

Title: String Theory Review-3

Date: Jan 28, 2015 10:15 AM

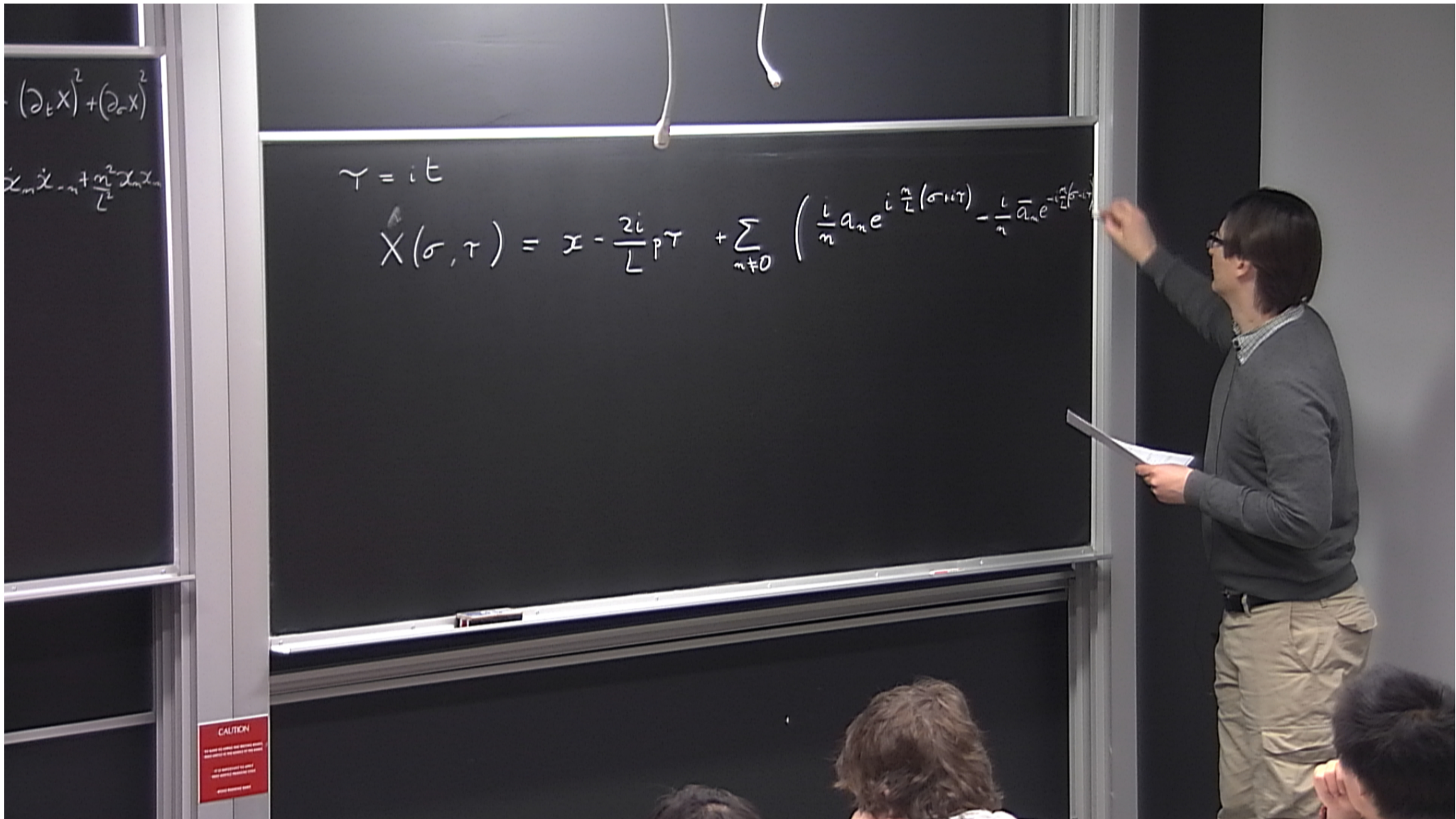
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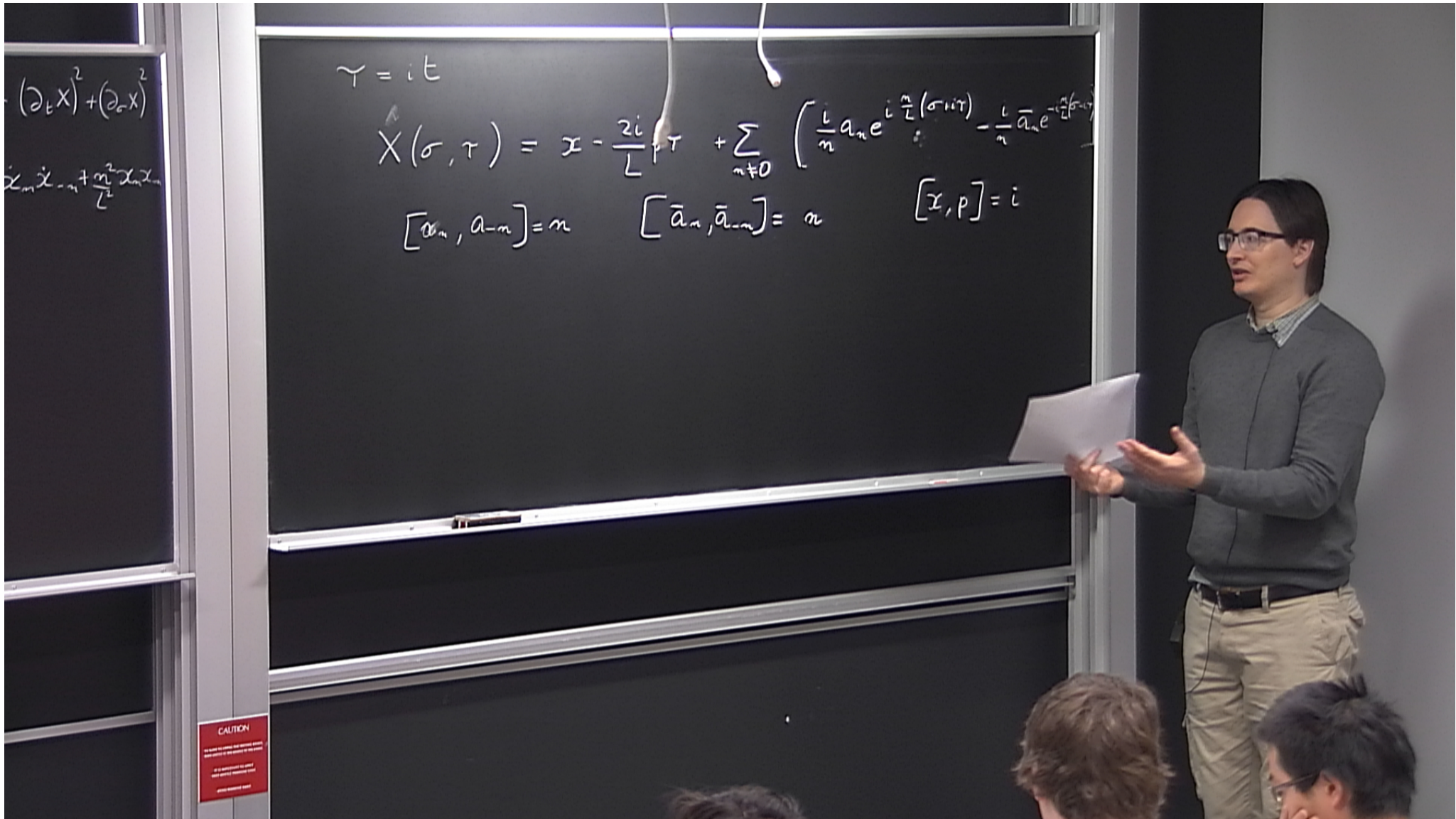
Abstract:

