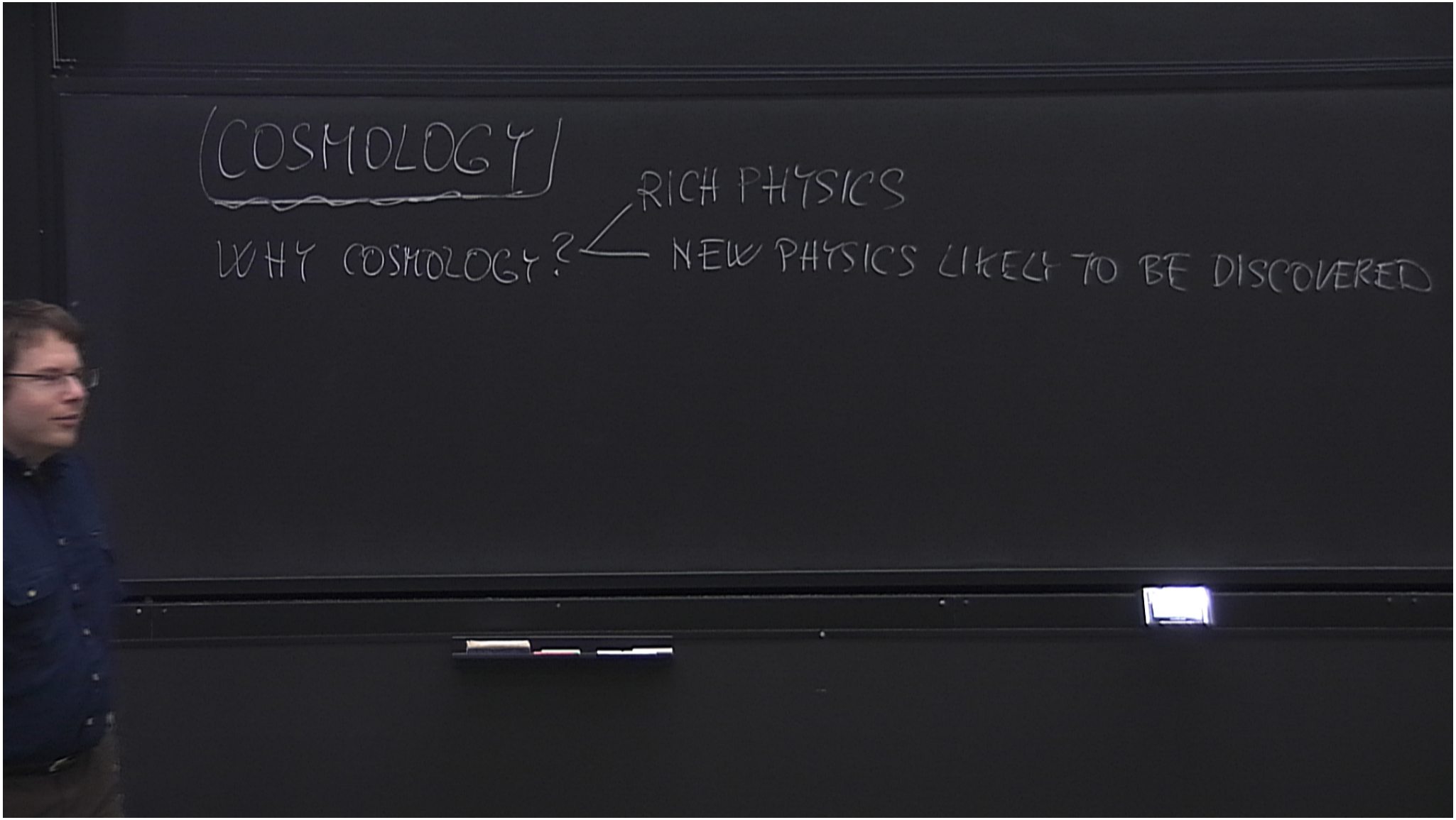
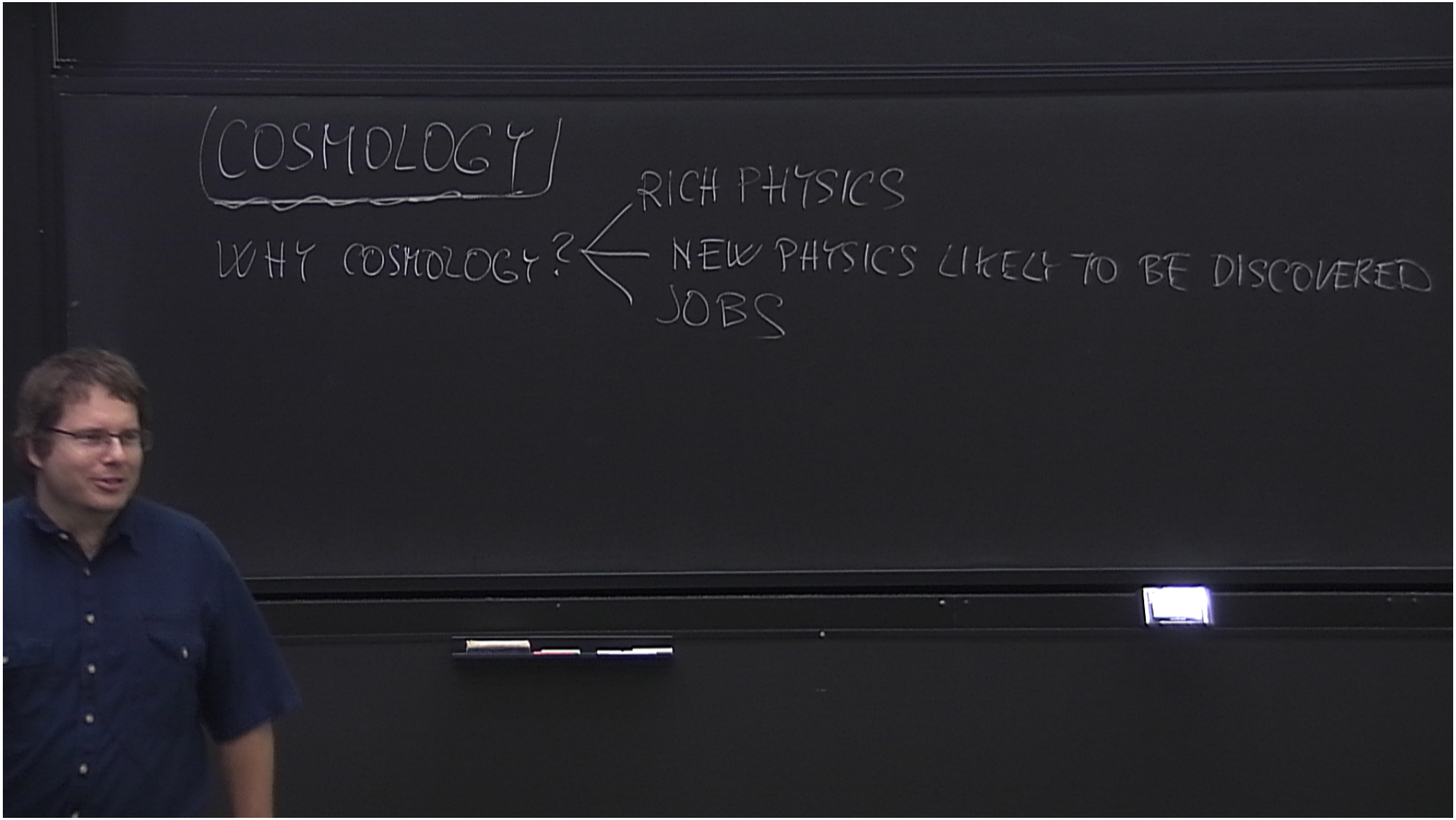
Title: Cosmology Review-1

