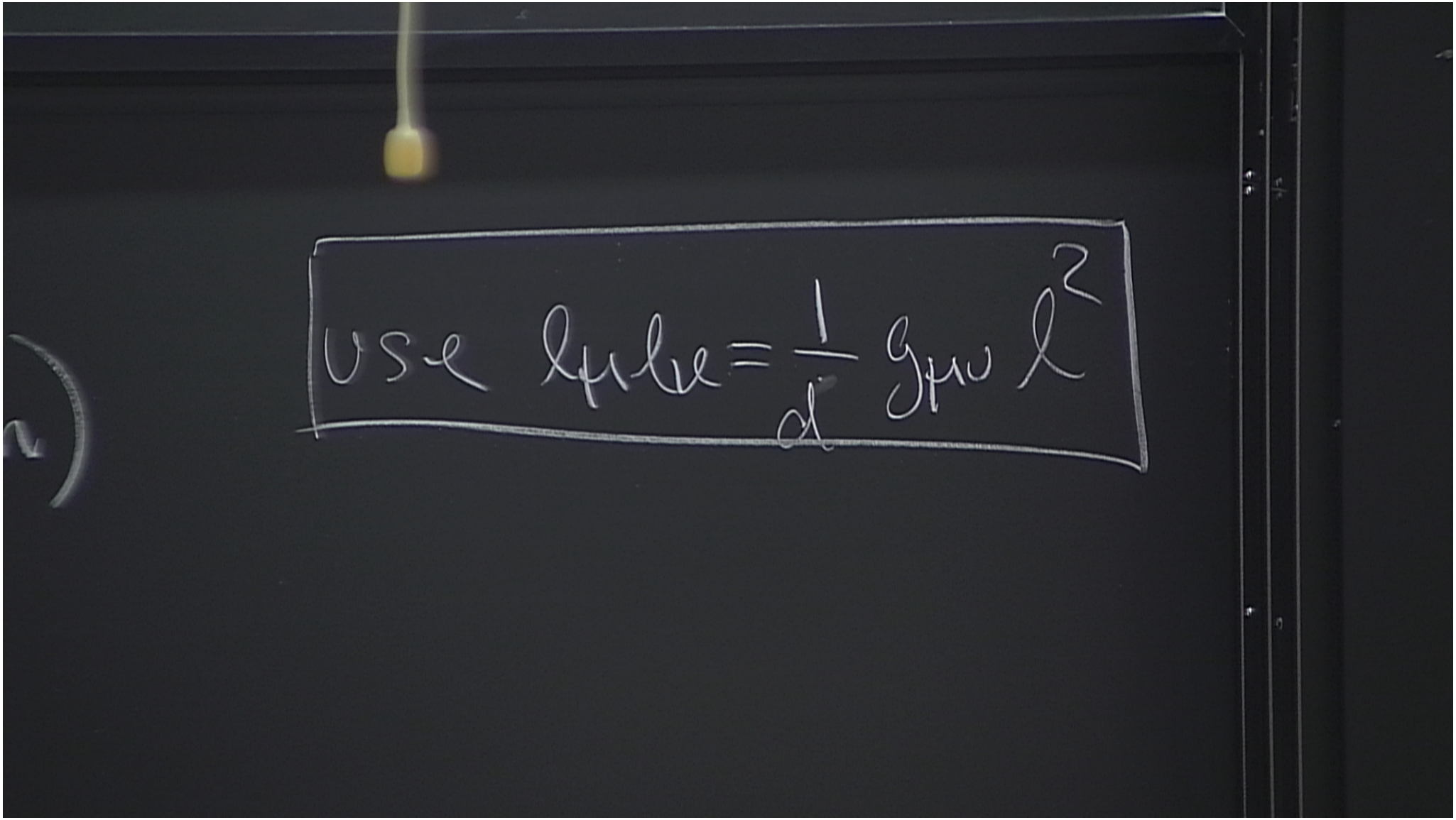
$\ell^2 = (\ell^0)^2 - (\vec{\ell})^2 \longrightarrow (\ell^{0})^2 + (\vec{\ell})^2$

