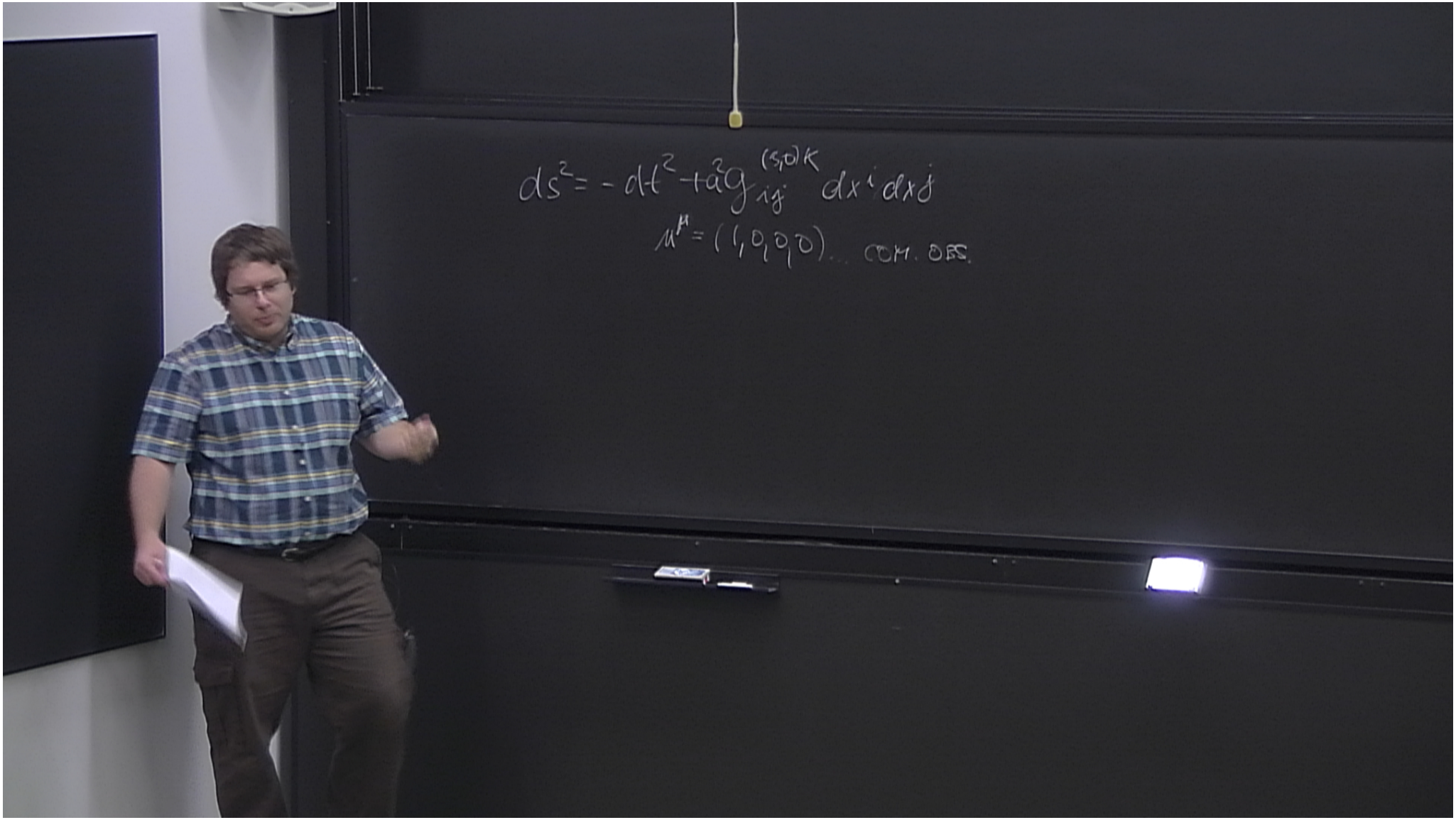
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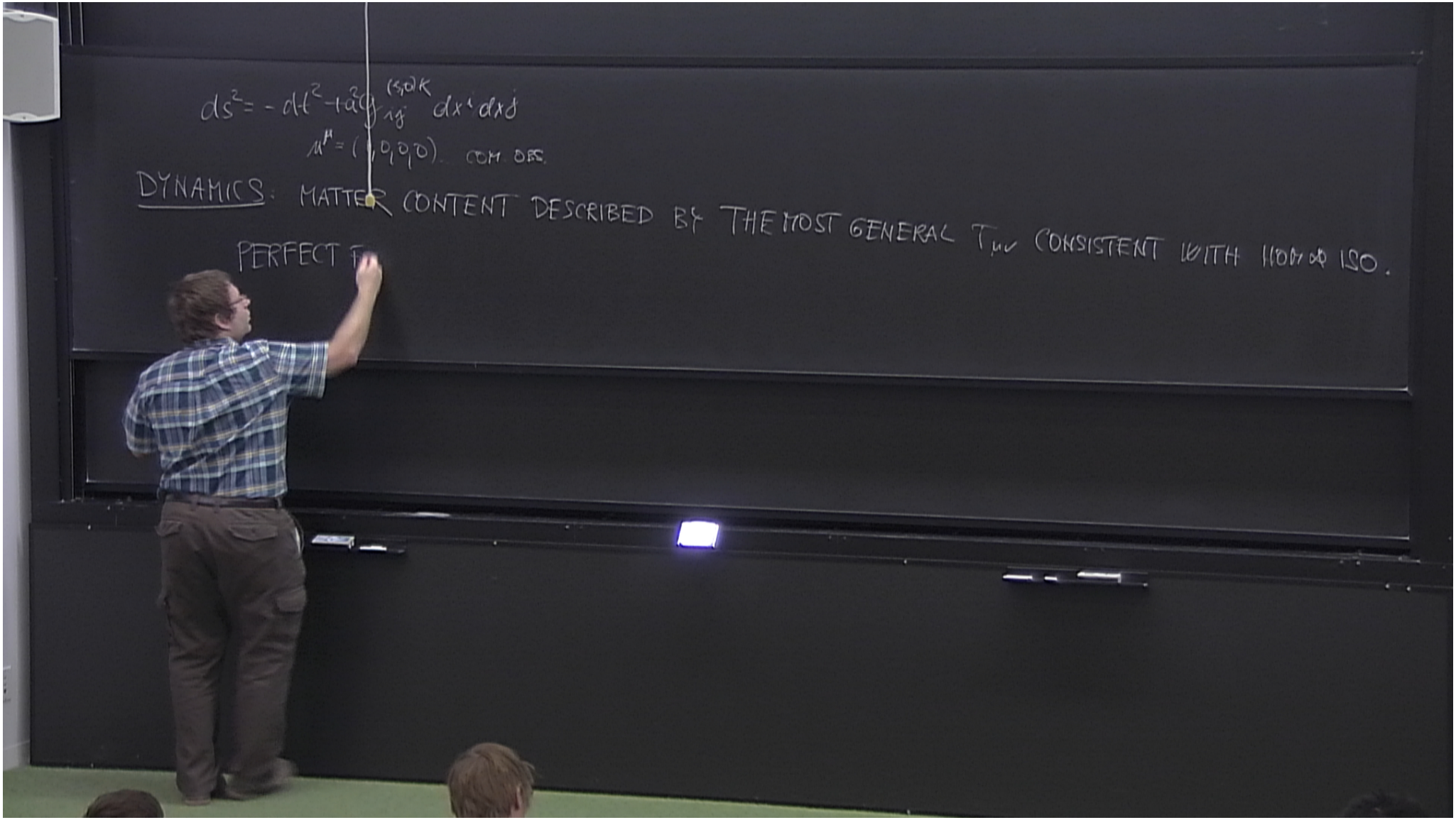
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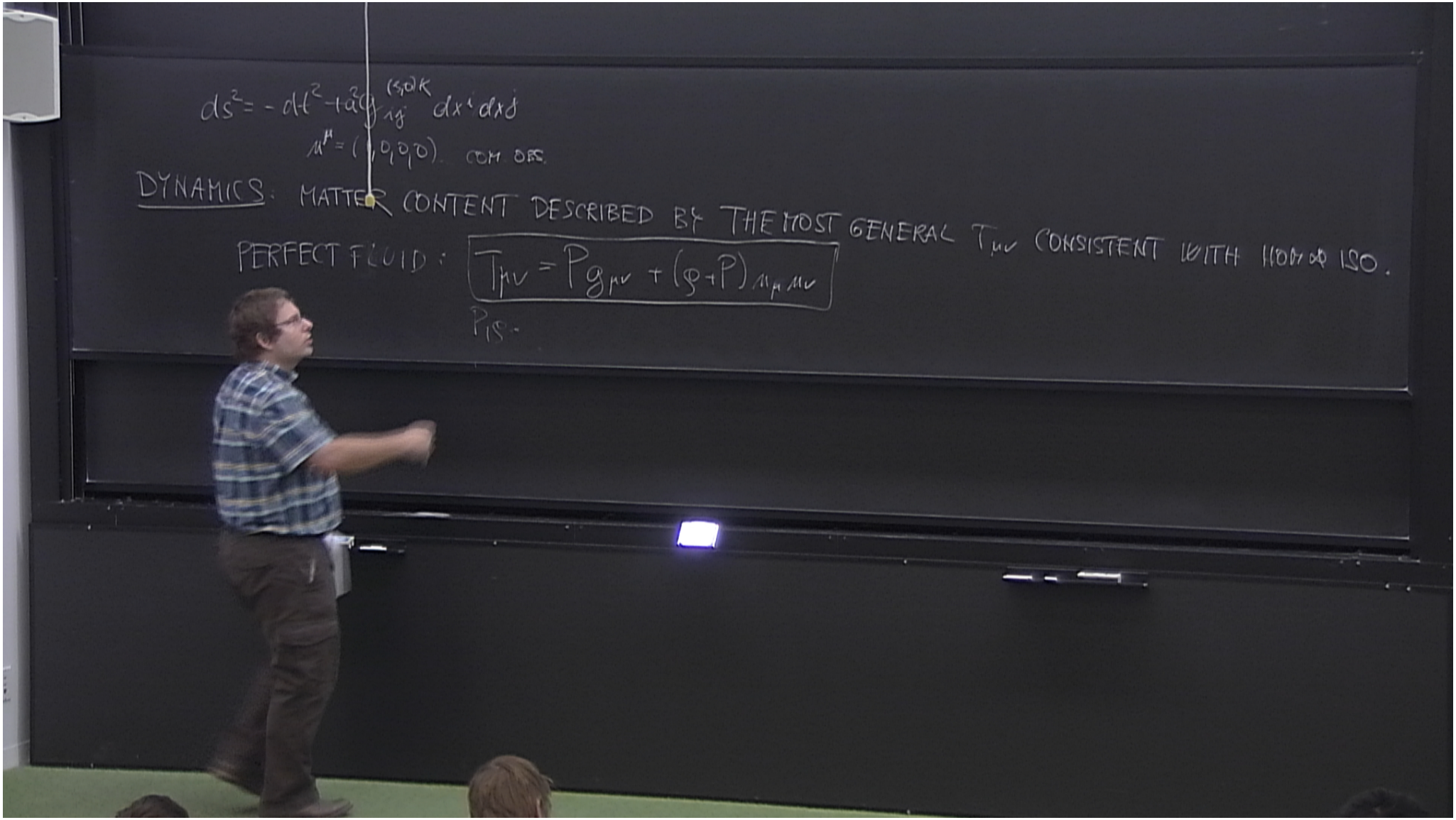
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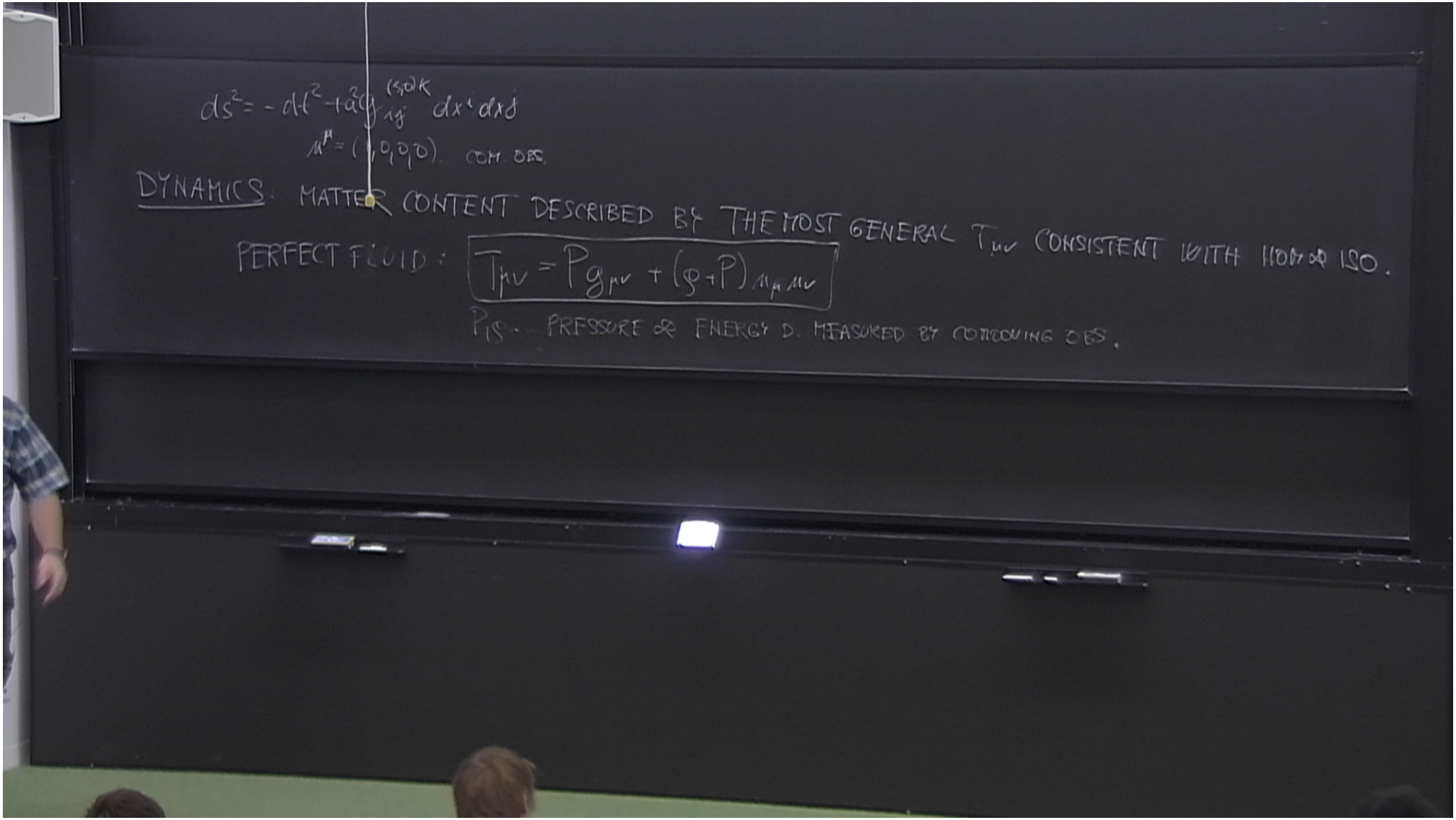
Abstract:

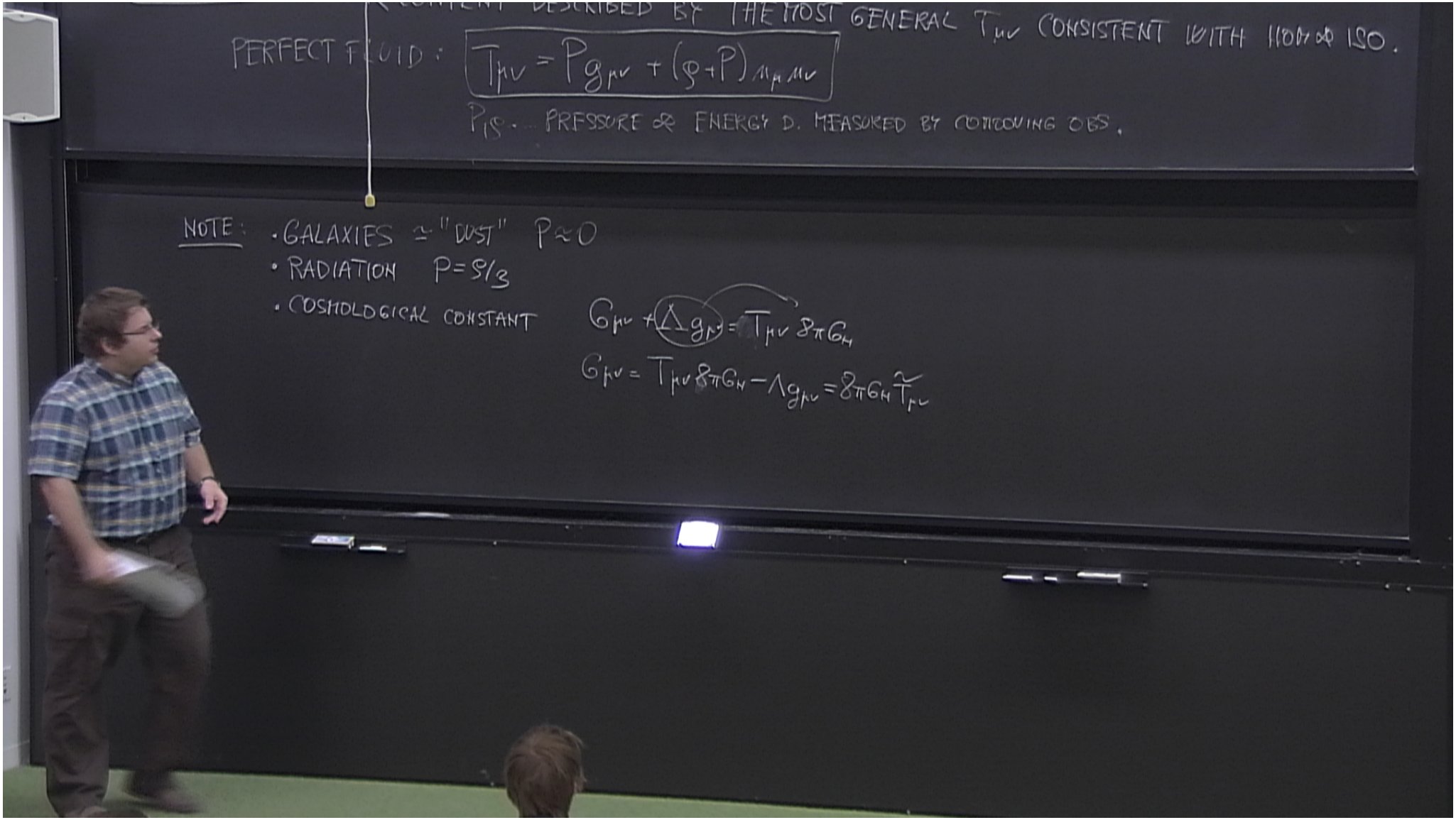


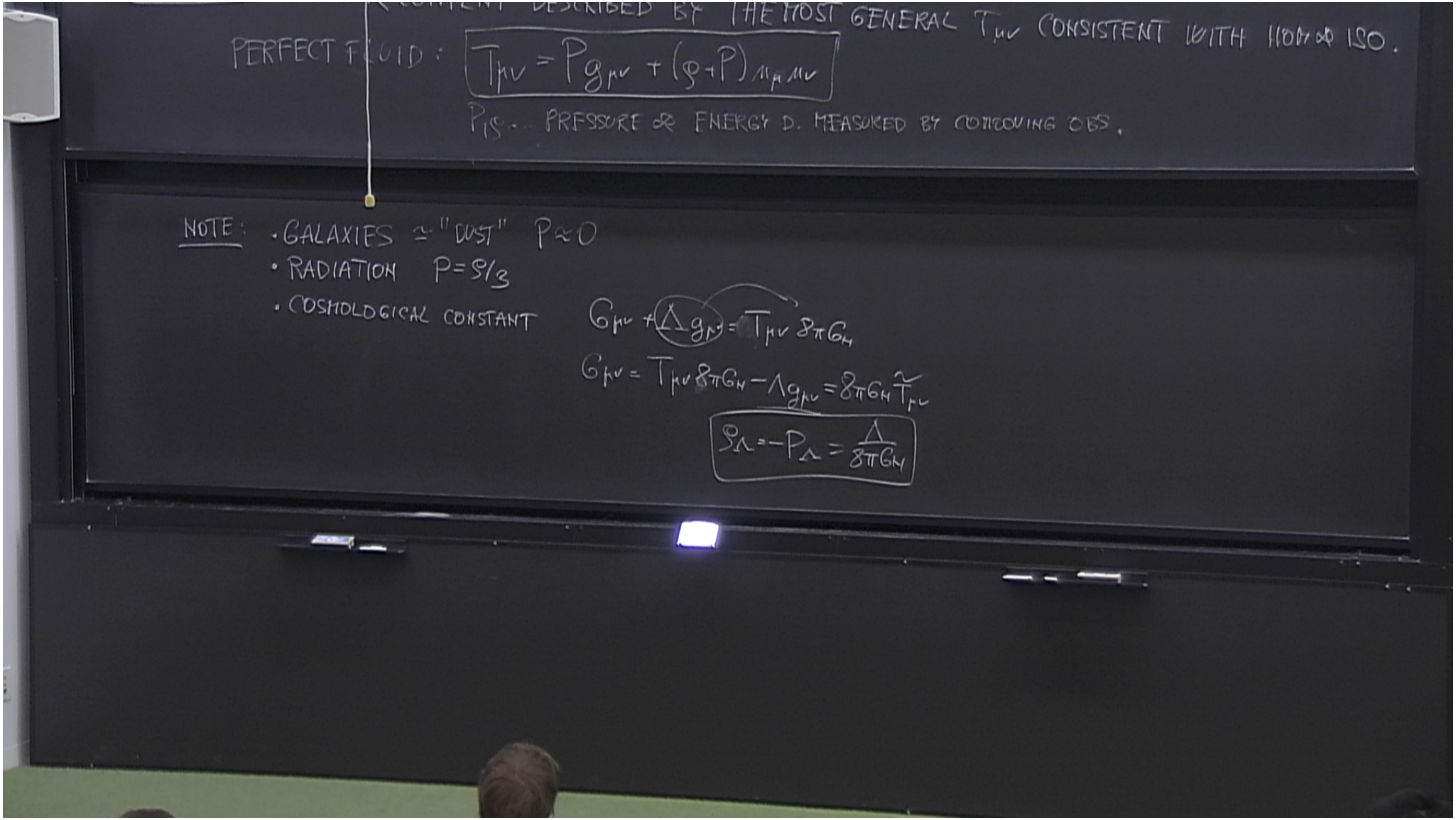

$ds^2 = -dt^2 + a^2 g^{(3,0)K}_{ij} dx^i dx^j$
$u^\mu = (1,0,0,0)$ COM. OBS.

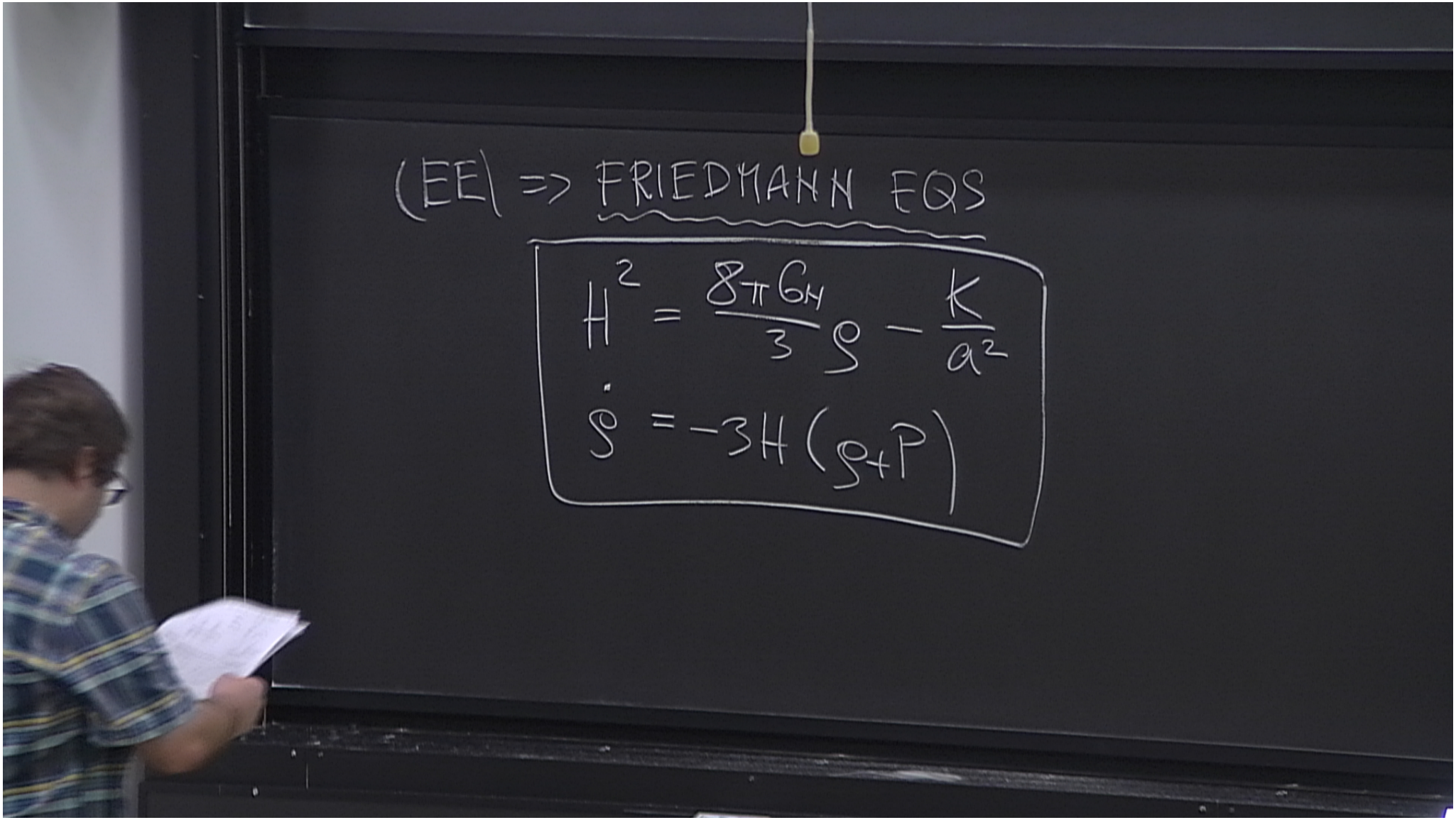

$ds^2 = -dt^2 + a^2(t)\,\gamma^{(3,0,K)}_{ij}\,dx^i dx^j$
$u^\mu = (1,0,0,0)$ COM OBS.
DYNAMICS: MATTER CONTENT DESCRIBED BY THE MOST GENERAL $T_{\mu\nu}$ CONSISTENT WITH HOM & ISO.
PERFECT F