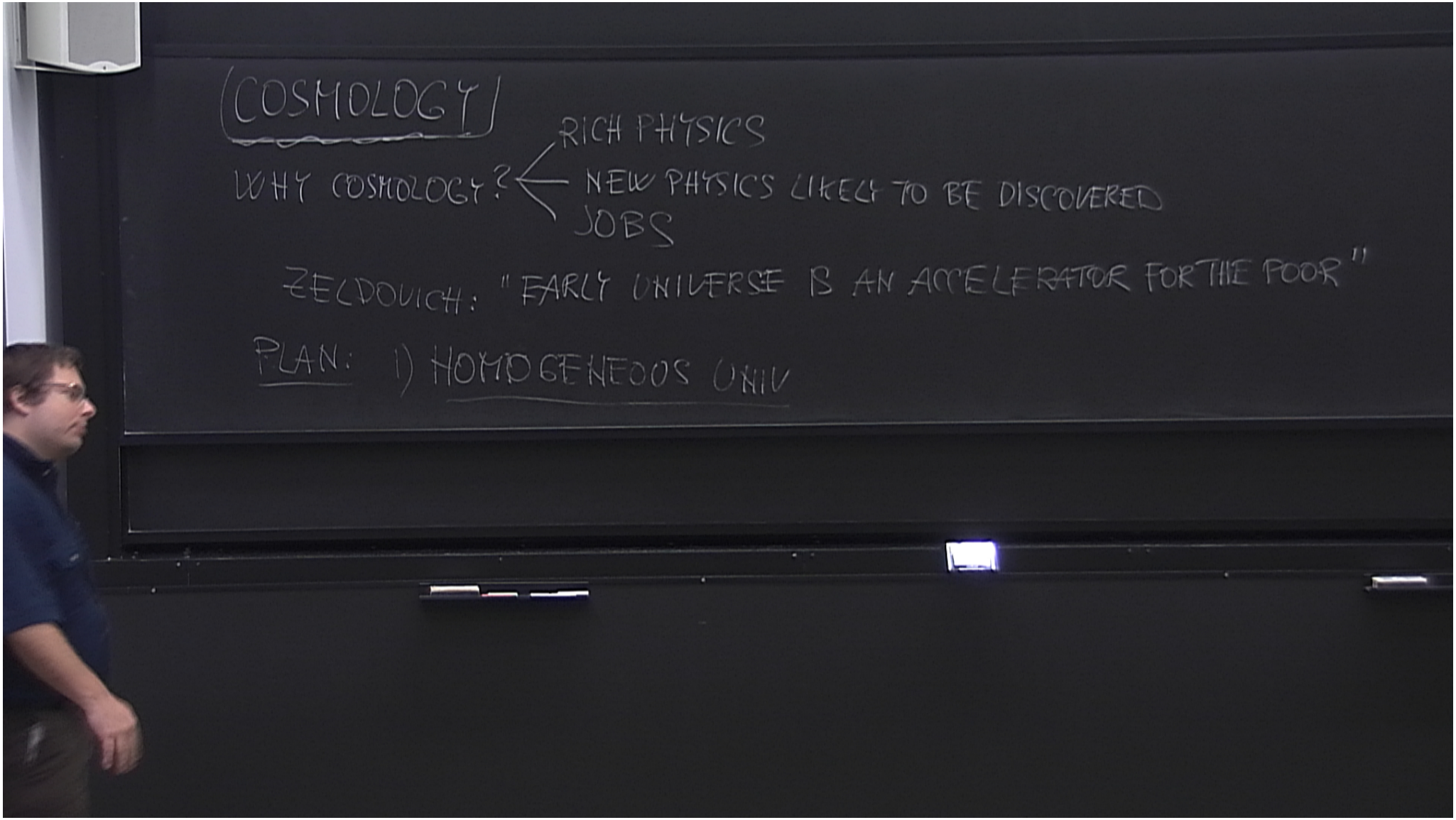
Date: Jan 26, 2015 11:30 AM

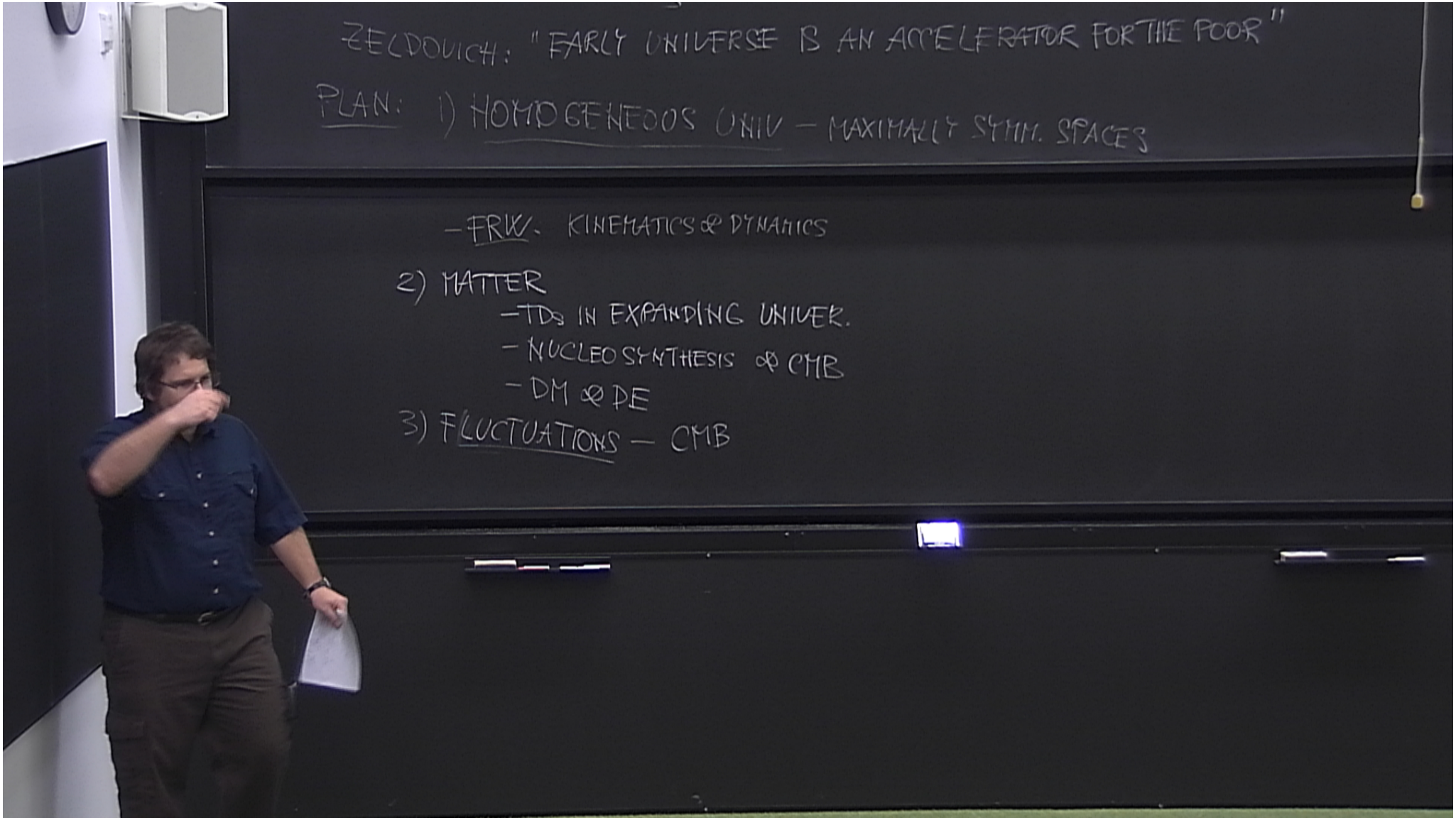
URL: http://pirsa.org/15010067

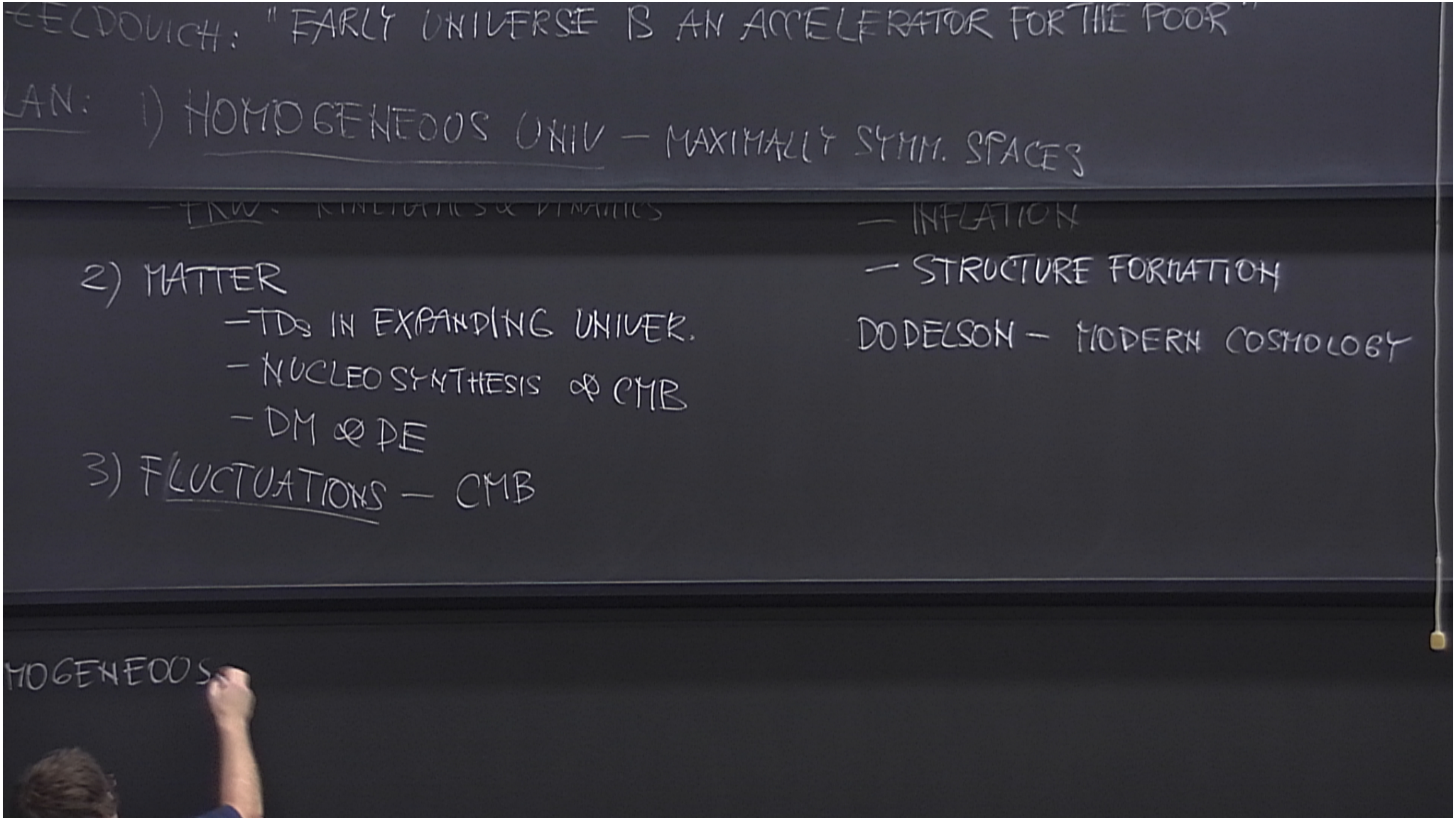
Abstract:

