Title: Standard Model-8

Date: Jan 15, 2015 10:15 AM

URL: http://pirsa.org/15010085

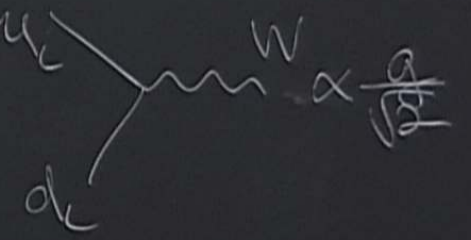
Abstract:

$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, (2, +1/6)$, $(u_R^c) \to (1, -2/3)$, $(d_R^c) \to (1, +1/3)$

$_R + h.c.)$

$R + h.c.)$

, Also Saw: $\mathcal{L}_{CW} \supset \frac{g}{\sqrt{2}} (u_L^\dagger \bar{\sigma}^\mu W_\mu^+ d_L + d_L^\dagger \bar{\sigma}^\mu W_\mu^- u_L)$



ration, (but there are 3!)

for W (as Exercise / Tutorial derive Z-interactions for 4-component Dirac spinors

Last time (15 Min Ago) Quarks: $Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$, (

Interactions w/ Higgs: $H^\dagger Q d_R^c \longrightarrow m_d (d_L d_R^c + h.c.)$

$\epsilon^{ij} H_i Q_j (u_R^c) \longrightarrow m_u (u_L u_R^c + h.c.)$

Define Dirac Spinors.

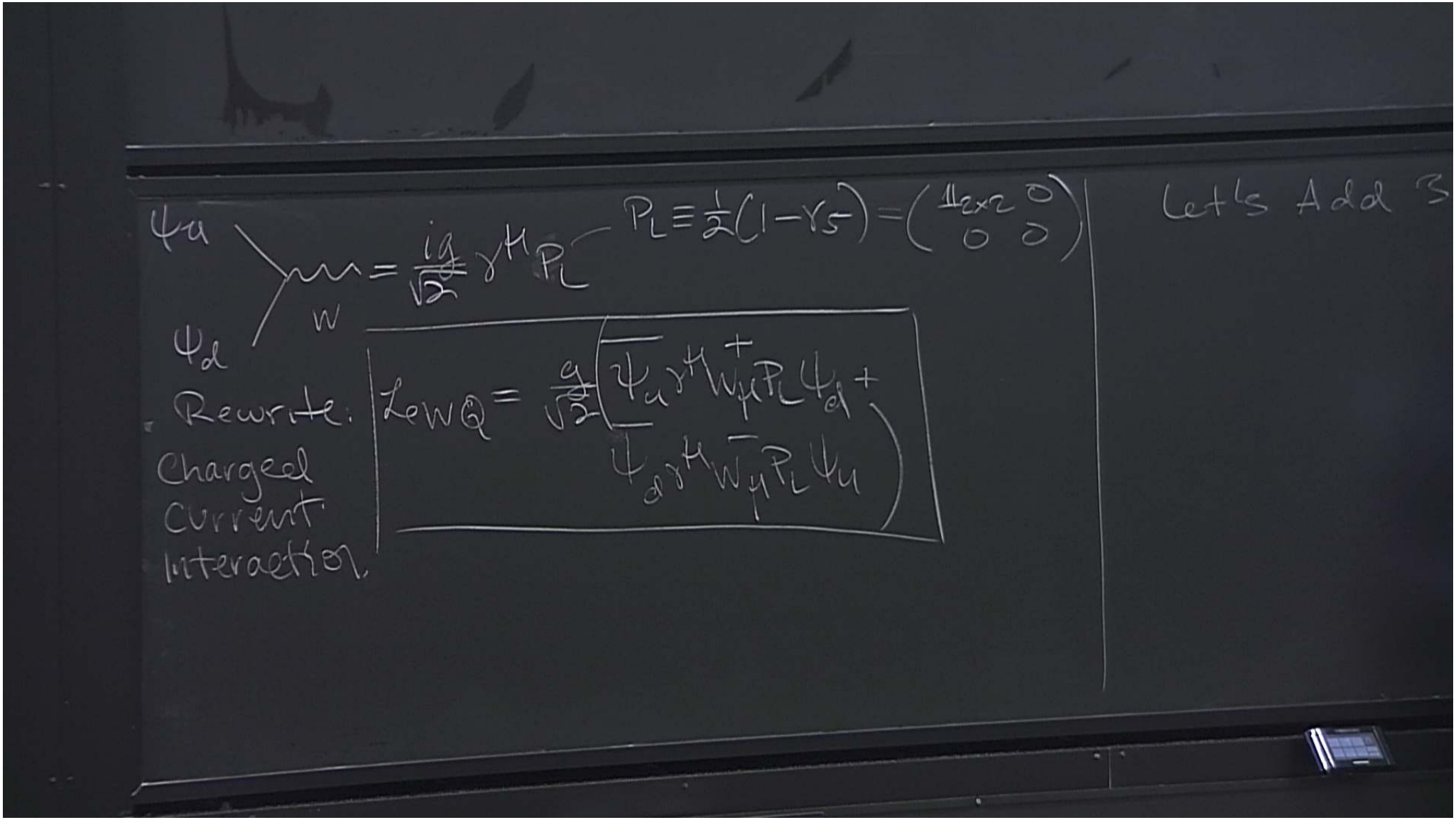
$$\psi_u \equiv \begin{pmatrix} u_L \\ (u_R^c)^\dagger \end{pmatrix}$$

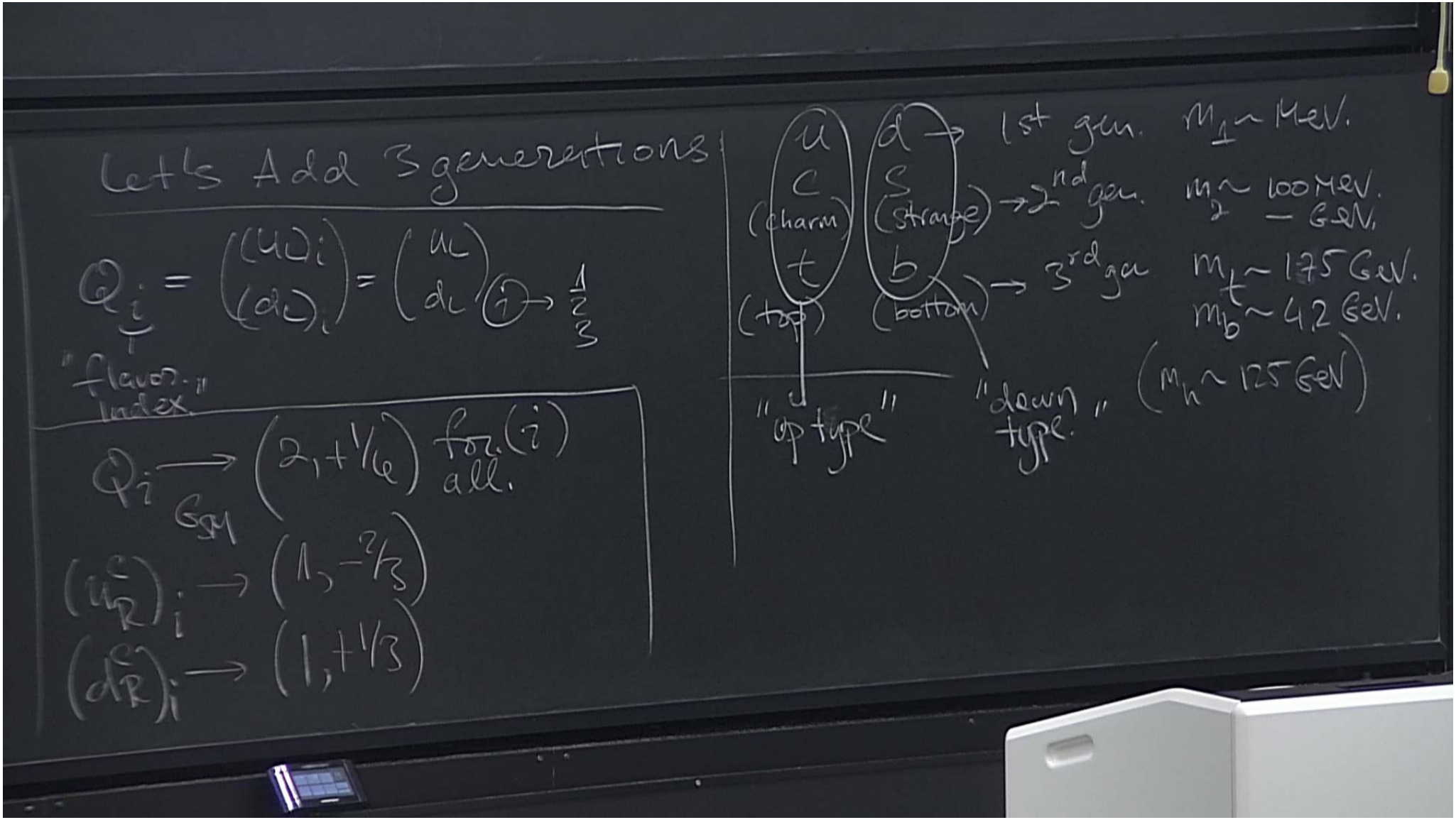
$$\psi_d \equiv \begin{pmatrix} d_L \\ (d_R^c)^\dagger \end{pmatrix}$$

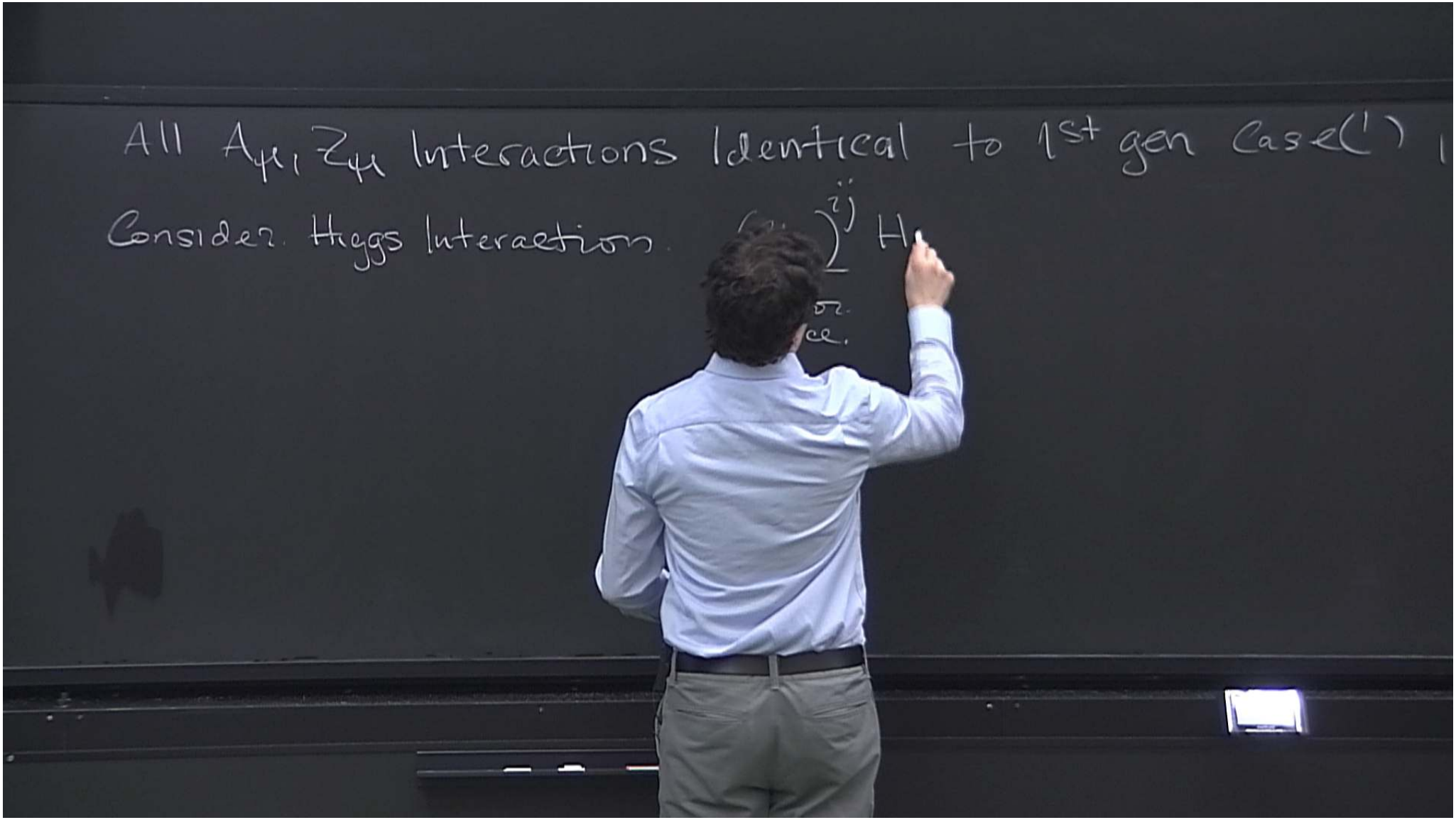
Dirac Masses $\Rightarrow$ $m_u \bar{\psi}_u \psi_u + m_d \bar{\psi}_d \psi_d$