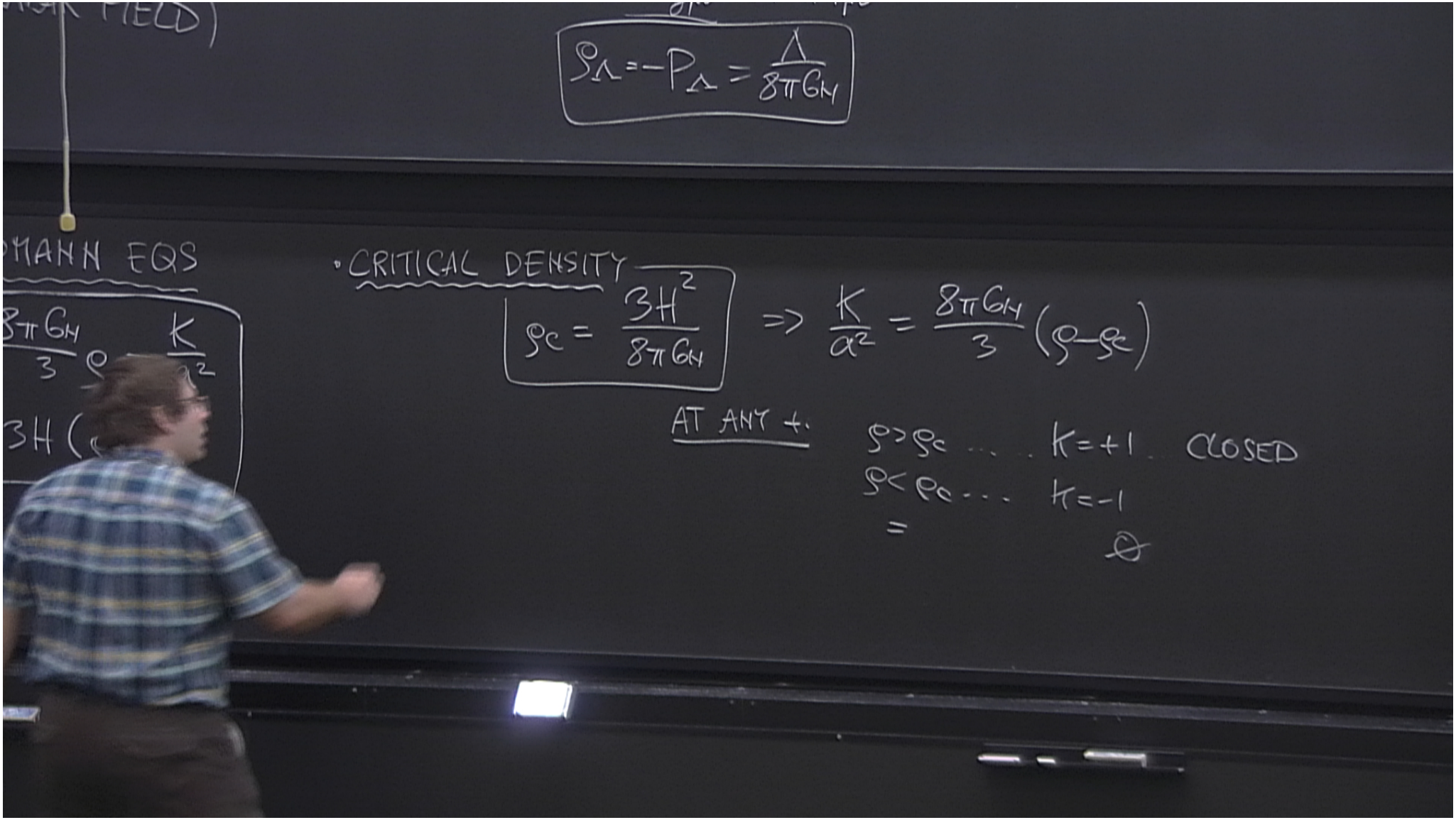

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P is -

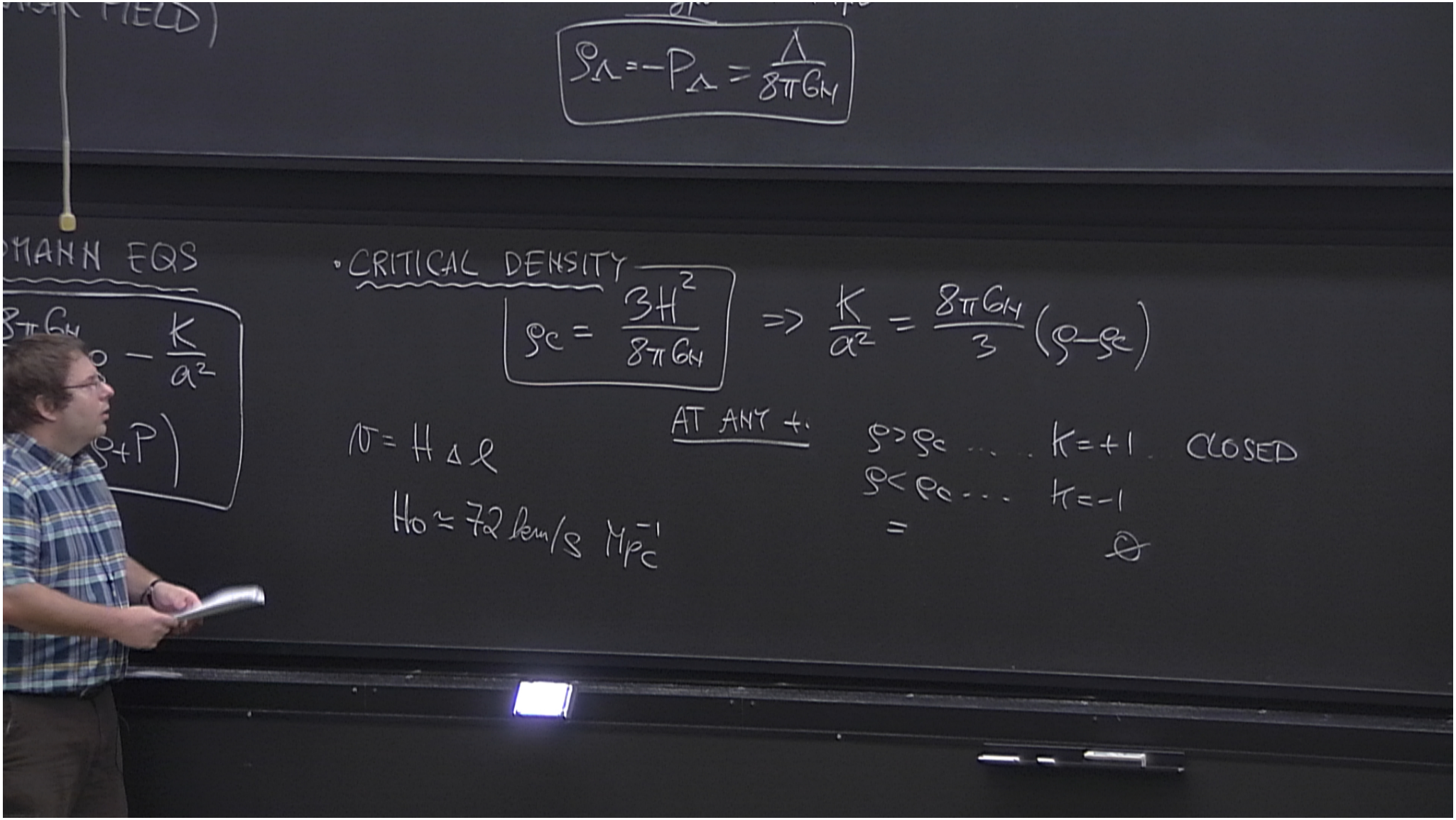

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P & ρ PRESSURE & ENERGY D. MEASURED BY COMOVING OBS.

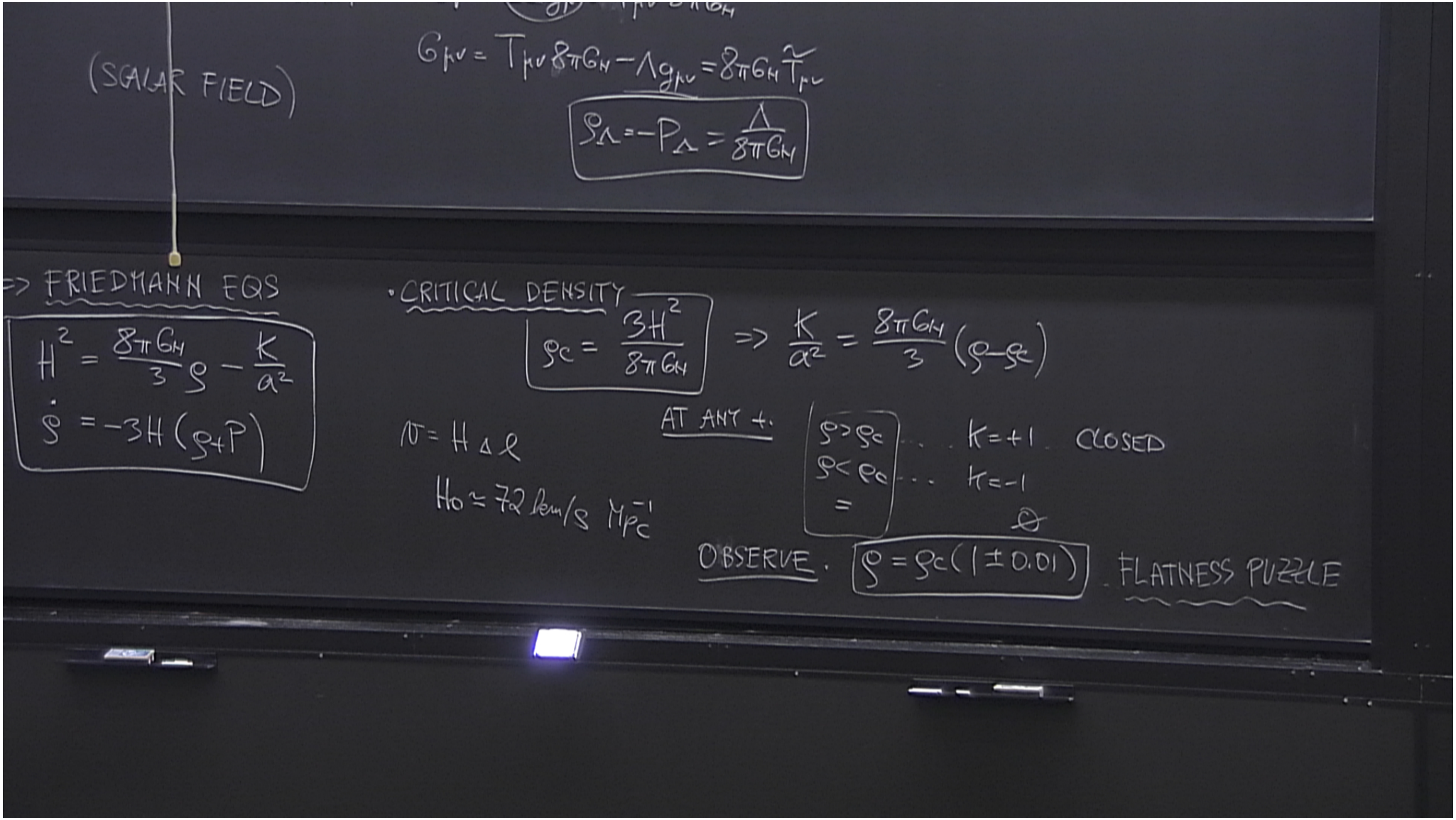

DESCRIBED BY THE MOST GENERAL $T_{\mu\nu}$ CONSISTENT WITH HOM. & ISO.
PERFECT FLUID: $T_{\mu\nu} = P g_{\mu\nu} + (\rho + P) u_\mu u_\nu$
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NOTE:
• GALAXIES $\simeq$ "DUST" $P \simeq 0$
• RADIATION $P = \rho/3$
• COSMOLOGICAL CONSTANT
$G_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} 8\pi G_N$
$G_{\mu\nu} = T_{\mu\nu} 8\pi G_N - \Lambda g_{\mu\nu} = 8\pi G_N \tilde{T}_{\mu\nu}$

