Title: Condensed Matter Review-5

Date: Jan 30, 2015 09:00 AM

URL: http://pirsa.org/15010061

Abstract:

$\gamma = |\langle\psi_1|\psi_2\rangle|$

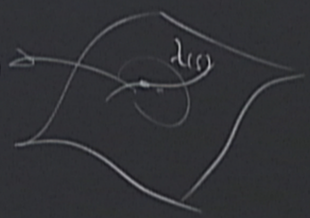
$d_{FS} = \cos^{-1}\gamma \simeq 2(1-\gamma)$

$\lambda = (\lambda^1, \dots, \lambda^n)$

$H = H(\lambda)$ smooth

$\Downarrow$

$\psi_0(\lambda)$



$\psi_1 = \otimes_i^N \psi_i^{(1)}$

$\psi_2 = \otimes_i^N \psi_i^{(2)}$

$|\langle\psi_i^{(1)}|\psi_i^{(2)}\rangle| = 1-\epsilon_i$

$\epsilon = \min_i \epsilon_i$

$|\langle\psi_1|\psi_2\rangle| \le (1-\epsilon)^N \xrightarrow{N} 0$

$|\langle\psi_0(s)\rangle|$

$$|\langle \psi_0(s) | \psi(s+ds)\rangle|$$

exercise

$$\psi(s+ds) \simeq \psi(s) + \partial_i \psi(s)\, ds_i + \frac{1}{2}\partial_i\partial_j \psi_s\, ds_i\, ds_j + \dots$$

$$\beta_i = -i\langle \psi | \partial_i \psi\rangle$$

$$\gamma_{ij} = \mathrm{Re}\langle \partial_i\psi | \partial_j \psi\rangle$$

$$d_{FS} \equiv ds^2 = \underbrace{(\gamma_{ij} - \beta_i\beta_j)}_{g_{ij}} ds_i\, ds_j$$

g_{ij} QUANTUM GEOMETRIC TENSOR

$$\mathrm{Im}\, g_{\mu\nu} = i(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \equiv F_{\mu\nu}$$

$\mathcal{F} = |\langle \psi_1 | \psi_2 \rangle|$

$d_{FS} = \cos^{-1} \mathcal{F} \simeq 2(1-\mathcal{F})$

$\lambda = (\lambda^1, \dots, \lambda^n)$

$H = H(\lambda)$ smooth

$\Downarrow$

$\psi_0(\lambda)$

$\lambda(s)$

$\psi_1 = \otimes_i^N \psi_i^{(1)}$

$\psi_2 = \otimes_i^N \psi_i^{(2)}$

$|\langle \psi_i^{(1)} | \psi_i^{(2)} \rangle| = 1 - \epsilon_i$

$\epsilon = \min_i \epsilon_i$

$|\langle \psi_1 | \psi_2 \rangle| \leq (1-\epsilon)^N \xrightarrow{N} 0$

$\lambda(s) = (\lambda^1(s), \dots, \lambda^n(s))$

$\frac{\partial}{\partial \lambda^i}$

$|\langle \psi_0(s) | \psi(s+ds) \rangle|$

$\psi(s+ds) \simeq \psi(s) + \partial_i$

$+\frac{1}{2} \partial_i \partial_j \psi_s$

$\beta_i = -i \langle \psi | \partial_i$

$g_{ij} = \mathrm{Re} \langle \partial_i \psi |$

$d_{FS} \equiv ds^2 = (g$

$$|\langle\psi_0(s)|\psi(s+ds)\rangle|$$

exercise

$$\psi(s+ds) \simeq \psi(s) + \partial_i\psi(s)\,ds_i + \frac{1}{2}\partial_i\partial_j\psi_s\,ds_i\,ds_j + \dots$$

$$\beta_i = -i\langle\psi|\partial_i\psi\rangle$$

$$\gamma_{ij} = \mathrm{Re}\langle\partial_i\psi|\partial_j\psi\rangle$$

$$d_{FS} \equiv ds^2 = \underbrace{(\gamma_{ij}-\beta_i\beta_j)}_{g_{ij}}\,ds_i\,ds_j$$

