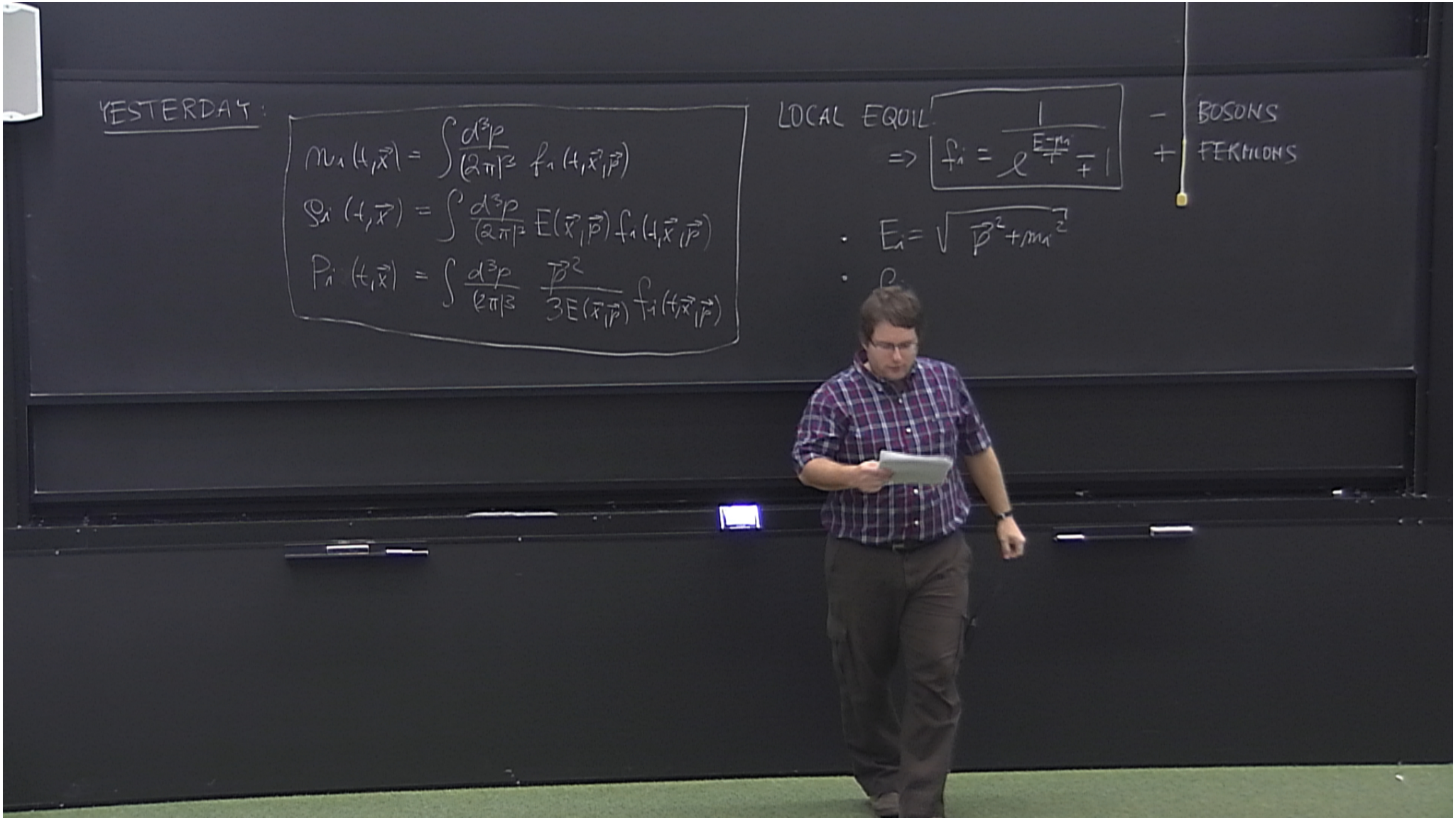
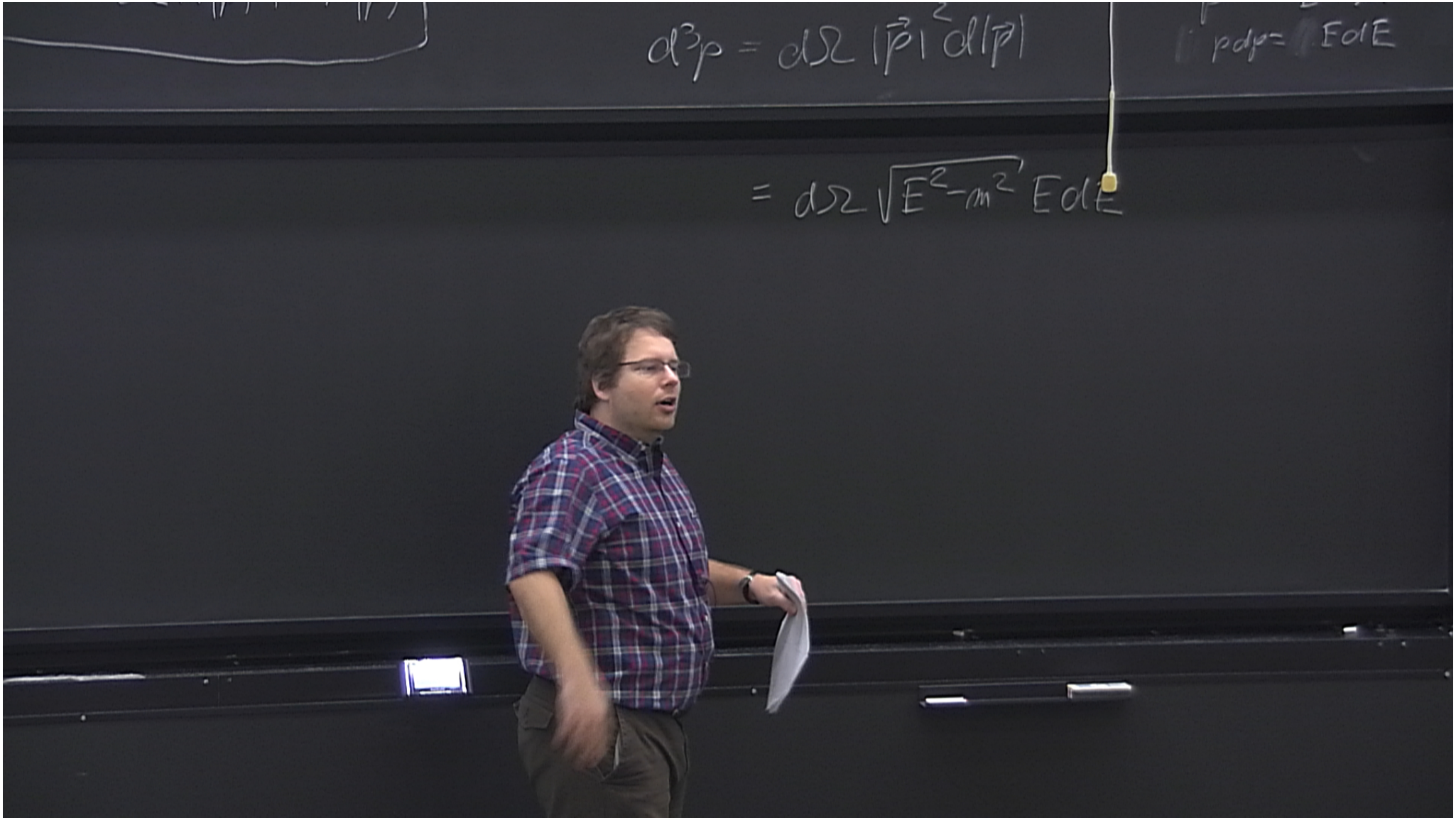
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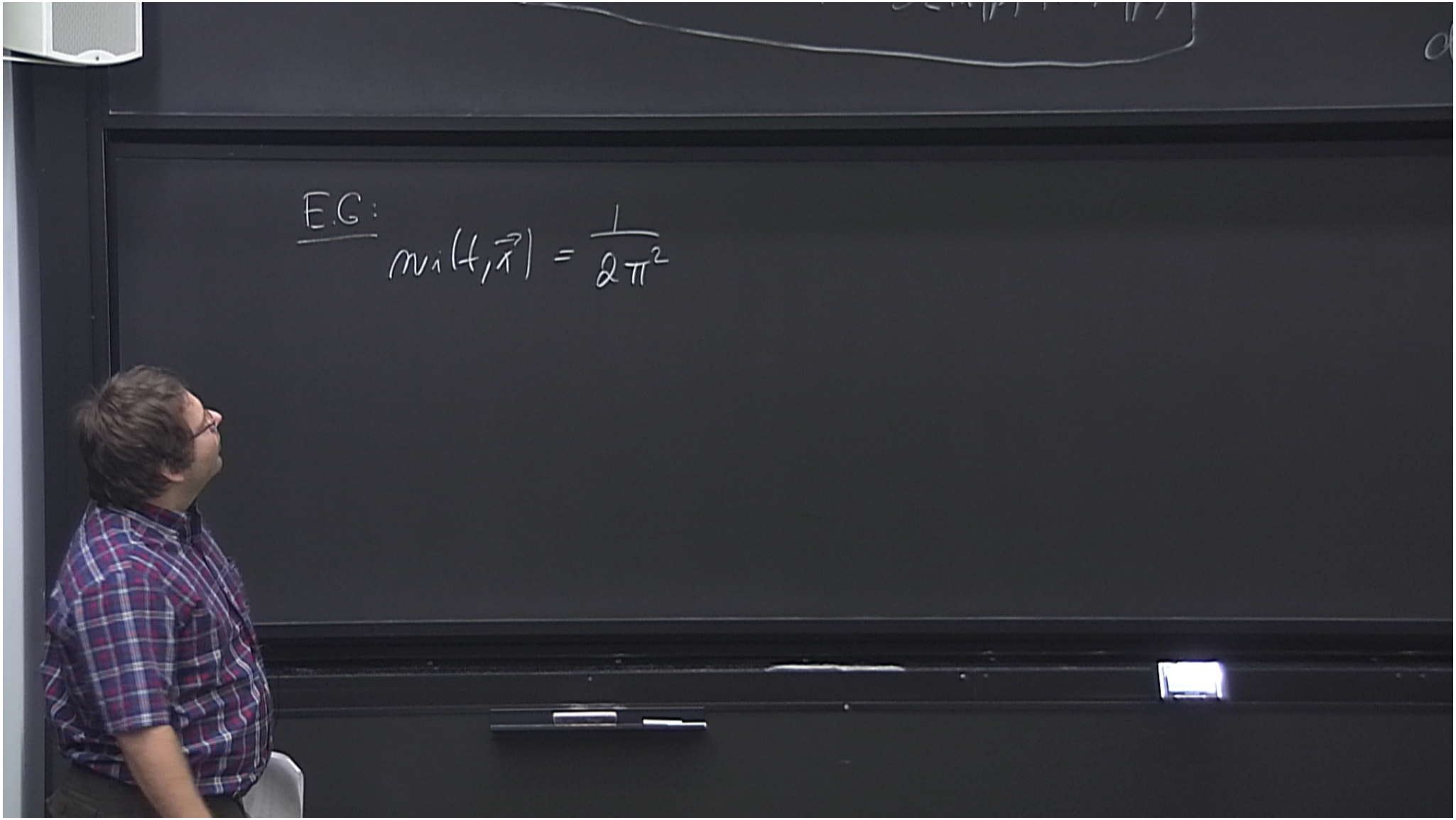
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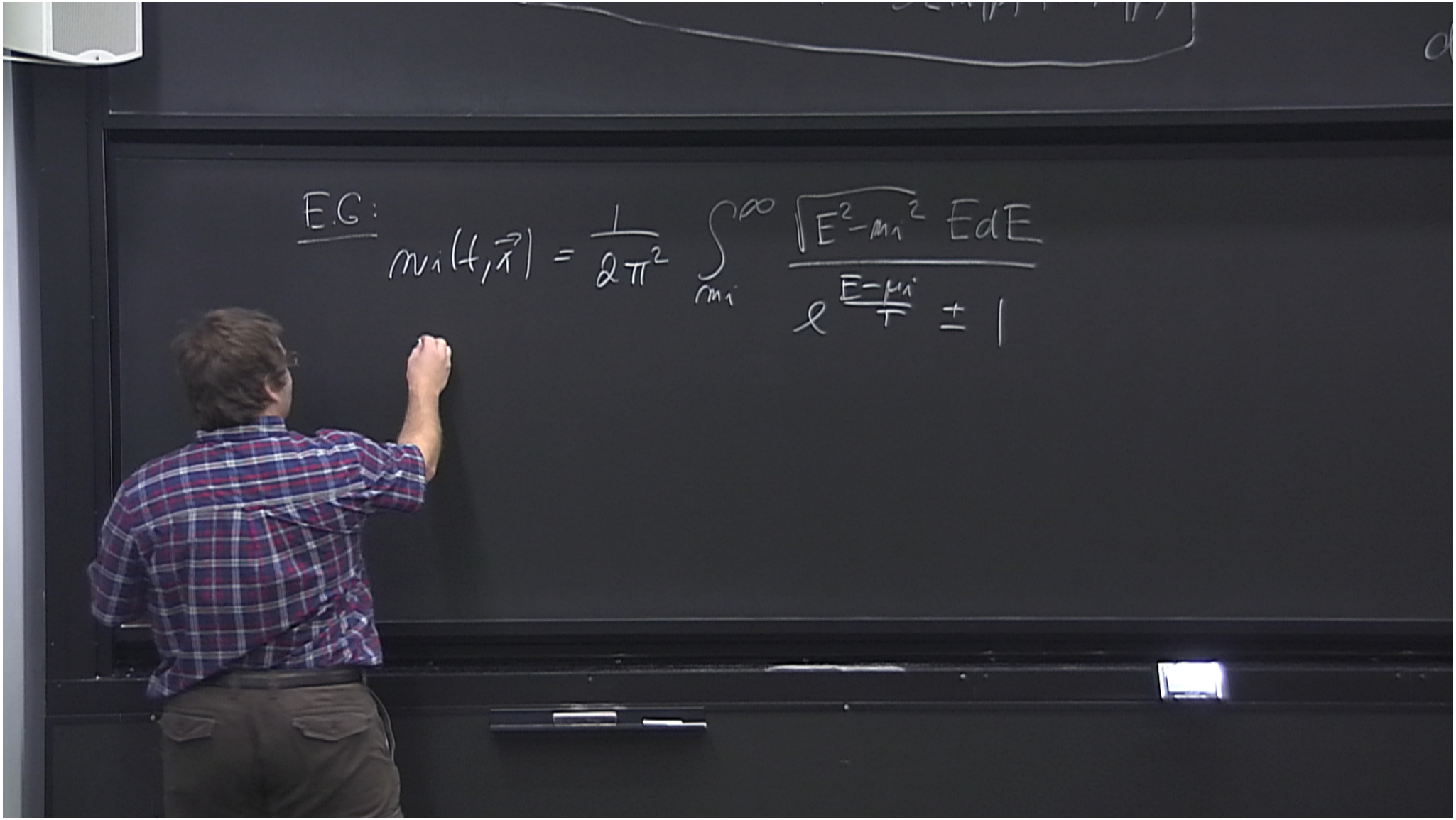
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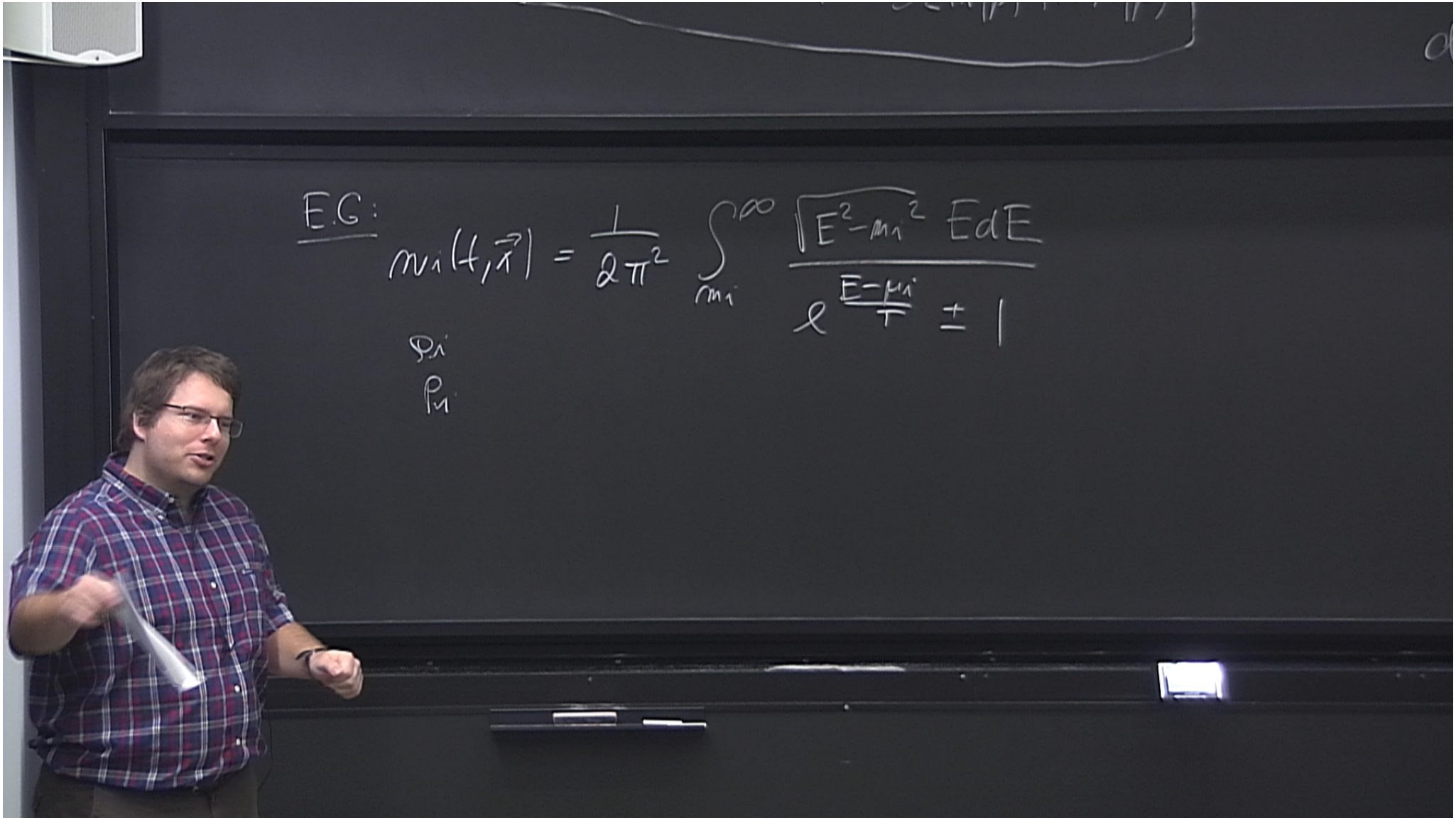
Abstract:

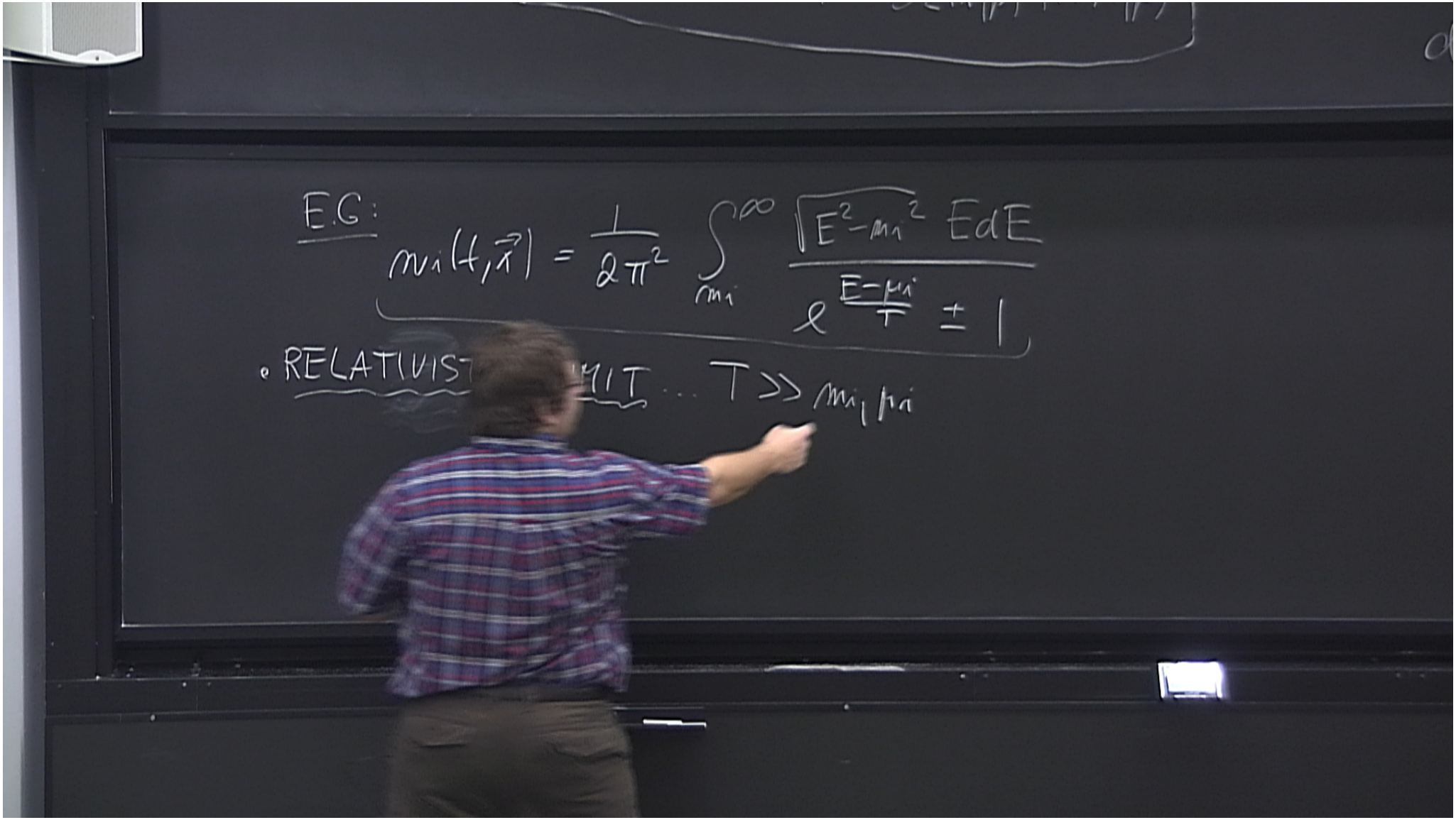

YESTERDAY:
$n_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} f_i(t,\vec{x},\vec{p})$
$\rho_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p}) f_i(t,\vec{x},\vec{p})$
$P_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E(\vec{x},\vec{p})} f_i(t,\vec{x},\vec{p})$
LOCAL EQUIL
$\Rightarrow f_i = \frac{1}{e^{\frac{E-\mu_i}{T}} \mp 1}$
$-$ BOSONS
$+$ FERMIONS
$E_i = \sqrt{\vec{p}^2 + m_i^2}$