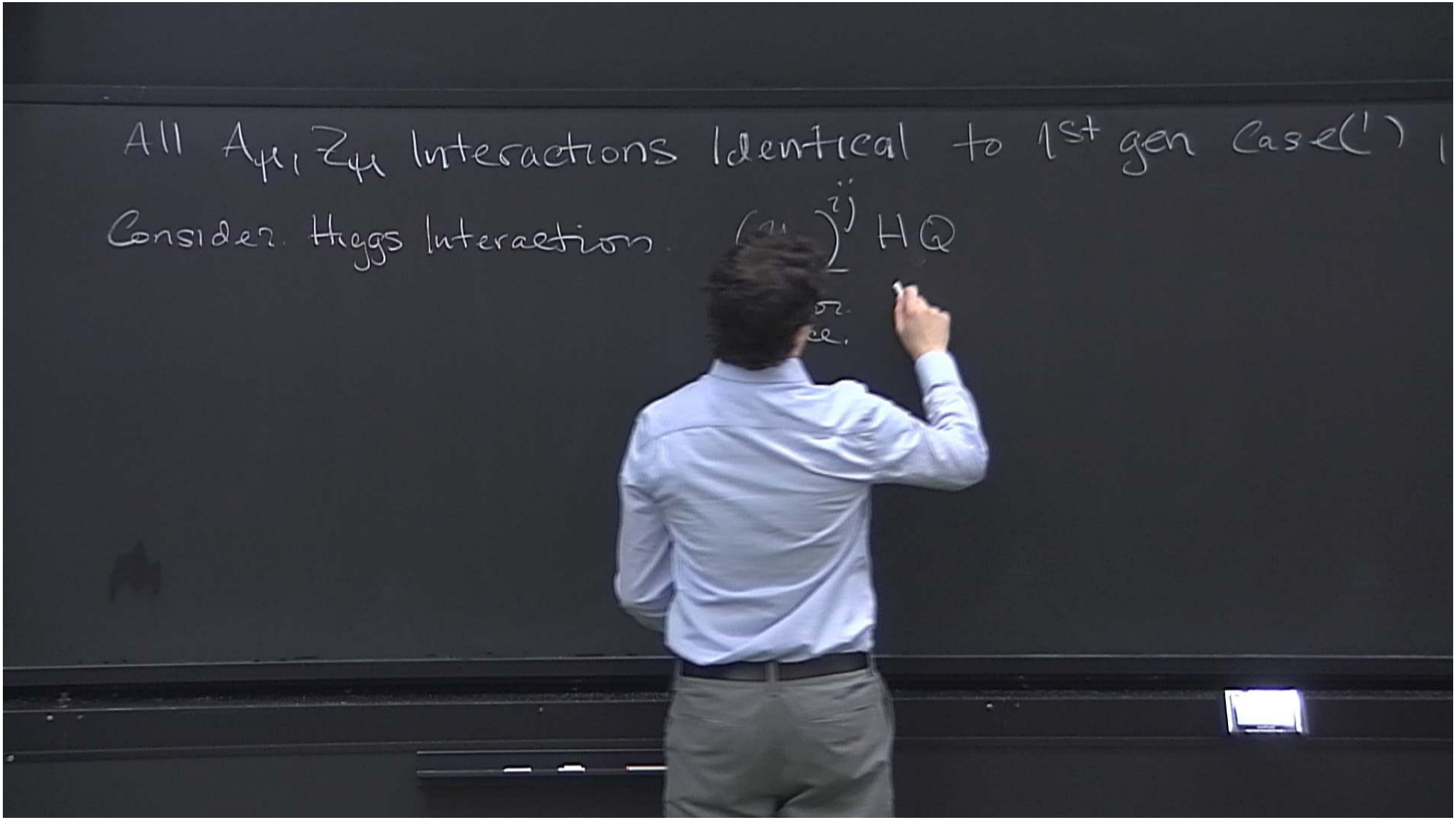
full Story for 1-generation, (b

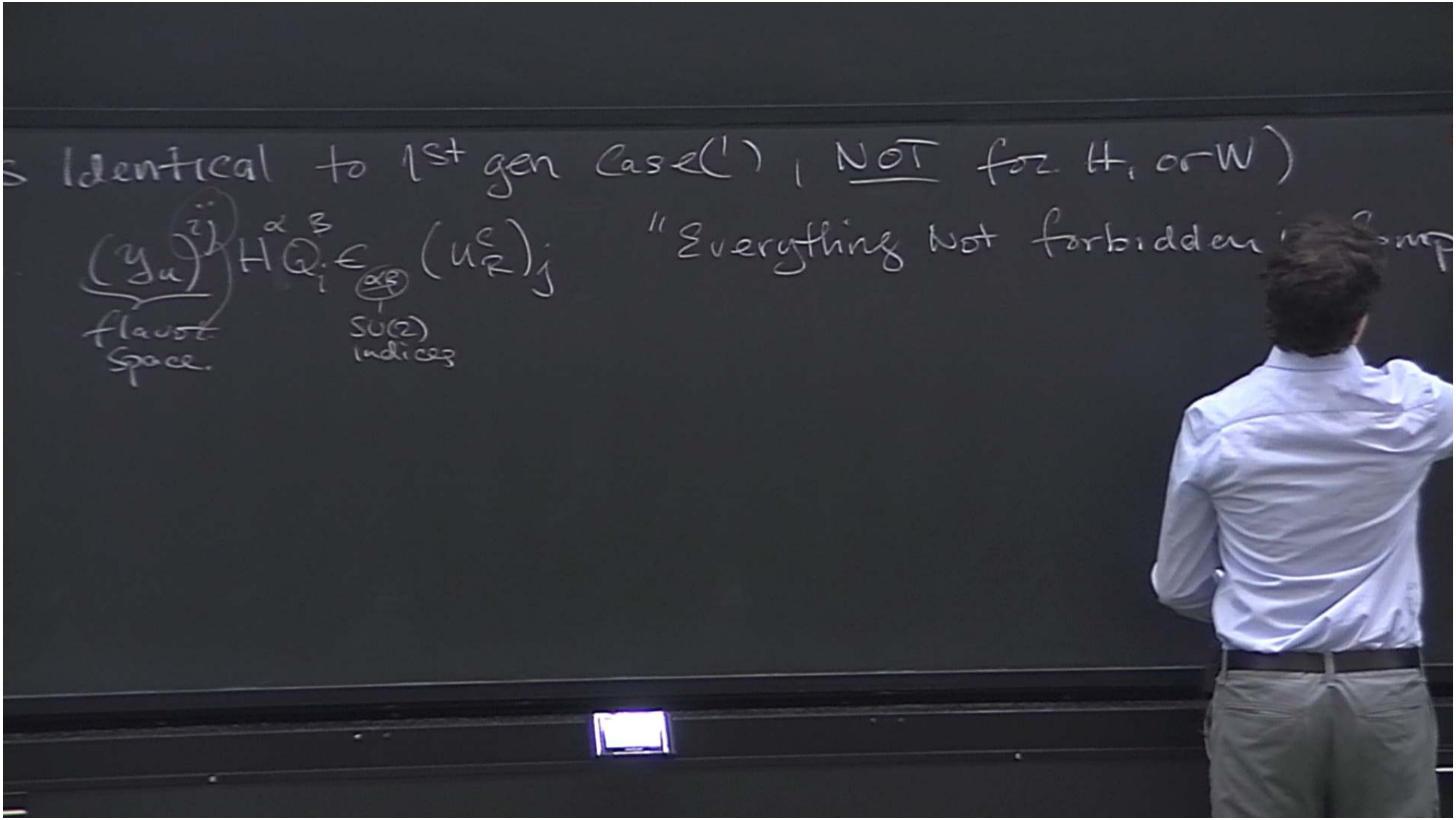
Feynman Rules for W (as

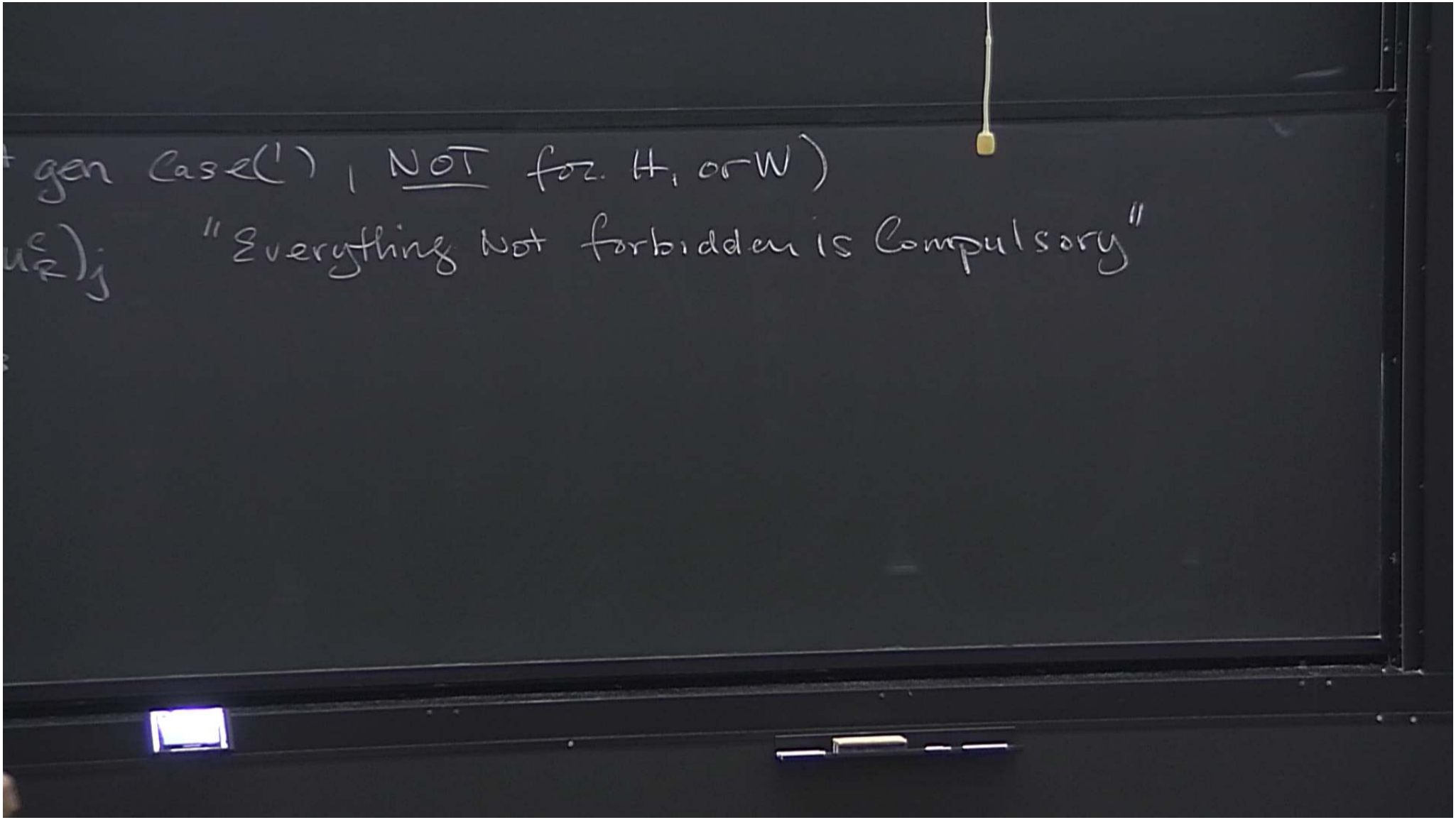

ψ_u
ψ_d
W
$\mu = \frac{ig}{\sqrt{2}} \gamma^\mu P_L$
$P_L \equiv \frac{1}{2}(1-\gamma_5) = \begin{pmatrix} \mathbb{1}_{2\times 2} & 0 \\ 0 & 0 \end{pmatrix}$
Let's Add S
Rewrite:
Charged Current Interaction.
$\mathcal{L}_{eWQ} = \frac{g}{\sqrt{2}} \left(\overline{\psi_u} \gamma^\mu W^+_\mu P_L \psi_d + \overline{\psi_d} \gamma^\mu W^-_\mu P_L \psi_u \right)$

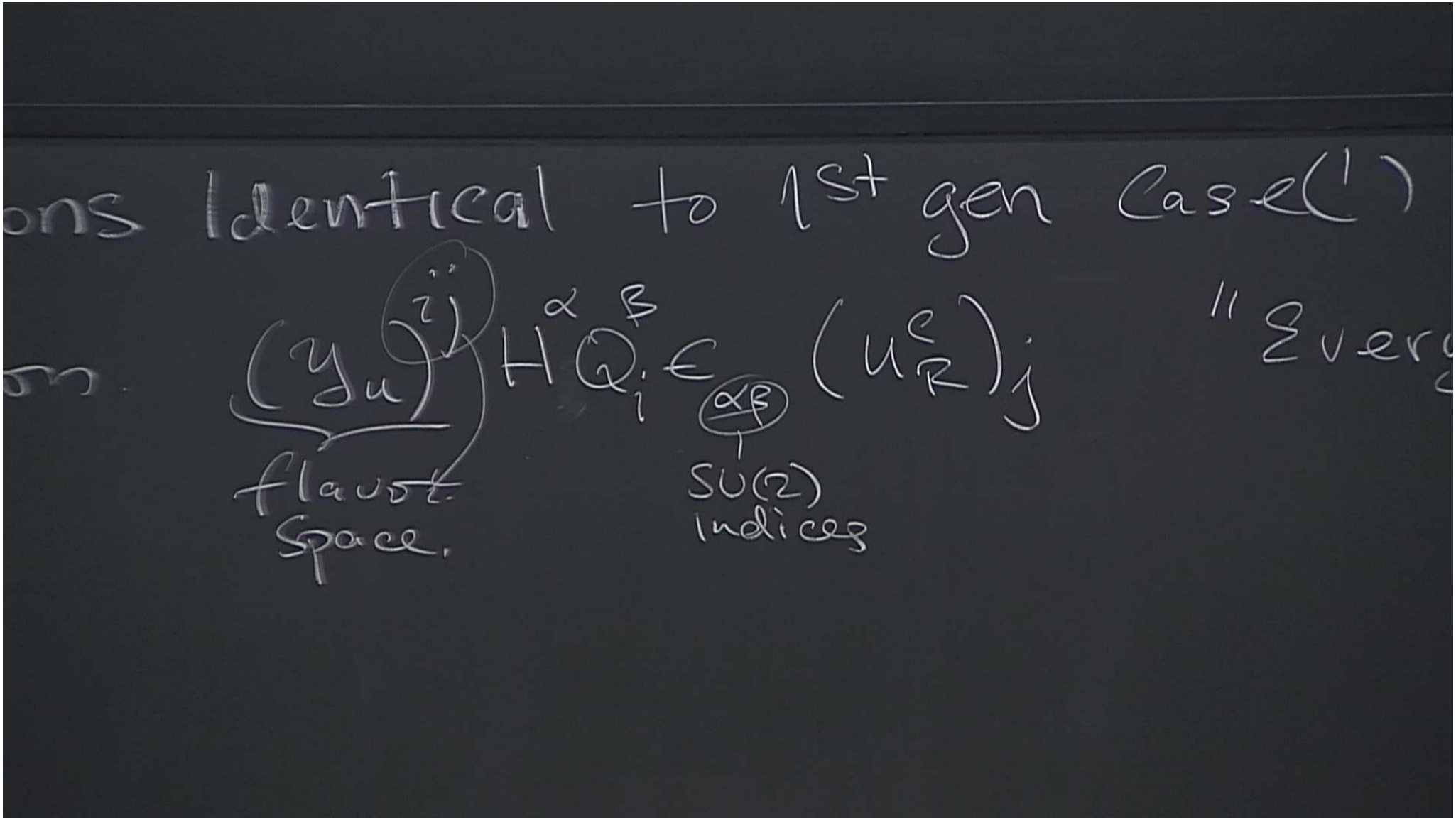

Let's Add 3 generations:
$Q_i = \begin{pmatrix} (u_L)_i \\ (d_L)_i \end{pmatrix} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$ ① → 1/2/3
"flavor index"
$Q_i \xrightarrow[SM]{} (2, +1/6)$ for all (i)
$(u_R^c)_i \to (1, -2/3)$
$(d_R^c)_i \to (1, +1/3)$
u c t (charm) (top)
d s b (strange) (bottom)
→ 1st gen. $m_1 \sim$ MeV.
→ 2nd gen. $m_2 \sim$ 100 MeV. — GeV.
→ 3rd gen. $m_t \sim 175$ GeV. $m_b \sim 4.2$ GeV.
$(m_h \sim 125$ GeV)
"up type"
"down type"


All A_4, Z_4 Interactions Identical to 1st gen Case(1)
Consider Higgs Interaction


All A_4, Z_4 Interactions Identical to 1st gen Case(1)
Consider Higgs Interaction $(a)^{ij}$ HQ


s Identical to 1st gen case(1), NOT for H, or W)
$(y_u)^{ij}$ $H^\alpha Q_i^\beta \epsilon_{\alpha\beta} (u_R^c)_j$
"Everything Not forbidden
flavor space.
SU(2) indices


gen Case(1), NOT for H, or W)
"Everything Not forbidden is Compulsory"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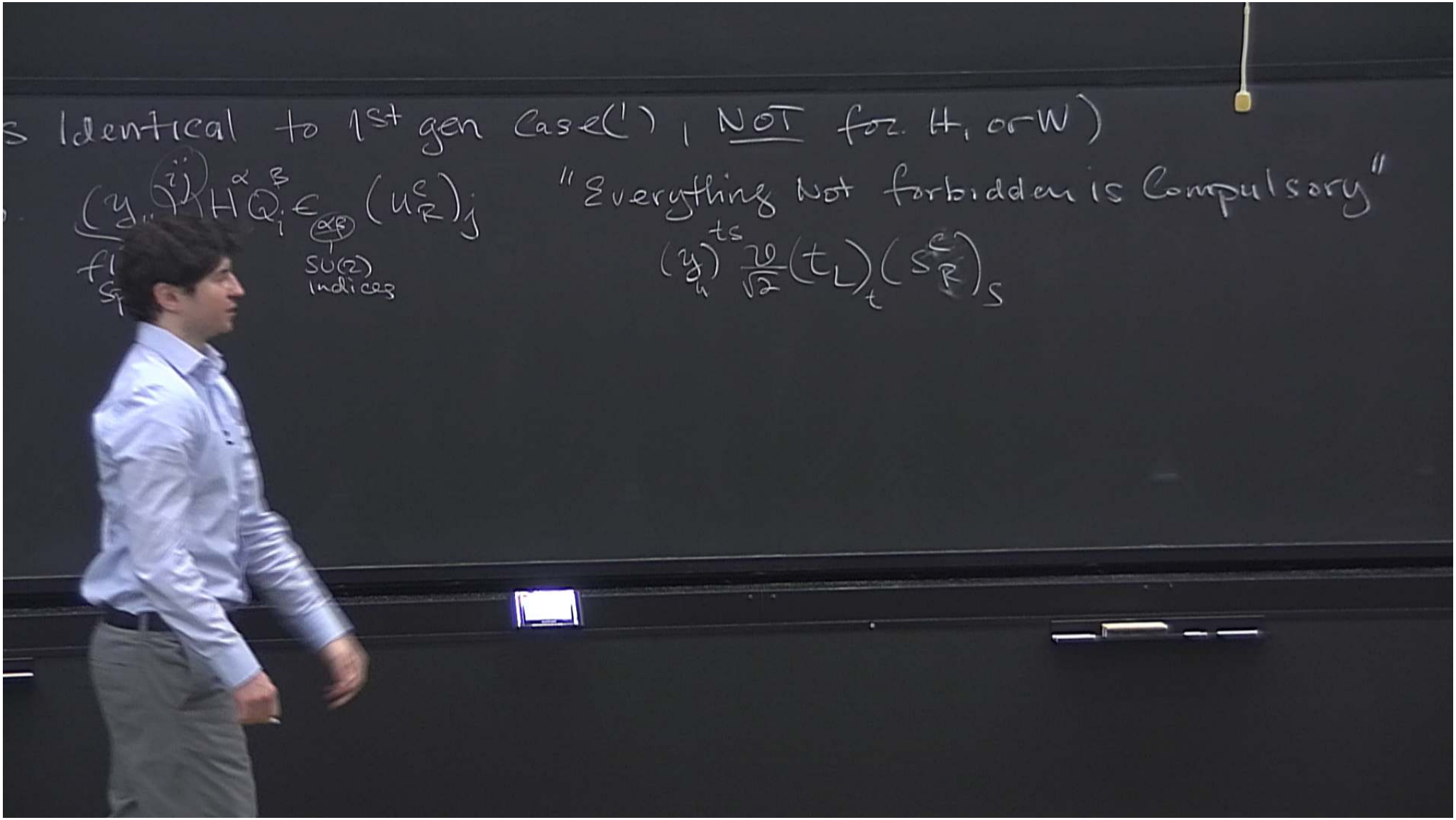
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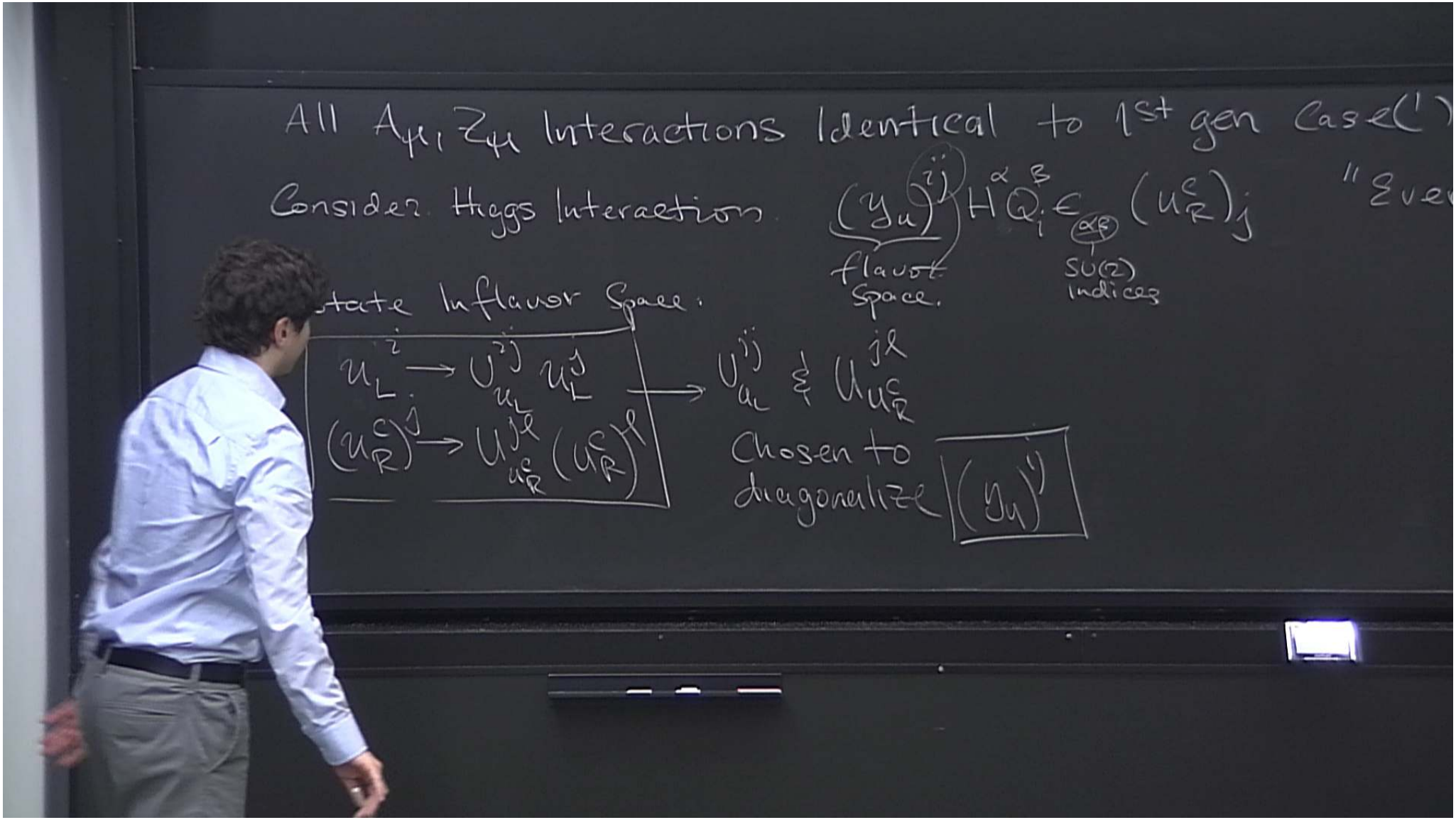
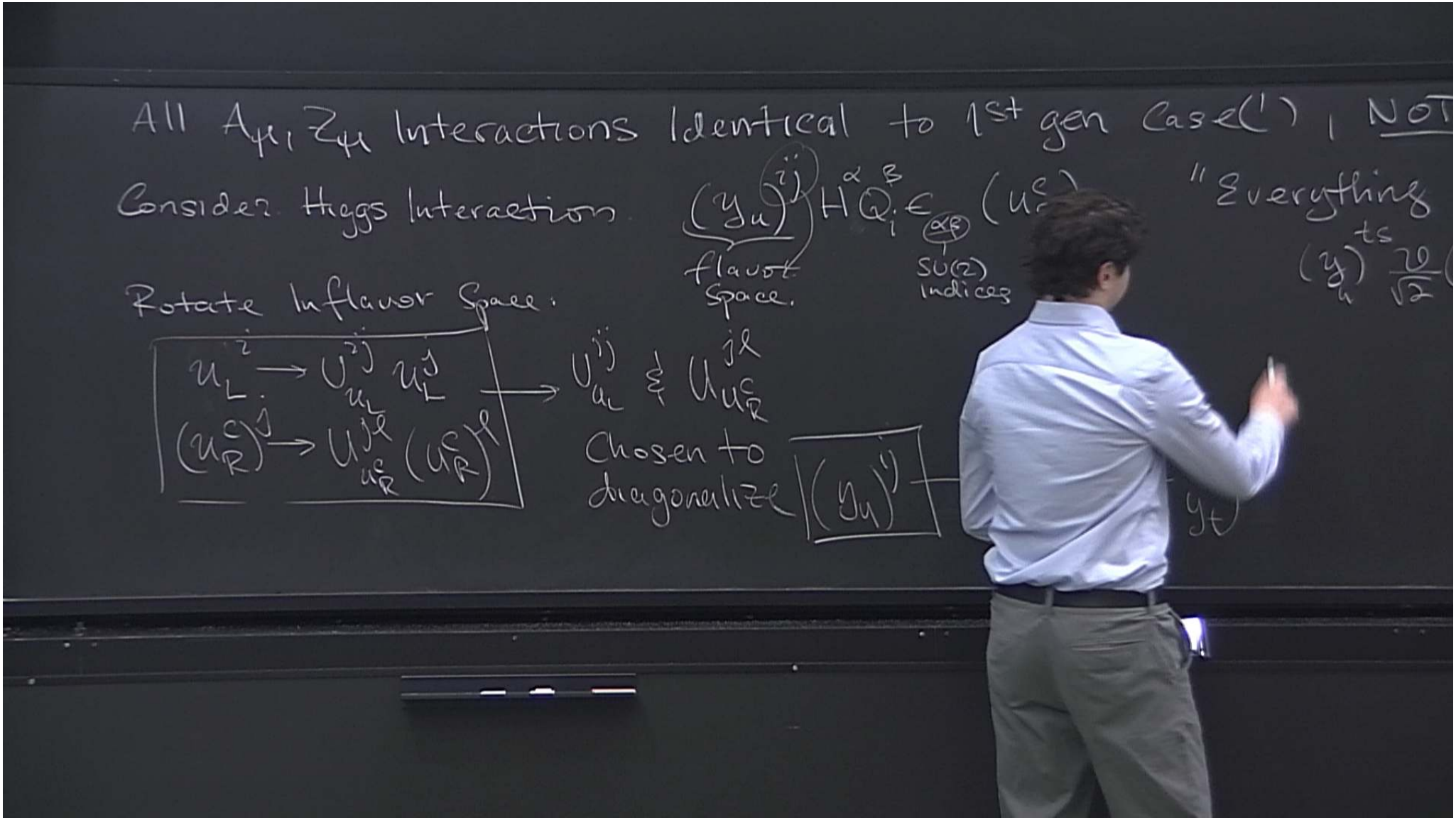
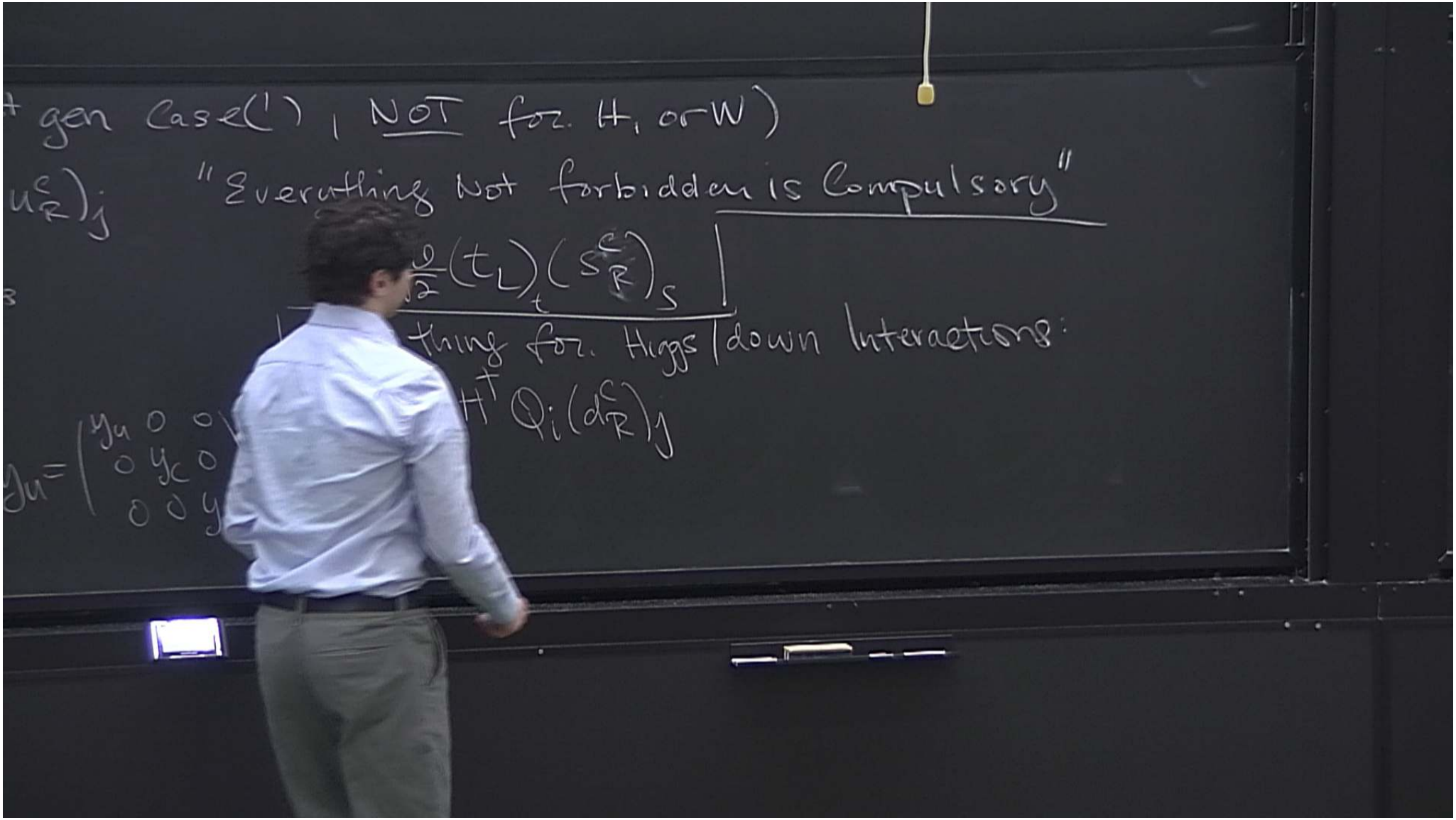
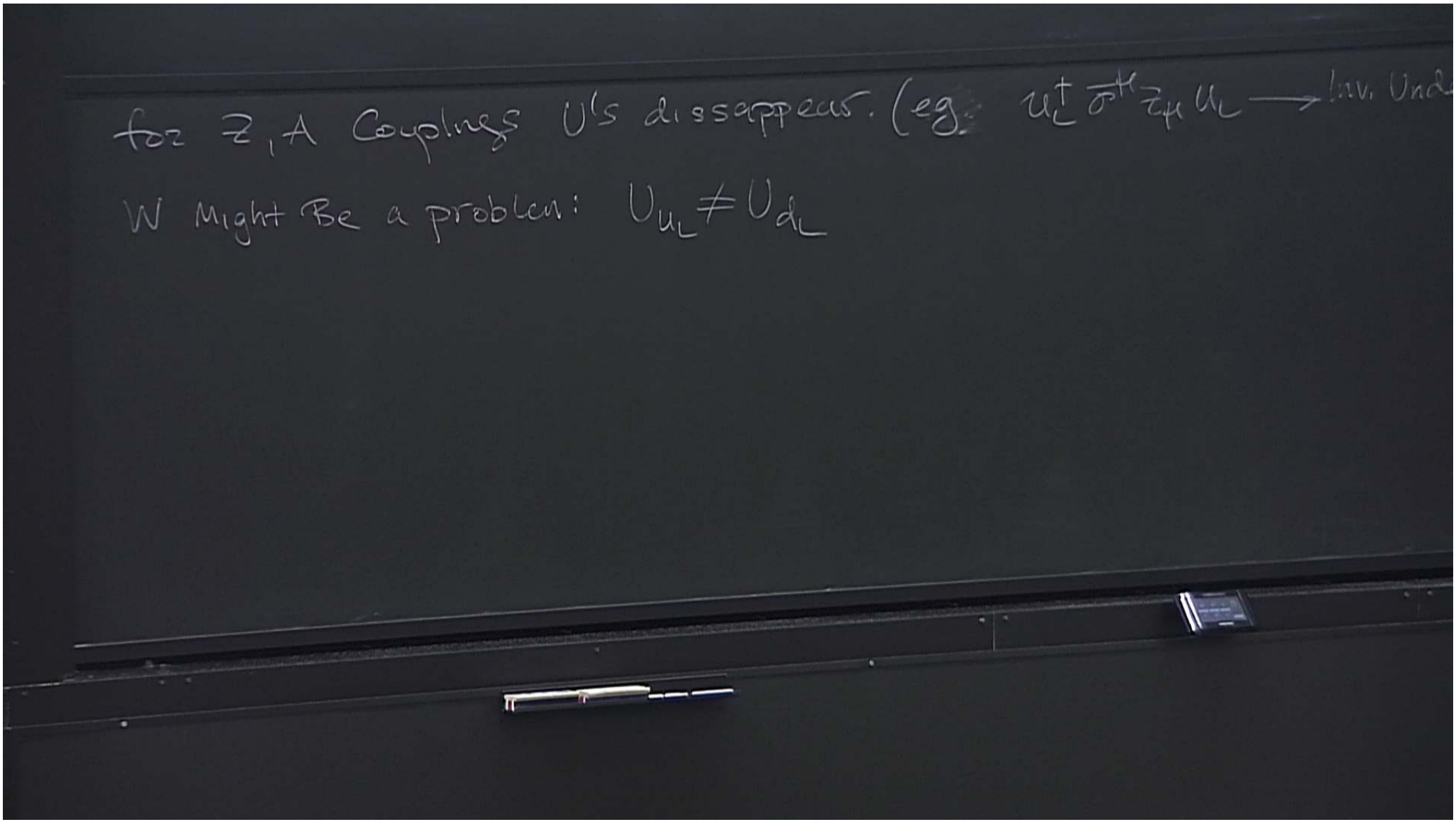

