Title: Effective potentials and morphological transitions for binary black-hole spin precession

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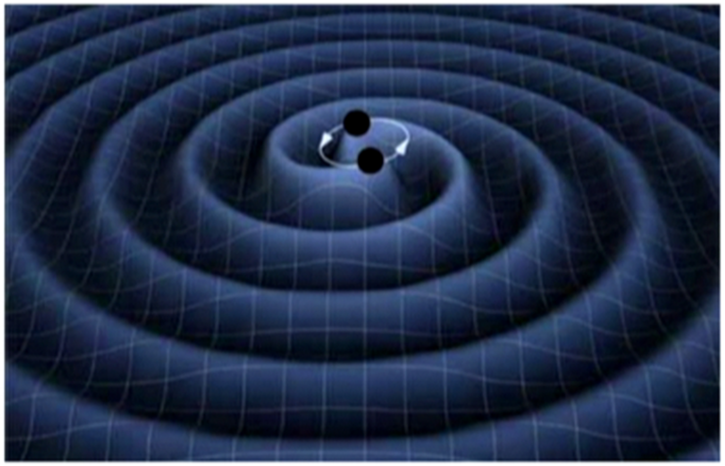
Abstract: <p>Generic binary black holes have spins that are misaligned with their orbital angular momentum. When the binary separation between the black holes is large compared to their gravitational radii, the timescale on which the spins precess is much shorter than the radiation-reaction time on which the orbital angular momentum decreases due to gravitational-wave emission. We use conservation of the total angular momentum and the projected effective spin on the precession time to derive an effective potential for BBH spin precession. This effective potential allows us to solve the orbit-averaged spin-precession equations analytically for arbitrary mass ratios and spins. These solutions are quasiperiodic functions of time: after a precessional period the spins return to their initial relative orientations. We classify black-hole spin precession into three distinct morphologies between which the black holes can transition during their inspiral. Our new solutions constitute fundamental progress in our understanding of black-hole spin precession and also have important applications to astrophysical black holes. We derive a precession-averaged evolution equation that can be numerically integrated on the radiation-reaction time, allowing us to statistically track black-hole spins from formation to merger. This will greatly help us predict the signatures of black-hole formation in the gravitational waves emitted near merger and the distributions of final spins and gravitational recoils.</p>

Effective potentials and morphological transitions for binary black-hole spin



Dr. Michael Kesden (UT Dallas)

Perimeter Institute

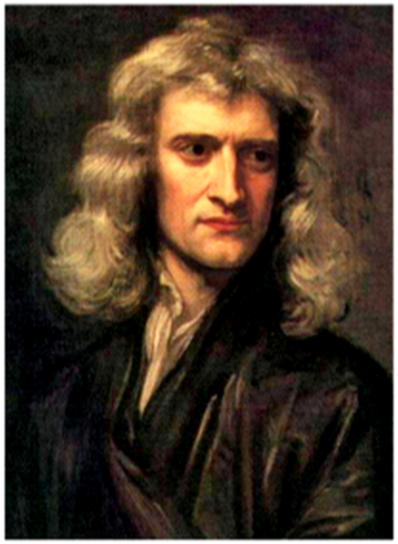
Waterloo, ON, Canada – Jan. 8, 2015

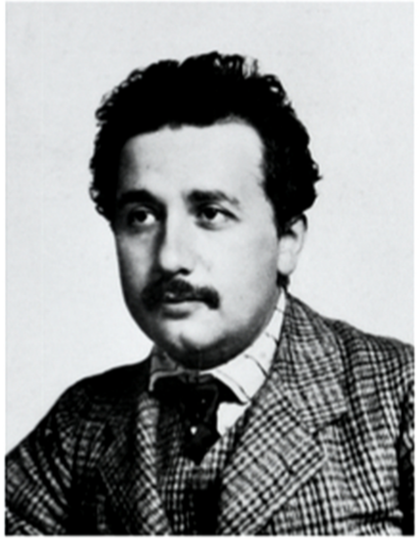
See our paper!!!