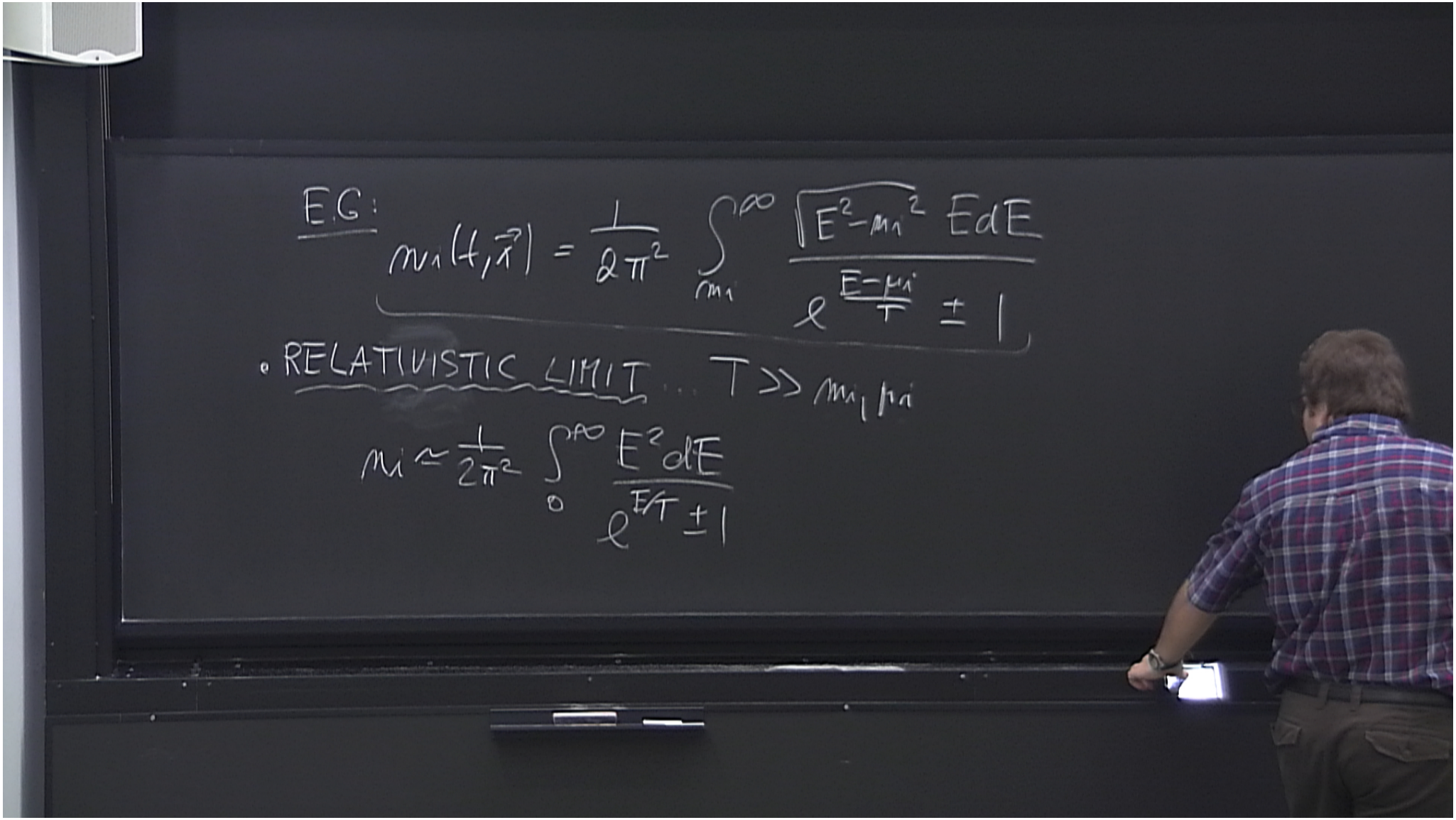

$d^3p = d\Omega\, |\vec{p}|^2 d|\vec{p}|$
$p\,dp = E\,dE$
$= d\Omega \sqrt{E^2 - m^2}\, E\,dE$


E.G:
$n_i(t,\vec{x}) = \frac{1}{2\pi^2}$


E.G:
$n_i(t,\vec{x}) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2-m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \pm 1}$


E.G:
$n_i(t,\vec{x}) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2-m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \pm 1}$
$\Phi_i$
$P_i$


E.G:
$n_i(t,\vec{x}) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2-m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \pm 1}$
• RELATIVIS
$\mu_i$ ... $T \gg m_i, \mu_i$


E.G:
$n_i(t,\vec{x}) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \pm 1}$
• RELATIVISTIC LIMIT ... $T \gg m_i, \mu_i$
$n_i \simeq \frac{1}{2\pi^2} \int_0^{\infty} \frac{E^2\, dE}{e^{E/T} \pm 1}$

E.G:

$$n_i(t,\vec{x}) = \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}} \pm 1} = d\Omega$$

- RELATIVISTIC LIMIT ... $T \gg m_i, \mu_i$

$$n_i \simeq \frac{1}{2\pi^2} \int_0^{\infty} \frac{E^2 dE}{e^{E/T} \pm 1} = \left| \begin{array}{l} x = E/T \\ dE = T dx \end{array} \right| = \frac{1}{2\pi^2} T^3 \int_0^{\infty} \frac{x^2 dx}{e^x \pm 1}$$

E.G:

$$n_i(t,\vec{x}) = \frac{1}{2\pi^2}\int_{m_i}^{\infty} \frac{\sqrt{E^2-m_i^2}\,E\,dE}{e^{\frac{E-\mu_i}{T}} \pm 1}$$

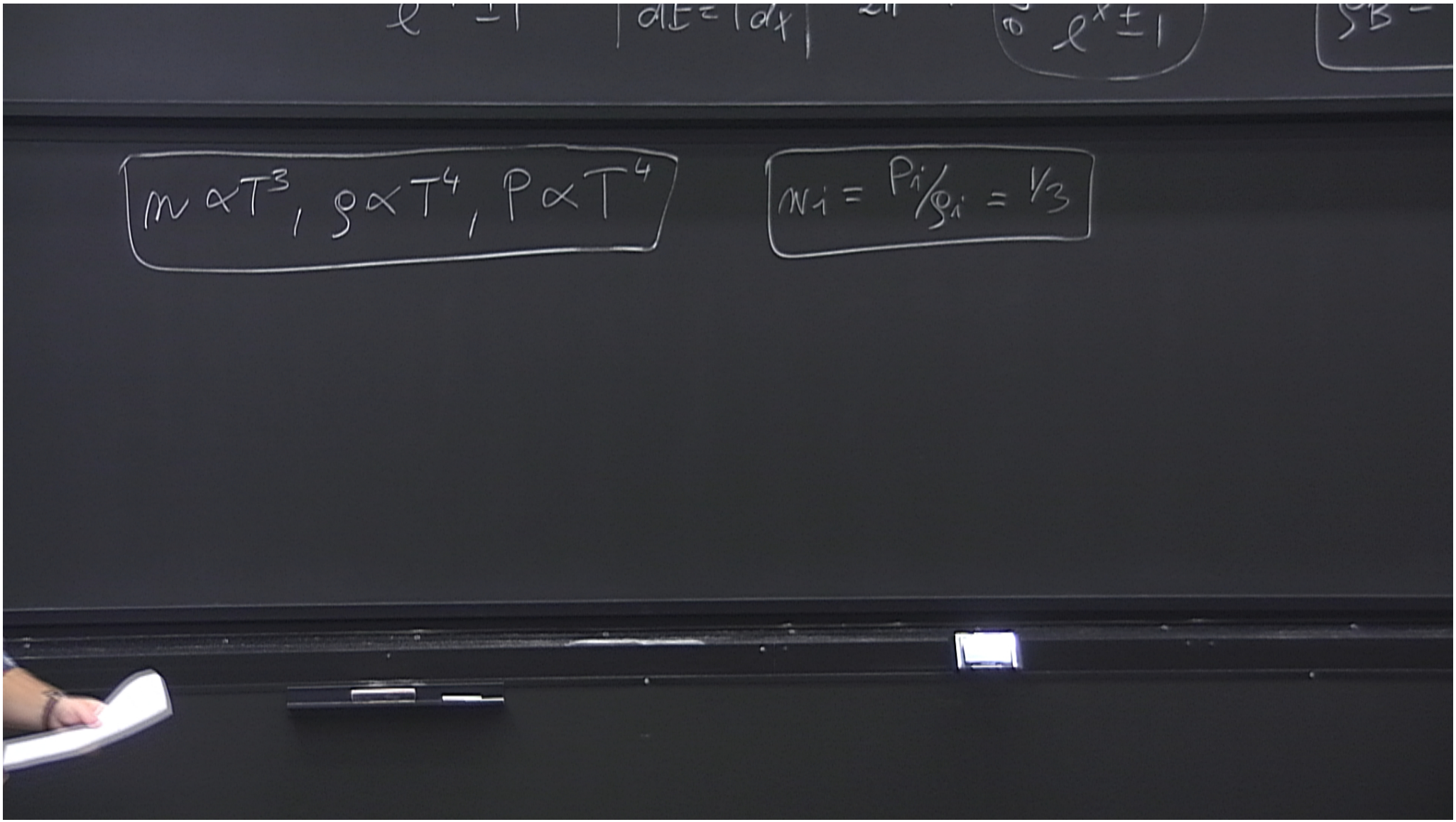
$= d\Omega$

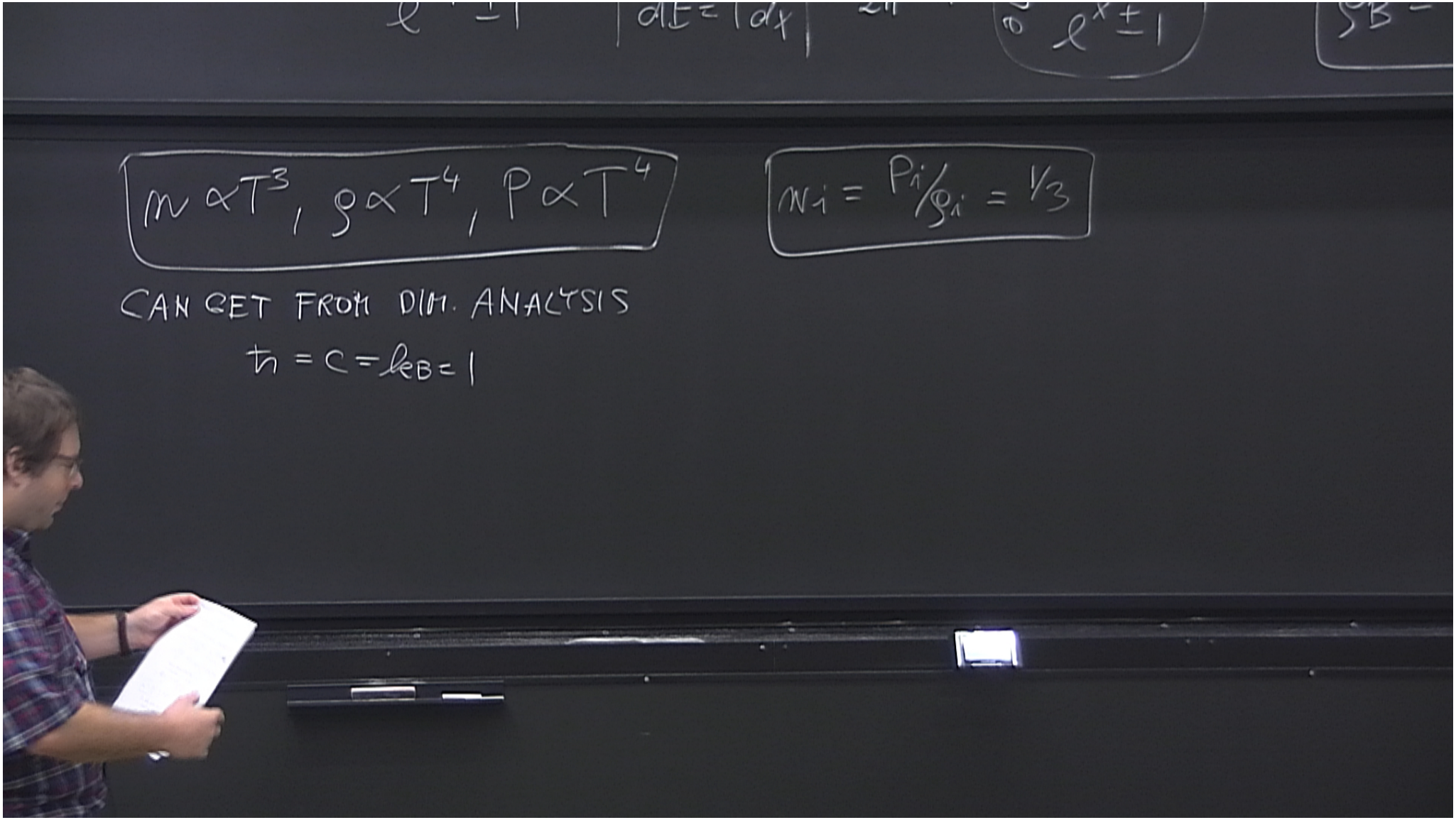
- RELATIVISTIC LIMIT .. $T \gg m_i, \mu_i$

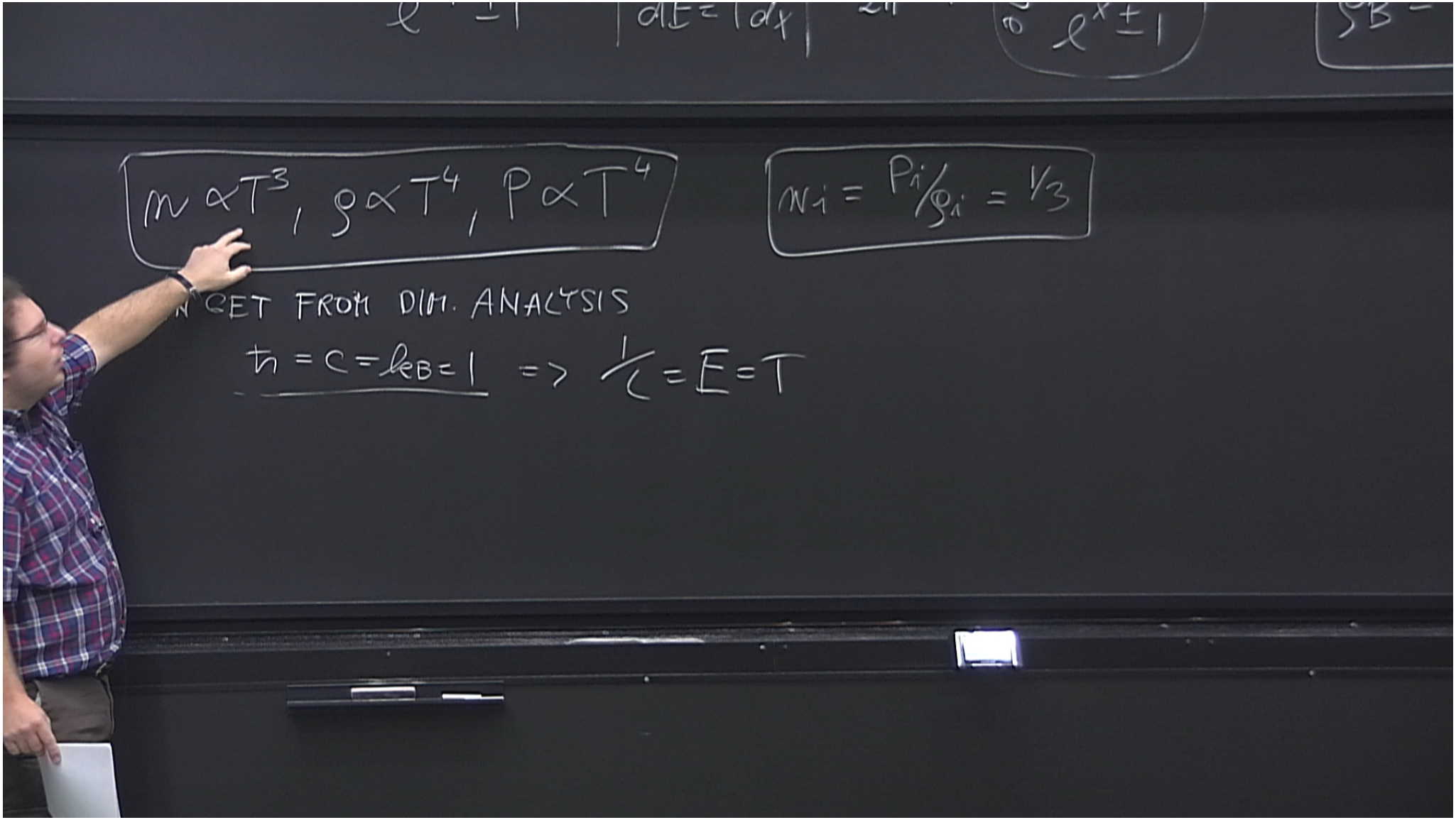
$$n_i \simeq \frac{1}{2\pi^2}\int_0^{\infty} \frac{E^2\,dE}{e^{E/T} \pm 1} = \left| \begin{array}{l} x = E/T \\ dE = T\,dx \end{array} \right| = \frac{1}{2\pi^2} T^3 \int_0^{\infty} \frac{x^2\,dx}{e^x \pm 1}$$

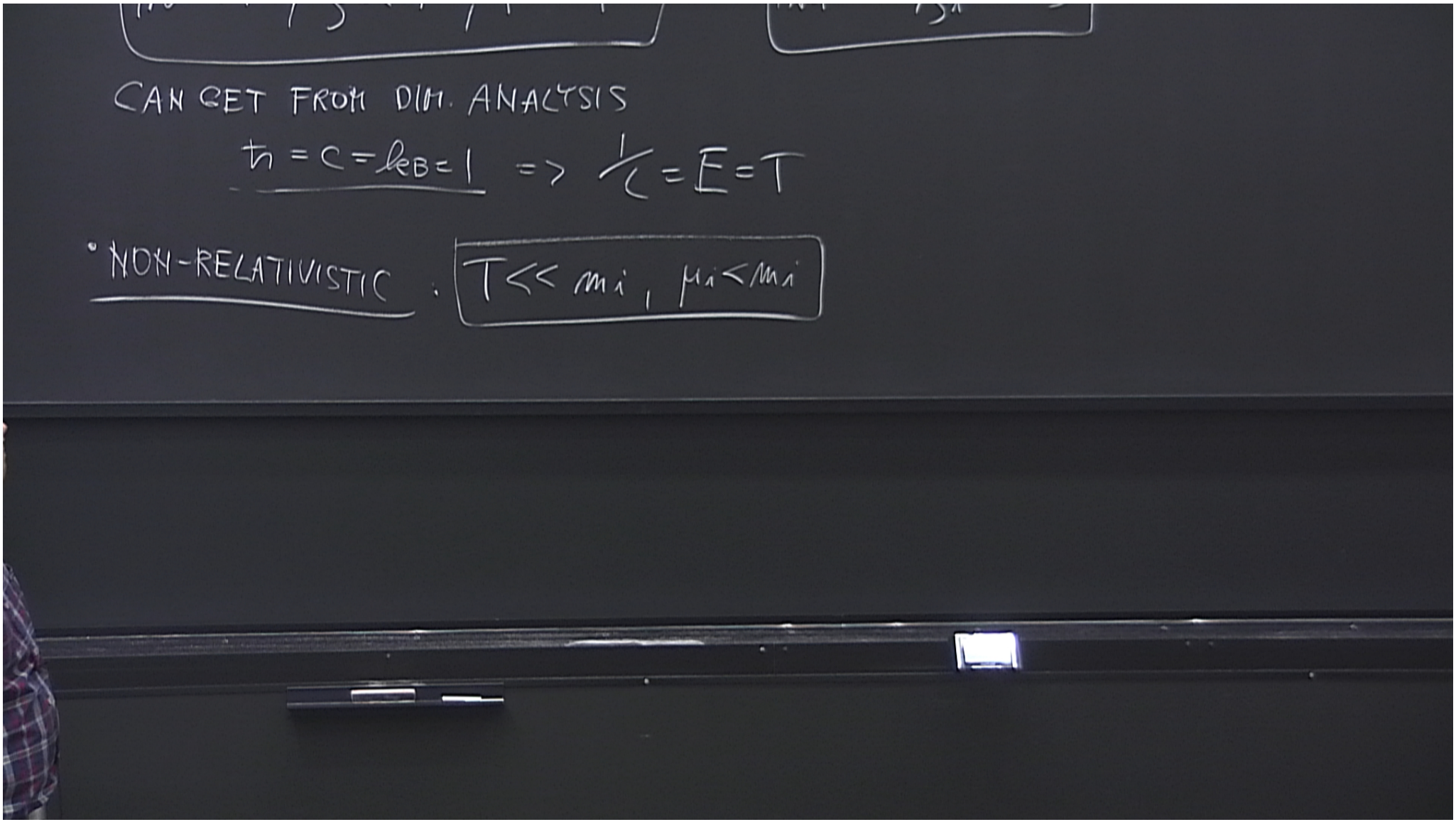
$2\zeta(3)$ $\qquad \frac{3}{2}\zeta(3)$




$n \propto T^3$, $\rho \propto T^4$, $P \propto T^4$
$w_i = P_i/\rho_i = 1/3$


$n \propto T^3$, $\rho \propto T^4$, $P \propto T^4$
$w_i = P_i/\rho_i = 1/3$
CAN GET FROM DIM. ANALYSIS
$\hbar = c = k_B = 1$


$n \propto T^3$, $\rho \propto T^4$, $P \propto T^4$
$w_i = P_i/\rho_i = 1/3$
GET FROM DIM. ANALYSIS
$\hbar = c = k_B = 1 \Rightarrow 1/\ell = E = T$


CAN GET FROM DIM. ANALYSIS
$\hbar = c = k_B = 1 \Rightarrow 1/\ell = E = T$
• NON-RELATIVISTIC : $T \ll m_i$, $\mu_i < m_i$