All A_4, Z_4 Interactions Identical to 1st gen Case(1)
Consider Higgs Interaction
$(y_u)^{ij} H^\alpha Q_i^\beta \epsilon_{\alpha\beta} (u_R^c)_j$
flavor Space.
SU(2) indices
tate Inflavor Space:
$u_L^i \to U_{u_L}^{ij} u_L^j$
$(u_R^c)^j \to U_{u_R^c}^{jk} (u_R^c)^k$
$U_{u_L}^{ij}$ & $U_{u_R^c}^{jk}$
Chosen to diagonalize
$(y_u)^{ij}$

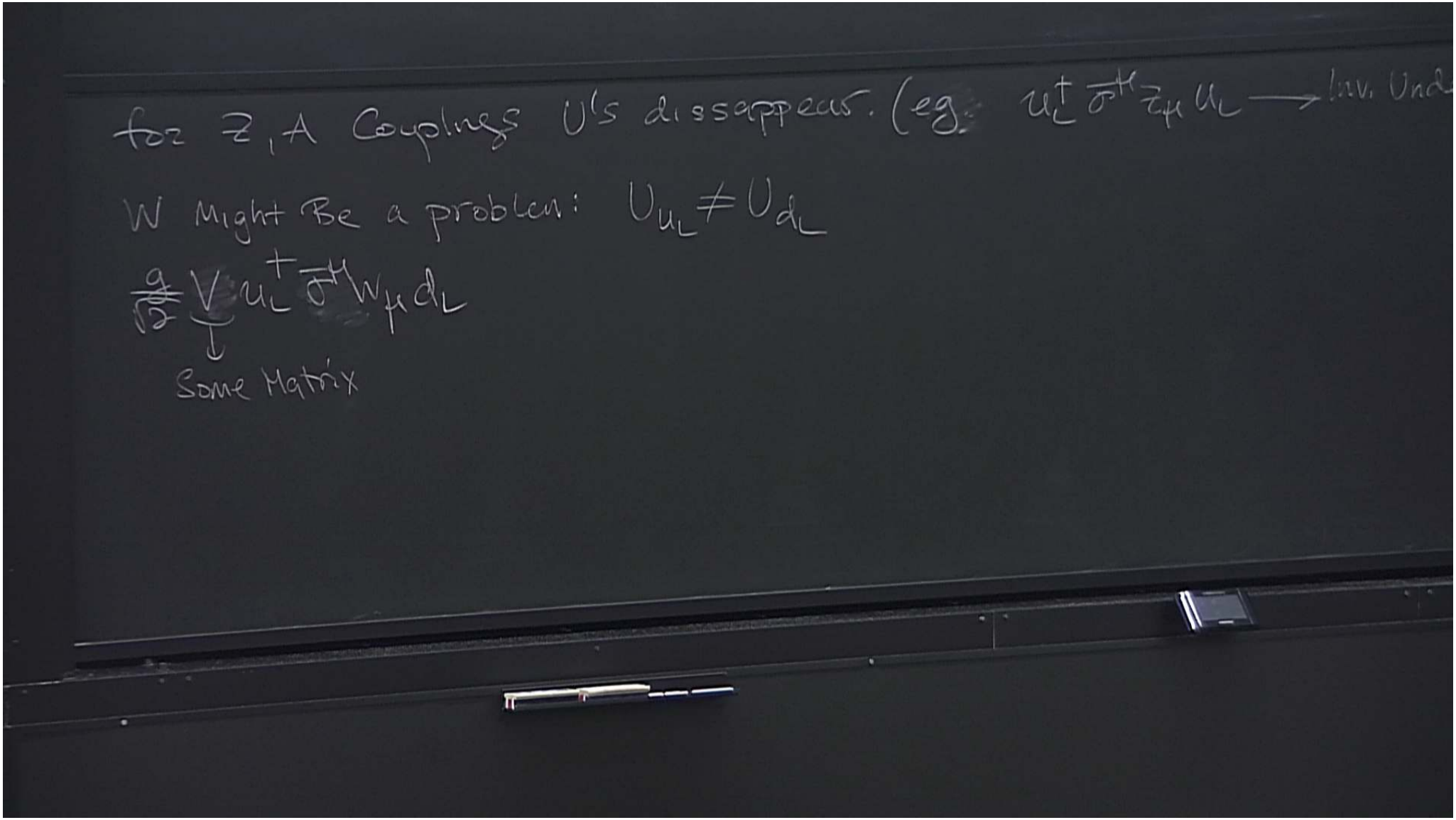

All A_4, Z_4 Interactions Identical to 1st gen Case(1), NOT
Consider Higgs Interaction $(y_u)^{ij} H^\alpha Q_i^\beta \epsilon_{\alpha\beta} (u_R^c)$
flavor space.
SU(2) indices
"Everything
$(y_u)^{ts} \frac{v}{\sqrt{2}}$
Rotate In flavor Space:
$u_L^i \to U_{u_L}^{ij} u_L^j$
$(u_R^c)^j \to U_{u_R}^{jk} (u_R^c)^k$
$\to U_{u_L}^{ij}$ & $U_{u_R}^{jk}$
Chosen to diagonalize $(y_u)^{ij}$
y_t


gen case(1), NOT for H, or W)
"Everything Not forbidden is Compulsory"
thing for. Higgs/down Interactions:

for Z, A Couplings U's dissappear. (eg: $u_L^\dagger \bar{\sigma}^\mu Z_\mu u_L \longrightarrow$ Inv. Und

W Might Be a problem: $U_{u_L} \neq U_{d_L}$

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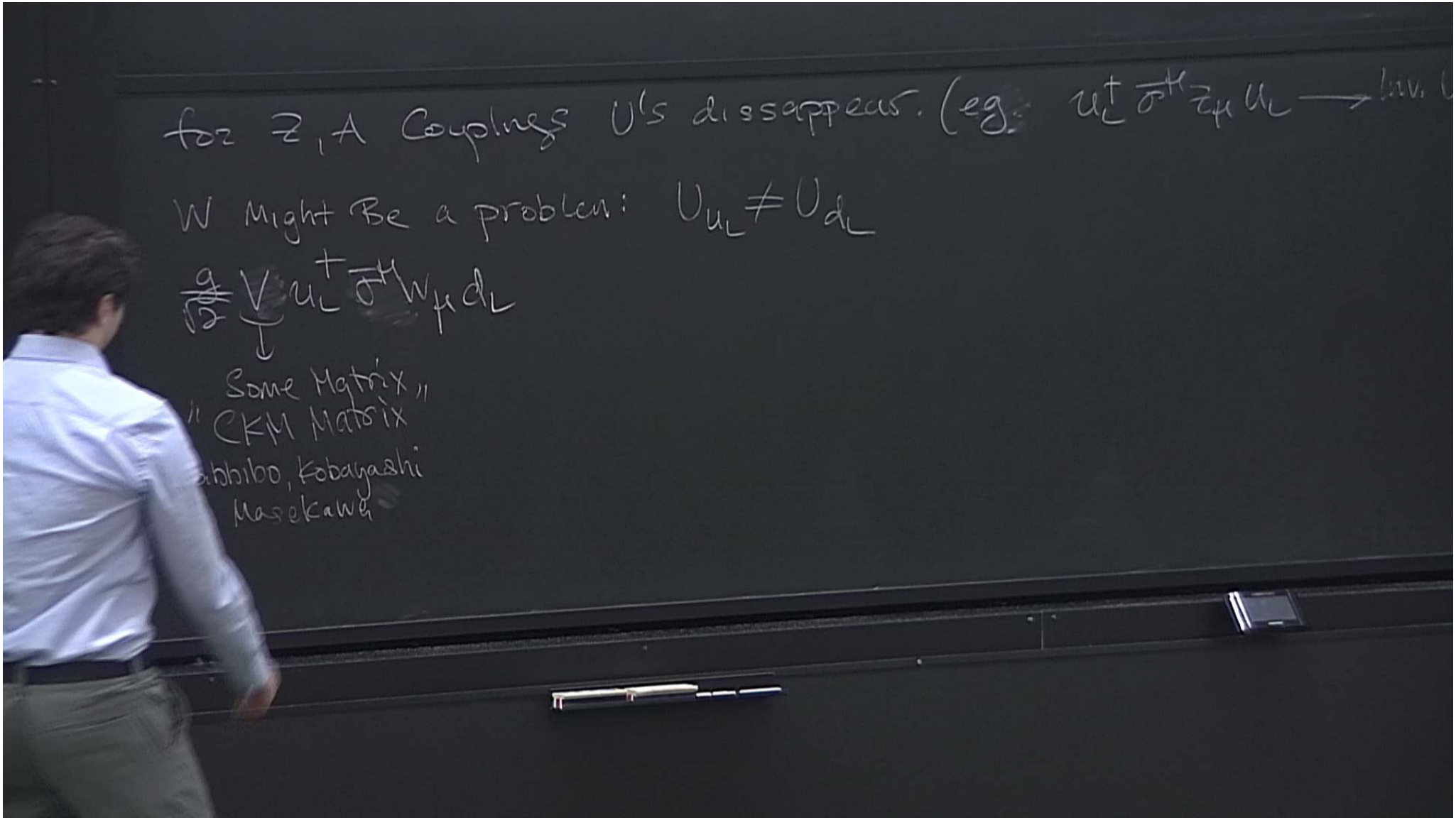
W Might Be a problem: $U_{u_L} \neq U_{d_L}$
$\frac{g}{\sqrt{2}} V u_L^\dagger \bar{\sigma}^\mu W_\mu d_L$
Some Matrix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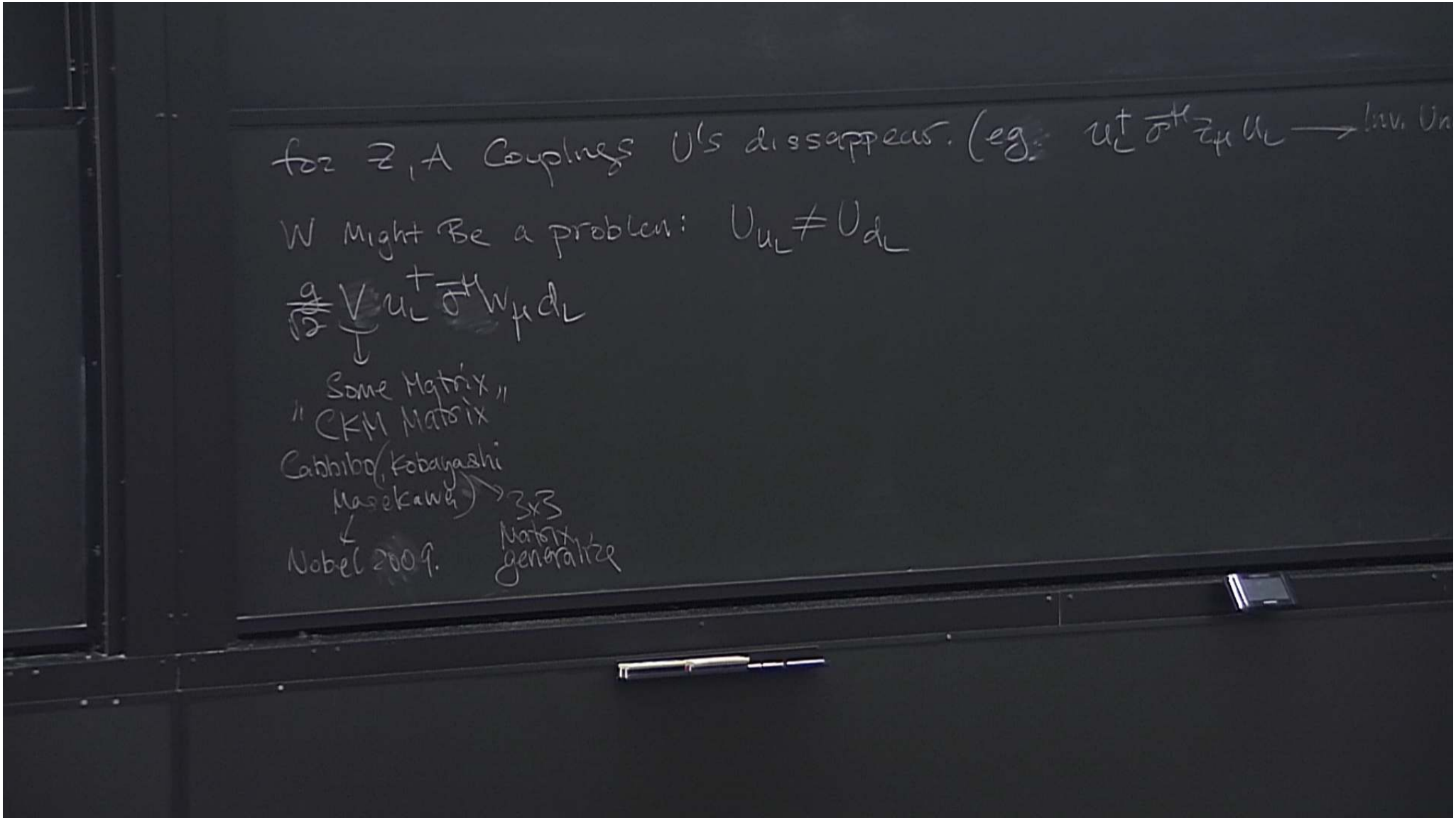
W Might Be a problem: $U_{u_L} \neq U_{d_L}$
$\frac{g}{\sqrt{2}} V u_L^\dagger \bar{\sigma}^\mu W_\mu d_L$
Some Matrix
" CKM Matrix
Kobayashi
Maskawa

for Z, A Couplings U's disappear. (eg: $u_L^\dagger \bar{\sigma}^\mu Z_\mu u_L \longrightarrow$ Inv.

