

Title: Condensed Matter Review-3

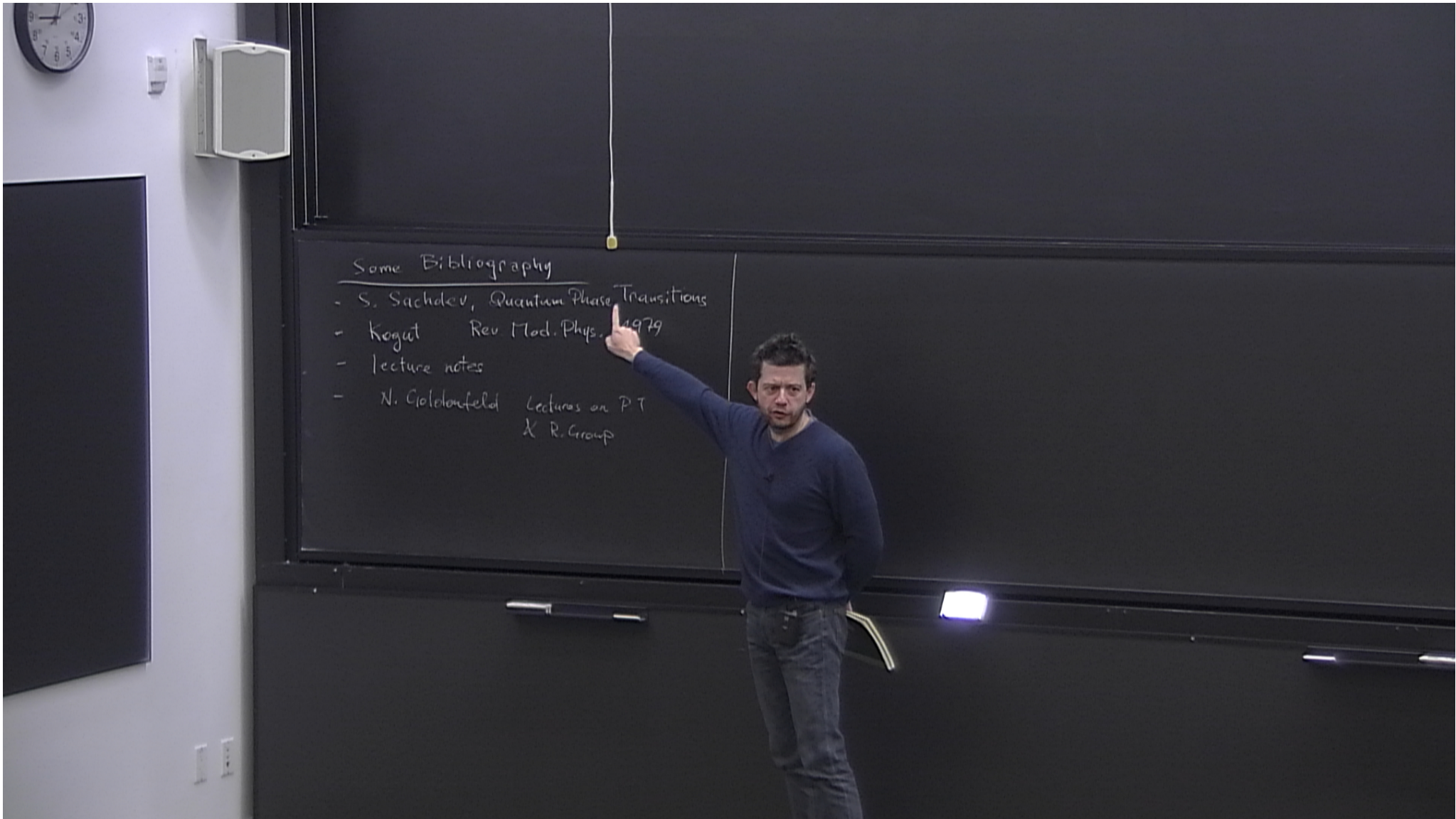
Date: Jan 28, 2015 09:00 AM

URL: <http://pirsa.org/15010059>

Abstract:

Some Bibliography

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev. Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on P.T
& R. Group



Some Bibliography

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev. Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on F & R.C.

