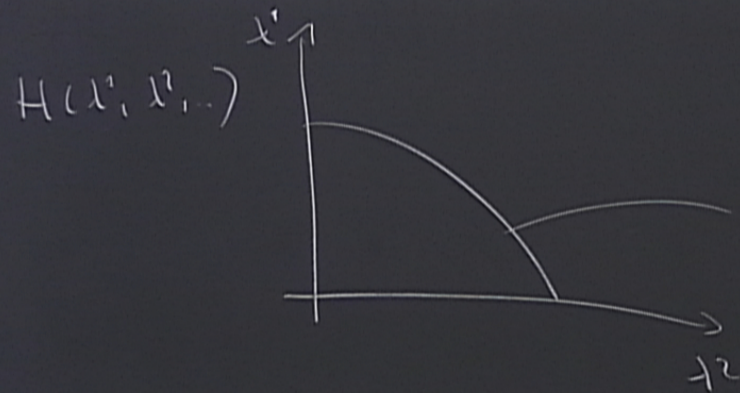


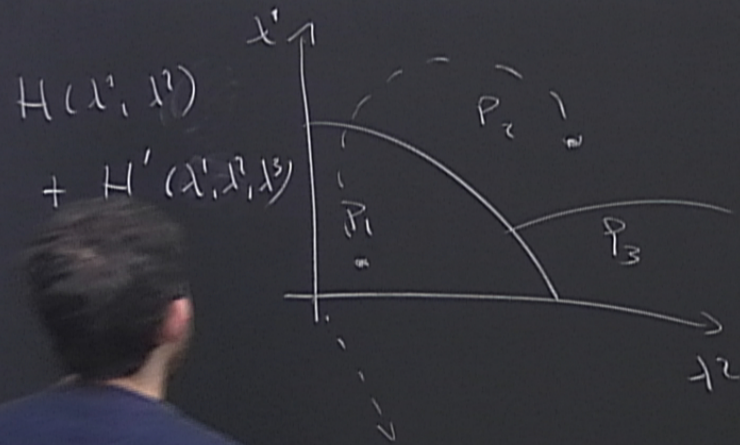
Title: Condensed Matter Review-2

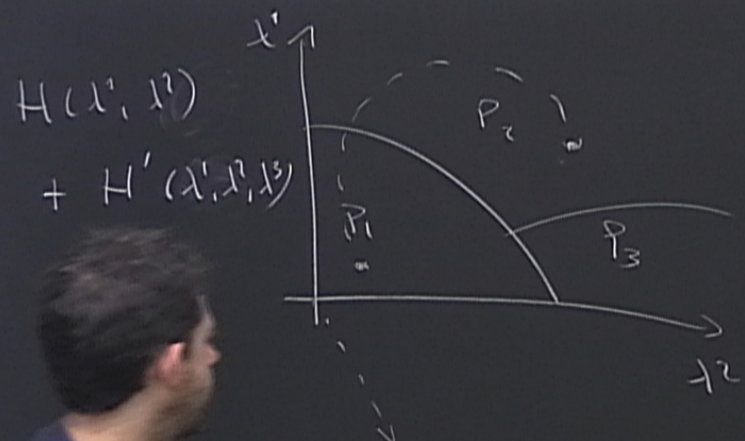
Date: Jan 27, 2015 09:00 AM

URL: <http://pirsa.org/15010058>

Abstract:



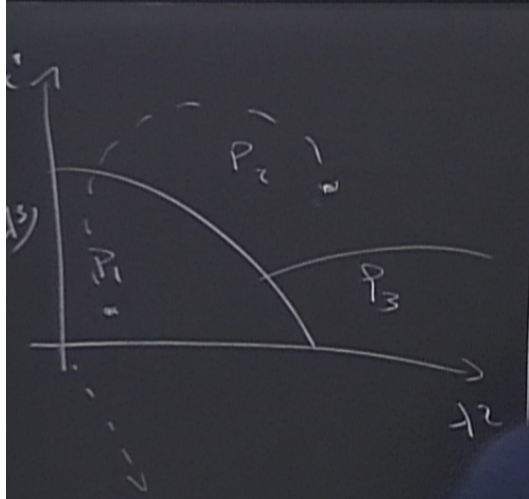




$$\Gamma = (\Lambda, E)$$

\uparrow set
 \nwarrow set of "edges"
 $E \subset \mathcal{P}(\Lambda)$





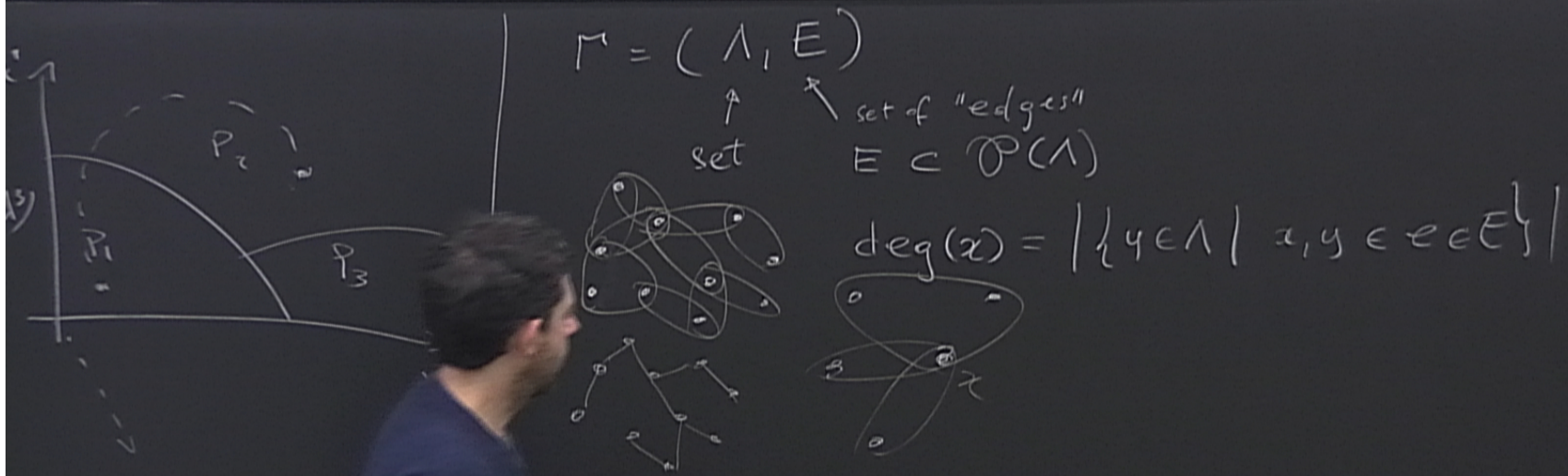
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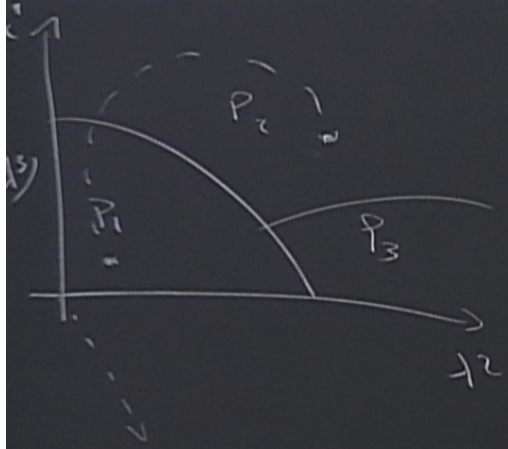
↑
set

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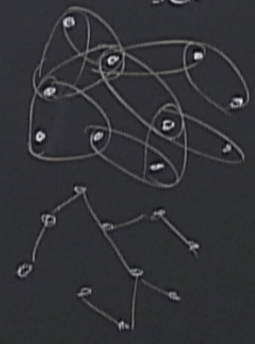
$$\text{deg}(x) = |\{y \in \Lambda \mid x, y \in e \in E\}|$$



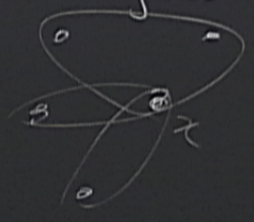


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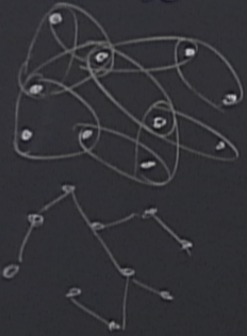


path(x, y)

$$\Gamma = (\Lambda, E)$$

↑
set

↖ set of "edges"
 $E \subset \mathcal{P}(\Lambda)$



$$\text{deg}(x) = |\{ e \in E \mid x \in e \}|$$

$$\text{path}(x, y) = \bigcap_{e_i \in I_e} e_i$$

$$e_i \cap e_{i+1} \neq \emptyset$$

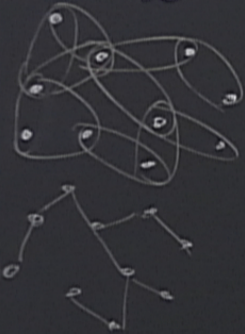
$x, y \in \text{path}$

L

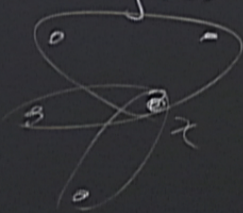
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$$\text{deg}(x) = |\{ e \in E \mid x \in e \}|$$



