Title: A Î>CDM Bounce Scenario

Date: Jan 21, 2015 04:00 PM

URL: http://pirsa.org/15010074

Abstract: We study a contracting universe composed of cold dark matter and radiation, and with a positive cosmological constant. Assuming that loop quantum cosmology captures the correct high-curvature dynamics of the space-time, we calculate the spectrum of scalar and tensor perturbations after the bounce, assuming initial quantum vacuum fluctuations. We find that the modes that exit the (sound) Hubble radius during matter-domination when the effective equation of state is slightly negative due to the cosmological constant will be nearly scale-invariant with a slight red tilt, in agreement with observations. The tensor perturbations are also nearly scale-invariant, and the predicted tensor-to-scalar ratio is small. Finally, as this scenario predicts a positive running of the scalar index, it can be differentiated from inflationary models.

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A \(\triangle\)CDM Bounce Scenario

Edward Wilson-Ewing

Albert Einstein Institute
Max Planck Institute for Gravitational Physics

Work with Yi-Fu Cai arXiv:1412.2914 [gr-qc]

Perimeter Institute Seminar

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Quantum Gravity Effects in the Early Universe

One of the main difficulties of any theory of quantum gravity is to obtain predictions (that are realistically testable) and confront them to experiment or observations.

The best hope in this direction appears to lie in the very early universe, where quantum gravity effects are expected to be strong and may have left some imprints on the cosmic microwave background (CMB). What form could these imprints have?

In this talk, I will focus on loop quantum cosmology (LQC) and some of its predictions concerning the CMB.

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Caveat: the dynamics of LQC (just like general relativity) depend on the matter content. Therefore the predictions of LQC will strongly depend on what the dominant matter field (radiation, inflaton, ...) is during the bounce.

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The CMB and the Matter Bounce

Precision measurements of the temperature anisotropies in the CMB indicate the perturbations are nearly scale-invariant, with a slight red tilt.

This is one of the main predictions that any cosmological model must offer. Inflation is one model known to generate scale-invariant perturbations, but it is not the only one.

One alternative to inflation is the matter bounce scenario: Fourier modes that are initially in the quantum vacuum state that exit the Hubble radius in a contracting matter-dominated Friedmann universe become scale-invariant. [Wands]

Then, if this contracting branch can be connected to our currently expanding universe via some sort of a bounce, these scale-invariant perturbations can provide suitable initial conditions for the expanding branch. [Finelli, Brandenberger]

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The ACDM Bounce

Here we shall consider a model where the matter fields are radiation and cold dark matter (CDM), and there is a positive cosmological constant Λ .

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The ACDM Bounce

Here we shall consider a model where the matter fields are radiation and cold dark matter (CDM), and there is a positive cosmological constant Λ .

We expect the modes that exit the (sound) horizon during matter-domination to have a scale-invariant spectrum.

Also, those that exit when the effective equation of state is slightly negative (due to Λ) will have a slight red tilt, in agreement with observations of the cosmic microwave background.



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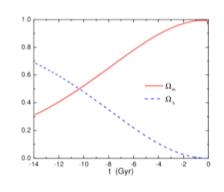
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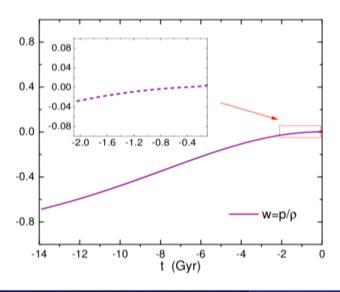
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Homogeneous Background I: ΛCDM Era

In the contracting branch, the dynamics of the space-time will initially be dominated by the cosmological constant Λ and afterwards by cold dark matter.





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During the transition between these two epochs, there will be a period of time where the effective equation of state will be slightly negative.

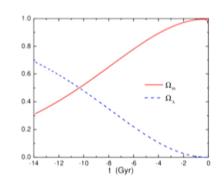
The Fourier modes that exit the sound horizon at this time will be nearly scale-invariant with a slight red tilt.