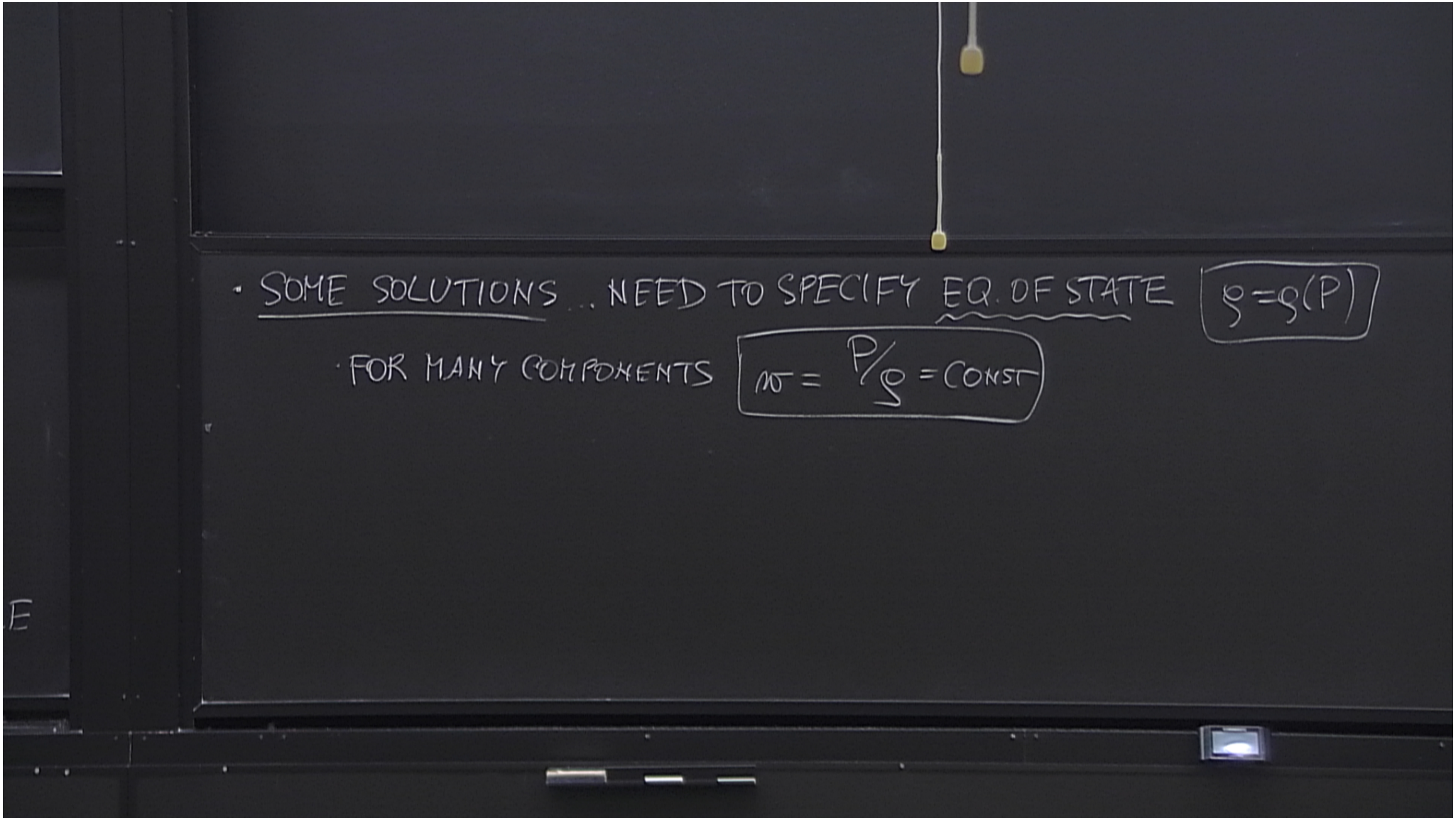

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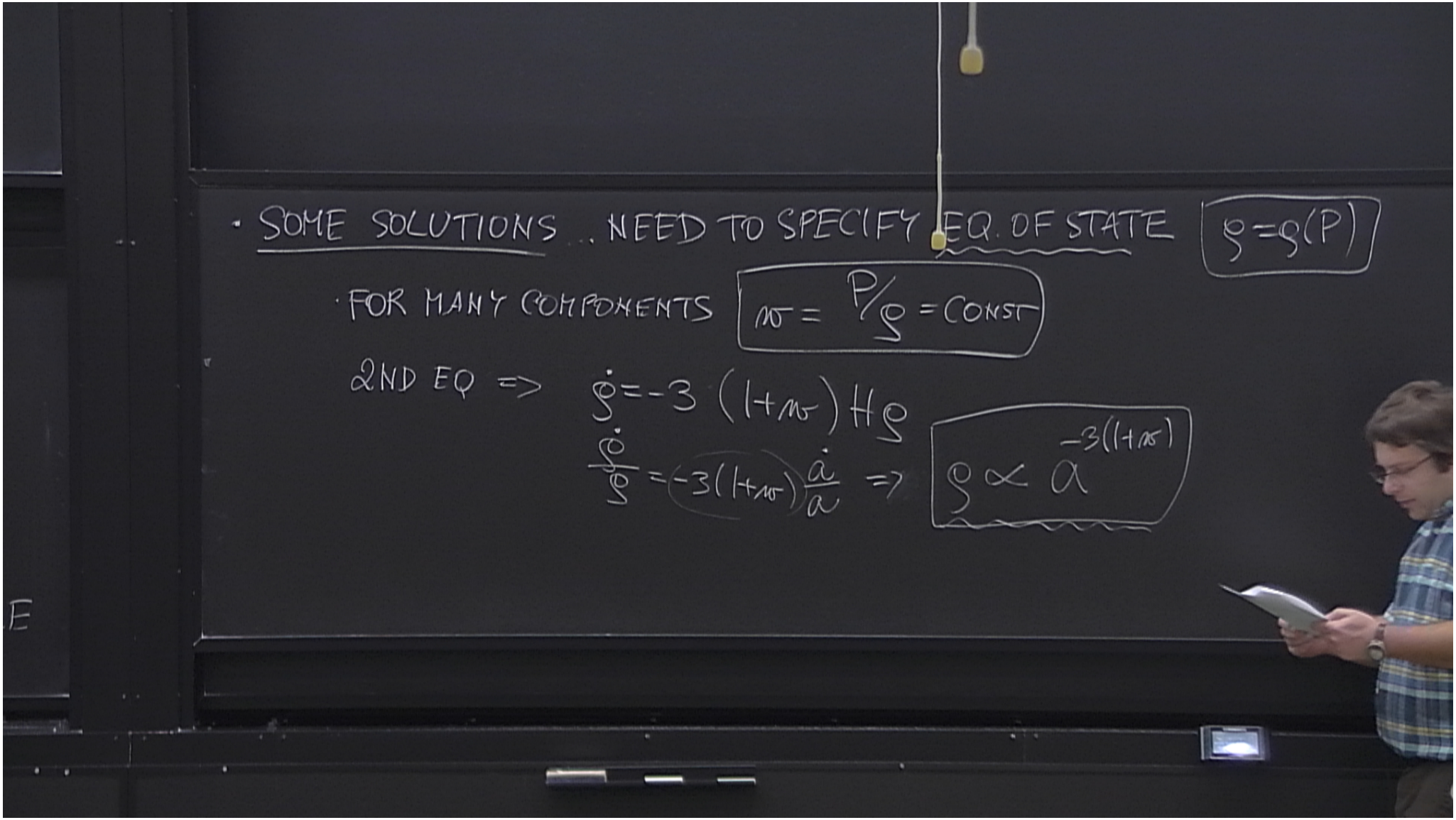

(EE) => FRIEDMANN EQS
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
$\dot{\rho} = -3H(\rho + P)$

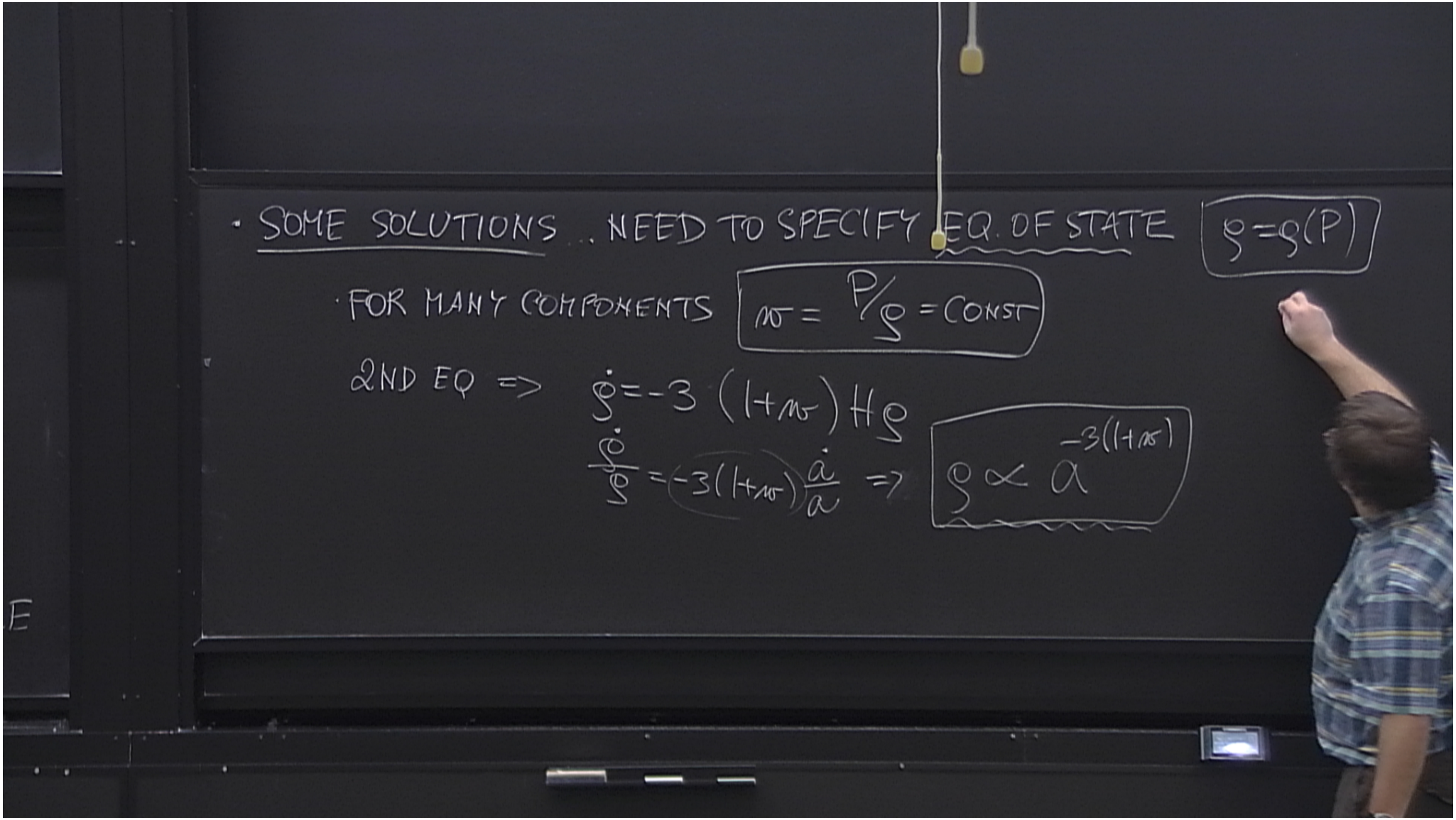

$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi G_N}$
MANN EQS
• CRITICAL DENSITY
$\rho_c = \frac{3H^2}{8\pi G_N}$
$\Rightarrow \frac{k}{a^2} = \frac{8\pi G_N}{3}(\rho - \rho_c)$
AT ANY t:
$\rho > \rho_c \ldots k = +1$ CLOSED
$\rho < \rho_c \ldots k = -1$
$\rho = \rho_c$

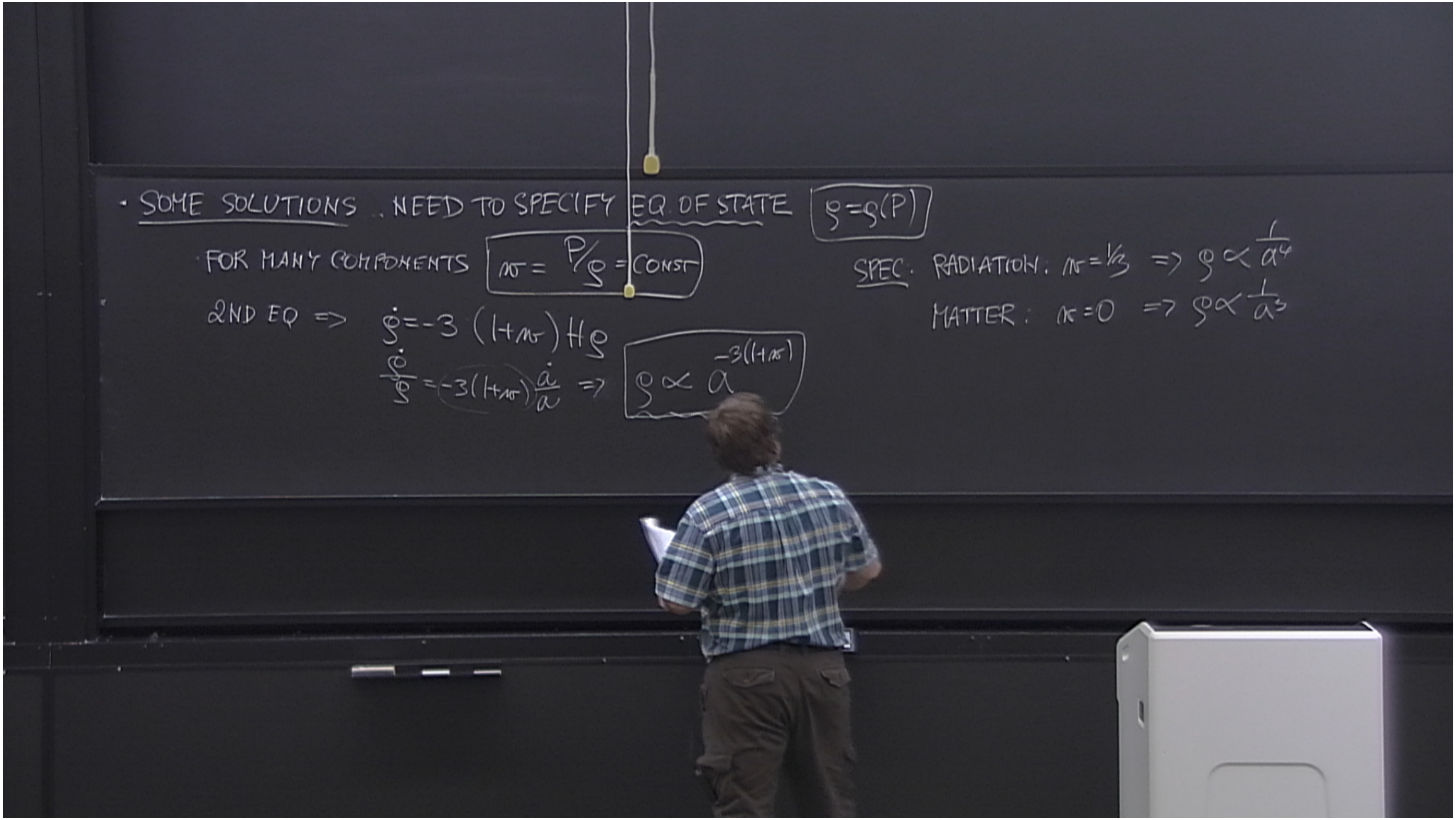

$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi G_N}$
MANN EQS
$\frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
$(\rho + P)$
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AT ANY t:
$\rho > \rho_c \ldots k = +1$ CLOSED
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$\rho = \rho_c \ldots k = 0$
$\Omega = H\Delta\ell$
$H_0 \simeq 72$ km/s Mpc^{-1}