Ferromagnet Sym. Break.	→	Paramagnet Preserves symm.
$g_c = 1$		g

$$[H(g), T_{Z_2}] = 0$$

small g $T_{Z_2} \psi_1^0 = \psi_2^0$

favorite
(sym. break.
basis)

Some Bibliography

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev. Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on P.T & R. Group

Ferromagnet Spn. Break.	+	Paramagnet Passover space.
	$g_c = 1$	g

$[H(g), T_{Z_2}] = 0$

small g $T_{Z_2} \psi_2^0 = \psi_2^0$ favorite
(spin-broken basis)

$$H(g) = -Jg \sum_i \hat{\sigma}_i^x - J \sum_i \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

$g \rightarrow \psi_0(g)$



Some Bibliography

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev. Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on P.T & R. Group

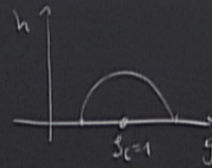
Ferromagnet
Spn. Break. $g_c=1$ g Paramagnet
Preserves spin.

$$[H(g), T_{Z_2}] = 0$$

small g $T_{Z_2} \psi_1^0 = \psi_2^0$ favorite
(spin broken
basis)

$$H(g) = -Jg \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z + h \sum_i \sigma_i^z$$

$$g \rightarrow \psi(g, h)$$



Ferromagnet
Sym. Break.
 $g_c = 1$
 Paramagnet
 Preserves symm.
 g

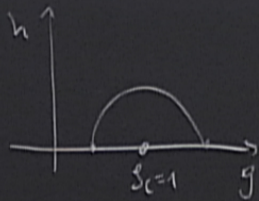
$$[H(g), T_{z_2}] = 0$$

small g $T_{z_2} \psi_1^0 = \psi_2^0$

favorite
(sym. break.
basis)

$$H(g) = -Jg \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z$$

$$g \rightarrow \psi_0(g, \mu)$$



Sym ¹	Sym ²	Sym ³
		Sym ⁿ

H_1
 ψ_{-1}^0
 $H(g)$
 H_2



positions

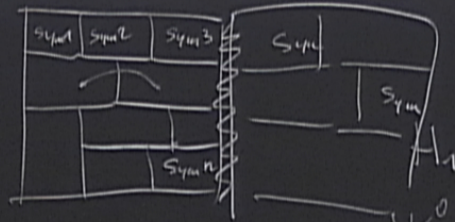
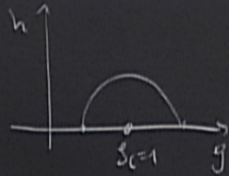
Ferromagnet
Sym Break: $g_c = 1$
Paramagnet
Preserves symm. g

$$[H(g), T_{2z}] = 0$$

small g T_{2z} ψ_2^0

$$H(g) = -J$$

$$g \rightarrow \psi_0$$



favorite
(Sym. break
basis)

$$\psi = \sum_{n=0}^{\infty} \frac{1}{\sqrt{n!}} \frac{(-1)^n}{\sigma^{n+1}} \psi_{n+1}$$

$$\psi_2^0 = T e^{-i \int_0^1 H(\lambda) d\lambda} \psi_1^0$$

H_2
 ψ_2^0

$$\lambda = 0 \quad H(0) = H_1$$

$$\lambda = 1 \quad H(1) = H_2$$

$H(\lambda)$

X R. Group

$$H(\psi) = -J \sum_i \sigma_i^x - J \sum_i \sigma_i^z \sigma_{i+1}^z - h \sum_i \sigma_i^z$$