$$\sigma, t \quad \sigma = \sigma + 2\pi L \quad \hat{H} = \int d\sigma (\partial_t X)^2 + (\partial_\sigma X)^2$$

$$X(\sigma) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i\frac{n}{L}\sigma} \quad \sum \alpha_n \alpha_{-n} + \frac{1}{2} \alpha_0^2$$

$$|P\rangle$$





$$\tau = it$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \left(\frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i\tau)} - \frac{i}{n} \bar{a}_n e^{-i \frac{n}{L} (\sigma - i\tau)} \right)$$

$$[a_n, a_{-n}] = n \quad [\bar{a}_n, \bar{a}_{-n}] = n \quad [x, p] = i$$

σ, t

$$\sigma \equiv \sigma + 2\pi L$$

$$\hat{H} = \int d\sigma (\partial_t X)^2 + (\partial_\sigma X)^2$$

$$X(\sigma) = x_0 + \sum_{n \neq 0} \alpha_n e^{i \frac{n}{L} \sigma}$$

$$\sum \alpha_m \alpha_{-m} + \frac{m^2}{L^2} \alpha_m \alpha_m$$

$|P\rangle$

$a_{-1} |P\rangle$

$\bar{a}_{-1} |P\rangle$

$a_{-1}^2 |P\rangle$

$a_{-1} \bar{a}_{-1} |P\rangle$

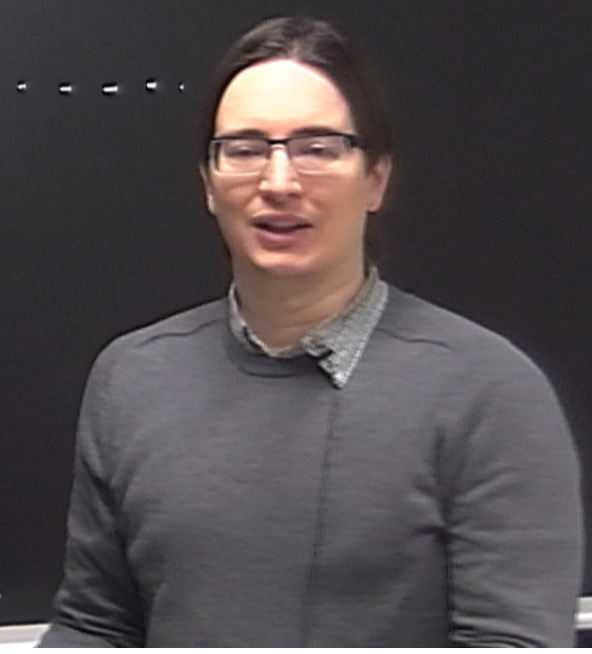
$\bar{a}_{-1}^2 |P\rangle$

$a_{-2} |P\rangle$

$\bar{a}_{-2} |P\rangle$

0

~~0~~







σ, t

$$\sigma \equiv \sigma + 2\pi L$$

$$\hat{H} = \int d\sigma (\partial_t X)^2 + (\partial_\sigma X)^2$$

$$X(\sigma) = \sum_{n=-\infty}^{\infty} \alpha_n e^{i \frac{n}{L} \sigma}$$

$$\sum \alpha_n \alpha_{-n} + \frac{n^2}{2} \alpha_n \alpha_n$$

$|p\rangle$

$a_{-1}|p\rangle$

$a_{-1}^2|p\rangle$

$a_{-1} \bar{a}_{-1}|p\rangle$

.....

