

Title: Condensed Matter Review-5

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URL: <http://pirsa.org/15010061>

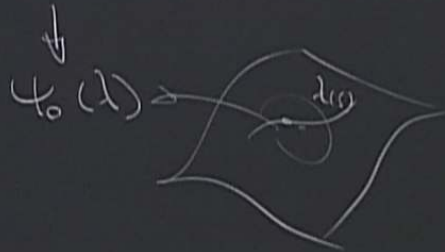
Abstract:

$$\gamma = |\langle \psi_1 | \psi_2 \rangle|$$

$$d_{FS} = \cos^{-1} \gamma \approx 2(1-\gamma)$$

$$\lambda = (\lambda^1, \dots, \lambda^N)$$

$$H = H(\lambda) \text{ smooth}$$



$$\psi_1 = \otimes_i^N \psi_i^{(1)}$$

$$\psi_2 = \otimes_i^N \psi_i^{(2)}$$

$$|\langle \psi_i^{(1)} | \psi_i^{(2)} \rangle| = 1 - \epsilon_i$$

$$\epsilon = \min_i \epsilon_i$$

$$|\langle \psi_1 | \psi_2 \rangle| \leq (1 - \epsilon)^N \xrightarrow{N} 0$$

$$|\langle \psi_0 | \psi \rangle|$$

$$|\langle \psi_0(s) | \psi(s+ds) \rangle|$$

$$\psi(s+ds) \simeq \psi(s) + \partial_i \psi(s) ds_i + \frac{1}{2} \partial_i \partial_j \psi(s) ds_i ds_j + \dots$$

exercise

$$\beta_i = -i \langle \psi | \partial_i \psi \rangle$$

$$\gamma_{ij} = \text{Re} \langle \partial_i \psi | \partial_j \psi \rangle$$

$$d_{FS} \equiv ds^2 = (\gamma_{ij} - \beta_i \beta_j) ds_i ds_j$$

QUANTUM
GEOMETRIC TENSOR

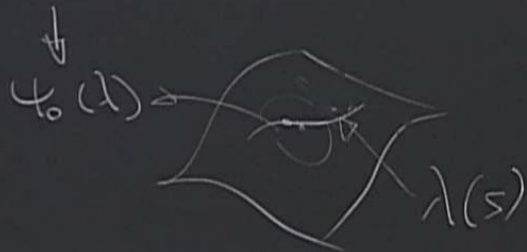
$$\text{Im } g_{\mu\nu} = i(\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \equiv F_{\mu\nu}$$

$$\gamma = |\langle \psi_1 | \psi_2 \rangle|$$

$$d_{FS} = \cos^{-1} \gamma \approx \sqrt{2(1-\gamma)}$$

$$\lambda = (\lambda^1, \dots, \lambda^N)$$

$$H = H(\lambda) \text{ smooth}$$



$$\psi_1 = \bigotimes_i^N \psi_i^{(1)}$$

$$\psi_2 = \bigotimes_i^N \psi_i^{(2)}$$

$$|\langle \psi_i^{(1)} | \psi_i^{(2)} \rangle| = 1 - \epsilon_i$$

$$\epsilon = \min_i \epsilon_i$$

$$|\langle \psi_1 | \psi_2 \rangle| \leq (1 - \epsilon)^N \xrightarrow{N} 0$$

$$\lambda(s) = (\lambda^1(s), \dots, \lambda^N(s))$$

$$\frac{\partial}{\partial \lambda^i}$$

$$|\langle \psi_0(s) | \psi(s+ds) \rangle|$$

$$\psi(s+ds) \approx \psi(s) + \partial_i \psi + \frac{1}{2} \partial_i \partial_j \psi ds^2$$

$$\beta_i = -i \langle \psi | \partial_i \psi \rangle$$

$$\gamma_{ij} = \text{Re} \langle \partial_i \psi | \partial_j \psi \rangle$$

$$d_{FS} \equiv ds^2 = (\gamma_{ij})$$

$$|\langle \psi_0(s) | \psi(s+ds) \rangle|$$

$$\psi(s+ds) \simeq \psi(s) + \partial_i \psi(s) ds^i + \frac{1}{2} \partial_i \partial_j \psi(s) ds^i ds^j + \dots$$