$\psi \rightarrow \psi_0(\psi, h)$

Classical \leftrightarrow Quantum mapping

Classical Model 1D Ising Model

$$H = -h \sum_i S_i$$

$$S_i = \pm 1$$

$$\sigma_i^z \rightarrow \pm 1$$

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on P.T & R. Group

$[H(g), T_z] = 0$
 small g $T_z \psi_2^0 = \psi_2^0$ favorite (sym. breaks basis)
 $H(g) = -J \sum \sigma_i^x - J \sum \sigma_i^z \sigma_{i+1}^z$
 $g \rightarrow \psi_0(g, h)$

Classical \leftrightarrow Quantum mapping $\begin{cases} \lambda = \beta h \\ K = \beta J \end{cases}$

Classical Model 1D Ising Model

$$H = -h \sum_i s_i - J \sum_i s_i s_{i+1}$$

$$s_i = \pm 1 \quad Z_1 = \text{Tr} e^{-\beta H} = \sum_{\{s_i\}} e^{-\beta H}$$

$$\beta h \rightarrow \pm 1$$

Some Bibliography

- S. Sachdev, Quantum Phase Transitions
- Kogut Rev Mod. Phys. 1979
- lecture notes
- N. Goldenfeld Lectures on P.T & R. Group

Transitions

Ferromagnet
Spin-Bose
 $g_c = 1$

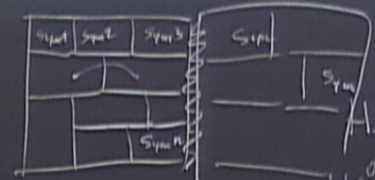
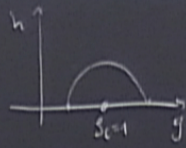
Paramagnet
Passes remain
 g

$$[H(g), T_z] = 0$$

small g $T_z \psi_2^0 = \psi_2^0$ favorite (spin-boson basis)

$$H(g) = -Jg \sum \sigma_i^x - J \sum \sigma_i^z \sigma_{i+1}^z - h \sum \sigma_i^z$$

$$g \rightarrow \psi_0(g, \omega)$$



$$\psi_{-1}^0 = T e^{-i \int_0^1 H(\lambda) d\lambda} \psi_1^0$$

Classical \leftrightarrow Quantum mapping $\left\{ \begin{array}{l} \lambda = \beta h \\ k = \beta J \end{array} \right.$

Classical Model 1D Ising Model

$$H = -h \sum_i s_i - J \sum_i s_i s_{i+1}$$

$$s_i = \pm 1$$

$$\beta h \rightarrow \pm 1$$

$$Z_1 = \text{Tr} e^{-\beta H} = \sum_{\{s_i\}} \prod_{i=1}^N e^{\frac{\lambda}{2}(s_i + s_{i+1}) + K s_i s_{i+1}}$$

$$= \left[e^{\frac{\lambda}{2}(s_1 + s_2) + K s_1 s_2} \right] \times \dots \times \left[e^{\frac{\lambda}{2}(s_N + s_1) + K s_N s_1} \right]$$

- Regular
- lecture notes
- N. Goldenfeld Lectures on P.T X R. Group

$[H(g), T_{Z_2}] = 0$

small g $T_{Z_2} \psi_2^0 = \psi_2^0$ favorite (super-extended basis)

$H(g) = -Jg \sum \sigma_i^x - J \sum \sigma_i^z \sigma_{i+1}^z$

$g \rightarrow \psi_0(g, \mu)$

ψ_1^0 ψ_2^0

$\psi_i^0 = T e^{-i \int_0^1 H(\lambda) d\lambda} \psi_1^0$

$\lambda=0$ $H(0) =$

$\lambda=1$ $H(1) =$

$H(g)$

Classical \leftrightarrow Quantum mapping $\begin{cases} \lambda = \beta h \\ k = \beta J \end{cases}$