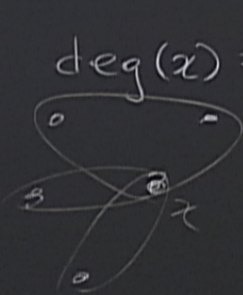
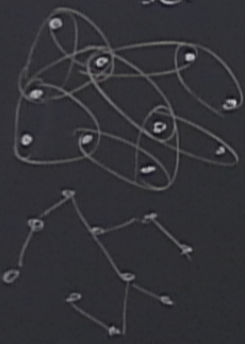
$$\text{path}(x, y) = \bigcap_{e_i \in I_e} e_i \quad \begin{array}{l} e_i \cap e_{i+1} \neq \emptyset \\ x, y \in \text{path} \end{array}$$

$$\text{dist}(x, y) = \min_{\text{path}(x, y)} L(\text{path}(x, y))$$

$$\Gamma = (\Lambda, E)$$

↑
set

↖ set of "edges"
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$$\text{deg}(x) = \left| \left\{ e \in E \mid x \in e \right\} \right|$$

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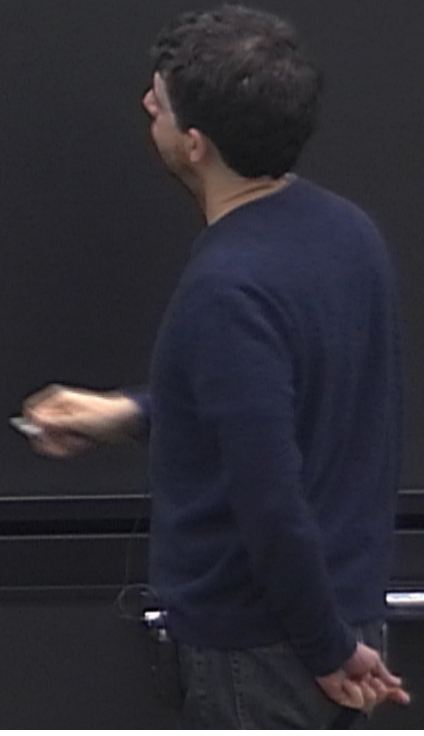
$x, y \in \text{path}$

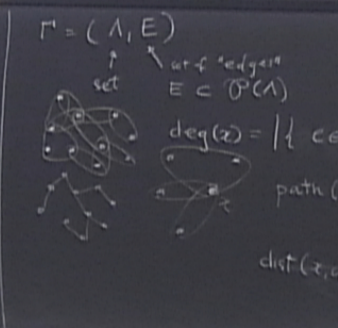
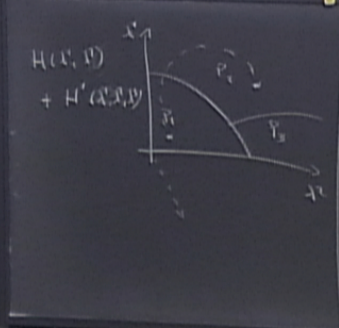
$$\text{dist}(x, y) = \min_{\text{path}(x, y)} L(L(\text{path}(x, y)))$$

$$X \subset \Lambda$$

$$\text{vol } X = |X|$$

$$\text{diam } X = \min_{x, y} d(x, y)$$





$X \subset \Lambda$

$\text{val } X = |X|$

$\text{diam } X = \max_{x, y \in X} d(x, y)$

$\text{deg}(x) = |\{e \in E \mid x \in e\}|$

$\text{path}(x, y) = \{e_c \mid e_c \cap e_{c+1} \neq \emptyset, x, y \in \text{path}\}$