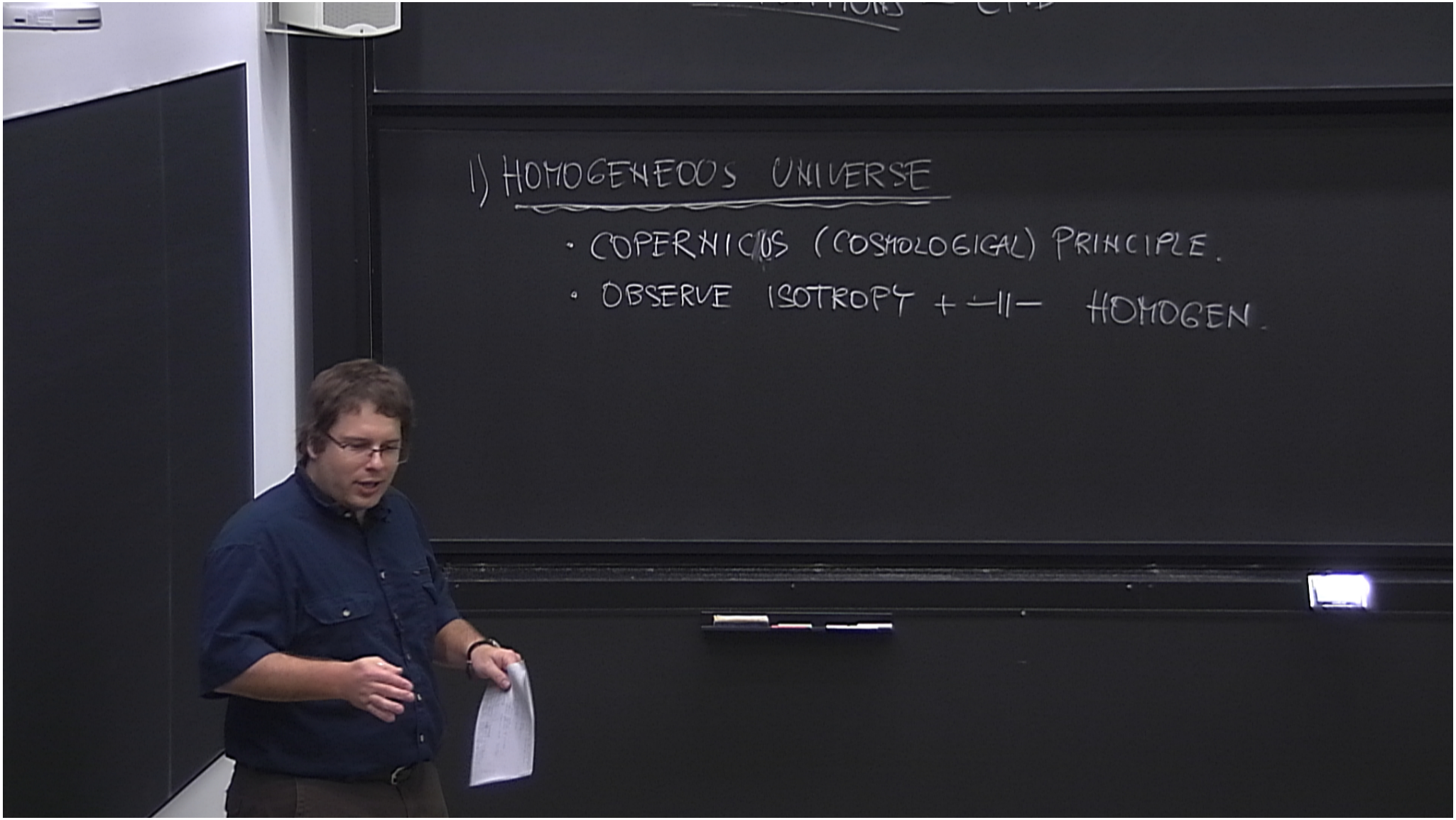

COSMOLOGY
WHY COSMOLOGY?
RICH PHYSICS
NEW PHYSICS LIKELY TO BE DISCOVERED

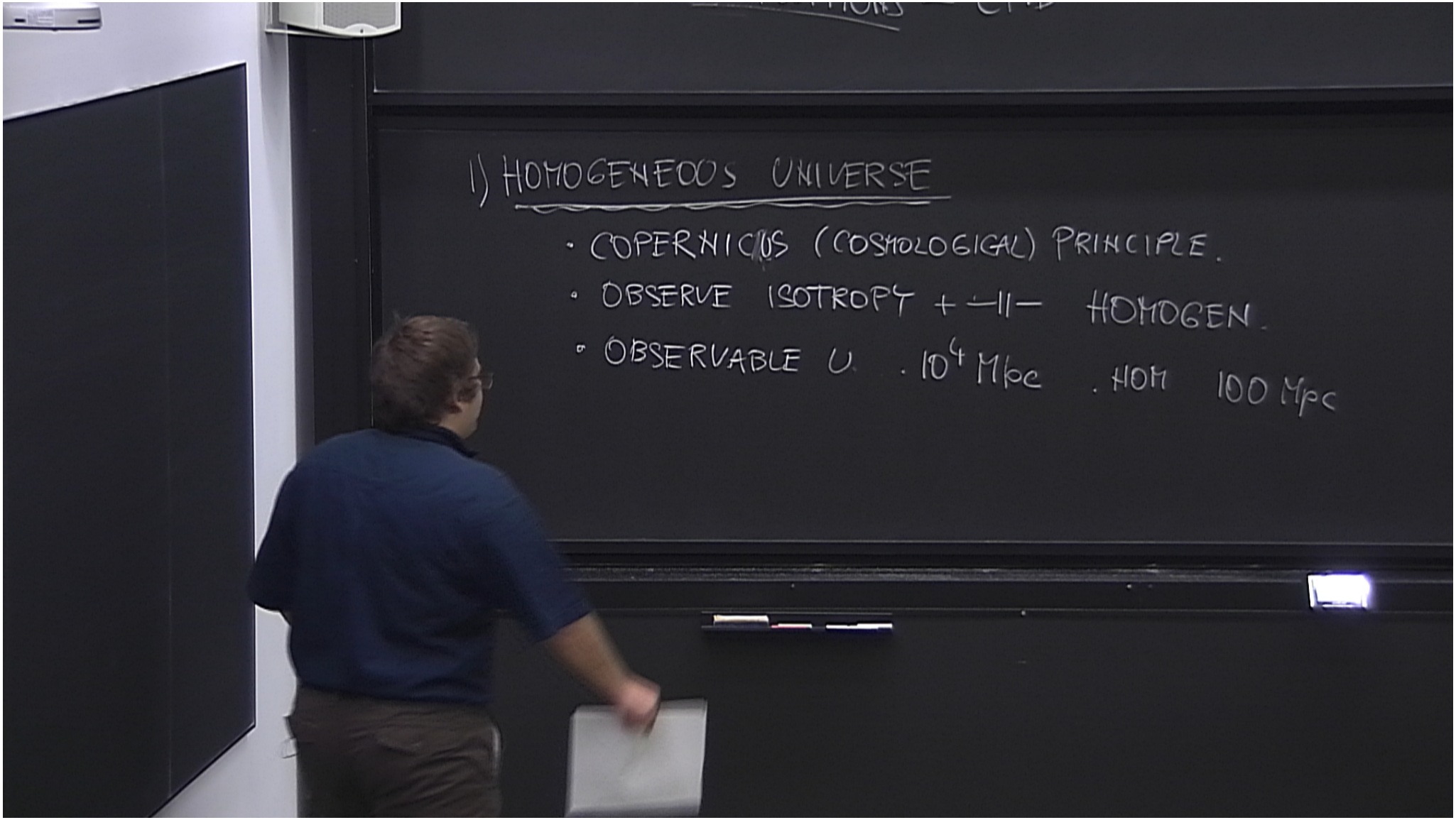

COSMOLOGY
WHY COSMOLOGY?
RICH PHYSICS
NEW PHYSICS LIKELY TO BE DISCOVERED
JOBS

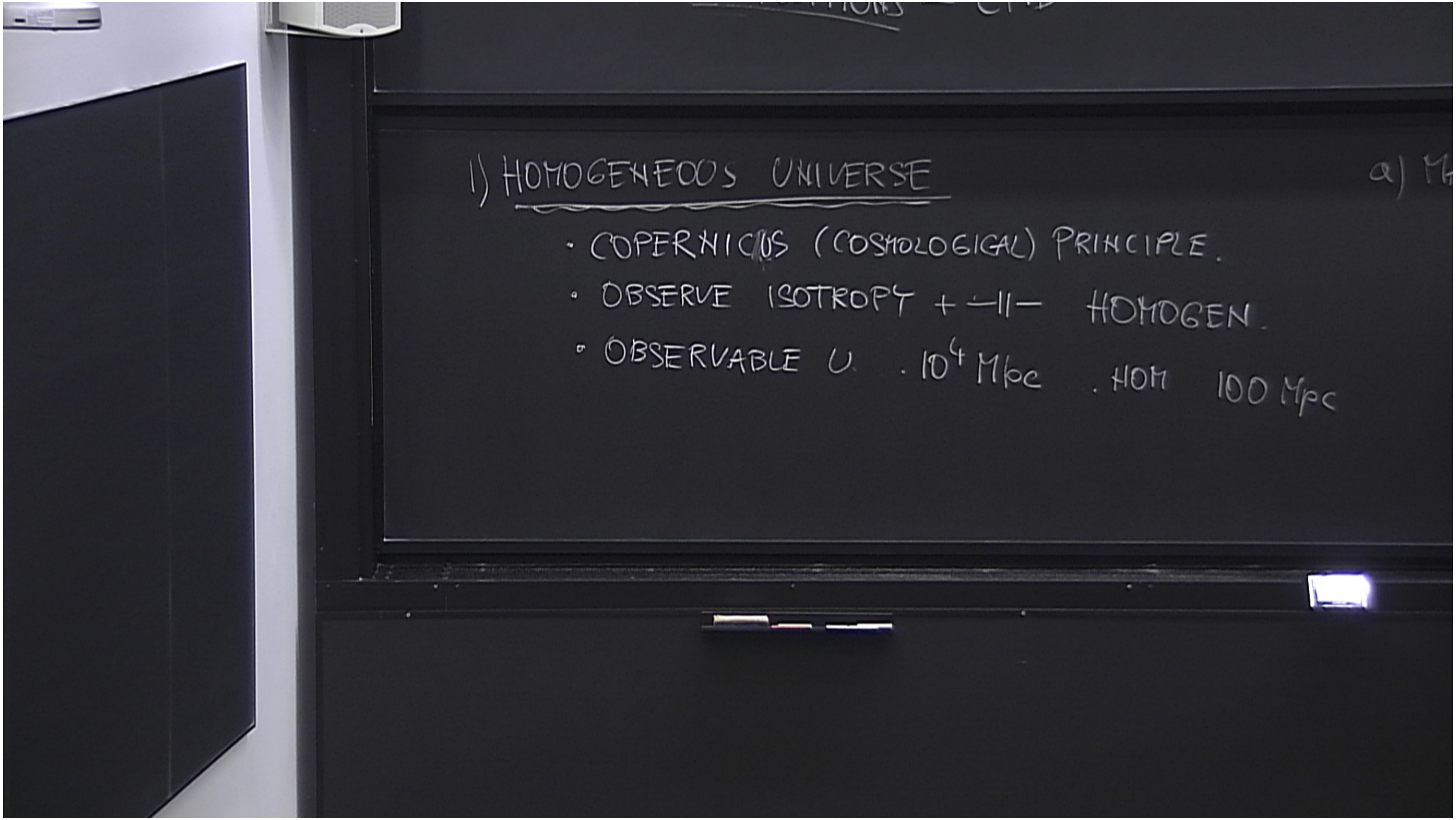

COSMOLOGY
WHY COSMOLOGY?
RICH PHYSICS
NEW PHYSICS LIKELY TO BE DISCOVERED
JOBS
ZELDOVICH: "EARLY UNIVERSE IS AN ACCELERATOR FOR THE POOR"
PLAN: 1) HOMOGENEOUS UNIV

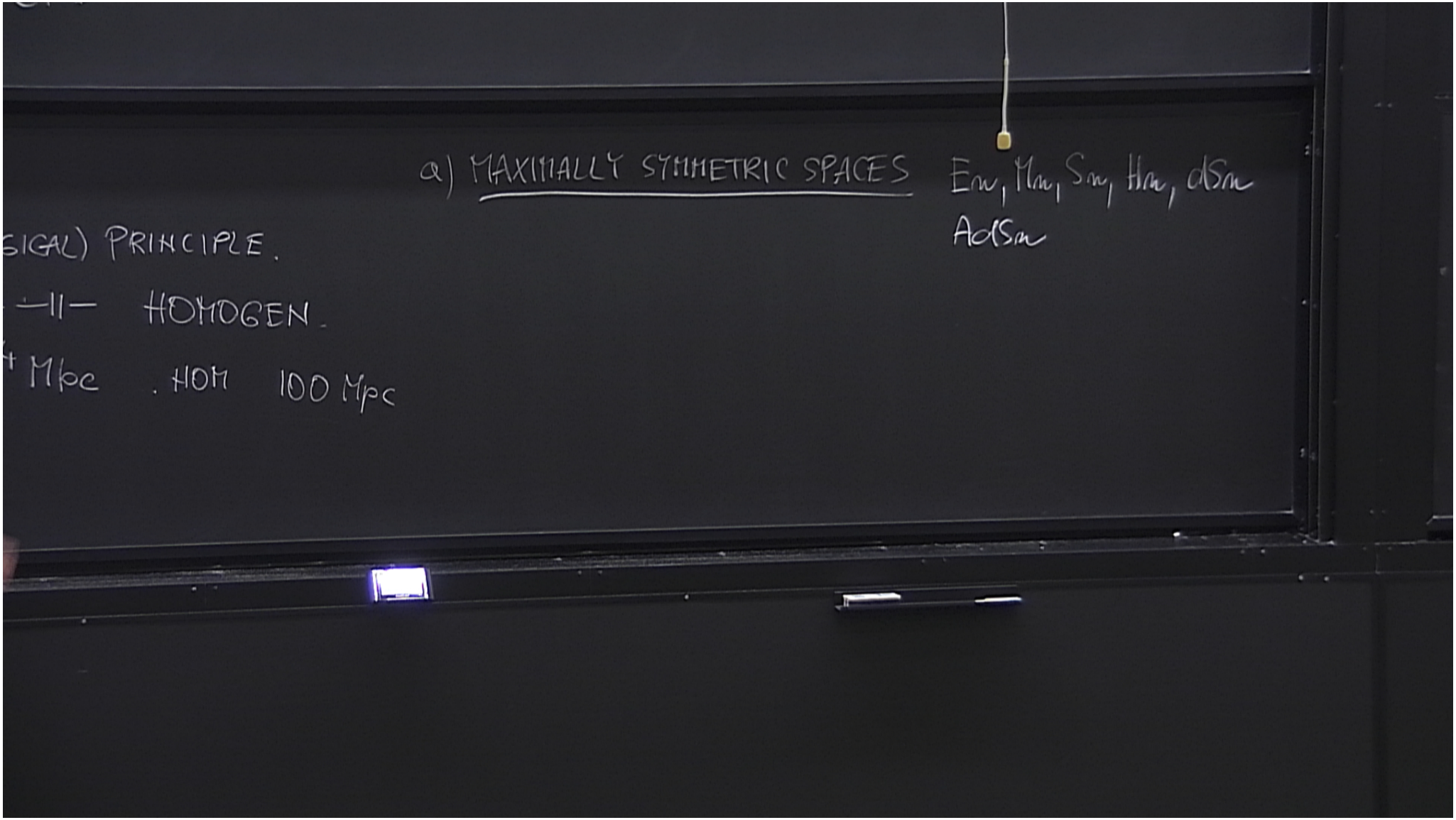

ZELDOVICH: "EARLY UNIVERSE IS AN ACCELERATOR FOR THE POOR"
PLAN: 1) HOMOGENEOUS UNIV – MAXIMALLY SYMM. SPACES
– FRW. KINEMATICS & DYNAMICS
2) MATTER
– TDs IN EXPANDING UNIVER.
– NUCLEOSYNTHESIS & CMB
– DM & DE
3) FLUCTUATIONS – CMB


EELDOVICH: "EARLY UNIVERSE IS AN ACCELERATOR FOR THE POOR"
LAN: 1) HOMOGENEOUS UNIV – MAXIMALLY SYMM. SPACES
– INFLATION
– STRUCTURE FORMATION
2) MATTER
– TDs IN EXPANDING UNIVER.
– NUCLEOSYNTHESIS & CMB
– DM & DE
3) FLUCTUATIONS – CMB
DODELSON – MODERN COSMOLOGY
MOGENEOUS


1) HOMOGENEOUS UNIVERSE
· COPERNICUS (COSMOLOGICAL) PRINCIPLE.
· OBSERVE ISOTROPY +⇒ HOMOGEN.


1) HOMOGENEOUS UNIVERSE
• COPERNICUS (COSMOLOGICAL) PRINCIPLE.
• OBSERVE ISOTROPY +-11- HOMOGEN.
• OBSERVABLE U. . 10^4 Mpc . HOM 100 Mpc

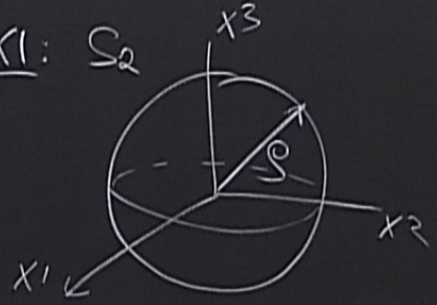

1) HOMOGENEOUS UNIVERSE
a)
· COPERNICUS (COSMOLOGICAL) PRINCIPLE.
· OBSERVE ISOTROPY + ⇒ HOMOGEN.
· OBSERVABLE U. . 10^4 Mpc . HOM 100 Mpc


a) MAXIMALLY SYMMETRIC SPACES
E_n, M_n, S_n, H_n, dS_n
AdS_n
SIGAL) PRINCIPLE.
—11— HOMOGEN.
Mpc . HOM 100 Mpc

a) MAXIMALLY SYMMETRIC SPACES E_n, M_n, S_n, H_n, dS_n AdS_n

ELEGANT CONSTRUCTION: EMBED IN $(n+1)$-DIM SPACE.

EX1: S_2



SIGAL) PRINCIPLE.

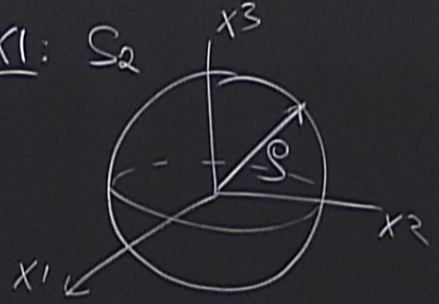
—11— HOMOGEN.

Mpc . HOM 100 Mpc

a) MAXIMALLY SYMMETRIC SPACES E_m, M_m, S_m, H_m, dS_m, AdS_m

ELEGANT CONSTRUCTION: EMBED IN $(m+1)$-DIM SPACE.

EX1: S_2



$$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$$

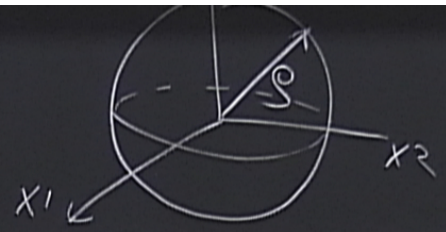
$$S^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$

SIGAL) PRINCIPLE.

—11— HOMOGEN.