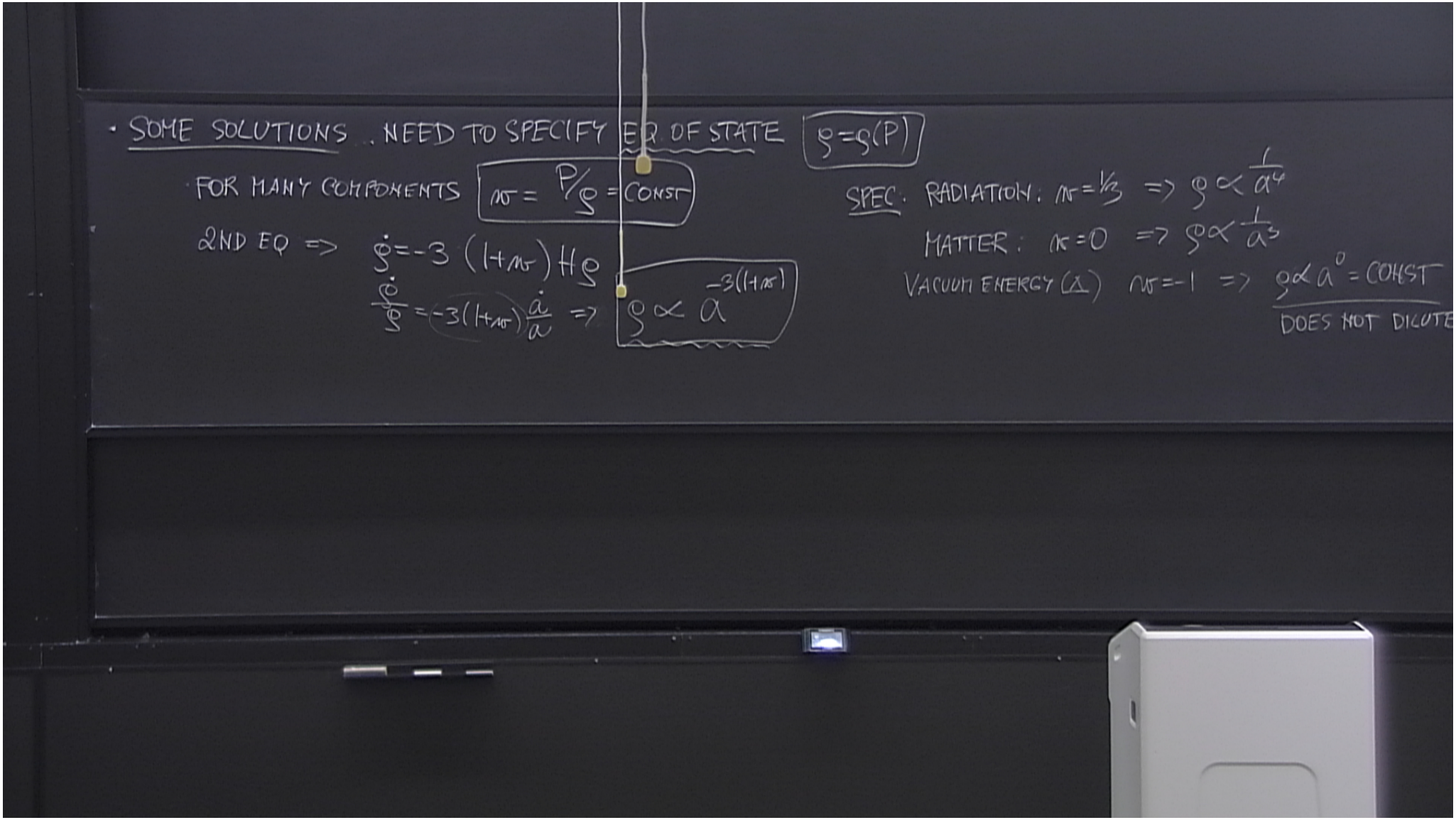

(SCALAR FIELD)
$G_{\mu\nu} = T_{\mu\nu} 8\pi G_N - \Lambda g_{\mu\nu} = 8\pi G_N \tilde{T}_{\mu\nu}$
$\rho_\Lambda = -P_\Lambda = \frac{\Lambda}{8\pi G_N}$
=> FRIEDMANN EQS
$H^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2}$
$\dot{\rho} = -3H(\rho + P)$
• CRITICAL DENSITY
$\rho_c = \frac{3H^2}{8\pi G_N}$ => $\frac{k}{a^2} = \frac{8\pi G_N}{3}(\rho - \rho_c)$
$\Omega = H \Delta \ell$
AT ANY t:
$\rho > \rho_c$ ... $k = +1$ CLOSED
$\rho < \rho_c$ ... $k = -1$
$=$ ... 0
$H_0 \simeq 72$ km/s Mpc^{-1}
OBSERVE: $\rho = \rho_c(1 \pm 0.01)$ FLATNESS PUZZLE

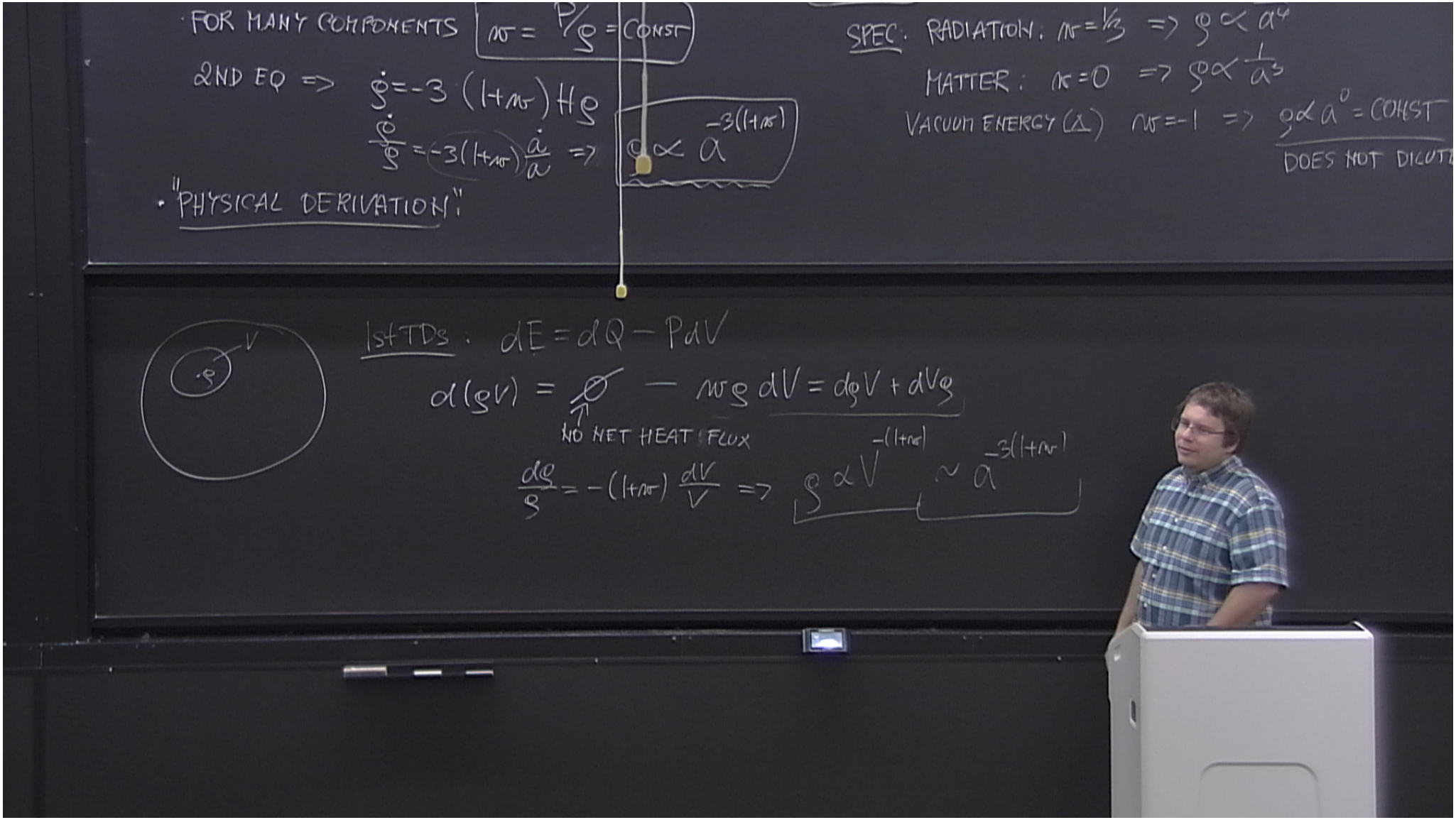

· SOME SOLUTIONS .. NEED TO SPECIFY EQ. OF STATE $\rho = \rho(P)$
· FOR MANY COMPONENTS $n = P/\rho = \text{CONST}$


· SOME SOLUTIONS .. NEED TO SPECIFY EQ. OF STATE $\rho = \rho(P)$
· FOR MANY COMPONENTS $w = P/\rho = \text{CONST}$
2ND EQ $\Rightarrow$ $\dot{\rho} = -3(1+w)H\rho$
$\frac{\dot{\rho}}{\rho} = -3(1+w)\frac{\dot{a}}{a} \Rightarrow \rho \propto a^{-3(1+w)}$


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SPEC. RADIATION: $w=1/3 \Rightarrow \rho \propto \frac{1}{a^4}$
MATTER: $w=0 \Rightarrow \rho \propto \frac{1}{a^3}$
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DOES NOT DILUTE

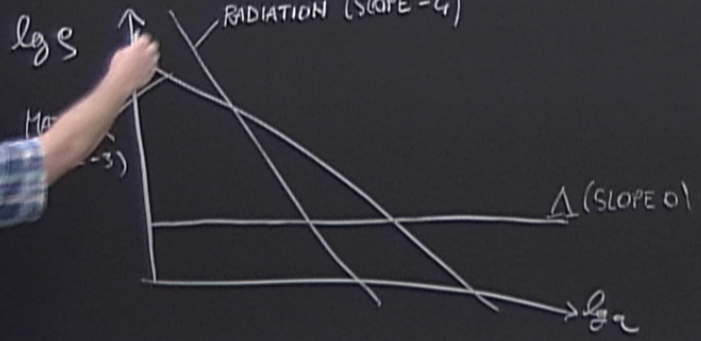

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"PHYSICAL DERIVATION"
SPEC: RADIATION: $w = 1/3 \Rightarrow \rho \propto a^{-4}$
MATTER: $w = 0 \Rightarrow \rho \propto \frac{1}{a^3}$
VACUUM ENERGY (Λ) $w = -1 \Rightarrow \rho \propto a^0 = \text{CONST}$
DOES NOT DILUTE
1st TDS: $dE = dQ - PdV$
$d(\rho V) = \cancel{0} - w\rho\, dV = d\rho V + dV\rho$
NO NET HEAT FLUX
$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow \rho \propto V^{-(1+w)} \sim a^{-3(1+w)}$

$d(\rho V) = 0 - w\rho\, dV = d\rho V + dV\rho$

NO NET HEAT FLUX

$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow \rho \propto V^{-(1+w)} \sim a^{-3(1+w)}$

- CURRENT COSMOLOGICAL MODEL

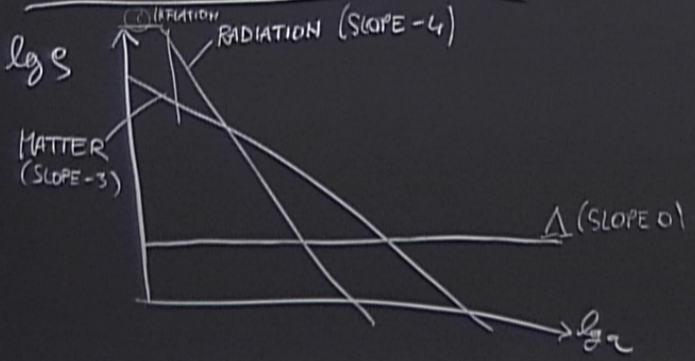


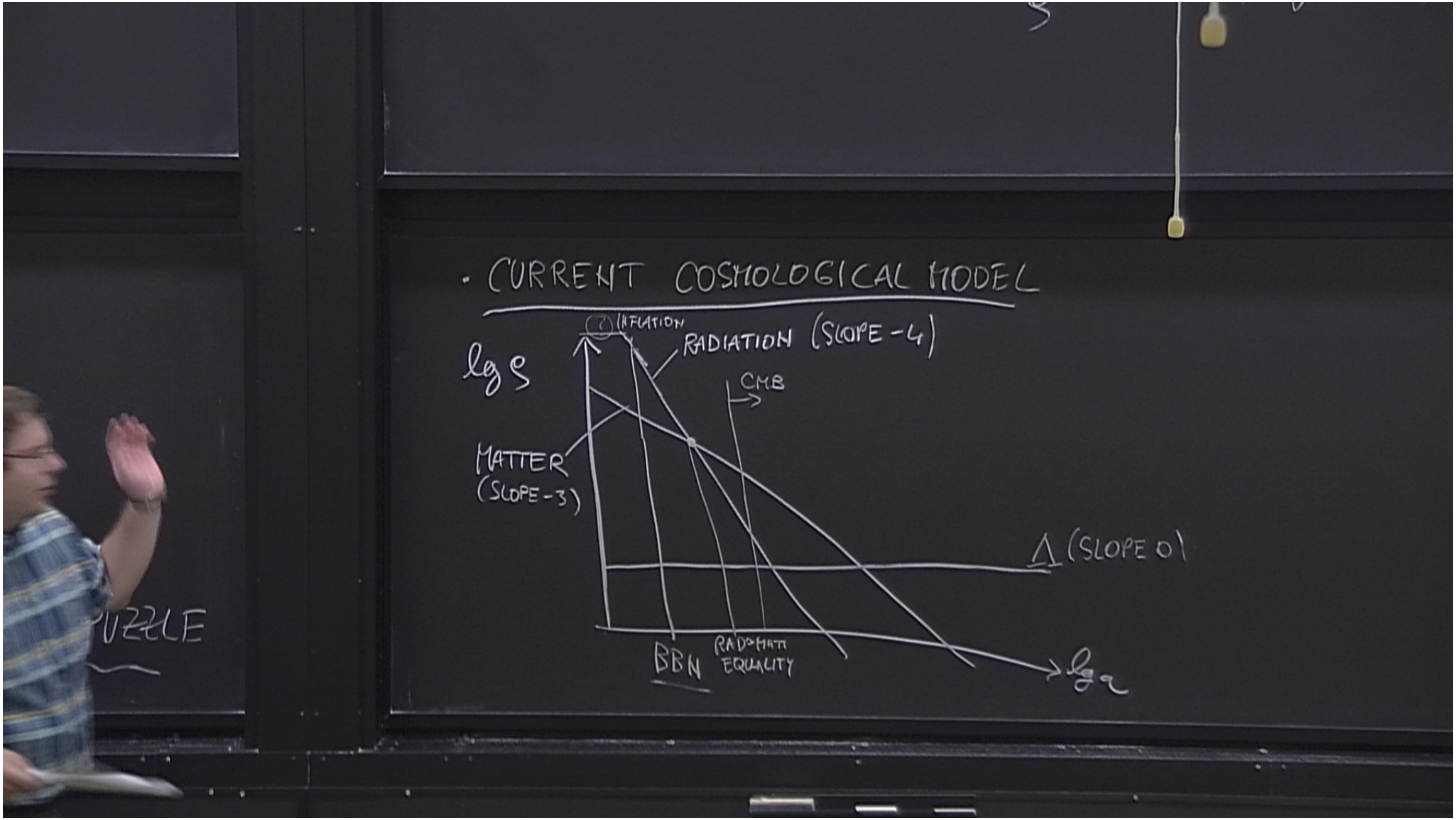
$d(\rho V) = \cancel{0} - nP\,dV = d\rho V + dV\rho$