$n \propto T^3$, $\rho \propto T^4$, $P \propto T^4$

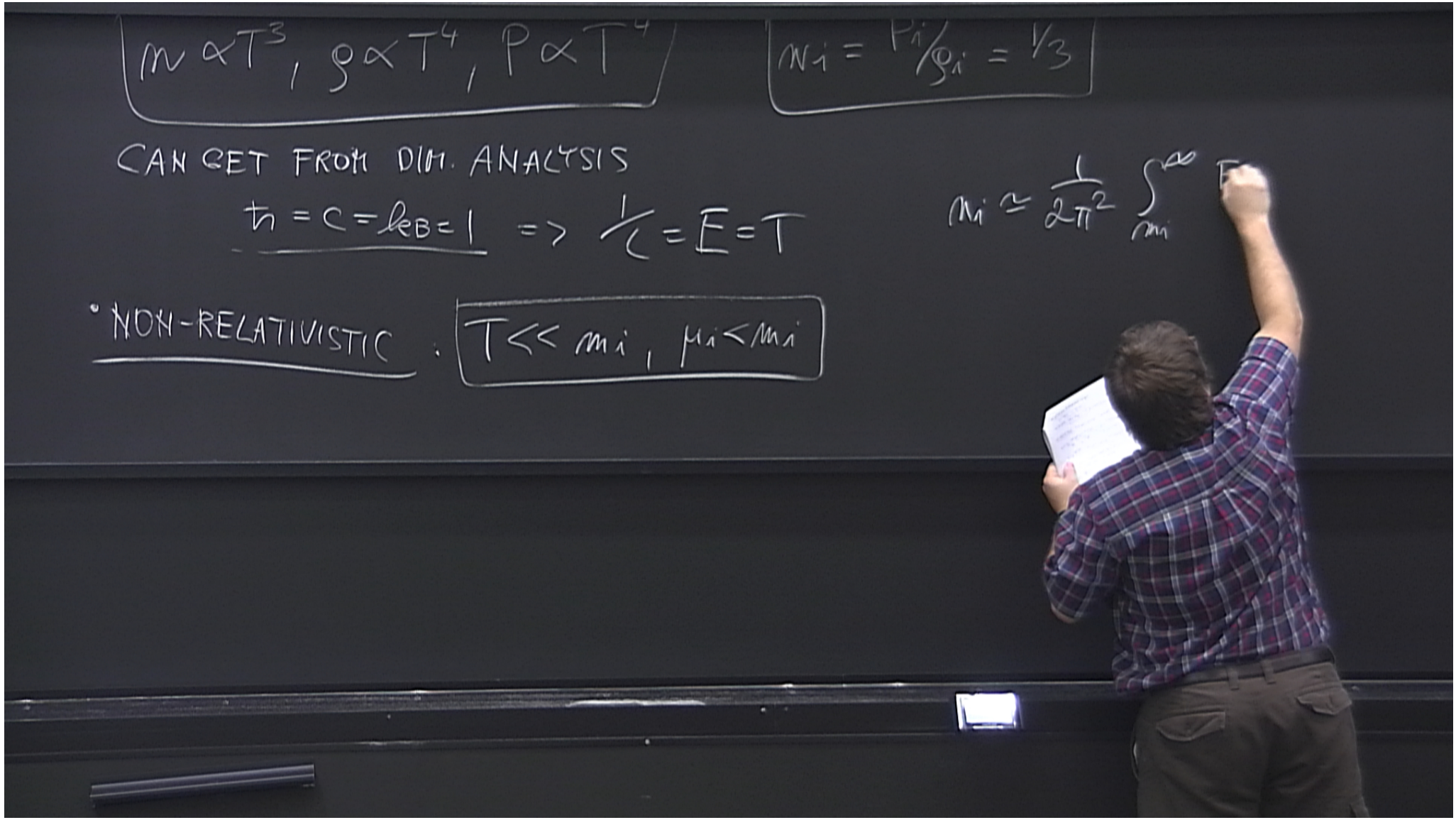
$w_i = P_i/\rho_i = 1/3$

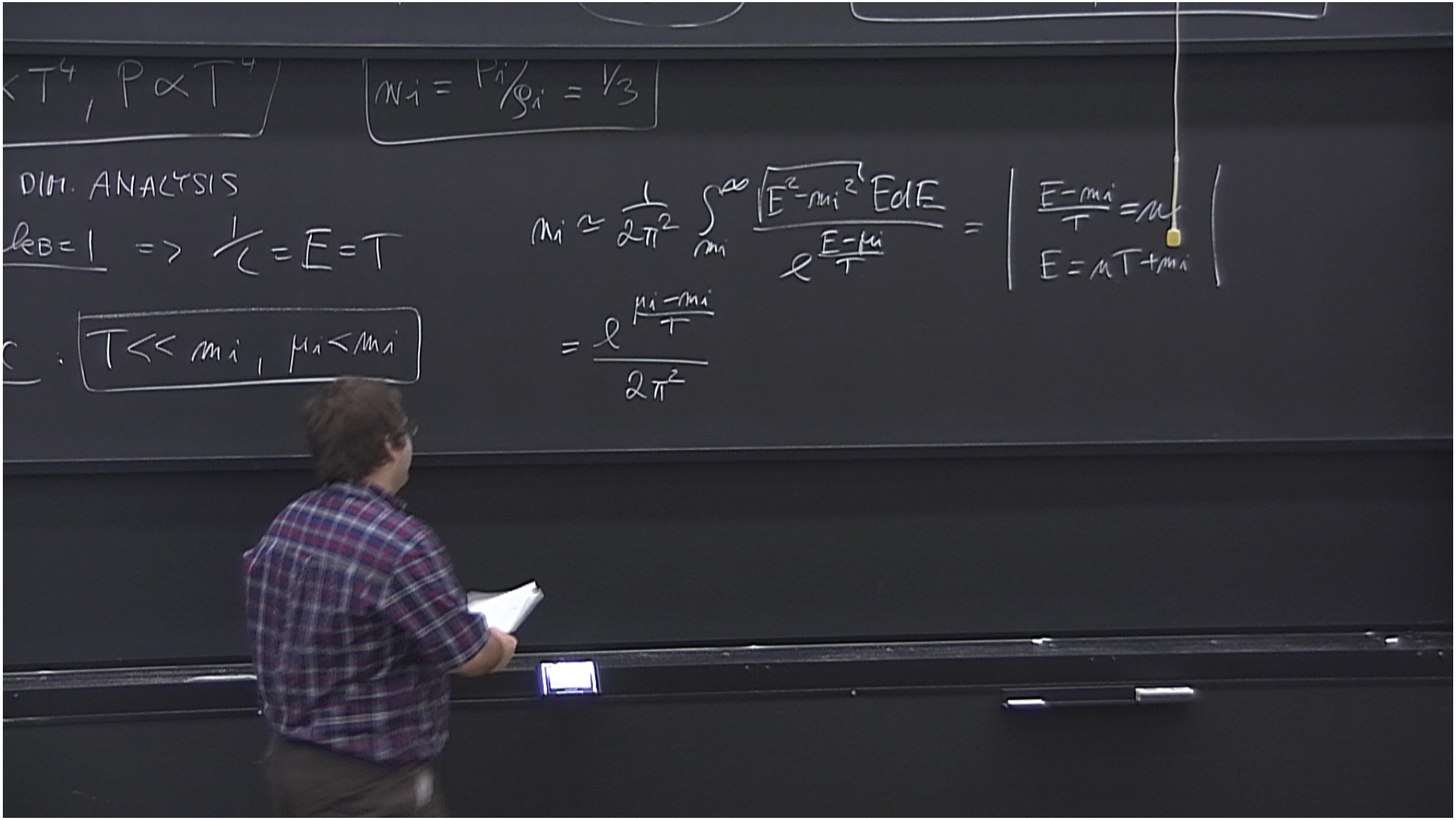
CAN GET FROM DIM. ANALYSIS

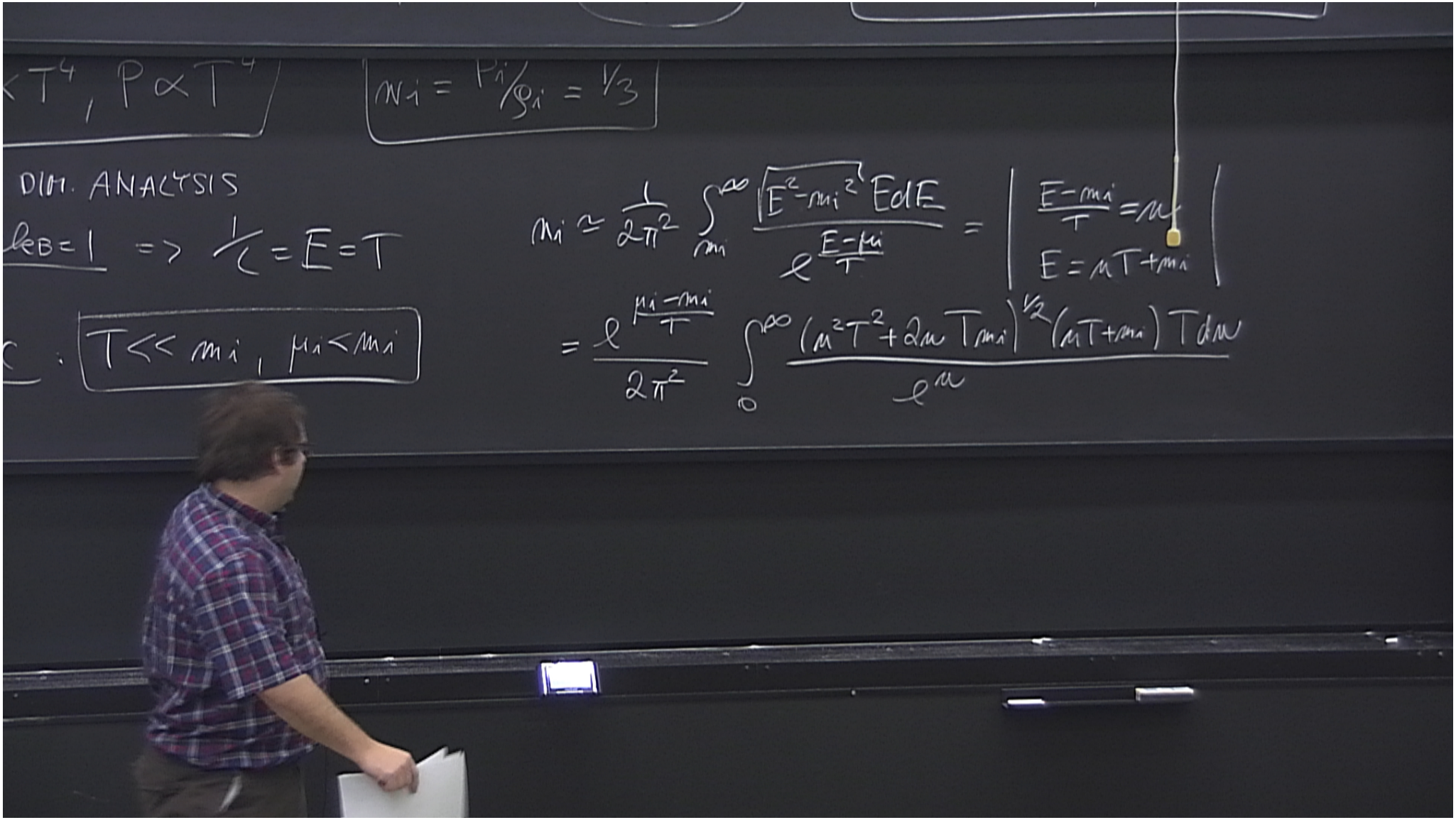
$\hbar = c = k_B = 1 \Rightarrow 1/\ell = E = T$

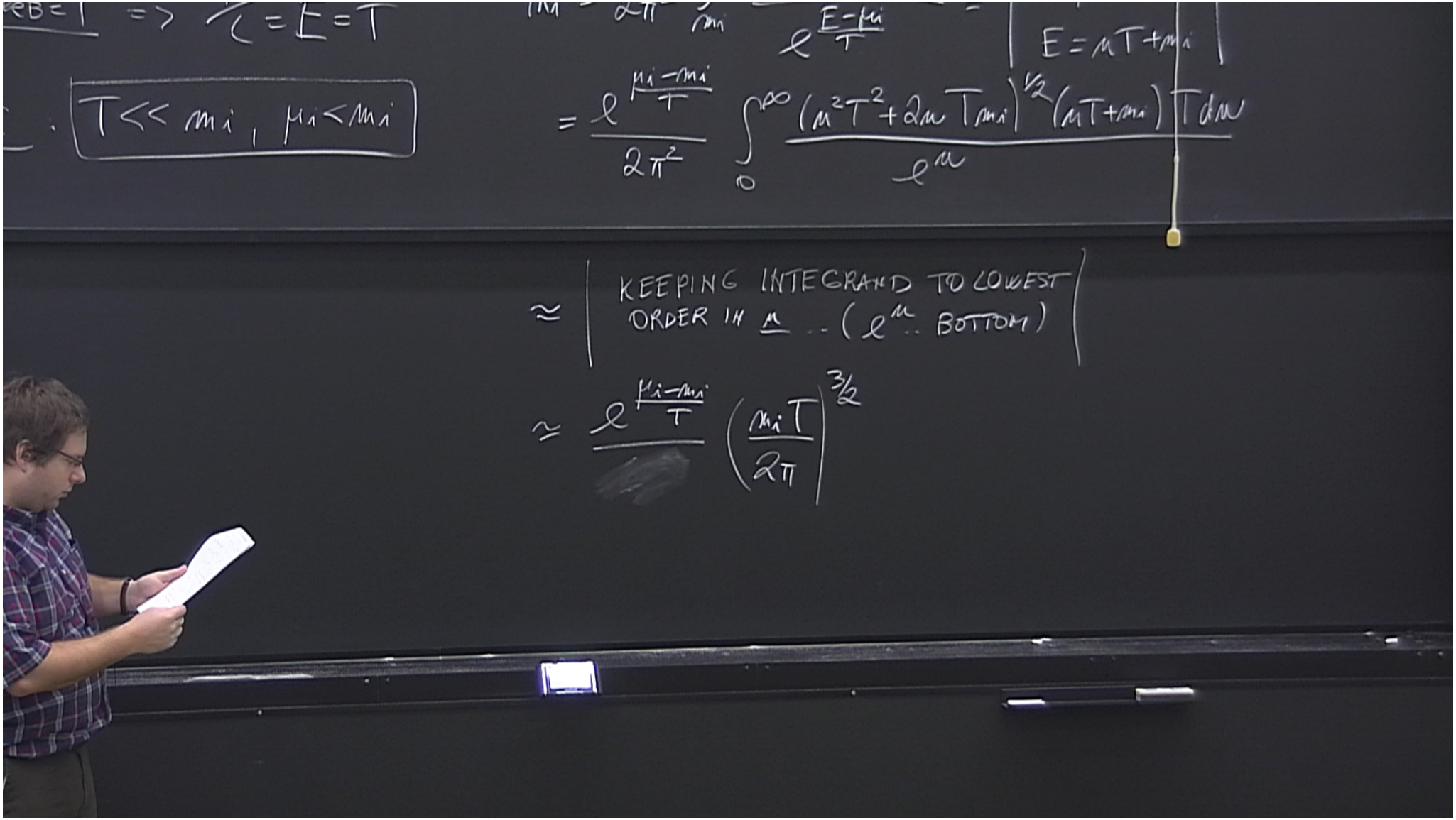
$n_i \simeq \frac{1}{2\pi^2} \int_{m_i}^{\infty}$

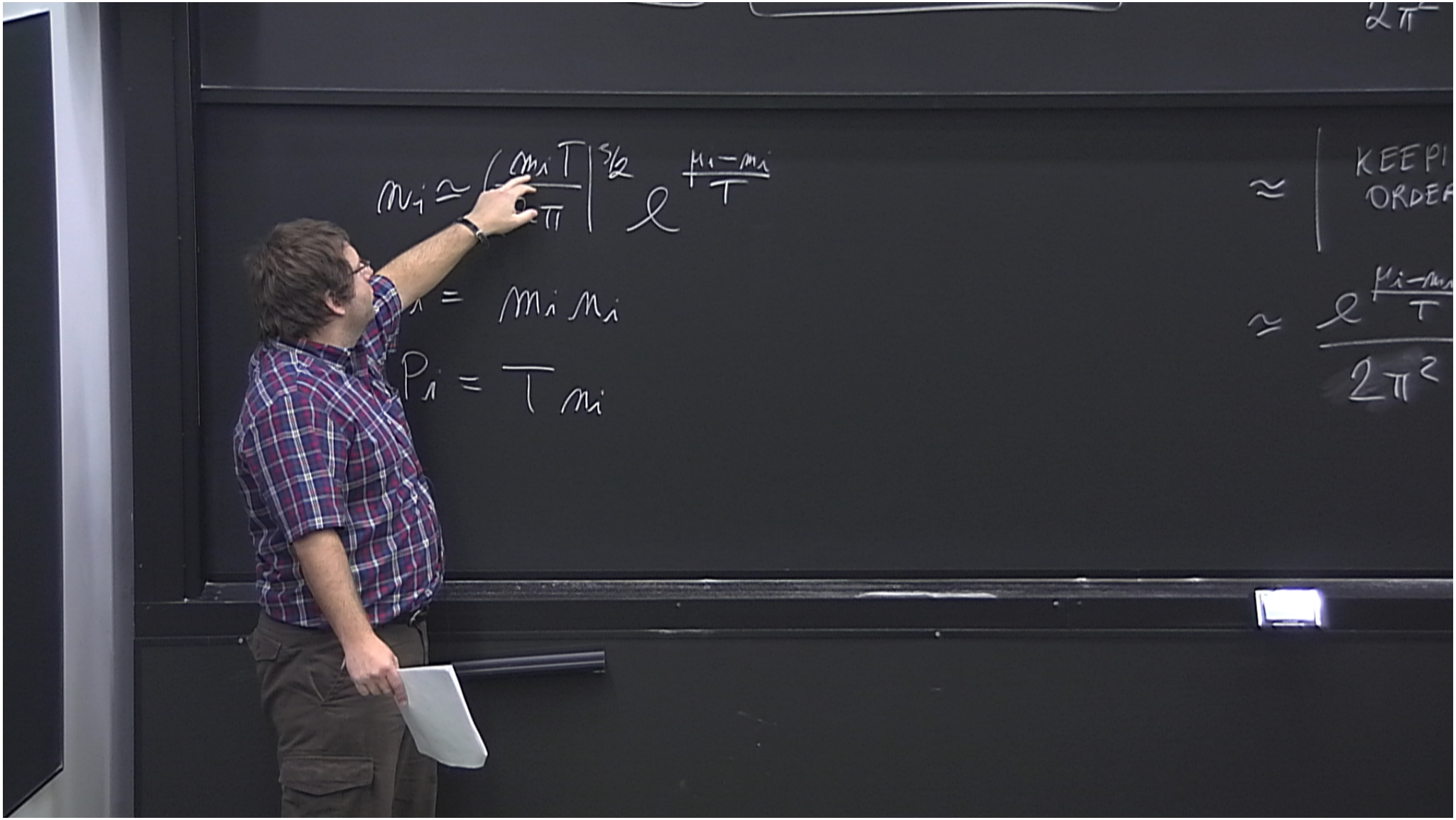
• NON-RELATIVISTIC : $T \ll m_i$, $\mu_i < m_i$



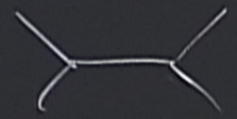

$\propto T^4, P \propto T^4$
$w_i = P_i/\rho_i = 1/3$
DIM. ANALYSIS
$\hbar = c = k_B = 1 \Rightarrow \frac{1}{L} = E = T$
$n_i \simeq \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}}} = \left| \begin{array}{l} \frac{E-m_i}{T} = x \\ E = xT + m_i \end{array} \right|$
$= \frac{e^{\frac{\mu_i - m_i}{T}}}{2\pi^2}$
$T \ll m_i, \mu_i < m_i$


$\propto T^4, P \propto T^4$
$w_i = P_i/\rho_i = 1/3$
DIM. ANALYSIS
$\ell_{eB} = 1 \Rightarrow \frac{1}{\ell} = E = T$
$\ell \cdot T \ll m_i, \mu_i < m_i$
$n_i \simeq \frac{1}{2\pi^2} \int_{m_i}^{\infty} \frac{\sqrt{E^2 - m_i^2}\, E\, dE}{e^{\frac{E-\mu_i}{T}}} = \left| \frac{E - m_i}{T} = u \right.$
$\left. E = uT + m_i \right|$
$= \frac{e^{\frac{\mu_i - m_i}{T}}}{2\pi^2} \int_0^{\infty} \frac{(u^2T^2 + 2uTm_i)^{1/2}(uT + m_i)\, T\, du}{e^u}$


$T \ll m_i, \mu_i < m_i$
$E = \mu T + m_i$
KEEPING INTEGRAND TO LOWEST ORDER IN $\mu$ .. ($e^{\mu}$ .. BOTTOM)

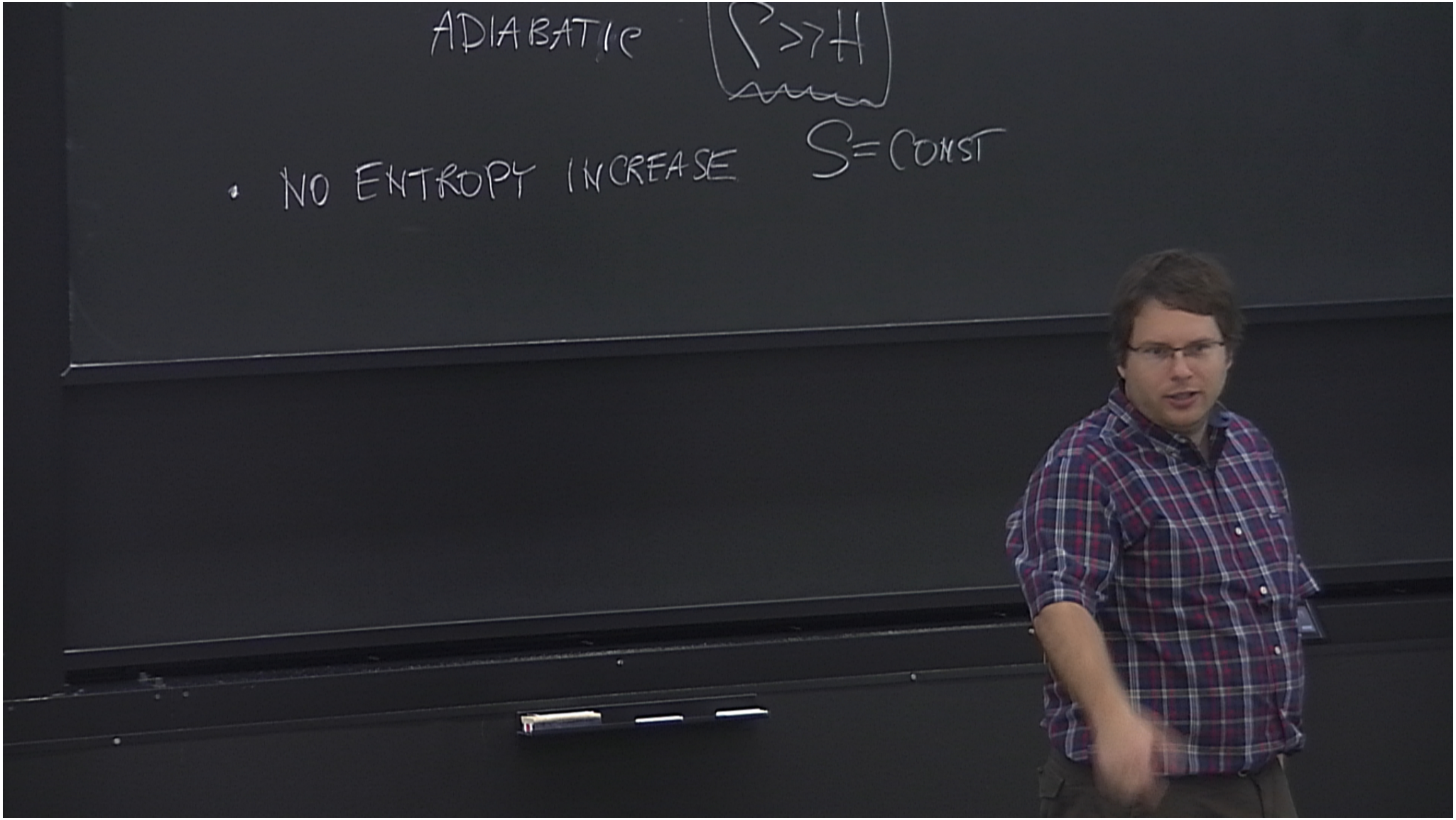

$n_i \simeq \left( \frac{m_i T}{\pi} \right)^{3/2} e^{\frac{\mu_i - m_i}{T}}$
$\simeq$
KEEPI
ORDER
$= m_i n_i$
$P_i = T m_i$
$\simeq \frac{e^{\frac{\mu_i - m_i}{T}}}{2\pi^2}$