- Online at arXiv:1411.0674 [gr-qc]
- Collaborators
  - Davide Gerosa (Cambridge)
  - Richard O'Shaughnessy (RIT)
  - Emanuele Berti (Mississippi)
  - Uli Sperhake (Cambridge)



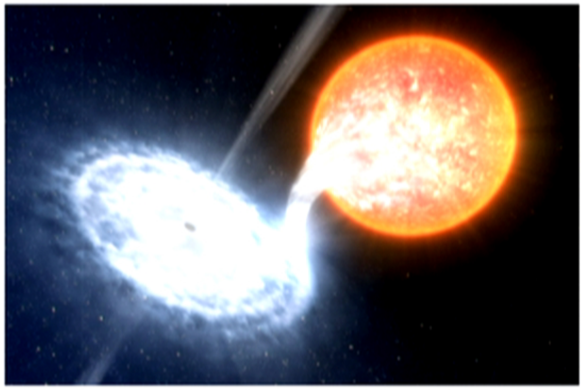

Theoretical motivation

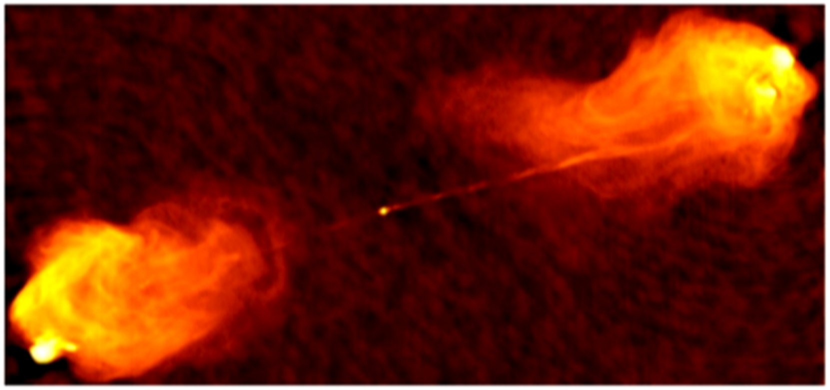




- Binary black hole (BBH) evolution is the relativistic generalization of the classic two-body problem.
- Some pretty smart guys recognized this as the benchmark for understanding gravity.

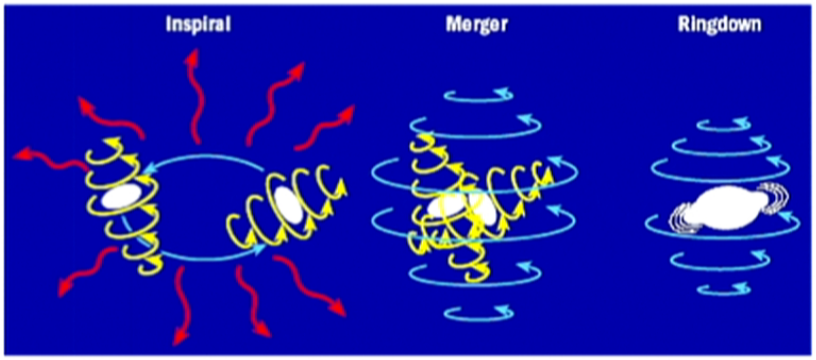
Astrophysical motivation



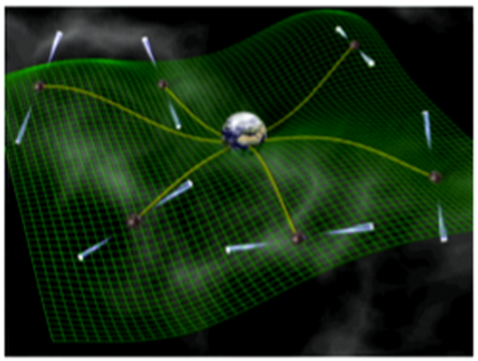


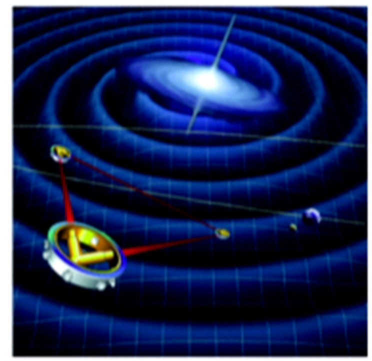
- Stellar-mass BHs
  - Massive stars collapse, accrete from companions.
  - Most massive binary stars evolve to BBHs.
- Supermassive BHs
  - Quasars/AGN seen in galactic centers.
  - Galaxies merge → SBHs should merge as well.

Gravitational waves!!!