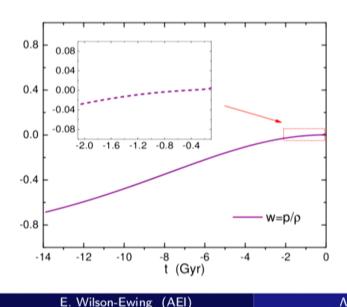
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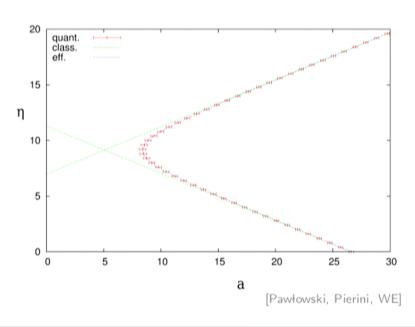
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Homogeneous Background II: Radiation Era

After this, the space-time will be dominated by radiation, and we will use LQC to model the dynamics in the high curvature regime.

In LQC, the big-bang singularity is replaced by a bounce:



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Furthermore, for sharply-peaked states, the effective equation

$$H^2 = \frac{8\pi G}{3} \rho \left(1 - \frac{\rho}{\rho_c} \right)$$

gives the dynamics of the expectation value of the scale factor at all times, including at the bounce point.

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In order to be able to analytically solve for the background and perturbations, some approximations are necessary.

- Split the evolution of the universe into two parts: ΛCDM and radiation,
- 2. Assume constant equation of state during each of these eras:
 - a) $\omega_{\mathrm{eff}} = -\delta$ during the $\Lambda \mathrm{CDM}$ era, with $\dot{\delta} = 0$ and $0 < \delta \ll 1$,
 - b) $\omega_{\rm eff}=rac{1}{3}$ during the radiation-dominated era.
- 3. Connect the two parts at the matter-raidiation time t_e by imposing continuity in the scale factor and in the Hubble rate.

This is a reasonable approximation for the Fourier modes that exit the sound horizon at a time when the effective equation of state is $\omega_{\rm eff} = -\delta$: this is a mode by mode calculation.

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The Scale Factor

Choosing the bounce time to be t=0 and setting the overall normalization of the scale factor so that a(t=0)=1, the approximations on the previous slide give

$$a(t) = \left(\frac{32\pi G\rho_c}{3}t^2 + 1\right)^{1/4}$$

during radiation domination, and during the ACDM era

$$a(\eta) = a_e \left(\frac{\eta - \eta_o}{\eta_e - \eta_o}\right)^{2/(1-3\delta)}, \quad \text{where} \quad \eta_o = \eta_e - \frac{2}{(1-3\delta)\mathcal{H}_e}.$$

Here the subscript 'e' denotes the instant of matter-radiation equality.



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Calculating the Spectrum of Scalar Perturbations

In order to determine the form of the scalar perturbations after the bounce, we follow this procedure:

- Solve the Mukhanov-Sasaki equation in a ΛCDM background, assuming a constant equation of state and that the perturbations are initially quantum vacuum fluctuations,
- 2. Assume a discontinuous transition (that occurs before any quantum gravity effects become important) between the ΛCDM era and radiation-domination,
- 3. Calculate the evolution of the long-wavelength perturbations through the bounce using loop quantum cosmology.

At each step, we impose continuity in v_k and v'_k . The result of this calculation can then be compared to observations.

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Scalar Perturbations I: ACDM Background

Scalar perturbations need only exit the sound horizon, which is much smaller than the Hubble radius by a factor of ϵ .

The Mukhanov-Sasaki equation in the ΛCDM background is

$$v_k'' + \epsilon^2 k^2 v_k - \frac{2(1+3\delta)}{(1-3\delta)^2 (\eta-\eta_o)^2} v_k = 0.$$

The solution (as usual for a constant equation of state) is a Hankel function. Assuming the quantum vacuum as the initial conditions,

$$v_k = \sqrt{\frac{-\pi\hbar(\eta - \eta_o)}{4}}H_n^{(1)}[-\epsilon k(\eta - \eta_o)],$$

where

$$n \approx \frac{3}{2} + 6 \,\delta + O(\delta^2).$$

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Scalar Perturbations II: Radiation Background