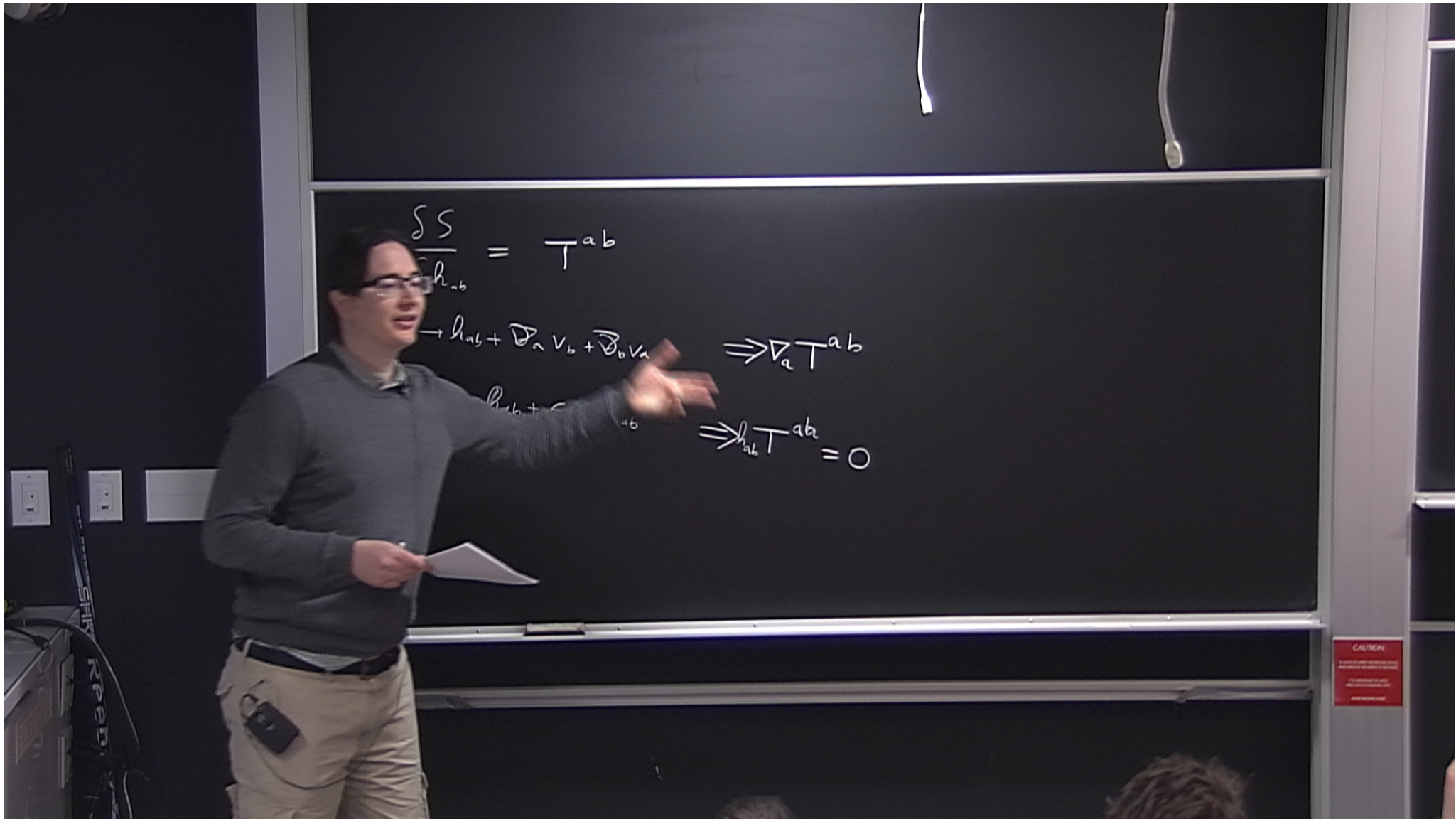
$\bar{a}_{-1}|p\rangle$

$\bar{a}_{-1}^2|p\rangle$

$a_{-2}|p\rangle$

$\bar{a}_{-2}|p\rangle$

0



$$\frac{\delta S}{\delta h_{ab}} = T^{ab}$$

$$h_{ab} \rightarrow h_{ab} + \nabla_a v_b + \nabla_b v_a \quad \Rightarrow \nabla_a T^{ab}$$

$$h_{ab} \rightarrow h_{ab} + \epsilon \delta h_{ab} \quad \Rightarrow \delta_{h_{ab}} T^{ab} = 0$$

$$\Rightarrow h_{ab} T^{ab} = 0$$

$$\int \sqrt{h} h^{ab} \frac{\partial X}{\partial u^a} \frac{\partial X}{\partial u^b} dz d\bar{z}$$

$$h_{z\bar{z}}$$

$$T_{ab} = h_{aa'} h_{bb'} T^{bb'}$$

$$T_{zz} = -\frac{1}{2} \partial_z X \partial_z X$$

$$T_{\bar{z}\bar{z}} = -\frac{1}{2} \partial_{\bar{z}} X \partial_{\bar{z}} X$$

$$T_{z\bar{z}} = 0$$

$$\Rightarrow h_{ab} T^{ab} = 0$$

$$\int \sqrt{h} h^{ab} \frac{\partial X}{\partial u^a} \frac{\partial X}{\partial u^b} dz d\bar{z}$$

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$$T_{\bar{z}\bar{z}} = -\frac{1}{2} \partial_{\bar{z}} X \partial_{\bar{z}} X$$

$$T_{z\bar{z}} = 0$$

$$\gamma = i t$$

$$s = \gamma - i \sigma$$

$$X(\sigma, \tau) = x - \frac{2i}{L} p \tau + \sum_{n \neq 0} \left(\frac{i}{n} a_n e^{i \frac{n}{L} (\sigma + i \tau)} - \frac{i}{n} \bar{a}_n e^{-i \frac{n}{L} (\sigma - i \tau)} \right)$$

$$[a_n, a_{-n}] = n \quad [\bar{a}_n, \bar{a}_{-n}] = n \quad [x, p] = i$$

$$[X(\sigma, 0), i \partial_\tau X(\sigma', 0)] = \frac{2i}{L} \sum_n e^{i \frac{n}{L} (\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

CAUTION

Do not use sharp objects to clean the board.
Do not use sharp objects to clean the board.
Do not use sharp objects to clean the board.

0

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$a_{-2}|p\rangle$
 $\bar{a}_{-2}|p\rangle$