W Might Be a problem: $U_{u_L} \neq U_{d_L}$

$$\frac{g}{\sqrt{2}} V u_L^\dagger \bar{\sigma}^\mu W_\mu d_L$$

V: Some Matrix
" CKM Matrix
Cabbibo, Kobayashi Maskawa
Nobel 1979.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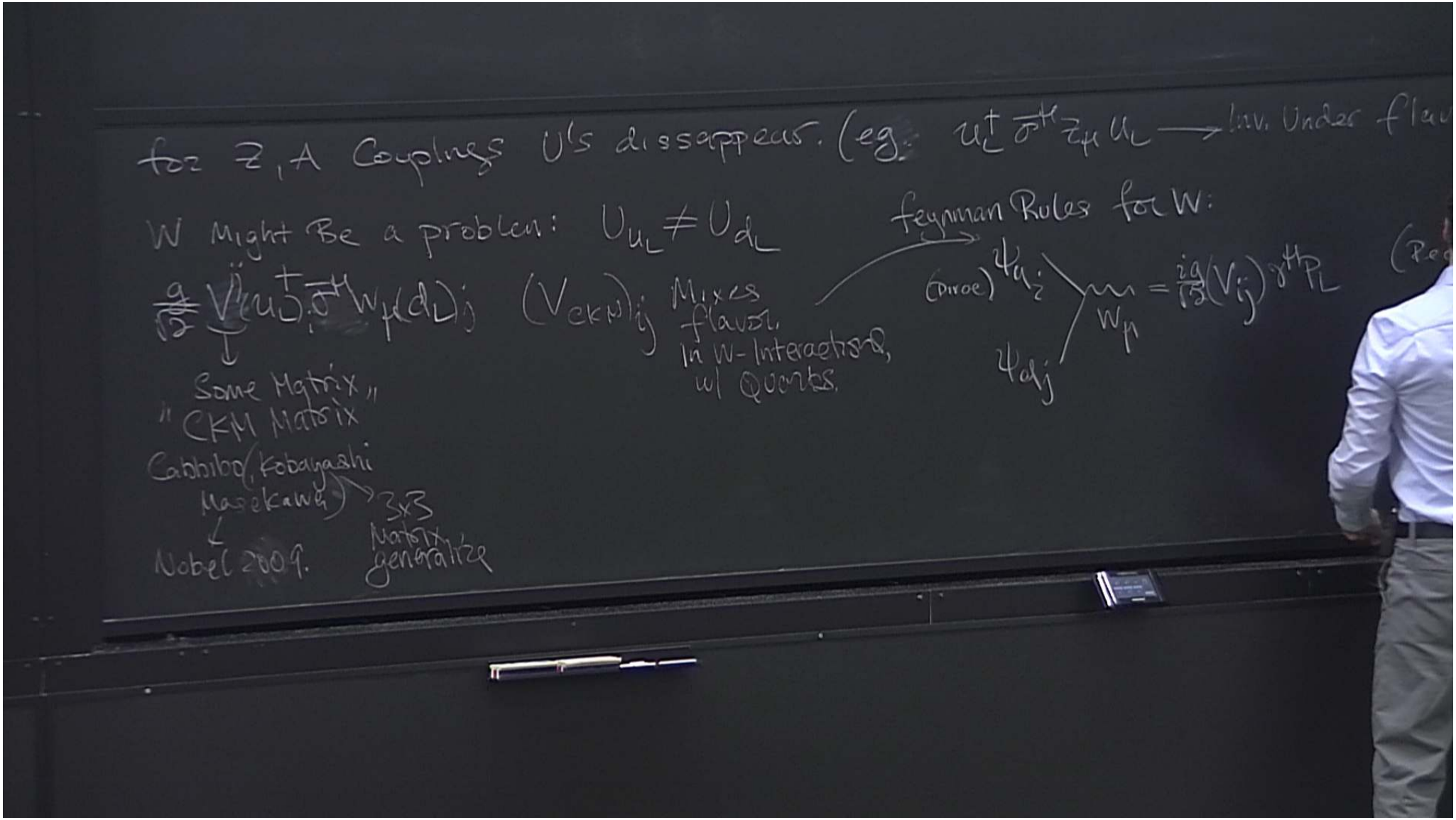
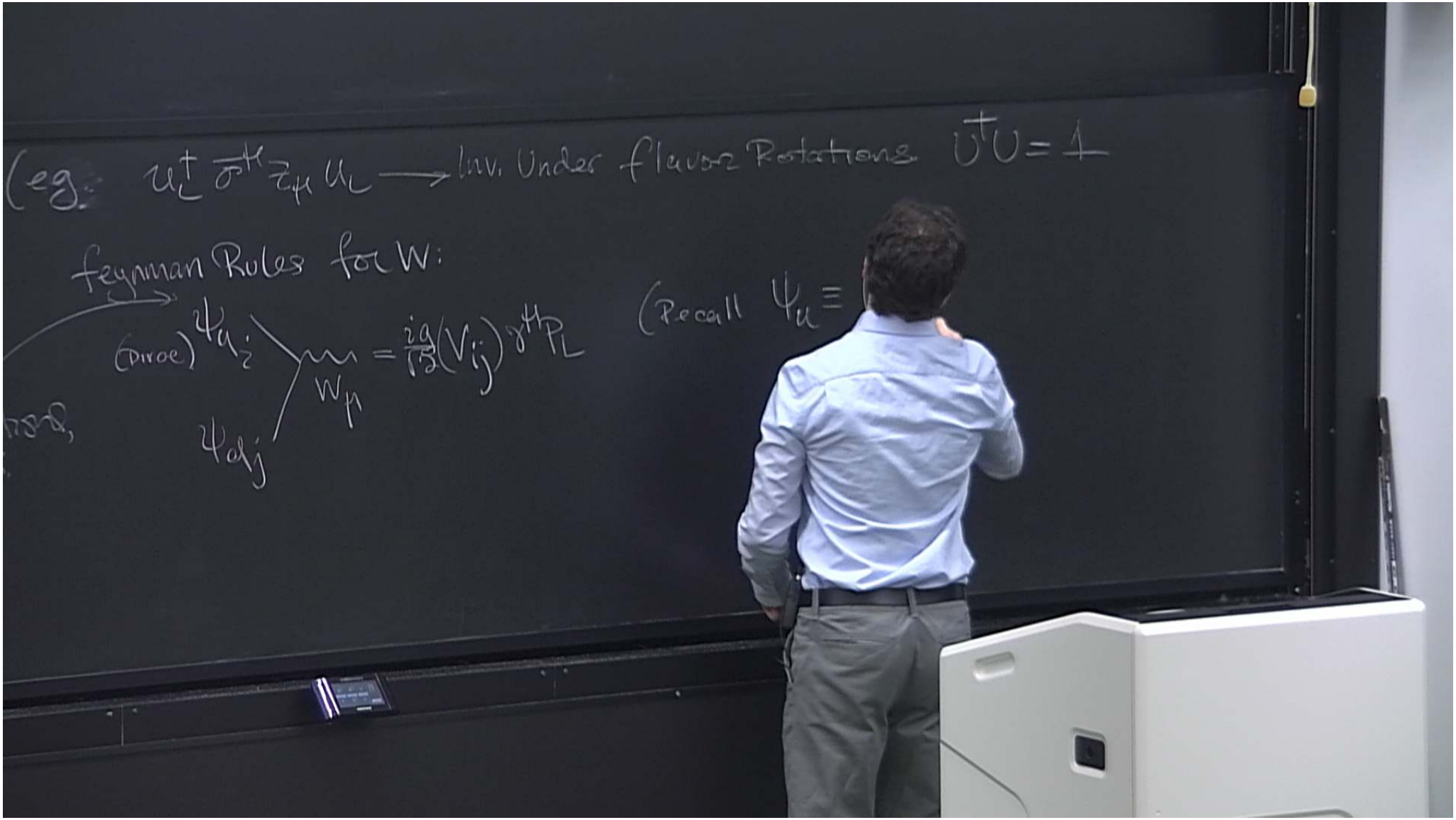
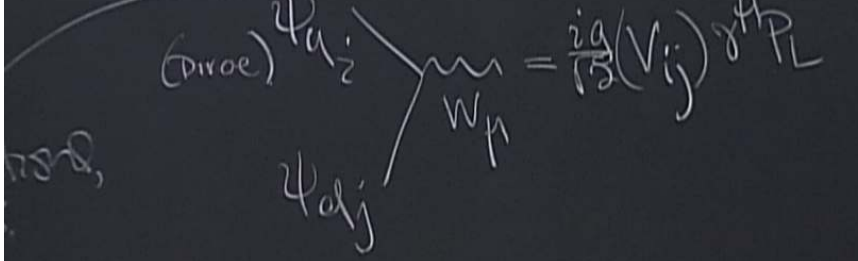
W Might Be a problem: $U_{u_L} \neq U_{d_L}$
$\frac{g}{\sqrt{2}} V u_L^\dagger \bar{\sigma}^\mu W_\mu d_L$
V
Some Matrix
"CKM Matrix
Cabbibo, Kobayashi
Maskawa
3x3
Matrix
generalize
Nobel 2009.


for Z, A Couplings U's disappear. (eg.
W Might Be a problem: $U_{uL} \neq U_{dL}$
Some Matrix
"CKM Matrix"
Cabibbo, Kobayashi
Maskawa
Nobel 2009.
3x3
Matrix
generalize
Mixes
flavors
in W-Interactions
w/ Quarks.
Feynman Rules for W:
(Dirac)

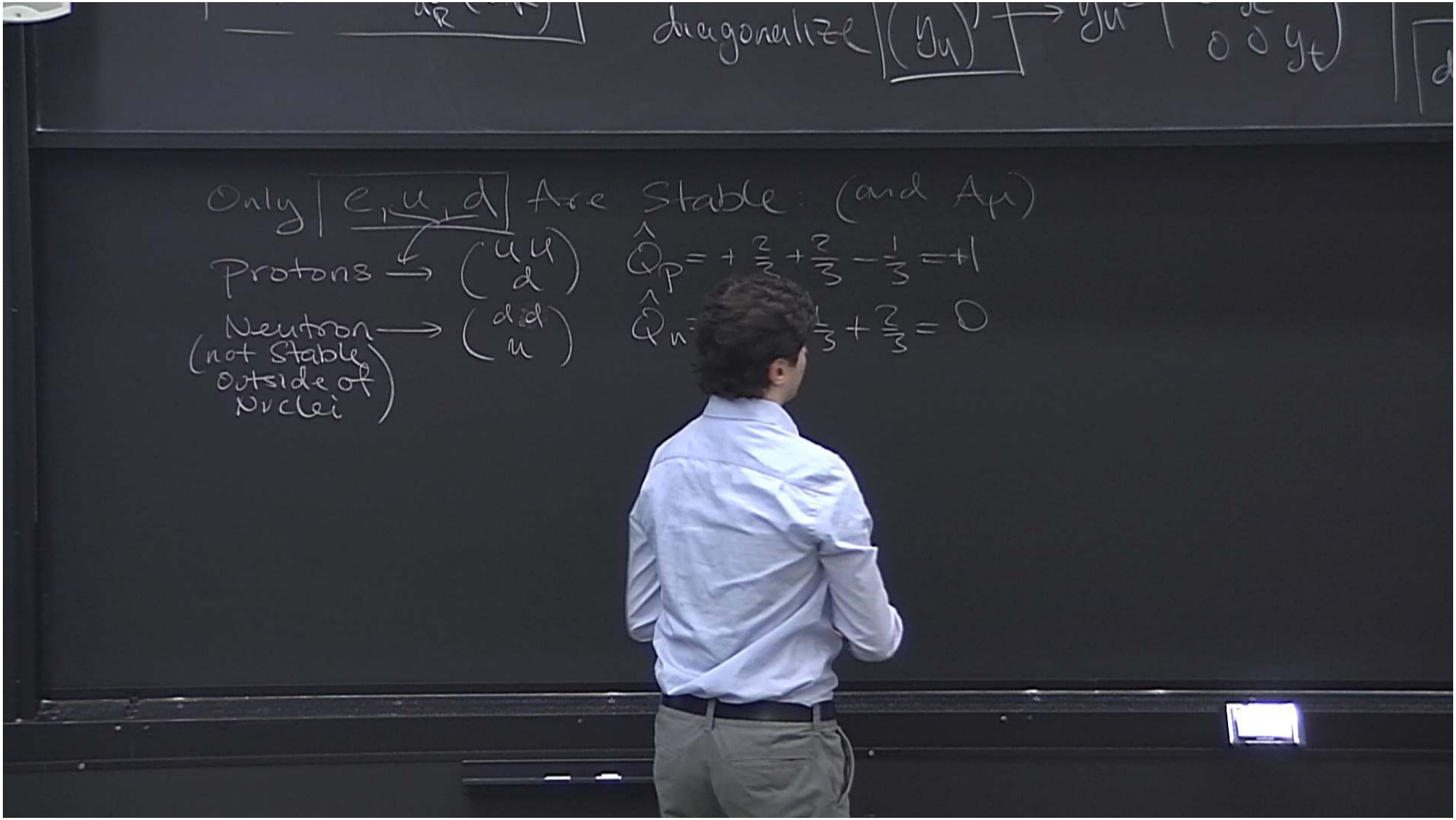

(eg. $u_L^\dagger \bar{\sigma}^\mu z_\mu u_L \longrightarrow$ Inv. Under flavor Rotations. $U^\dagger U = 1$
Feynman Rules for W:
(Dirac) ψ_{u_i}
ψ_{d_j}
W_μ
$= \frac{ig}{\sqrt{2}}(V_{ij})\gamma^\mu P_L$
(Recall $\psi_u \equiv$

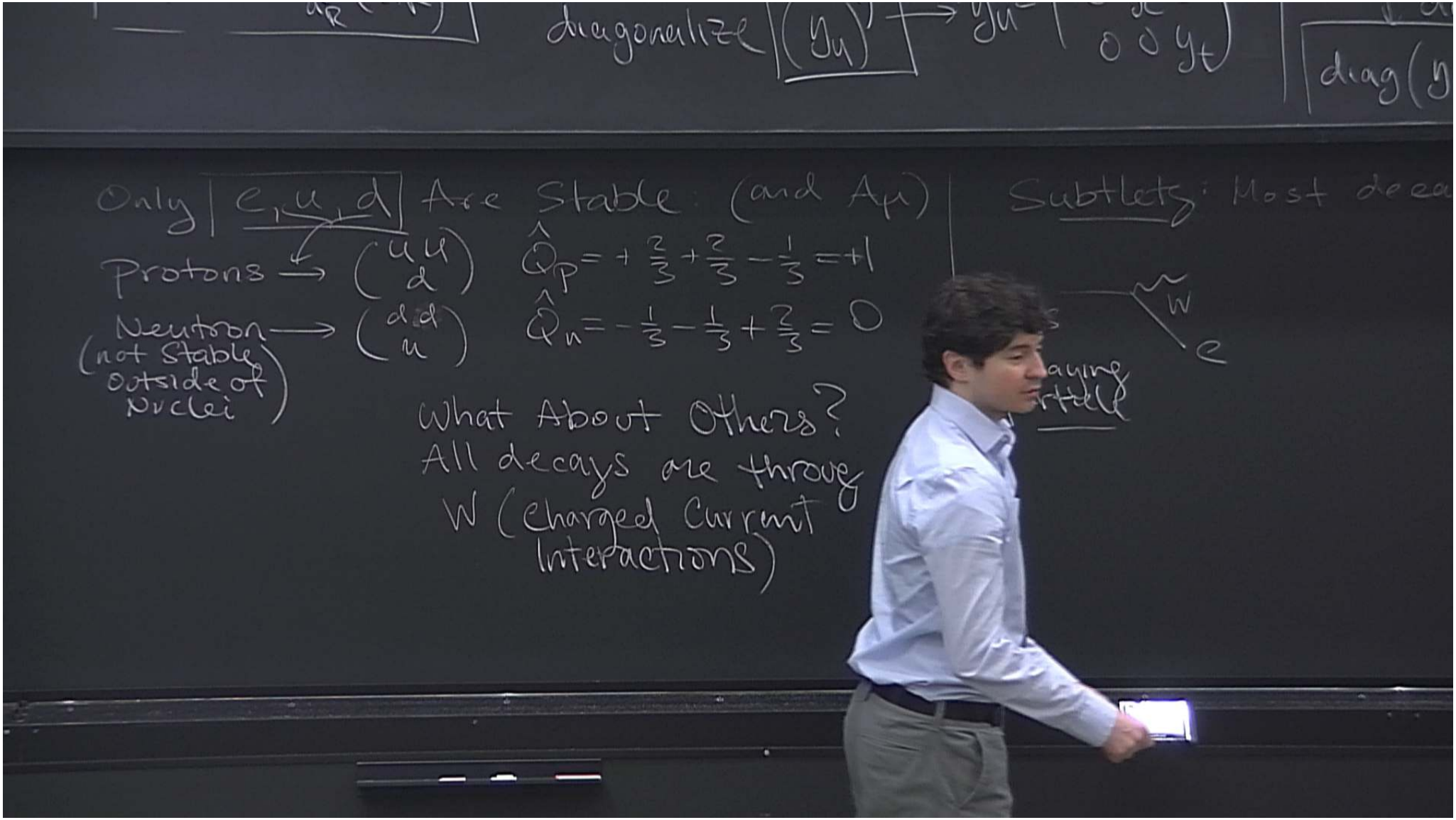
(eg. $\bar{u}_L^\dagger \bar{\sigma}^\mu z_\mu u_L \longrightarrow$ Inv. Under flavor Rotations. $U^\dagger U = 1$

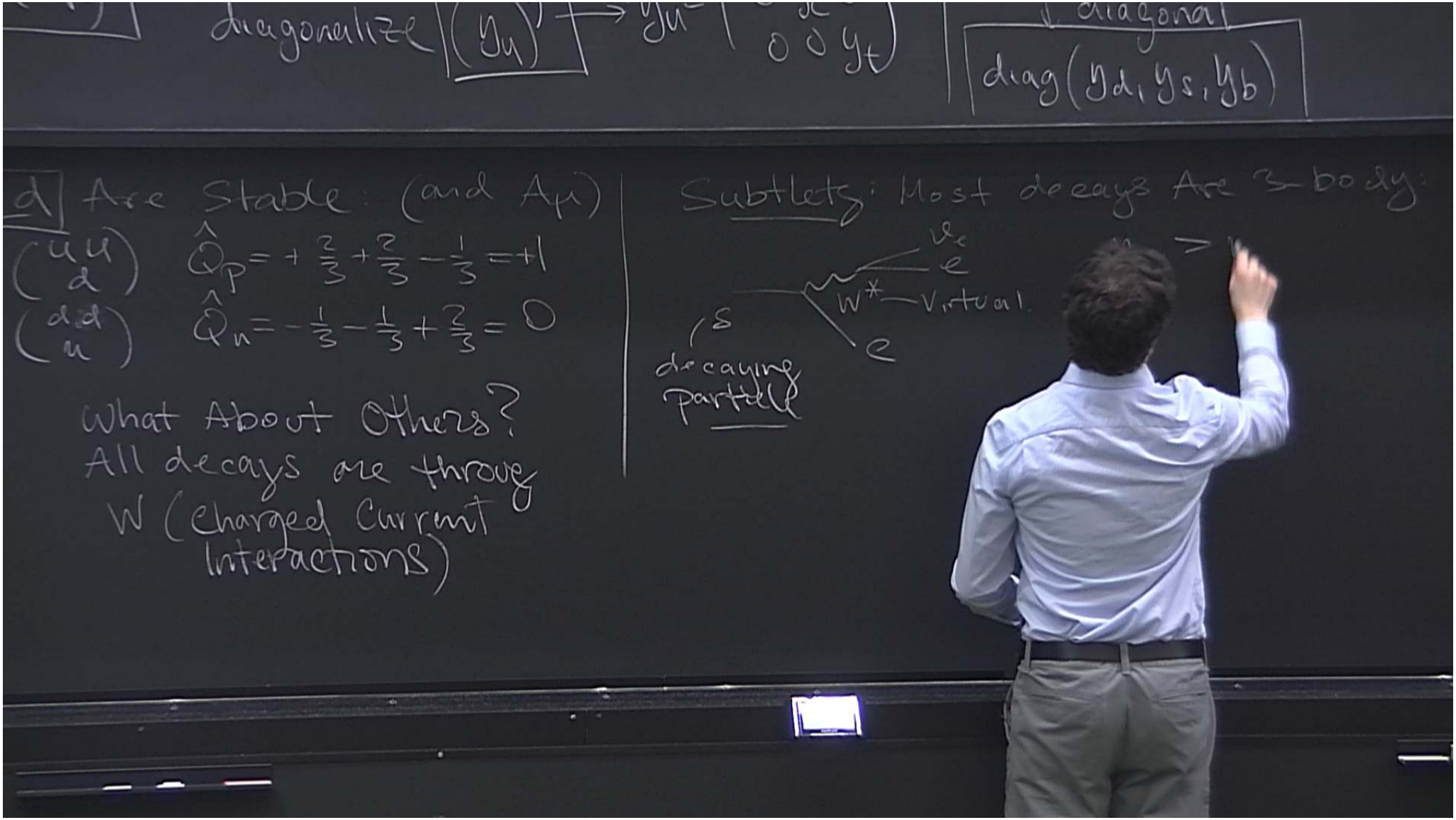
Feynman Rules for W:

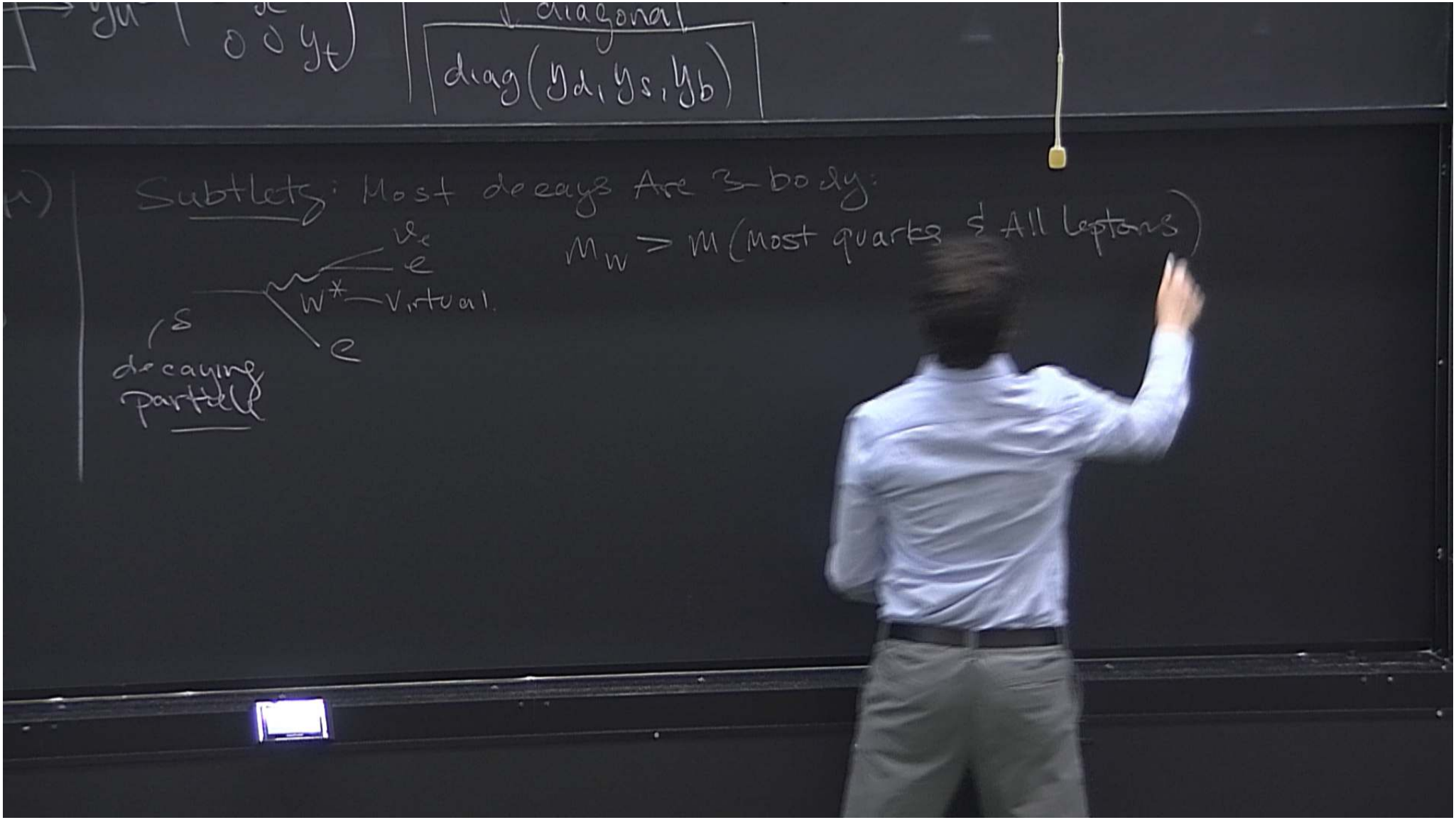


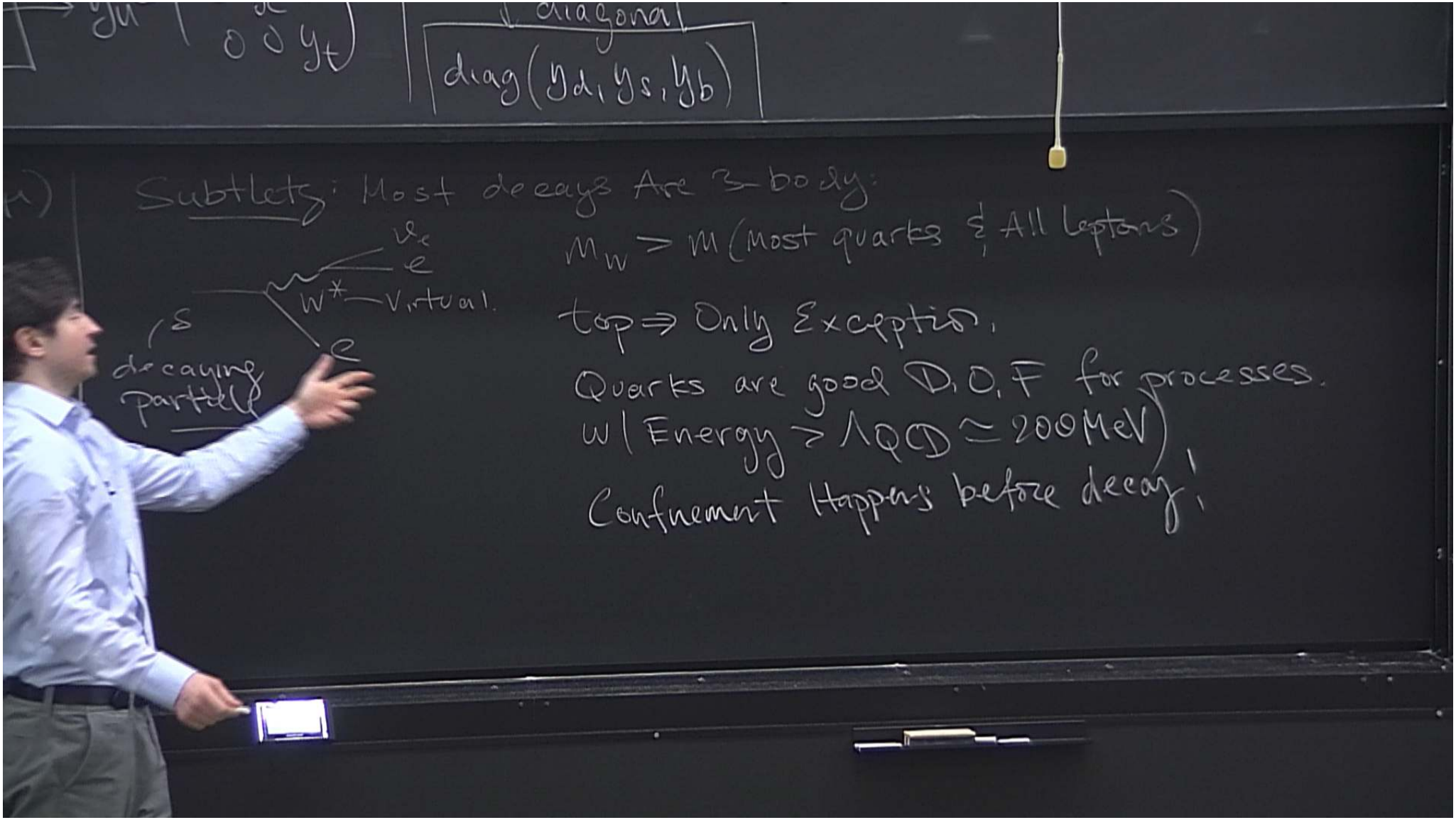
(Recall $\psi_{u_i} \equiv \begin{pmatrix} u_L \\ (u_R^c)^\dagger \end{pmatrix}_i$, $\psi_{d_j} = \begin{pmatrix} d_L \\ (d_R^c)^\dagger \end{pmatrix}_j$


diagonalize
Only | e, u, d | Are Stable (and Aμ)
Protons → (u u d)
Neutron → (d d u)
(not Stable Outside of Nuclei)
$\hat{Q}_p = +\frac{2}{3}+\frac{2}{3}-\frac{1}{3} = +1$
$\hat{Q}_n = -\frac{1}{3}+\frac{2}{3} = 0$


diagonalize
Only | e, u, d | Are Stable (and Aμ)
Protons → (u u d)
Neutron → (d d u)
(not Stable Outside of Nuclei)
What About Others?
All decays are through W (Charged Current Interactions)
Subtlety: Most
W
e

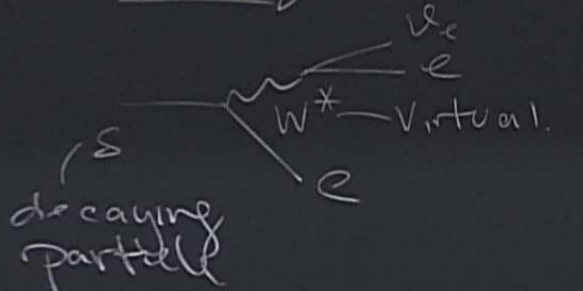

diagonalize
diagonal
diag(yd, ys, yb)
d
Are Stable (and Aμ)
(u u d) $\hat{Q}_p = +\frac{2}{3}+\frac{2}{3}-\frac{1}{3} = +1$
(d d u) $\hat{Q}_n = -\frac{1}{3}-\frac{1}{3}+\frac{2}{3} = 0$
What About Others?
All decays are through
W (Charged Current
Interactions)
Subtlety: Most decays Are 3-body:
W* — Virtual
e
e
s
decaying
particle


diagonal
$\mathrm{diag}(y_d, y_s, y_b)$
Subtlety: Most decays Are 3-body:
$m_W > m$ (Most quarks & All leptons)
ν_e
e
W* — Virtual
s
e
decaying particle


diagonal
diag(y_d, y_s, y_b)
Subtlety: Most decays Are 3-body:
ν_e
e
W* — Virtual.
s
decaying particle
e
$m_W > m$ (Most quarks & All Leptons)
top ⇒ Only Exception.
Quarks are good D.O.F for processes.
w/ Energy > $\Lambda_{QCD} \simeq 200$MeV)
Confinement Happens before decay!

$\begin{pmatrix} & & \\ & & \\ 0 & 0 & y_t \end{pmatrix}$ | diagonal | $\mathrm{diag}(y_d, y_s, y_b)$

Subtlety: Most decays Are 3-body:



$m_W > m$ (Most quarks & All leptons)

top ⇒ Only Exception.

Quarks are good D.O.F for processes. w/ Energy $> \Lambda_{QCD} \simeq 200\,\mathrm{MeV}$)

Confinement Happens before decay!

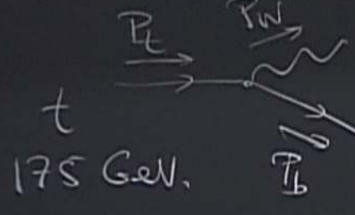
Nobel 2009. Natori generalize

Top decay: Bonus: V_{CKM} (~~off~~ diag. top interactions is tiny!!)

Amplitude:

Nobel 2009. Natax generalize

Top decay: Bonus: V_{CKM} (~~off~~ diag. top interactions is tiny!)

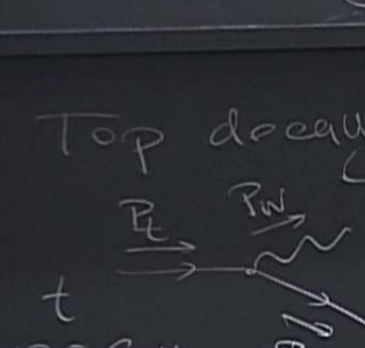


Amplitude: $A(t \to Wb) = \frac{ig}{\sqrt{2}} \overline{u}(p_b)$

$P_W^\mu \equiv q^\mu$

Nobel 2009. Natalx generalize

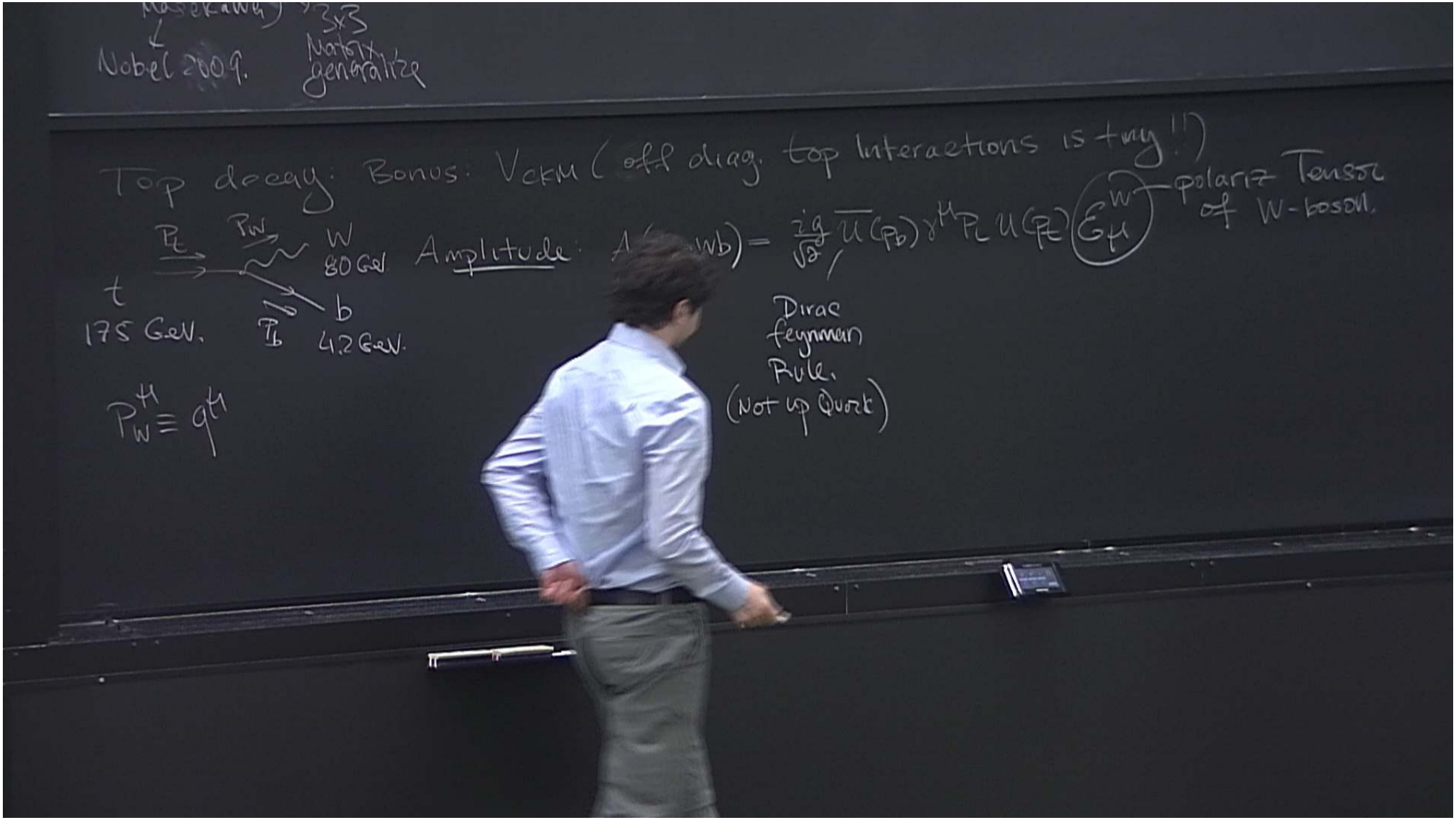
Top decay: Bonus: V_{CKM} (~~off~~ diag. top interactions is tiny!)

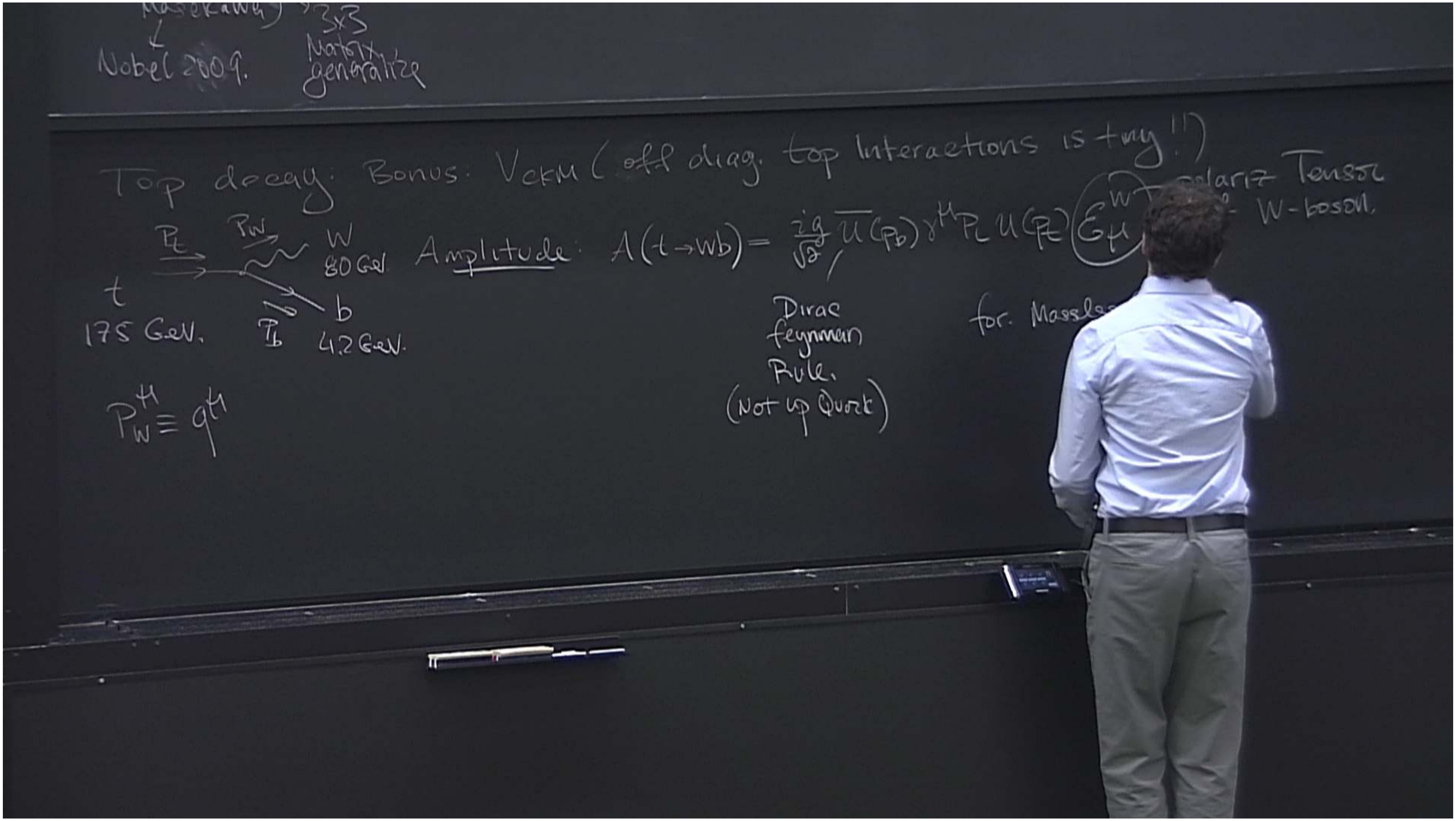


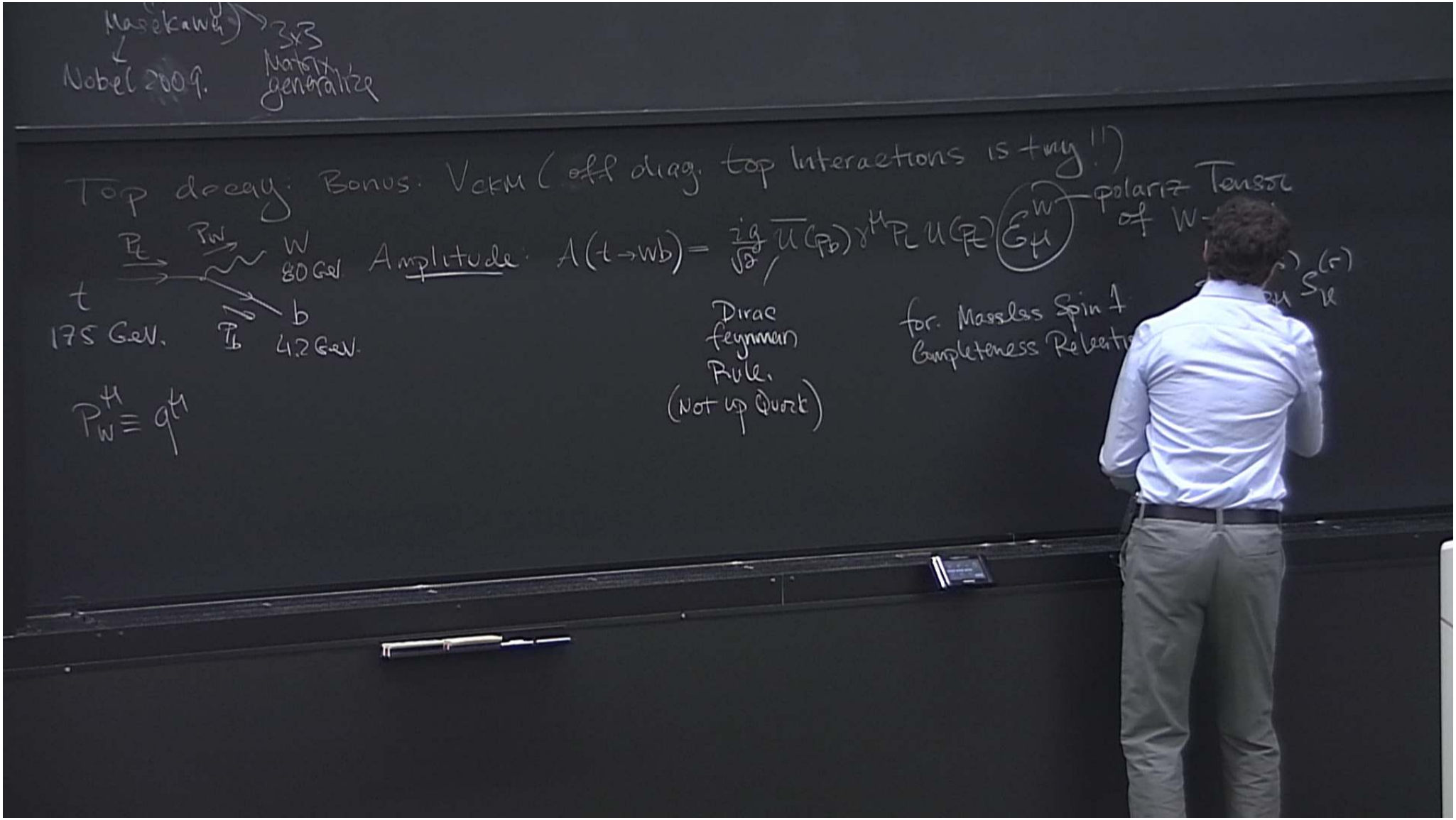
Amplitude: $A(t \to Wb) = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu P_L u(p_t$