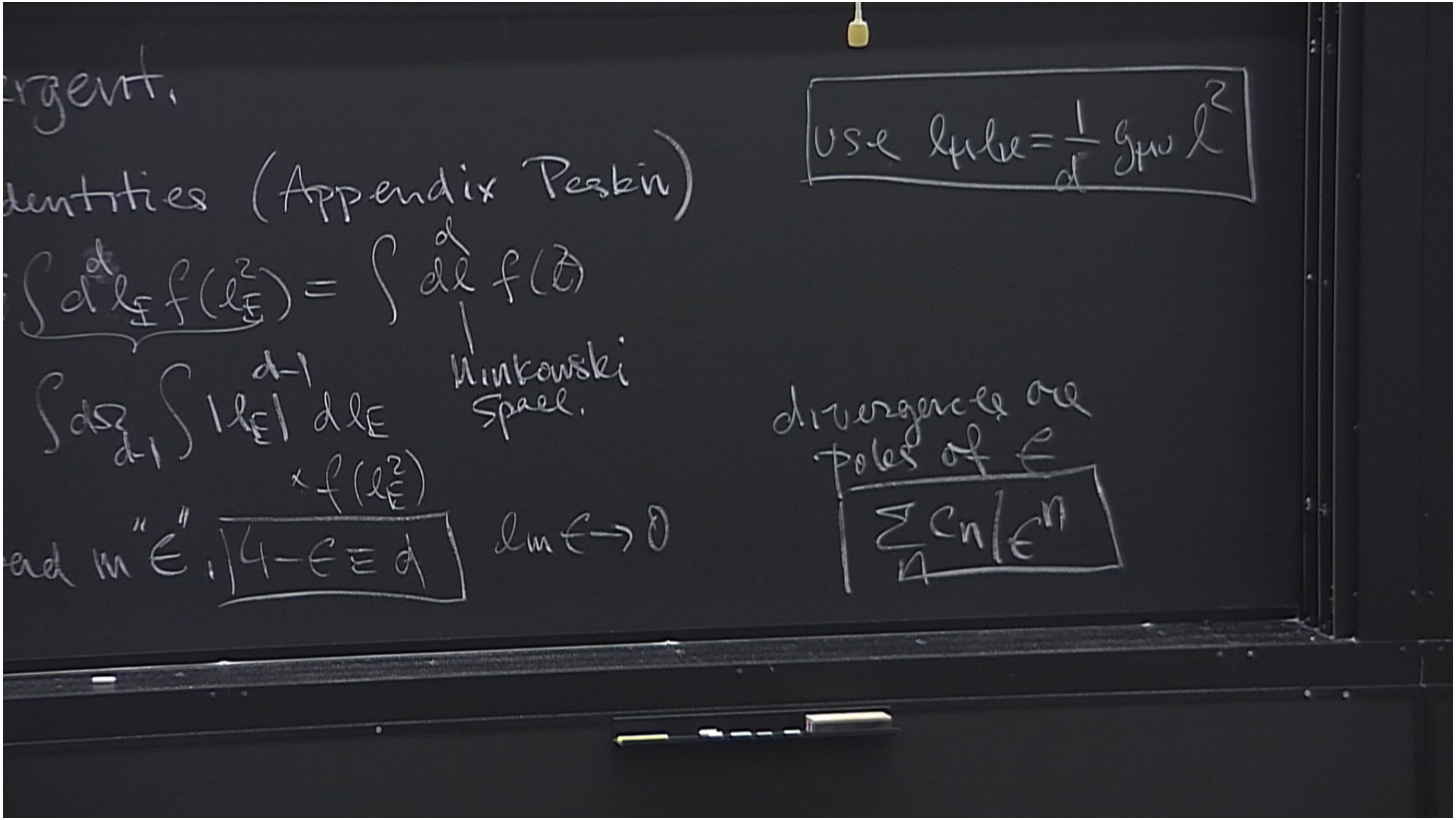
Spherical Coord. in d-dim.

$\frac{1}{(p^2-\Delta)+i\varepsilon}$ $\ell_0 = +\sqrt{|\vec{\ell}|^2+\Delta}-i\varepsilon$

Identi (Appendix B

$\int d^d\ell$ $_E) = \int$


use $l_\mu l_\nu = \frac{1}{d} g_{\mu\nu} l^2$


rgent.
dentities (Appendix Peskin)
use $k_\mu k_\nu = \frac{1}{d} g_{\mu\nu} k^2$
$\int d^d k_E f(k_E^2) = \int d\Omega_d f(k)$
Minkowski space.
$\int d\Omega_{d-1} \int |k_E|^{d-1} dk_E \times f(k_E^2)$
divergences are poles of ϵ
$\sum_n c_n / \epsilon^n$
ed in "ϵ": $4 - \epsilon = d$
$\lim \epsilon \to 0$