$$T_a = i\partial_a X$$

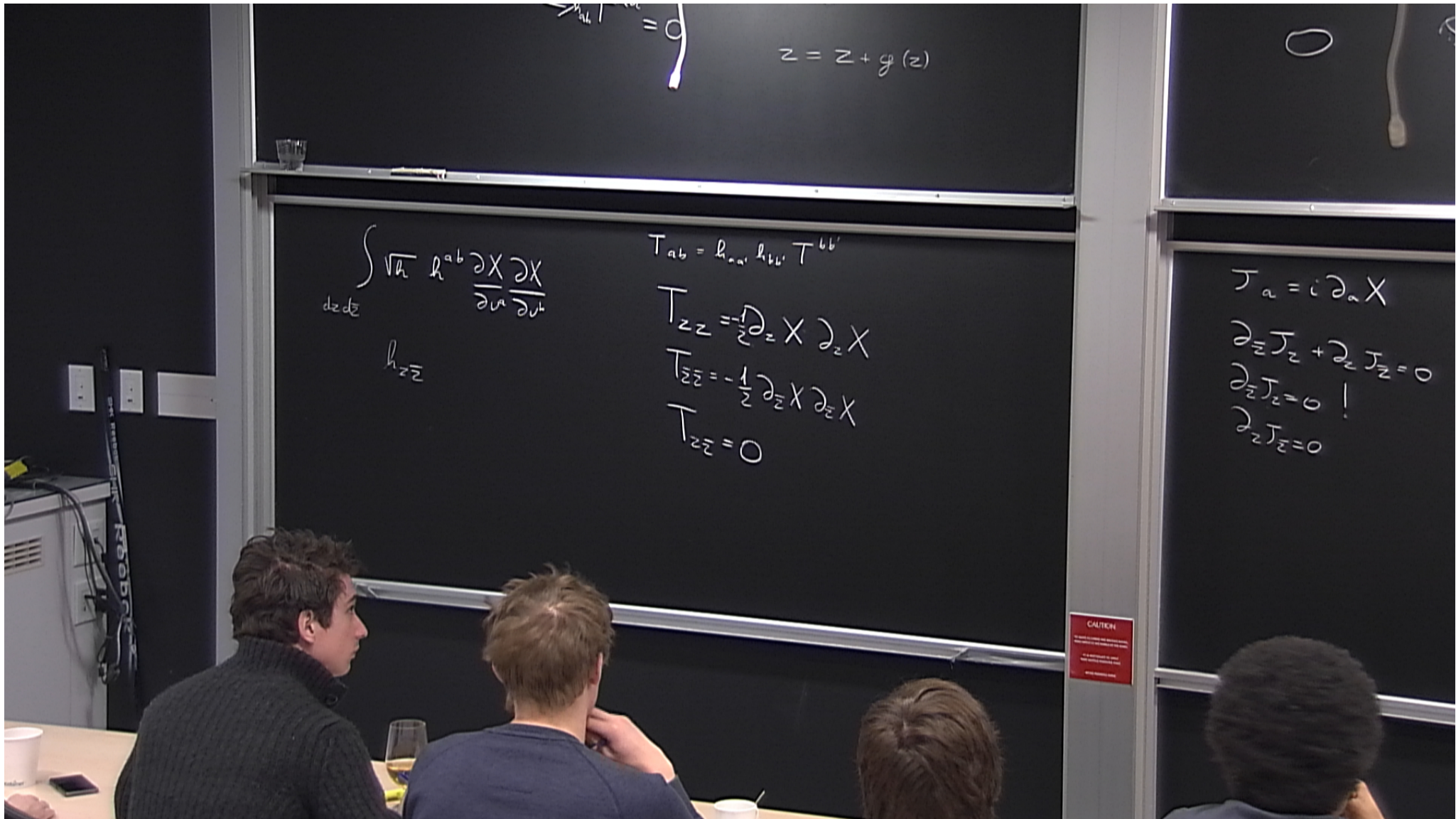
$$T_s = i\partial_a X = \frac{p}{L} + \sum_{n \neq 0} \frac{1}{L} a_n e^{-nz}$$

$$\partial_{\bar{z}} T_z + \partial_z T_{\bar{z}} = 0$$

$$\partial_{\bar{z}} T_z = 0 !$$

$$\partial_z T_{\bar{z}} = 0$$

$$\partial_{\bar{z}} f(z) T_z = 0$$



$$z = z + g(z)$$

$$\int \sqrt{h} h^{ab} \frac{\partial X}{\partial u^a} \frac{\partial X}{\partial u^b} dz d\bar{z}$$
$$h_{z\bar{z}}$$