NO NET HEAT FLUX

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- CURRENT COSMOLOGICAL MODEL




· CURRENT COSMOLOGICAL MODEL
INFLATION
RADIATION (SLOPE -4)
log ρ
CMB
MATTER
(SLOPE -3)
Λ (SLOPE 0)
PUZZLE
BBN
RAD=MATT
EQUALITY
log a

$d(\rho V) = \cancel{0} - w\rho\, dV = d\rho V + dV\rho$

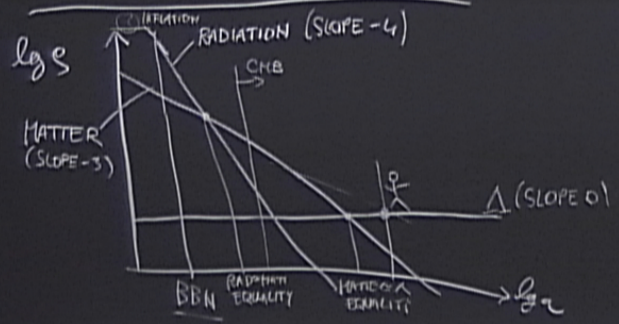
NO NET HEAT FLUX

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- CURRENT COSMOLOGICAL MODEL

RAD DOM $\rightarrow$ MATTER DOM $\rightarrow$ ~~CURVATURE DOM~~ $\rightarrow$ Λ dom

$\frac{1}{a^4}$ $\frac{1}{a^3}$ ~~$\frac{1}{a^2}$~~ a^0

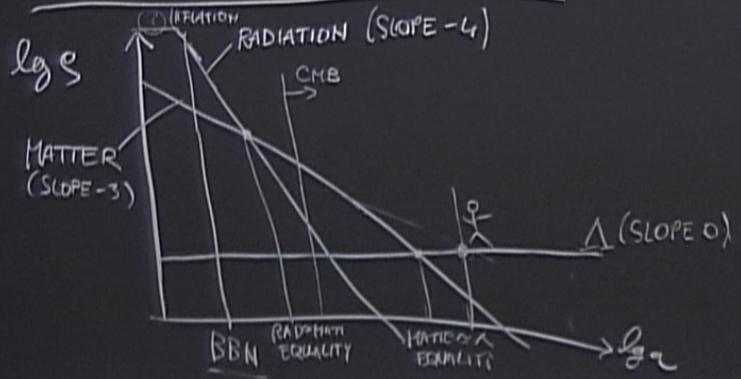


$d(\rho V) = \delta Q - w\rho\, dV = d\rho V + dV\rho$

NO NET HEAT FLUX

$\frac{d\rho}{\rho} = -(1+w)\frac{dV}{V} \Rightarrow \rho \propto V^{-(1+w)} \sim a^{-3(1+w)}$

- CURRENT COSMOLOGICAL MODEL



RAD DOM → MATTER DOM → CURVATURE DOM → Λ dom

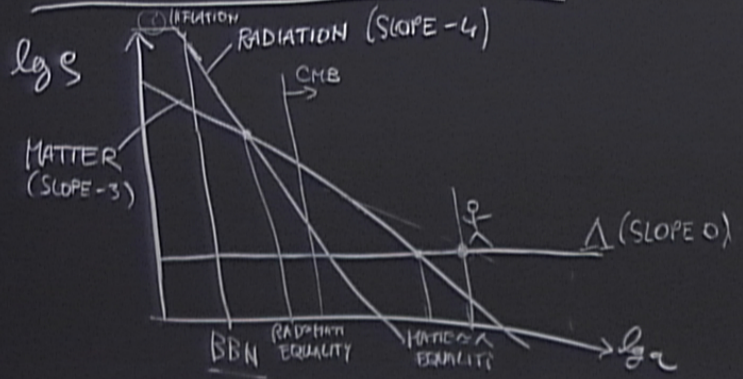
$\frac{1}{a^4}$ $\frac{1}{a^3}$ $\frac{1}{a^2}$ a^0

$d(\rho V) = \cancel{\delta Q} - w\rho\, dV = d\rho V + dV\rho$

NO NET HEAT FLUX

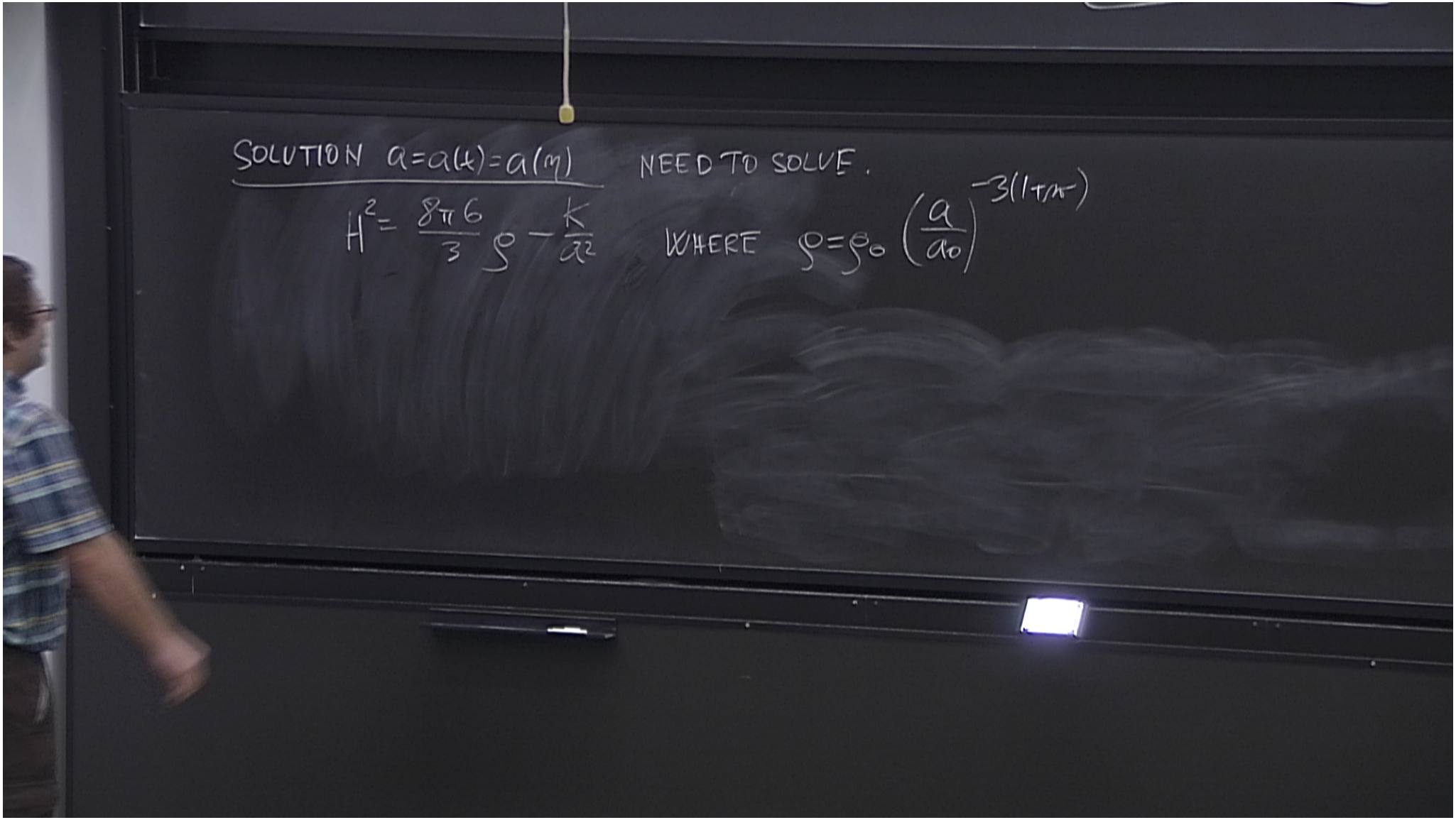
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- CURRENT COSMOLOGICAL MODEL



RAD DOM → MATTER DOM → CURVATURE DOM → Λ dom

$\frac{1}{a^4}$ $\frac{1}{a^3}$ $\frac{1}{a^2}$ a^0


SOLUTION $a=a(t)=a(\eta)$ NEED TO SOLVE.
$H^2=\frac{8\pi G}{3}\rho-\frac{k}{a^2}$ WHERE $\rho=\rho_0\left(\frac{a}{a_0}\right)^{-3(1+w)}$

NOTE: • GALAXIES ≃ "DUST" $P \approx 0$

• RADIATION $P = \rho/3$

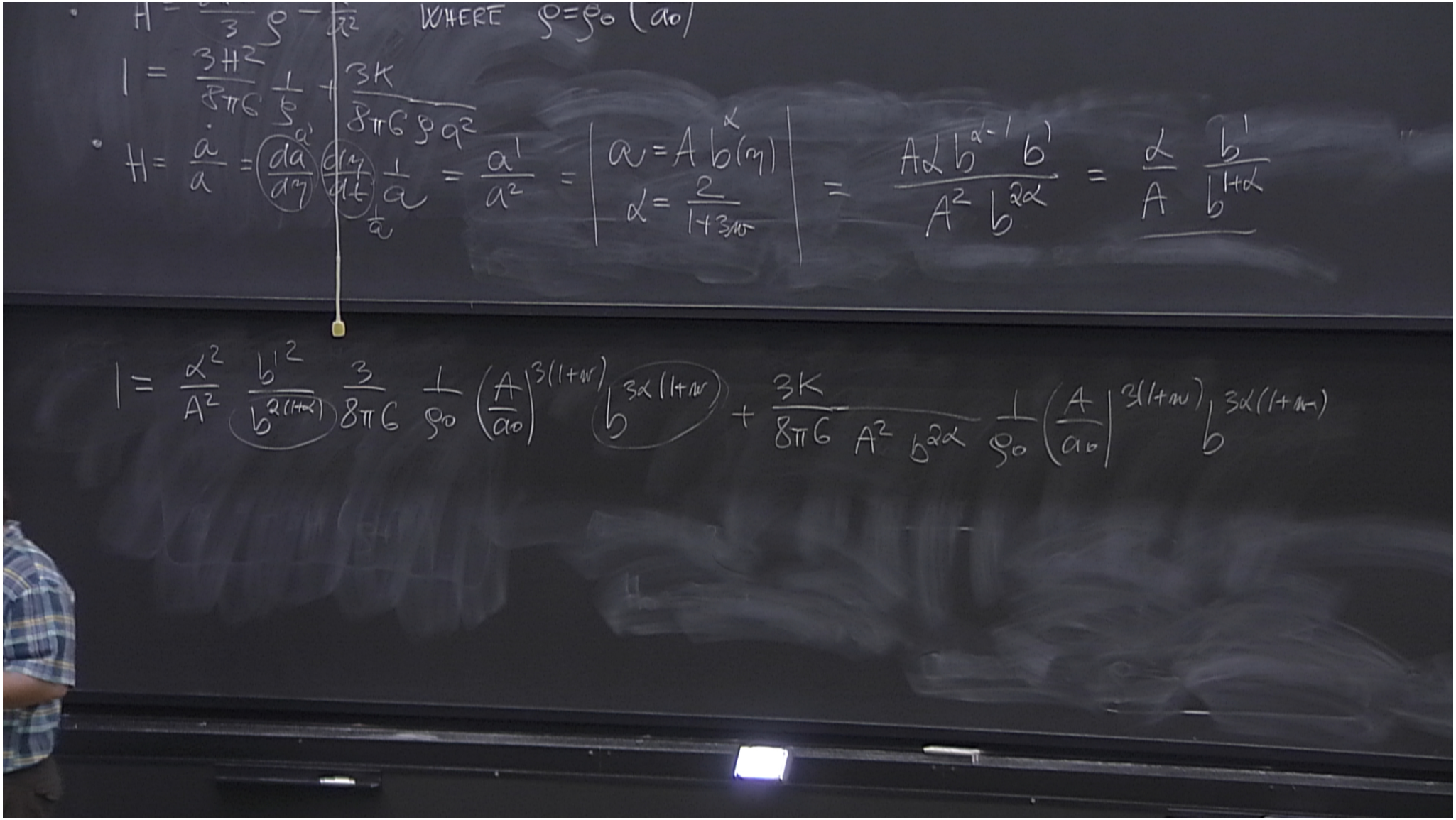
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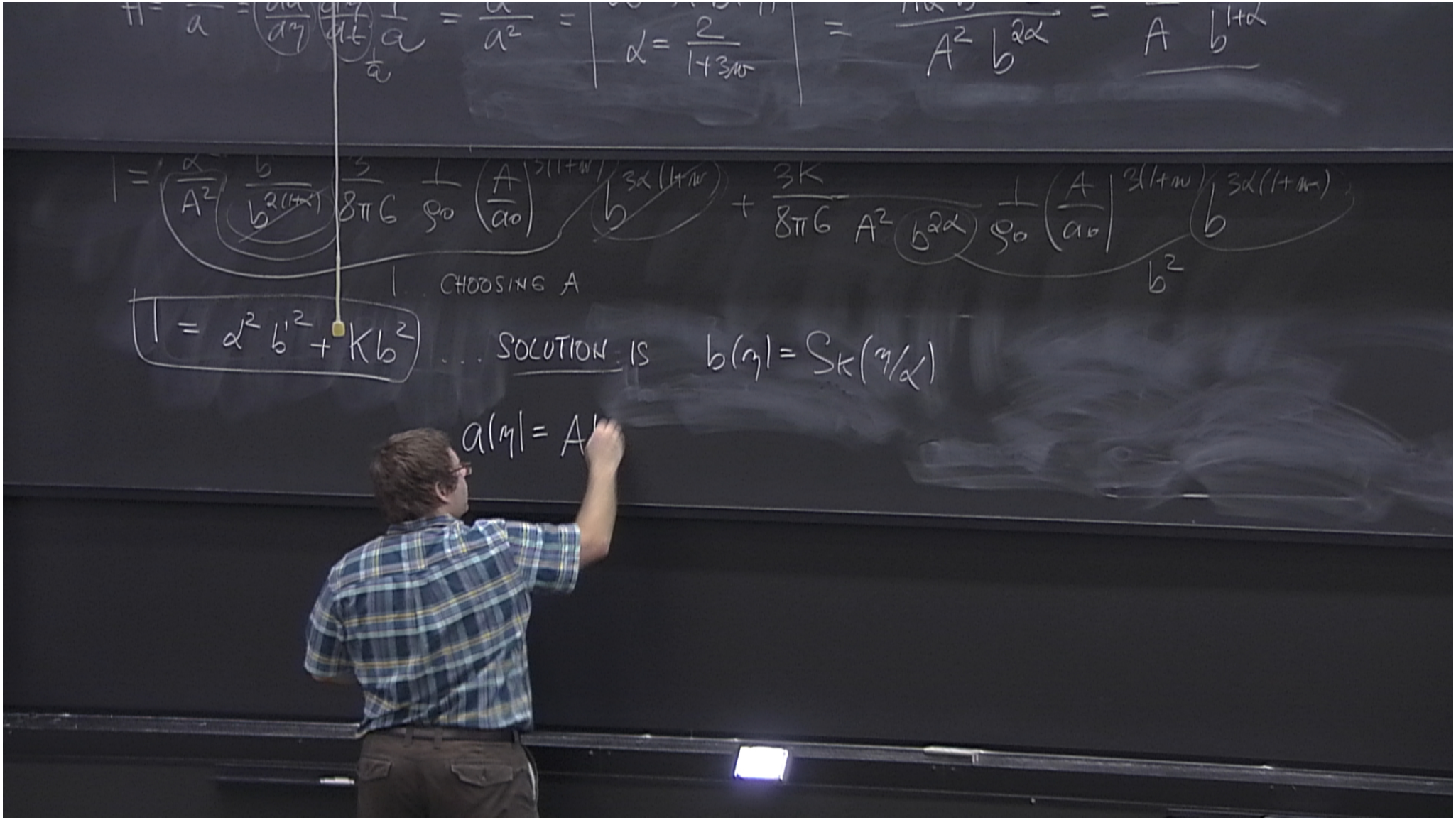
• $H^2 = \frac{8\pi G}{3}\rho - \frac{K}{R^2}$ WHERE $\rho = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)}$