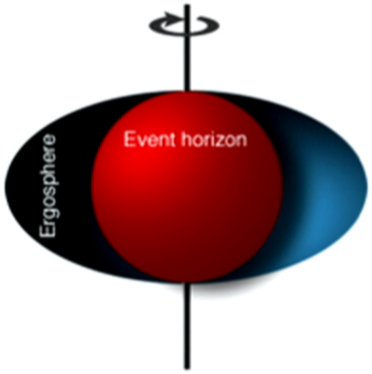
Hanford, Washington   Livingston, Louisiana





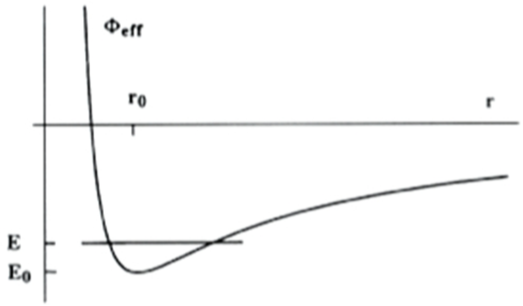
- BBHs emit GWs that extract energy and angular momentum.
- Stellar-mass BBHs are GW source for detectors like LIGO.
- Supermassive BBHs are GW source for PTAs, LISA.

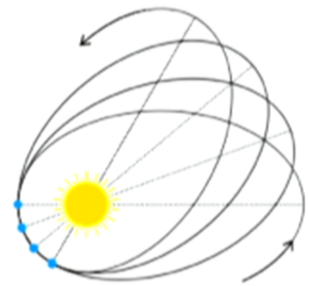
Black holes have spin



- BH spins are relics of formation and growth
  - Natal spins of stellar-mass BHs???
  - SBHs grow through mergers, coherent/chaotic accretion
- BH spins affect EM emission
  - Radiative efficiency of thin disks.
  - Spinning BHs launch jets through Blandford-Znajek
- BH spins affect GW emission
  - Misaligned spins modulate GW waveform

Spherical Newtonian potential - a simpler problem?





- Test particle in spherical potential: $d^2\mathbf{r}/dt^2 = -\nabla\Phi(r)$
- 3 coupled, 2nd order ODEs $\Rightarrow$ 6D phase space
- 4 conserved quantities E, $\mathbf{L}$ $\Rightarrow$ 2D hypersurface (r, φ)
- Effective potential: $E = \frac{1}{2}\mathbf{v}^2 + \Phi = \frac{1}{2}(dr/dt)^2 + \Phi_{eff}(r, L)$
- Radial period τ, pericenter precession for $\alpha \neq 2\pi$:

$$\tau(E,L) = 2\int_{r_-}^{r_+} \frac{dr}{dr/dt} \qquad \alpha(E,L) = 2\int_{r_-}^{r_+} \frac{\Omega_z\ dr}{dr/dt}$$

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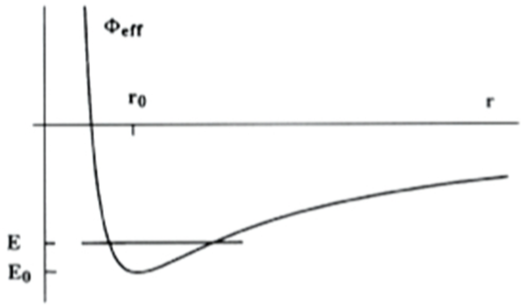

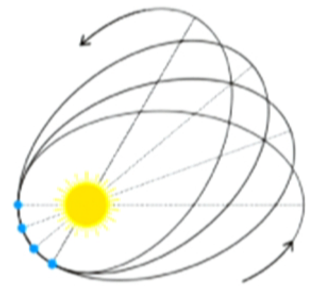

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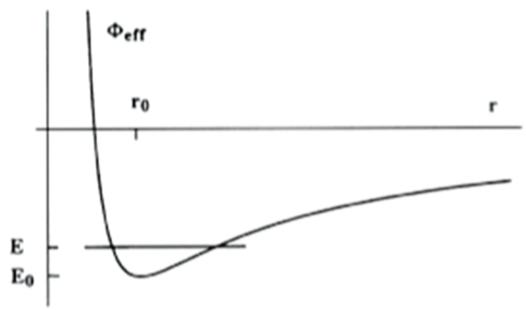