$$T_{ab} = h_{aa'} h_{bb'} T^{a'b'}$$

$$T_{zz} = -\frac{1}{2} \partial_z X \partial_z X$$

$$T_{\bar{z}\bar{z}} = -\frac{1}{2} \partial_{\bar{z}} X \partial_{\bar{z}} X$$

$$T_{z\bar{z}} = 0$$

$$T_a = i \partial_a X$$

$$\partial_{\bar{z}} T_z + \partial_z T_{\bar{z}} = 0$$

$$\partial_{\bar{z}} T_z = 0 !$$

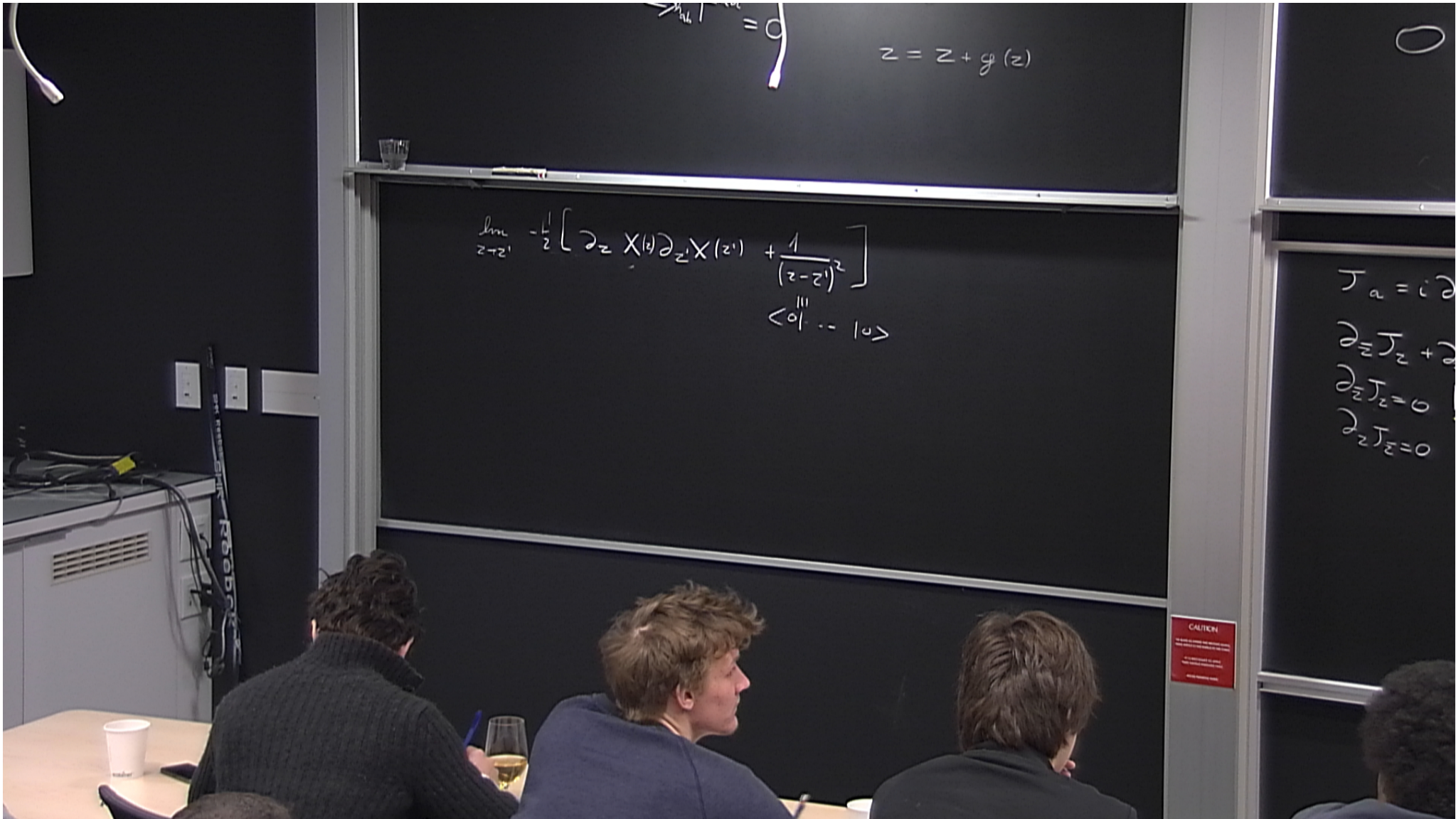
$$\partial_z T_{\bar{z}} = 0$$

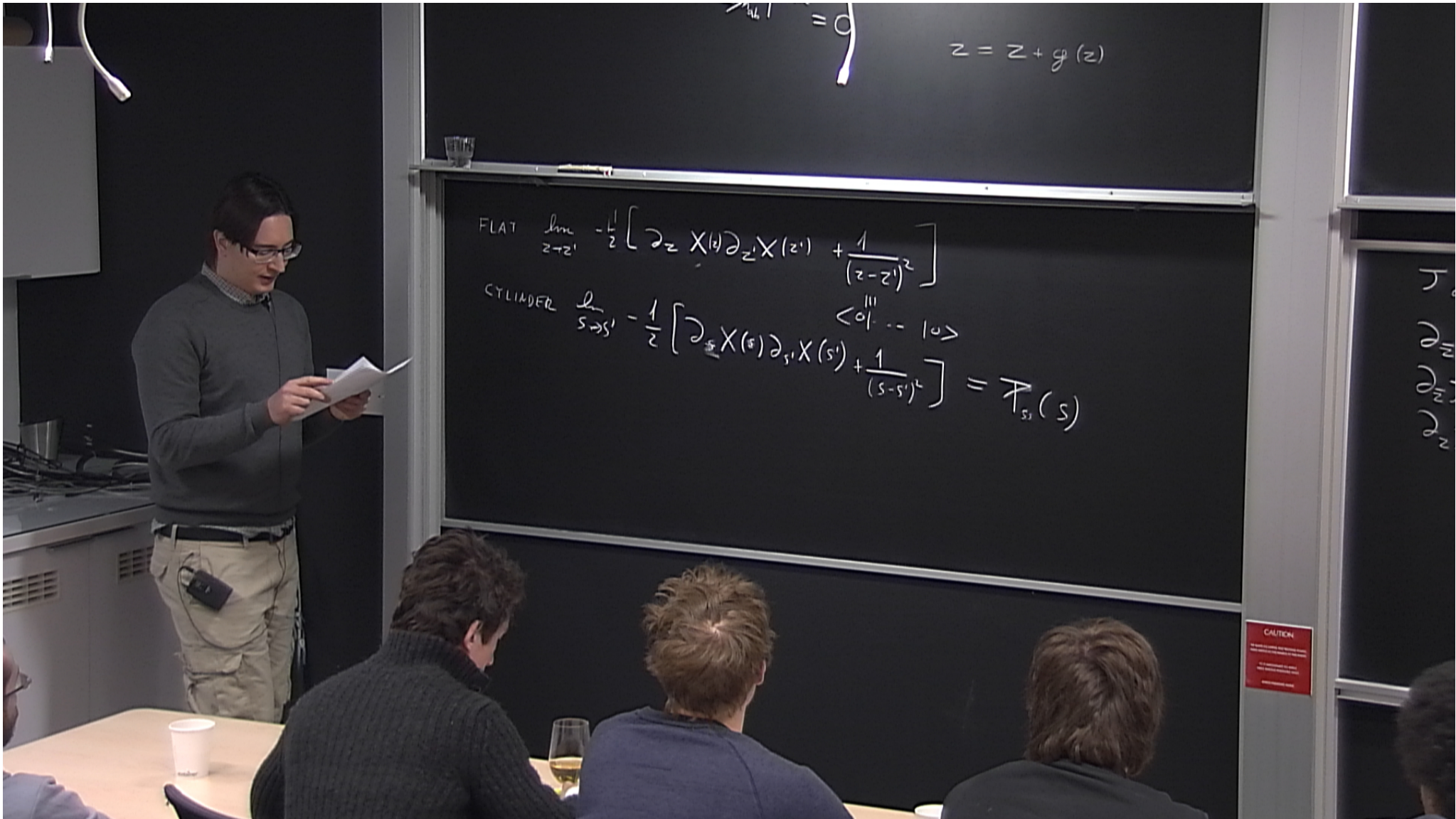
$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_{n>0} n e^{2i\pi n s} e^{-2i\pi n s'}$$

$$= -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$$

$$\underbrace{s+s'}_{\text{pole}} \rightarrow -\frac{1}{(s-s')^2}$$







$$[a_n, a_{-n}] = n$$

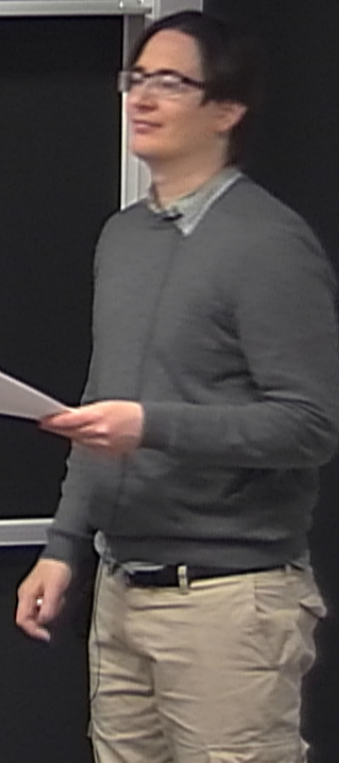
$$[X(\sigma, 0), \partial_s X(\sigma', 0)] = \frac{2\pi}{L} \sum_n e^{iL\sigma n} = 4\pi i \delta(\sigma - \sigma')$$

$$\partial_s X(\sigma) \partial_{s'} X(\sigma') + \frac{1}{L^2} \frac{e^{iL(\sigma + \sigma')}}{e^{iL(\sigma - \sigma')}} = \dots$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_{n \neq 0} e^{2iLns}$$

$$= -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$$

$$\langle 0 | T_{ss} | 0 \rangle = \lim_{s \rightarrow s'} -\frac{1}{L} \left[-\frac{1}{L^2} \frac{e^{iL(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2} + \frac{1}{(s-s')^2} \right] = -\frac{1}{24L^2}$$