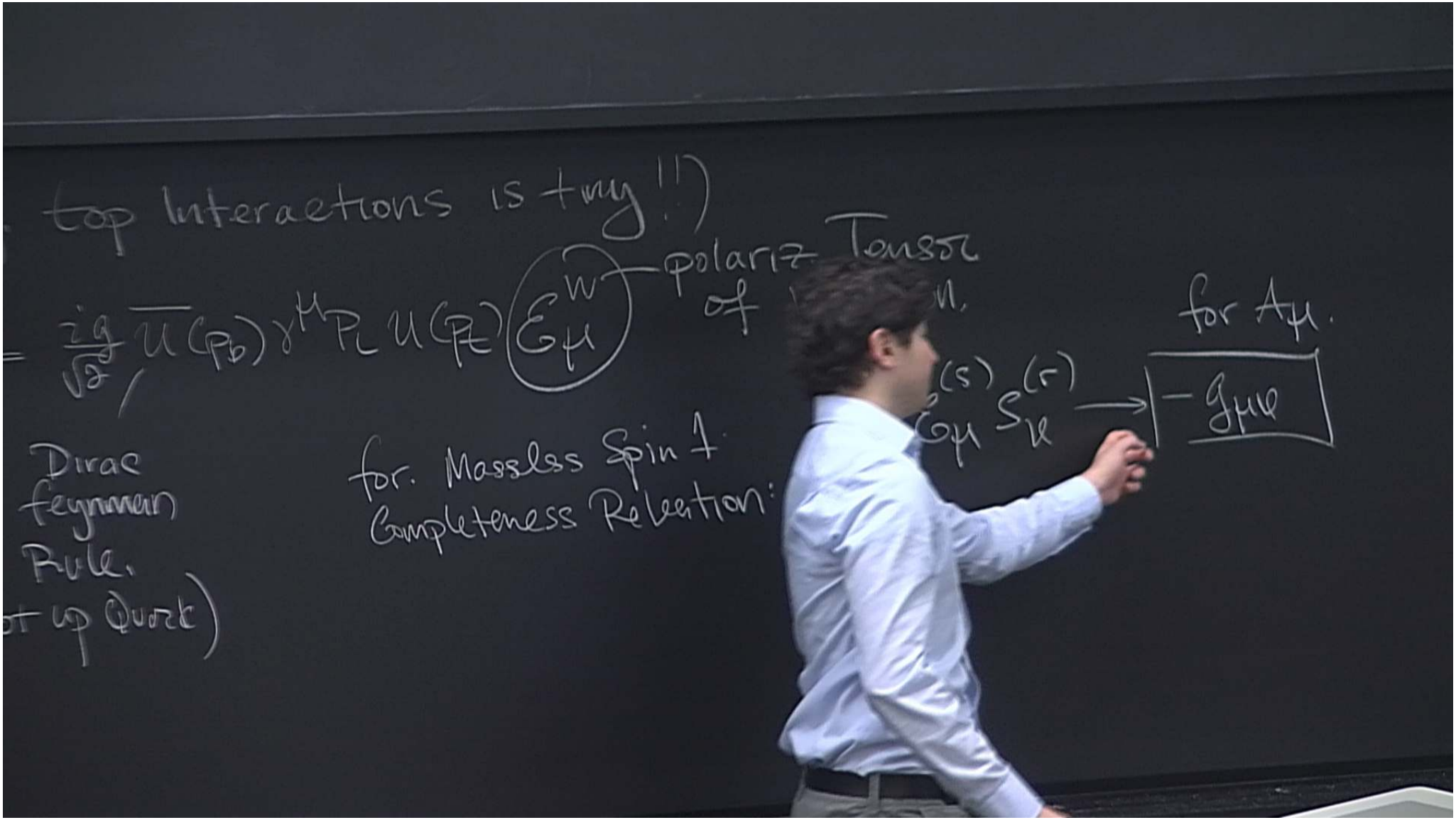
Dirac feynman Rule. (Not up Quark)

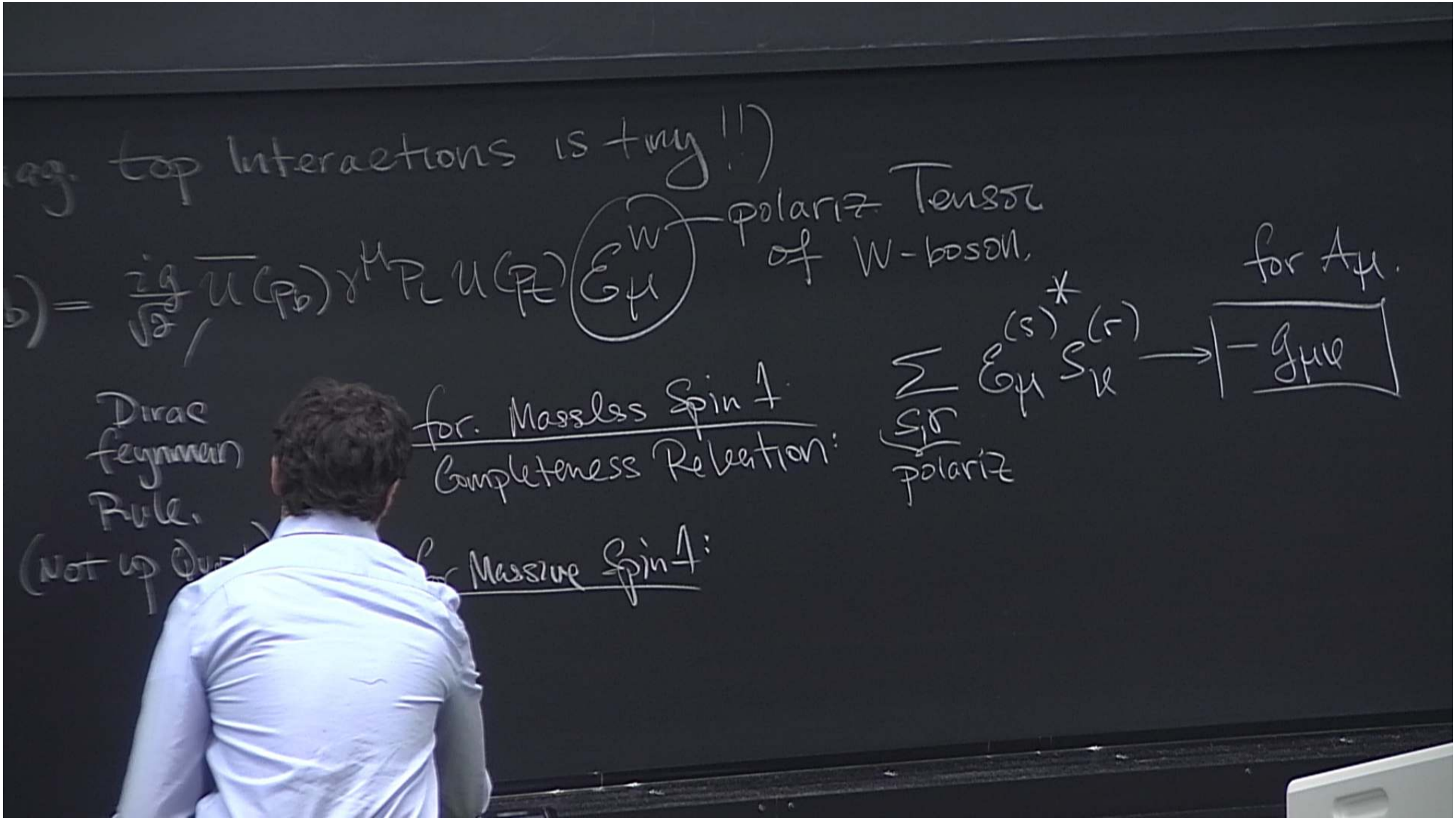
$P_W^\mu \equiv q^\mu$

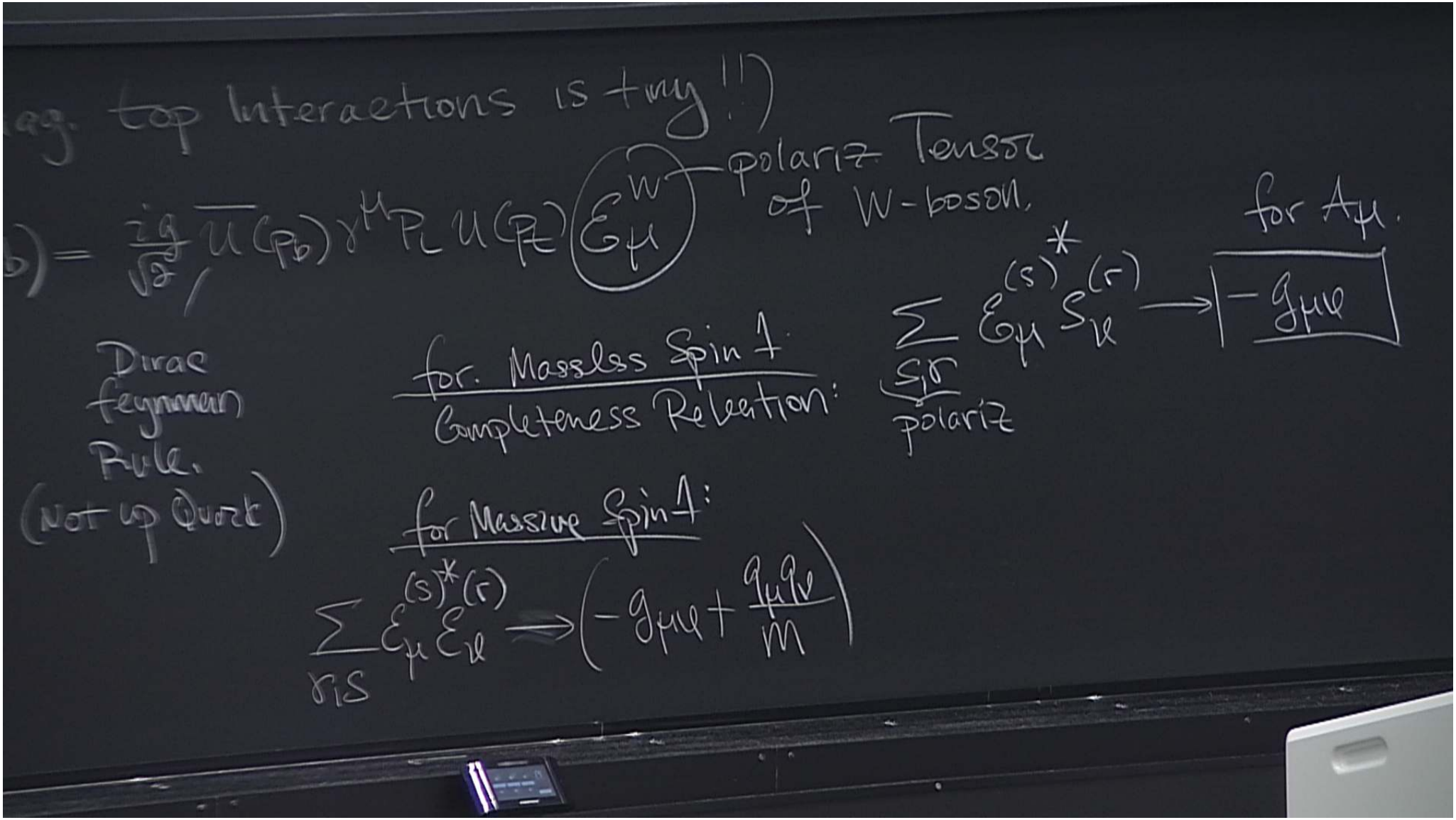

Nobel 2009.
3x3
Matrix
generalize
Top decay: Bonus: V_{CKM} (off diag. top interactions is tiny!)
P_L
P_W
W
80 GeV
t
175 GeV.
b
P_b
4.2 GeV.
Amplitude:
Polariz Tensor of W-boson.
Dirac
feynman
Rule.
(Not up Quark)
$P_W^\mu \equiv q^\mu$

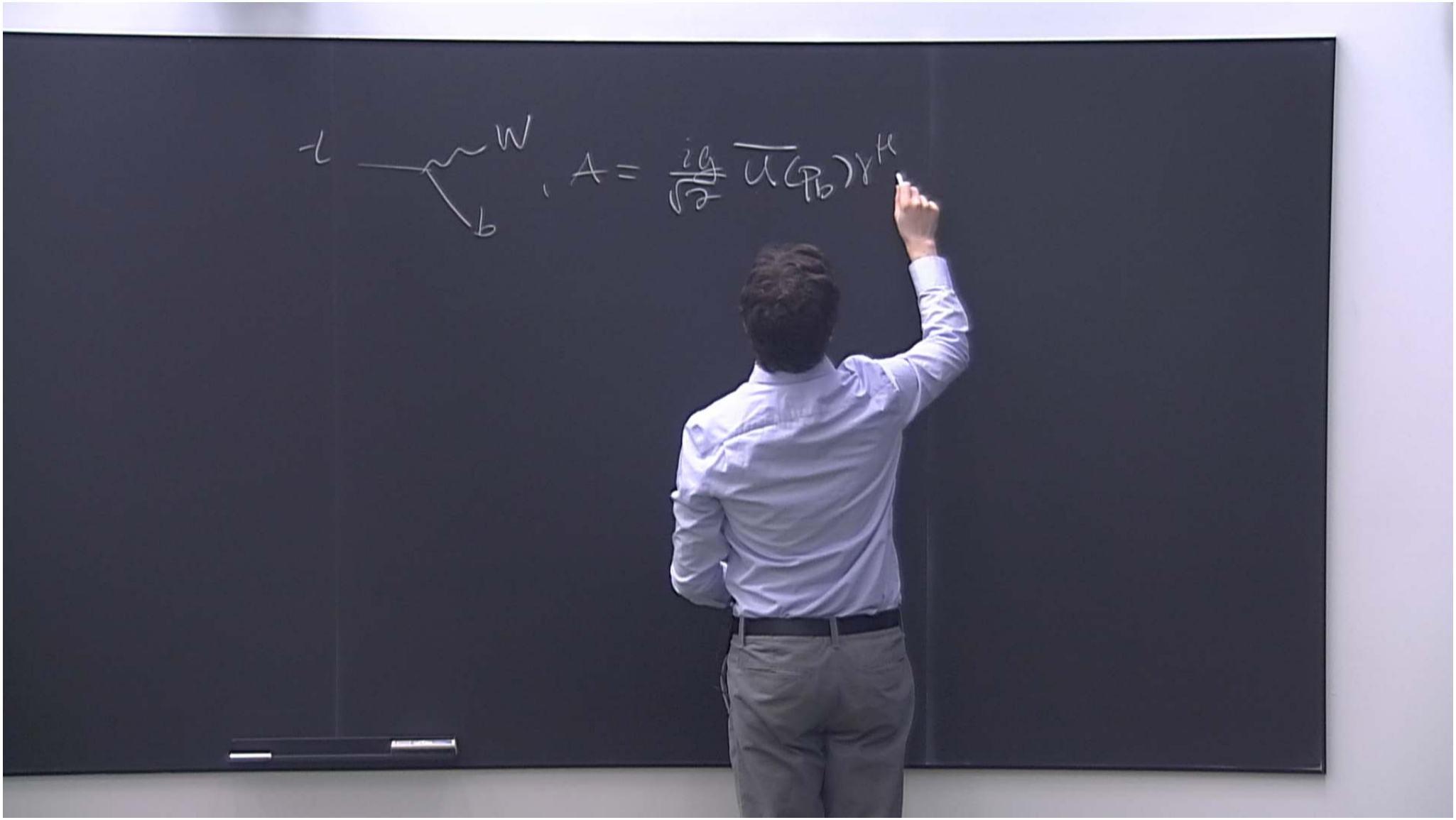

Nobel 2009.
3x3
Matrix
generalize
Top decay: Bonus: VCKM (off diag. top Interactions is tiny !!)
Polariz Tensor
W-boson.
W
80 GeV
Amplitude: A(t→Wb) = ig/√2 ū(pb) γ^μ PL u(pt) εμ
t
175 GeV.
b
4.2 GeV.
Dirac
feynman
Rule.
(Not up Quark)
for. Massless
P^μ_W ≡ q^μ

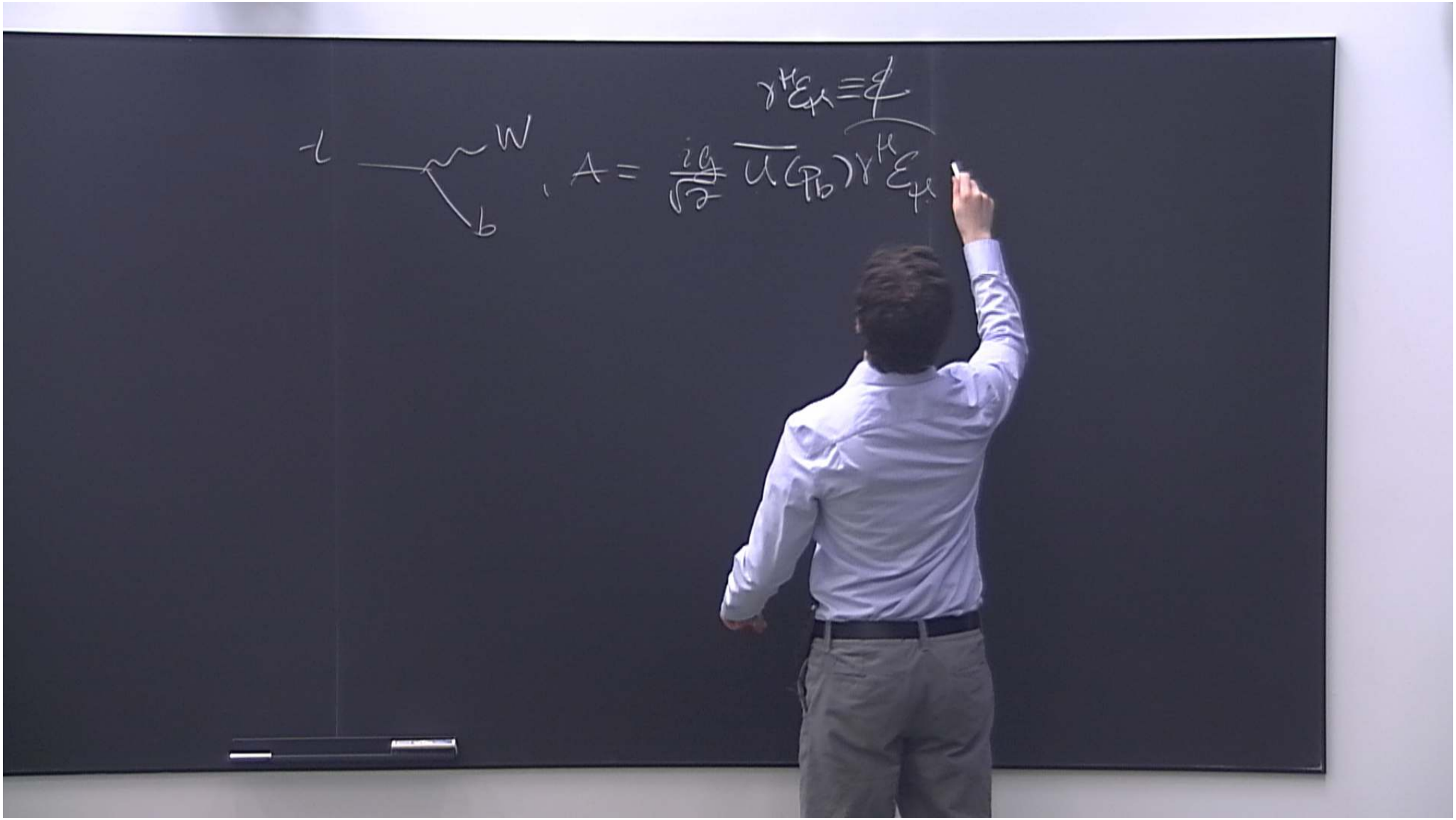

Nobel 2009.
3x3 Matrix generalize
Top decay: Bonus: V_{CKM} (off diag. top interactions is tiny !!)
P_t P_W W 80 GeV
t 175 GeV.
P_b b 4.2 GeV.
Amplitude: $A(t \to Wb) = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu P_L u(p_t) \epsilon^W_\mu$
Dirac feynman Rule. (Not up Quark)
polariz Tensor of W
for. Massless Spin 1
Completeness Relation
$P_W^\mu \equiv q^\mu$