QUANTUM GEOMETRIC TENSOR

$$\mathrm{Im}\, g_{\mu\nu} = i(\partial_\mu\beta_\nu - \partial_\nu\beta_\mu) \equiv F_{\mu\nu}$$

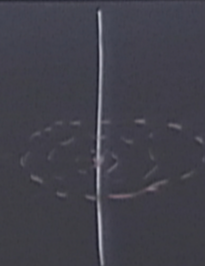
$$\mathrm{Re}\, g_{\mu\nu} = \frac{1}{2}(g_{\mu\nu}+g_{\nu\mu}) = G_{\mu\nu}$$

$$\psi_0(\lambda) \longrightarrow \psi_0(\lambda+\delta\lambda)$$

$$H(\lambda) \rightarrow H(\lambda+\delta\lambda) \simeq H(\lambda) + \delta H(\lambda)$$

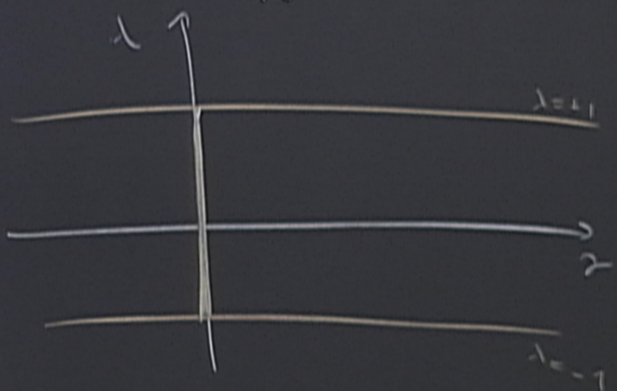
$$|\psi_0(\lambda+\delta\lambda)\rangle \simeq |\psi_0(\lambda)\rangle + \sum_{n\neq 0} |\psi_n(\lambda)\rangle \frac{\langle\psi_n(\lambda)|\delta H|\psi_0(\lambda)\rangle}{\Delta_{0n}}$$

$$G_{\mu\nu} = \mathrm{Re}\left[\sum_{n\neq 0} \frac{\langle\psi_0|\partial_\mu H|\psi_n\rangle\langle\psi_n|\partial_\nu H|\psi_0\rangle}{\Delta_{0n}^2}\right]$$



$$H(\gamma,\lambda) = -\sum_{i=-M}^{+M}\left(\frac{1+\gamma}{2}\hat{\sigma}_i^x\hat{\sigma}_{i+1}^x + \frac{1-\gamma}{2}\hat{\sigma}_i^y\hat{\sigma}_{i+1}^y - \lambda\,\hat{\sigma}_i^z\right)$$

$N = 2M+1$



Jordan-Wigner
Fourier Tr.
Bogoliubov Tr. $d_k \to b_k$

$$\to H(\gamma,\lambda) = \sum_{k=-M}^{M} \Lambda_k \left(b_k^\dagger b_k - 1\right)$$

$$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2\frac{2\pi k}{N}} \qquad \cos\theta_k := \frac{\epsilon_k}{\Lambda_k}$$

$$\epsilon_k = \cos\frac{2\pi k}{N} - \lambda$$

$$\epsilon_k = \cos\frac{2\pi k}{N} - \lambda$$

$$G_{\mu\nu} \qquad |\phi\rangle = |\phi^1, \ldots, \phi^n\rangle \qquad \delta H = \sum_j \delta V_j$$

$$\|G_{\mu\nu}\| = \langle \phi | G_{\mu\nu} | \phi \rangle \lesssim \epsilon_1^{-2} L^d K$$

$$\langle \partial_\mu H | \phi \rangle = \sum_\mu \partial_\mu H \, \phi^\mu \equiv \delta H \qquad \sum_\mu \partial_\mu H \cdot \phi^\mu$$