Mpc . HOM 100 Mpc

Mpc . HOM 100 Mpc



$$S^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$$

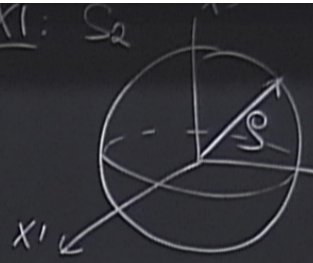
$$2S = x^1 dx^1 + x^2 dx^2 + x^3 dx^3$$

$$(dx^3)^2 = \frac{(x^1 dx^1 + x^2 dx^2)^2}{S^2 - (x^1)^2 - (x^2)^2}$$

OBSERVABLE U. $\cdot 10^4$ Mpc . HOM 100 Mpc

HOMOGEN.

EXT: S_2



$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$

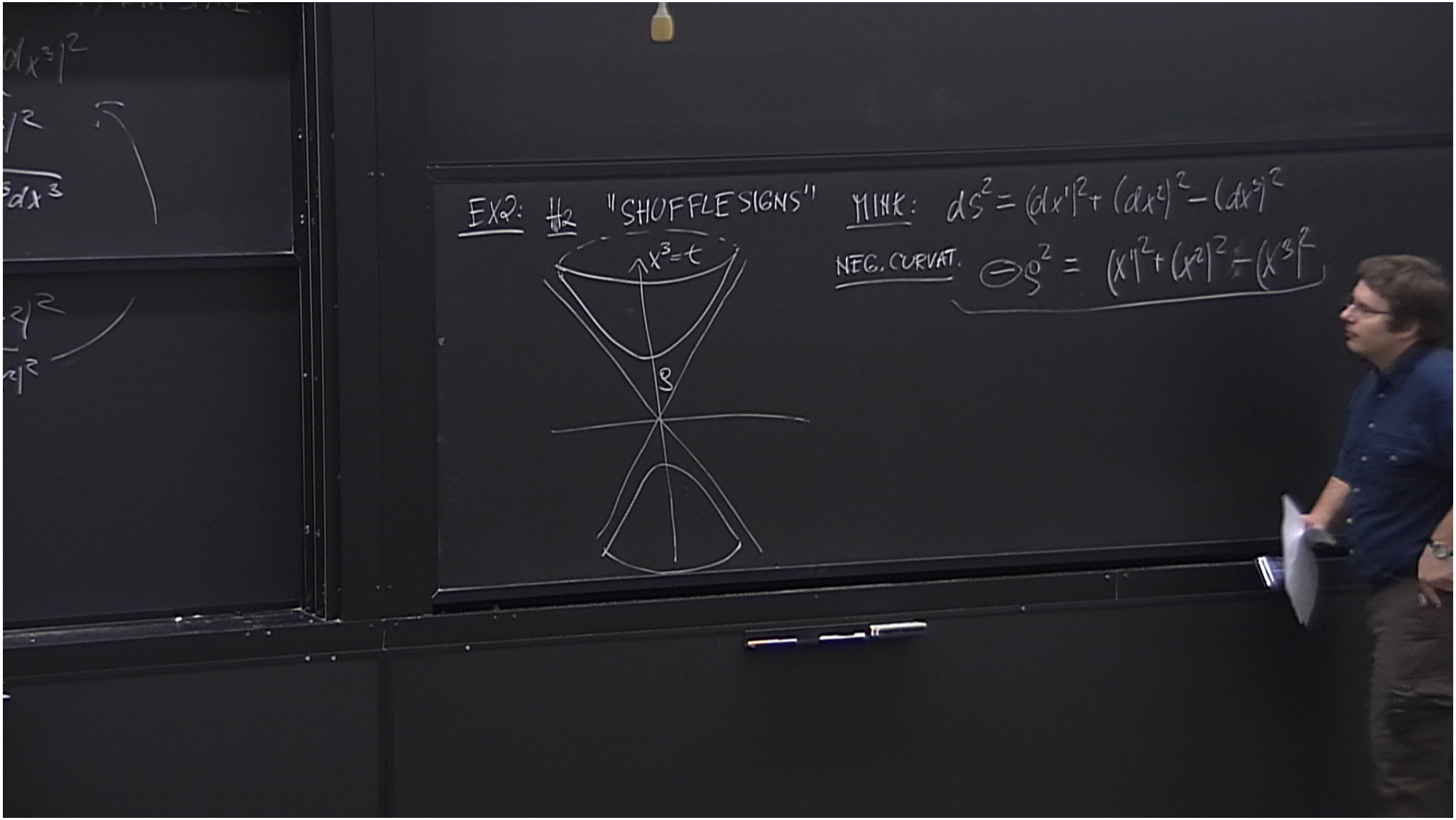
$\rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$

$\rho = x^1 dx^1 + x^2 dx^2 + x^3 dx^3$

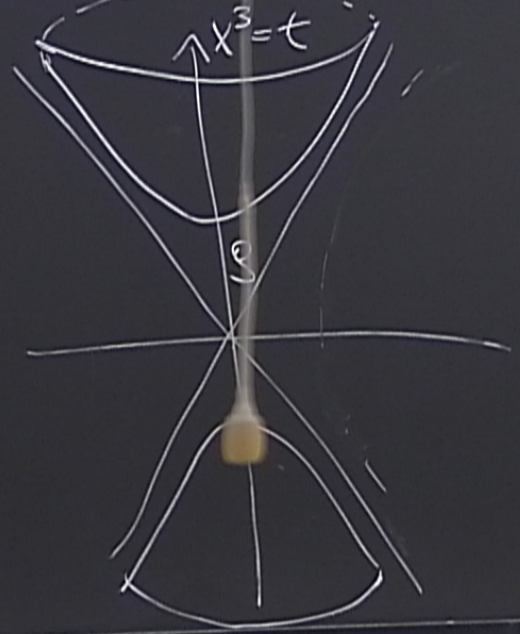
CHANGE TO SPHER.

$$\left.\begin{aligned} x^1 &= \rho \sin\theta \cos\varphi \\ x^2 &= \rho \sin\theta \sin\varphi \\ x^3 &= \rho \cos\theta \end{aligned}\right\} \Rightarrow ds^2 = \rho^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)$$

$$(dx^3)^2 = \frac{(x^1 dx^1 + x^2 dx^2)^2}{\rho^2 - (x^1)^2 - (x^2)^2}$$


EX2: #2 "SHUFFLE SIGNS"
MINK: $ds^2 = (dx^1)^2 + (dx^2)^2 - (dx^3)^2$
NEG. CURVAT. $-\rho^2 = (x^1)^2 + (x^2)^2 - (x^3)^2$
$x^3 = t$
ρ

EX2: $\mathbb{H}^2$ "SHUFFLESIGNS"



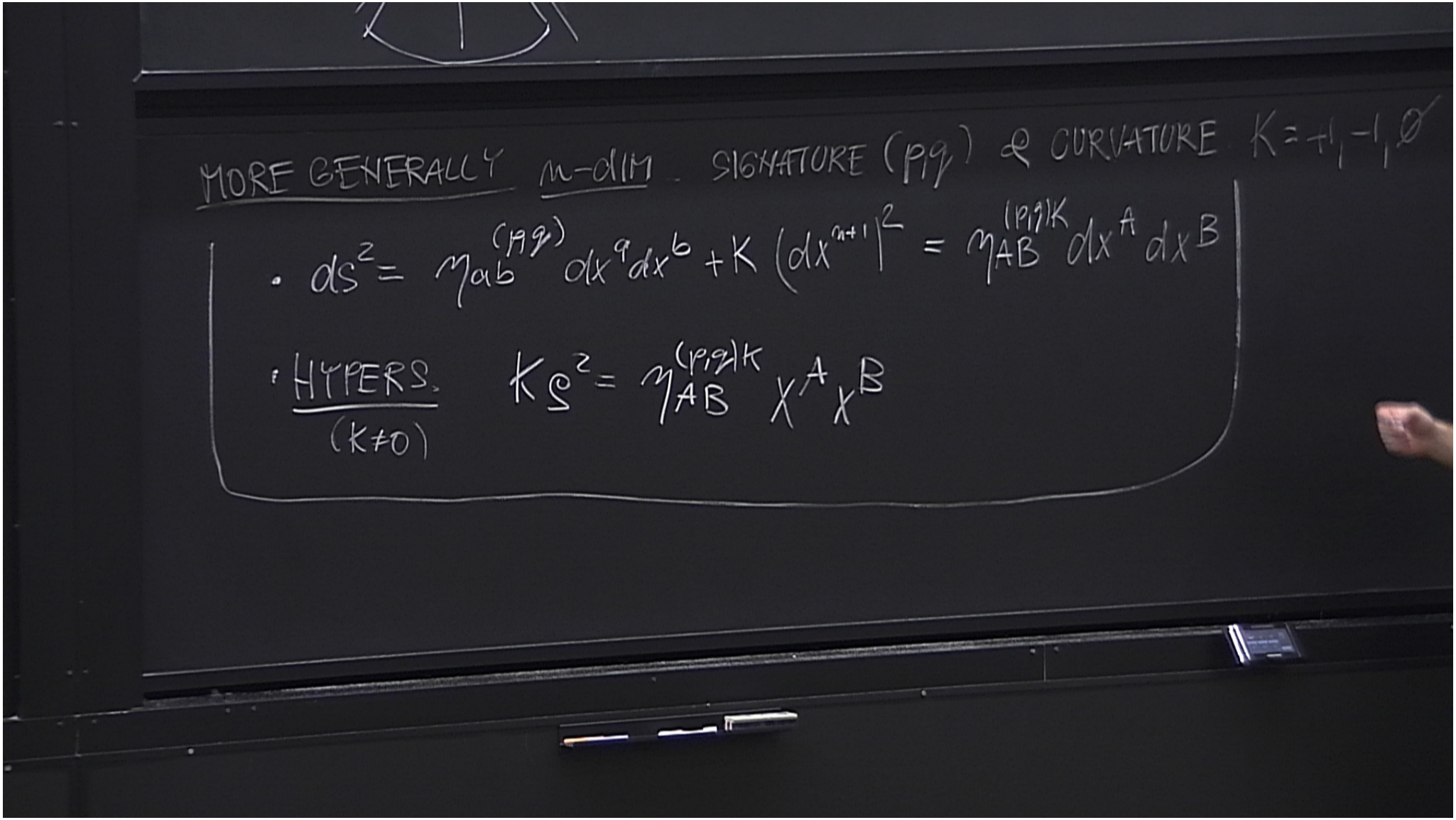
MINK: $ds^2 = (dx^1)^2 + (dx^2)^2 - (dx^3)^2$

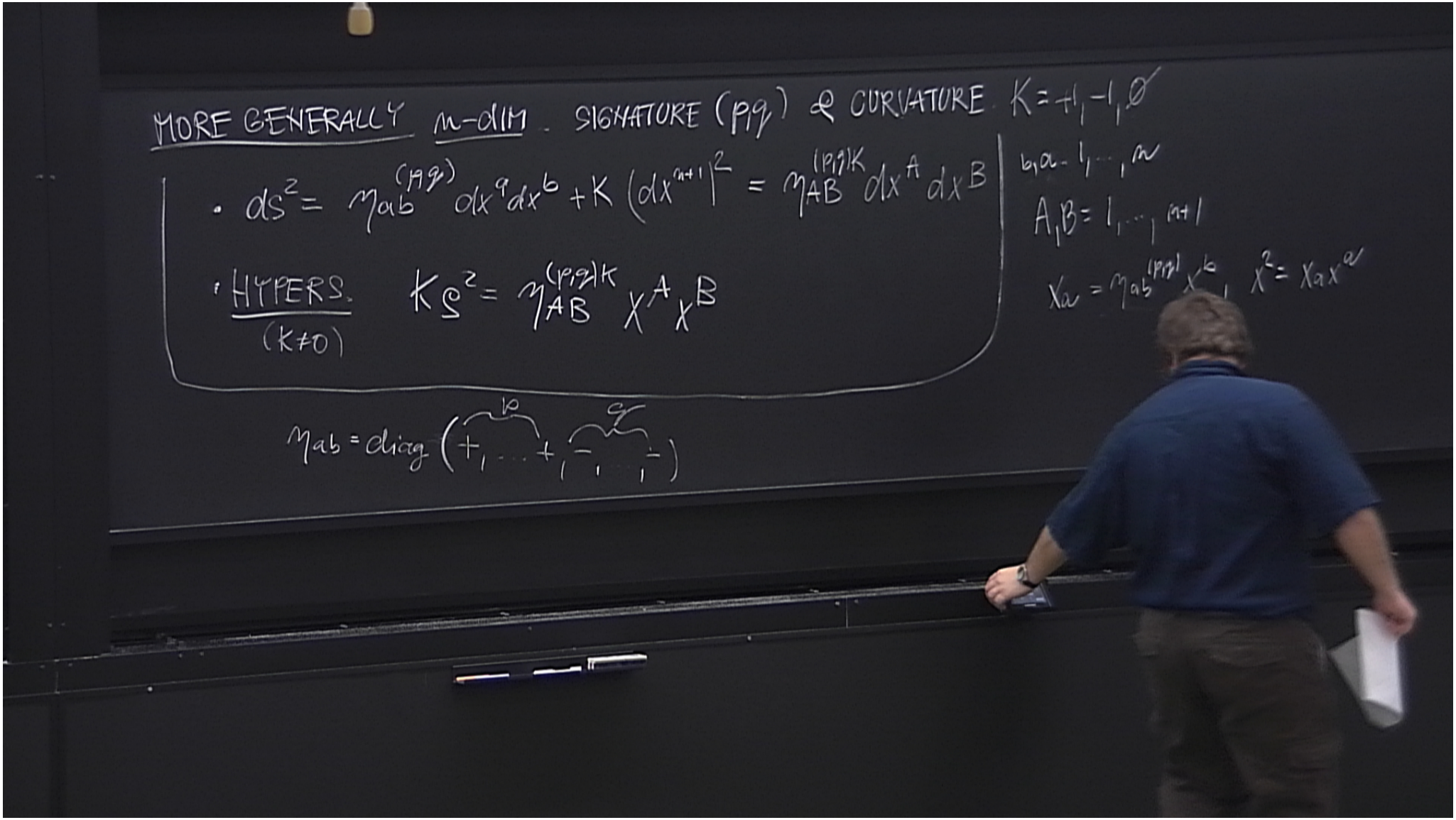
NEG. CURVAT. $-\rho^2 = (x^1)^2 + (x^2)^2 - (x^3)^2$