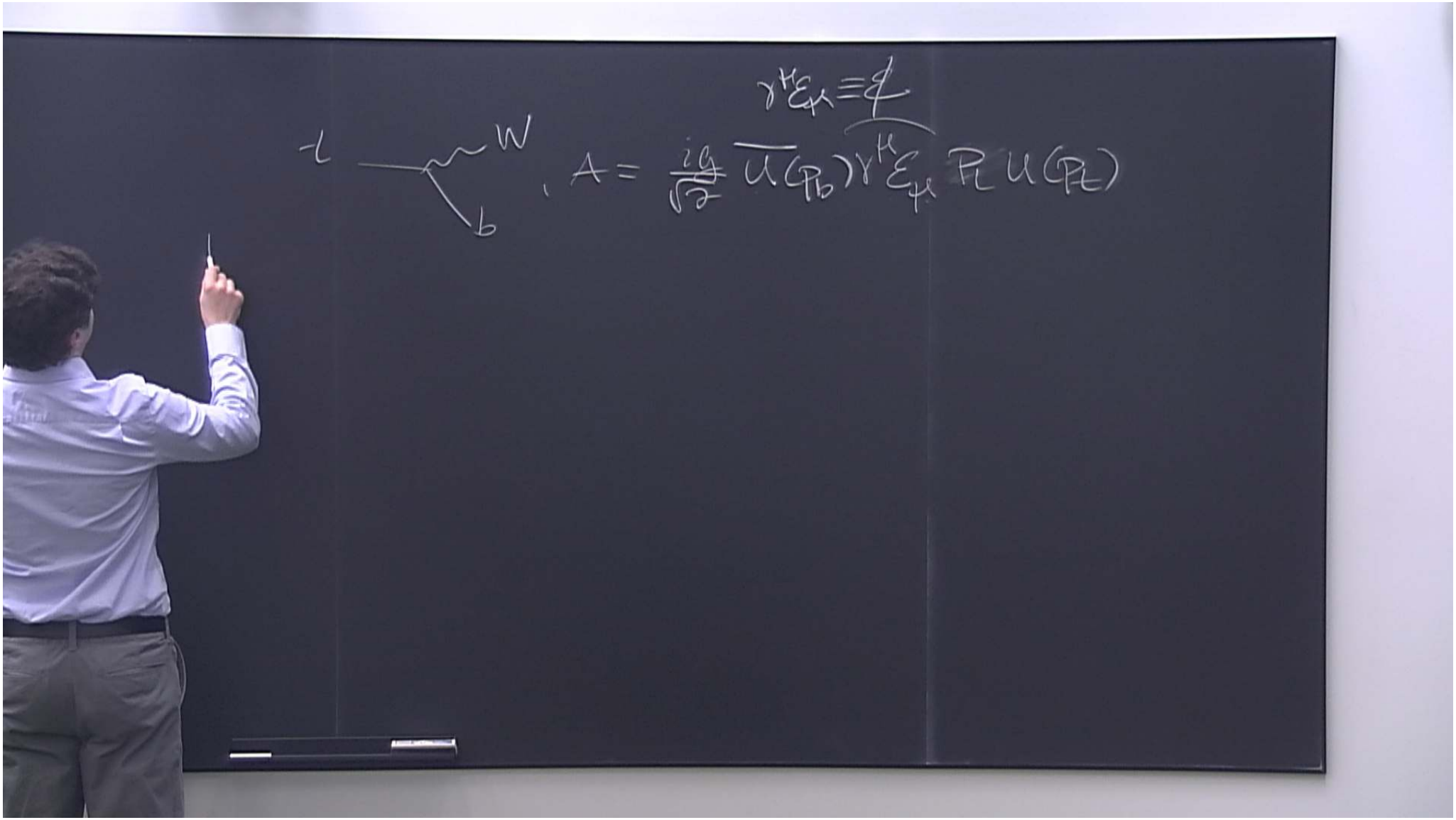

top Interactions is tiny!!)
Polariz Tensor
for A_μ.
$-g_{\mu\nu}$
for. Massless Spin 1
Completeness Relation:
Dirac
feynman
Rule.

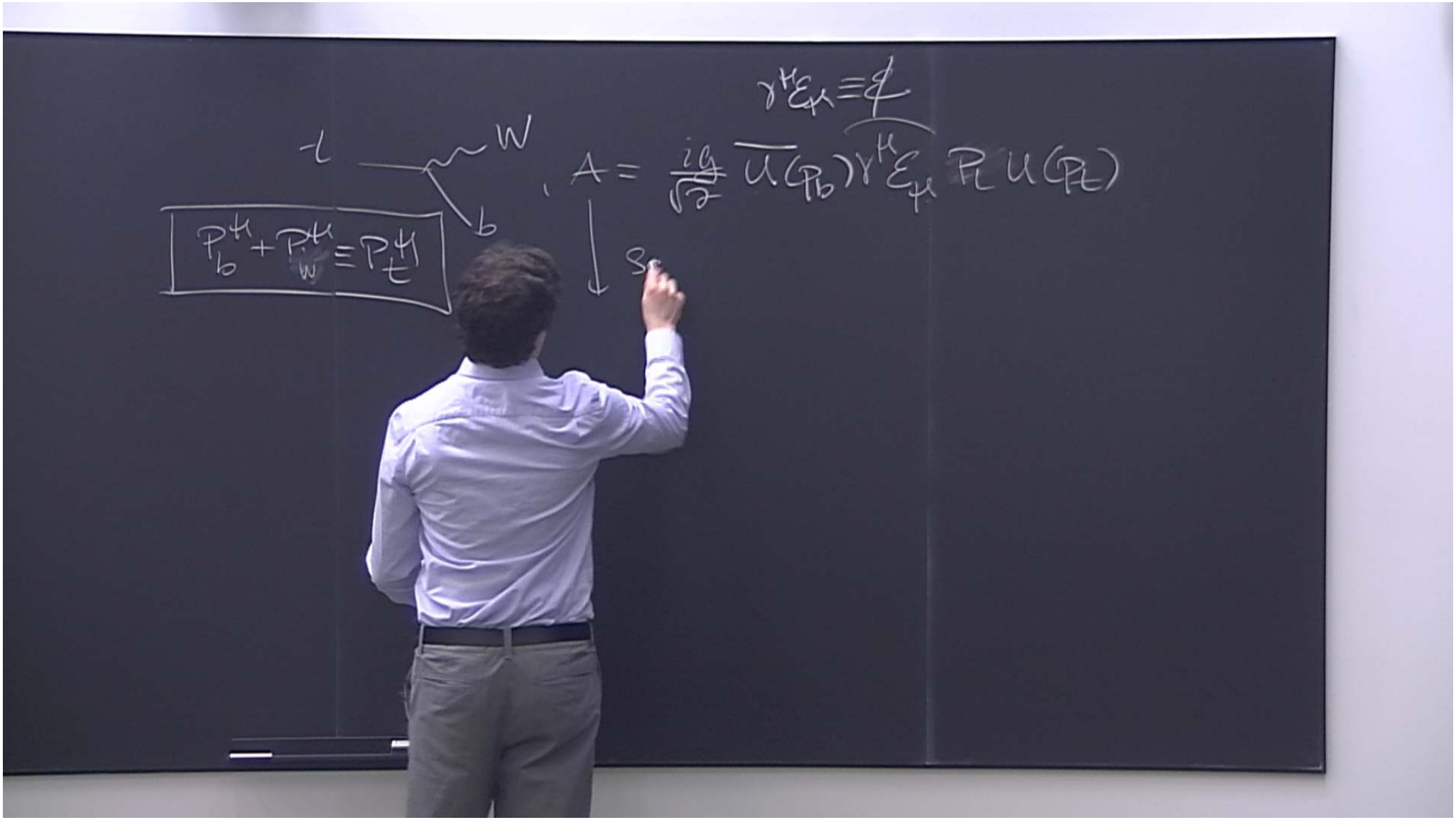

top Interactions is tiny!!)
polariz Tensor of W-boson.
for A_μ.
$-g_{\mu\nu}$
Dirac feynman Rule.
for. Massless Spin 1
Completeness Relation:
polariz
Massive Spin 1:

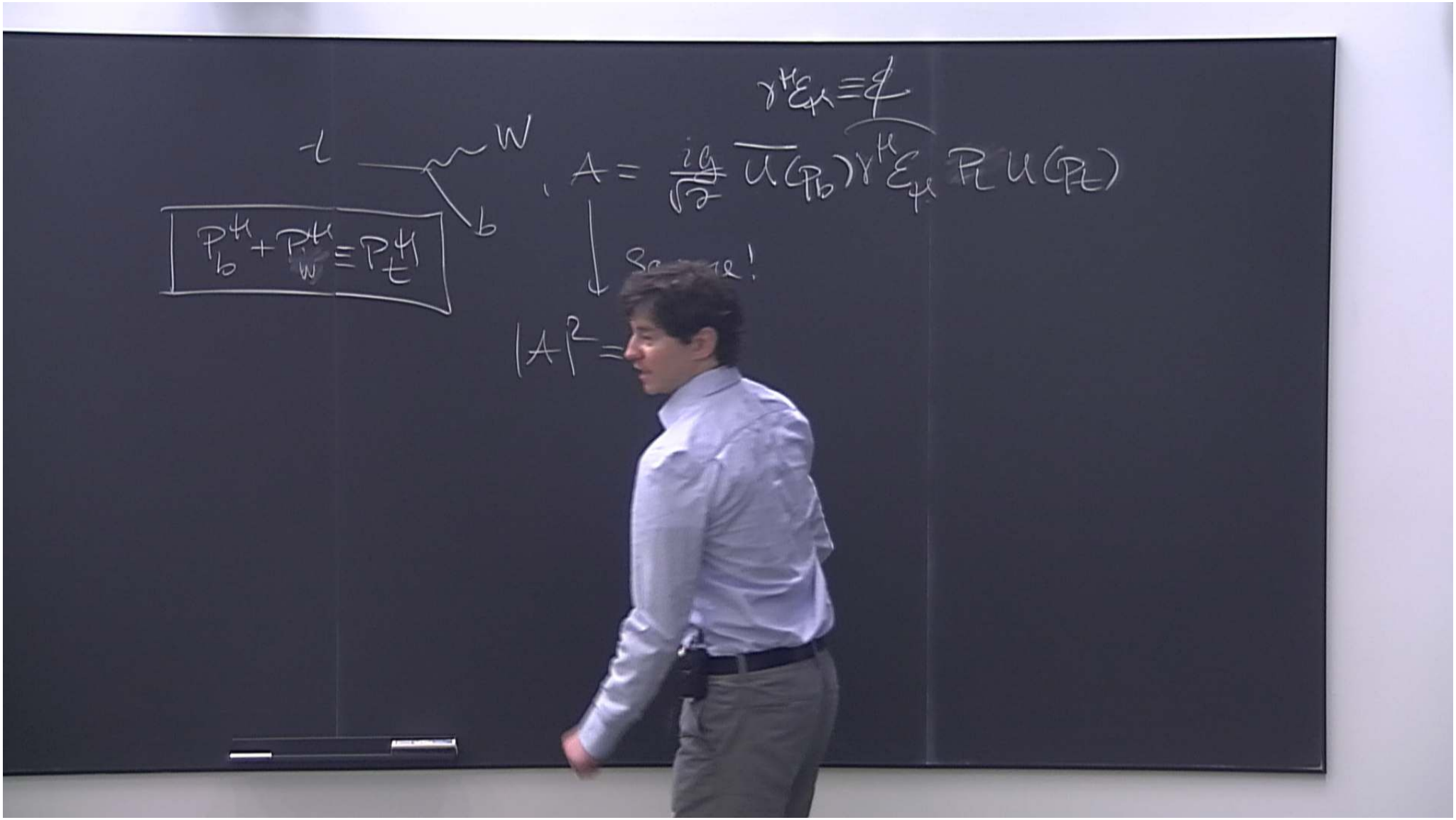

top Interactions is tiny!!)
polariz Tensor of W-boson.
Dirac feynman Rule. (Not up Quark)
for Massless Spin 1
Completeness Relation:
for A_μ.
for Massive Spin 1:

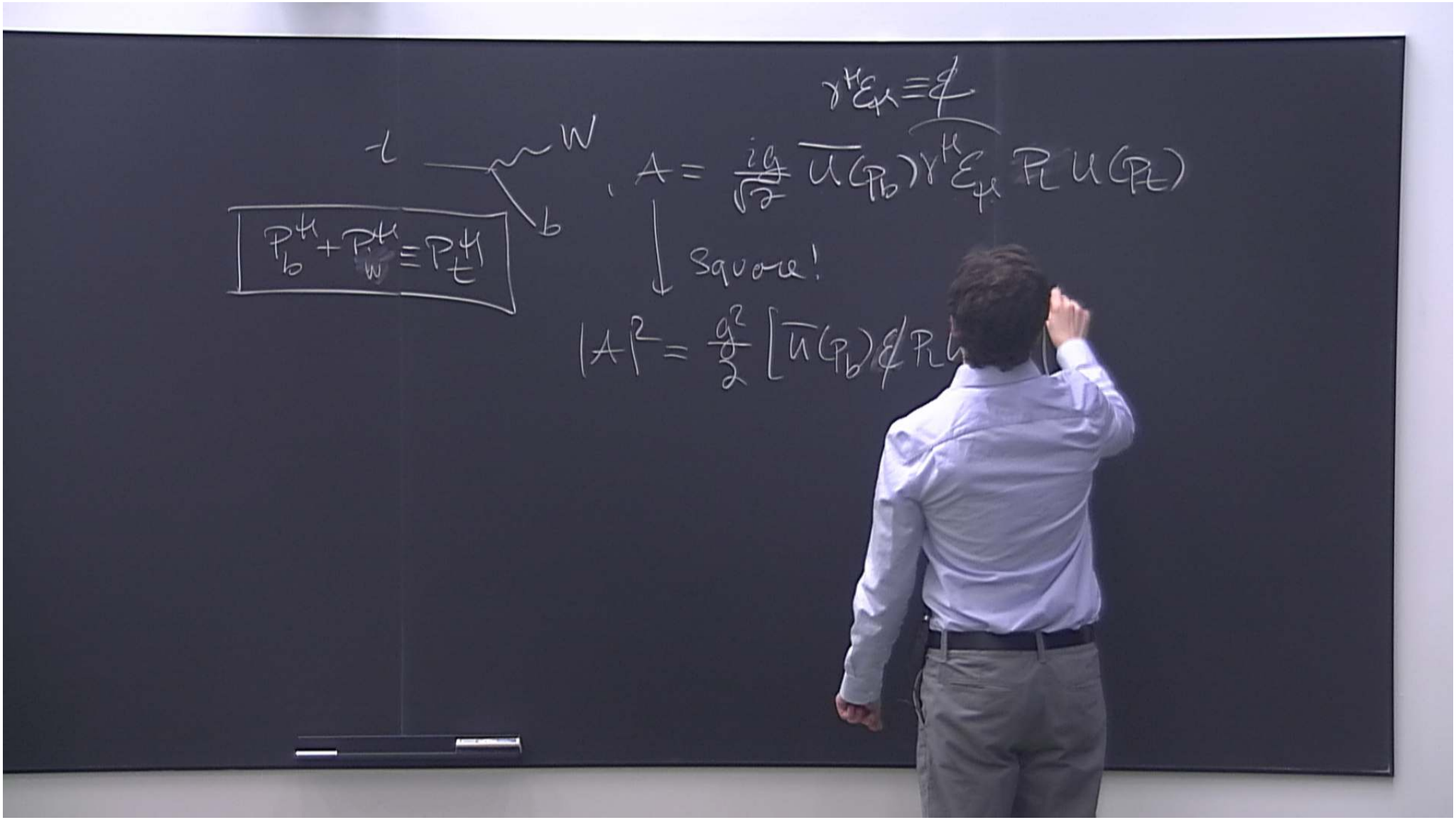

-l
W
b
, A = $\frac{ig}{\sqrt{2}}$ $\overline{u}(p_b)\gamma^\mu$

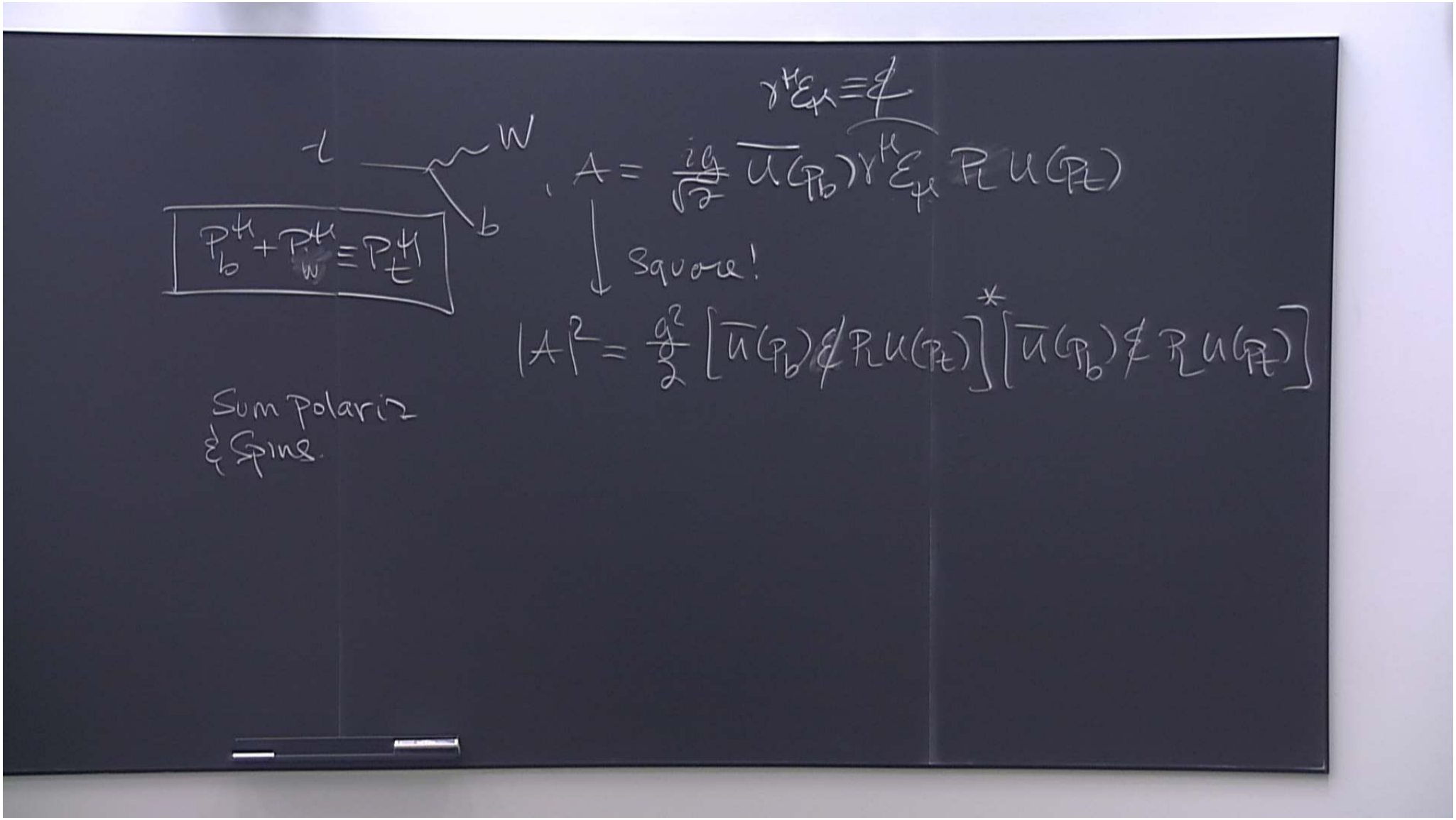

$\gamma^\mu \varepsilon_{\mu\lambda} = \not{\varepsilon}$
$-i$
W
b
$A = \frac{ig}{\sqrt{2}} \overline{u}(p_b) \gamma^\mu \varepsilon_{\mu}$