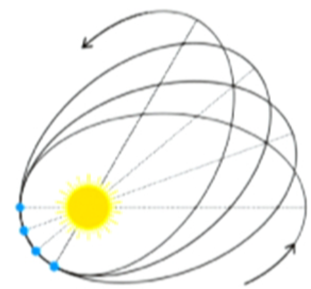

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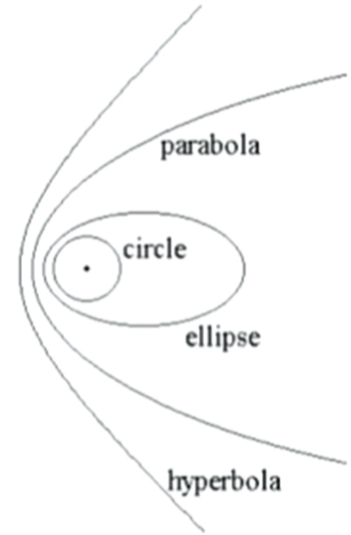
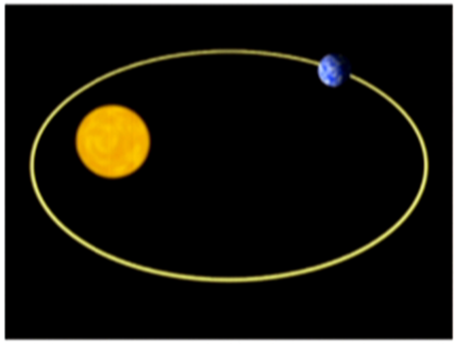
Analytic solutions – even better!

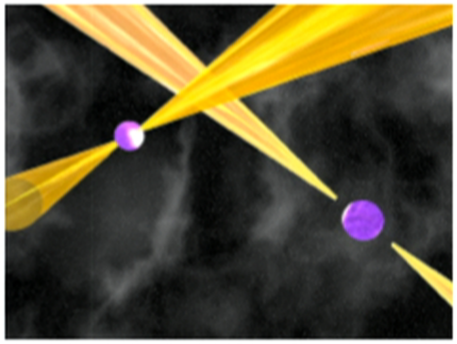
$$r(f) = \frac{a(1-e^2)}{1+e\cos f}$$

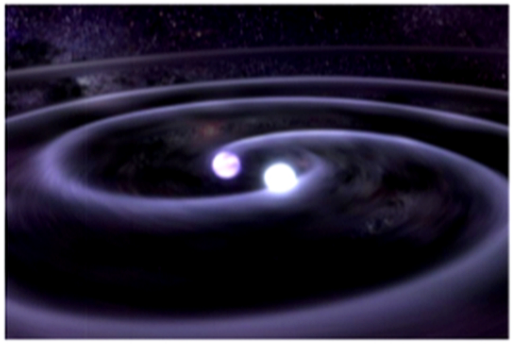


- We have analytic solutions for the Kepler potential *Φ(r) = -GM/r.*
- Perturbations (like a 3rd body) cause orbital elements *a*, *e*, etc. to evolve on timescale t $\gg$ τ.