Classical Model 1D Ising Model

$H = -h \sum_i s_i - J \sum_i s_i s_{i+1}$

$s_i = \pm 1$

$\sigma_i^z \rightarrow s_i$

$Z = \text{Tr} e^{-\beta H} = \sum_{\{s_i\}} \prod_{i=1}^N$

$\left[e^{\frac{\lambda}{2}(s_1+s_2) + K s_1 s_2} \right] \times \dots \times \left[e^{\frac{\lambda}{2}(s_N+s_{N+1}) + K s_N s_{N+1}} \right] = \text{Tr} T^N$

$T_{s_i s_{i+1}} = e^{\frac{\lambda}{2}(s_i+s_{i+1}) + K s_i s_{i+1}} = \begin{pmatrix} e^{\frac{\lambda+K}{2}} & e^{-K} \\ e^{-K} & e^{\frac{-\lambda+K}{2}} \end{pmatrix}$ Transfer Matrix

$$[H(s), T_2] = 0$$

small g $T_2 \psi_2^0 = \psi_2^0$

favorite
(super-branch
basis)

$$H(s) = -Jg \sum \hat{\sigma}_i^x - J \sum \hat{\sigma}_i^z \hat{\sigma}_{i+1}^z$$

$$g \rightarrow \psi_0(s, \omega)$$



$$\psi_1^0 = T e^{-i \int_0^1 H(\lambda) d\lambda} \psi_0^0$$

$$\psi_2^0$$

$$\lambda=0 \quad H(0) = H_1$$

$$\lambda=1 \quad H(1) = H_2$$

$$H(s)$$

$$\begin{cases} \lambda = \beta h \\ k = \beta J \end{cases}$$

$$\beta H = \sum_{\{s_i\}} \left[e^{\frac{\lambda}{2}(s_1+s_2) + K s_1 s_2} \right] \times \left[e^{\frac{\lambda}{2}(s_2+s_3) + K s_2 s_3} \right] \dots \times \left[e^{\frac{\lambda}{2}(s_{N-1}+s_N) + K s_{N-1} s_N} \right] = \text{Tr } T^N = \epsilon_1^N + \epsilon_2^N$$

$$T_{s_i, s_{i+1}} = e^{\frac{\lambda}{2}(s_i + s_{i+1}) + K s_i s_{i+1}} = \begin{pmatrix} e^{\frac{\lambda+K}{2}} & e^{-K} \\ e^{-K} & e^{\frac{\lambda-K}{2}} \end{pmatrix}$$

Transfer Matrix

$$\epsilon_{1,2} = e^{K \cosh \lambda} \pm \sqrt{e^{2K \sinh^2 \lambda} + e^{-2K}}$$

$\epsilon_1 > \epsilon_2$

$$= \epsilon_1^N \left(1 + \left(\frac{\epsilon_2}{\epsilon_1} \right)^N \right) \xrightarrow{N \rightarrow \infty} \epsilon_1^N$$

$$C(l-l') := \langle S_l S_{l'} \rangle = \frac{1}{Z} \sum_{\{s_l\}} e^{-H} S_l S_{l'}$$

$$= \frac{1}{Z} \text{Tr} \left[T^{N-l'} \frac{1}{\sigma_l} T^{l'-l} \frac{1}{\sigma_{l'}} T^l \right]$$

$$\lambda=0 \quad T = \begin{pmatrix} e^k & e^{-k} \\ e^{-k} & e^k \end{pmatrix} \quad \begin{aligned} E_1 &= 2 \cosh k \\ E_2 &= 2 \sinh k \end{aligned}$$

$$C(l-l') = \frac{E_1^{N-l'-l} E_2^{l'-l} + E_2^{N-l'+l} E_1^{l'-l}}{E_1^N + E_2^N}$$

$$\frac{\epsilon_{1,2} - \epsilon \cosh k}{\epsilon_1 > \epsilon_2}$$

$$= \epsilon_1^N \left(1 + \left(\frac{\epsilon_2}{\epsilon_1} \right)^N \right) \xrightarrow{N \rightarrow \infty} \epsilon_1^N$$

