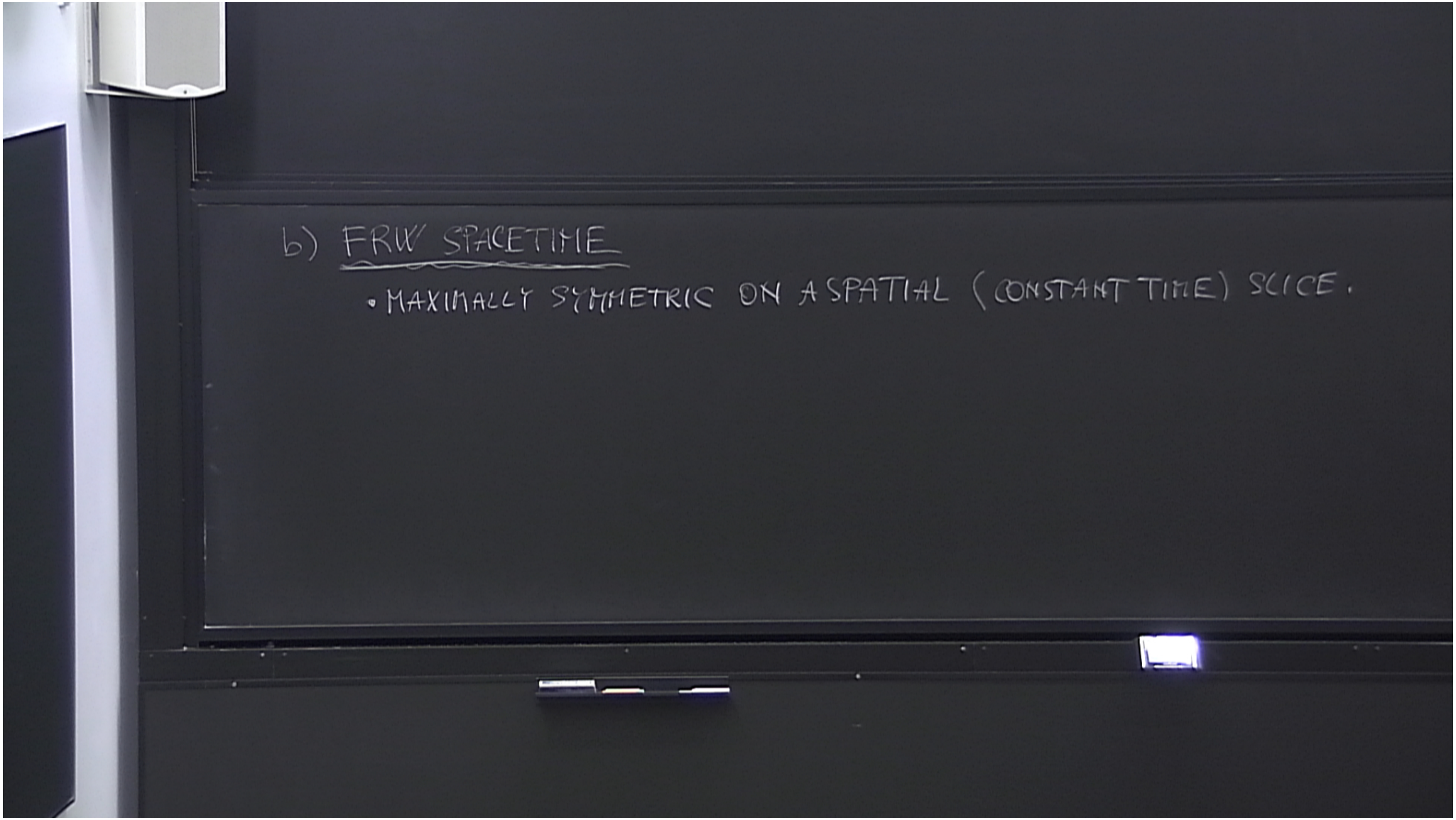
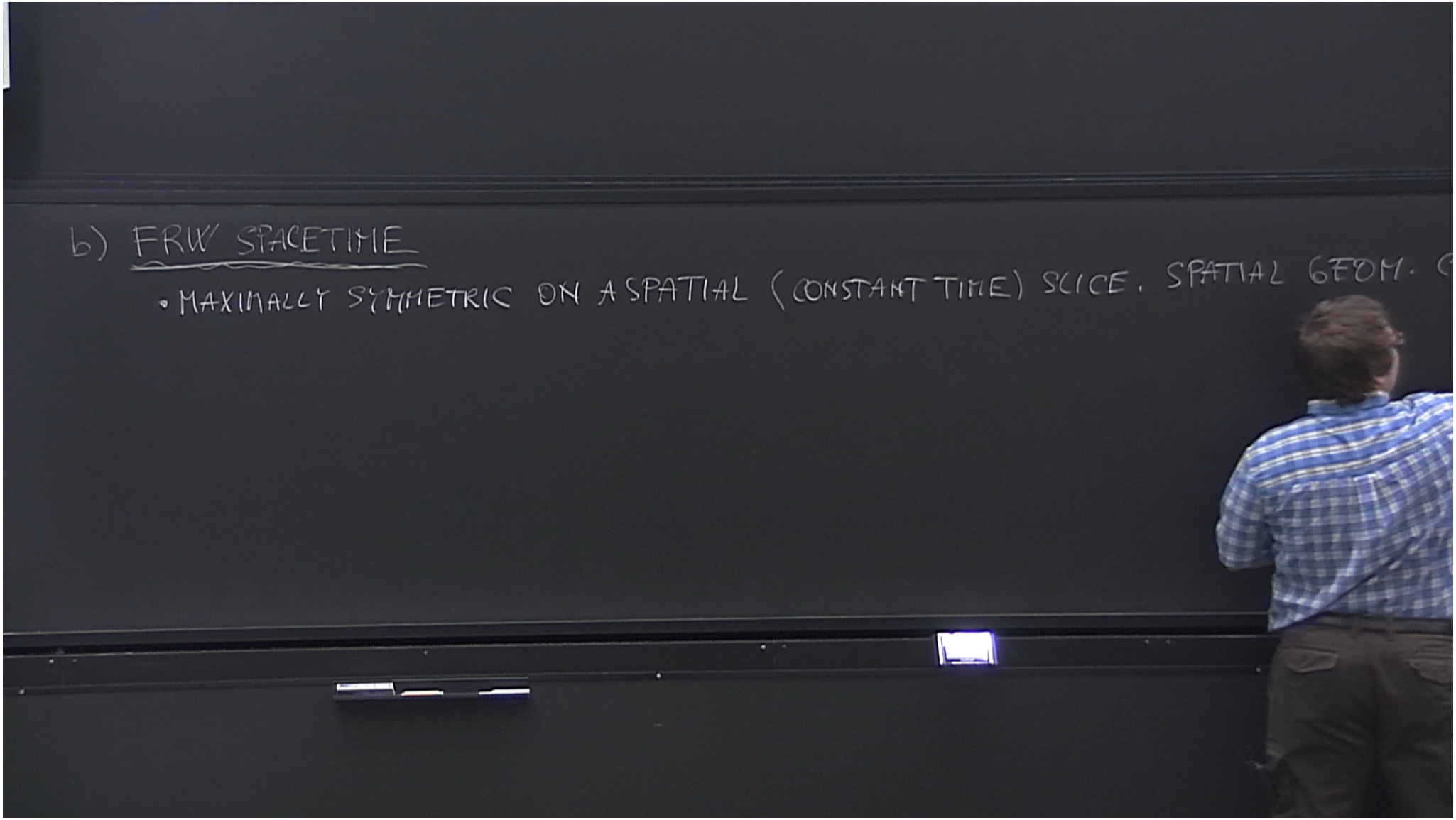
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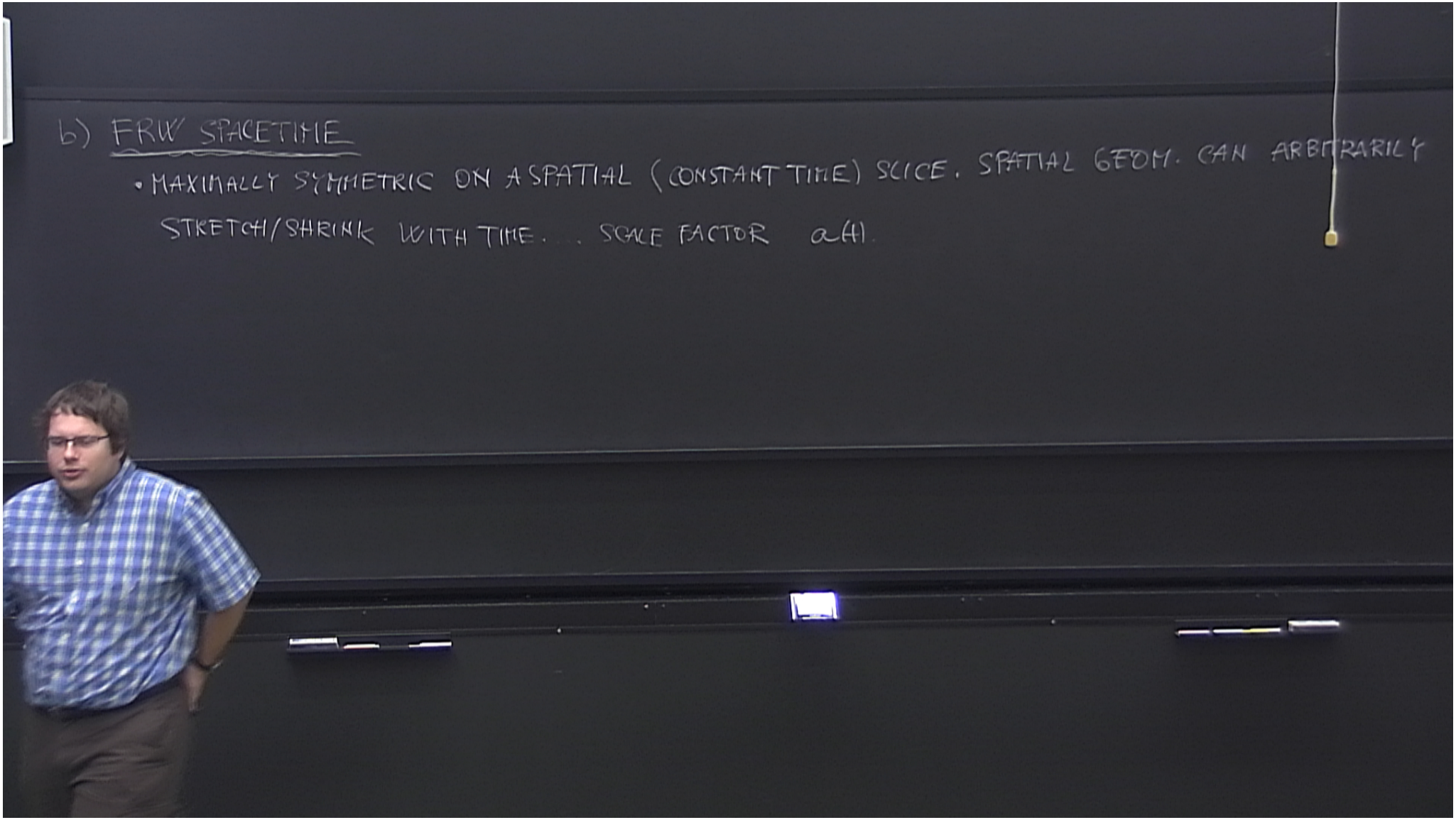
Date: Jan 27, 2015 11:30 AM

URL: http://pirsa.org/15010068

Abstract:


b) FRW SPACETIME
• MAXIMALLY SYMMETRIC ON A SPATIAL (CONSTANT TIME) SLICE.


b) FRW SPACETIME
• MAXIMALLY SYMMETRIC ON A SPATIAL (CONSTANT TIME) SLICE. SPATIAL GEOM.


b) FRW SPACETIME
• MAXIMALLY SYMMETRIC ON A SPATIAL (CONSTANT TIME) SLICE. SPATIAL GEOM. CAN ARBITRARILY
STRETCH/SHRINK WITH TIME. . SCALE FACTOR a(t).

$= -dt^2 + a^2(t)\left[\delta_{ij} + \frac{K x_i x_j}{\rho^2 - K x^2}\right] dx^i dx^j$

H_3. $K=-1$ "OPEN" $\int$ $(x^i = \rho \tilde{x}^i,\ a = \frac{a}{\rho})$

$\Rightarrow a(t)$: RADIUS OF CURVATURE OF THE SPATIAL SLICE AT

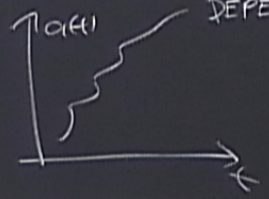
MMETRY: • SS (ISOTROPIC) + SPATL TRANS S. (HOM) = 3+3 = 6 KVs

• NO BOOST INVARIANCE. $\Rightarrow$ ∃ PREFERRED FRAME .. COMOVING OBSERVERS ... t ... PROPER TIME

x^i = CONST

.. ONLY SUCH OBSERVERS SEE HOM & ISOTROPY.

SCALE FACTOR



DEPENDS ON MATTER CONTENT.

$H = \frac{\dot{a}}{a}$ HUBBLE EXPANSION RATE

$q = \frac{\ddot{a}}{a H^2}$

$= -dt^2 + a^2(t)\left[\delta_{ij} + \frac{K x_i x_j}{\rho^2 - K x^2}\right] dx^i dx^j$

H3. $K=-1$ "OPEN"

$(x^i = \rho x^i, \; a = \frac{a}{\rho})$

$\Rightarrow a(t)$ RADIUS OF CURVATURE OF THE SPATIAL SLICE AT t.

- SS (ISOTROPIC) + SPAT. TRANS. S. (HOM) = 3+3 = 6 KVs
- NO BOOST INVARIANCE $\Rightarrow$ ∃ PREFERRED FRAME ... COMOVING OBSERVERS ... t ... PROPER TIME
  x^i = CONST

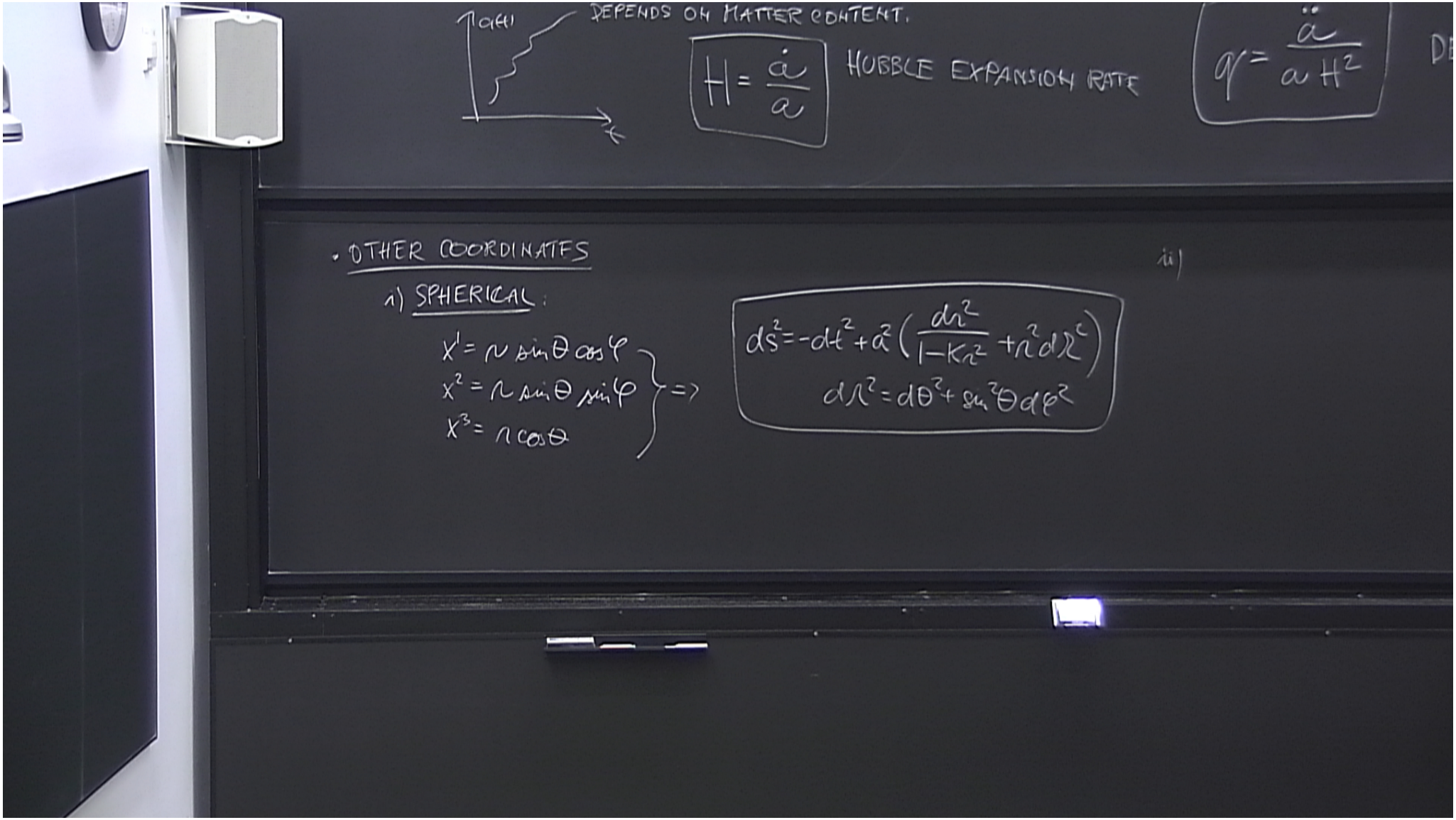
... ONLY SUCH OBSERVERS SEE HOM & ISOTROPY.