Post-Newtonian spin precession

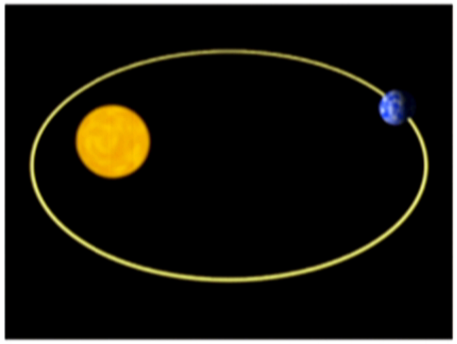


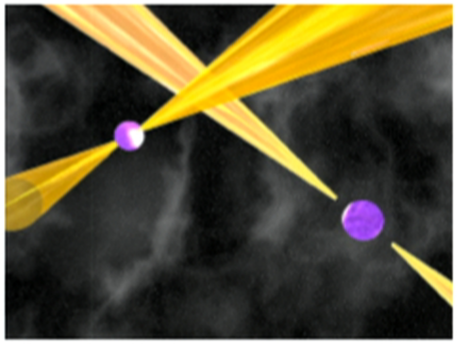


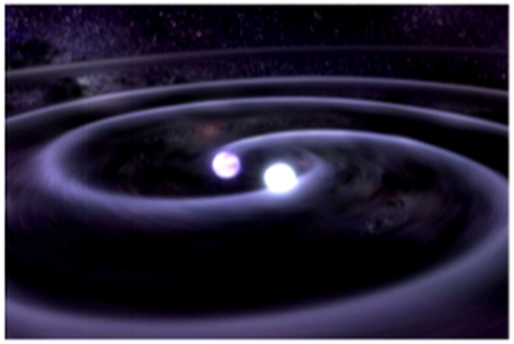


- EOM can be expanded about Newtonian solution when:
  - $v \ll c$, $r \gg r_g = GM/c^2$.
- Very good approximation for astrophysics:
  - $r_g/r \simeq 10^{-7}\,(M/10\,M_\odot)(r/1\text{ a.u.})^{-1}$
  - $r_g/r \simeq 10^{-7}\,(M/2 \times 10^6\,M_\odot)(r/1\text{ pc})^{-1}$
- 3 timescales for BBH evolution
  - Orbital time $t_{\rm orb} \sim (r^3/GM)^{1/2}$
  - Precession time $t_{\rm pre} \sim (t_{\rm orb}/\eta)(r/r_g) \gg t_{\rm orb}$
  - Radiation-reaction time $t_{\rm RR} \sim (t_{\rm orb}/\eta)(r/r_g)^{5/2} \gg t_{\rm pre}$

Post-Newtonian spin precession







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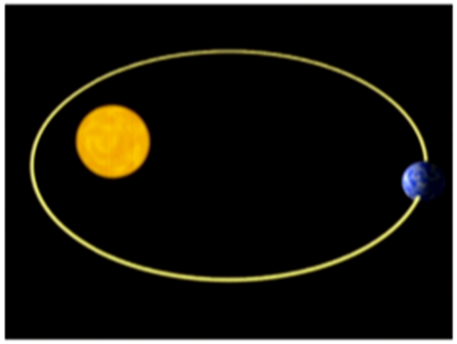 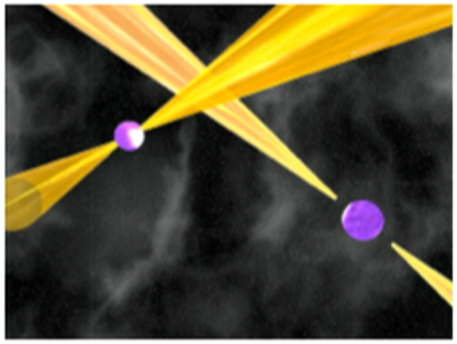 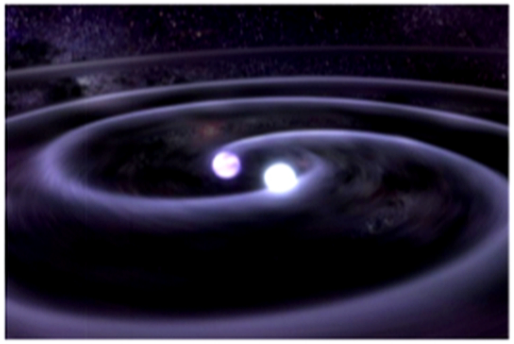

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PN spin precession seems hard!

$$\dot{\mathbf{S}}_1 = \mathbf{\Omega}_1 \times \mathbf{S}_1 \qquad \mathbf{\Omega}_1 = \frac{1}{r^3}\left\{\left(2+\frac{3q}{2}\right)\mathbf{L} + 3[\hat{\mathbf{r}}\cdot(\mathbf{S}_2 + q\mathbf{S}_1)]\hat{\mathbf{r}} - \mathbf{S}_2\right\}$$

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$$\dot{\mathbf{L}} = -\dot{\mathbf{S}}_1 - \dot{\mathbf{S}}_2$$

- 3 vectors $\mathbf{S}_1$, $\mathbf{S}_2$, $\mathbf{L} \Rightarrow$ 9 degrees of freedom
- Conservative equations complete to 2 PN
- Hierarchy $t_{\mathrm{orb}} \ll t_{\mathrm{pre}}$ allows *orbit averaging:*

$$\bar{\mathbf{\Omega}}_1 = \frac{1}{a^3(1-e^2)^{3/2}}\left\{\left[2+\frac{3q}{2}-\frac{3}{2}\frac{(\mathbf{S}_2+q\mathbf{S}_1)\cdot\mathbf{L}}{L^2}\right]\mathbf{L} + \frac{1}{2}\mathbf{S}_2\right\}$$

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I see the light!!!

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- Total angular momentum $\mathbf{J} = \mathbf{L} + \mathbf{S}_1 + \mathbf{S}_2$ conserved: 6 – 3 = 3.
- Projected effective spin:

$$\xi \equiv M^{-2}[(1+q)\mathbf{S}_1 + (1+q^{-1})\mathbf{S}_2] \cdot \hat{\mathbf{L}}$$

  is conserved (Racine 2008): 3 – 1 = 2.
- 2 degrees of freedom remain: S (magnitude of total spin $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$) and φ_L (azimuthal angle of $\mathbf{L}$ in frame with $\mathbf{J}$ on z axis).