CAUTION
DO NOT TOUCH THE BOARD OR THE PROJECTOR
IF YOU NEED TO USE THE BOARD OR THE PROJECTOR
PLEASE ASK THE TA

$$[a_m, a_{-m}] = m$$

$$[X(\sigma, 0), \partial_t X(\sigma', 0)] = \frac{2\pi}{L} \sum_n e^{iL(\sigma - \sigma')n} = 4\pi i \delta(\sigma - \sigma')$$

$$X^2 + (\partial_t X)^2$$

$$-\frac{m^2}{L^2} X_n X_{-n}$$

$$\partial_s X(s) \partial_{s'} X(s') + \frac{1}{L^2} \frac{e^{-iL(s-s')}}{s-s'}$$

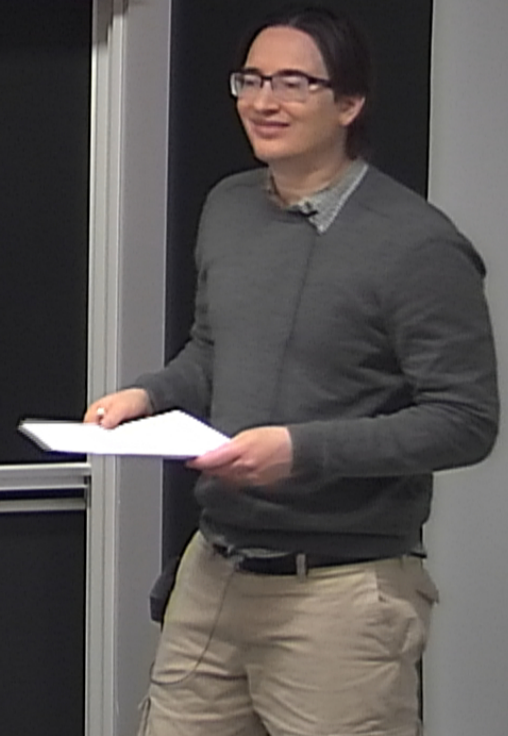
$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_{n>0} n e^{2iL(s-s')n}$$

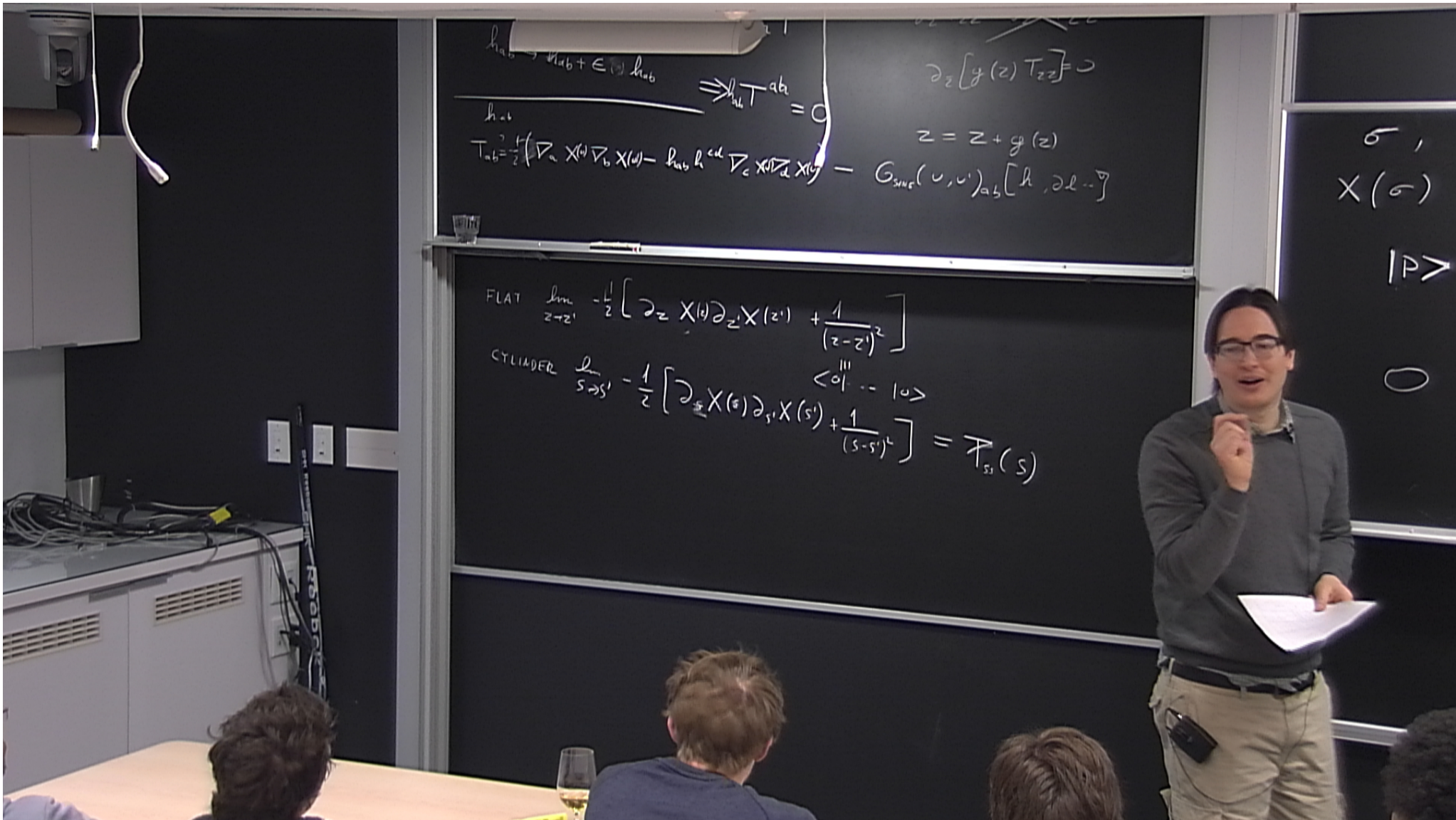
$$= -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s-s')}}{(e^{\frac{2}{L}(s-s')} - e^{-\frac{2}{L}(s-s')})^2}$$



$$\langle 0 | T_{ss} | 0 \rangle = \lim_{s \rightarrow s'} -\frac{1}{L} \left[-\frac{1}{L^2} \frac{e^{iL(s-s')}}{(e^{\frac{2}{L}(s-s')} - e^{-\frac{2}{L}(s-s')})^2} + \frac{1}{(s-s')^2} \right] = -\frac{1}{24L^2}$$