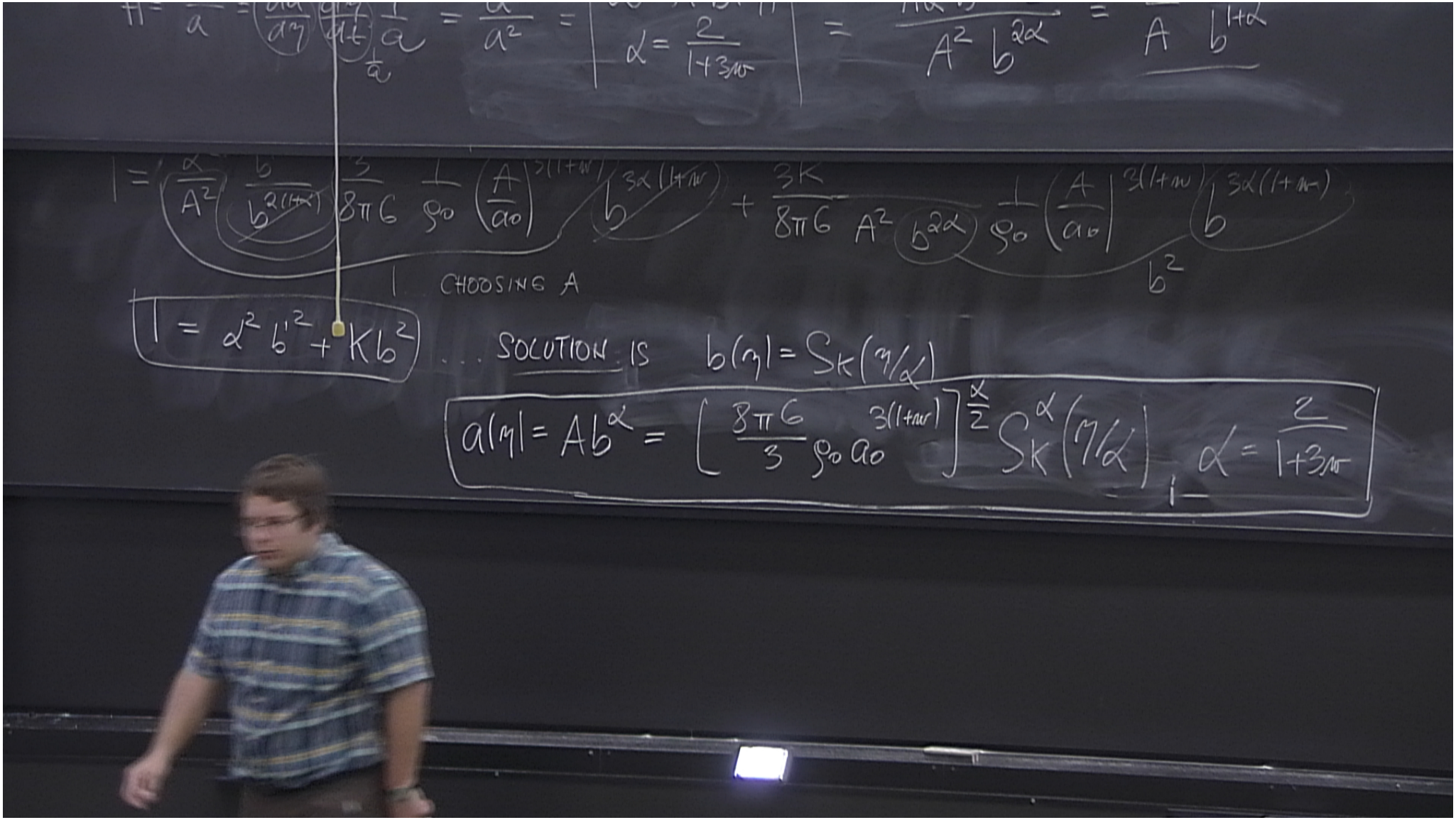
$1 = \frac{3H^2}{8\pi G}\frac{1}{\rho} + \frac{3K}{8\pi G \rho a^2}$

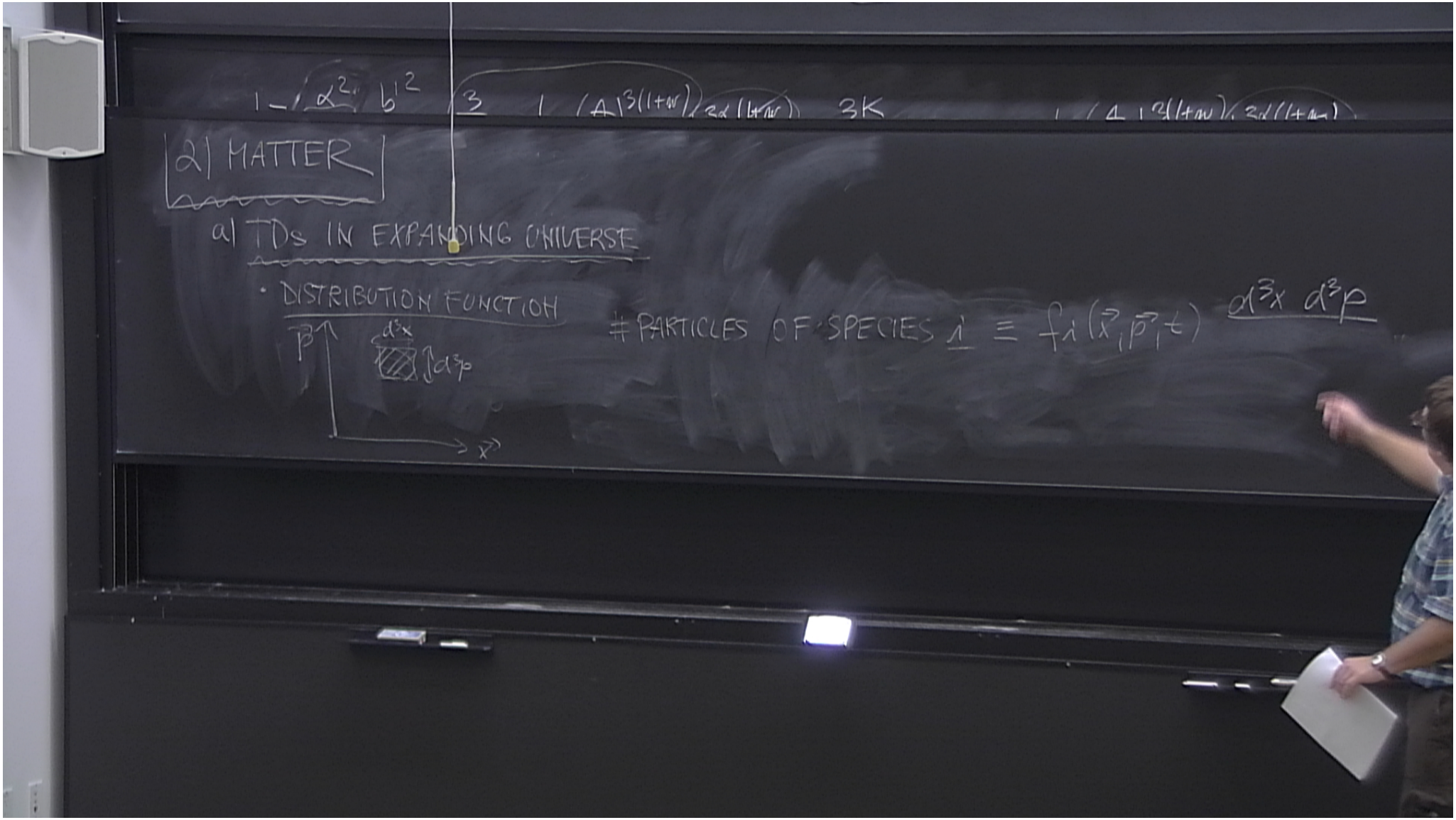
• $H = \frac{\dot{a}}{a} = \frac{da}{d\eta}\frac{d\eta}{dt}\frac{1}{a} = \frac{a'}{a^2} = \left| \begin{array}{l} a = A\,b(\eta)^{\alpha} \\ \alpha = \frac{2}{1+3w} \end{array} \right| = \frac{A\alpha\, b^{\alpha-1} b'}{A^2 b^{2\alpha}} = \frac{\alpha}{A}\frac{b'}{b^{1+\alpha}}$

(EE) => FRIEDMANN EQS • CRITICAL DENSIT


WHERE $\rho=\rho_0\left(\frac{a_0}{a}\right)^{3(1+w)}$
$1=\frac{3H^2}{8\pi G}\frac{1}{\rho}+\frac{3K}{8\pi G\rho a^2}$
$H=\frac{\dot a}{a}=\frac{da}{d\eta}\frac{d\eta}{dt}\frac{1}{a}=\frac{a'}{a^2}=\left|\begin{array}{l}a=A\,b^{\alpha}(\eta)\\ \alpha=\frac{2}{1+3w}\end{array}\right|=\frac{A\alpha b^{\alpha-1}b'}{A^2b^{2\alpha}}=\frac{\alpha}{A}\frac{b'}{b^{1+\alpha}}$
$1=\frac{\alpha^2}{A^2}\frac{b'^2}{b^{2(1+\alpha)}}\frac{3}{8\pi G}\frac{1}{\rho_0}\left(\frac{A}{a_0}\right)^{3(1+w)}b^{3\alpha(1+w)}+\frac{3K}{8\pi G}\frac{1}{A^2b^{2\alpha}}\frac{1}{\rho_0}\left(\frac{A}{a_0}\right)^{3(1+w)}b^{3\alpha(1+w)}$


$\alpha = \frac{2}{1+3w}$
$A^2 b^{2\alpha}$
$A\, b^{1+\alpha}$
CHOOSING A
$1 = \alpha^2 b'^2 + Kb^2$
... SOLUTION IS $b(\eta) = S_K(\eta/\alpha)$
$a(\eta) = A$


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CHOOSING A
$1 = \alpha^2 b'^2 + Kb^2$
... SOLUTION IS $b(\eta) = S_K(\eta/\alpha)$
$a(\eta) = Ab^\alpha = \left[\frac{8\pi G}{3}\rho_0 a_0^{3(1+w)}\right]^{\frac{\alpha}{2}} S_K^\alpha(\eta/\alpha)$, $\alpha = \frac{2}{1+3w}$

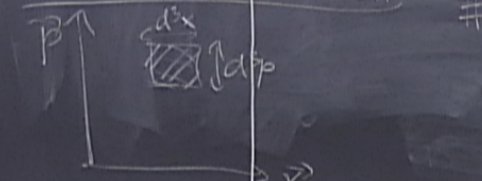

2| MATTER
a| TDs IN EXPANDING UNIVERSE
• DISTRIBUTION FUNCTION
PARTICLES OF SPECIES i ≡ $f_i(\vec{x}, \vec{p}, t)$ $d^3x\, d^3p$
d^3x
d^3p
$\vec{p}$
$\vec{x}$