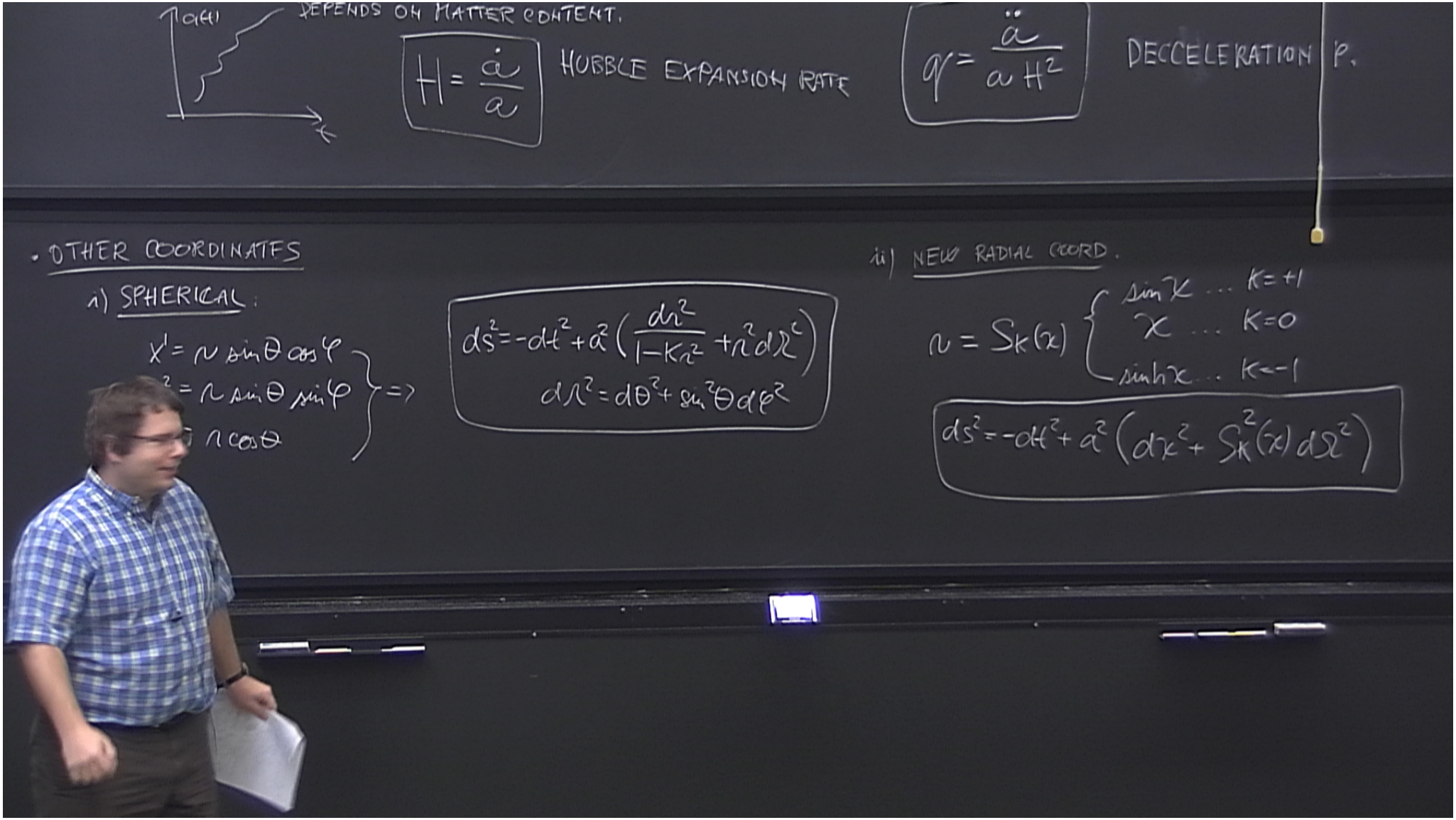
TOR

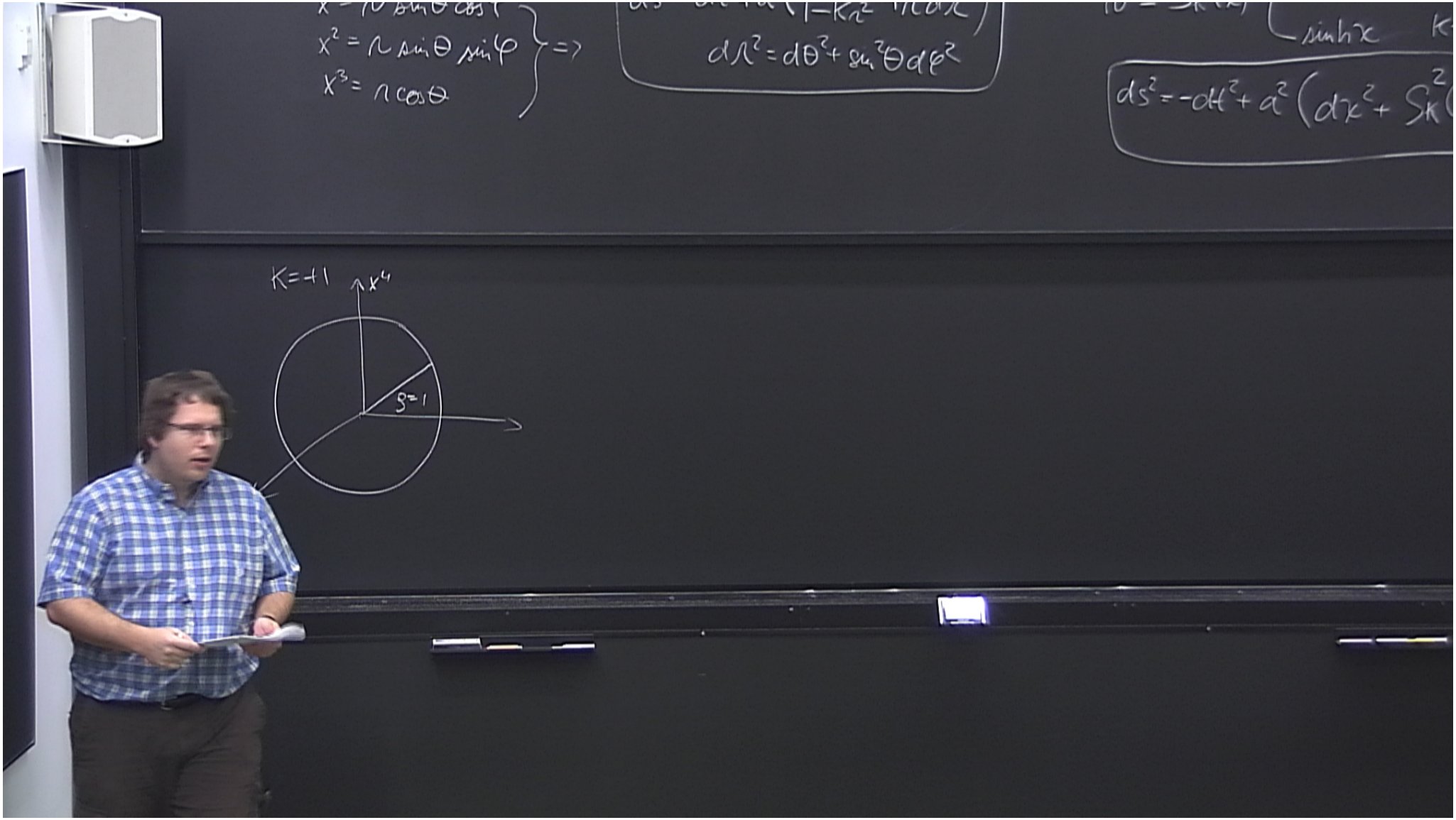
DEPENDS ON MATTER CONTENT.

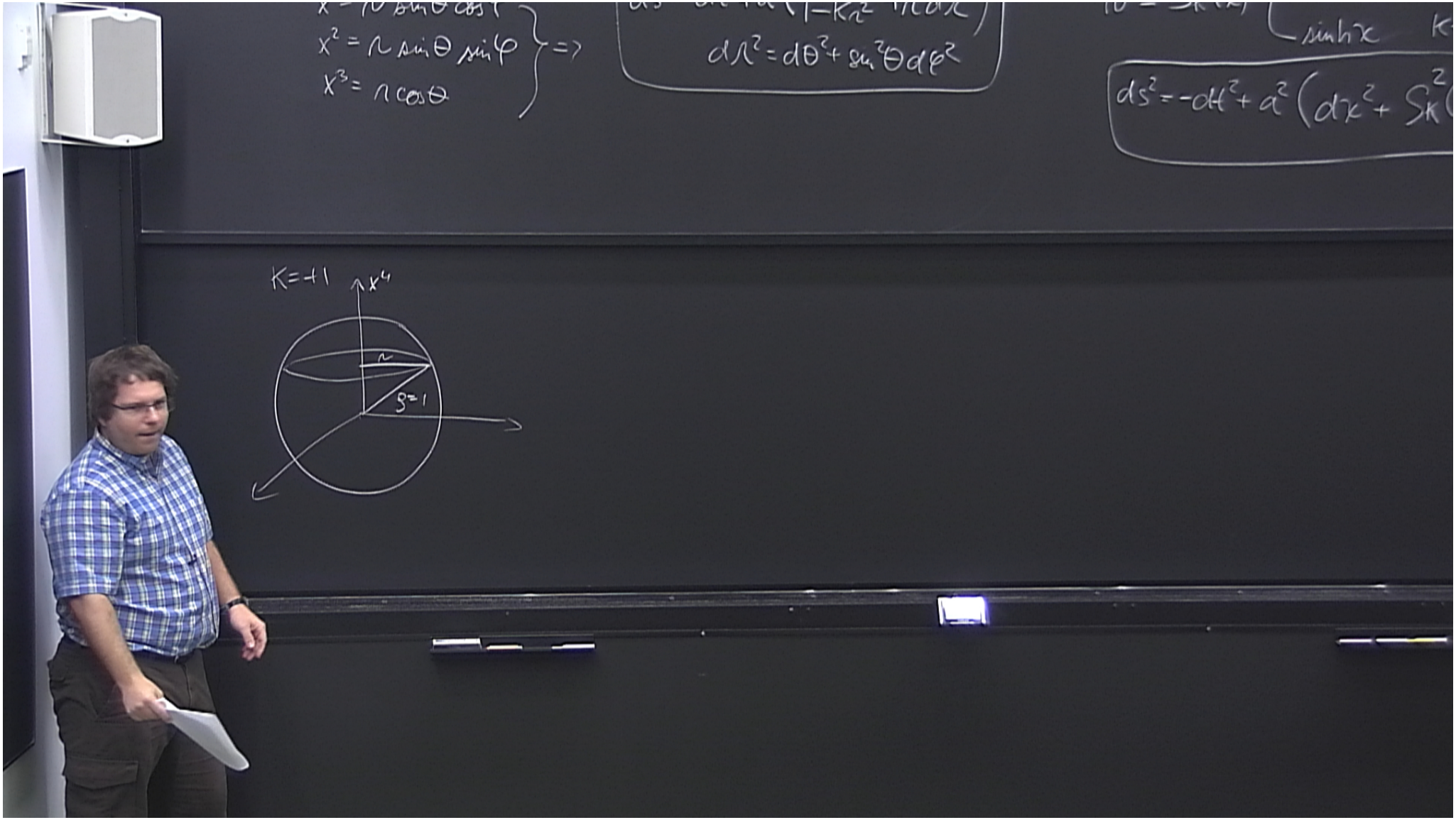
$H = \frac{\dot{a}}{a}$ HUBBLE EXPANSION RATE

$q = \frac{\ddot{a}}{a H^2}$ DECCELERATION P.


a(t)
t
DEPENDS ON MATTER CONTENT.
$H = \frac{\dot{a}}{a}$
HUBBLE EXPANSION RATE
$q = \frac{\ddot{a}}{a H^2}$
• OTHER COORDINATES
i) SPHERICAL:
$x^1 = r \sin\theta \cos\varphi$
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Rightarrow$
$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
ii)


a(t)
t
DEPENDS ON MATTER CONTENT.
$H = \frac{\dot{a}}{a}$
HUBBLE EXPANSION RATE
$q = \frac{\ddot{a}}{a H^2}$
DECCELERATION P.
• OTHER COORDINATES
i) SPHERICAL:
$x' = r \sin\theta \cos\varphi$
$= r \sin\theta \sin\varphi$
$= r \cos\theta$
$\Rightarrow$
$ds^2 = -dt^2 + a^2\left(\frac{dr^2}{1-Kr^2} + r^2 d\Omega^2\right)$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
ii) NEW RADIAL COORD.
$r = S_K(\chi)$ { $\sin\chi$ ... $K=+1$; χ ... $K=0$; $\sinh\chi$ ... $K=-1$ }
$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$


$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
$ds^2 = -dt^2 + a^2(dx^2 +$
$\sinh x$
K=+1
x^4
S=1


$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$d\sigma^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
$ds^2 = -dt^2 + a^2(dx^2 + S_k^2$
$K=+1$
x^4
r
$S=1$

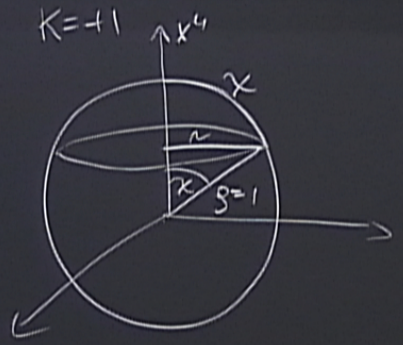
$x^2 = r \sin\theta \sin\varphi$

$x^3 = r \cos\theta$ $\Big\} \Rightarrow$

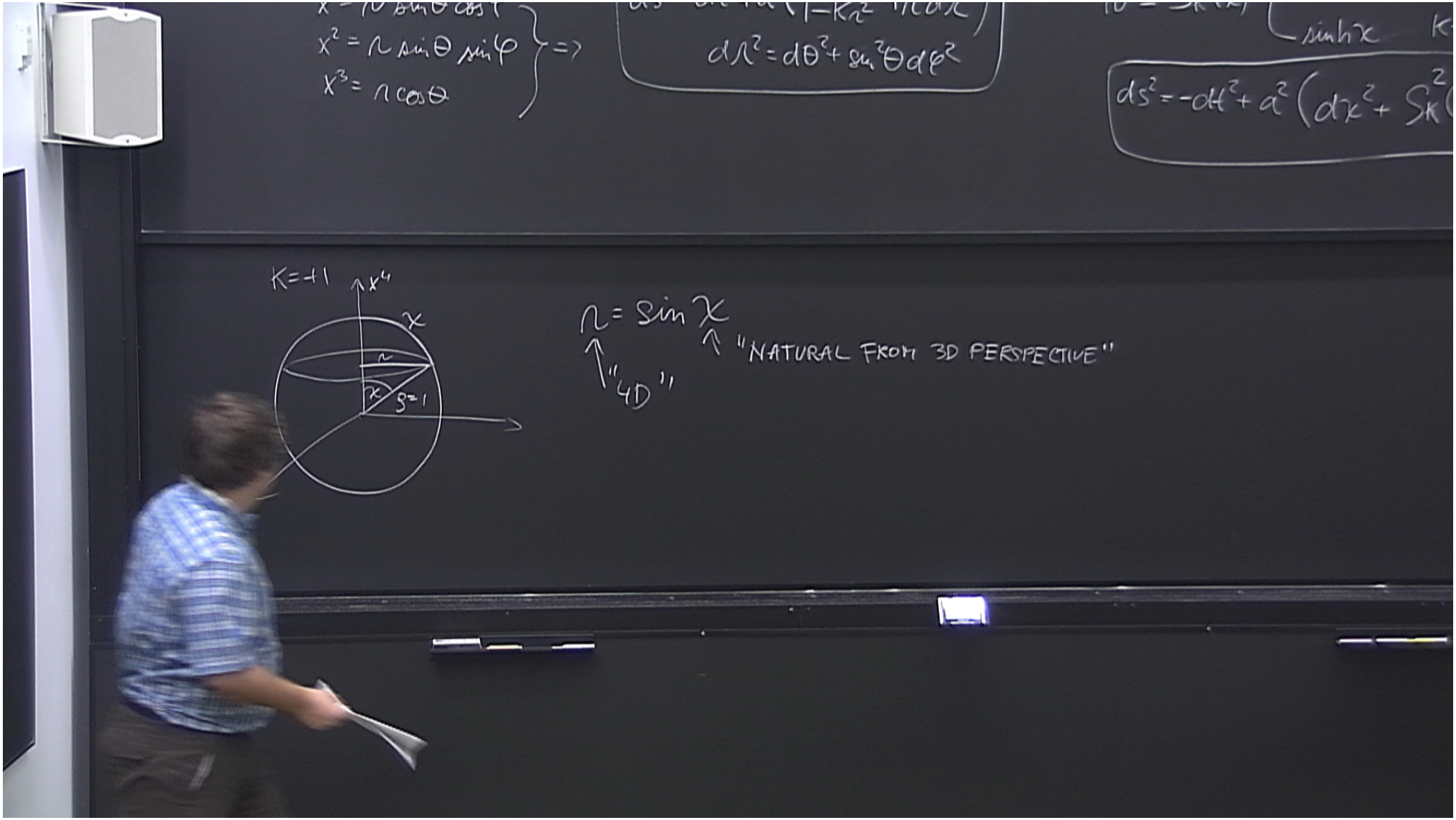
$d\Lambda^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh\chi$

$ds^2 = -dt^2 + a^2 \left(d\chi^2 + S_K^2 \right.$



$r = \sin\chi$


$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
$ds^2 = -dt^2 + a^2(dx^2 + S_K^2$
$\sinh x$
K=+1
x^4
x
r
χ
$S=1$
$r = \sin\chi$
"4D"
"NATURAL FROM 3D PERSPECTIVE"

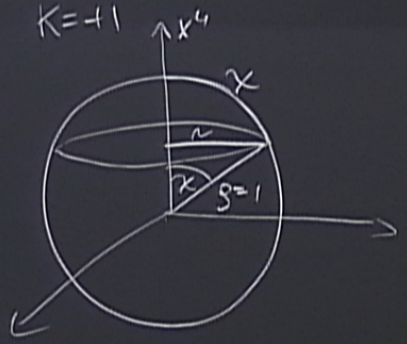
$x^2 = r \sin\theta \sin\varphi$ } $\Rightarrow$