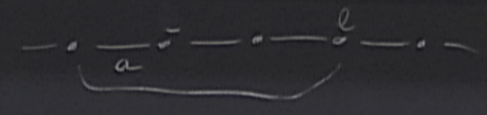
$$= \frac{1}{2} \sum_{\{s_l\}} e^{-H \sum s_l s_{l+1}}$$

$$\left[\frac{\partial}{\partial \epsilon} T^{l'-l} \frac{\partial}{\partial \epsilon'} T^l \right]$$

$$a = 2 \cosh k$$

$$l = 2 \sinh k$$

$$\frac{\epsilon_2^{l'-l} + \epsilon_2^{N-l'+l} \epsilon_1^{l'-l}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N \rightarrow \infty}$$



For large k ,

$$\tau = al$$

$$\left(\tanh k \right)^{l'-l} = e^{-|l|/\xi}$$

$$l=0 \quad \xi^{-1} = \frac{1}{a} \log \coth k$$

$$\epsilon_{1,2} = 2 \cos \dots$$

$$\epsilon_1 > \epsilon_2$$

$$= \epsilon_1^N \left(1 + \left(\frac{\epsilon_2}{\epsilon_1} \right)^N \right) \xrightarrow{N \rightarrow \infty} \epsilon_1^N$$

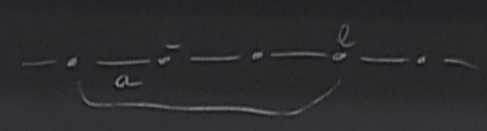
$$= \frac{1}{2} \sum_{\{s_l\}} e^{-H[s_l]} \dots$$

$$\left[\frac{1}{\sigma_l} T^{l'-l} \dots \right]$$

$$a = 2 \cosh k$$

$$z = 2 \sinh k$$

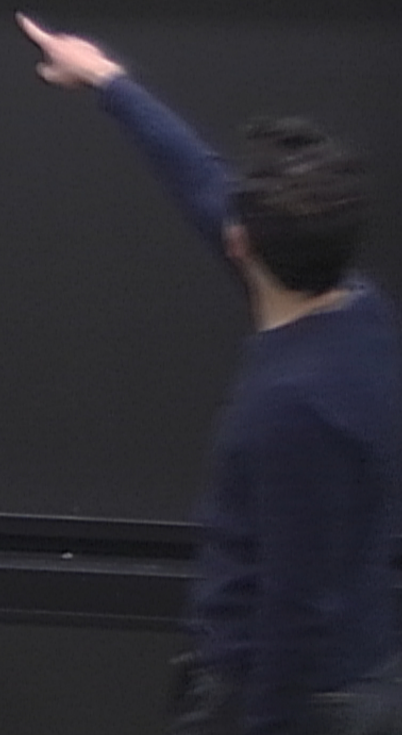
$$\frac{\epsilon_2^{l'-l} + \epsilon_2^{N-l'+l} \epsilon_1^{l'-l}}{\epsilon_1^N + \epsilon_2^N} \xrightarrow{N \rightarrow \infty} (\tanh k)^{l'-l} = e^{-|l|/3}$$



$$\tau = a l$$

For large k , $z \sim 2e^{-2k}$

$$z^{-1} = \frac{1}{a} \log \coth k$$



$$T_{S_i, S_{i+2}} = e^{\lambda/2 (S_i + S_{i+2}) + K S_i S_{i+2}} = \begin{pmatrix} e^{-K} & e^{-\lambda/2} \\ e^{-\lambda/2} & e^{-K} \end{pmatrix}$$

Transfer Matrix

$$E_{1,2} = e^K \cosh \lambda \pm \left| e^{2K} \sinh^2 \lambda \right|^{1/2} e^{-K}$$

$E_1 > E_2$

$$= E_1^N \left(1 + \left(\frac{E_2}{E_1} \right)^N \right) \xrightarrow{N \rightarrow \infty} E_1^N$$