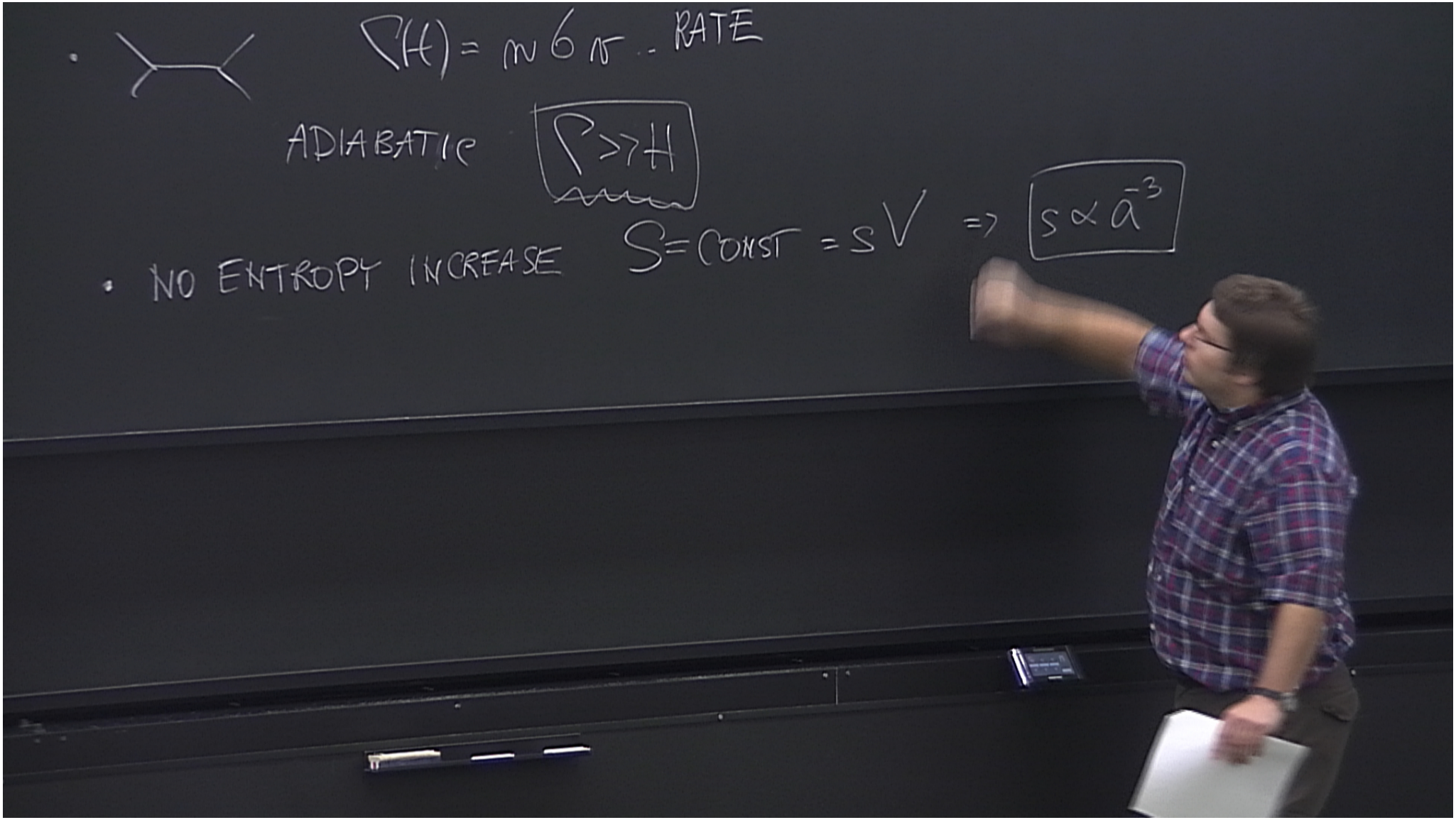
# ADIABATIC EXPANSION

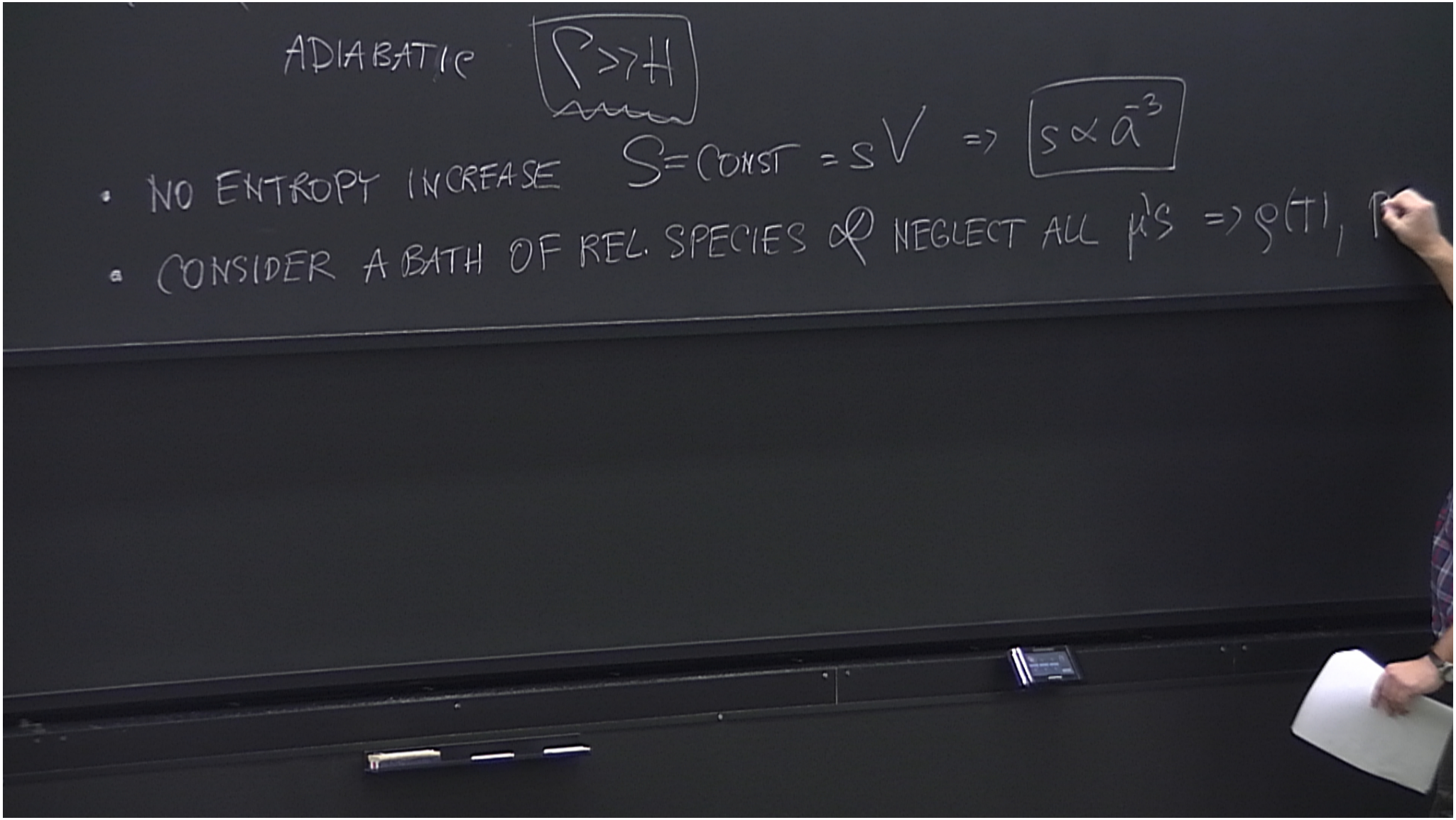


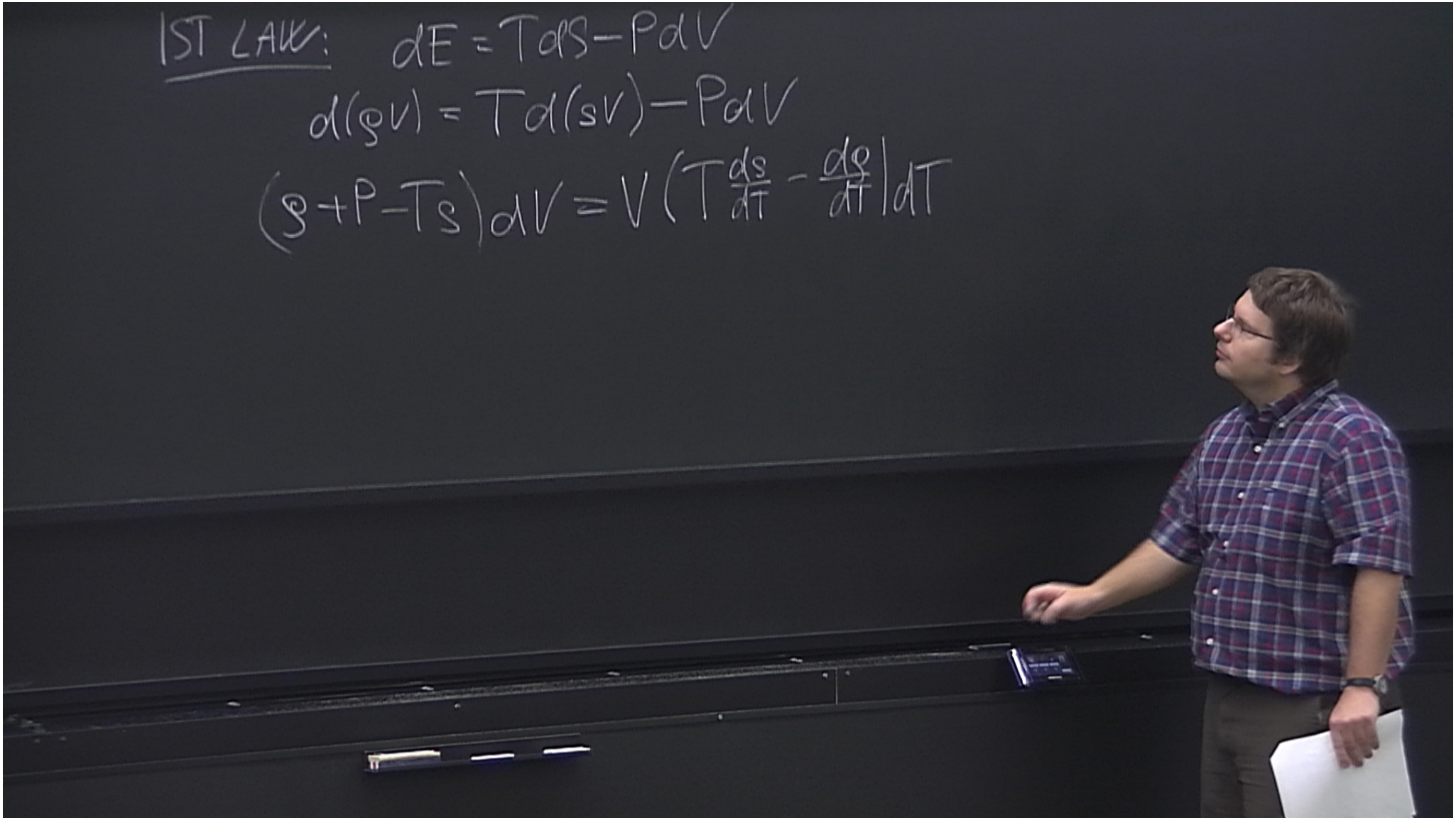
$\Gamma(H) = n\,6\pi \ldots$ RATE

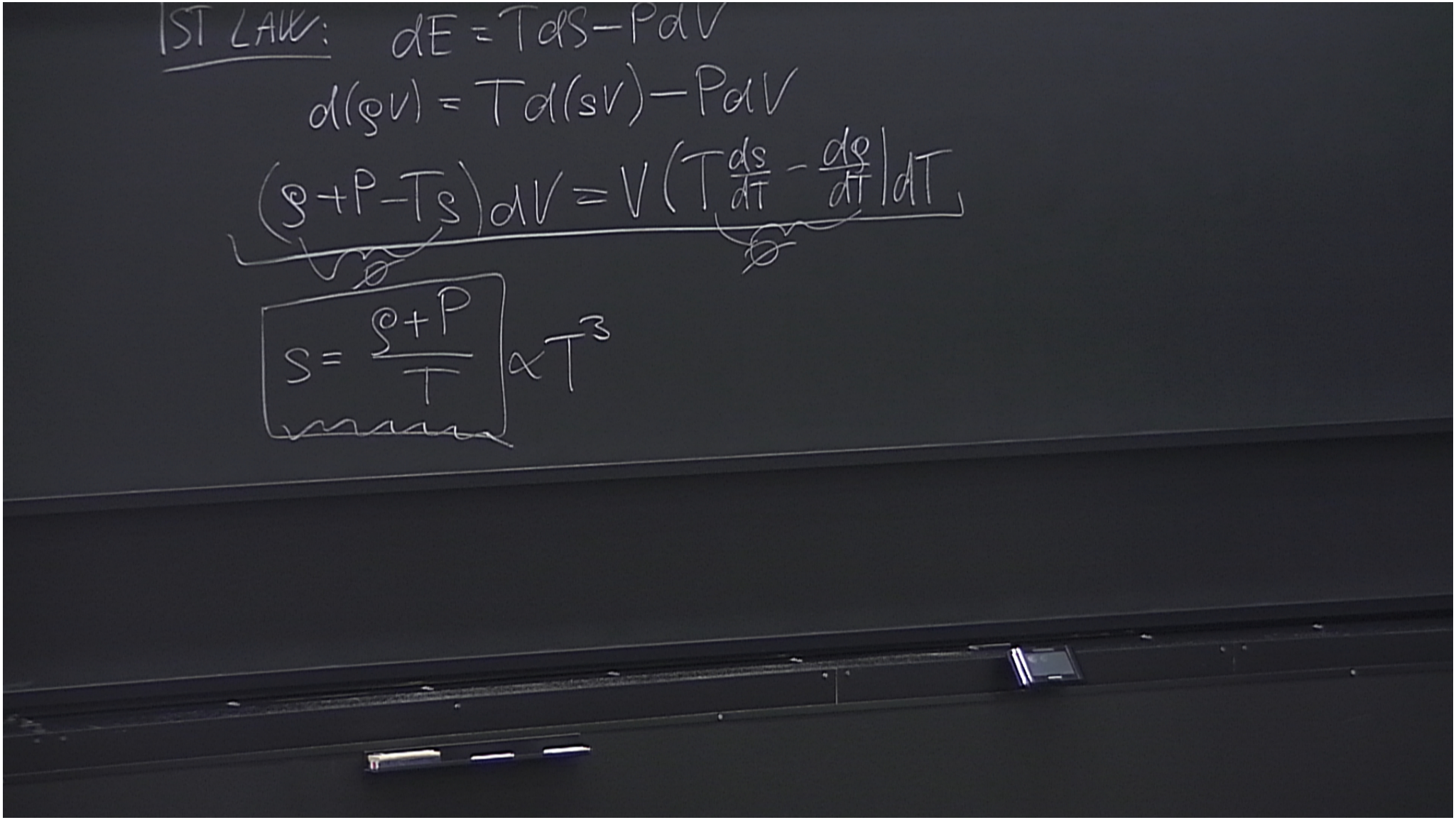
ADIABATIC $\Gamma \gg H$

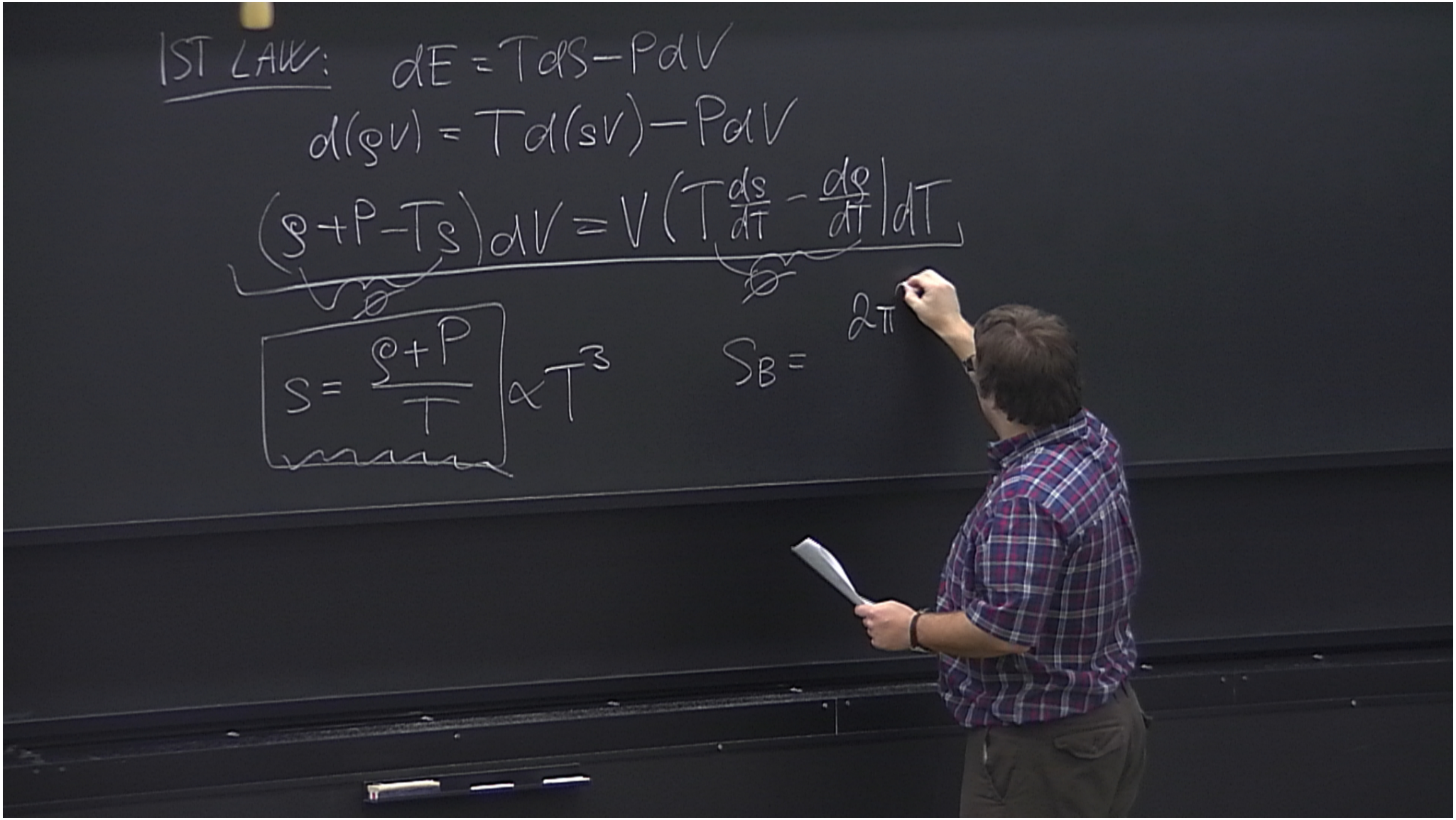

ADIABATIC
P >> H
• NO ENTROPY INCREASE
S = CONST

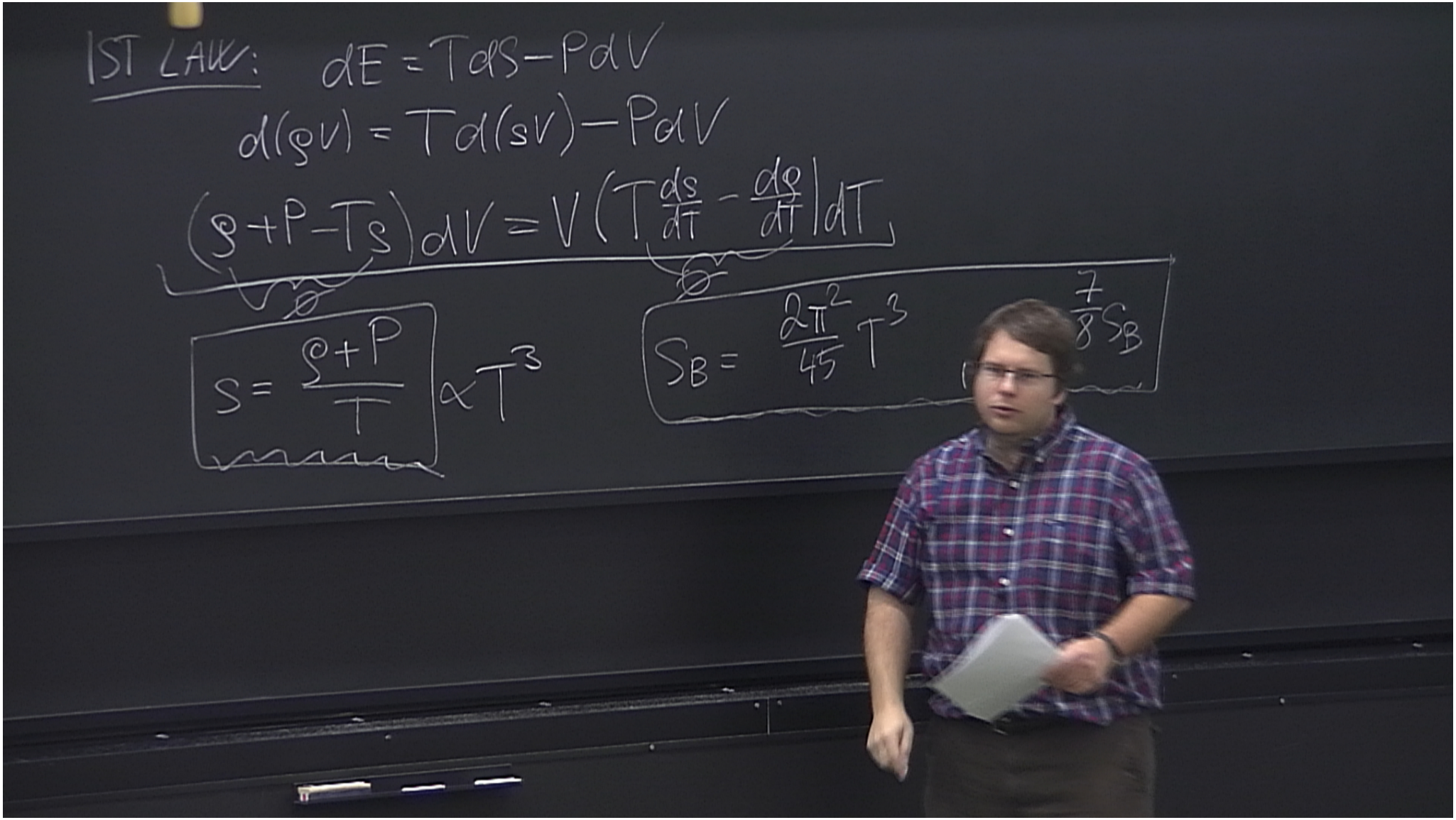

Γ(t) = n 6 π . RATE
ADIABATIC
P >> H
NO ENTROPY INCREASE
S = CONST = s V =>
s ∝ a^-3

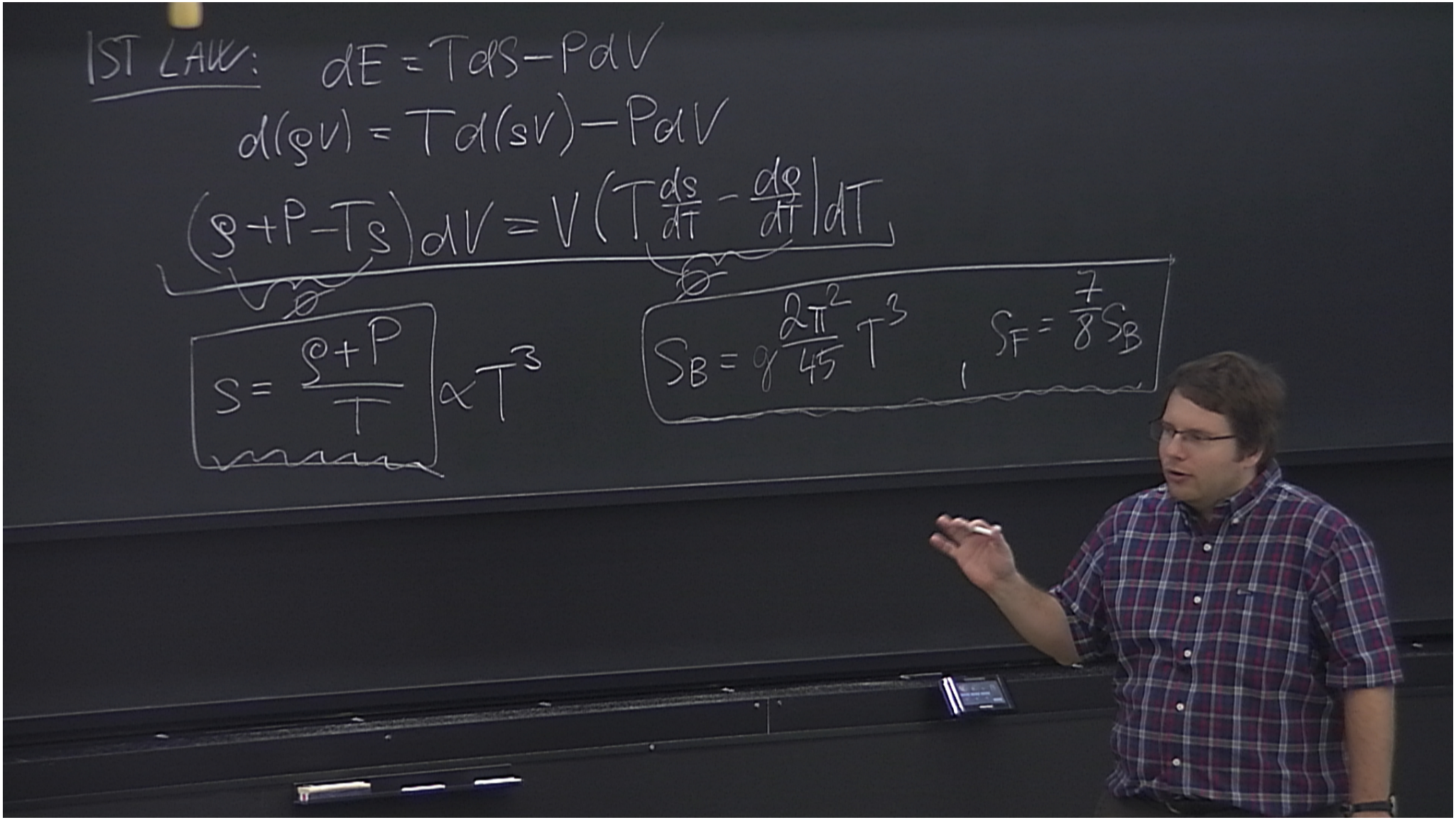

ADIABATIC
$\Gamma \gg H$
• NO ENTROPY INCREASE $S = \text{CONST} = sV \Rightarrow s \propto a^{-3}$
• CONSIDER A BATH OF REL. SPECIES & NEGLECT ALL $\mu$'S $\Rightarrow \rho(T)$,

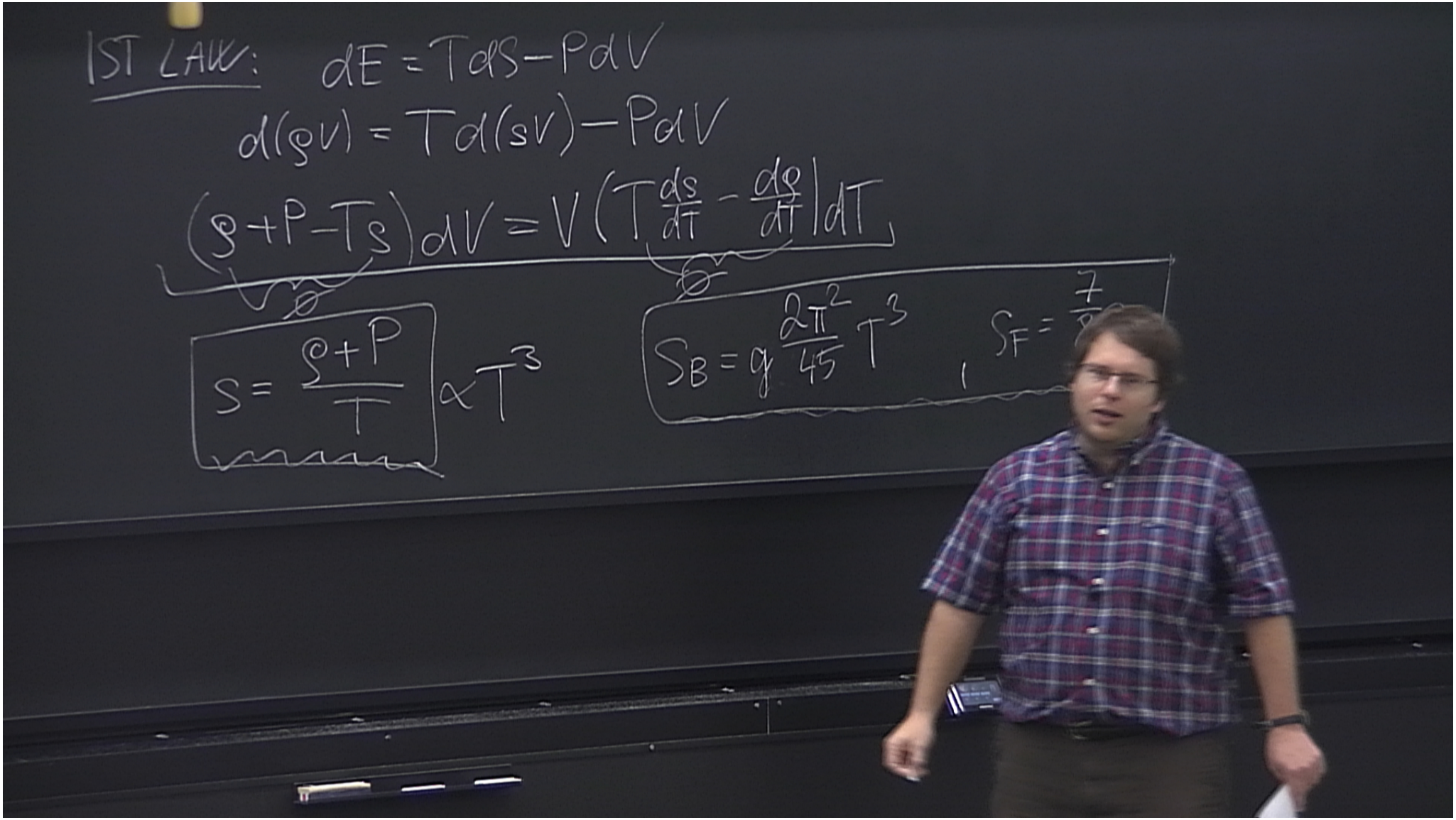