$x^3 = r \cos\theta$

$dl^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

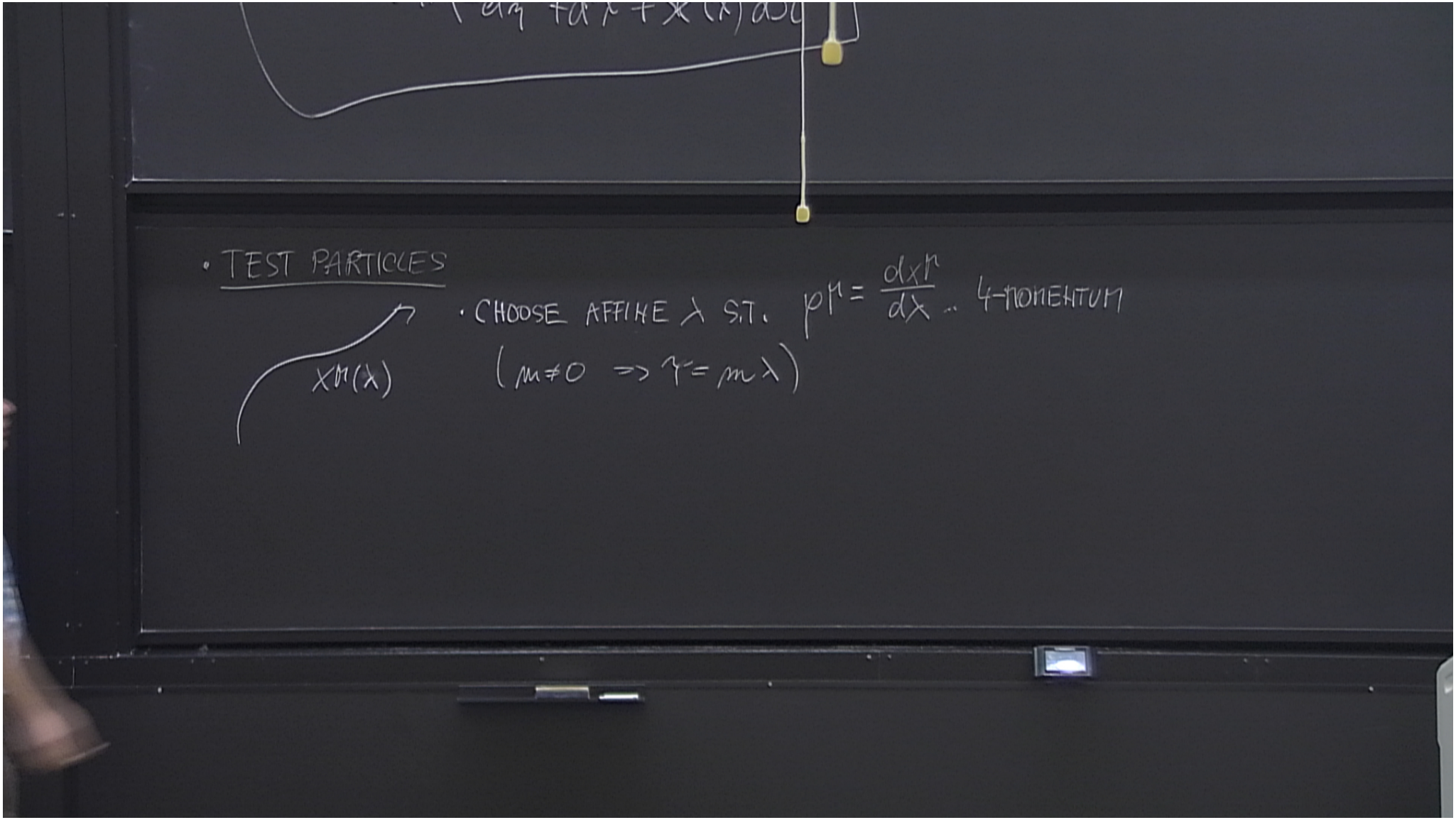
$\sinh x \quad K<-1$

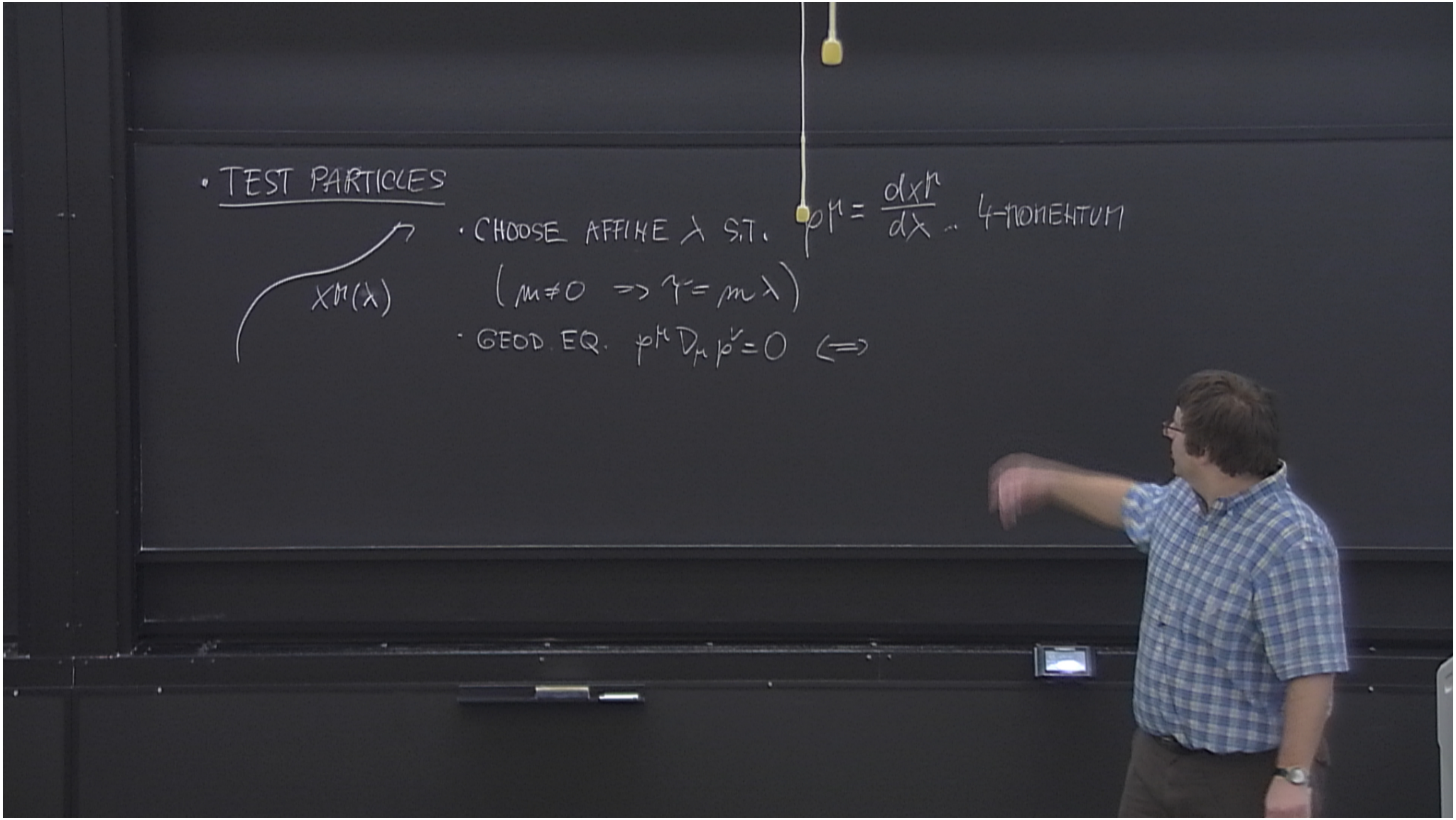
$ds^2 = -dt^2 + a^2\left(dx^2 + S_K^2(x)\, d\Omega^2\right)$

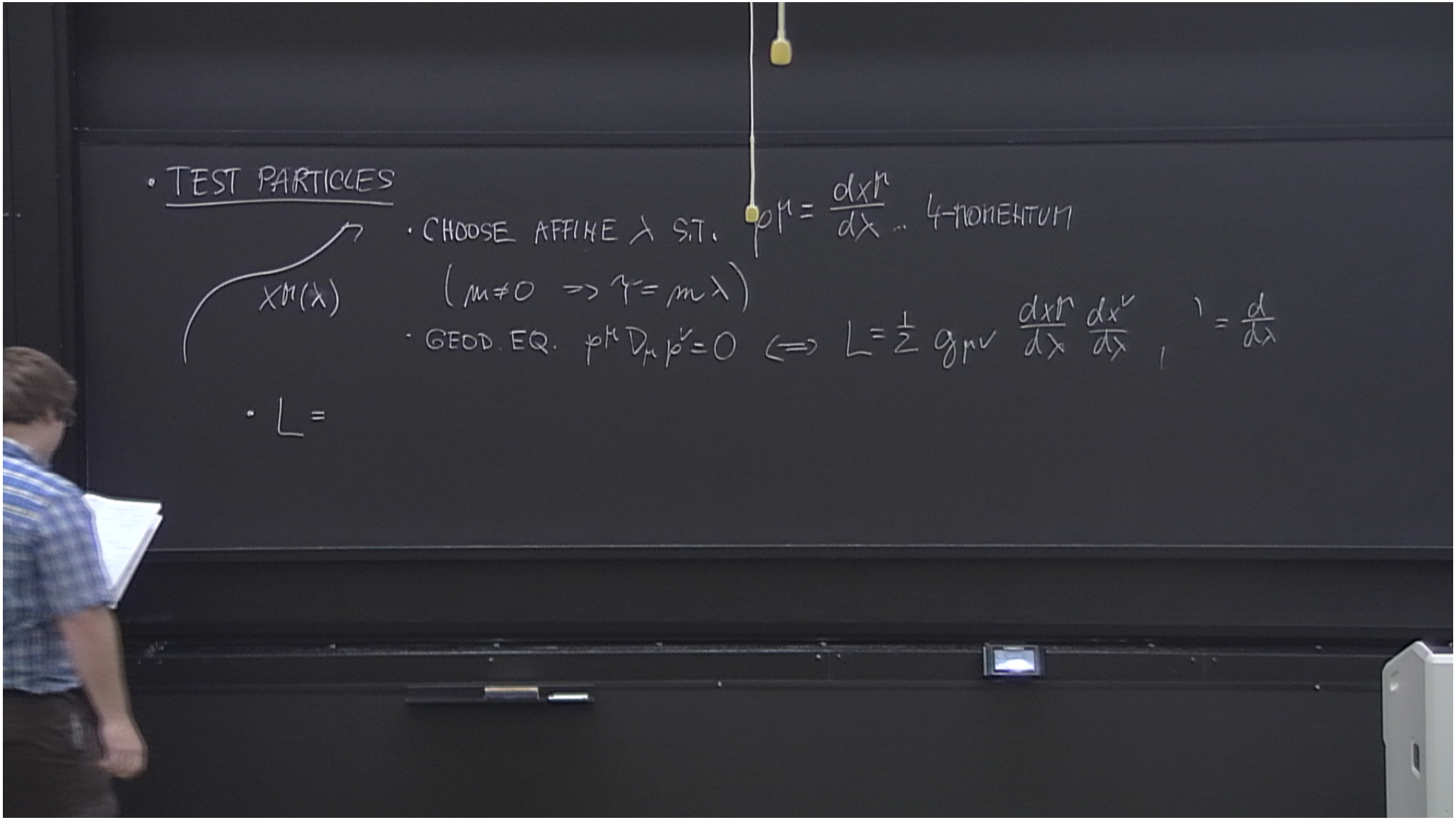


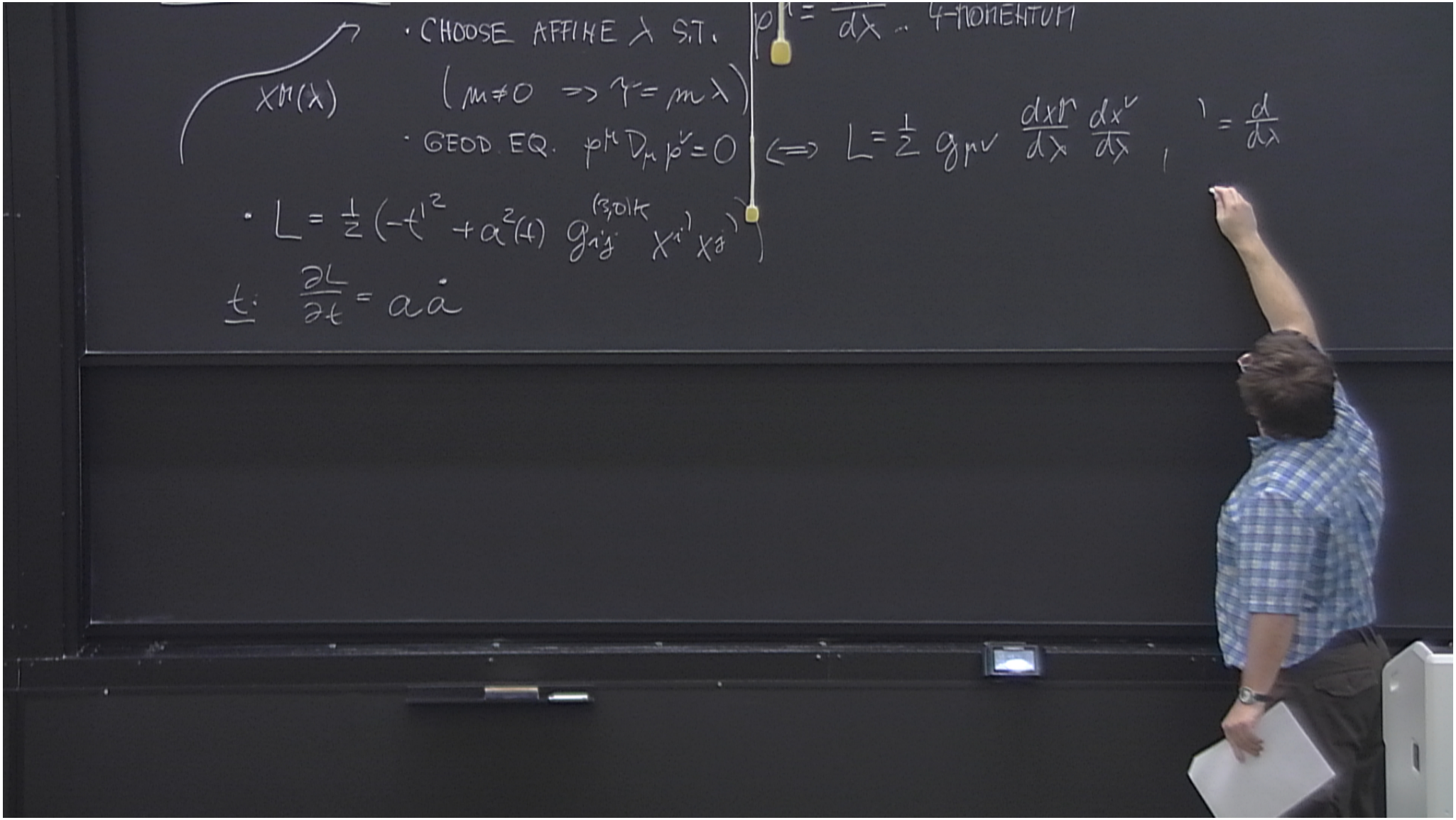
$r = \sin\chi$ ↑ "NATURAL FROM 3D PERSPECTIVE"

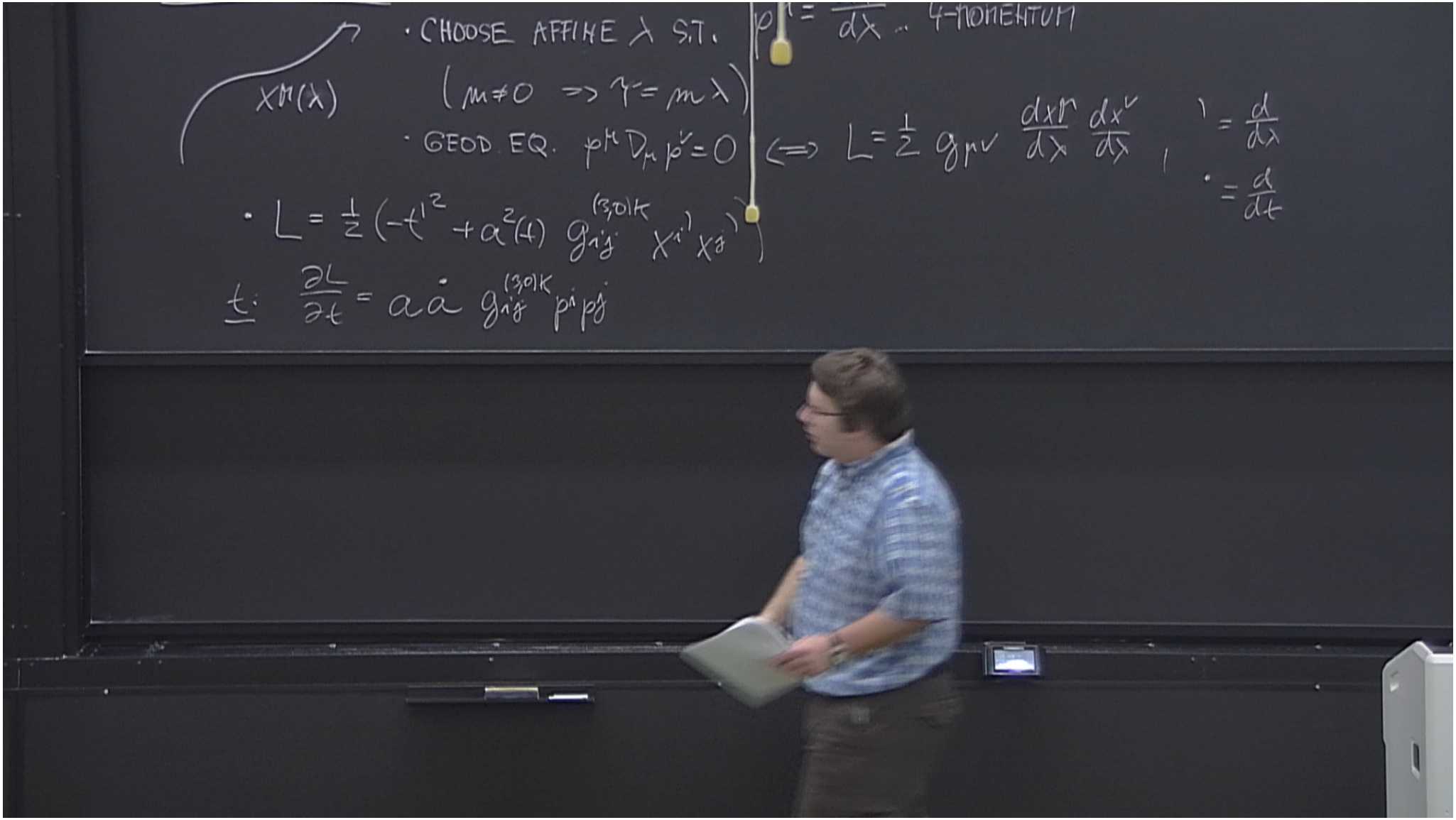
↑ "4D"

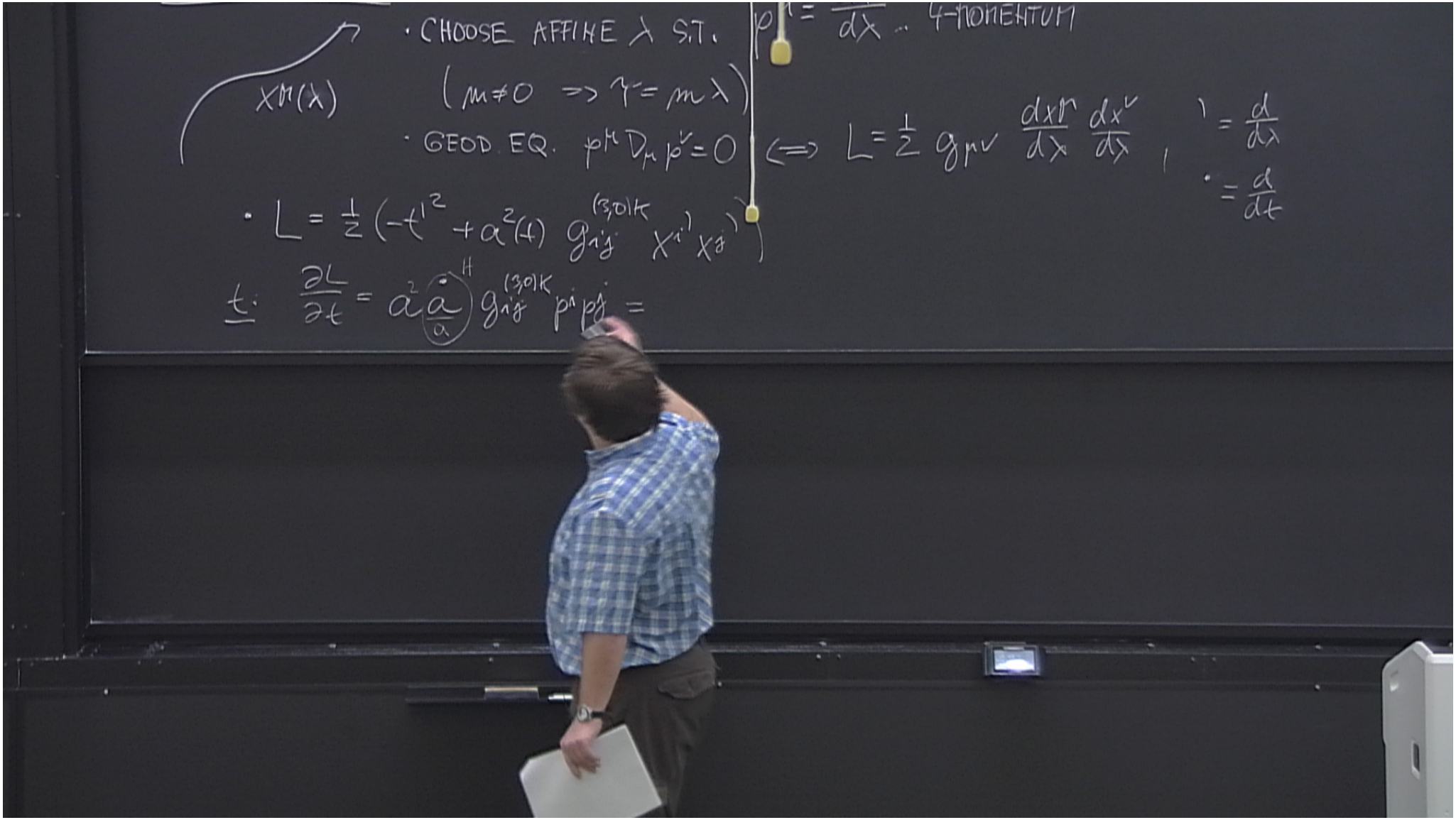

• TEST PARTICLES
$x^\mu(\lambda)$
• CHOOSE AFFINE λ S.T. $p^\mu = \frac{dx^\mu}{d\lambda}$ .. 4-MOMENTUM
$(m \neq 0 \Rightarrow \tau = m\lambda)$