exercise

$$\beta_i = -i \langle \psi | \partial_i \psi \rangle$$

$$\alpha_{ij} = \text{Re} \langle \partial_i \psi | \partial_j \psi \rangle$$

$$d_{FS} \equiv ds^2 = (\alpha_{ij} - \beta_i \beta_j) ds^i ds^j$$

QUANTUM
GEOMETRIC TENSOR
 g_{ij}

$$\text{Im } g_{\mu\nu} = i (\partial_\mu \beta_\nu - \partial_\nu \beta_\mu) \equiv F_{\mu\nu}$$

$$\text{Re } g_{\mu\nu} = \frac{1}{2} (g_{\mu\nu} + g_{\nu\mu}) = G_{\mu\nu}$$

$$\psi_0(\lambda) \rightarrow \psi_0(\lambda + s\lambda)$$

$$H(\lambda) \rightarrow H(\lambda + s\lambda) \simeq H(\lambda) + sH'(\lambda)$$

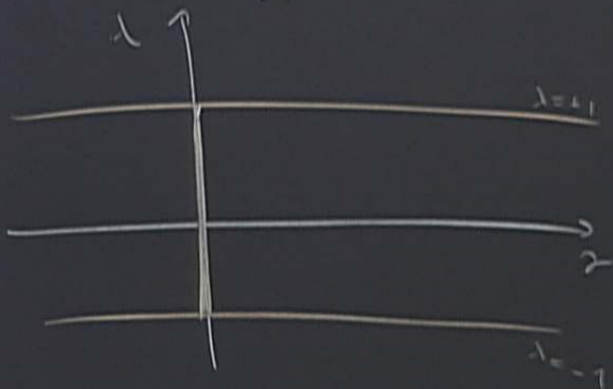
$$|\psi_0(\lambda + s\lambda)\rangle \simeq |\psi_0(\lambda)\rangle + \sum_{n \neq 0} |\psi_n(\lambda)\rangle \frac{\langle \psi_n(\lambda) | sH | \psi_0(\lambda) \rangle}{\Delta_{0n}}$$

$$G_{\mu\nu} = \text{Re} \left[\frac{\sum_{n \neq 0} \langle \psi_0 | \partial_\mu H | \psi_n \rangle \langle \psi_n | \partial_\nu H | \psi_0 \rangle}{\Delta_{0n}^2} \right]$$



$$H(\gamma, \lambda) = - \sum_{i=-M}^{+M} \left(\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y - \lambda \sigma_i^z \right)$$

$$N = 2M + 1$$



Jordan-Wigner
Fourier Tr.
Bogoliubov Tr. $d_k \rightarrow b_k$

$$\rightarrow H(\gamma, \lambda) = \sum_{k=-M}^{+M} \Lambda_k (b_k^\dagger b_k - 1)$$

$$\Lambda_k = \sqrt{\epsilon_k^2 + \gamma^2 \sin^2 \frac{2\pi k}{N}} \quad \cos \theta_k := \frac{\epsilon_k}{\Lambda_k}$$

$$\epsilon_k = \cos \frac{2\pi k}{N} - \lambda$$

$$G_{\mu\nu} \quad |\phi\rangle = |\phi^1, \dots, \phi^n\rangle$$

$$\delta H = \bar{z}_j \delta V$$

$$\|G_{\mu\nu}\| = \langle \phi | G_{\mu\nu} | \phi \rangle \lesssim \epsilon_1^{-2} L^d K$$

$$\langle \partial_\mu H | \phi \rangle = \sum_n \partial_\mu H \phi^n \equiv \delta H$$

$$\bar{z}_\mu \partial_\mu H \cdot \phi^n$$

$$\begin{aligned} \|G_{\mu\nu}\| &\leq \sum_{n>0} \Delta_{n_0}^{-2} |\langle \psi_0 | \delta H | \psi_n \rangle|^2 \\ &\leq \epsilon_{n_0}^{-2} \sum_{n>0} |\langle \psi_0 | \delta H | \psi_n \rangle|^2 \end{aligned}$$

$$\xrightarrow{\text{Res. Id}} \epsilon_{n_0}^{-2} \left\{ \langle \delta H \delta H^\dagger \rangle - K \delta \right\}$$