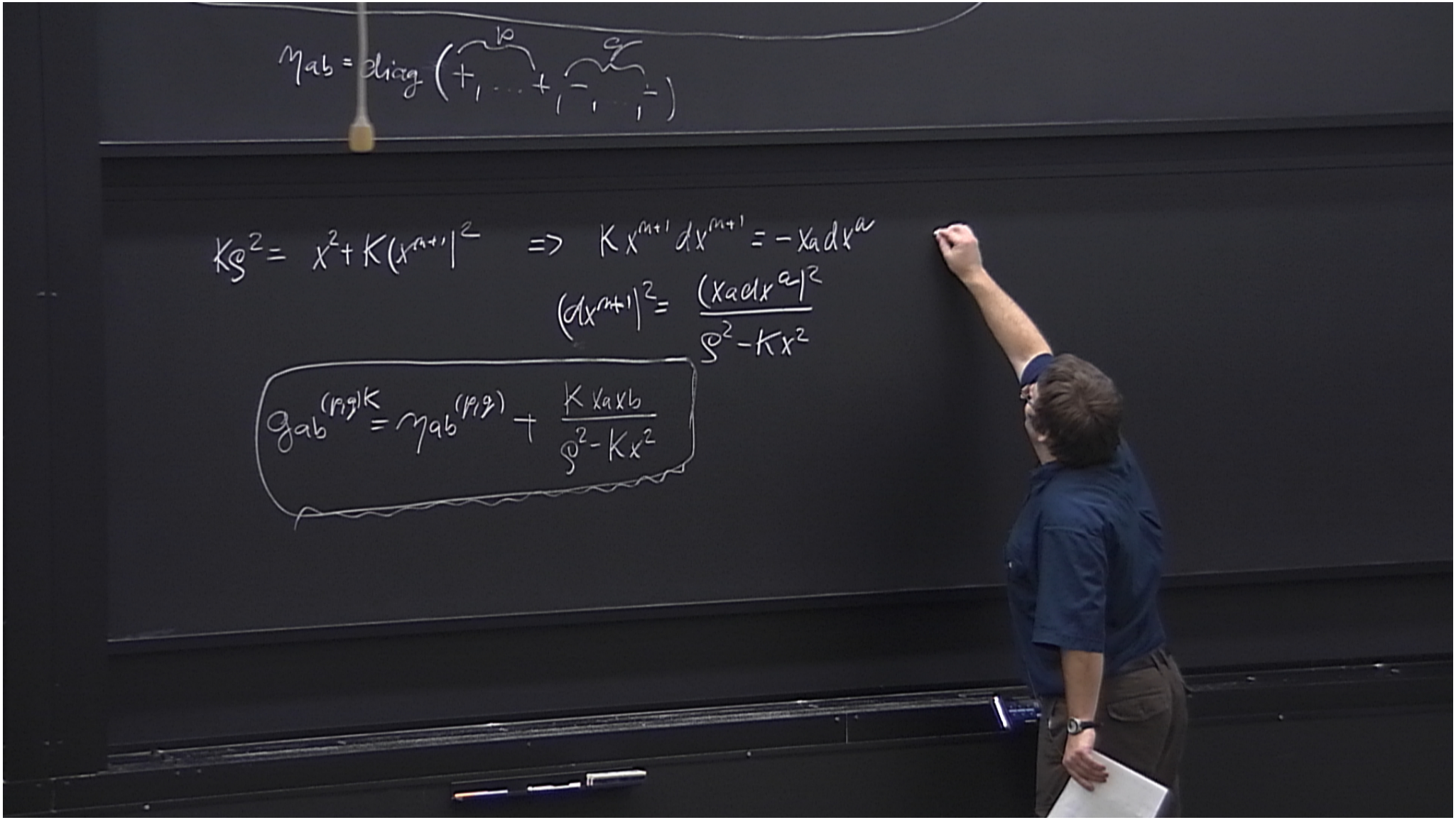
TWO SHEET HYP. EMBED. IN MINK.

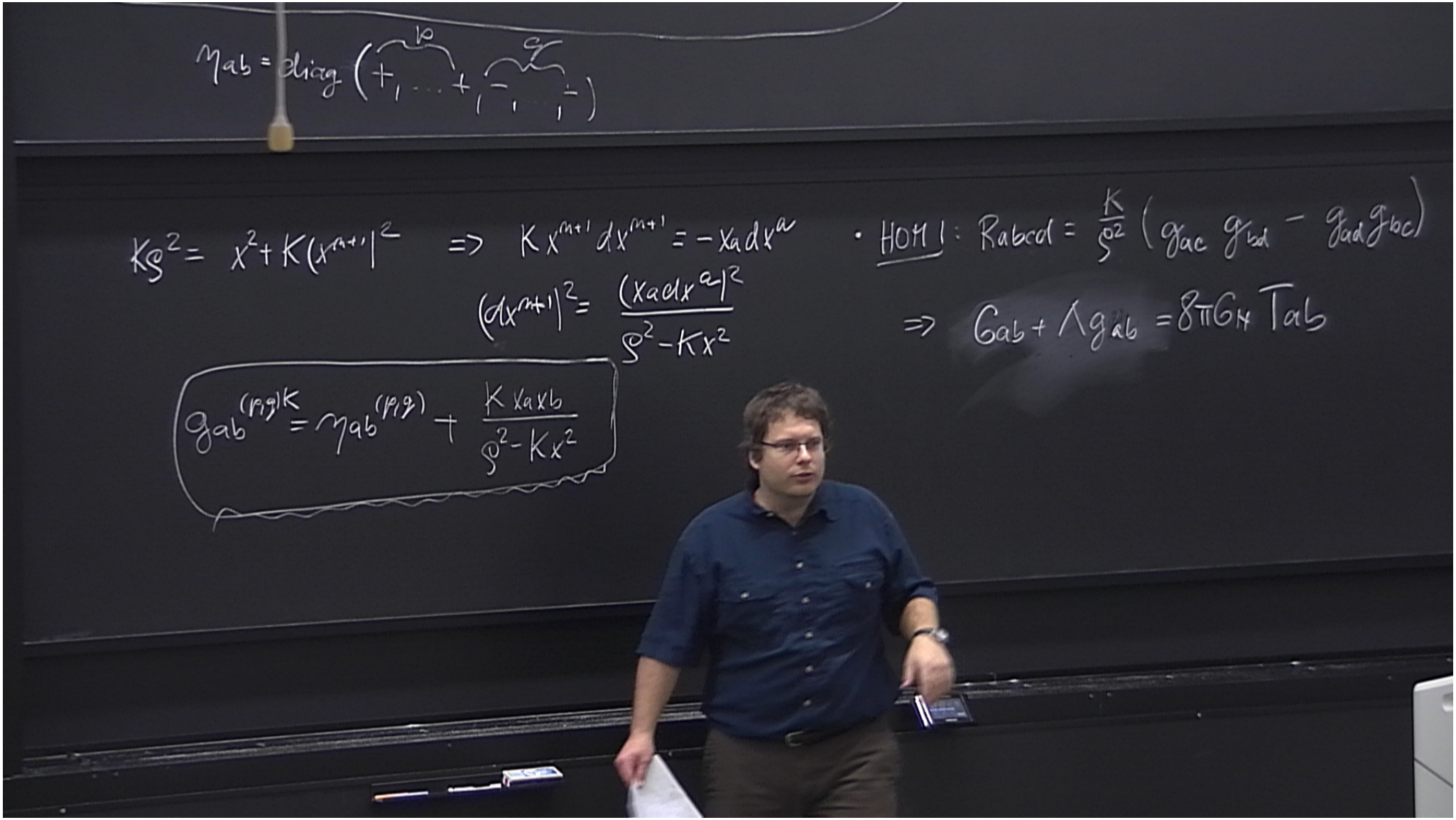
"WITHIN LIGHT CONE"