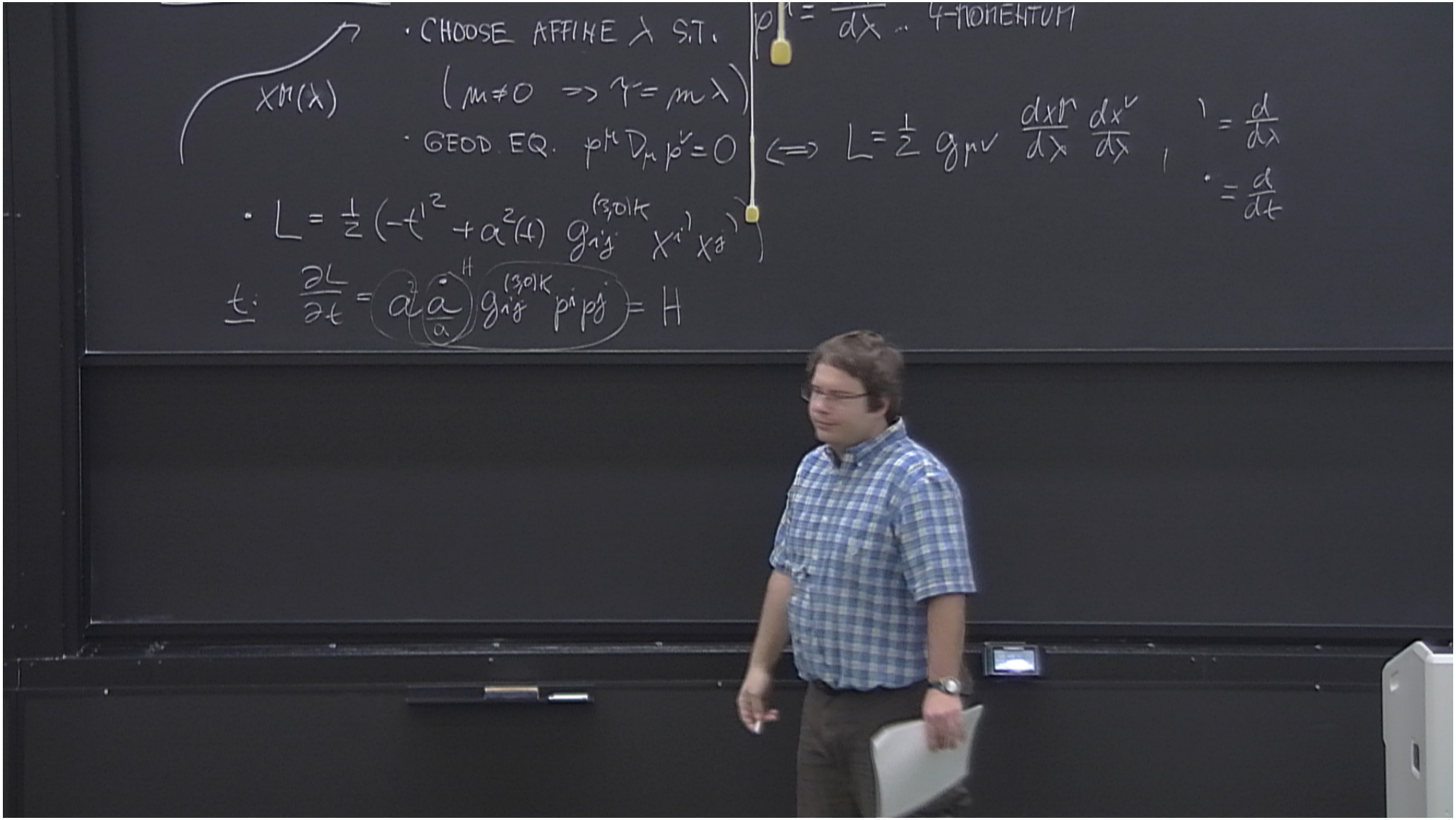

• TEST PARTICLES
$x^\mu(\lambda)$
• CHOOSE AFFINE λ S.T. $p^\mu = \frac{dx^\mu}{d\lambda}$ .. 4-MOMENTUM
$(m \neq 0 \Rightarrow \tau = m\lambda)$
• GEOD. EQ. $p^\mu D_\mu p^\nu = 0 \iff$

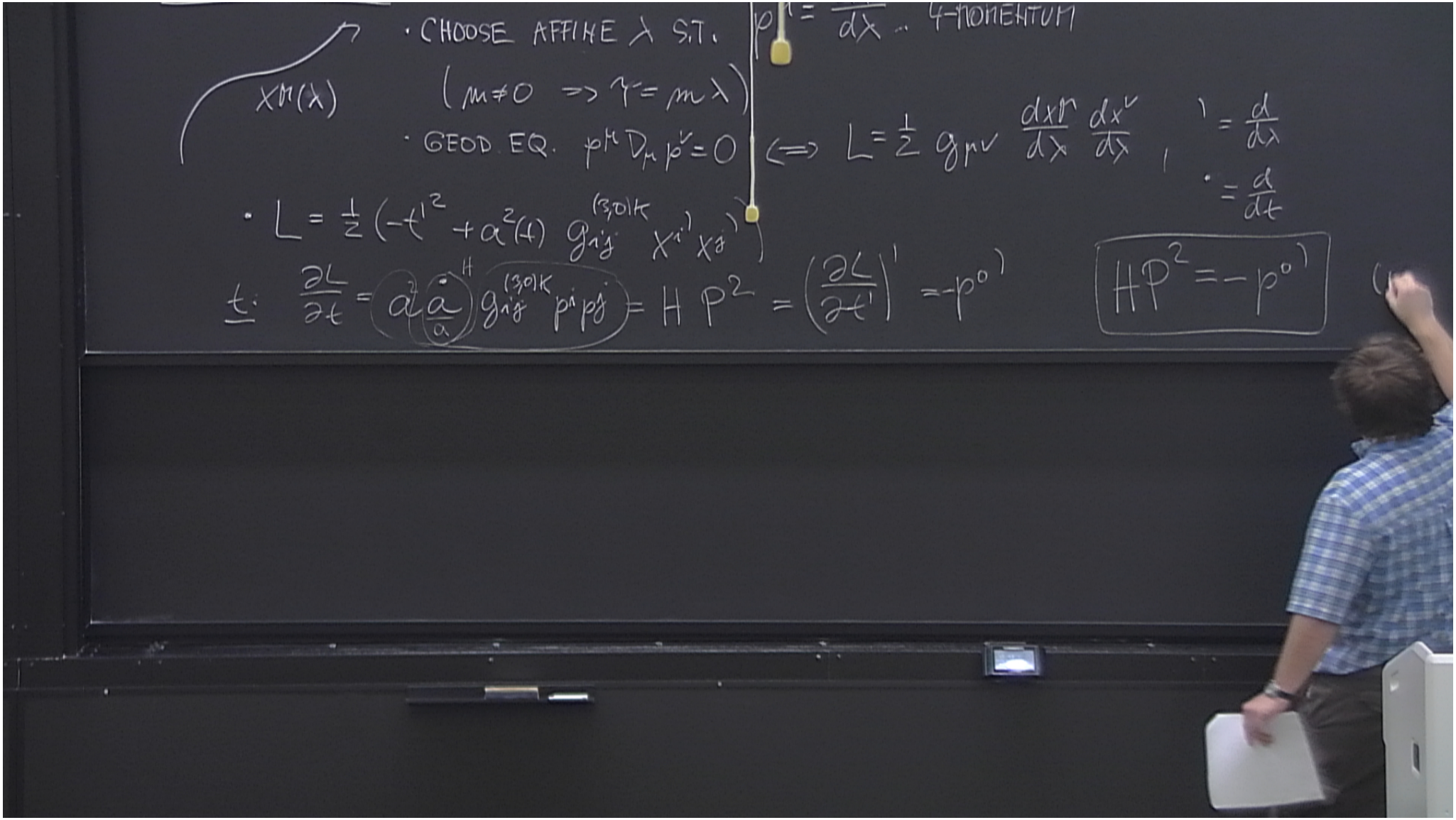

• TEST PARTICLES
• CHOOSE AFFINE λ S.T. $p^\mu = \frac{dx^\mu}{d\lambda}$ .. 4-MOMENTUM
$x^\mu(\lambda)$
($m \neq 0 \Rightarrow \tau = m\lambda$)
• GEOD. EQ. $p^\mu D_\mu p^\nu = 0 \iff L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$, $' = \frac{d}{d\lambda}$
• L =

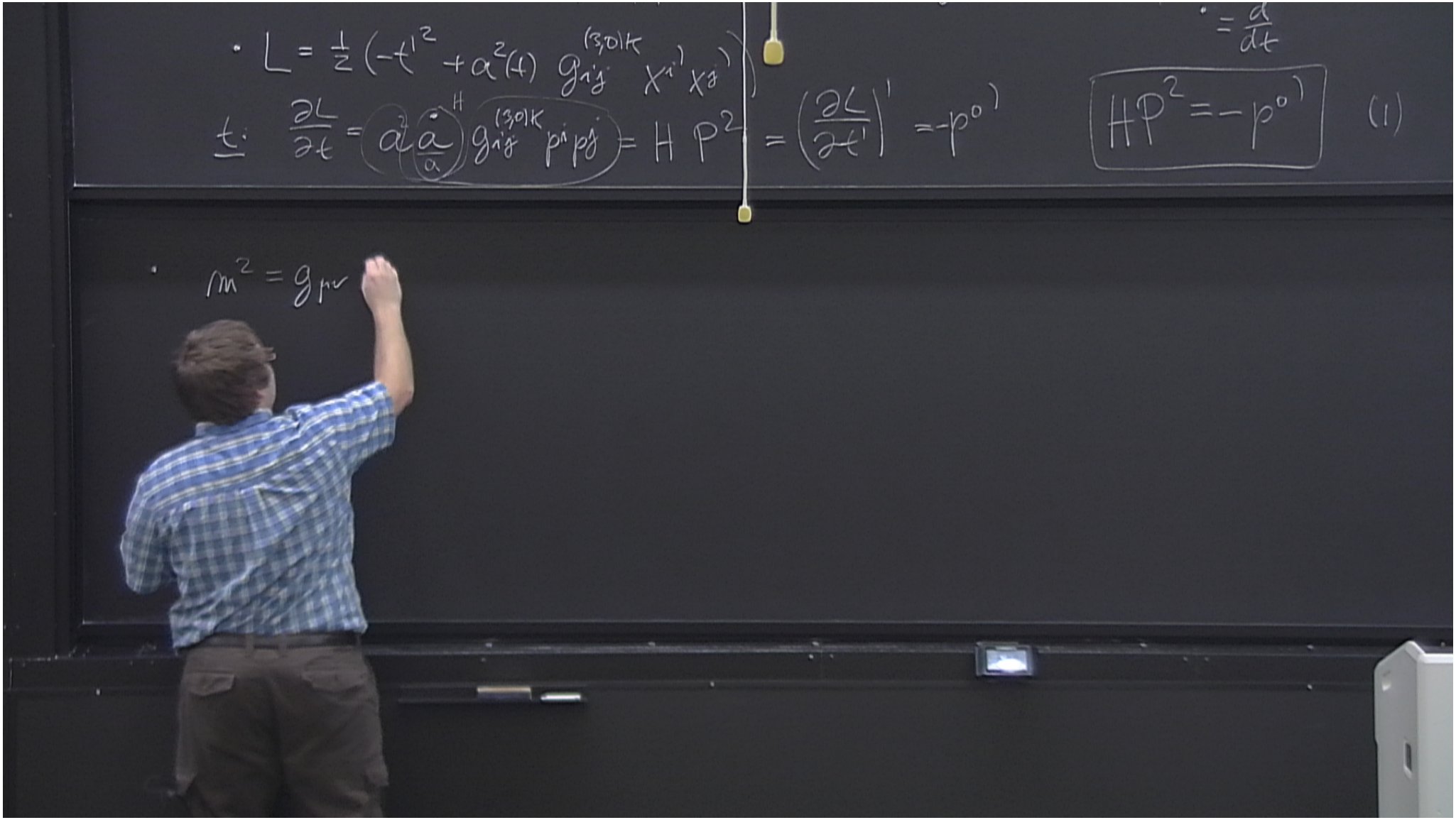

· CHOOSE AFFINE λ S.T.
$x^\mu(\lambda)$
$(m \neq 0 \Rightarrow \tau = m\lambda)$
· GEOD. EQ. $p^\mu D_\mu p^\nu = 0$
$\Longleftrightarrow L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$, $' = \frac{d}{d\lambda}$
4-MOMENTUM
$\cdot L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij} x^{i\prime} x^{j})$
t: $\frac{\partial L}{\partial t} = a\dot{a}$

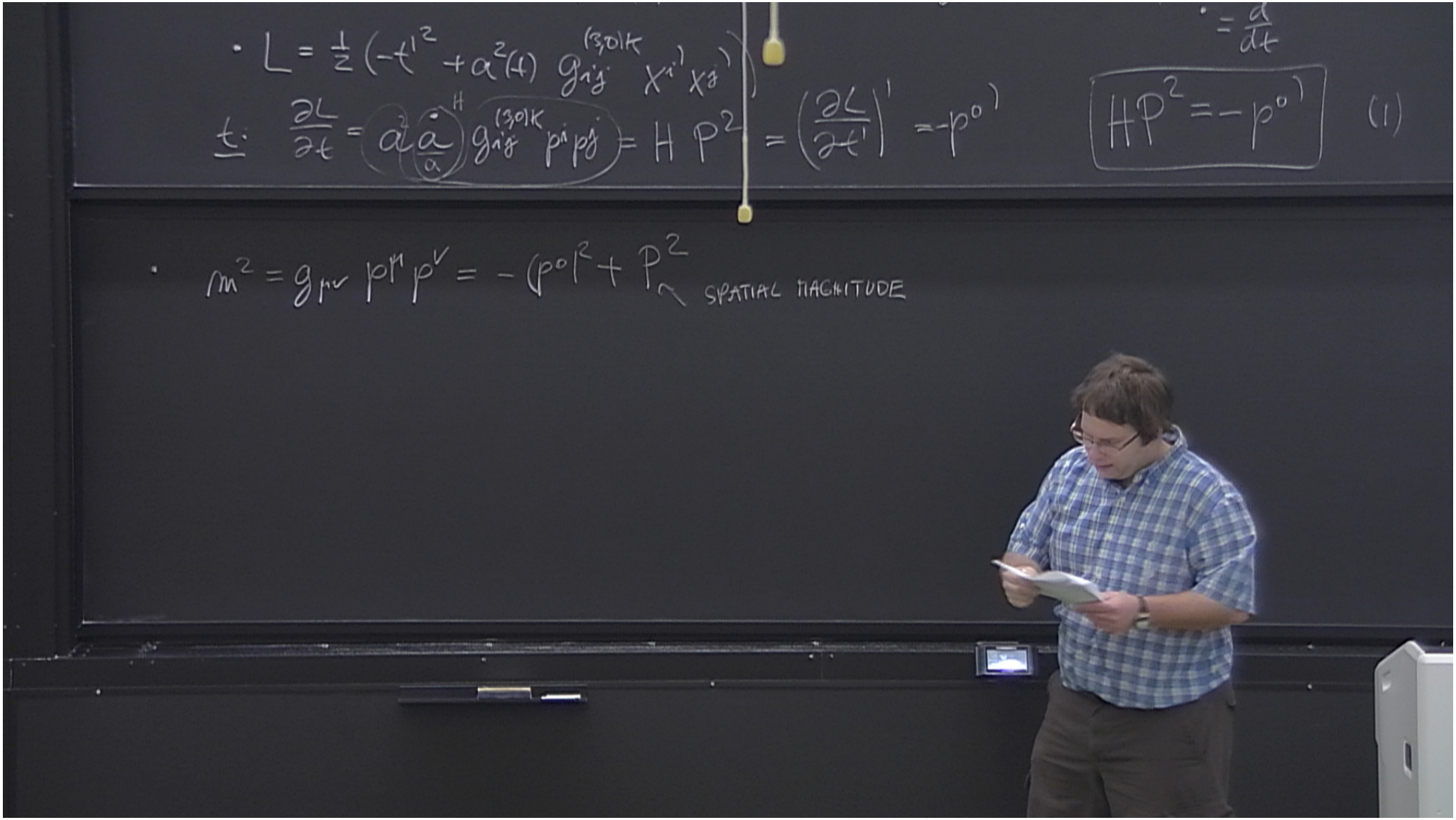

· CHOOSE AFFINE λ S.T.
$x^\mu(\lambda)$
$(m \neq 0 \Rightarrow \tau = m\lambda)$
· GEOD. EQ. $p^\mu D_\mu p^\nu = 0$
$\Longleftrightarrow L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
$p^\mu = \frac{dx^\mu}{d\lambda}$ .. 4-MOMENTUM
$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\cdot L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij} x^i x^j)$
t: $\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij} p^i p^j$


· CHOOSE AFFINE λ S.T.
$x^\mu(\lambda)$
$(m \neq 0 \Rightarrow \tau = m\lambda)$
· GEOD. EQ. $p^\mu D_\mu p^\nu = 0$
$\Longleftrightarrow L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
4-MOMENTUM
$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
$\cdot L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij} x^i x^j)$
t: $\frac{\partial L}{\partial t} = a^2 \frac{\dot{a}}{a} H\, g^{(3,0)k}_{ij} p^i p^j =$


· CHOOSE AFFINE λ S.T.
$x^\mu(\lambda)$
$(m \neq 0 \Rightarrow \tau = m\lambda)$
· GEOD. EQ. $p^\mu D_\mu p^\nu = 0$
$\iff L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
· $L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^i x^j)$
t: $\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij}\, p^i p^j = H$