The Mukhanov-Sasaki equation for a radiation-dominated space-time (in the absence of quantum gravity effects) is

$$v_k''+\frac{k^2}{3}v_k=0,$$

and the solutions are simply plane waves. Requiring that v_k and v'_k be continuous during the transition between the Λ CDM era and the radiation-dominated period gives

$$v_k \sim \left(k^{-n}\cos\frac{k\eta_e}{\sqrt{3}} + k^{-n-1}\sin\frac{k\eta_e}{\sqrt{3}}\right)\cos\frac{k\eta}{\sqrt{3}} + \sin\frac{k\eta}{\sqrt{3}}.$$

Note that we have not imposed the condition that $k|\eta_e|/\sqrt{3}\ll 1$. However, as we can see here already, this condition will be necessary for scale-invariance.

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In loop quantum cosmology, scalar perturbations can be studied using the 'separate universe' approach. [Salopek, Bond; Wands, Malik, Lyth, Liddle]

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This is done by working on a lattice and assuming that each cell is homogeneous and isotropic. An LQC quantization can be performed in each cell, and then the interactions between neighbouring cells are relatively easy to handle.

We can derive effective equations for the scalar perturbations that are expected to be valid for Fourier modes whose wavelength remains much larger than $\ell_{\rm Pl}$. [Rovelli, WE] In the long-wavelength limit, the effective LQC Mukhanov-Sasaki equation is

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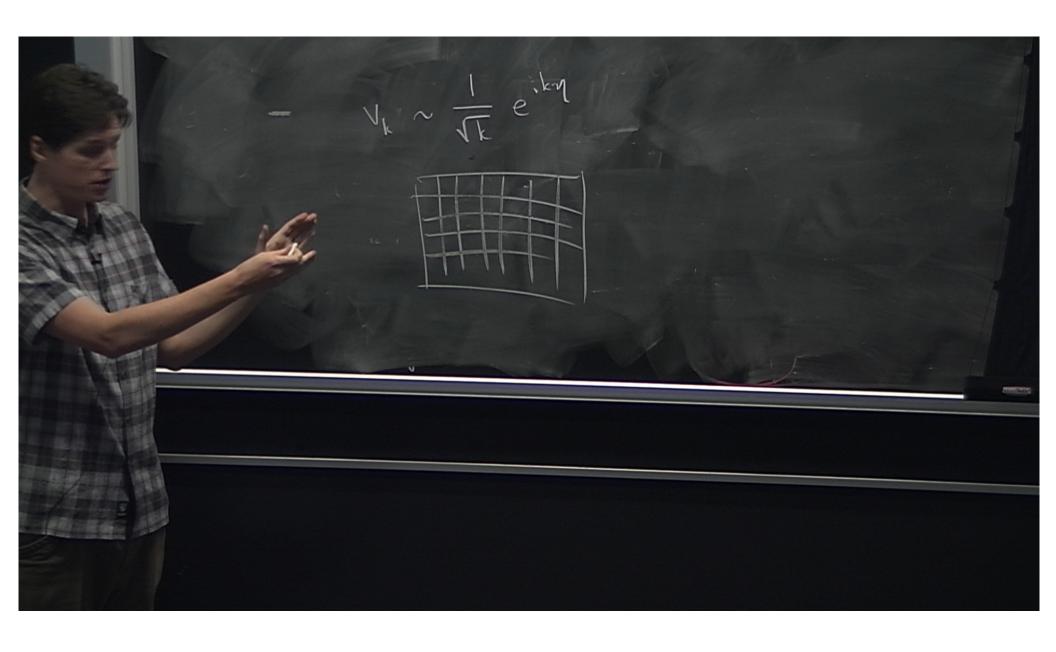
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Scalar Perturbations III: Bounce

Knowing the background evolution, the effective LQC Mukhanov-Sasaki equation can be solved, giving a hypergeometric function. The numerical pre-factors can be obtained by matching this function with the solution obtained earlier for the pre-bounce radiation-dominated epoch.

Then, by taking the limit of $t \gg t_{\rm Pl}$, we obtain the form of the scalar perturbations after the bounce:

$$\Delta_{\mathcal{R}}^2(k) = rac{k^3}{2\pi} |\mathcal{R}_k|^2 \sim \sqrt{rac{
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where the co-moving curvature perturbation is $\mathcal{R}_k \sim v_k/a$.