$$|G_{\mu\nu}| \leq \sum_{n>0} \Delta_{n0}^{-2} \left| \langle \psi_0 | \delta H | \psi_n \rangle \right|^2$$

$$\leq \epsilon_{10}^{-2} \sum_{n>0} \left| \langle \psi_0 | \delta H | \psi_n \rangle \right|^2 \xrightarrow{\text{Res. Id}} \epsilon_{01}^{-2} \left\{ \langle \delta H \delta H^\dagger \rangle - K\delta \right.$$

$$|\phi\rangle = |\phi^1, \dots, \phi^n\rangle \qquad \delta H = \sum_j \delta V_j$$

$$= \langle \phi | G_{\mu\nu} | \phi \rangle \lesssim \epsilon_1^{-2} L^d K$$

$$|\phi\rangle = \sum_\mu \partial_\mu H \, \phi^\mu \equiv \delta H \qquad \sum_\mu \partial_\mu H \cdot \phi^\mu$$

$$\leq \sum_{n>0} \Delta_{n0}^{-2} |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

$$\leq \epsilon_{n0}^{-2} \sum_{n>0} |\langle \psi_0 | \delta H | \psi_n \rangle|^2 \xrightarrow{\text{Res. Id}} \epsilon_{01}^{-2} \left\{ \langle \delta H \, \delta H^\dagger \rangle - |\langle \delta H \rangle|^2 \right\} = \epsilon_0^{-2}$$

g_{ij} QUANTUM GEOMETRIC TENSOR

$$G_{\mu\nu} = \mathrm{Re}\left[\sum_{n\neq 0} \frac{\quad}{\Delta_{0n}^2}\right]$$

$$|\Psi_0(\gamma,\lambda)\rangle = \bigotimes_{k=1}^{M}\left[\cos\frac{\theta_k}{2}|0\rangle_k|0\rangle_{-k} - i\sin\frac{\theta_k}{2}|1\rangle_k|1\rangle_{-k}\right]$$

$$G_{\mu\nu} = \frac{1}{4}\sum_{k=1}^{M}\frac{\partial\theta_k}{\partial\lambda^\mu}\frac{\partial\theta_k}{\partial\lambda^\nu} \xrightarrow{M\to\infty}$$

$$\sum_k^M \to \frac{L}{2\pi}\int_0^\pi dx$$

$$|\lambda|<1 \qquad G_{\mu\nu} = \frac{L}{16|\gamma|}\begin{pmatrix}\frac{1}{1-\lambda^2} & 0\\ 0 & \frac{1}{(1+|\gamma|)^2}\end{pmatrix}$$