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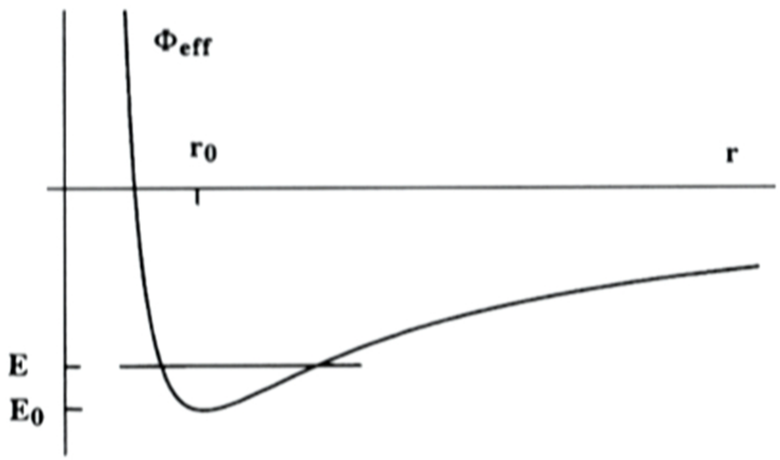
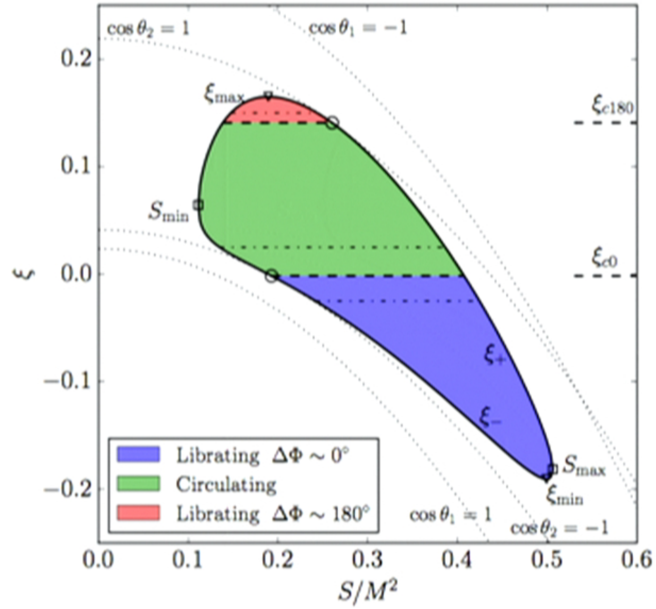
Effective potential for spin precession



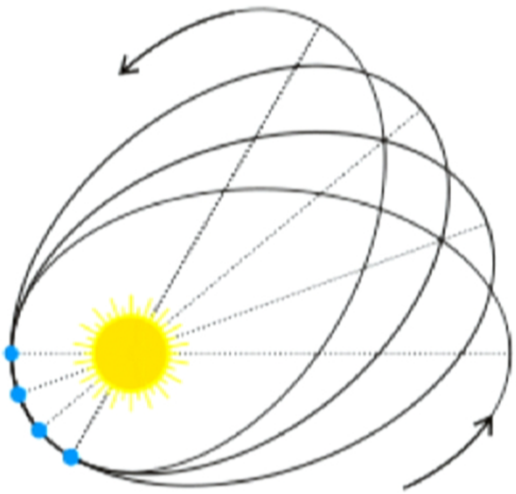


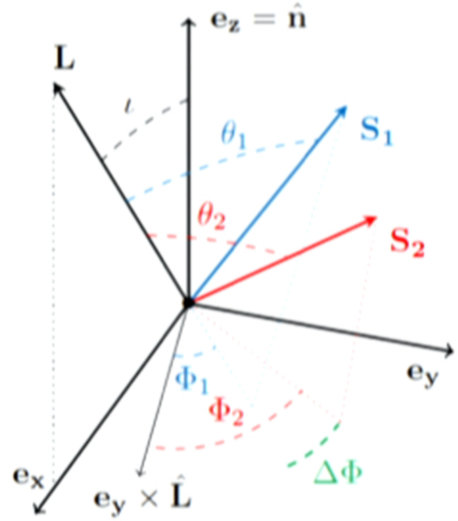
- $L = \eta(GM^3r)^{1/2}]$, $L - S_1 - S_2 \leq J \leq L + S_1 + S_2$
- $E = \Phi_{\text{eff}}(r_\pm)$ sets limits of radial motion
- $\xi = \xi_\pm(S_\pm)$ sets limits on S, where