NOTE: • GALAXIES ≃ "DUST" $P \approx 0$
• RADIATION $P = \rho/3$

2) MATTER

a) TDs IN EXPANDING UNIVERSE

• DISTRIBUTION FUNCTION



PARTICLES OF SPECIES $i \equiv f_i(\vec{x}, \vec{p}, t) \dfrac{d^3x\, d^3p}{(2\pi\hbar)^3}$

∴ SMALLEST CELL PARTICLE CAN BE LOCALIZED

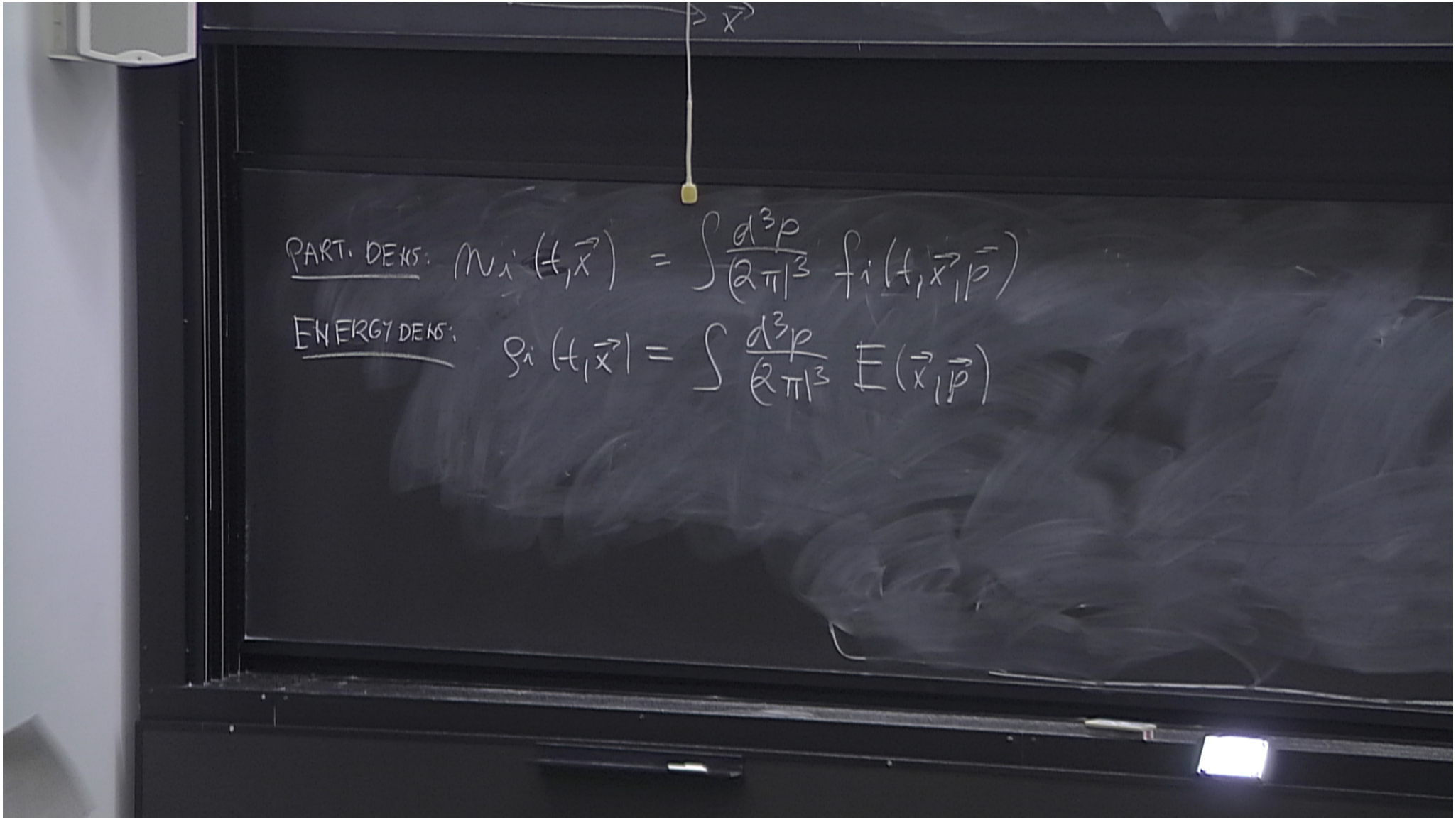
$\hbar = 1$

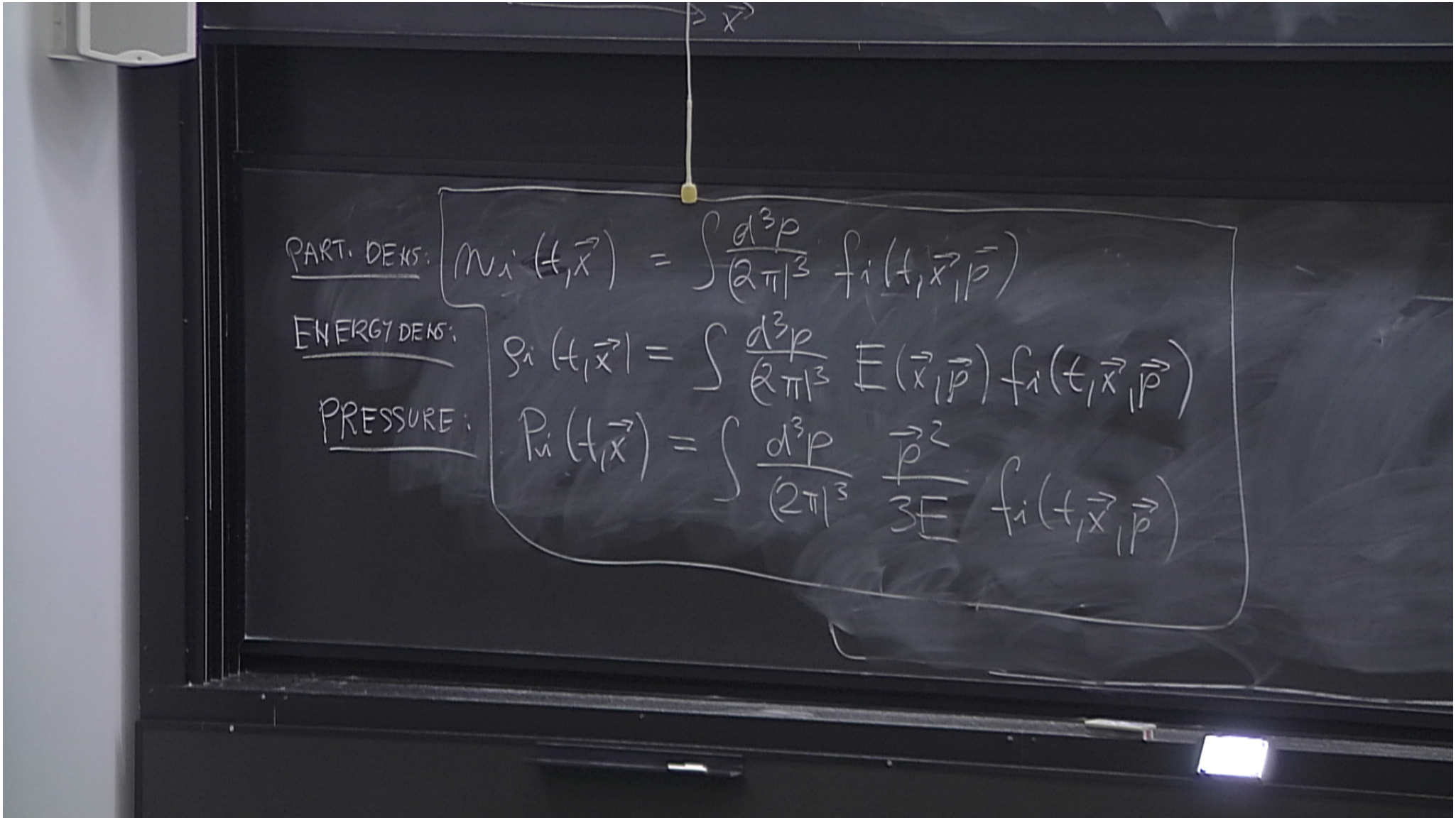
$$\frac{3}{8\pi G}\frac{1}{\rho_0}\left(\frac{A}{a_0}\right)^{3(1+w)} b^{3\alpha(1+w)} + \frac{3K}{8\pi G} A^2 b^{2\alpha} \frac{1}{\rho_0}\left(\frac{A}{a_0}\right)^{3(1+w)} b^{3\alpha(1+w)}$$

CHOOSING A $\qquad b^2$

SOLUTION IS $b(\eta) = S_k(\eta/\alpha)$

$$a(\eta) = A^{1/\alpha} \left[\frac{8\pi G}{3(1+w)\rho_0}\right]^{\frac{1}{3\alpha(1+w)}} S_k^{\alpha}(\eta/\alpha)$$


$\vec{x}$
PART. DENS: $n_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} f_i(t,\vec{x},\vec{p})$
ENERGY DENS: $\rho_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p})$

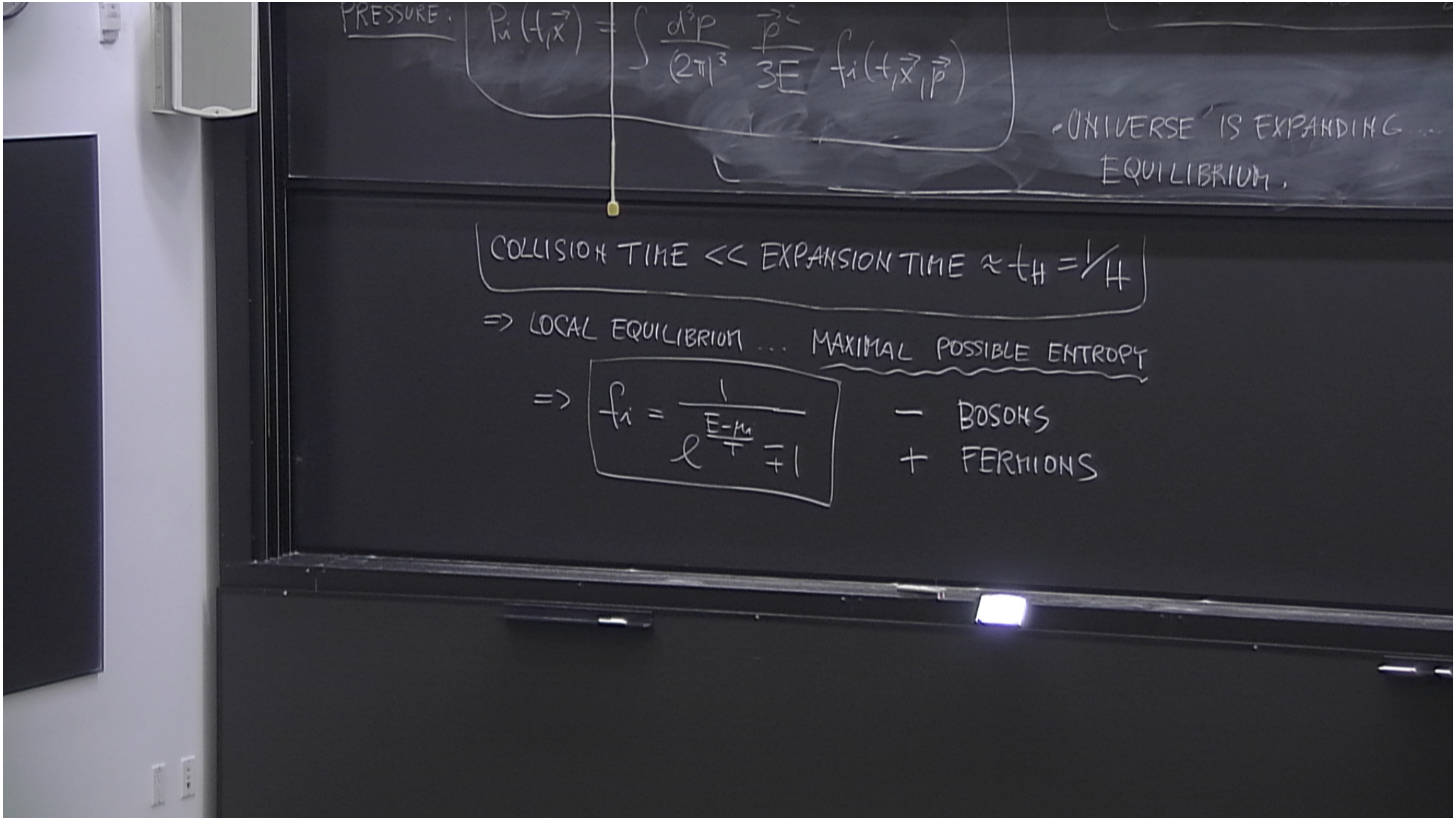

PART. DENS: $n_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} f_i(t,\vec{x},\vec{p})$
ENERGY DENS: $\rho_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} E(\vec{x},\vec{p}) f_i(t,\vec{x},\vec{p})$
PRESSURE: $P_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E} f_i(t,\vec{x},\vec{p})$

- OUT OF EQUILIBRIUM f_i GOVERNED BY THE BOLTZMANN

$$\frac{df_i}{dt} = \frac{\partial f_i}{\partial t} + \frac{\partial f_i}{\partial x_j}\dot{x}_j + \frac{\partial f_i}{\partial p_j}\dot{p}_j = Col[f_i]$$

$(t, \vec{x}, \vec{p})$

$(t, \vec{x}, \vec{p})$


PRESSURE:
$P_i(t,\vec{x}) = \int \frac{d^3p}{(2\pi)^3} \frac{\vec{p}^2}{3E} f_i(t,\vec{x},\vec{p})$
- UNIVERSE IS EXPANDING ... EQUILIBRIUM.
COLLISION TIME << EXPANSION TIME $\approx t_H = 1/H$
=> LOCAL EQUILIBRIUM ... MAXIMAL POSSIBLE ENTROPY
=> $f_i = \frac{1}{e^{\frac{E-\mu_i}{T}} \mp 1}$
- BOSONS
+ FERMIONS