For large K ,
 $\frac{a}{\xi} \sim 2e^{-2K}$

$$Z = \text{Tr} T^N = \text{Tr} (T_1 T_2)^N$$

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix}$$

$$T_2 = \begin{pmatrix} e^\lambda & \\ & e^{-\lambda} \end{pmatrix}$$

$$T = T_1^{1/2} T_2 T_1^{1/2}$$

$$\tau = a l$$

$$\tanh \left(\frac{\tau}{a} \right) = e^{-|\tau|/a}$$

$$\xi^{-1} = \frac{1}{a} \log \coth K$$

$$T_{S_i, S_{i+1}} = e^{\lambda/2 (S_i + S_{i+1}) + K S_i S_{i+1}} = \begin{pmatrix} e^{-K} & e^{\lambda/2} \\ e^{\lambda/2} & e^K \end{pmatrix}$$

Transfer Matrix

$$E_{1,2} = e^K \cosh \lambda \pm \sqrt{e^{2K} \sinh^2 \lambda + e^{-2K}}$$

$E_1 > E_2$

$$= E_1^N \left(1 + \left(\frac{E_2}{E_1} \right)^N \right) \xrightarrow{N \rightarrow \infty} E_1^N$$



$$\tau = al$$

$$\tanh \left(\frac{\tau}{2} \right) = e^{-\tau/2}$$

$$\frac{\tau}{2} = \frac{1}{a} \log \coth k$$

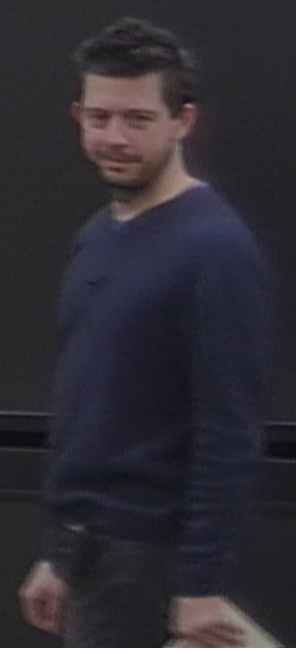
For large K ,
 $\frac{a}{\xi} \sim 2e^{-2K}$