CAUTION
DO NOT TOUCH THE BOARD OR THE PROJECTOR
IF YOU NEED TO STOP THE BOARD OR THE PROJECTOR
PLEASE CONTACT THE STAFF





$$h_{ab} \rightarrow h_{ab} + \epsilon h_{ab} \Rightarrow h_{ab} T^{ab} = 0$$

$$T_{ab} = \frac{1}{2} (\nabla_a X^c \nabla_b X_c - h_{ab} h^{cd} \nabla_c X^e \nabla_d X_e) - G_{\text{Synch}}(u, v)_{ab} [h, g]$$

$$\partial_z [g(z) T_{zz}] = 0$$

$$z = z + g(z)$$

FLAT $\lim_{z \rightarrow z'} -\frac{1}{2} \left[\partial_z X^c \partial_{z'} X_c + \frac{1}{(z-z')^2} \right]$

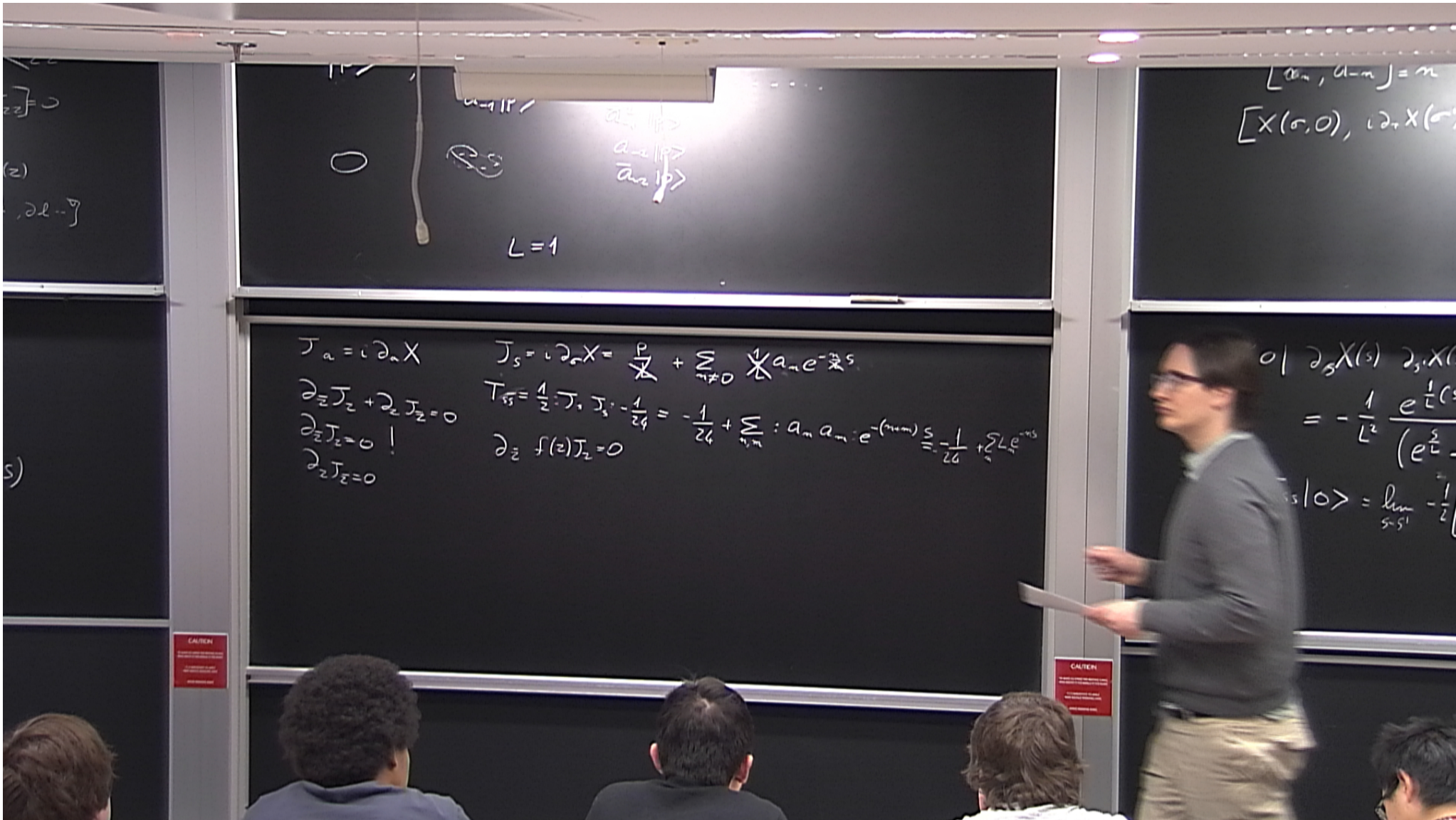
CYLINDER $\lim_{s \rightarrow s'} -\frac{1}{2} \left[\partial_s X^c \partial_{s'} X_c + \frac{1}{(s-s')^2} \right] = T_{s'}(s)$

$$\sigma,$$

$$X(\sigma)$$

$$|P\rangle$$

$$0$$



$$[z] = 0$$

$$\partial_z \dots$$

$$s)$$

$$L=1$$

$$a_{-1} | \dots$$

$$a_{-2} | \dots$$

$$a_{-1} | \dots$$

$$T_a = i \partial_z X$$

$$T_s = i \partial_z X = \frac{p}{2\alpha'} + \sum_{n \neq 0} \frac{1}{2} \alpha_n \alpha_{-n}$$

$$\partial_z T_z + \partial_{\bar{z}} T_{\bar{z}} = 0$$

$$\partial_z T_z = 0 !$$

$$\partial_z T_{\bar{z}} = 0$$

$$T_{\bar{z}} = \frac{1}{2} T_z, T_{\bar{z}} = -\frac{1}{2\alpha'} = -\frac{1}{2\alpha'} + \sum_{n \neq 0} \alpha_n \alpha_{-n} e^{-in\sigma} = -\frac{1}{2\alpha'} + \sum_{n \neq 0} L_n e^{-in\sigma}$$