· CHOOSE AFFINE λ S.T.
$x^\mu(\lambda)$
$(m \neq 0 \Rightarrow \tau = m\lambda)$
· GEOD. EQ. $p^\mu D_\mu p^\nu = 0$
$\Longleftrightarrow L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$
$' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
4-MOMENTUM
$\cdot L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^{i\prime} x^{j\prime})$
$\underline{t}: \frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$
$HP^2 = -p^{0\prime}$


$L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^{i\prime} x^{j\prime})$
$= \frac{d}{dt}$
t: $\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$
$HP^2 = -p^{0\prime}$ (1)
$m^2 = g_{\mu\nu}$


$= \frac{d}{dt}$
$L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3)}_{ij}\, x^{i\prime} x^{j\prime})$
$t: \quad \frac{\partial L}{\partial t} = a\dot{a}\, g^{(3)}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$
$H P^2 = -p^{0\prime}$ (1)
$m^2 = g_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + P^2$
SPATIAL MAGNITUDE

$= \frac{d}{dt}$

- $L = \frac{1}{2}\left(-\dot{t}^2 + a^2(t)\, g^{(3,0)k}_{ij}\, \dot{x}^i \dot{x}^j\right)$

t: $\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$ $\qquad \boxed{H P^2 = -p^{0\prime}}$ (1)

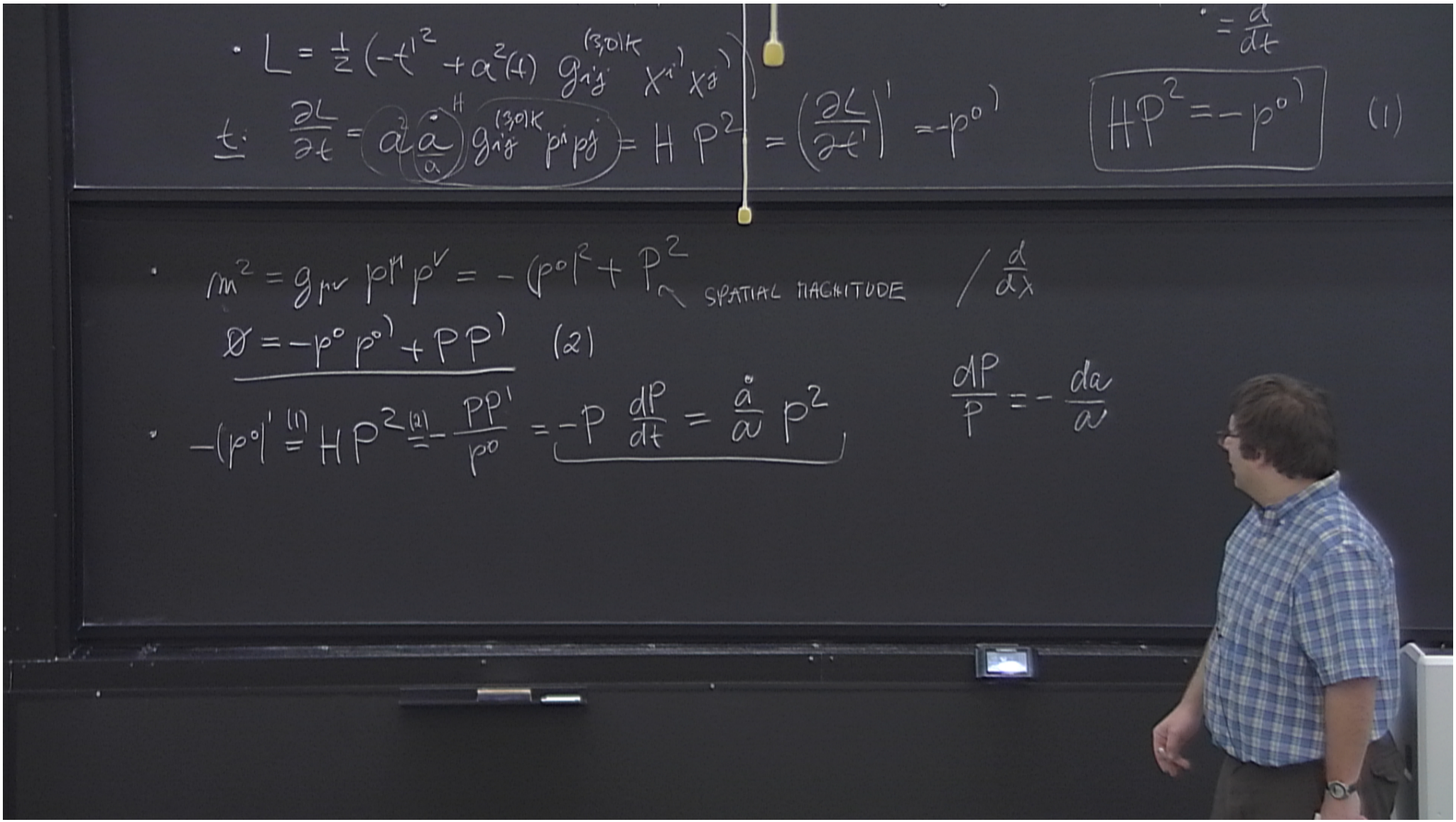
$\left(\frac{\dot{a}}{a} = H\right)$

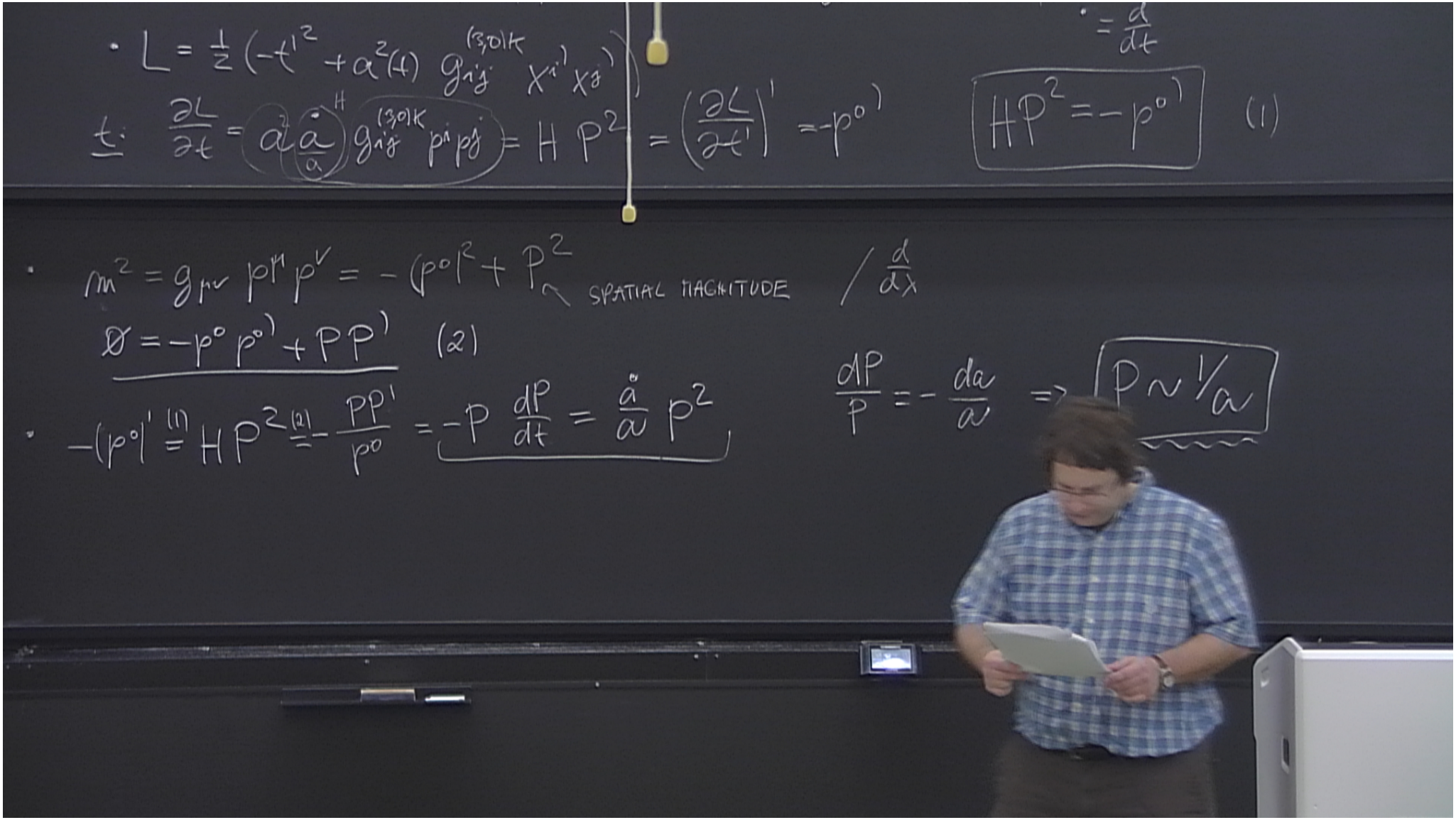
- $m^2 = g_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + P^2$ $\qquad$ SPATIAL MAGNITUDE $\qquad / \frac{d}{dx}$

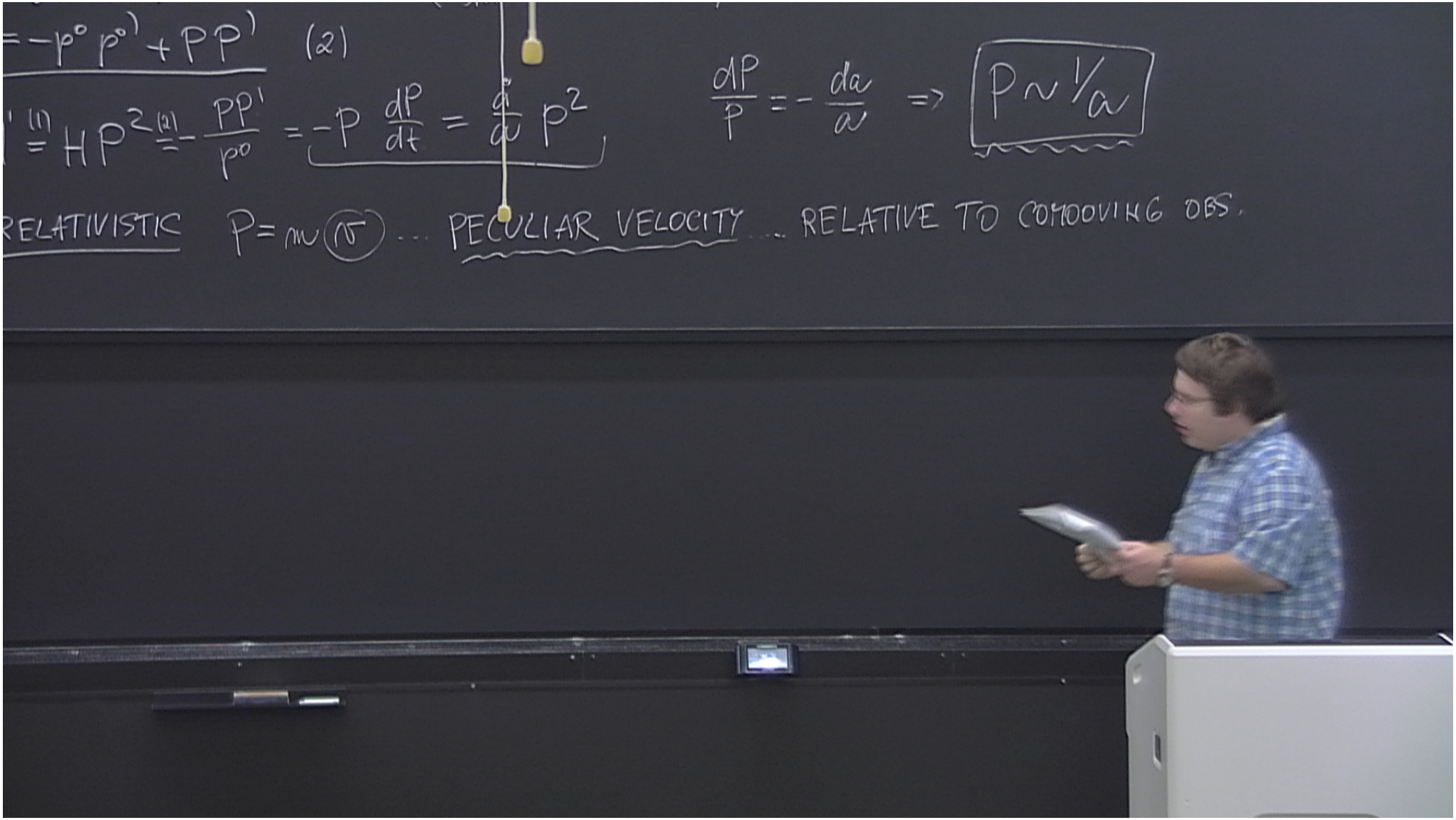
$0 = -p^0 p^{0\prime} + P P'$ (2)

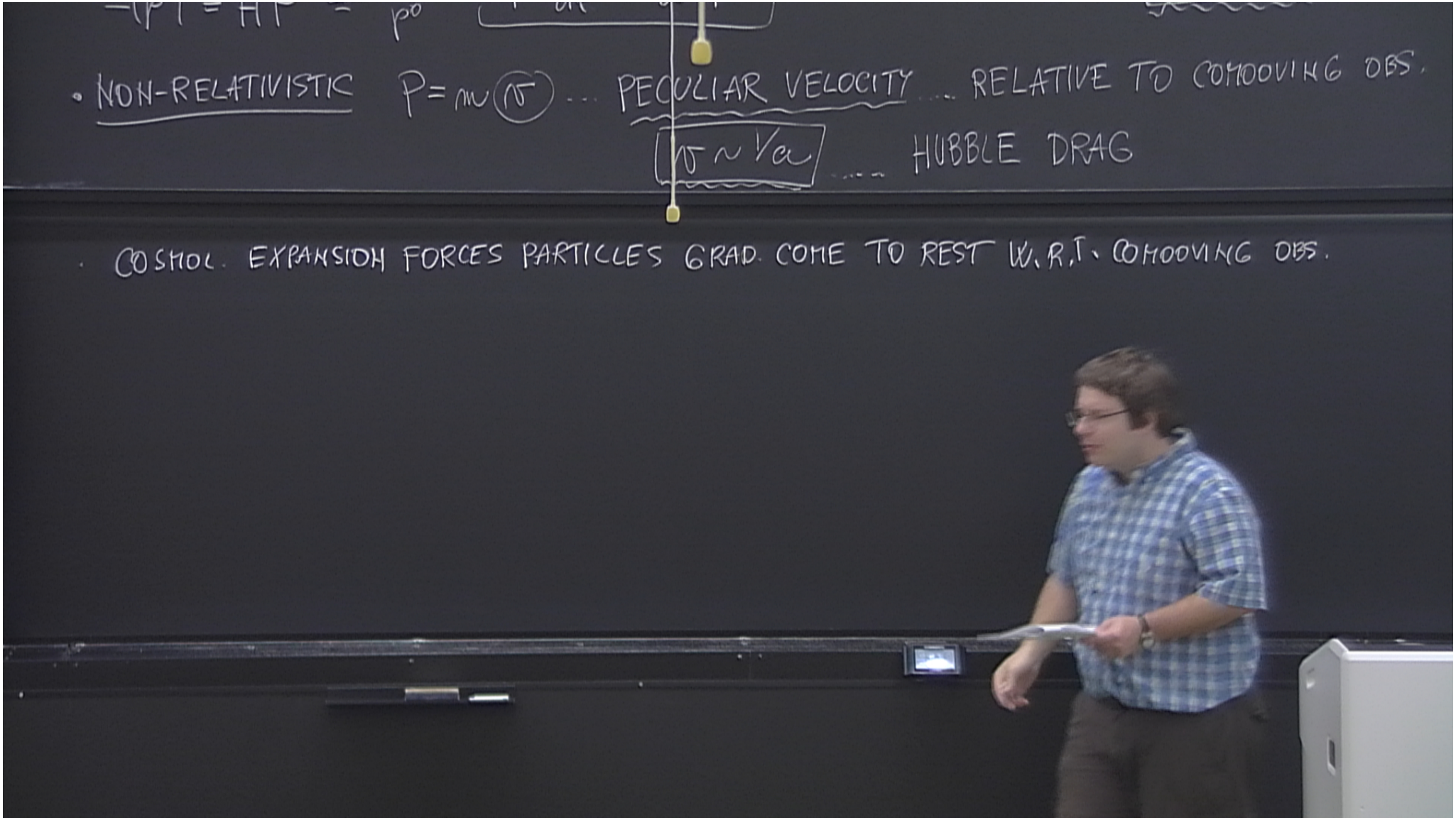
- $-(p^0)' \overset{(1)}{=} H P^2 \overset{(2)}{=} -\frac{P P'}{p^0} = \underline{-P \frac{dP}{dt} = \frac{\dot{a}}{a} p^2}$