1ST LAW: dE = TdS − PdV
d(ρV) = Td(sV) − PdV
(ρ + P − Ts)dV = V(T ds/dT − dρ/dT)dT

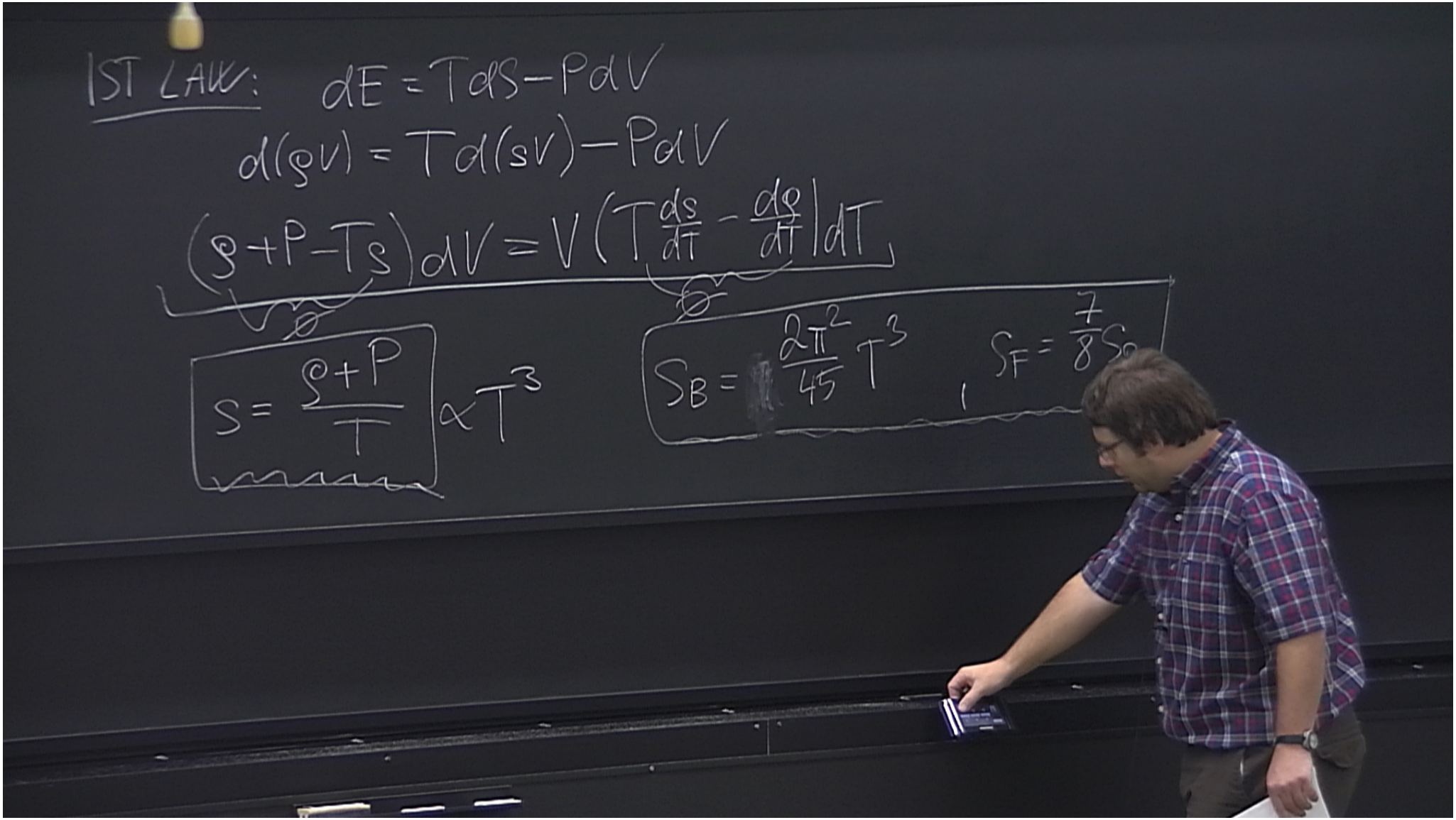

1ST LAW: $dE = TdS - PdV$
$d(\rho V) = T\,d(sV) - P\,dV$
$(\rho + P - Ts)\,dV = V\left(T\frac{ds}{dT} - \frac{d\rho}{dT}\right)dT$
$s = \frac{\rho + P}{T} \propto T^3$

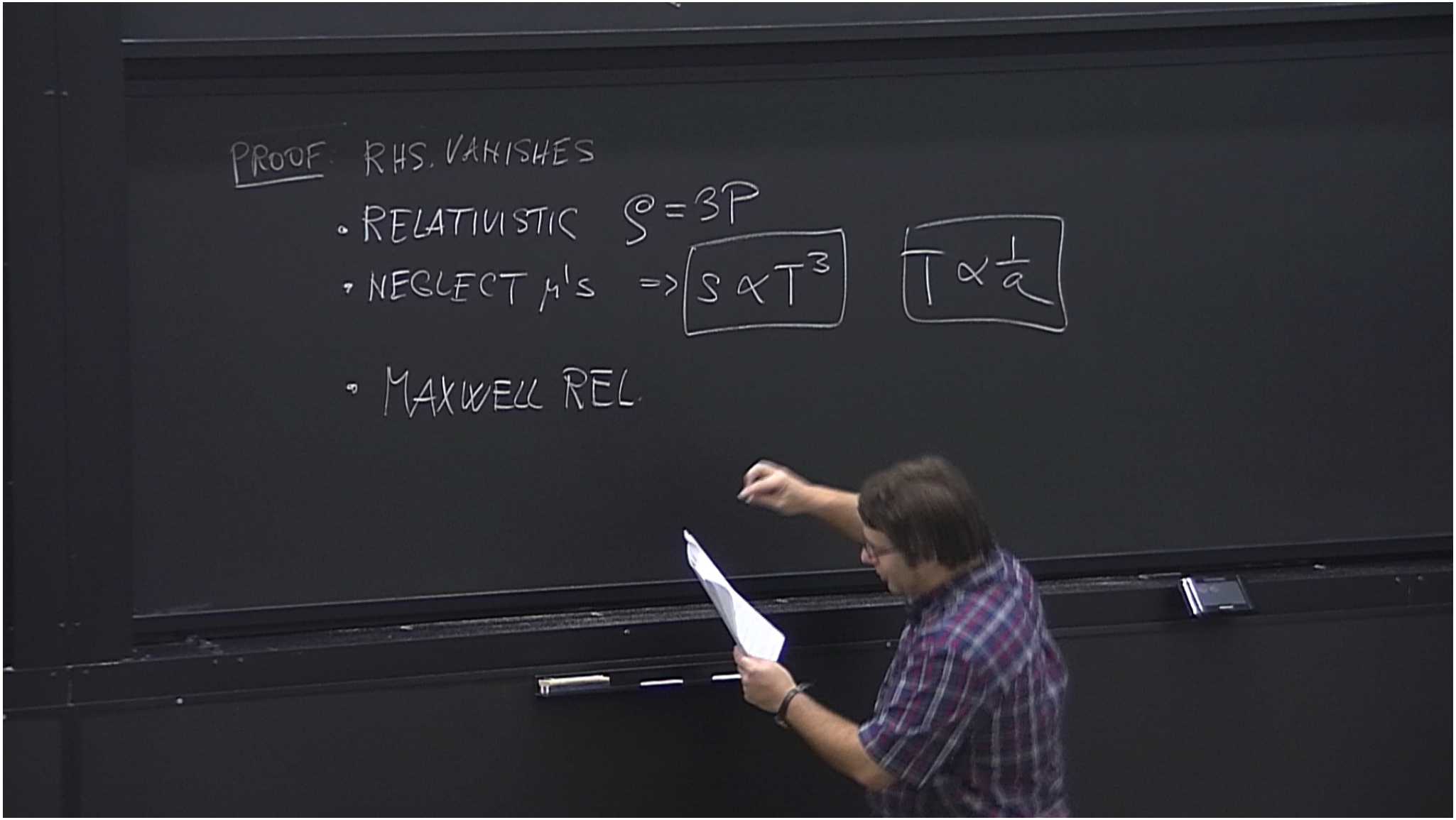

1ST LAW: $dE = TdS - PdV$
$d(\rho V) = T\,d(sV) - P\,dV$
$(\rho + P - Ts)\,dV = V\left(T\frac{ds}{dT} - \frac{d\rho}{dT}\right)dT$
$s = \frac{\rho + P}{T} \propto T^3$
$S_B =$
$2\pi$

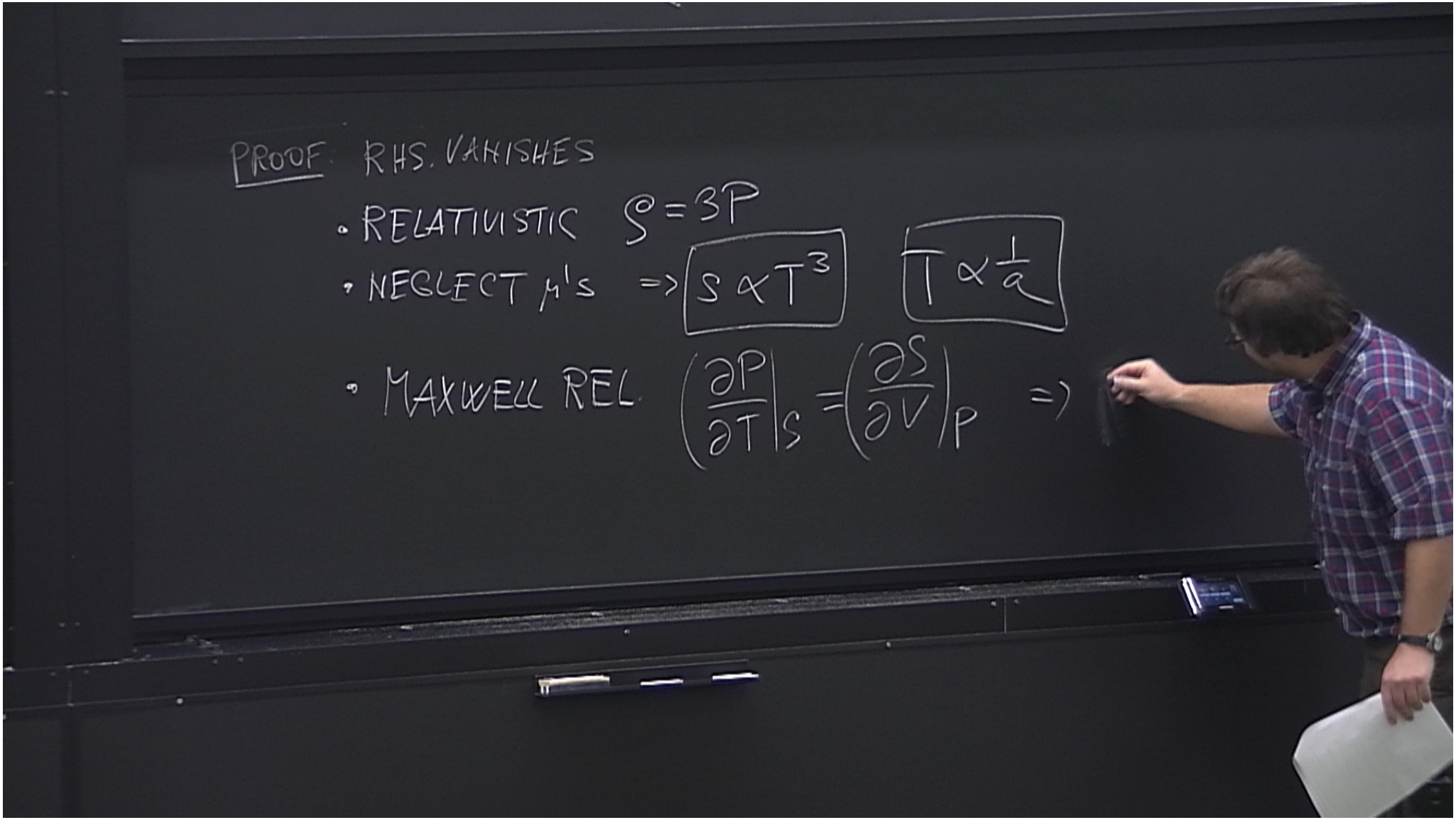

1ST LAW: $dE = TdS - PdV$
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$s = \frac{\rho + P}{T} \propto T^3$
$s_B = \frac{2\pi^2}{45}T^3$
$\frac{7}{8}s_B$