$$A_1 \equiv [J^2 - (L-S)^2]^{1/2}\,,$$
$$A_2 \equiv [(L+S)^2 - J^2]^{1/2}\,,$$
$$A_3 \equiv [S^2 - (S_1 - S_2)^2]^{1/2}\,,$$
$$A_4 \equiv [(S_1+S_2)^2 - S^2]^{1/2}\,.$$

$$\xi_\pm(S) = \frac{(J^2 - L^2 - S^2)[S^2(1+q)^2 - (S_1^2 - S_2^2)(1-q^2)] \pm (1-q^2)A_1A_2A_3A_4}{4qM^2S^2L}$$

BBH spin precession is *regular*





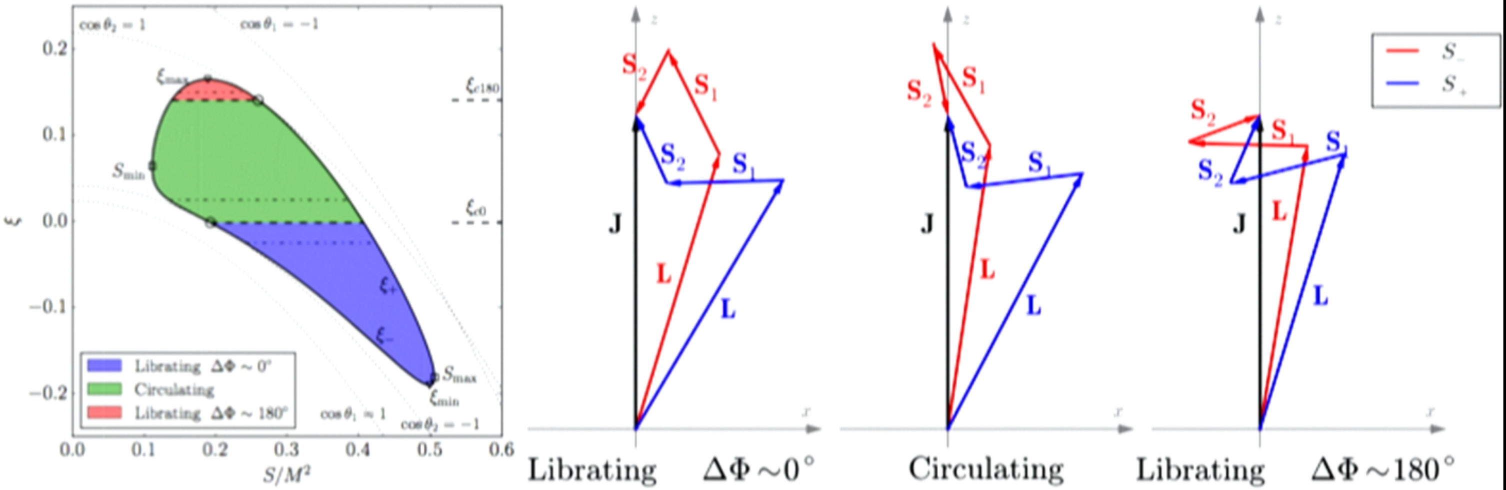
- Orbits in a spherical potential are regular (quasi-periodic)

$$\tau(E, L) = 2\int_{r_-}^{r_+} \frac{dr}{dr/dt} \qquad \alpha(E, L) = 2\int_{r_-}^{r_+} \frac{\Omega_z \; dr}{dr/dt}$$

- BBH spin precession is regular (quasi-periodic)

$$\tau(L, J, \xi) = 2\int_{S_-}^{S_+} \frac{dS}{dS/dt} \qquad \alpha(L, J, \xi) = 2\int_{S_-}^{S_+} \frac{\Omega_z \; dS}{dS/dt}$$

Morphological transitions



- Keplerian orbits can be circular *(e = 0)*, elliptical *(e < 1)*, parabolic *(e = 1)*, or hyperbolic *(e > 1)*.
- *S* is constant at $\xi = \xi_{min}$, $\xi = \xi_{max}$ which are *spin-orbit resonances* (Schnittman 2004).
- $\Delta\Phi$ can librate about 0° $(\xi < \xi_{c0})$, circulate $(\xi_{c0} < \xi < \xi_{c180})$, librate about 180° $(\xi > \xi_{c180})$.