Here we have assumed that $k|\eta_e|/\sqrt{3}\ll 1$ in order to ensure that the resulting spectrum is nearly scale-invariant.

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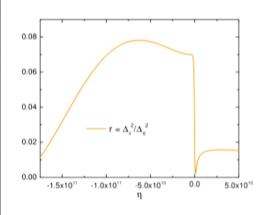
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Tensor Perturbations

A similar calculation can be done for tensor modes, and the result is that their resulting amplitude is predicted to be significantly smaller than the amplitude of the scalar perturbations for two reasons:

- 1. The amplitude of the scalar perturbations are boosted by a factor of ϵ^{-3} ,
- 2. The amplitude of the tensor perturbations is damped by a factor of 1/4 during the bounce due to LQC effects.



The predicted tensor-to-scalar ratio is

$$r = \frac{\Delta_h^2(k)}{\Delta_R^2(k)} = 24\epsilon^3,$$

which is in agreement with the current observational bound of r < 0.11 [Planck+WMAP] for small ϵ .

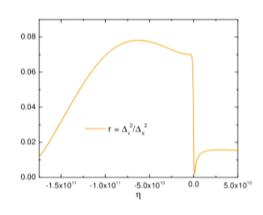
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An Asymmetric Bounce

To have scale-invariance, we required that

$$\frac{k|\eta_e|}{\sqrt{3}} \ll 1.$$

It is easy to check that, defining η_e^+ to be the time of matter-radiation equality in the expanding branch, for k observed in the CMB today $k\eta_e^+\sim 1$.

Therefore, in order for the first condition to hold, $|\eta_e| \ll \eta_e^+$. This requires an asymmetric bounce where the radiation-dominated era lasts longer in the expanding branch than in the contracting branch.



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The Effective Equation of State is not Constant

In the contracting branch during the Λ CDM epoch, the effective equation of state is not constant. Rather,

$$\frac{\mathrm{d}\omega_{\mathrm{eff}}}{\mathrm{d}\eta} > 0, \qquad \Rightarrow \qquad \frac{\mathrm{d}\omega_{\mathrm{eff}}}{\mathrm{d}k_h} > 0,$$

where the implication follows due to the fact that small k exit the sound horizon first.

Recall that the k-dependence of the scalar power spectrum depends directly on the effective equation of state when the Fourier mode exits the sound horizon. The same relation holds for tensor modes.

Since the sound speed for tensor modes is larger $(1 \gg \epsilon)$, the tensor modes exit their sound horizon later than their corresponding scalar modes. Therefore, by the above inequalities, the tensor index n_t must satisfy the relation

$$n_t > n_s - 1$$
.

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The Running of the Scalar Index

Furthermore,

$$\frac{\mathrm{d} n_s}{\mathrm{d} k} = \frac{\mathrm{d} n_s}{\mathrm{d} \omega_{\mathrm{eff}}} \cdot \frac{\mathrm{d} \omega_{\mathrm{eff}}}{\mathrm{d} k} = \frac{\mathrm{d} (1 - 12\delta)}{\mathrm{d} (-\delta)} \cdot \frac{\mathrm{d} \omega_{\mathrm{eff}}}{\mathrm{d} k} > 0.$$

This is an important prediction, as inflation predicts the opposite: therefore this effect can allow observations to differentiate between inflation and the Λ CDM bounce.



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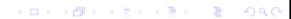
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Are There Any LQC Effects?

In the model studied here, the bounce occurs due to LQC quantum-gravity effects. But what happens if we consider a different Λ CDM bounce, where the bounce occurs due to some other effect?

Most of the predictions are very robust. As we saw, the majority of the features of the spectrum are determined by the physics of the contracting branch far before the bounce and therefore the bounce will not affect these predictions.



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However, there is one exception: in LQC, we found that the amplitude of the tensor modes is suppressed by a factor of 1/4 during the bounce. Therefore, in other bouncing models (unless there is a similar suppression of the tensor modes), we would expect a larger value of r (for a given CDM sound speed ϵ).

This effect is a potential test for LQC versus other bouncing cosmology models.