$$|\phi\rangle = |\phi^1, \dots, \phi^n\rangle$$

$$\delta H = \bar{z}_j \delta v_j$$

$$= \langle \phi | G_{mn} | \phi \rangle \lesssim \epsilon_1^{-2} L^d K$$

$$|\phi\rangle = \sum_n \partial_{\mu H} \phi^n \equiv \delta H$$

$$\bar{z}_\mu \partial_{\mu H} \cdot \phi^n$$

$$\leq \sum_{n>0} \Delta_{n_0}^{-2} |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

$$\xrightarrow{\text{Res. Id}} \epsilon_{n_1}^{-2} \left\{ \langle \delta H \delta H^\dagger \rangle - |\langle \delta H \rangle|^2 \right\} = \epsilon_{n_1}^{-2}$$

$$\leq \epsilon_{n_0}^{-2} \sum_{n>0} |\langle \psi_0 | \delta H | \psi_n \rangle|^2$$

QUANTUM
 δ_{ij} GEOMETRIC TENSOR

$$G_{\mu\nu} = \text{Re} \left[\begin{matrix} \leftarrow n \neq 0 \end{matrix} \right]$$

$$\Delta_{on}^2$$

$$|\psi_0(\gamma, \lambda)\rangle = \bigotimes_{k=1}^n \left[\cos \frac{\theta_k}{2} |0\rangle_k |0\rangle_k - i \sin \frac{\theta_k}{2} |1\rangle_k |1\rangle_k \right]$$

$$G_{\mu\nu} = \frac{1}{4} \sum_{k=1}^n \frac{\partial \theta_k}{\partial \lambda^\mu} \frac{\partial \theta_k}{\partial \lambda^\nu} \xrightarrow{n \rightarrow \infty} \frac{-n}{2} \rightarrow \frac{L}{2\pi} \int_0^\pi d\alpha$$

$$|\lambda| < 1 \quad G_{\mu\nu} = \frac{L}{16|\sigma|} \begin{pmatrix} \frac{1}{1-\lambda^2} & 0 \\ 0 & \frac{1}{1+\lambda^2 R^2} \end{pmatrix}$$

$$R = R_a^a = \begin{matrix} -\frac{16}{L} \frac{1+|\sigma|}{|\sigma|} & \frac{16}{L} \frac{1+\sqrt{\lambda^2 + \sigma^2 - 1}}{\sqrt{\lambda^2 + \sigma^2 - 1}} \end{matrix}$$

$|\lambda| < 1$ $|\lambda| > 1$

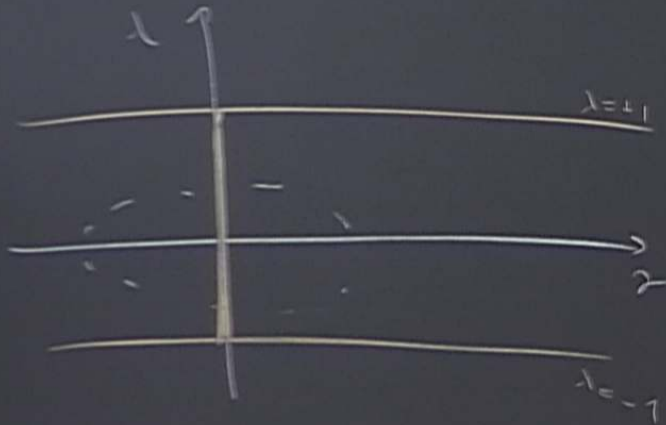
Side exercise

$$\phi \quad R(\phi) = \prod_{i=1}^n e^{i \frac{\sigma_i}{2} \phi}$$

$$H(\alpha, \gamma) \rightarrow \underbrace{R(\phi) H R^\dagger(\phi)}_{H(\phi, \lambda, \gamma)}$$

$$H(\gamma, \lambda) = - \sum_{i=-M}^{+M} \left(\frac{1+\gamma}{2} \sigma_i^x \sigma_{i+1}^x + \frac{1-\gamma}{2} \sigma_i^y \sigma_{i+1}^y - \lambda \sigma_i^z \right)$$

$$N = 2M + 1$$



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$$\cos \theta_k := \frac{\epsilon_k}{\Lambda_k}$$