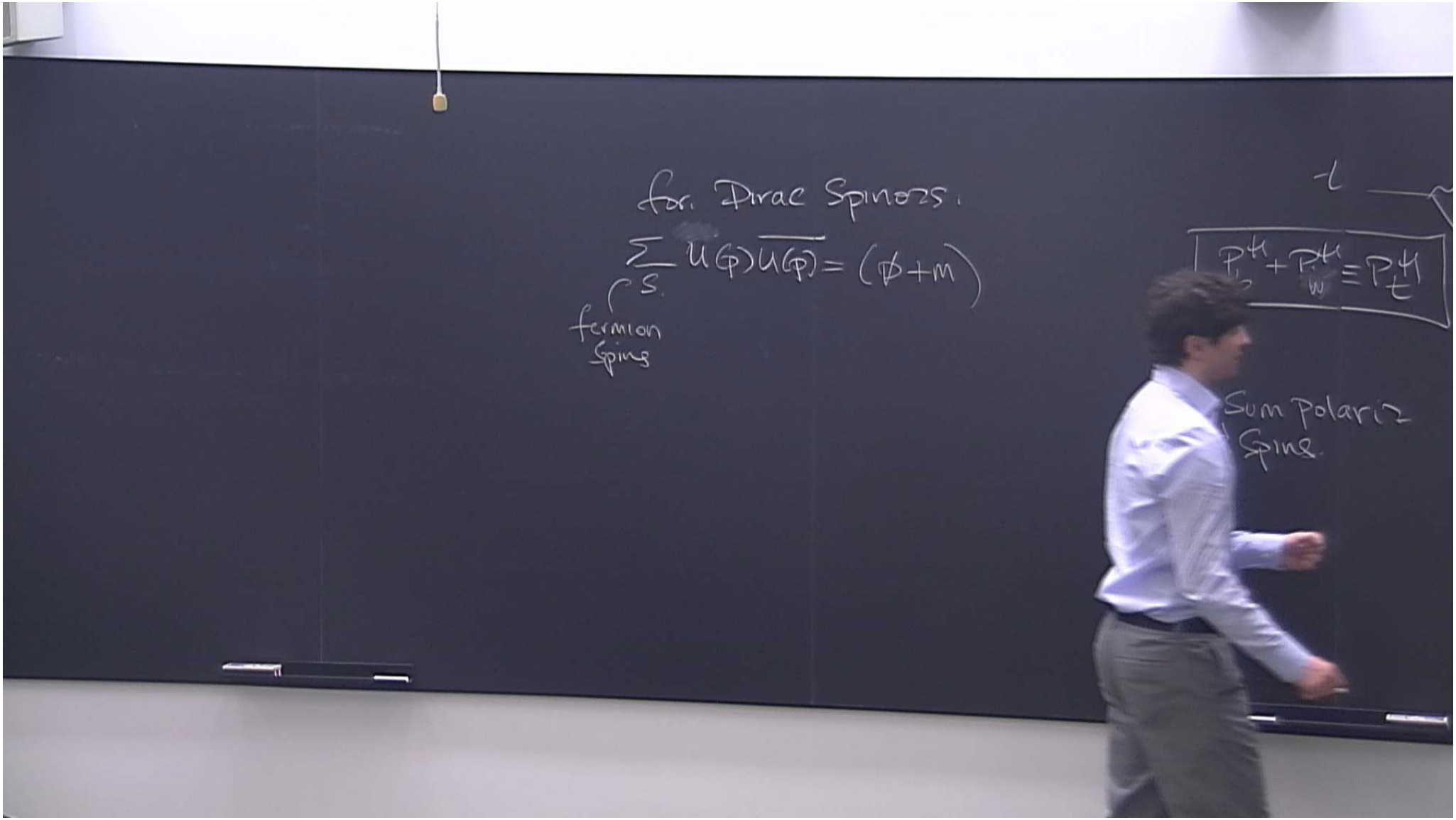

$\gamma^\mu \varepsilon_{\mu\lambda} = \not{\varepsilon}$
$-i$
W
b
$A = \frac{ig}{\sqrt{2}} \overline{u(p_b)} \gamma^\mu \varepsilon_{\mu\lambda} P_L u(p_L)$

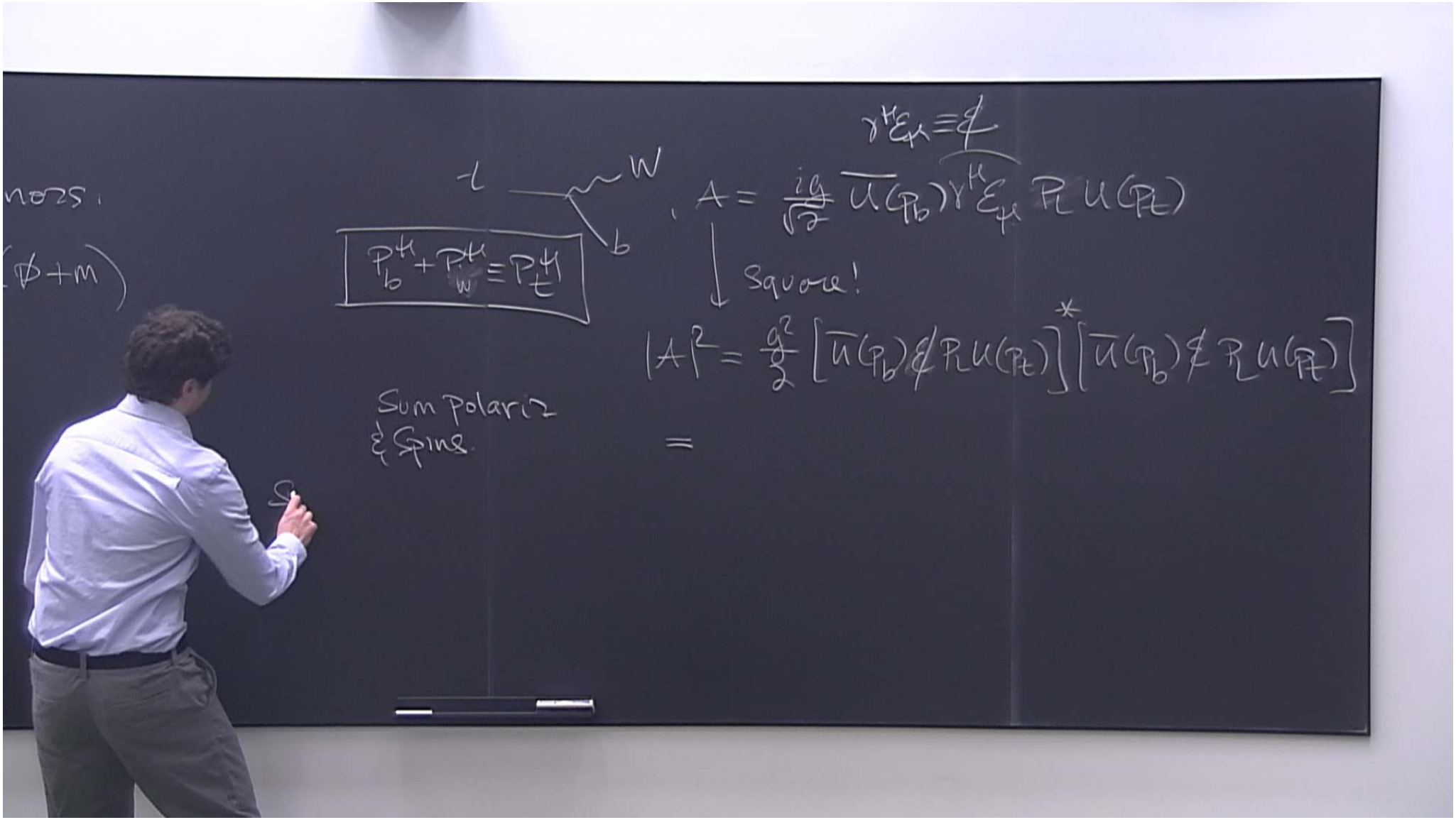

$\gamma^\mu \varepsilon_\mu \equiv \not{\varepsilon}$
t
W
b
$A = \frac{ig}{\sqrt{2}} \overline{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
$\boxed{p_b^\mu + p_W^\mu \equiv p_t^\mu}$


$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
$-i \mathcal{M} = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
$p_b^\mu + p_W^\mu = p_t^\mu$
$|\mathcal{M}|^2 =$

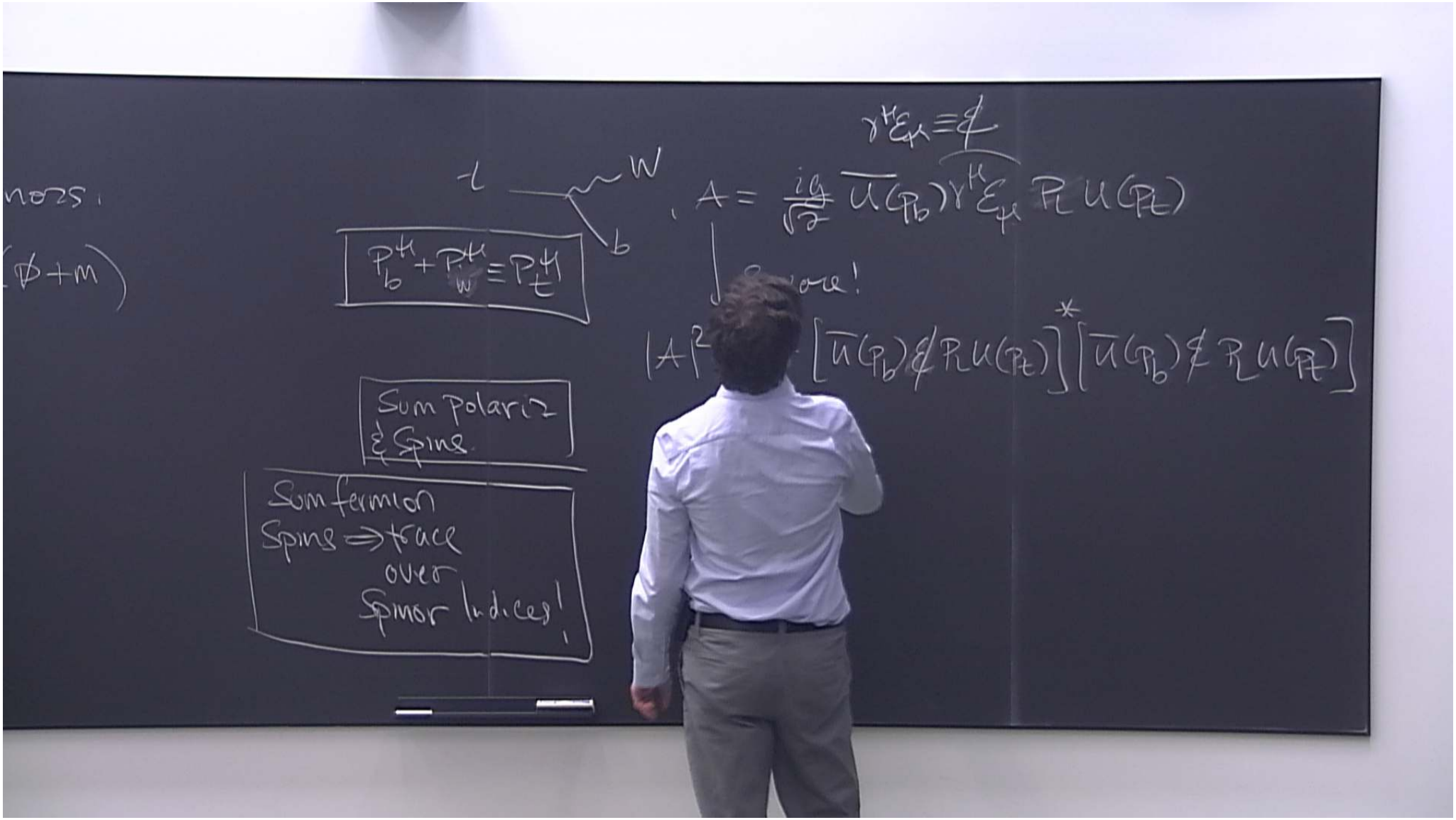

$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
t
W
b
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
$p_b^\mu + p_W^\mu = p_t^\mu$
Square!
$|A|^2 = \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L$

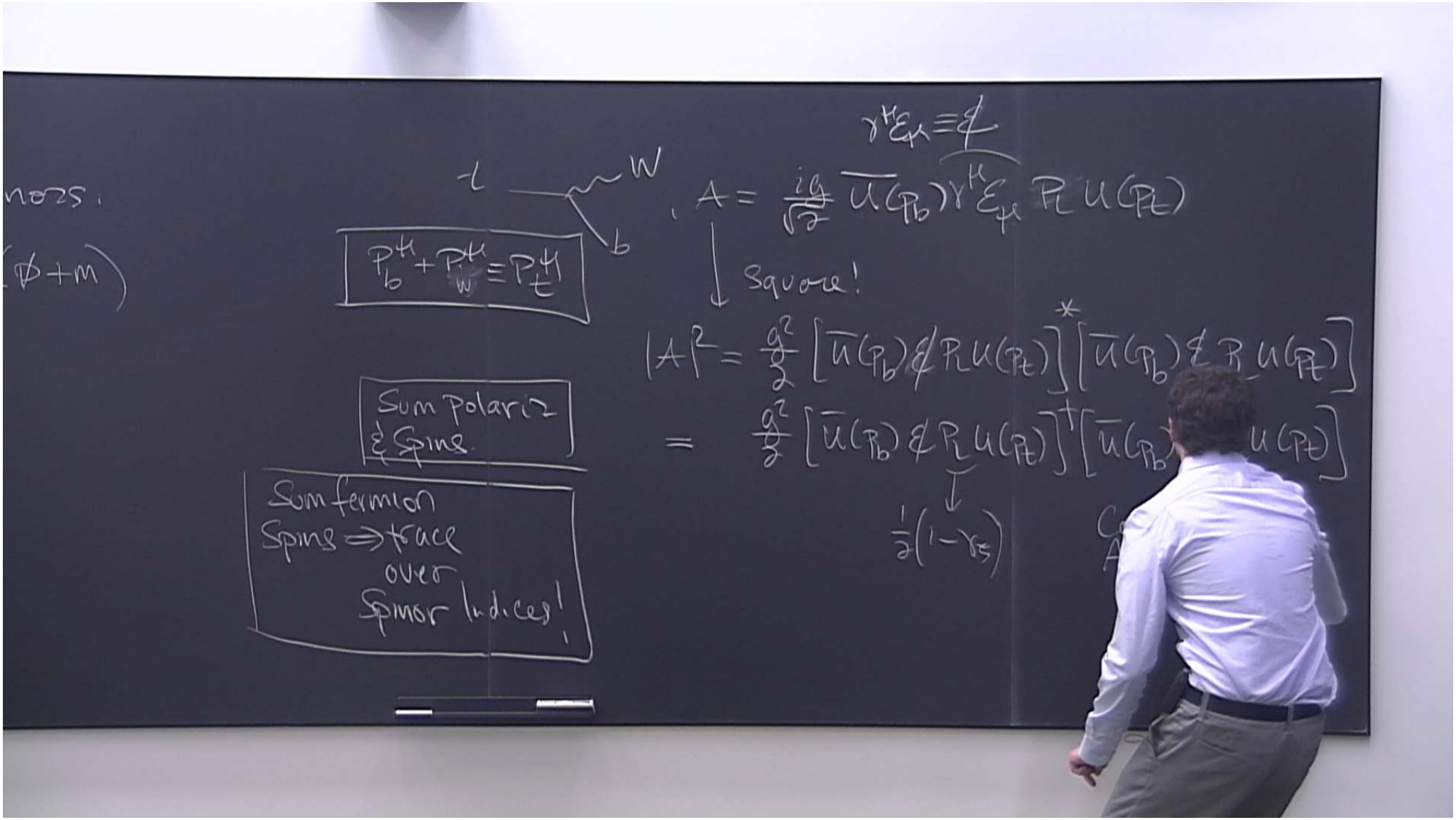

$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
t
W
b
$P_b^\mu + P_W^\mu = P_t^\mu$
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
Square!
$|A|^2 = \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^* [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]$
Sum Polariz & Spins.

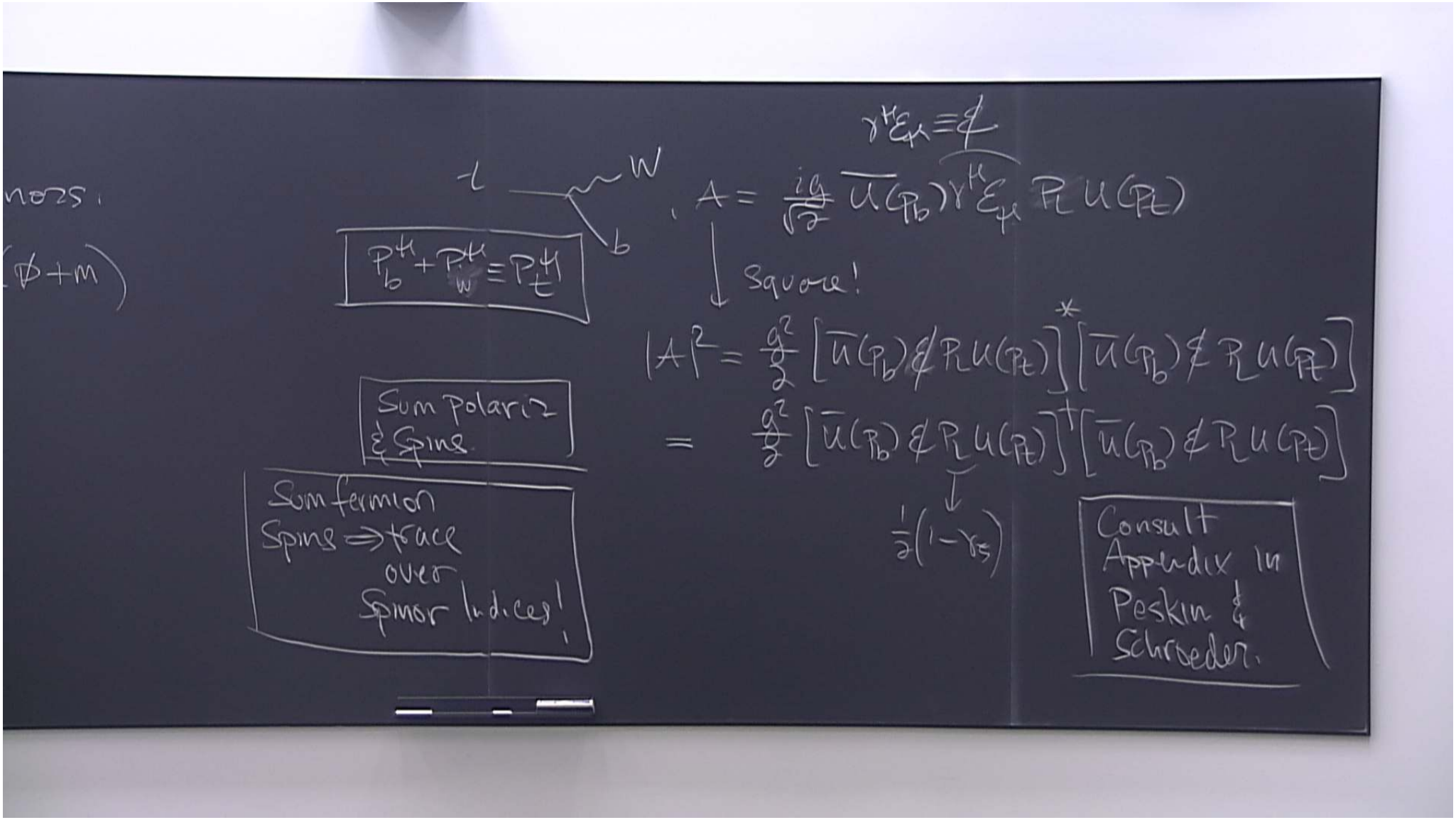

for. Dirac Spinors.
$\sum_s u(p)\overline{u(p)} = (\not{p}+m)$
fermion Spins
$P_i^\mu + P_w^\mu \equiv P_t^\mu$
Sum polariz
Spins.



$\gamma^\mu \varepsilon_\mu \equiv \not{\varepsilon}$
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
Square!
$|A|^2 = \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^* [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]$
$=$
$p_b^\mu + p_W^\mu \equiv p_t^\mu$
Sum polariz
& Spins.


$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
$p_b^\mu + p_W^\mu = p_t^\mu$
Sum Polariz & Spins
Sum fermion Spins ⇒ trace over Spinor Indices!


$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
$-i$ W b
$P_b^\mu + P_W^\mu = P_t^\mu$
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
Square!
$|A|^2 = \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^* [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]$
$= \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^\dagger [\bar{u}(p_b)$ $u(p_t)]$
$\frac{1}{2}(1-\gamma_5)$
Sum Polariz & Spins
Sum fermion Spins ⇒ trace over Spinor Indices!


$\gamma^\mu \varepsilon_\mu = \not{\varepsilon}$
t
W
b
$A = \frac{ig}{\sqrt{2}} \bar{u}(p_b) \gamma^\mu \varepsilon_\mu P_L u(p_t)$
$p_b^\mu + p_W^\mu = p_t^\mu$
Square!
$|A|^2 = \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^* [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]$
$= \frac{g^2}{2} [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]^\dagger [\bar{u}(p_b) \not{\varepsilon} P_L u(p_t)]$
$\frac{1}{2}(1-\gamma_5)$
Sum Polariz & Spins
Sum fermion Spins ⇒ trace over Spinor Indices!
Consult Appendix in Peskin & Schroeder.