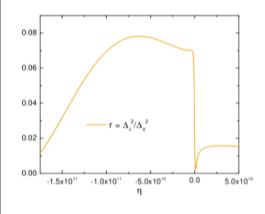
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Tensor Perturbations

A similar calculation can be done for tensor modes, and the result is that their resulting amplitude is predicted to be significantly smaller than the amplitude of the scalar perturbations for two reasons:

- 1. The amplitude of the scalar perturbations are boosted by a factor of ϵ^{-3} ,
- 2. The amplitude of the tensor perturbations is damped by a factor of 1/4 during the bounce due to LQC effects.



The predicted tensor-to-scalar ratio is

$$r = \frac{\Delta_h^2(k)}{\Delta_R^2(k)} = 24\epsilon^3,$$

which is in agreement with the current observational bound of r < 0.11 [Planck+WMAP] for small ϵ .

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Conclusions

In the Λ CDM bounce scenario, quantum vacuum fluctuations become nearly scale-invariant with a slight red tilt for the modes that exit the sound horizon when the effective equation of state is slightly negative.

This model requires an asymmetric bounce, and makes three important predictions beyond scale-invariance:

- A small tensor-to-scalar ratio,
- A positive running of n_s , and
- $n_t > n_s 1$.

There is an LQC-specific effect of an extra damping of the tensor-to-scalar by a factor of 1/4 which is a potential test for the theory.

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Outlook

There remain three main open questions in the ΛCDM bounce scenario:

- Amplitude of the running of n_s : We showed that $\mathrm{d} n_s/\mathrm{d}(\ln k) > 0$, but to calculate the amplitude, more must be known about the details of the pre-bounce era.
- Particle production during the bounce: It has been pointed out that particle production may be important during the LQC bounce, but can it provide sufficient asymmetry?
- Anisotropies during the bounce: In the absence of an ekpyrotic phase, we expect anisotropies to dominate the dynamics during the bounce. How might the presence of anisotropies change the predictions?



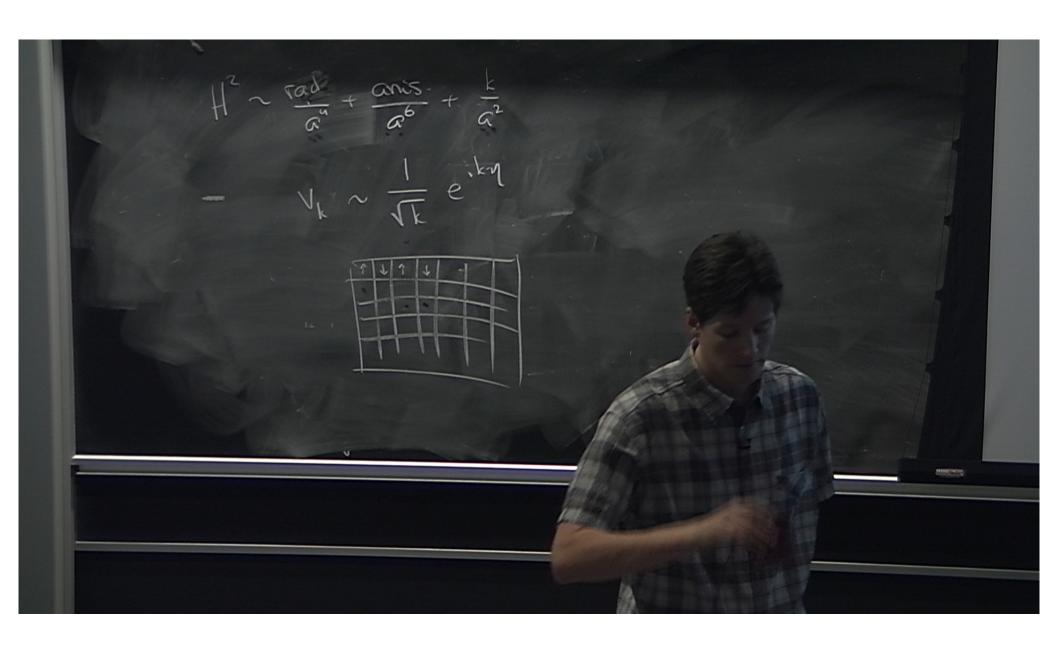
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