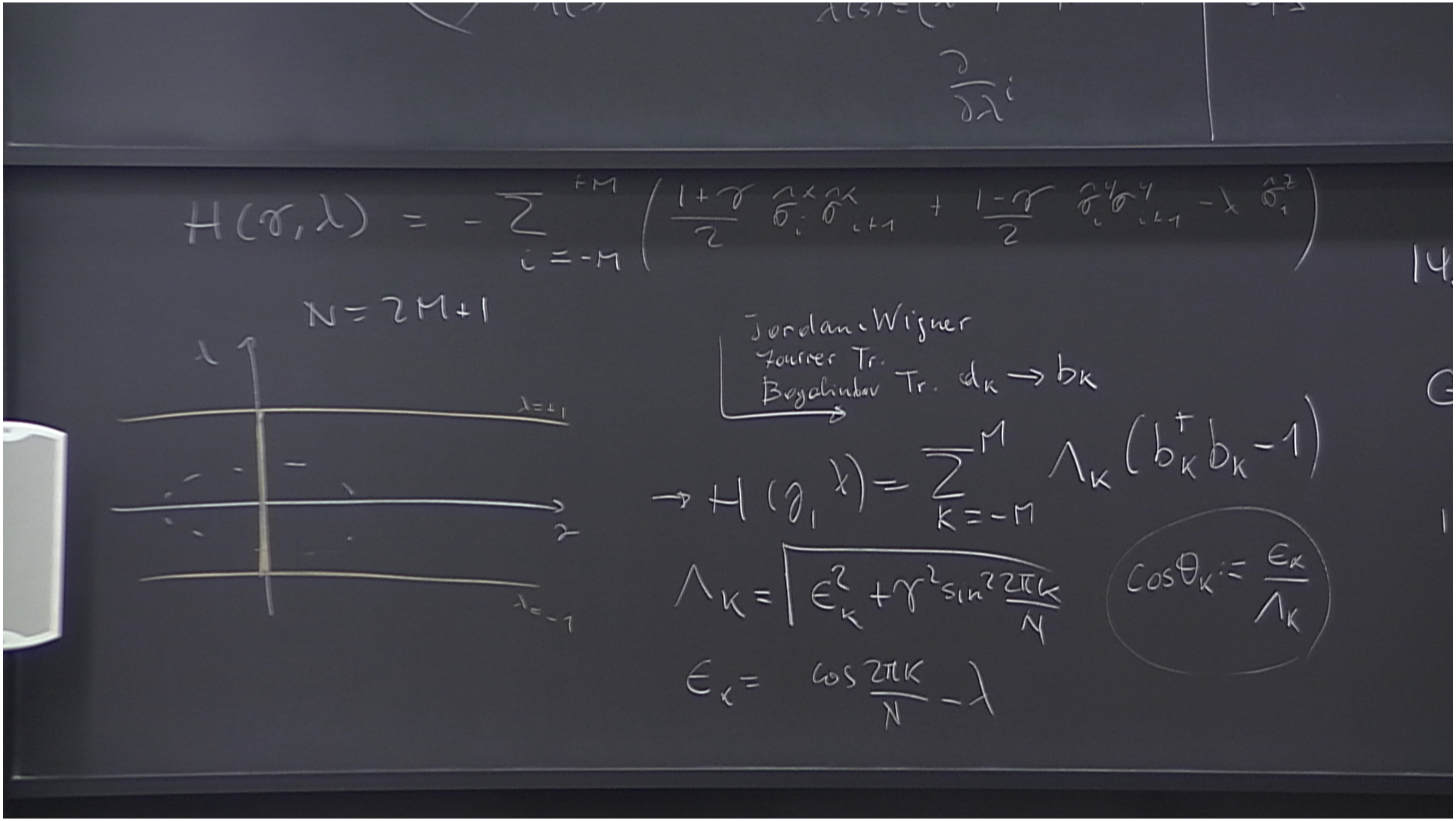
$$R = R^\alpha_\alpha = -\frac{16}{L}\frac{1+|\gamma|}{|\gamma|} \qquad \frac{16}{L}\frac{|\lambda|+\sqrt{\lambda^2+\gamma^2-1}}{\sqrt{\lambda^2+\gamma^2-1}}$$

$|\lambda|<1$ $\qquad$ $|\lambda|>1$

Side exercise

$\phi \qquad R(\phi) = \prod_{l=-M}^{M} e^{i\frac{\sigma_l^z}{2}\phi}$

$$H(\lambda,\gamma) \to \underbrace{R(\phi)\,H\,R^\dagger(\phi)}_{H(\phi,\lambda,\gamma)}$$


$H(\gamma,\lambda) = -\sum_{i=-M}^{+M} \left(\frac{1+\gamma}{2} \hat\sigma^x_i \hat\sigma^x_{i+1} + \frac{1-\gamma}{2} \hat\sigma^y_i \hat\sigma^y_{i+1} - \lambda \hat\sigma^z_i \right)$
$N = 2M+1$
$\lambda = +1$
$\lambda = -1$
Jordan-Wigner
Fourier Tr.
Bogoliubov Tr. $d_k \to b_k$
$\to H(\gamma, \lambda) = \sum_{k=-M}^{M} \Lambda_k (b_k^\dagger b_k - 1)$
$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{N}}$
$\cos\theta_k := \frac{\epsilon_k}{\Lambda_k}$
$\epsilon_k = \cos \frac{2\pi k}{N} - \lambda$