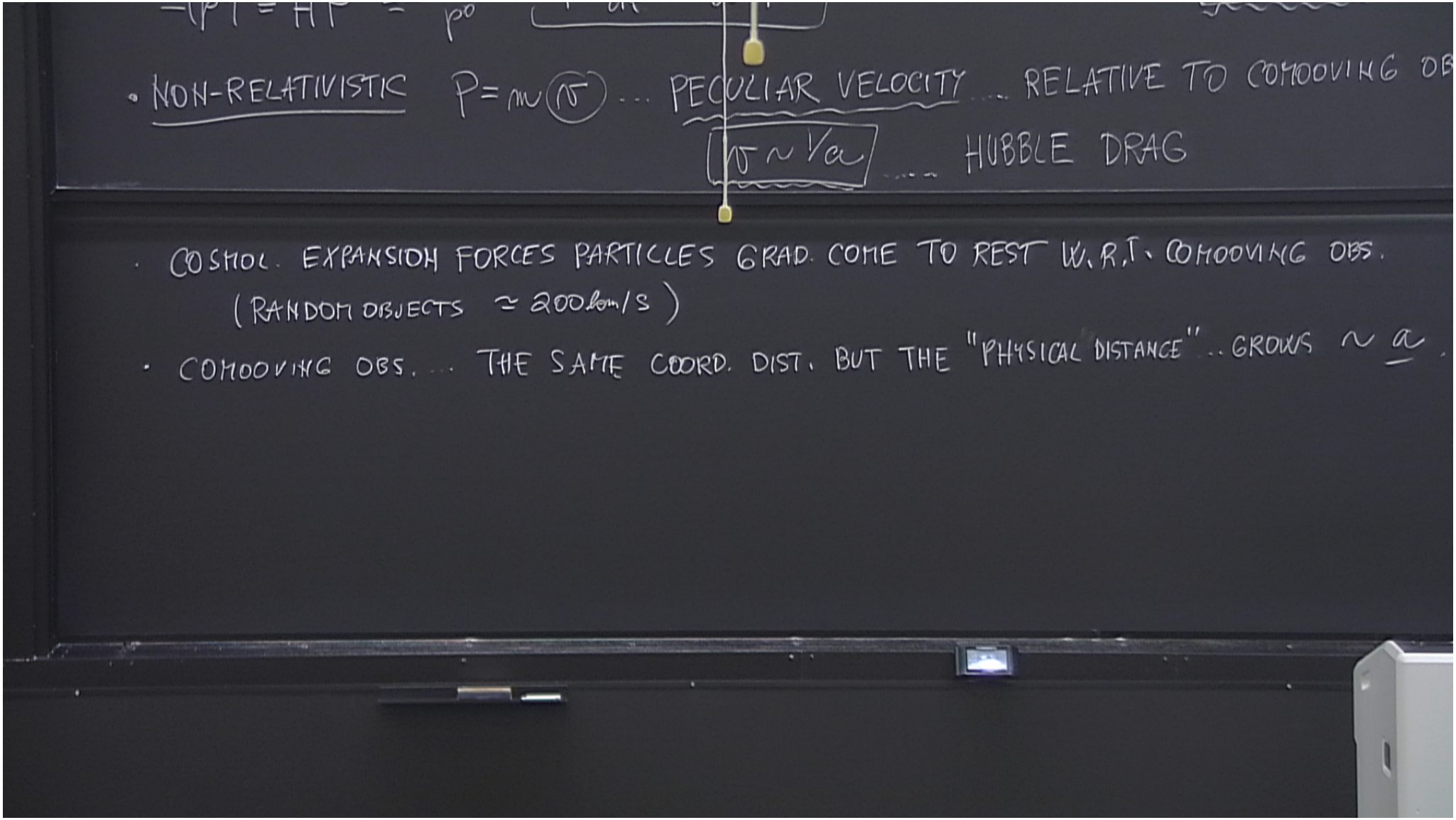
$\frac{dP}{P} = -\frac{da}{a}$

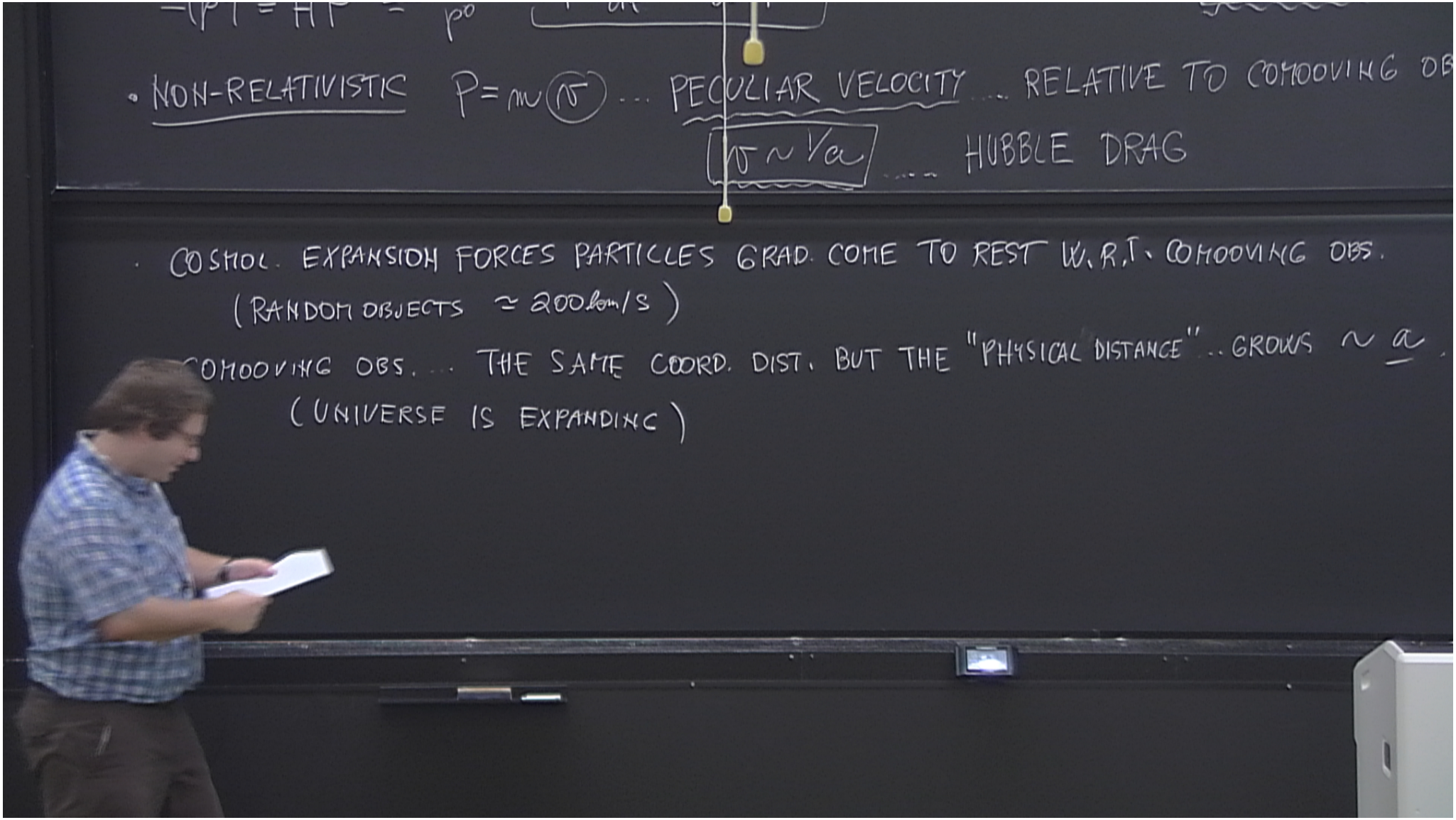


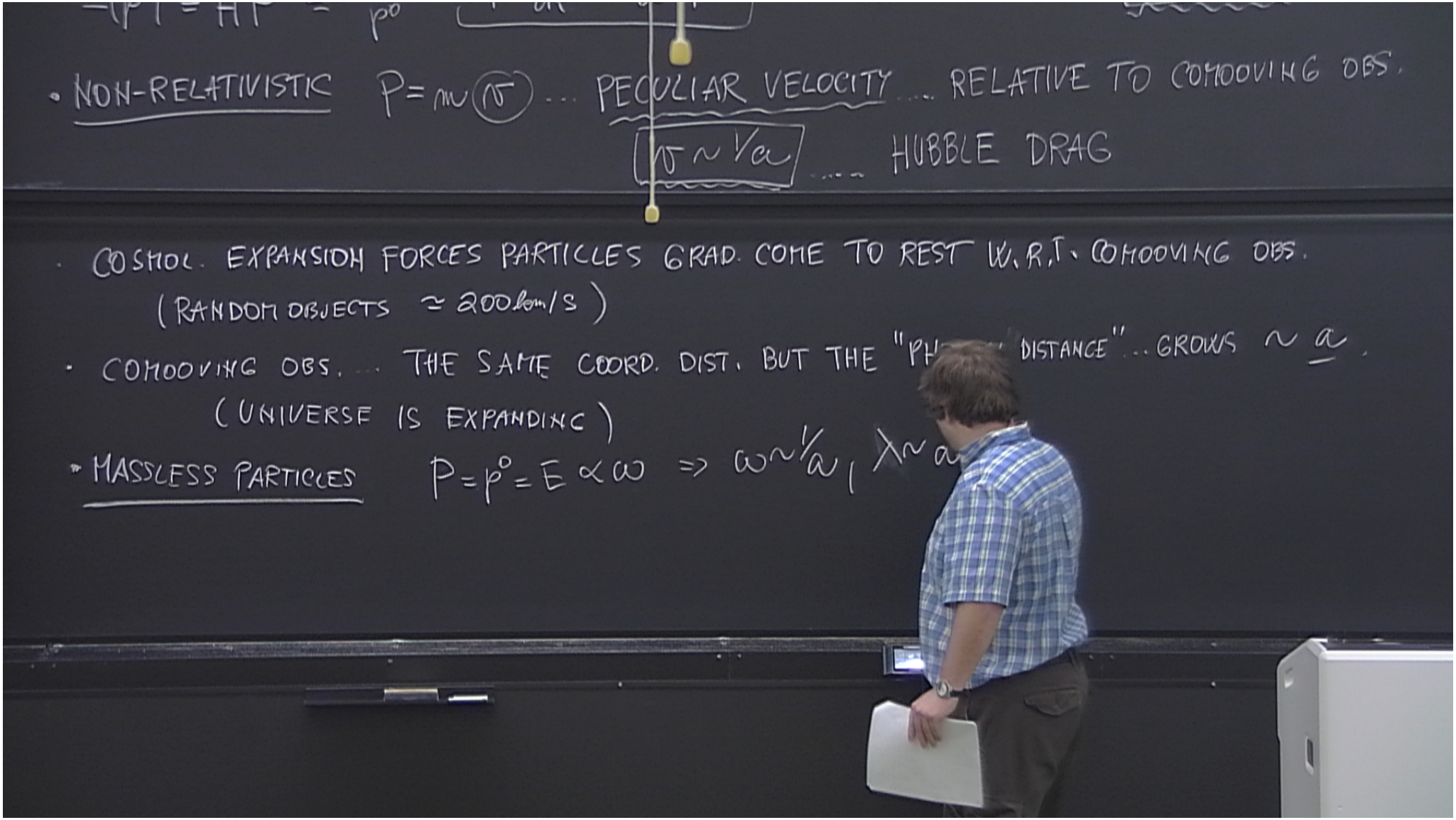

$\cdot = \frac{d}{dt}$
$\cdot L = \frac{1}{2}(-t'^2 + a^2(t)\, g^{(3,0)k}_{ij}\, x^{i\prime} x^{j\prime})$
$t: \frac{\partial L}{\partial t} = a\dot{a}\, g^{(3,0)k}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial t'}\right)' = -p^{0\prime}$
$HP^2 = -p^{0\prime}$ (1)
$m^2 = g_{\mu\nu} p^\mu p^\nu = -(p^0)^2 + P^2$
SPATIAL MAGNITUDE
$/\frac{d}{dx}$
$0 = -p^0 p^{0\prime} + P P'$ (2)
$-(p^0)' \overset{(1)}{=} HP^2 \overset{(2)}{=} -\frac{PP'}{p^0} = -P\frac{dP}{dt} = \frac{\dot{a}}{a} p^2$
$\frac{dP}{P} = -\frac{da}{a} \Rightarrow P \sim 1/a$

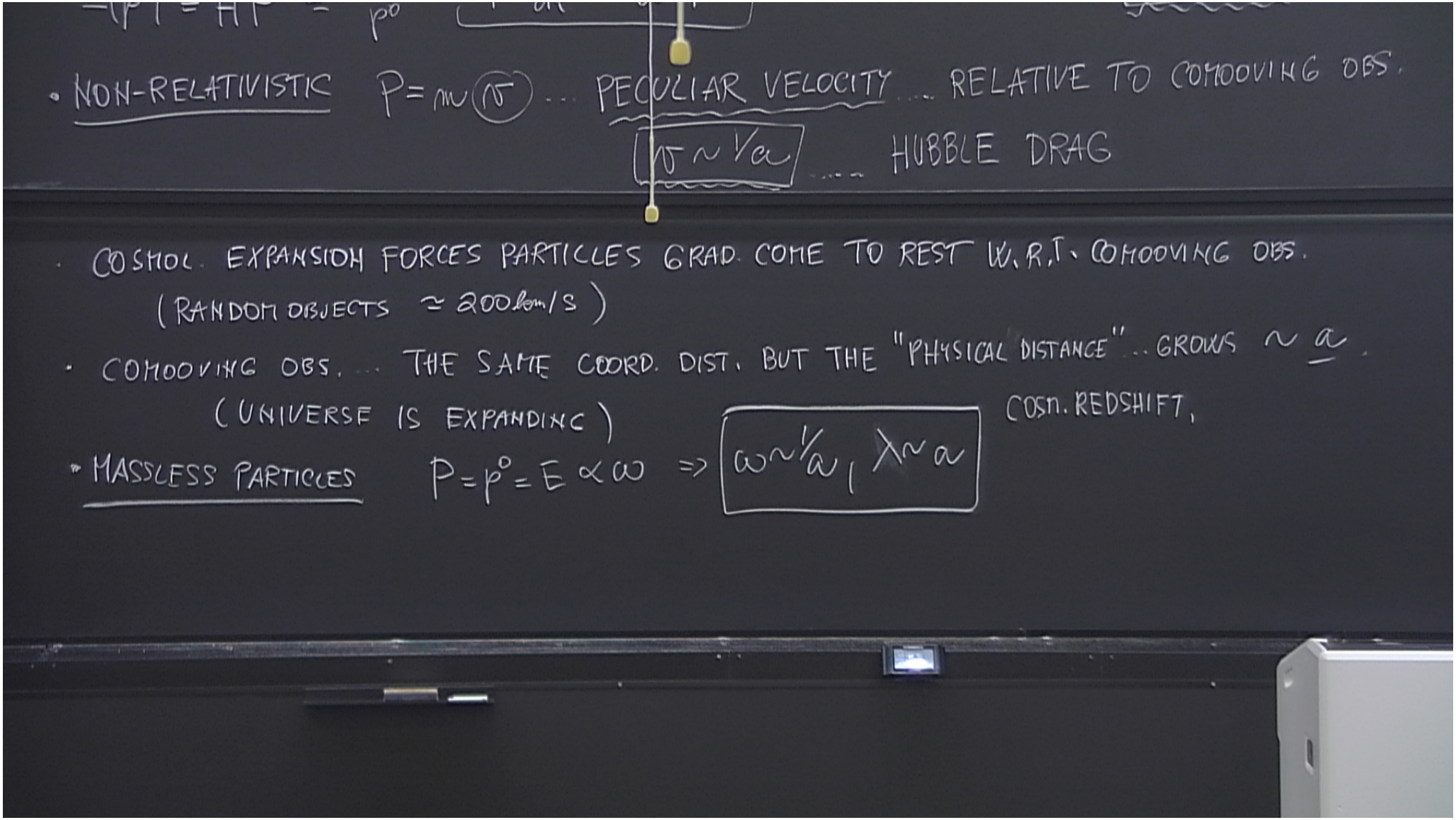

$= -p^0 p^{0\prime} + \vec{p}\vec{p}'$ (2)
$\dot{p} \overset{(1)}{=} Hp^2 \overset{(2)}{=} -\frac{\vec{p}\vec{p}'}{p^0} = -p\frac{dp}{dt} = \frac{\dot{a}}{a}p^2$
$\frac{dp}{p} = -\frac{da}{a} \Rightarrow p \sim 1/a$
RELATIVISTIC $p = mv\gamma$ ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OBS.

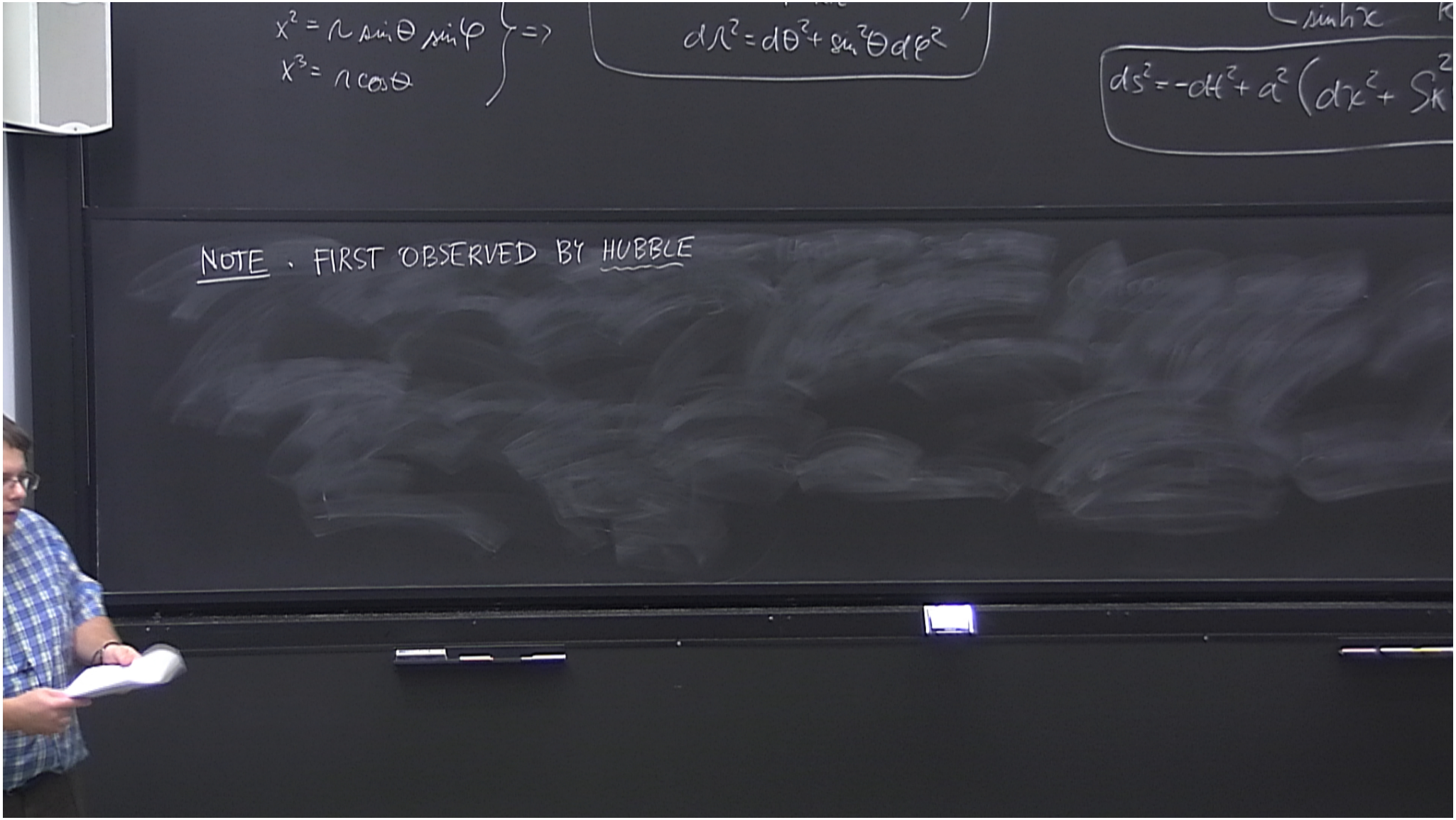

NON-RELATIVISTIC
PECULIAR VELOCITY ... RELATIVE TO COMOVING OBS.
HUBBLE DRAG
COSMOL. EXPANSION FORCES PARTICLES GRAD. COME TO REST W.R.T. COMOVING OBS.


• NON-RELATIVISTIC P = mv (v) ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OB
v ~ 1/a ... HUBBLE DRAG
· COSMOL. EXPANSION FORCES PARTICLES GRAD. COME TO REST W.R.T. COMOVING OBS.
(RANDOM OBJECTS ≃ 200km/s)
· COMOVING OBS. ... THE SAME COORD. DIST. BUT THE "PHYSICAL DISTANCE" .. GROWS ~ a .