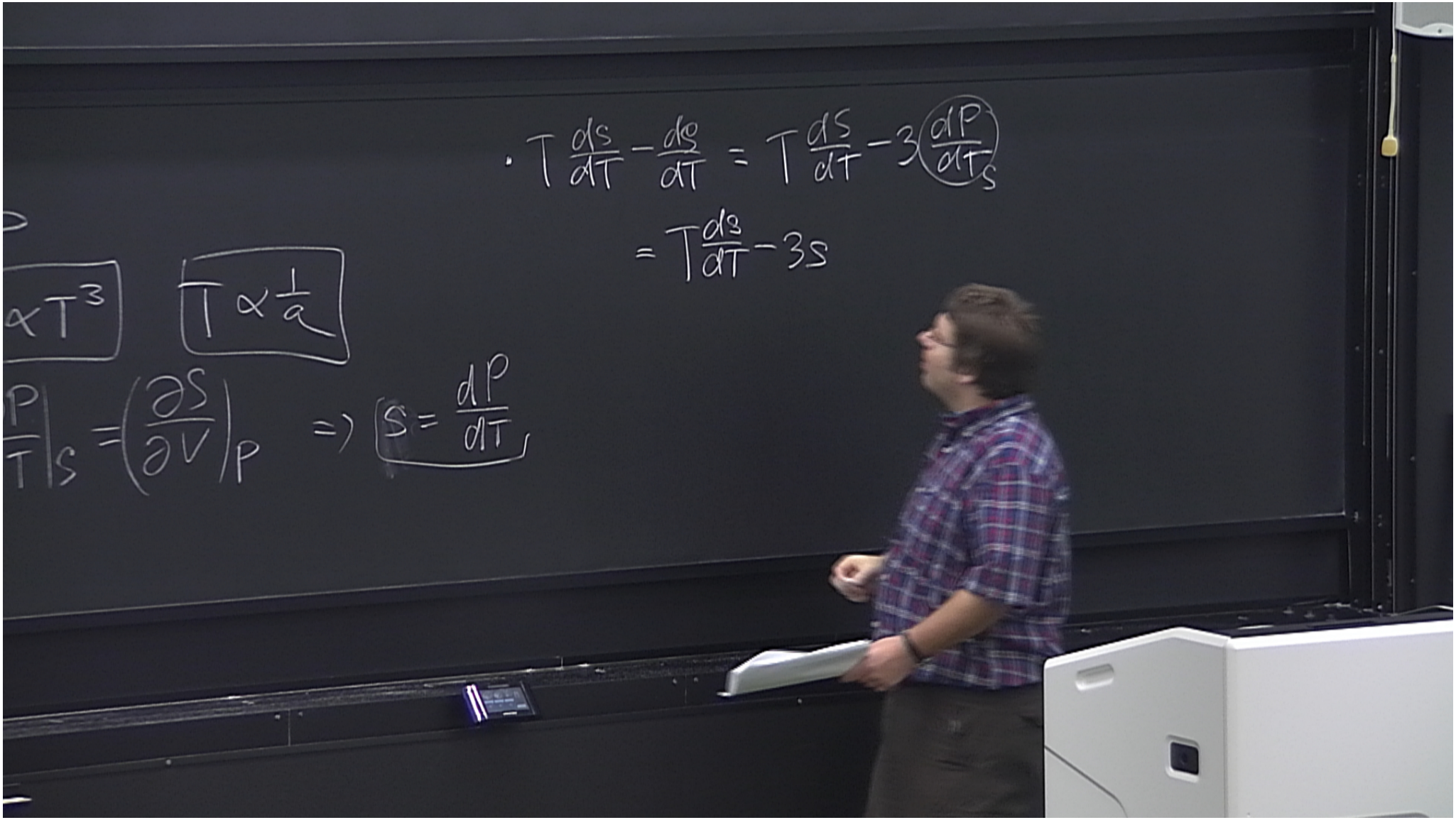

1ST LAW: $dE = TdS - PdV$
$d(\rho V) = Td(sV) - PdV$
$(\rho + P - Ts)dV = V(T\frac{ds}{dT} - \frac{d\rho}{dT})dT$
$s = \frac{\rho + P}{T} \propto T^3$
$s_B = g\frac{2\pi^2}{45}T^3$, $s_F = \frac{7}{8}s_B$

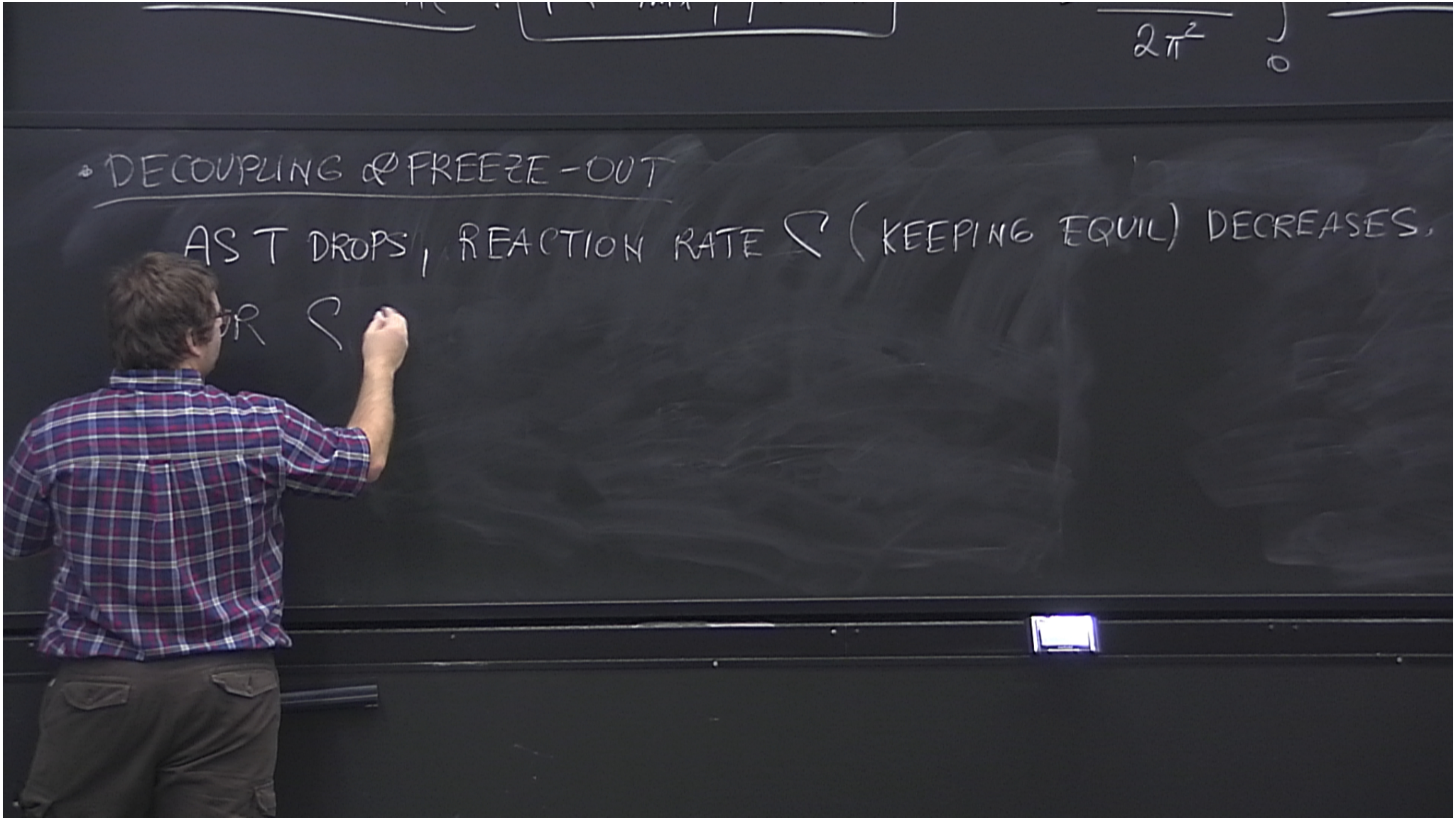

1ST LAW: $dE = TdS - PdV$
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$s = \frac{\rho + P}{T} \propto T^3$
$s_B = g\frac{2\pi^2}{45}T^3$, $s_F = \frac{7}{8}$

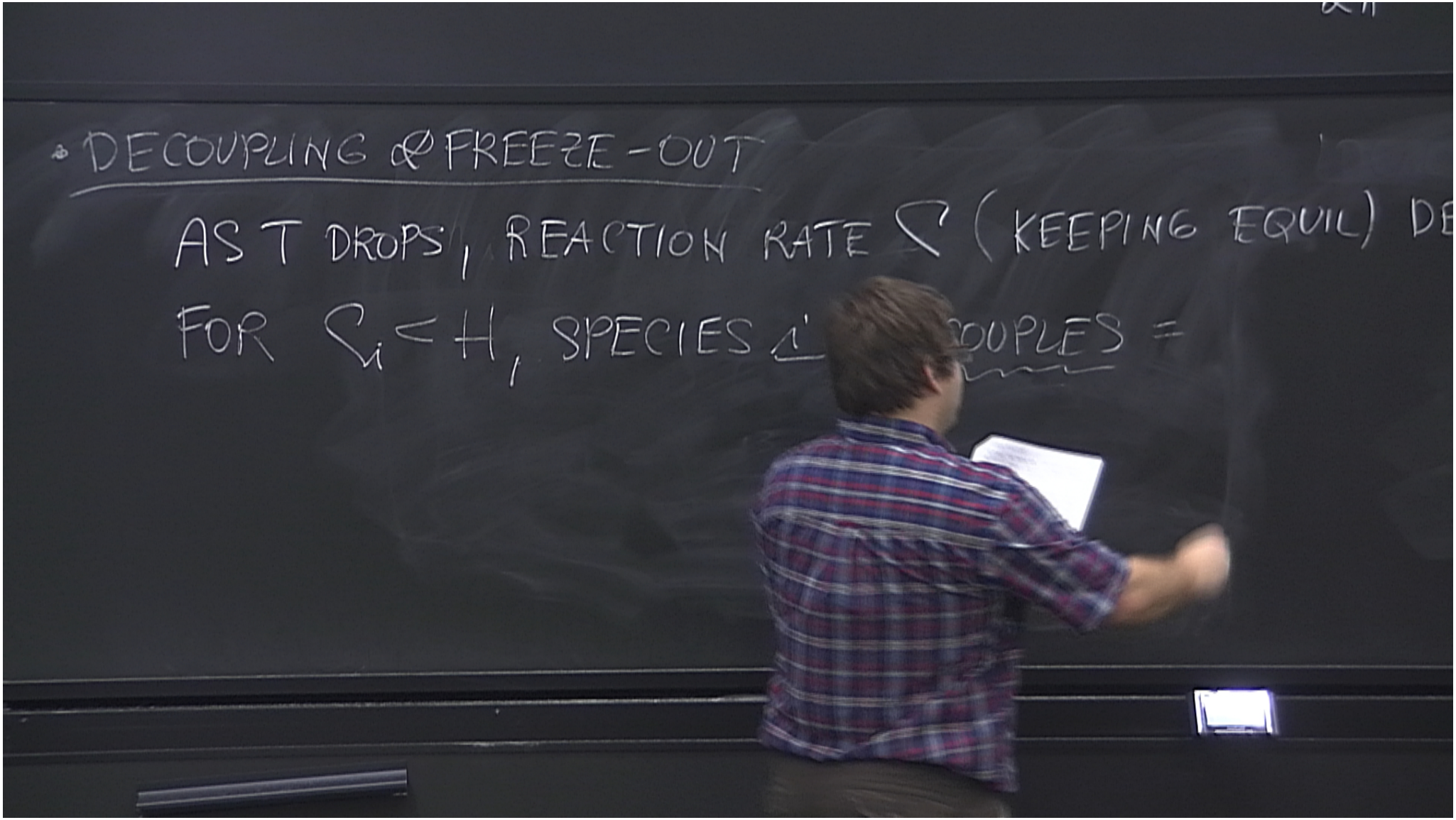

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$s = \frac{\rho + P}{T} \propto T^3$
$s_B = \frac{2\pi^2}{45}T^3$, $s_F = \frac{7}{8}s_B$

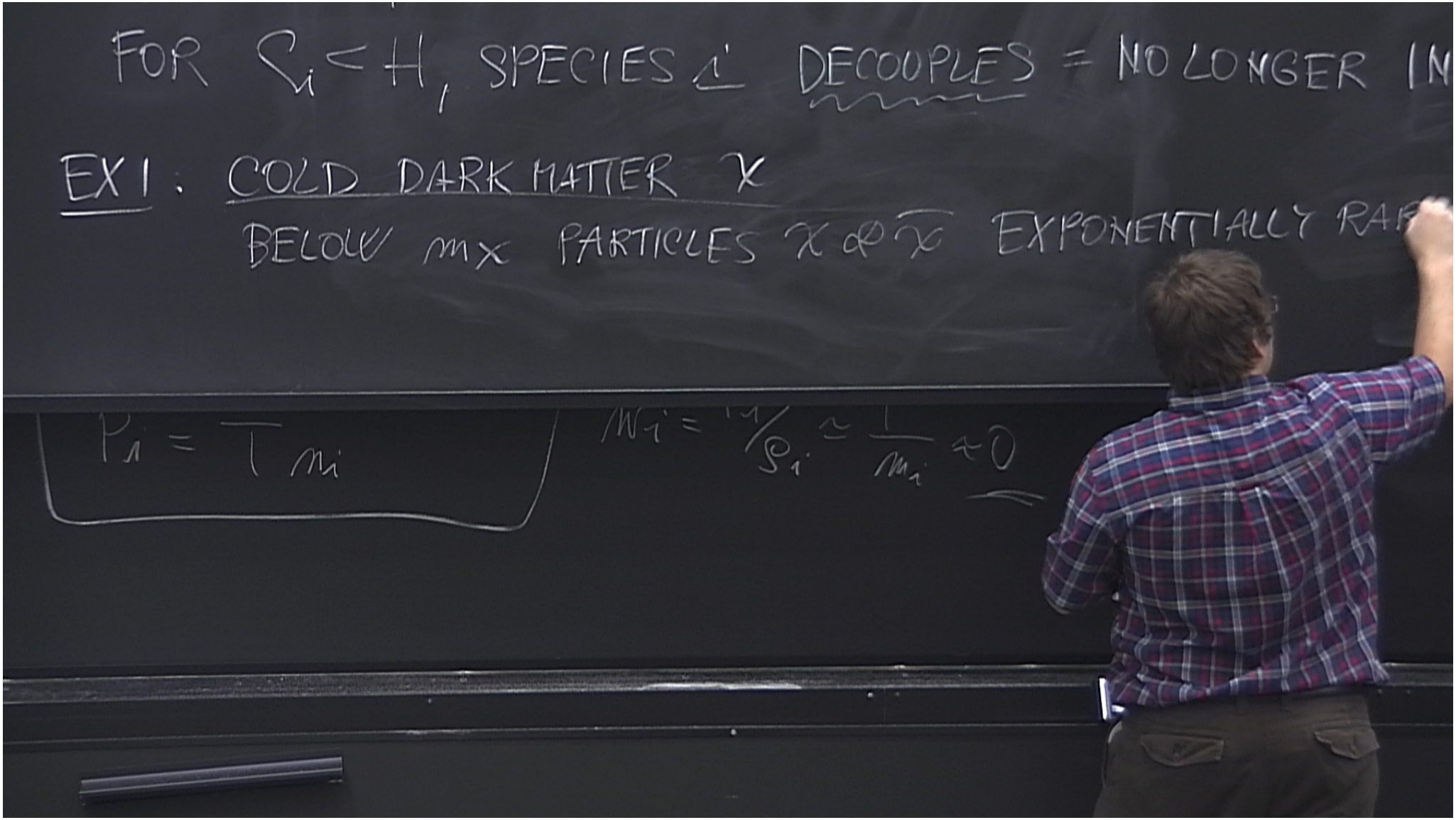

PROOF: RHS. VANISHES
• RELATIVISTIC ρ = 3P
• NEGLECT μ's => S ∝ T³
T ∝ 1/a
• MAXWELL REL.

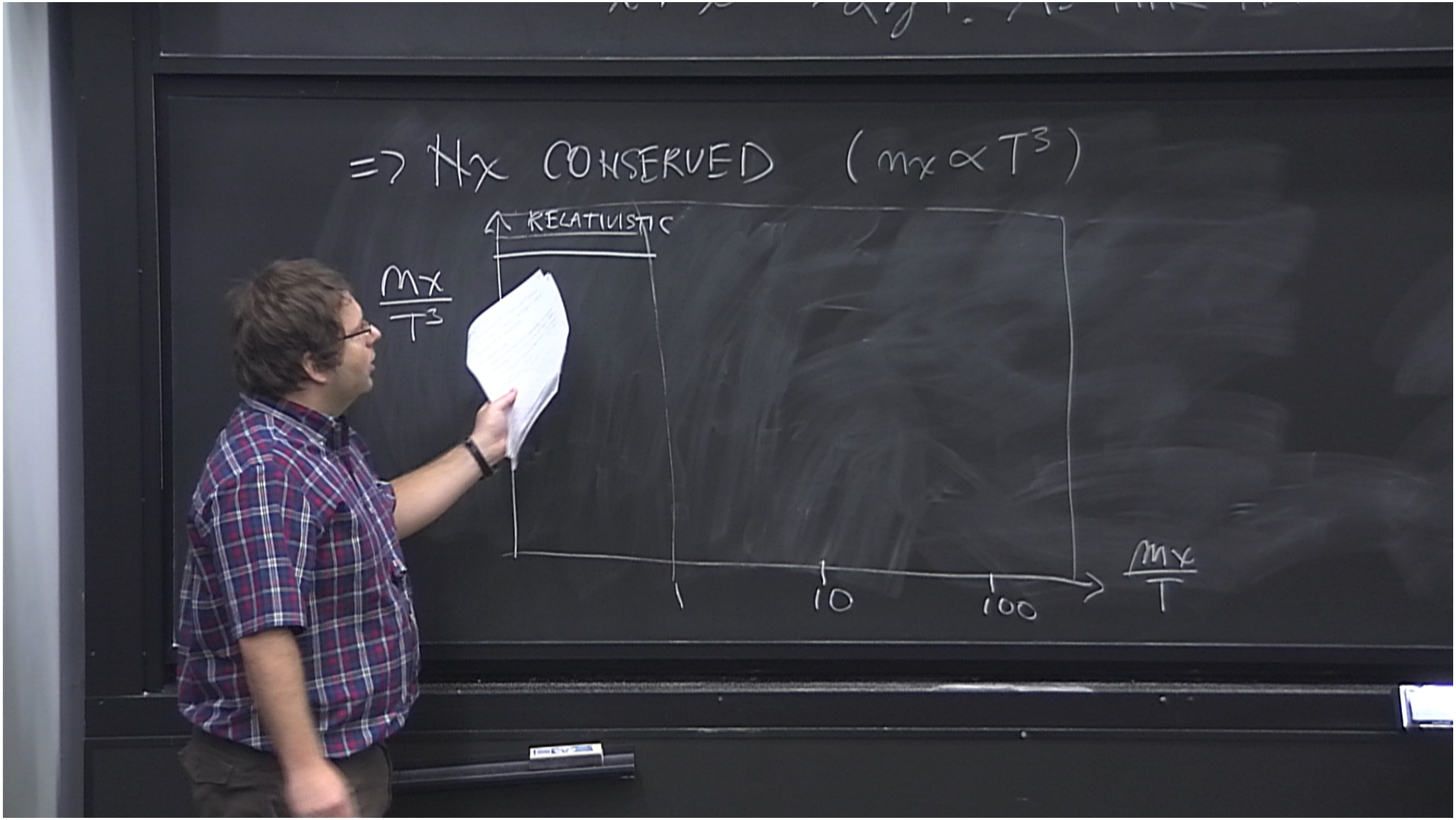

PROOF: RHS. VANISHES
• RELATIVISTIC $\rho = 3P$
• NEGLECT $\mu$'s $\Rightarrow$ $S \propto T^3$ $T \propto \frac{1}{a}$
• MAXWELL REL $\left(\frac{\partial P}{\partial T}\right)_S = \left(\frac{\partial S}{\partial V}\right)_P \Rightarrow$

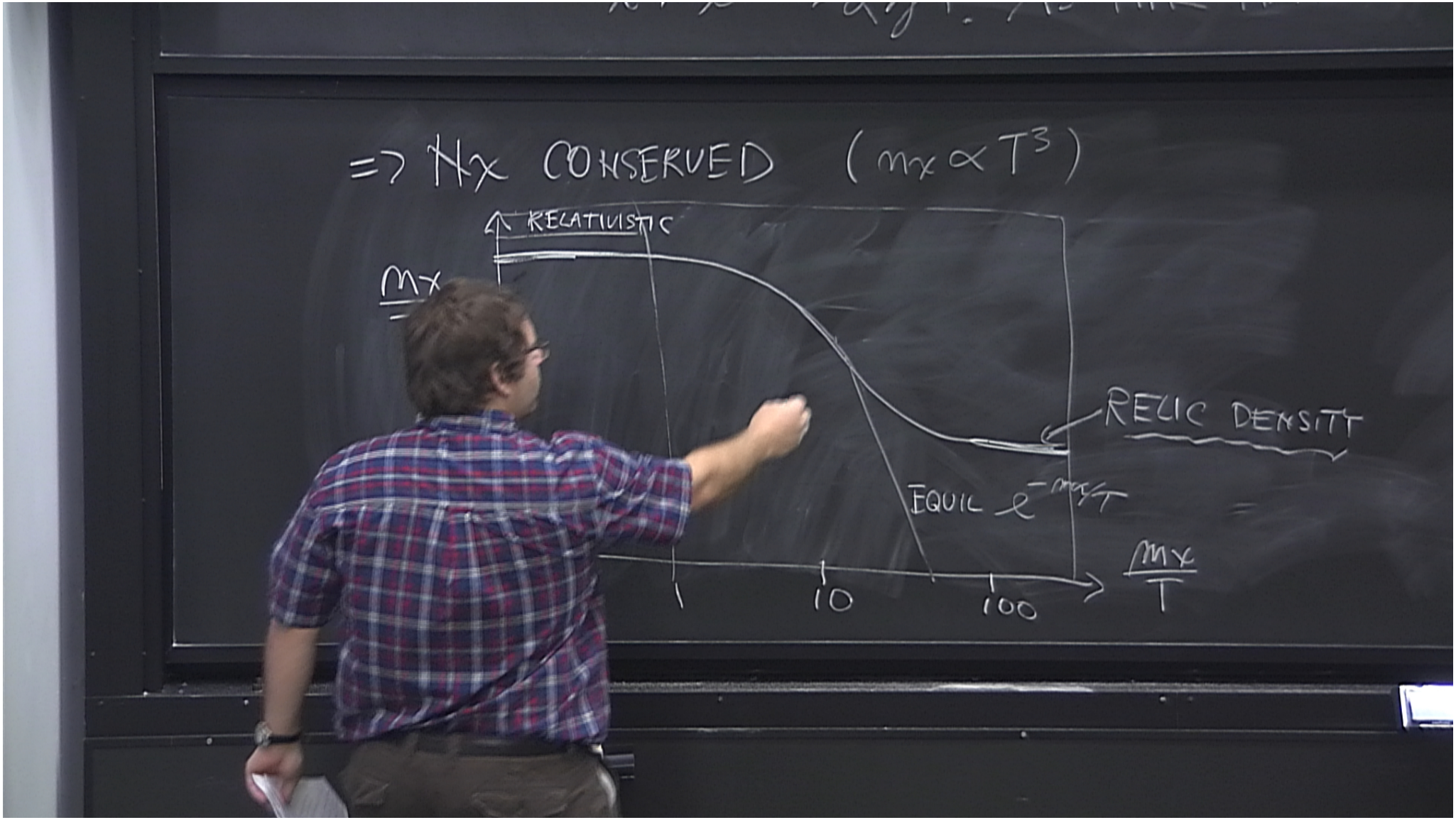

$\cdot T\frac{ds}{dT} - \frac{de}{dT} = T\frac{ds}{dT} - 3\left(\frac{dP}{dT}\right)_S$
$= T\frac{d3}{dT} - 3s$
$\propto T^3$
$T \propto \frac{1}{a}$
$\left(\frac{P}{T}\right)_S = \left(\frac{\partial S}{\partial V}\right)_P \Rightarrow s = \frac{dP}{dT}$