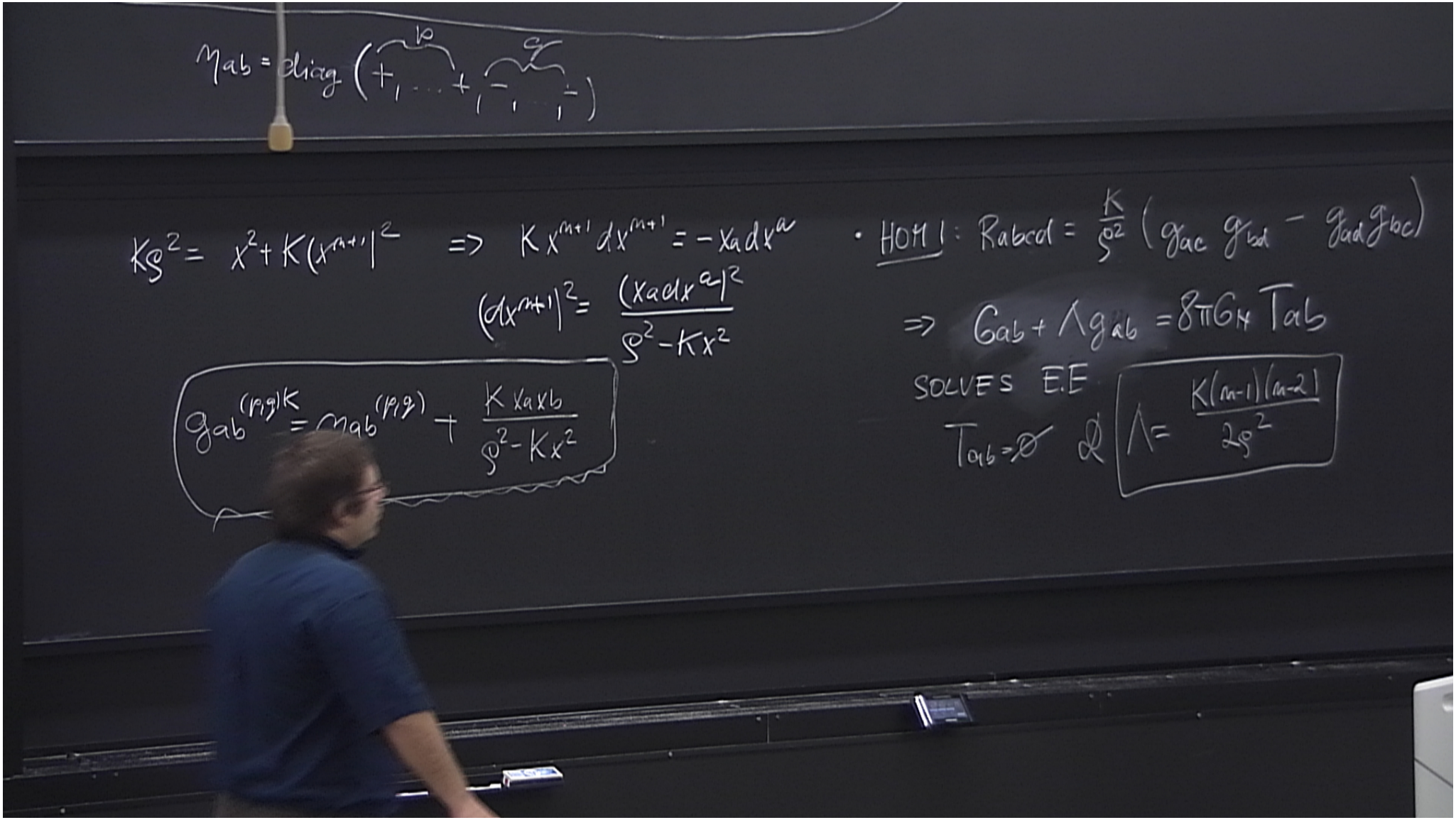
$\Rightarrow$ EUCLIDEAN INDUCED METRIC

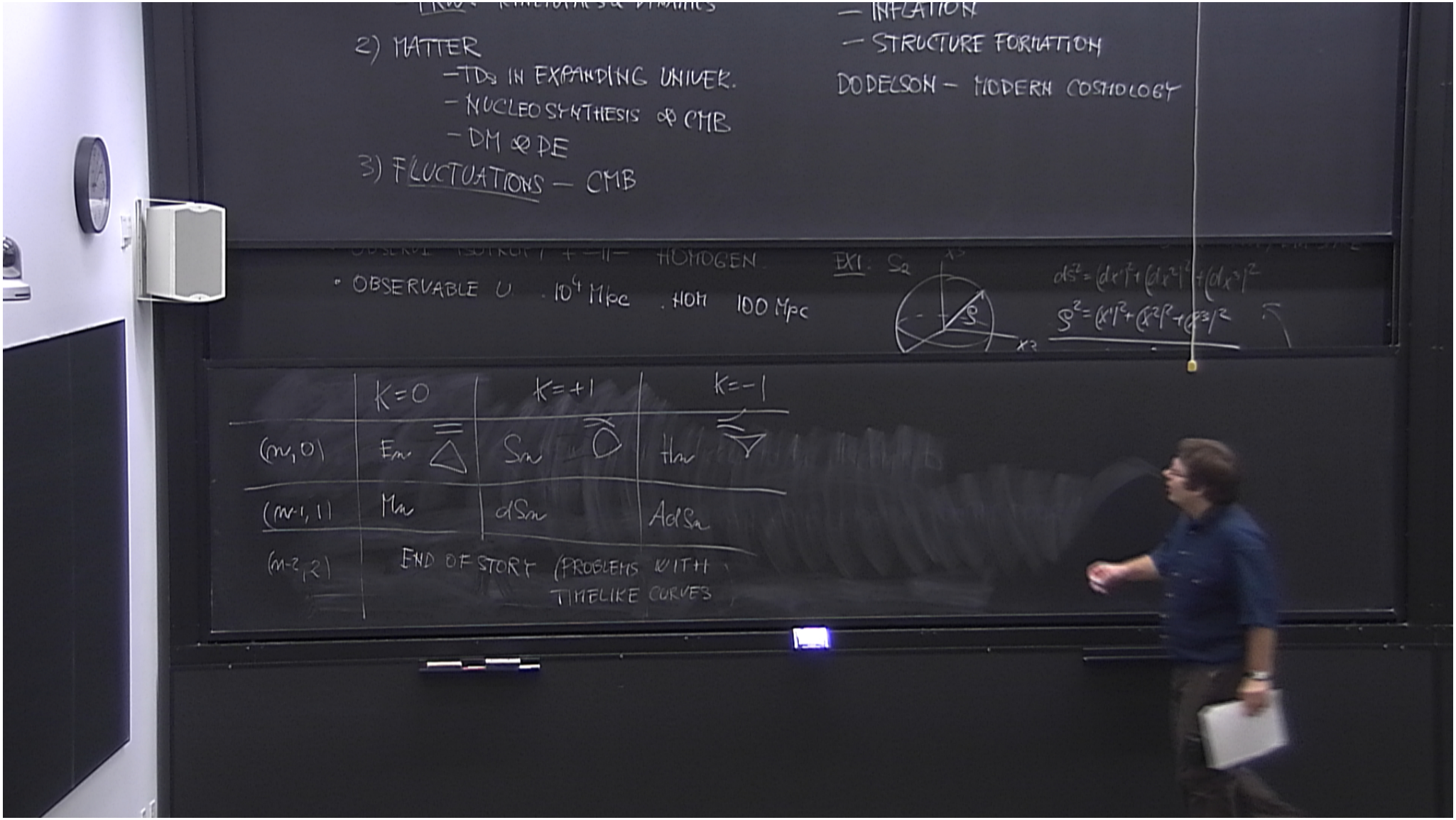

MORE GENERALLY n-dim. SIGNATURE (p,q) & CURVATURE K = +1, -1, 0
• $ds^2 = \eta_{ab}^{(p,q)} dx^a dx^b + K (dx^{n+1})^2 = \eta_{AB}^{(p,q),K} dx^A dx^B$
• HYPERS. $(K \neq 0)$ $K s^2 = \eta_{AB}^{(p,q),K} X^A X^B$

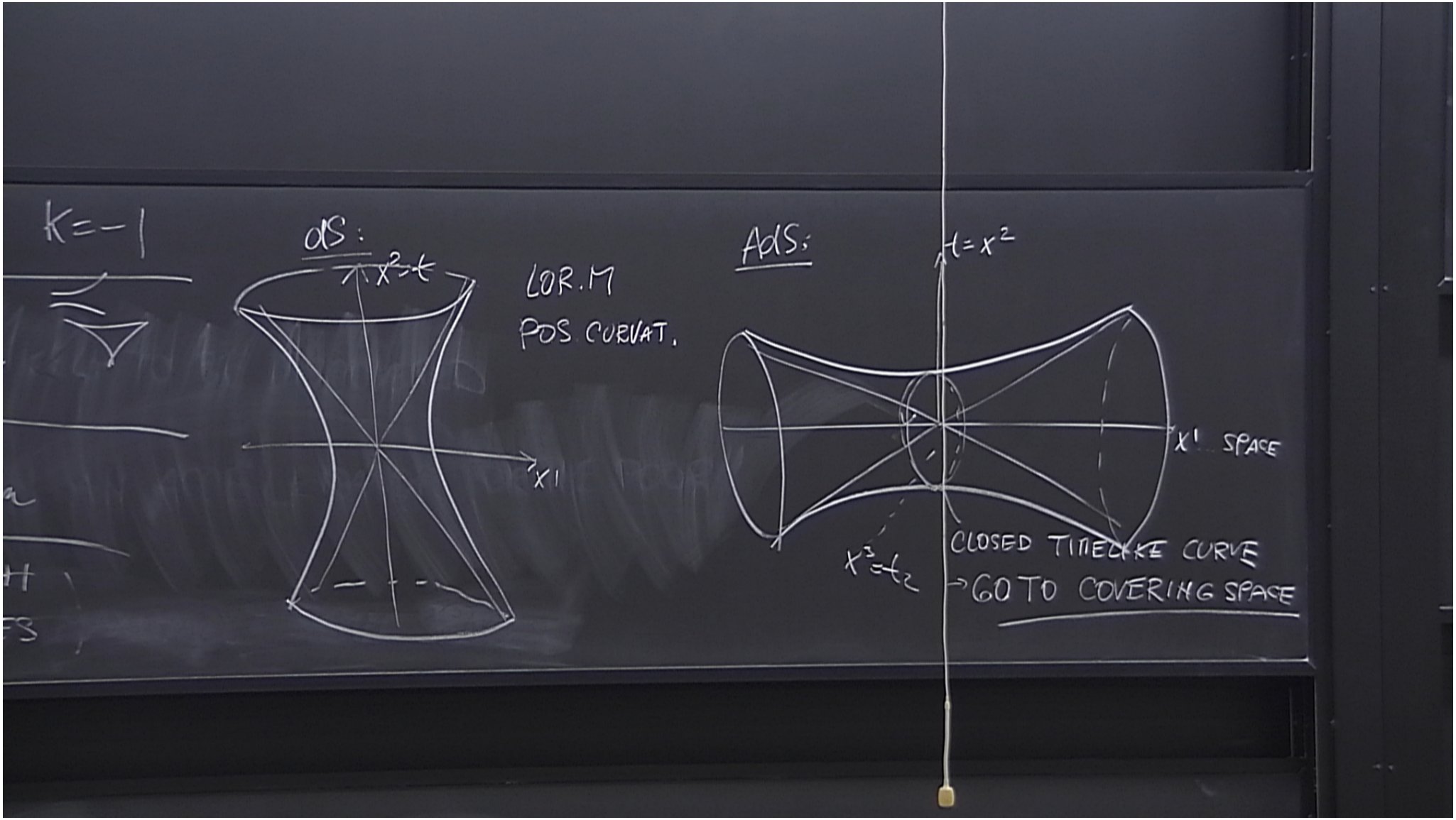

MORE GENERALLY n-dim. SIGNATURE (p,q) & CURVATURE K = +1, -1, 0
• $ds^2 = \eta^{(p,q)}_{ab}\, dx^a dx^b + K\,(dx^{n+1})^2 = \eta^{(p,q),K}_{AB}\, dx^A dx^B$
• HYPERS. (K≠0) $K s^2 = \eta^{(p,q),K}_{AB}\, X^A X^B$
a, b = 1, ..., n
A, B = 1, ..., n+1
$X_a = \eta_{ab}^{(p,q)} x^b$, $x^2 = X_a x^a$
$\eta_{ab} = \mathrm{diag}(\underbrace{+,\dots,+}_{p},\underbrace{-,\dots,-}_{q})$

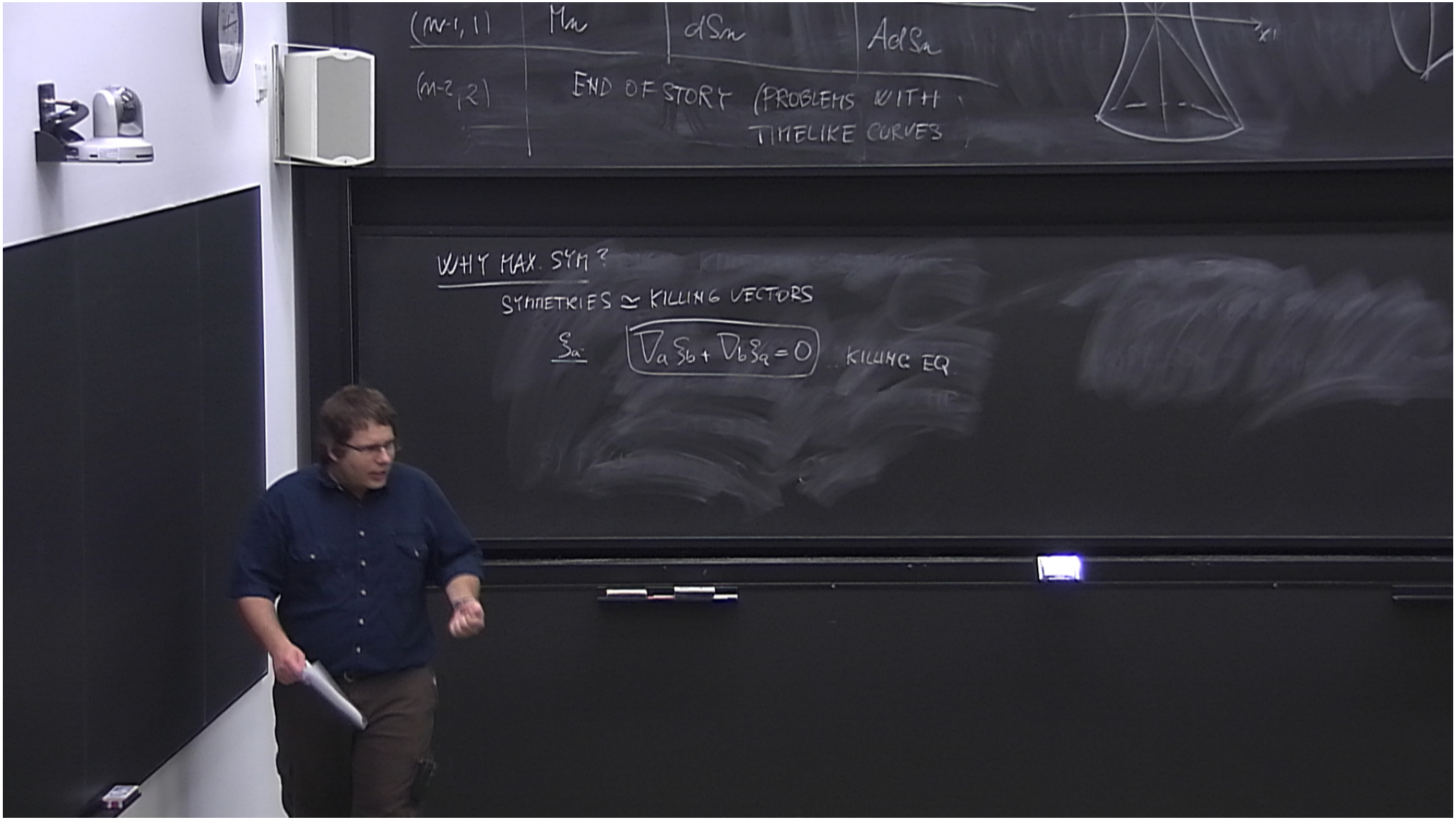

$\eta_{ab} = \mathrm{diag}(\overbrace{+,\dots,+}^{p}, \overbrace{-,\dots,-}^{q})$
$K s^2 = x^2 + K(x^{n+1})^2 \Rightarrow K x^{n+1} dx^{n+1} = -x_a dx^a$
$(dx^{n+1})^2 = \frac{(x_a dx^a)^2}{s^2 - K x^2}$
$g_{ab}^{(p,q)|K} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{s^2 - K x^2}$