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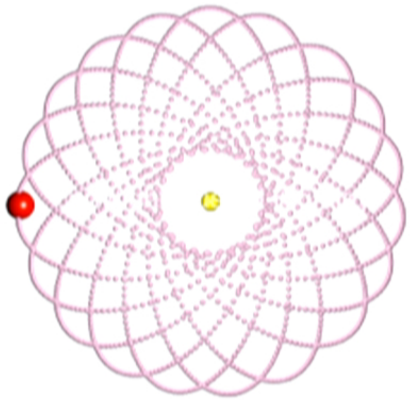
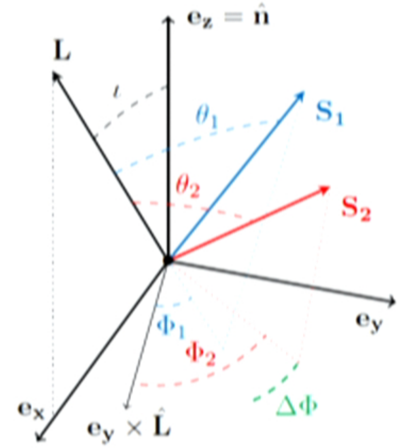
New spin-precession resonances



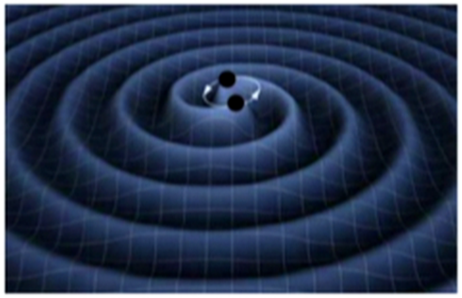
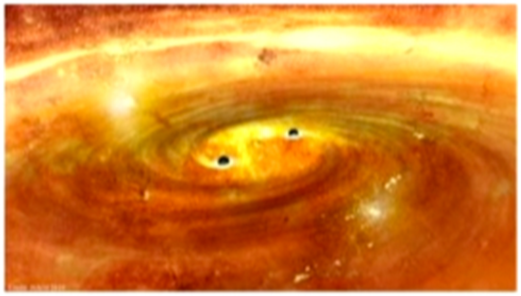
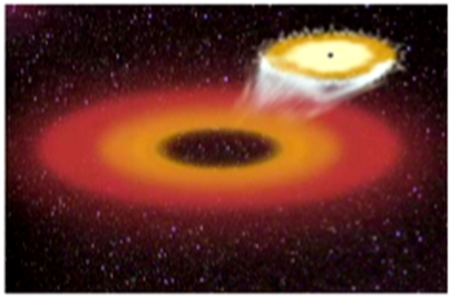


$$\frac{d\mathbf{J}}{dt} = -\frac{32}{5}\left(\frac{\eta M^2}{L}\right)^8 \frac{\eta \mathbf{L}}{M}$$

$$\cos\iota = \frac{J^2 + L^2 - S^2}{2JL}$$

- Orbits are *resonant* (closed) if $\alpha = n\pi/m$.
- BBH spin precession *resonant* (periodic) if $\alpha = n\pi/m$.
- Resonances with $m = 1$, $n = 2j$ have $d\mathbf{J}/dt \nparallel \mathbf{J}$.
  - Direction of **J** is "kicked" as BBHs inspiral through this resonance.
  - Possible EM/GW signature if it happens near merger.

Astrophysical applications

- BBH spin alignments set when they decouple
  - Stellar-mass BBHs decouple at $r \sim 10^6\, r_g$
  - Supermassive BBHs decouple at $r \sim 10^2 - 10^3\, M$
- Spins must be evolved from decoupling to merger
- $< dJ/dL >_{\mathrm{pre}}$ can be integrated on $t_{\mathrm{RR}} \Rightarrow$ *much* cheaper
- Gravitational recoils sensitive to BBH spin direction

Conclusions

- BBH spins precess on a timescale t_{pre} much shorter than the inspiral time t_{RR}.
- We have derived analytic solutions to the orbit-averaged spin-precession equations at 2PN, the highest order yet published.
- BBH spin precession is quasi-periodic: spins return to initial relative orientation after a time τ during which they precess through an angle α.
- New class of resonances discovered for $\alpha = 2\pi n/m$.
- Radiation reaction can be *precession averaged* to provide faster evolution from large separation.
- Applications to astrophysical BBHs and GW analysis.