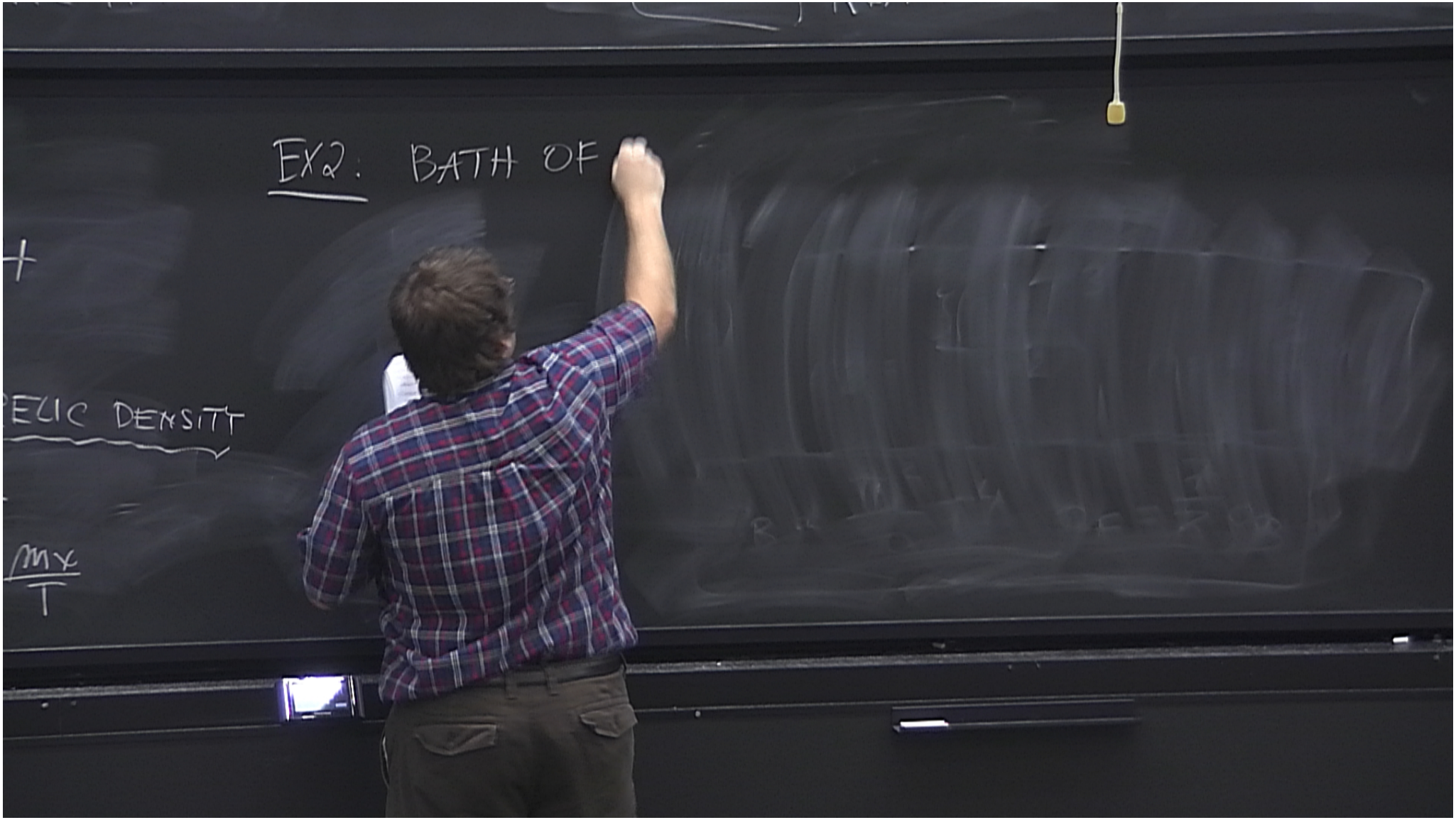

$2\pi^2$
• DECOUPLING & FREEZE-OUT
AS T DROPS, REACTION RATE ↘ (KEEPING EQUIL) DECREASES.
OR ↘

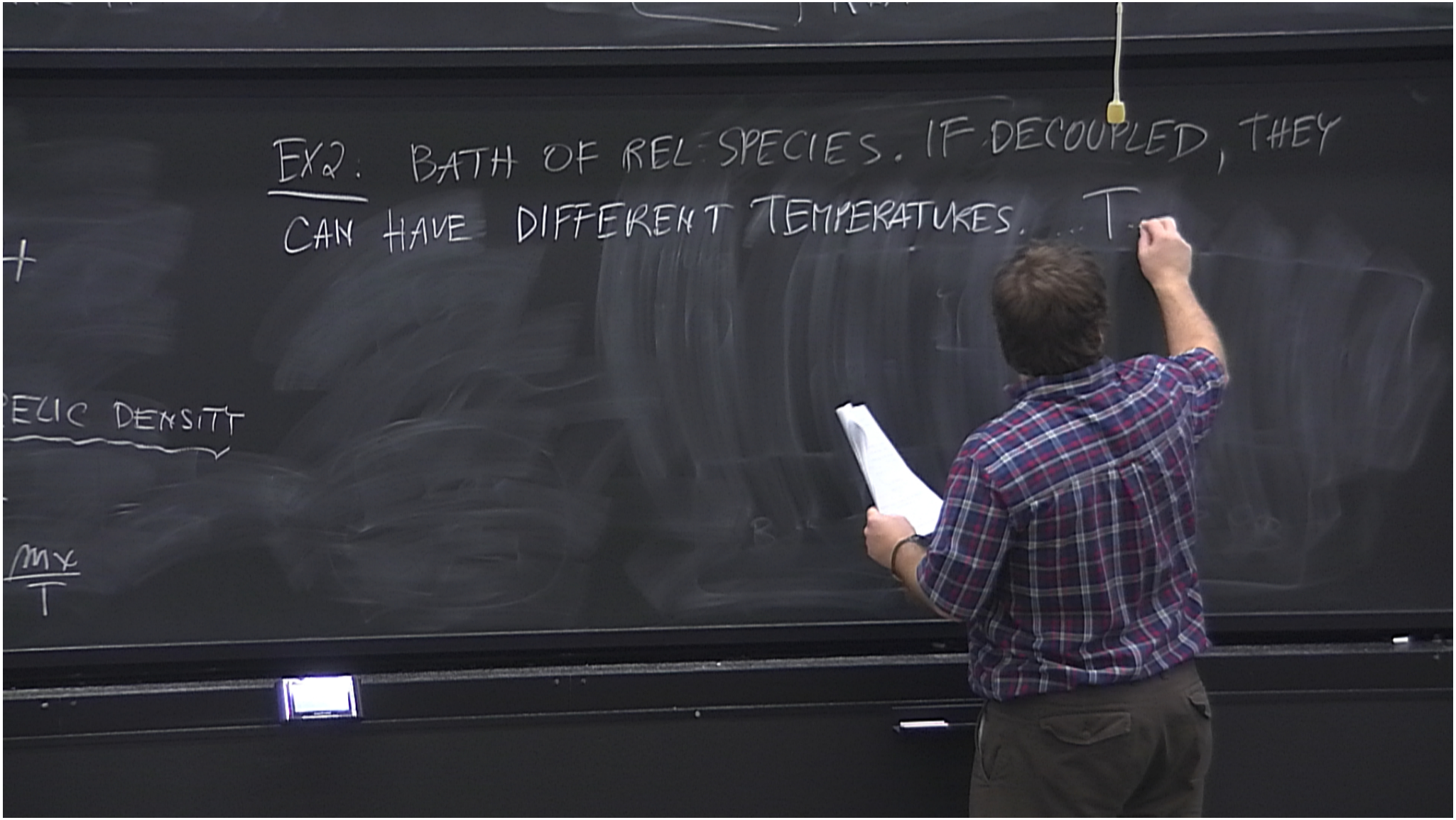

• DECOUPLING & FREEZE-OUT
AS T DROPS, REACTION RATE
(KEEPING EQUIL)
FOR
SPECIES

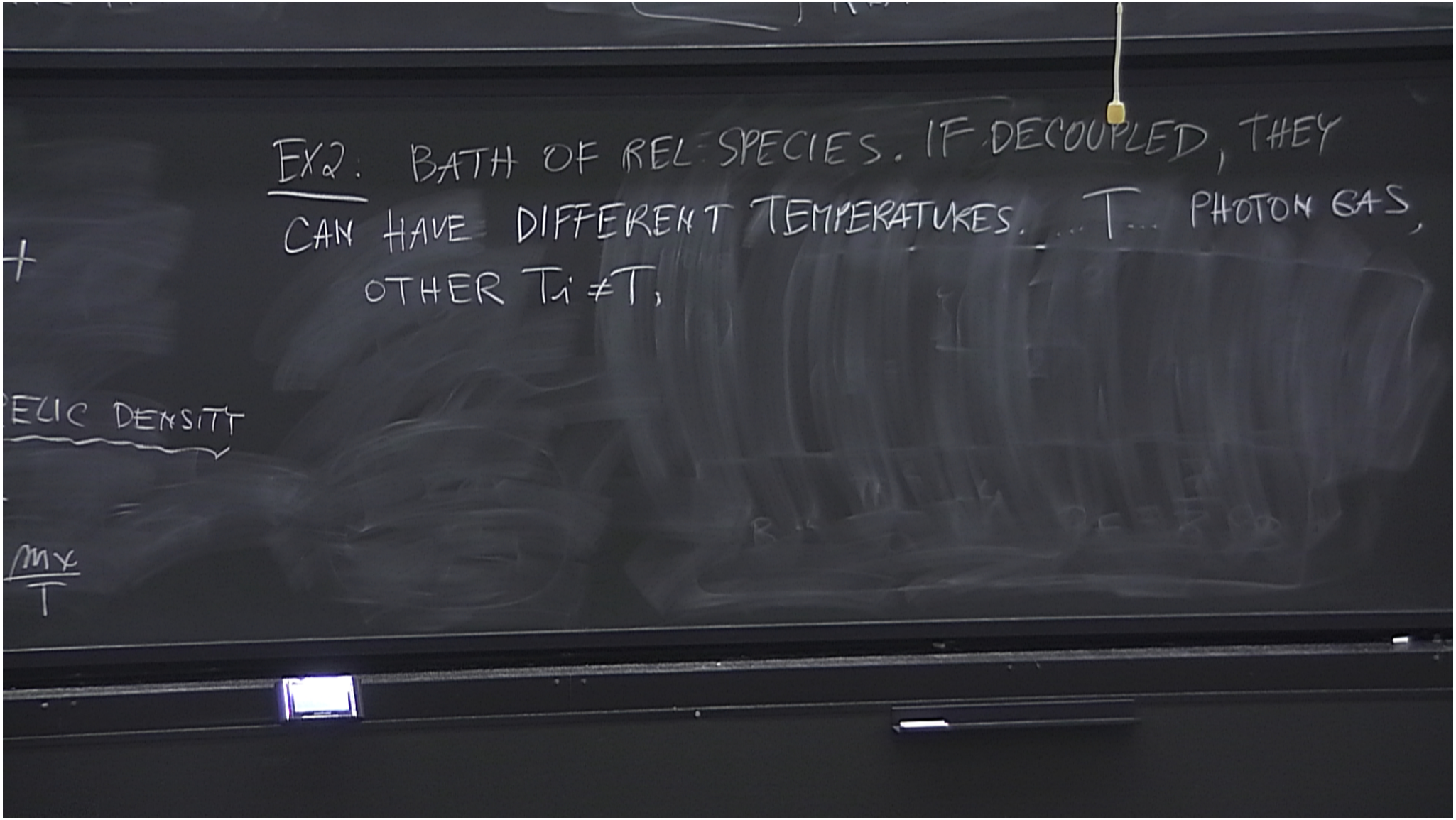

FOR $\Gamma_i < H$, SPECIES $i$ DECOUPLES = NO LONGER IN
EX 1: COLD DARK MATTER $\chi$
BELOW $m_\chi$ PARTICLES $\chi \& \bar{\chi}$ EXPONENTIALLY RA
$P_i = T m_i$
$N_i = n/s_i \simeq \frac{1}{m_i} \to 0$

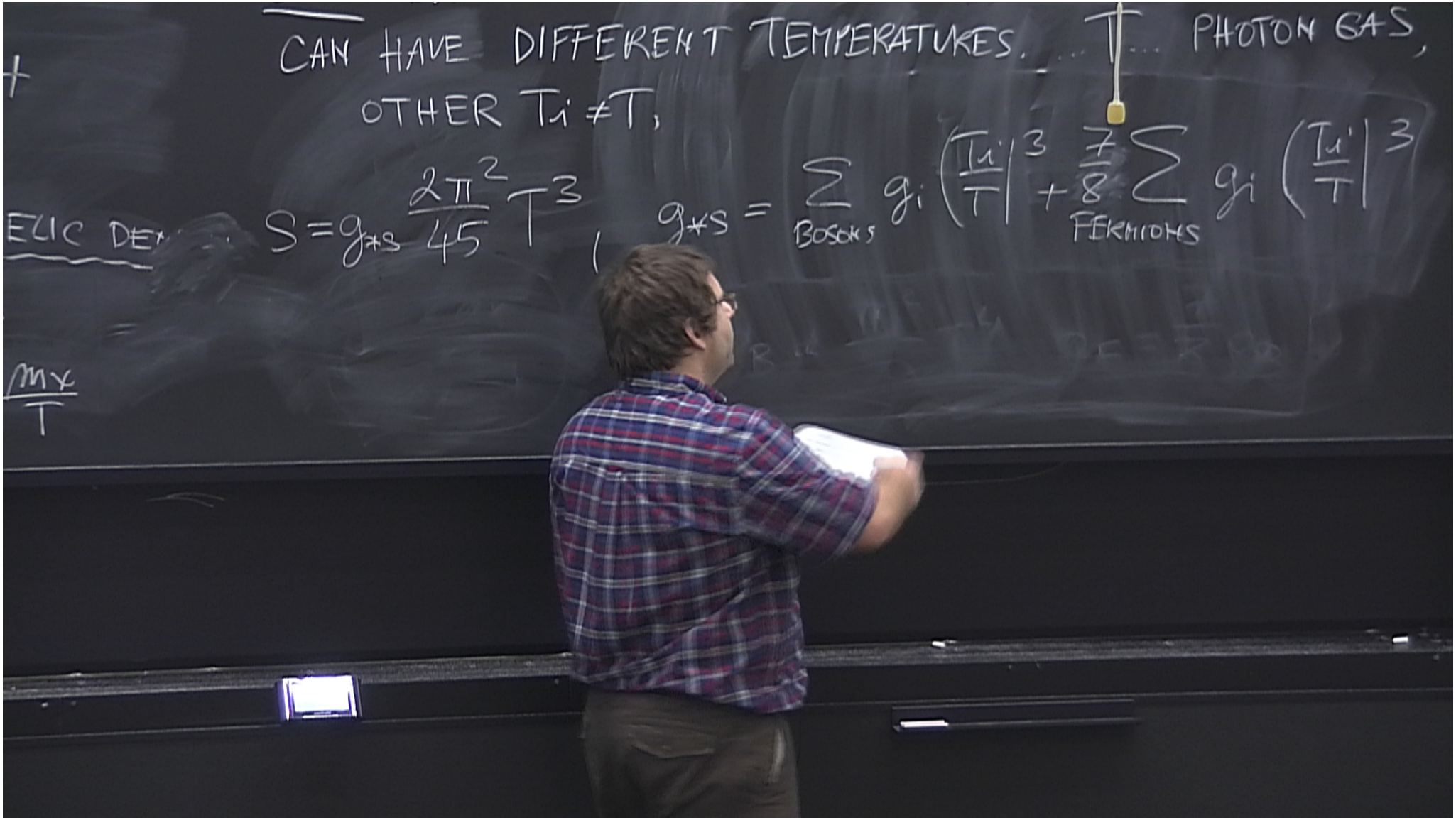

$\Rightarrow N_\chi$ CONSERVED $(m_\chi \propto T^3)$
RELATIVISTIC
$\frac{m_\chi}{T^3}$
1
10
100
$\frac{m_\chi}{T}$

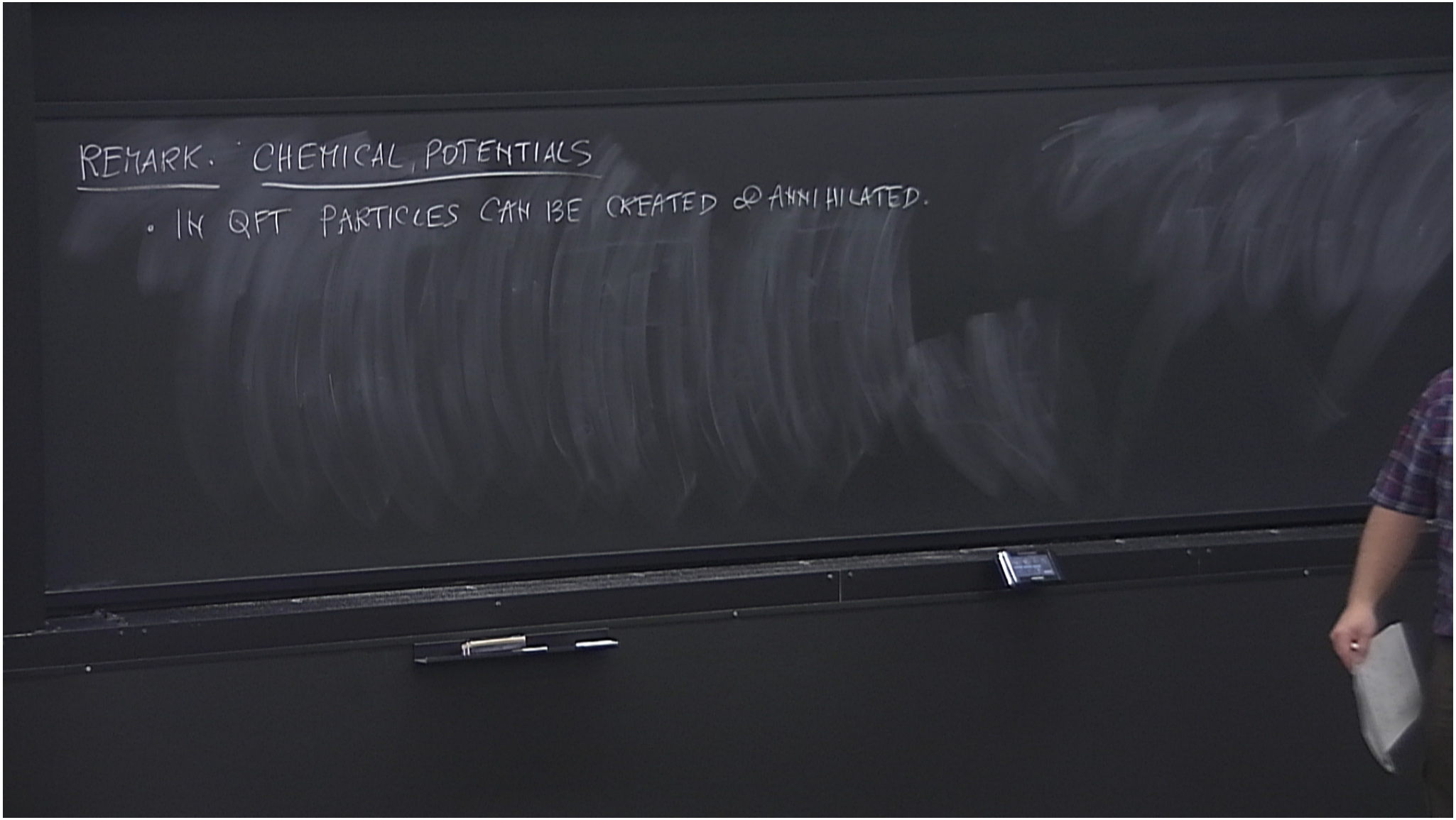

$\Rightarrow N_\chi$ CONSERVED $(n_\chi \propto T^3)$
RELATIVISTIC
$m_\chi$
RELIC DENSITY
EQUIL $e^{-m_\chi/T}$
1
10
100
$\frac{m_\chi}{T}$