$$\partial_z f(z) T_z = 0$$

$$[a_m, a_{-m}] = m$$

$$[X(\sigma, 0), \partial_z X(\sigma, 0)]$$

$$0 | \partial_z X(\sigma) \partial_z X(\sigma) = -\frac{1}{L^2} \frac{e^{\frac{1}{2}(\sigma - \sigma')}}{(e^{\frac{1}{2}(\sigma - \sigma')})^2}$$

$$|s\rangle = \lim_{s \rightarrow s'} -\frac{1}{2}$$

$$2\pi L \hat{H} = \int ds (\partial_t X)^2 + (\partial_s X)^2$$

$$\sum \alpha_n \dot{\alpha}_{-n} + \frac{\alpha_n^2}{L}$$

$$[L_n, \alpha_m] = m \alpha_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{ns} \partial_s X)$$


$$[\alpha_n, \alpha_{-n}] = n$$

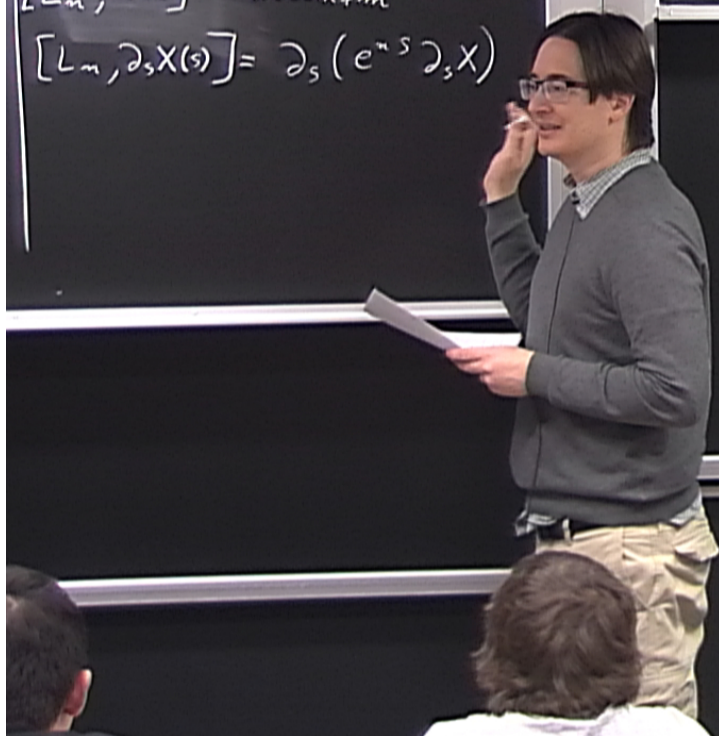
$$[X(\sigma, 0), i\partial_s X(\sigma', 0)] = \frac{2}{L} \sum_n e^{iL\sigma\omega_n} = 4\pi i \delta(\sigma - \sigma')$$

$$\partial_s X(s) \partial_{s'} X(s') + \frac{1}{L^2} \frac{e^{-}}{-}$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_{n \neq 0} n e^{2i\sigma\omega_n}$$

$$= -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$$

$$\langle 0 | T_{ss} | 0 \rangle = \lim_{s \rightarrow s'} -\frac{1}{2} \left[-\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2} + \frac{1}{(s-s')^2} \right] = -\frac{1}{24L^2}$$




$$H = \int ds (\partial_t X + \partial_s X)$$

$$\sum \alpha_n \dot{\alpha}_{-n} + \frac{\alpha_n^2}{2}$$

$$[L_n, a_m] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{ns} \partial_s X)$$

$$[L_n, \partial_s X(s) \partial_{s'} X(s')] =$$

$$= \partial_s (e^{ns} \partial_s X \partial_{s'} X(s')) + \partial_{s'} (e^{ns'} \partial_{s'} X \partial_s X(s))$$

$$[a_n, a_{-m}] = n$$

$$[X(\sigma, 0), \partial_t X(\sigma', 0)] = \frac{2}{L} \sum_n e^{iL(n\sigma - \sigma')} = 4\pi i \delta(\sigma - \sigma')$$

$$\partial_s X(s) \partial_{s'} X(s') + \frac{1}{L^2} \frac{e^{-s-s'}}{s-s'}$$

$$\langle 0 | \partial_s X(s) \partial_{s'} X(s') | 0 \rangle = -\frac{1}{L^2} \sum_{n \neq 0} n e^{2i(n\sigma - \sigma')}$$

$$= -\frac{1}{L^2} \frac{e^{\frac{1}{2}(s+s')}}{(e^{\frac{s}{2}} - e^{\frac{s'}{2}})^2}$$

$$\langle 0 | T_{ss} | 0 \rangle = \lim_{s \rightarrow s'} -\frac{1}{2} \left[-\frac{1}{L^2} \frac{e^{\frac{1}{2}(s+s')}}{(e^{\frac{s}{2}} - e^{\frac{s'}{2}})^2} + \frac{1}{(s-s')^2} \right] = -\frac{1}{24L^2}$$

$$[a_m, a_n] = m a_{n+m}$$

$$[L_n, \partial_s X(s)] = \partial_s (e^{-s} \partial_s X)$$

$$[L_n, \partial_s X(s) \partial_s X(s')] = \partial_s (e^{-s} \partial_s X \partial_s X(s')) + \partial_{s'} (e^{-s'} \partial_{s'} X(s') \partial_s X(s))$$

$$[L_n, T_2(s)] = e^{-s} (2n+2) (T(s) + \frac{1}{24}) + \frac{1}{12} n(n-1) e^{-s}$$

$$[a_n, a_{-n}] = n$$

$$[X(s, 0), \partial_s X(s', 0)] = \partial_s X(s)$$

$$J_a = i \partial_n X$$

$$J_s = i \partial_n X = \frac{p}{X} + \sum_{n \neq 0} \frac{1}{2} a_n e^{-2ns}$$

$$\partial_z J_z + \partial_z J_z = 0$$

$$\partial_z J_z = 0!$$

$$\partial_z J_z = 0$$

$$T_{33} = \frac{1}{2} J_z J_z - \frac{1}{24} = -\frac{1}{24} + \sum_{n,m} a_n a_m e^{-(n+m)s} = -\frac{1}{24} + \sum_n L_n e^{-ns}$$

$$\partial_z f(z) J_z = 0$$

$$\partial_s X(s) \partial_s X(s') \Big|_0 = -\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2}$$

$$\Rightarrow = \lim_{s \rightarrow s'} -\frac{1}{L} \left[-\frac{1}{L^2} \frac{e^{\frac{1}{L}(s+s')}}{(e^{\frac{s}{L}} - e^{\frac{s'}{L}})^2} \right]$$