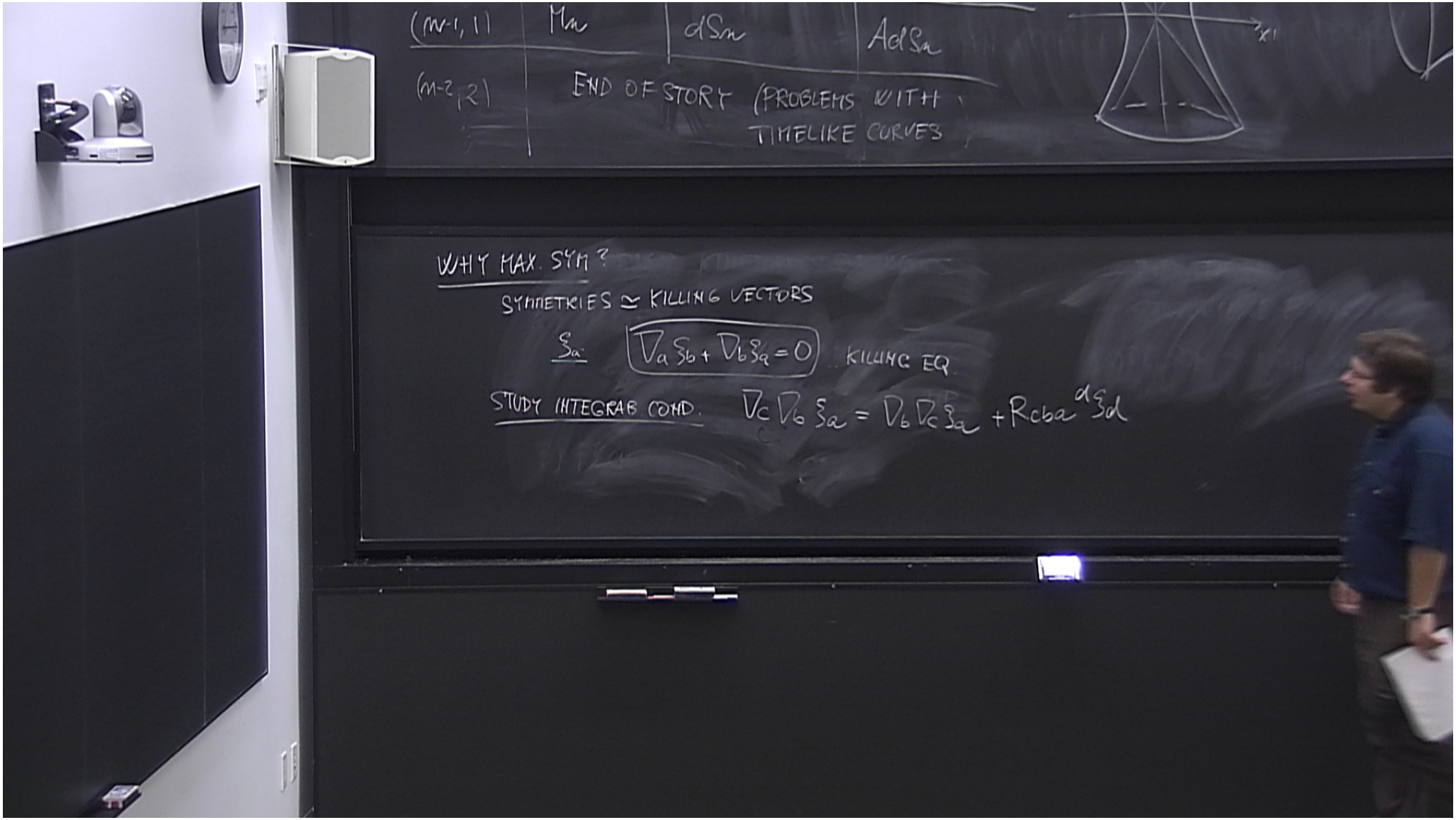

$\eta_{ab} = \mathrm{diag}(\overbrace{+,\dots,+}^{p},\overbrace{-,\dots,-}^{q})$
$ds^2 = dx^2 + K(x^{n+1})^2 \Rightarrow K x^{n+1} dx^{n+1} = -x_a dx^a$
$(dx^{n+1})^2 = \frac{(x_a dx^a)^2}{s^2 - Kx^2}$
$g_{ab}^{(p,q|K)} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{s^2 - Kx^2}$
• HOM I: $R_{abcd} = \frac{K}{s^2}(g_{ac}g_{bd} - g_{ad}g_{bc})$
$\Rightarrow G_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab}$

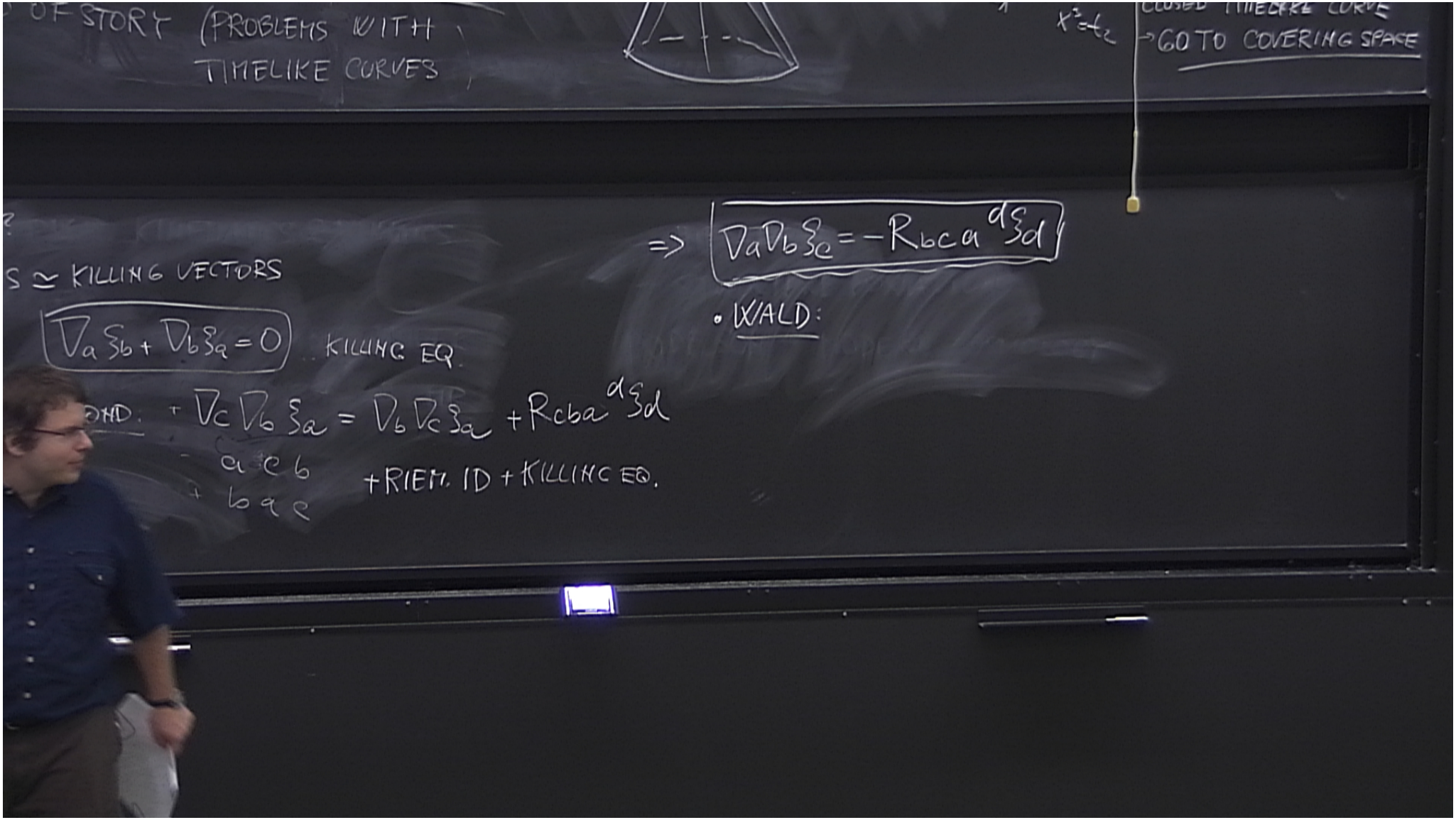

$\eta_{ab} = \mathrm{diag}(+,\dots,+,-,\dots,-)$
p
q
$K s^2 = x^2 + K(x^{n+1})^2 \Rightarrow K x^{n+1} dx^{n+1} = -x_a dx^a$
$(dx^{n+1})^2 = \frac{(x_a dx^a)^2}{s^2 - K x^2}$
$g_{ab}^{(p,q)K} = \eta_{ab}^{(p,q)} + \frac{K x_a x_b}{s^2 - K x^2}$
• HOM I: $R_{abcd} = \frac{K}{s^2}(g_{ac} g_{bd} - g_{ad} g_{bc})$
$\Rightarrow G_{ab} + \Lambda g_{ab} = 8\pi G_N T_{ab}$
SOLVES E.E
$T_{ab} = 0$ &
$\Lambda = \frac{K(n-1)(n-2)}{2s^2}$

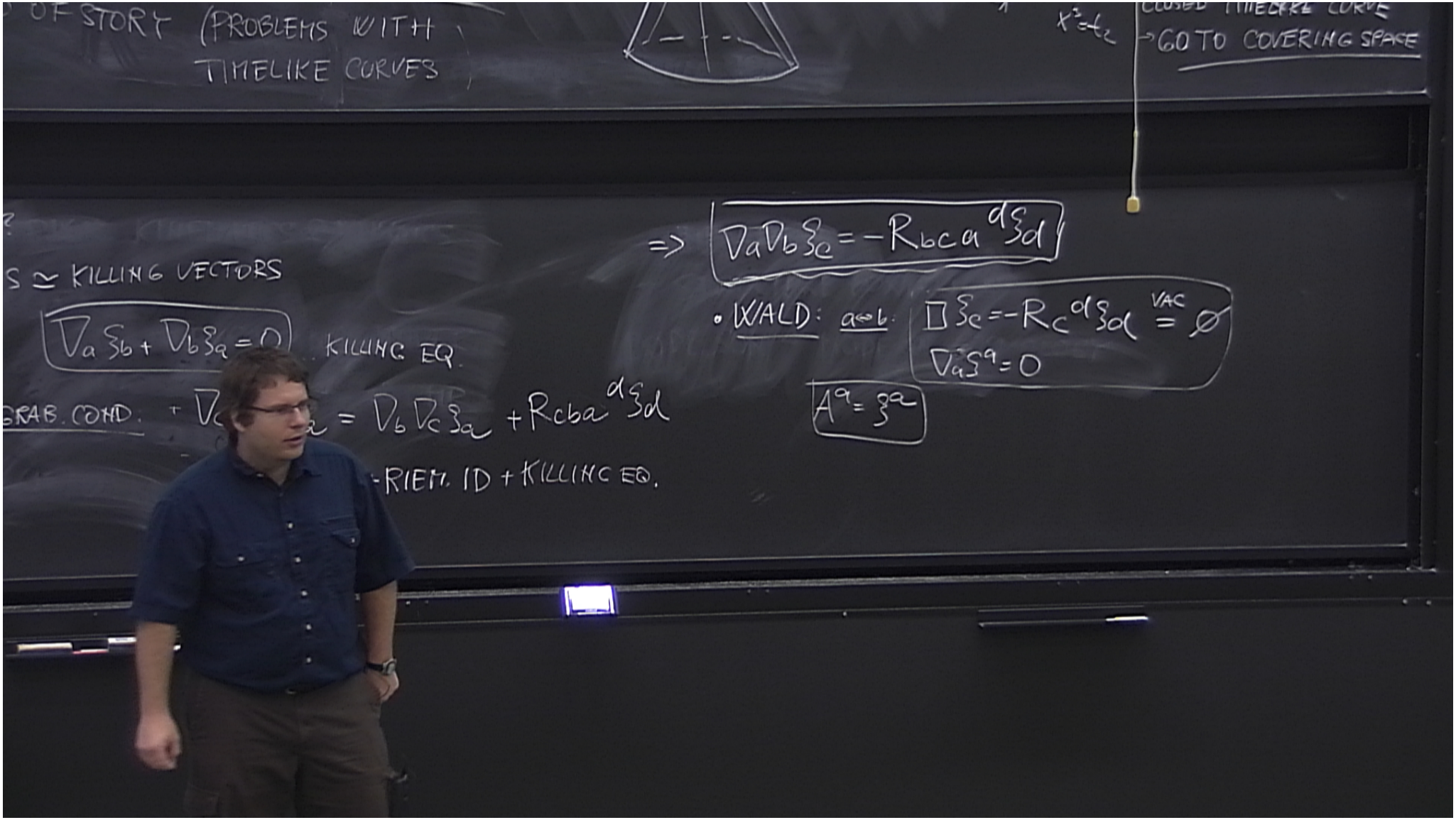

2) MATTER
– TDs IN EXPANDING UNIVER.
– NUCLEOSYNTHESIS & CMB
– DM & DE
3) FLUCTUATIONS – CMB
– INFLATION
– STRUCTURE FORMATION
DODELSON – MODERN COSMOLOGY
• OBSERVABLE U . 10^4 Mpc . HOM 100 Mpc
EX: S_2
$ds^2 = (dx^1)^2 + (dx^2)^2 + (dx^3)^2$
$S^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$
K=0
K=+1
K=-1
(n,0)
E_n
S_n
H_n
(n-1,1)
M_n
dS_n
AdS_n
(n-2,2)
END OF STORY (PROBLEMS WITH TIMELIKE CURVES)