• NON-RELATIVISTIC P = nv (v) ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OB
v ~ 1/a ... HUBBLE DRAG
• COSMOL. EXPANSION FORCES PARTICLES GRAD. COME TO REST W.R.T. COMOVING OBS.
(RANDOM OBJECTS ≃ 200km/s)
COMOVING OBS. ... THE SAME COORD. DIST. BUT THE "PHYSICAL DISTANCE" .. GROWS ~ a
(UNIVERSE IS EXPANDING)


• NON-RELATIVISTIC $p = mv$ (v) ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OBS.
$v \sim 1/a$ ... HUBBLE DRAG
• COSMOL. EXPANSION FORCES PARTICLES GRAD. COME TO REST W.R.T. COMOVING OBS.
(RANDOM OBJECTS $\simeq$ 200 km/s)
• COMOVING OBS. ... THE SAME COORD. DIST. BUT THE "PH... DISTANCE" .. GROWS $\sim a$.
(UNIVERSE IS EXPANDING)
• MASSLESS PARTICLES $p = p^0 = E \propto \omega \Rightarrow \omega \sim 1/a$, $\lambda \sim a$


NON-RELATIVISTIC $p = mv$ ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OBS.
$v \sim 1/a$ ... HUBBLE DRAG
COSMOL. EXPANSION FORCES PARTICLES GRAD. COME TO REST W.R.T. COMOVING OBS.
(RANDOM OBJECTS $\simeq$ 200 km/s)
COMOVING OBS. ... THE SAME COORD. DIST. BUT THE "PHYSICAL DISTANCE" .. GROWS $\sim a$.
(UNIVERSE IS EXPANDING)
COSM. REDSHIFT,
MASSLESS PARTICLES $p = p^0 = E \propto \omega \Rightarrow \omega \sim 1/a, \lambda \sim a$


$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Rightarrow$
$dr^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$
$\sinh^2\chi$
$ds^2 = -dt^2 + a^2(d\chi^2 + S_k$
NOTE . FIRST OBSERVED BY HUBBLE

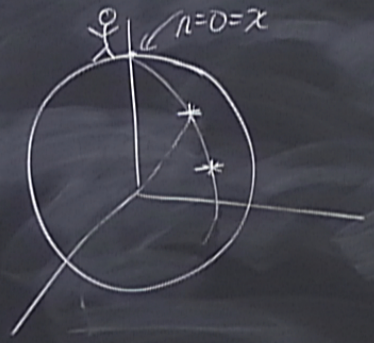
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Rightarrow$

$dr^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh^2\chi$

$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_k^2\right.$

NOTE . FIRST OBSERVED BY HUBBLE . SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE .



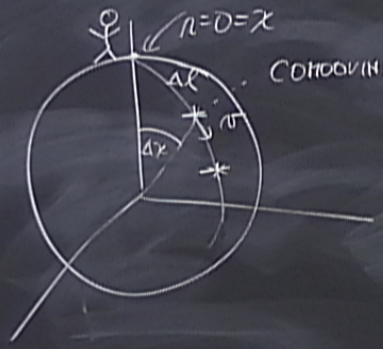
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Big\} \Rightarrow$

$d\ell^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh x$ $k=-1$

$ds^2 = -dt^2 + a^2\left(dx^2 + S_k^2(x)\, d\Omega^2\right)$

NOTE . FIRST OBSERVED BY HUBBLE . SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE .



SPATIAL DISTANCE $\Delta\ell = a\,\Delta x$

$v = \frac{\Delta\ell}{dt} = \dot{a}\,\Delta x = \frac{\dot{a}}{a}\, a\,\Delta x \Rightarrow$ H $\Delta\ell$

$\boxed{v = H\Delta\ell}$

$d\chi^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh\chi$ $k=-1$

$$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_k^2(\chi)\, d\Omega^2\right)$$

SERVED BY HUBBLE. SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE.

χ

COMOVING AT $\Delta\chi$

SPATIAL DISTANCE $\Delta\ell = a\,\Delta\chi$

$$v = \frac{\Delta\ell}{dt} = \dot{a}\,\Delta\chi = \frac{\dot{a}}{a}\, a\,\Delta\chi \Rightarrow v = H\,\Delta\ell$$

(H, $\Delta\ell$) HUBBLE'S LAW

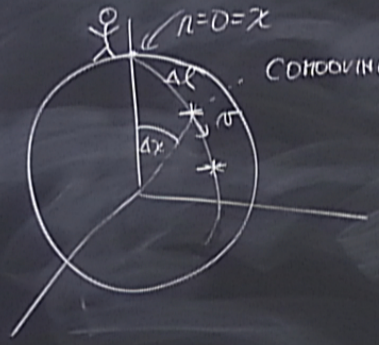
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Big\} \Rightarrow$

$dr^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh\chi$ $k=-1$

$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_k^2(\chi)\, d\Omega^2\right)$

NOTE . FIRST OBSERVED BY HUBBLE . SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE .



COMOVING AT $\Delta\chi$

SPATIAL DISTANCE $\Delta\ell = a\,\Delta\chi$

$$\nu = \frac{\Delta\ell}{dt} = \dot{a}\,\Delta\chi = \left(\frac{\dot{a}}{a}\right)\left(a\,\Delta\chi\right) \Rightarrow \boxed{\nu = H\,\Delta\ell}$$

(H, $\Delta\ell$)

$\nu \Leftrightarrow$ DOPPLER SHIFT OF ATOMIC SPECTRA

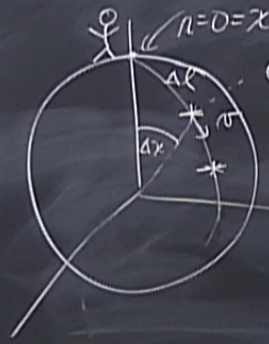
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$ $\Rightarrow$

$dr^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh\chi$ $K=-1$

$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$

NOTE . FIRST OBSERVED BY HUBBLE . SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE .



COMOVING AT $\Delta\chi$

SPATIAL DISTANCE $\Delta l = a\,\Delta\chi$

$$v = \frac{\Delta l}{dt} = \dot{a}\,\Delta\chi = \left(\frac{\dot{a}}{a}\right)\left(a\,\Delta\chi\right) \Rightarrow \boxed{v = H\,\Delta l}$$

(H, Δl)

$v \leftrightarrow$ DOPPLER SHIFT OF ATOMIC SPECTRA

$\Delta l \leftrightarrow$ STANDARD CANDLES

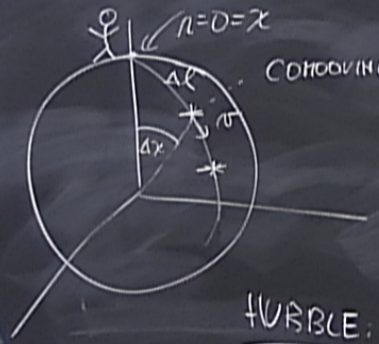
$x^2 = r \sin\theta \sin\varphi$
$x^3 = r \cos\theta$
$\Big\} \Rightarrow$

$dr^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$

$\sinh\chi$ $K<-1$

$ds^2 = -dt^2 + a^2\left(d\chi^2 + S_K^2(\chi)\, d\Omega^2\right)$

NOTE. FIRST OBSERVED BY HUBBLE. SPECTRA OF OBJECTS REDSHIFTED ~ DISTANCE.



COMOVING AT $\Delta\chi$

SPATIAL DISTANCE $\Delta r = a\,\Delta\chi$

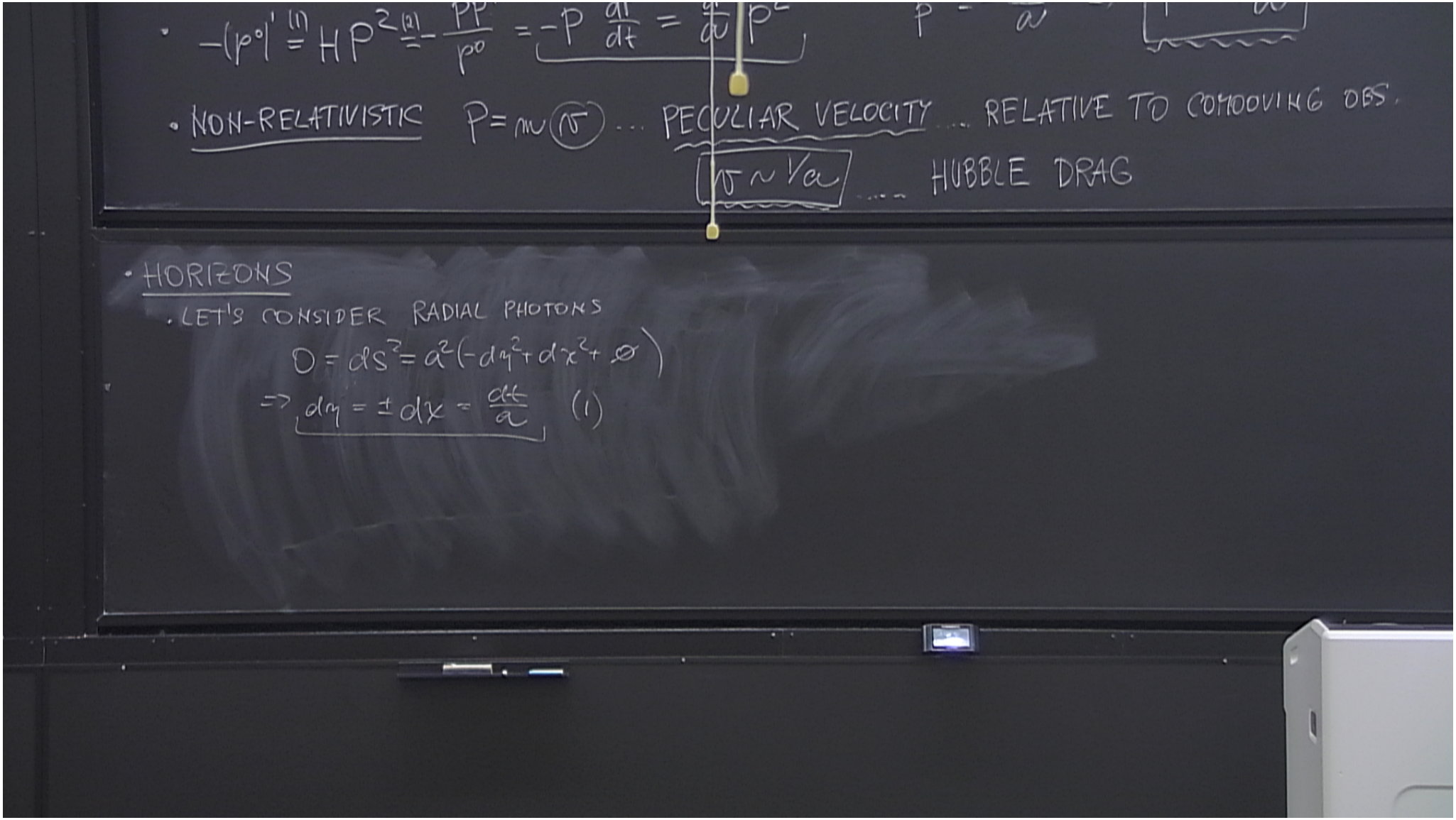
$$\nu = \frac{\Delta r}{dt} = \dot{a}\,\Delta\chi = \left(\frac{\dot{a}}{a}\right)\left(a\,\Delta\chi\right) \Rightarrow \boxed{\nu = H\,\Delta r}$$

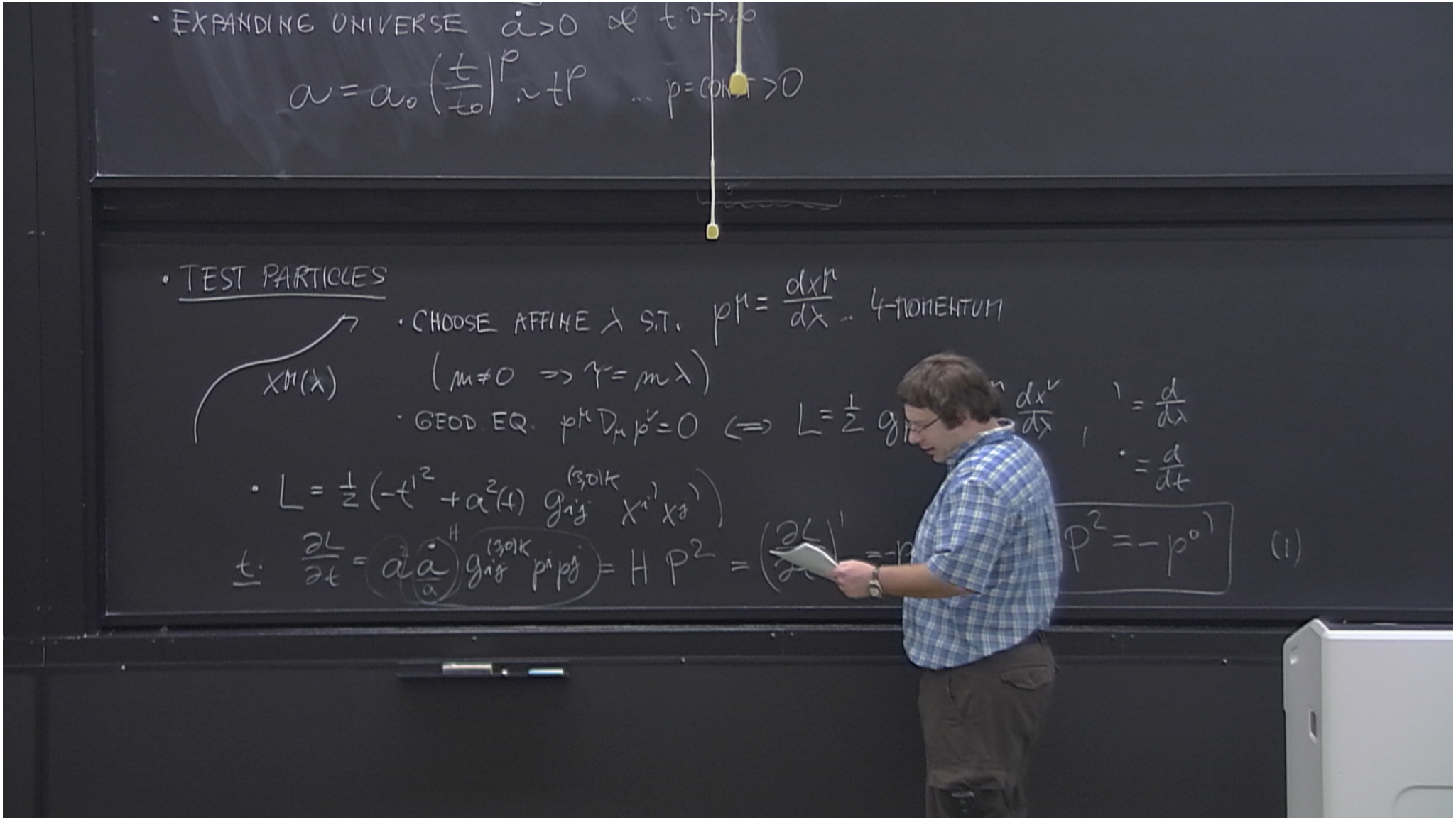
(H, Δr)

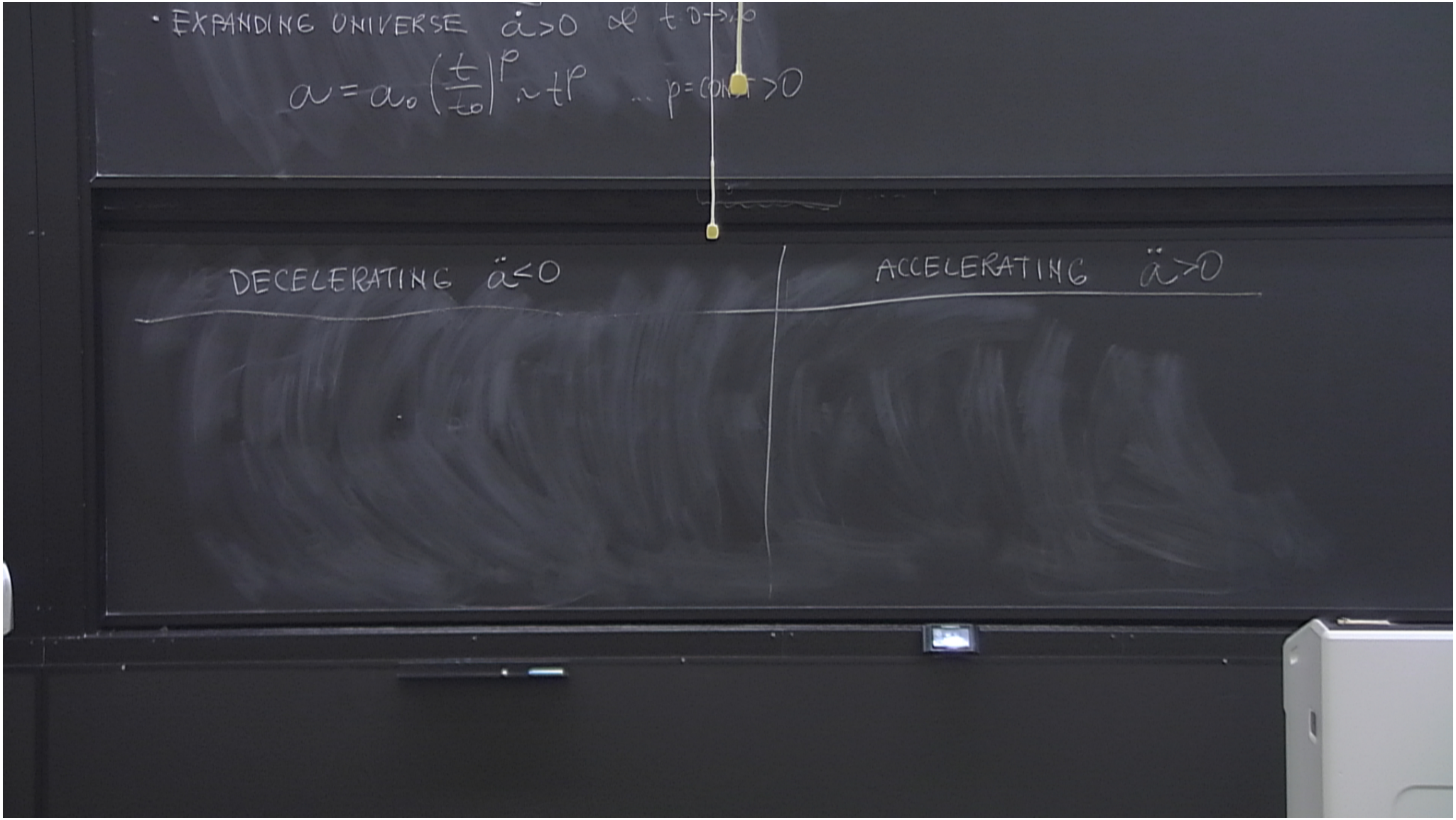
$\nu \leftrightarrow$ DOPPLER SHIFT OF ATOMIC SPECTRA