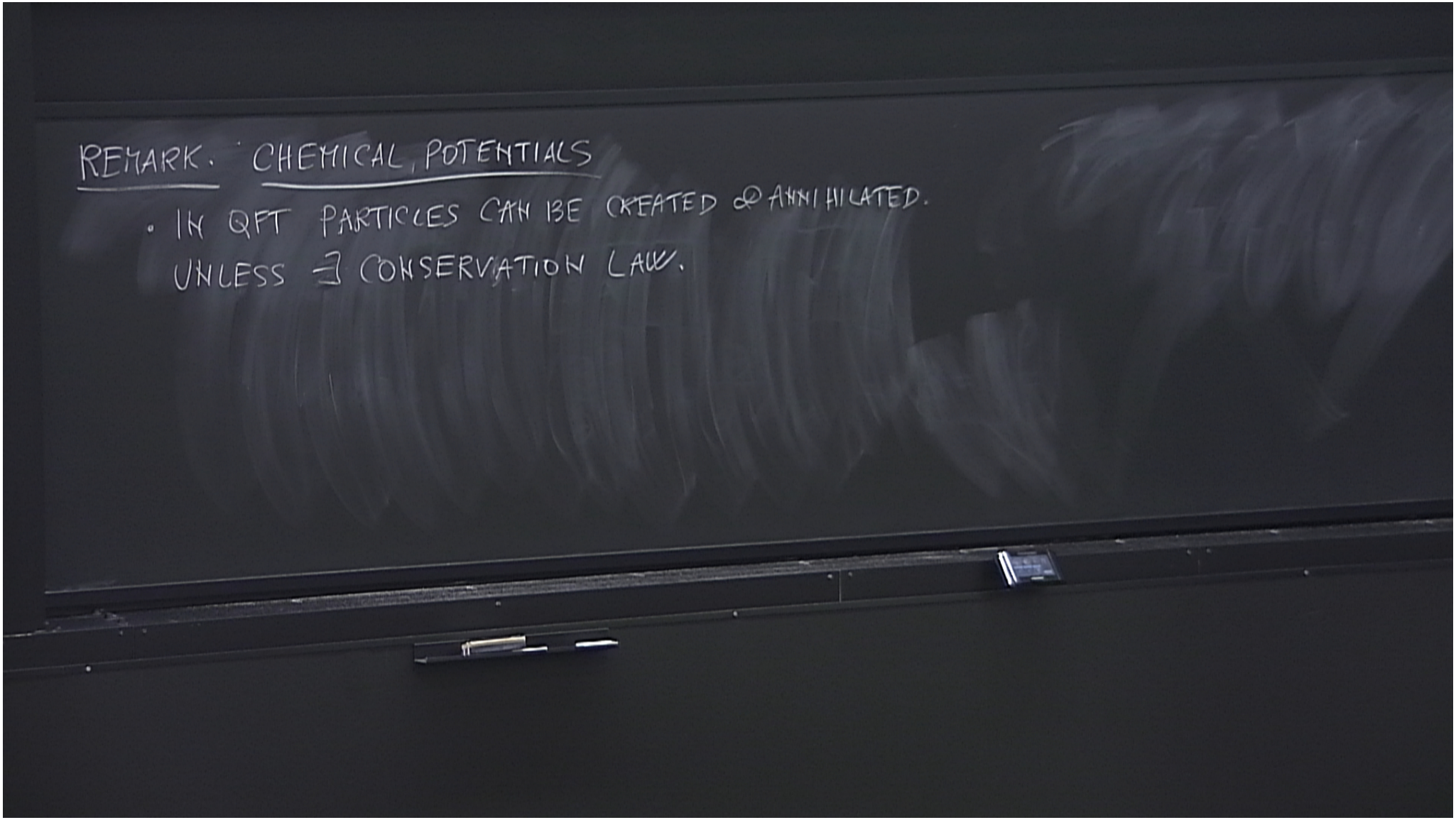

EX2. BATH OF
ELIC DENSITY

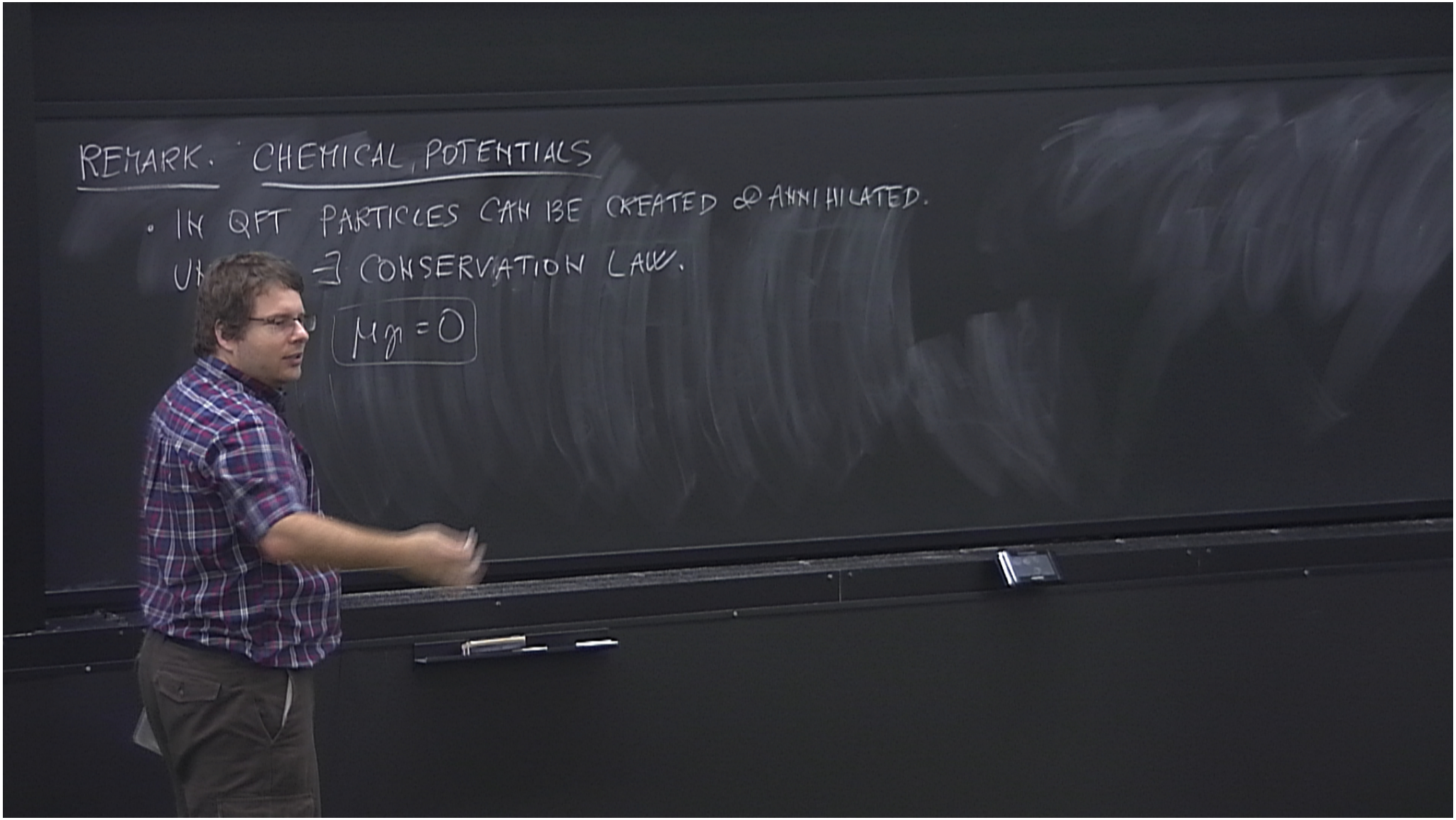

EX2. BATH OF REL SPECIES. IF DECOUPLED, THEY
CAN HAVE DIFFERENT TEMPERATURES. . . T
ELIC DENSITY

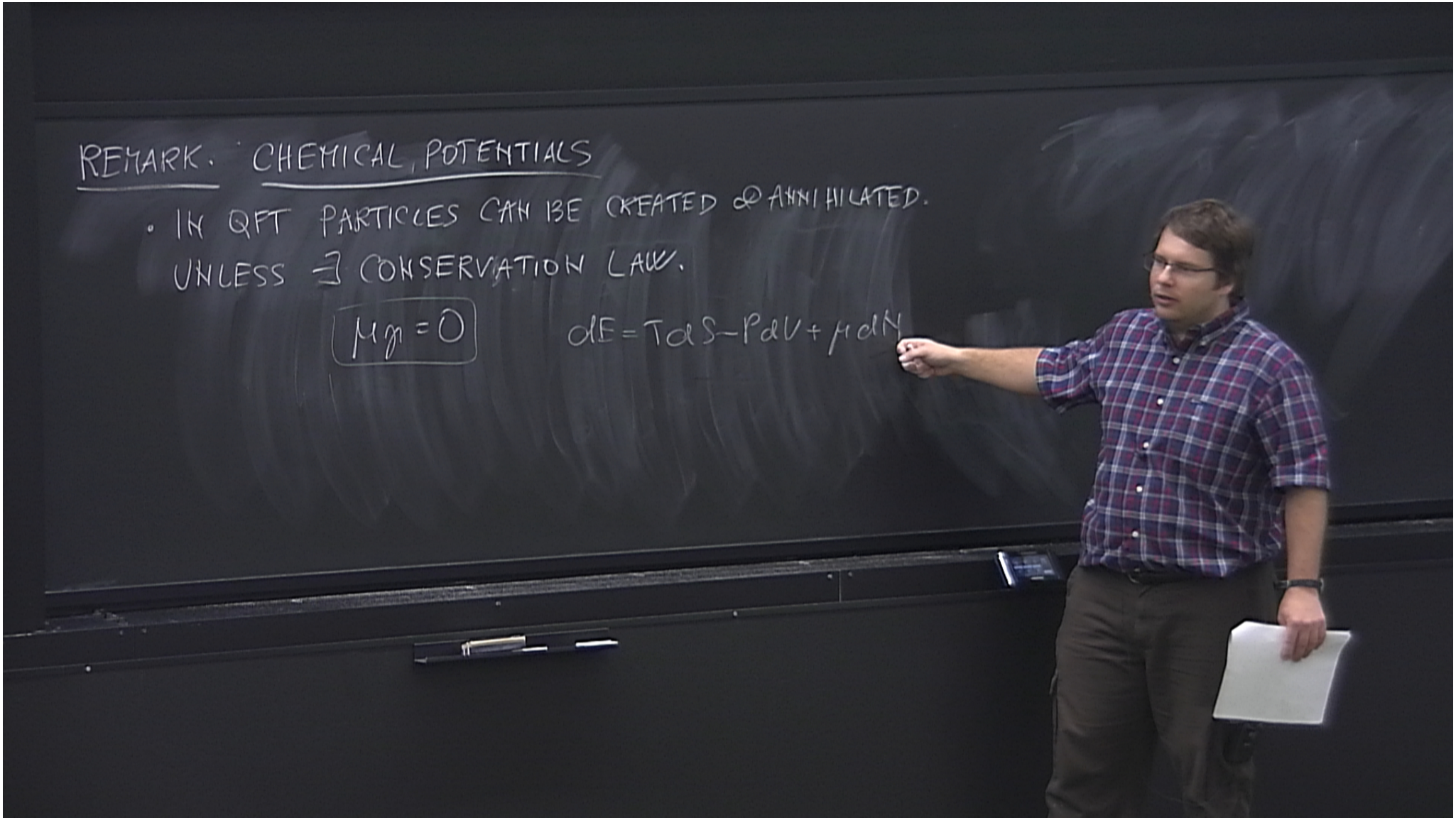

EX2. BATH OF REL SPECIES. IF DECOUPLED, THEY
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OTHER Ti ≠ T,
ELIC DENSITY
mx
T

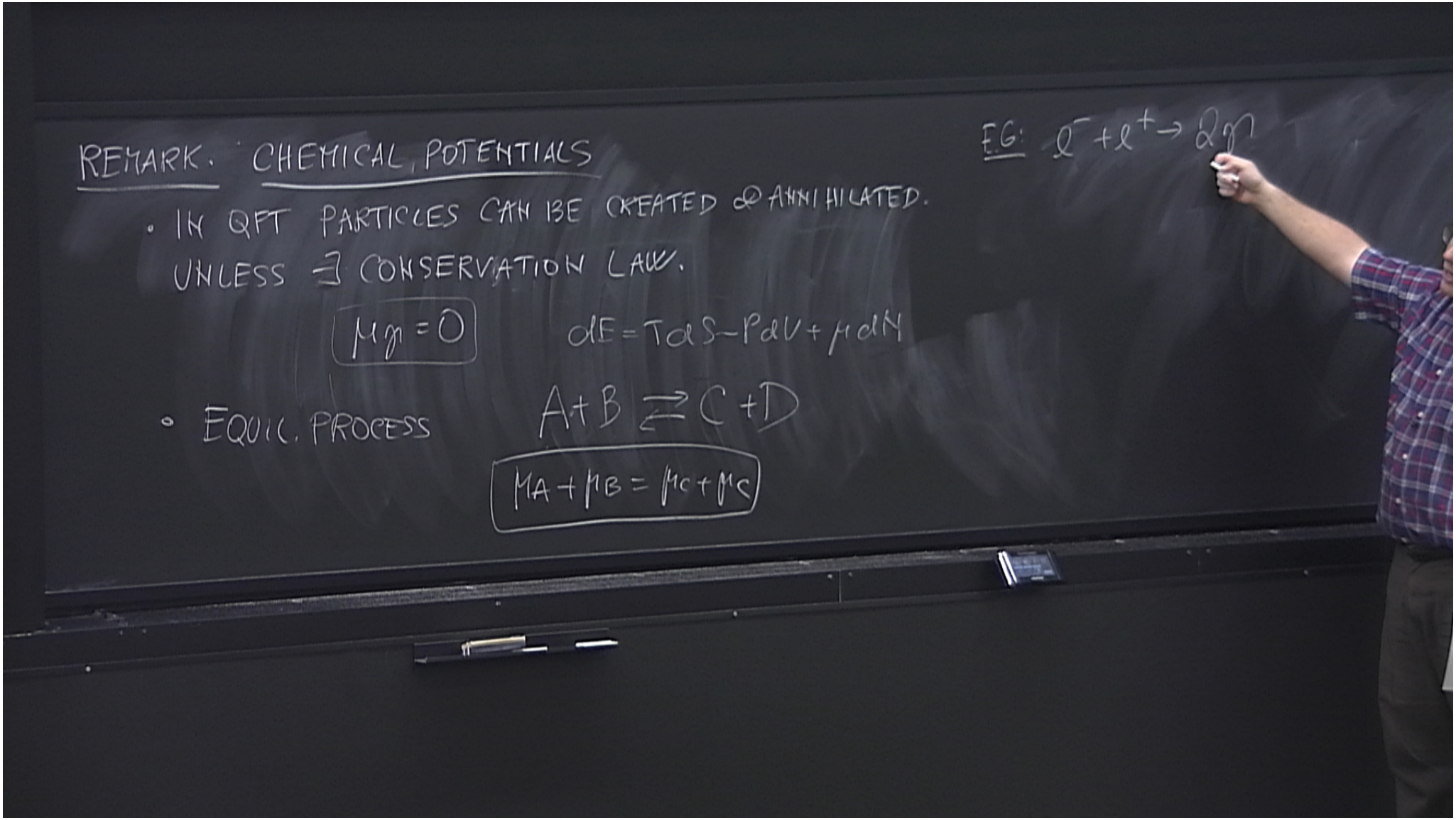

CAN HAVE DIFFERENT TEMPERATURES. T PHOTON GAS,
OTHER Ti ≠ T,
$S = g_{*S} \frac{2\pi^2}{45} T^3$, $g_{*S} = \sum_{\text{BOSONS}} g_i \left(\frac{T_i}{T}\right)^3 + \frac{7}{8} \sum_{\text{FERMIONS}} g_i \left(\frac{T_i}{T}\right)^3$
$\frac{m_\chi}{T}$

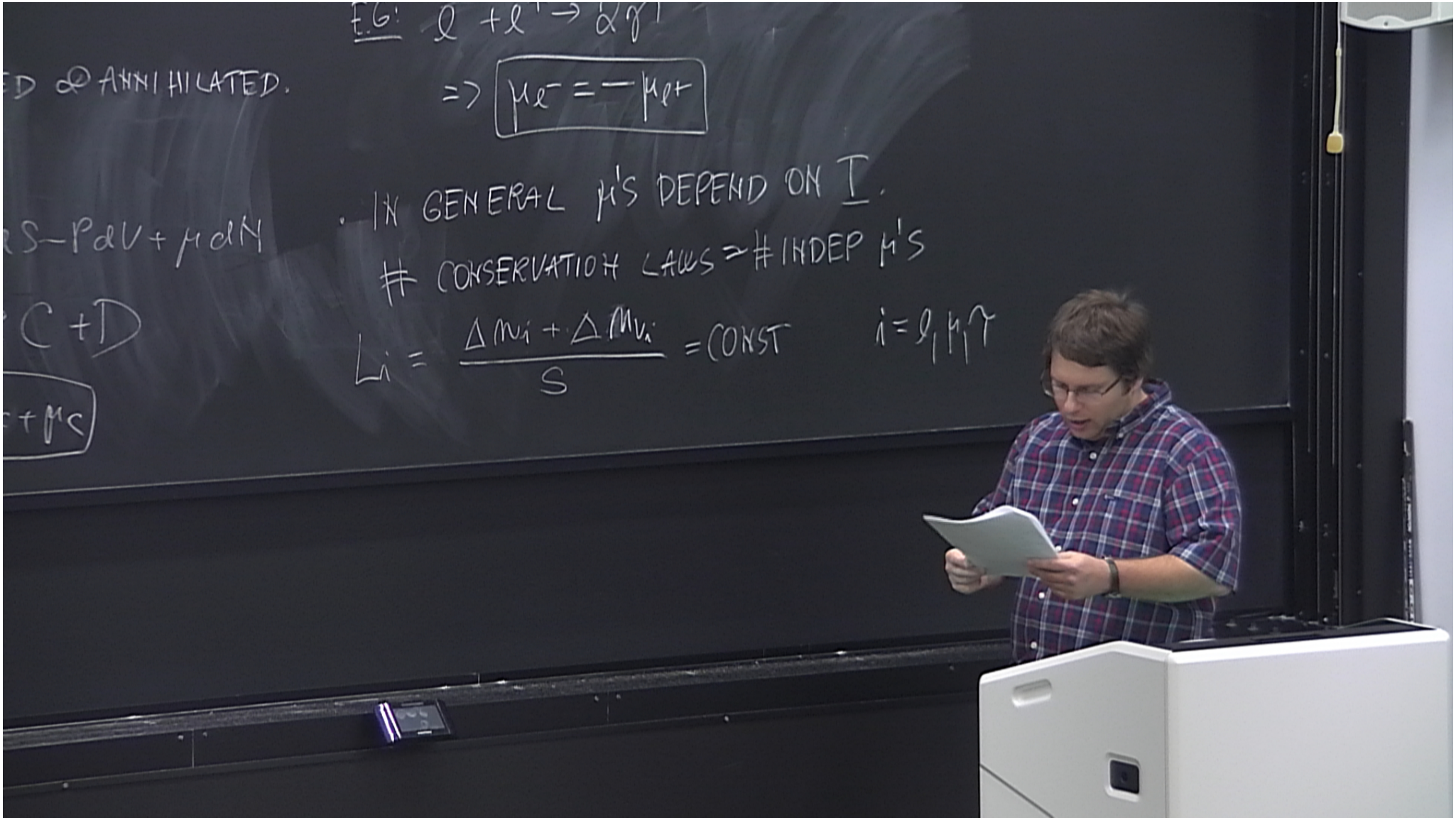

REMARK. CHEMICAL POTENTIALS
• IN QFT PARTICLES CAN BE CREATED & ANNIHILATED.

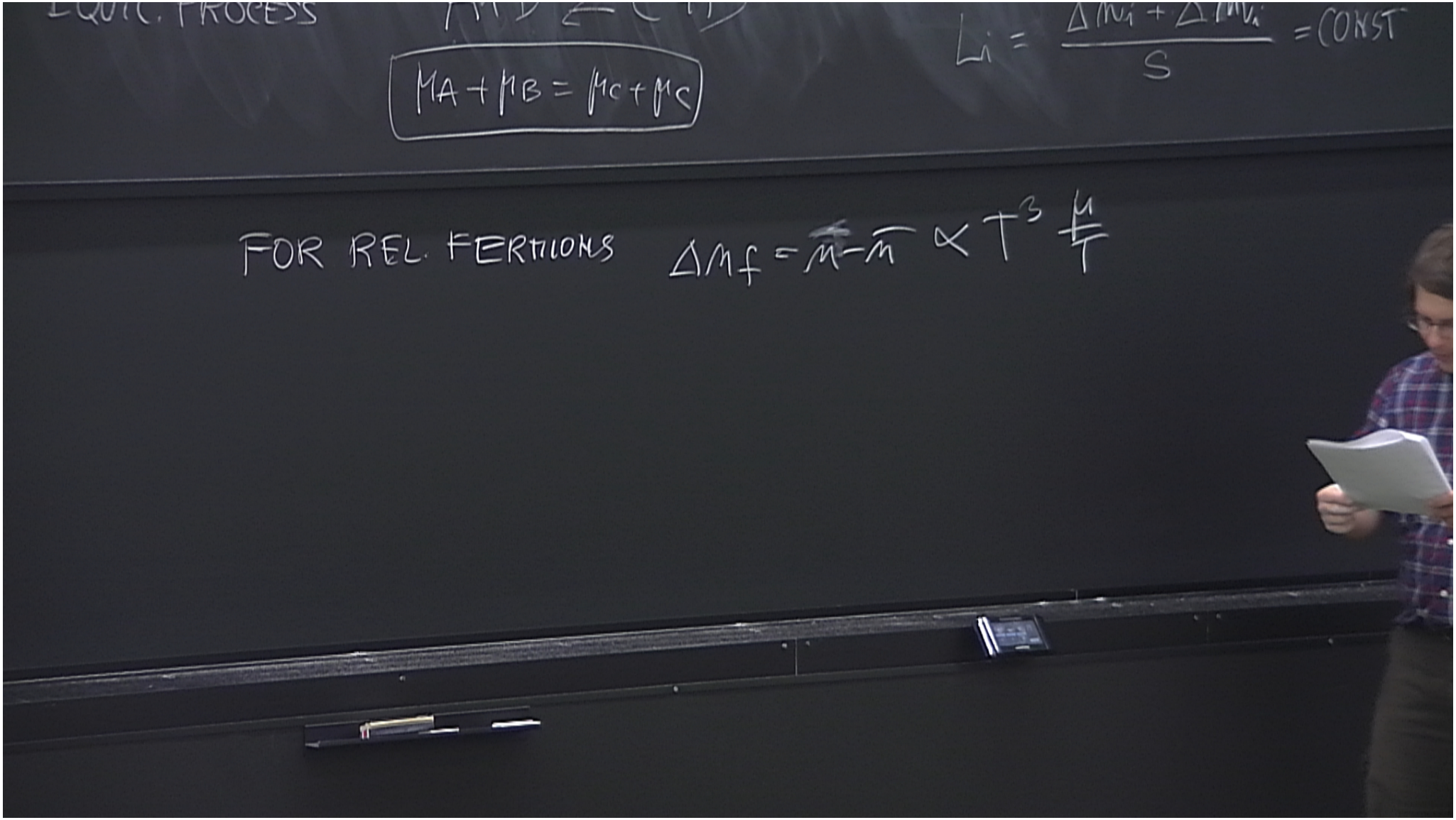

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UNLESS ∄ CONSERVATION LAW.


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$\mu_\gamma = 0$
$dE = TdS - PdV + \mu dN$
• EQUIL. PROCESS
$A + B \rightleftarrows C + D$
$\mu_A + \mu_B = \mu_C + \mu_D$
E.G: $e^- + e^+ \to 2\gamma$


E.G:
ED & ANNIHILATED.
$\Rightarrow \boxed{\mu_{e^-} = -\mu_{e^+}}$
IN GENERAL $\mu$'S DEPEND ON I.
# CONSERVATION LAWS = # INDEP $\mu$'S
$L_i = \frac{\Delta N_i + \Delta N_{\bar{i}}}{S} = \text{CONST}$
$i = e, \mu, \tau$
RS − PdV + μdN
C + D
+ μc


EQUIL. PROCESS
$\mu_A + \mu_B = \mu_C + \mu_S$
$L_i = \frac{\Delta N_i + \Delta \bar{N}_i}{S} = \text{CONST}$
FOR REL. FERMIONS
$\Delta n_f = \bar{n} - \bar{n} \propto T^3 \frac{\mu}{T}$