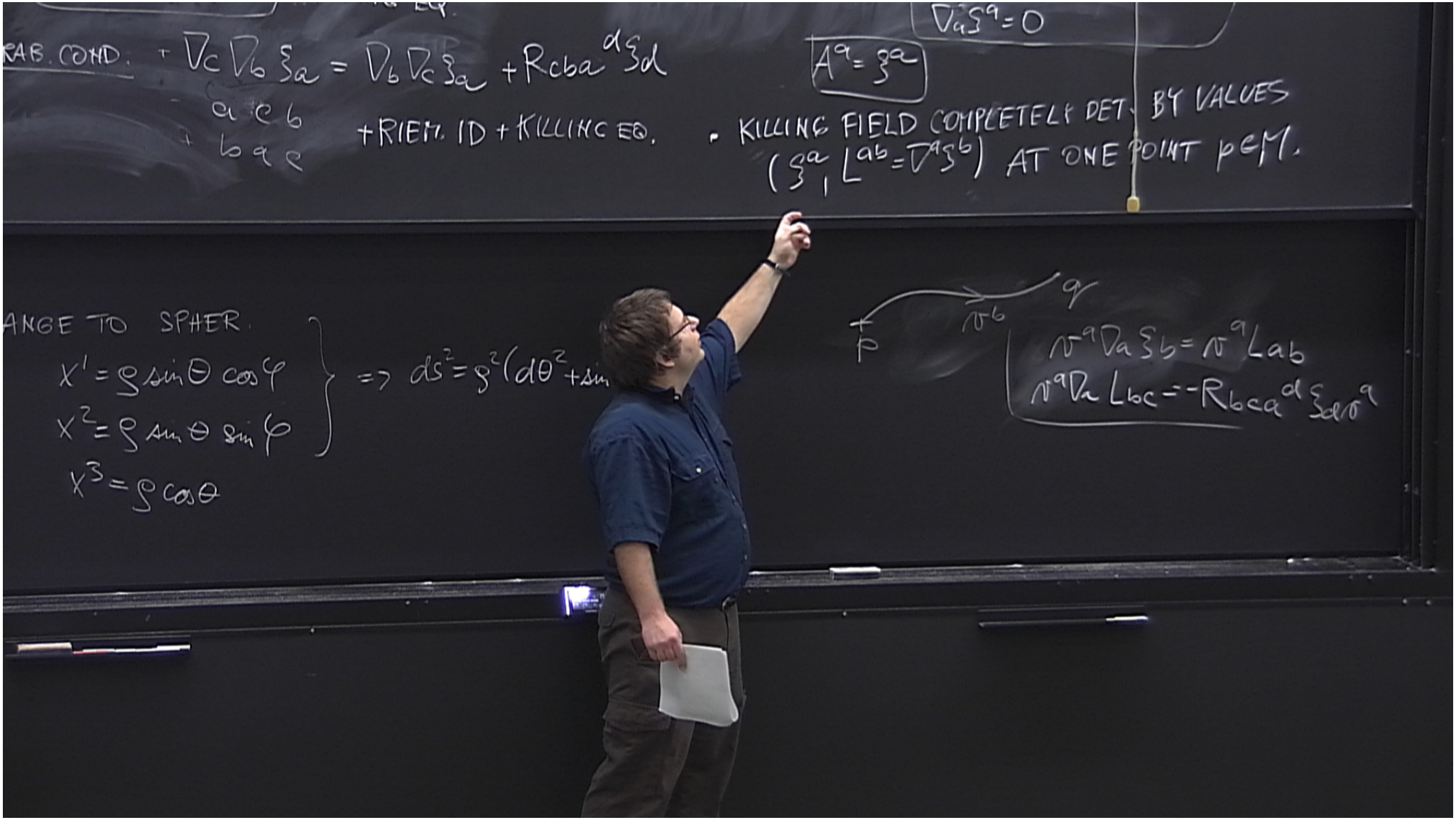

k=-1
dS:
$x^2=t$
x^1
LOR. M
POS CURVAT.
AdS:
$t=x^2$
x^1 SPACE
$x^2=t_2$
CLOSED TIMELIKE CURVE
→GO TO COVERING SPACE


(M-1, 1)
Mw
dSw
AdSw
(M-2, 2)
END OF STORY (PROBLEMS WITH TIMELIKE CURVES)
x1
WHY MAX. SYM?
SYMMETRIES ≃ KILLING VECTORS
ξa
∇a ξb + ∇b ξa = 0
KILLING EQ


(N-1, 1)
Mn
dSn
AdSn
(N-2, 2)
END OF STORY (PROBLEMS WITH TIMELIKE CURVES)
x1
WHY MAX. SYM?
SYMMETRIES ≃ KILLING VECTORS
ξa
∇aξb + ∇bξa = 0
KILLING EQ.
STUDY INTEGRAB COND.
∇c∇bξa = ∇b∇cξa + Rcba^d ξd


OF STORY (PROBLEMS WITH TIMELIKE CURVES
$x^3 = t_2$
CLOSED TIMELIKE CURVE
→GO TO COVERING SPACE
$\Rightarrow \boxed{\nabla_a \nabla_b \xi_c = -R_{bca}{}^d \xi_d}$
• WALD:
S ≃ KILLING VECTORS
$\boxed{\nabla_a \xi_b + \nabla_b \xi_a = 0}$ KILLING EQ.
IND. $+ \nabla_c \nabla_b \xi_a = \nabla_b \nabla_c \xi_a + R_{cba}{}^d \xi_d$
a c b
+ b a c
+RIEM. ID + KILLING EQ.


OF STORY (PROBLEMS WITH TIMELIKE CURVES
$x^3 = t_2$
CLOSED TIMELIKE CURVE
→ GO TO COVERING SPACE
S ≃ KILLING VECTORS
$\nabla_a \xi_b + \nabla_b \xi_a = 0$ KILLING EQ.
$\Rightarrow \nabla_a \nabla_b \xi_c = -R_{bca}{}^d \xi_d$
• WALD: a=b: $\Box \xi_c = -R_c{}^d \xi_d \overset{VAC}{=} \emptyset$
$\nabla_a \xi^a = 0$
$A^a = \xi^a$
GRAB. COND. + ∇_c $= \nabla_b \nabla_c \xi_a + R_{cba}{}^d \xi_d$
-RIEM. ID + KILLING EQ.


RAB. COND.
$+\nabla_c\nabla_b\xi_a = \nabla_b\nabla_c\xi_a + R_{cba}{}^d\xi_d$
a c b
+ b a c
+RIEM. ID + KILLING EQ.
$\nabla_a\xi^a = 0$
$A^a = \xi^a$
• KILLING FIELD COMPLETELY DET. BY VALUES
$(\xi^a, L^{ab}=\nabla^a\xi^b)$ AT ONE POINT $p\in M$.
ANGE TO SPHER.
$x^1 = \varrho\sin\theta\cos\varphi$
$x^2 = \varrho\sin\theta\sin\varphi$
$x^3 = \varrho\cos\theta$
$\Rightarrow ds^2 = \varrho^2(d\theta^2 + \sin$
p
n^b
q
$n^a\nabla_a\xi_b = n^a L_{ab}$
$n^a\nabla_a L_{bc} = -R_{bca}{}^d\xi_d n^a$

SYMMETRIES ≃ KILLING VECTORS

ξ_a $\quad \nabla_a \xi_b + \nabla_b \xi_a = 0$ KILLING EQ

STUDY INTEGRAB COND. $\nabla_c \nabla_b \xi_a = \nabla_b \nabla_c \xi_a + R_{cba}{}^d \xi_d$

$a \neq b \neq c \neq a$ + RIEM. ID + KILLING EQ.

$\Rightarrow \nabla_a \nabla_b \xi_c = -R_{bca}{}^d \xi_d$

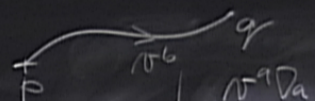
- WALD: $a=b$ $\quad \Box \xi_c = -R_c{}^d \xi_d$ $\quad$ VAC $= \emptyset$
  $\nabla_a \xi^a = 0$
  $A^a = \xi^a$
- KILLING FIELD COMPLETELY DET. BY VALUES $(\xi^a, L^{ab} = \nabla^a \xi^b)$ AT ONE POINT $p \in M$.

NOTE:

CHANGE TO SPHER.

$x^1 = \rho \sin\theta \cos\varphi$
$x^2 = \rho \sin\theta \sin\varphi$
$x^3 = \rho \cos\theta$

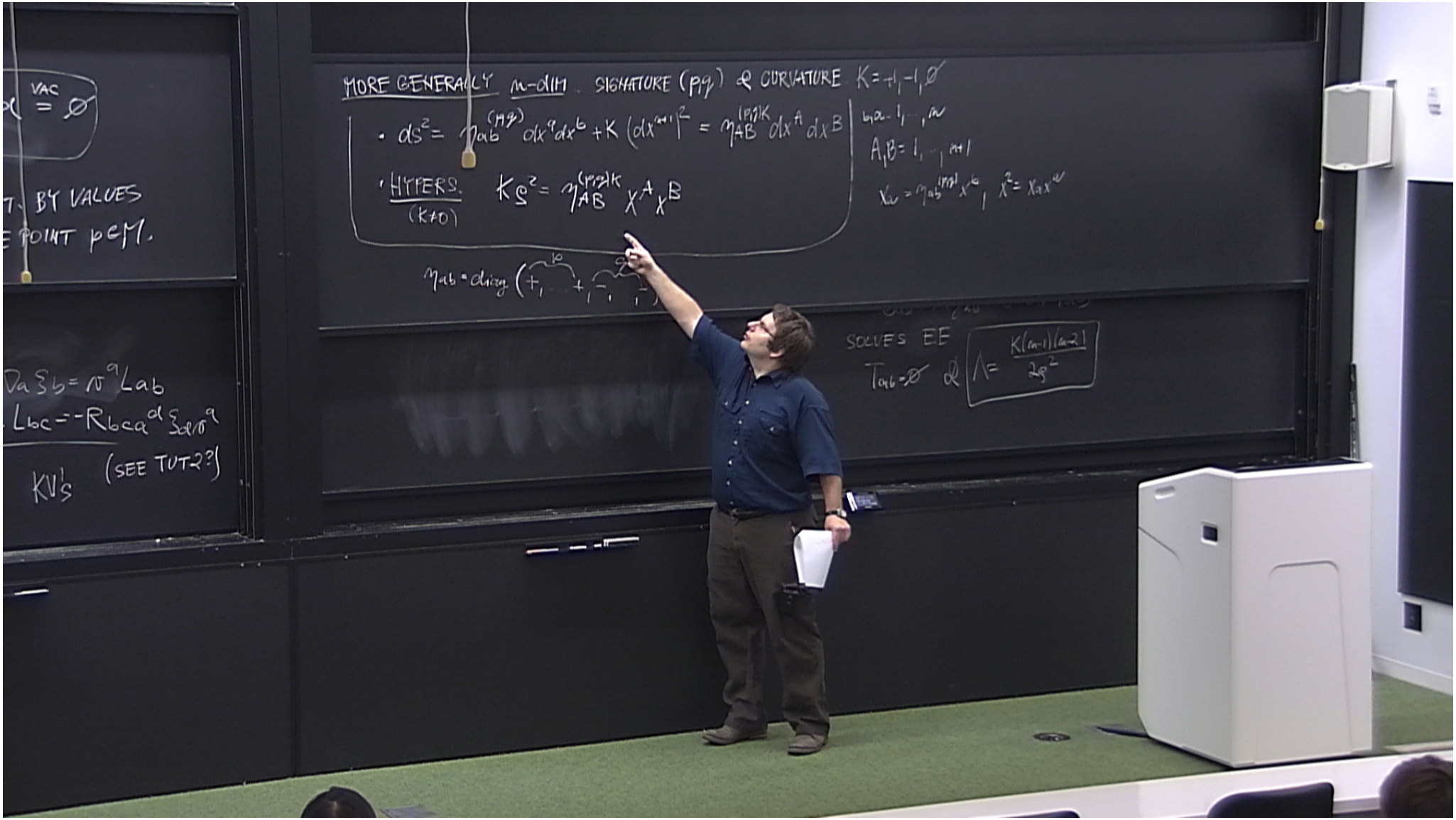
$\Rightarrow ds^2 = \rho^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$



$\pi^a \nabla_a \xi_b = \pi^a L_{ab}$
$\pi^a \nabla_a L_{bc} = -R_{bca}{}^d \xi_d \pi^a$

ON M OF DIM n

$n + \binom{n}{2} = \frac{n(n+1)}{2}$ KV's (SEE TUT 2?)


MORE GENERALLY n-dim SIGNATURE (p,q) & CURVATURE K = +1, -1, 0
SOLVES EE
BY VALUES
(SEE TUT 2?)
KV's