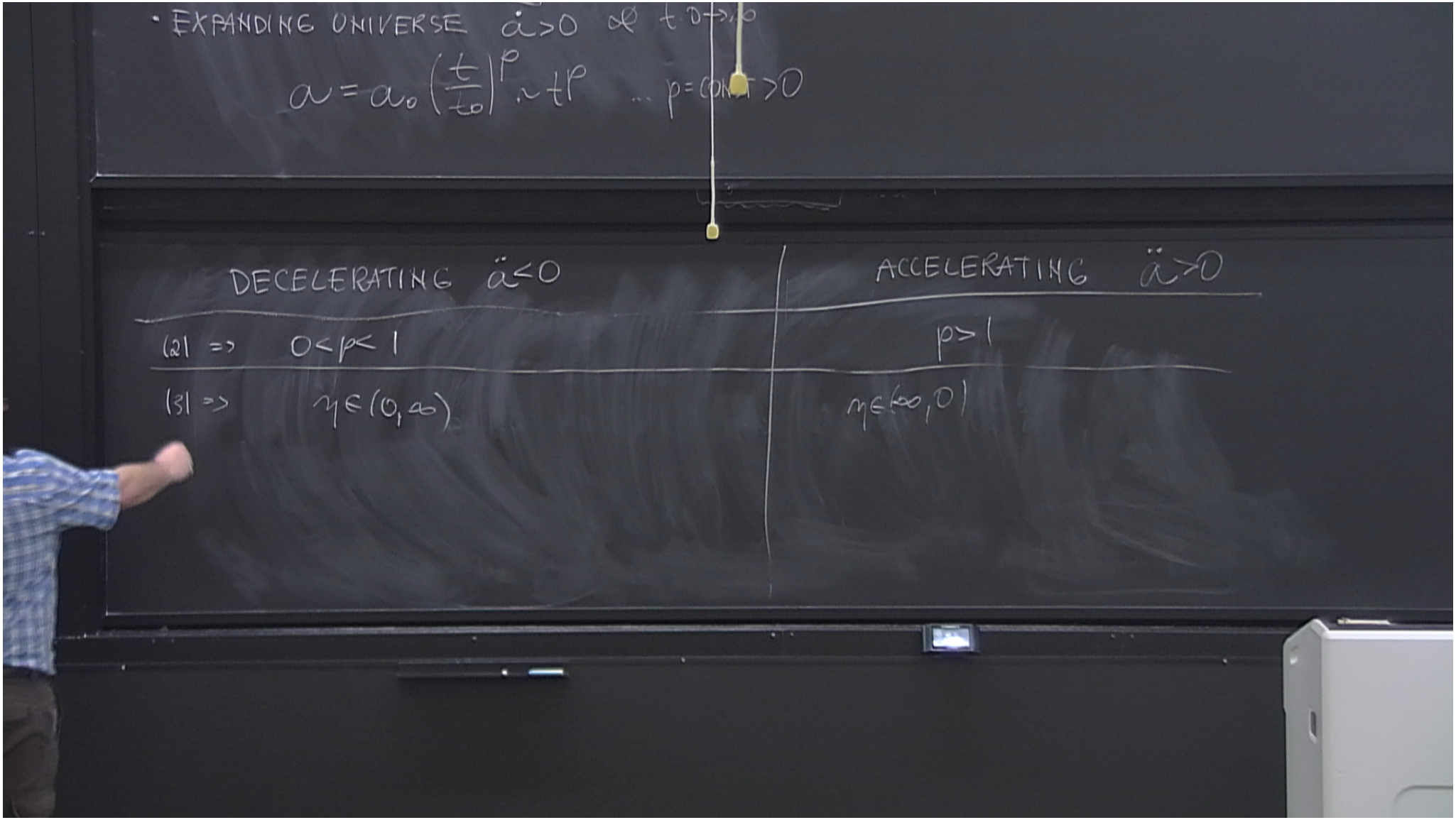
$\Delta r \leftrightarrow$ STANDARD CANDLES

HUBBLE: DOPPLER SHIFT $\leftrightarrow$ GR: SCALE FACTOR.


$-(p^0)' \overset{(1)}{=} Hp^2 \overset{(2)}{=} -\frac{pp'}{p^0} = -p\frac{dt}{dt} = \frac{d}{dt}p^2$
NON-RELATIVISTIC $P = mv$ (v) ... PECULIAR VELOCITY ... RELATIVE TO COMOVING OBS.
$v \sim 1/a$ ... HUBBLE DRAG
HORIZONS
LET'S CONSIDER RADIAL PHOTONS
$0 = ds^2 = a^2(-d\eta^2 + dx^2 + \not{\sigma})$
$\Rightarrow d\eta = \pm dx = \frac{dt}{a}$ (1)


• EXPANDING UNIVERSE $\dot{a}>0$ ∝ $t \to \infty$
$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p$ ... $p = \text{CONST} > 0$
• TEST PARTICLES
$x^\mu(\lambda)$
• CHOOSE AFFINE λ S.T. $p^\mu = \frac{dx^\mu}{d\lambda}$ ... 4-MOMENTUM
$(m \neq 0 \Rightarrow \tau = m\lambda)$
• GEOD EQ. $p^\mu D_\mu p^\nu = 0 \iff L = \frac{1}{2} g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}$, $' = \frac{d}{d\lambda}$
$\cdot = \frac{d}{dt}$
• $L = \frac{1}{2}\left(-t'^2 + a^2(t)\, g^{(3)}_{ij}\, x^i x^j\right)$
t: $\frac{\partial L}{\partial t} = a\dot{a}\, g^{(3)}_{ij}\, p^i p^j = H P^2 = \left(\frac{\partial L}{\partial \dot{x}^0}\right)' = -p^{0\prime}$
$P^2 = -p^0$ (1)

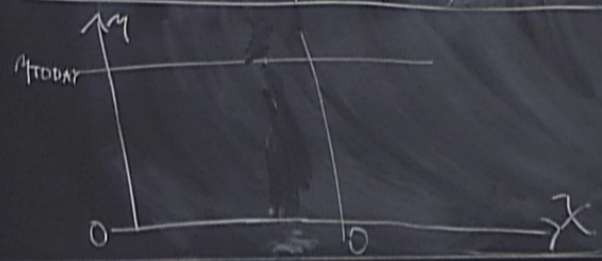

• EXPANDING UNIVERSE $\dot{a}>0$ $\propto$ $t\,0\to\infty$
$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p$ ... p=CONST >0
DECELERATING $\ddot{a}<0$
ACCELERATING $\ddot{a}>0$

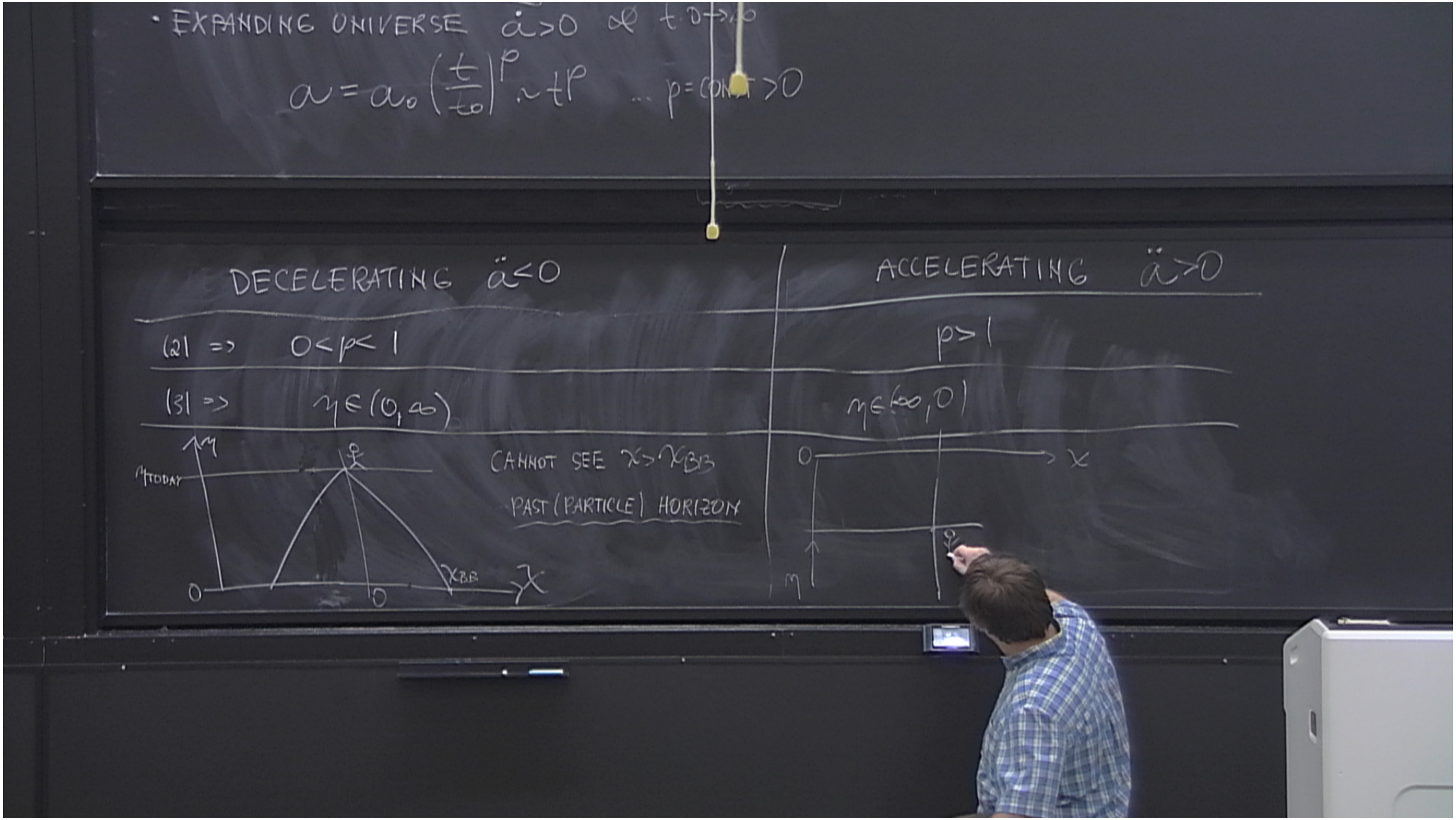

• EXPANDING UNIVERSE $\dot{a}>0$ as $t \to \infty$
$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p$ ... $p = \text{const} > 0$
DECELERATING $\ddot{a}<0$
ACCELERATING $\ddot{a}>0$
(2) => $0<p<1$
$p>1$
(3) => $\eta \in (0, \infty)$
$\eta \in (-\infty, 0)$

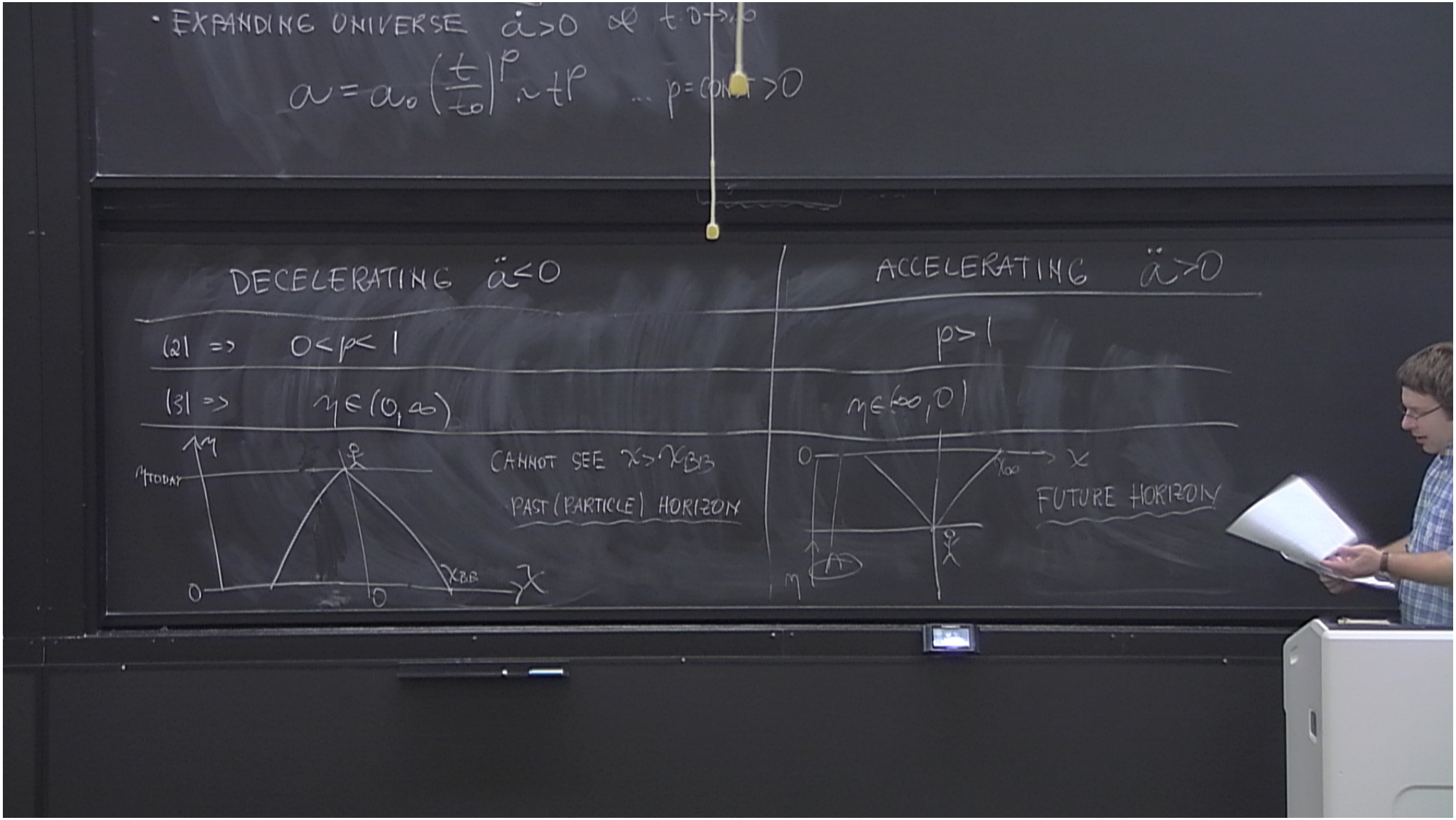
• EXPANDING UNIVERSE $\dot{a} > 0$ $\propto$ t $0 \to \infty$

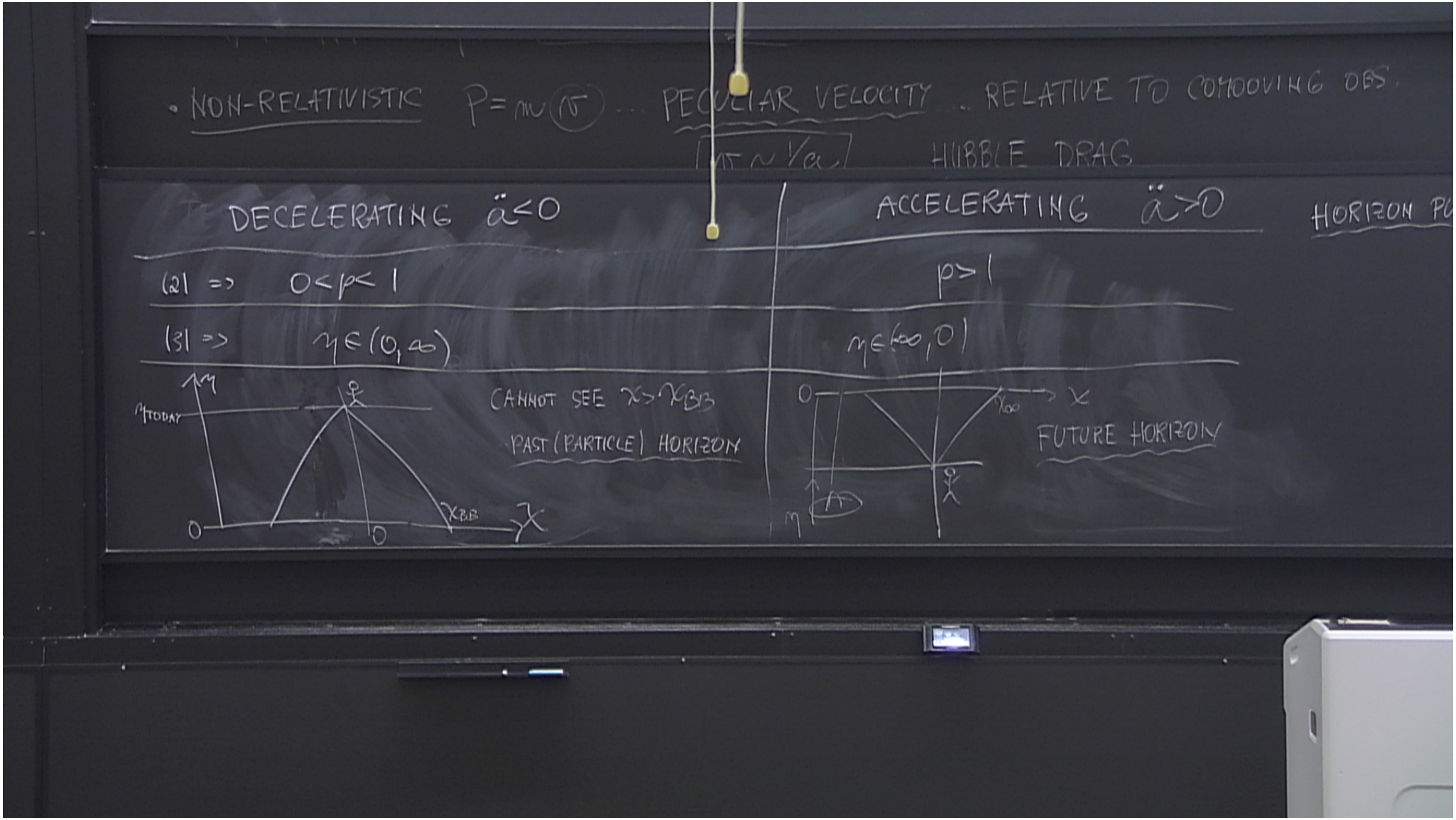
$$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p \quad \dots \; p = \text{CONST} > 0$$

DECELERATING $\ddot{a} < 0$	ACCELERATING $\ddot{a} > 0$
(2) $\Rightarrow$ $0 < p < 1$	$p > 1$
(3) $\Rightarrow$ $\eta \in (0, \infty)$	$\eta \in (-\infty, 0)$




• EXPANDING UNIVERSE $\dot{a}>0$ as $t \to \infty$
$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p$ ... $p = \text{CONST} > 0$
DECELERATING $\ddot{a}<0$
ACCELERATING $\ddot{a}>0$
(2) => $0<p<1$
$p>1$
(3) => $\eta \in (0, \infty)$
$\eta \in (-\infty, 0)$
η_{TODAY}
η
0
χ_{BB}
χ
CANNOT SEE $\chi > \chi_{BB}$
PAST (PARTICLE) HORIZON
0
χ
η


• EXPANDING UNIVERSE $\ddot{a}>0$ $\propto$ $t \to \infty$
$a = a_0 \left(\frac{t}{t_0}\right)^p \sim t^p$ ... $p = \text{CONST} > 0$
DECELERATING $\ddot{a}<0$
ACCELERATING $\ddot{a}>0$
(2) => $0<p<1$
$p>1$
(3) => $\eta \in (0, \infty)$
$\eta \in (-\infty, 0)$
η
η_{TODAY}
0
χ_{BB}
χ
CANNOT SEE $\chi > \chi_{BB}$
PAST (PARTICLE) HORIZON
χ_{∞}
FUTURE HORIZON


• NON-RELATIVISTIC p = mv (√) ... PECULIAR VELOCITY .. RELATIVE TO COMOVING OBS.
[ṁ ~ 1/a] HUBBLE DRAG
DECELERATING ä<0
ACCELERATING ä>0
HORIZON PU
(2) => 0<p<1
p>1
(3) => η∈(0,∞)
η∈(-∞,0)
η
η_TODAY
0
χ_BB
χ
CANNOT SEE χ>χ_BB
PAST (PARTICLE) HORIZON
0
χ_∞
χ
FUTURE HORIZON
η
A