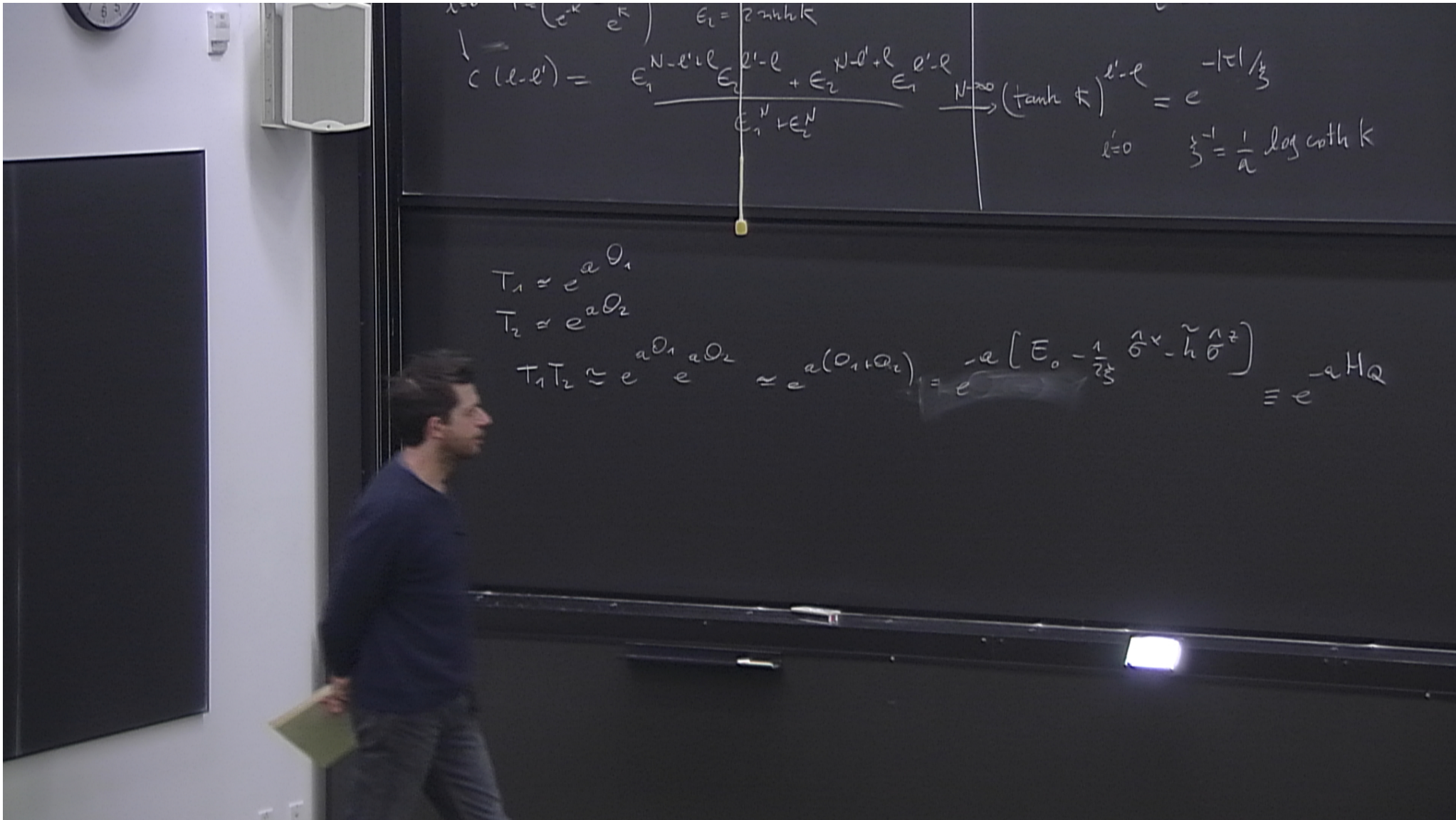
$$Z = \text{Tr} T^N = \text{Tr} (T_1 T_2)^N$$

$$T_1 = \begin{pmatrix} e^K & e^{-K} \\ e^{-K} & e^K \end{pmatrix} = e^K \begin{pmatrix} 1 & e^{-2K} \\ e^{-2K} & 1 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} e^{\lambda/2} & \\ & e^{\lambda/2} \end{pmatrix} = e^{\lambda/2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$= e^{\lambda/2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \approx \exp \left[\frac{\lambda}{2} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} \right]$$





$$C(l-l') = \frac{E_1^{N-l-l'} E_2^{l-l'} + E_2^{N-l'+l} E_1^{l-l'}}{E_1^N + E_2^N} \xrightarrow{N \rightarrow \infty} (\tanh k)^{l-l'} = e^{-|l-l'|/a}$$

$$E_i = \pm \cosh k$$

$$\frac{1}{3} = \frac{1}{a} \log \coth k$$

$$T_1 \approx e^{aQ_1}$$

$$T_2 \approx e^{aQ_2}$$

$$T_1 T_2 \approx e^{aQ_1} e^{aQ_2} \approx e^{a(Q_1 + Q_2)} = e^{-a \left(E_0 - \frac{1}{2\epsilon} \tilde{\sigma}^x - \tilde{h} \tilde{\sigma}^z \right)} = e^{-aHQ}$$

$$Z = \text{Tr} [T_1 T_2]^N = \text{Tr} e^{-H_Q L} \equiv \text{Tr} e^{-\frac{H_Q}{T} L} \quad T = \frac{1}{L}$$