$\text{dist}(x, y) = \min_{\text{paths}(x, y)} L(\text{path}(x, y))$

$$X \subset \Lambda$$

$$\text{vol } X = |X|$$

$$\text{diam } X = \max_{x, y \in X} d(x, y)$$

$$e \in E \mid x \in e$$

$$(x, y) = \bigcap_{e \in I_{x,y}} e \quad \begin{array}{l} e_i \cap e_{i+1} \neq \emptyset \\ x, y \in \text{path} \end{array}$$

$$d(x, y) = \min_{\text{path}(x, y)} L(\text{path}(x, y))$$

Local Quantum System
(generalized spin system)

$$X \subset \Lambda$$

$$\text{vol } X = |X|$$

$$\text{diam } X = \max_{x, y \in X} d(x, y)$$

$$e \in E \mid x \in e$$

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$x, y \in \text{path}$

$$d(x, y) = \min_{\text{path}(x, y)} L(\text{path}(x, y))$$

Local Quantum System
(generalized spin system)

$$\mathcal{H}_\Lambda = \bigotimes_x \mathcal{H}_x$$

$x \in \Lambda \rightarrow \mathcal{H}_x$ local Hilbert space

$c \Lambda$
 $ol X = |X|$
 $min X = \max_{x,y} d(x,y) \quad x,y \in X$
 $e \in e$
 $\bigcap e_c$
 $e_c \in I_e$
 $e_c \cap e_{c+1} \neq \emptyset$
 $x,y \in \text{path}$
 L
 $L(\text{path}(x,y))$
 h
 $h(x,y)$

Local Quantum System
(generalized spin system)

$\mathcal{H}_\Lambda = \bigotimes_{x \in \Lambda} \mathcal{H}_x$ Total Hilbert space
 $x \in \Lambda \rightarrow \mathcal{H}_x$ local Hilbert space $\left(\begin{array}{l} \mathbb{C} \text{SS} \\ \dim \mathcal{H}_x < \infty \end{array} \right)$

$X \subset \Lambda \rightarrow \mathcal{H}_X = \bigotimes_{x \in X} \mathcal{H}_x$
 $\hat{O} \in \mathcal{H}_\Lambda$ If $\hat{O} = \hat{O}_X \otimes \mathbb{1}_{\bar{X}}$ $\text{supp}(\hat{O}) = \mathcal{H}_X$
 $\Lambda = X \cup \bar{X}$



$c \Lambda$
 $|\Lambda| = |\Lambda|$
 $\max_{x,y} d(x,y) \quad x,y \in X$
 $e \in \mathcal{E}$
 $\bigcap e_i$
 $e_i \in \mathcal{I}e$
 $e_i \cap e_{i+1} \neq \emptyset$
 $x,y \in \text{path}$
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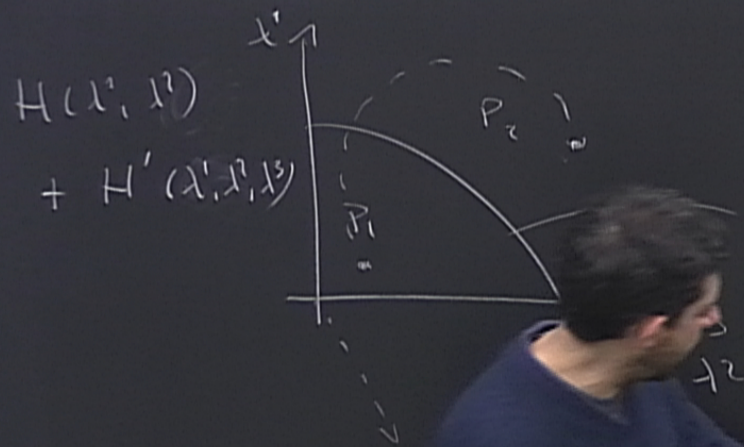


$c \Lambda$
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Local Quantum System
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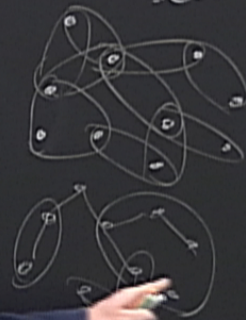
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 $\Lambda = X \cup \bar{X}$ R-local if $\dim \text{supp}(\hat{O}) < R$



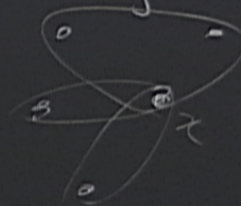
$$\Gamma = (\Lambda, E)$$

↑
set

↖ set of "edges"
 $E \subset \mathcal{P}(\Lambda)$



$$\deg(x) = |\{ e \in E \mid x \in e \}|$$



$$\text{path}(x, y) = \bigcap_{e_i \in I_e}^L e_i$$

$$\text{dist}(x, y) = \min_{\text{path}(x, y)} L(\text{path}(x, y))$$

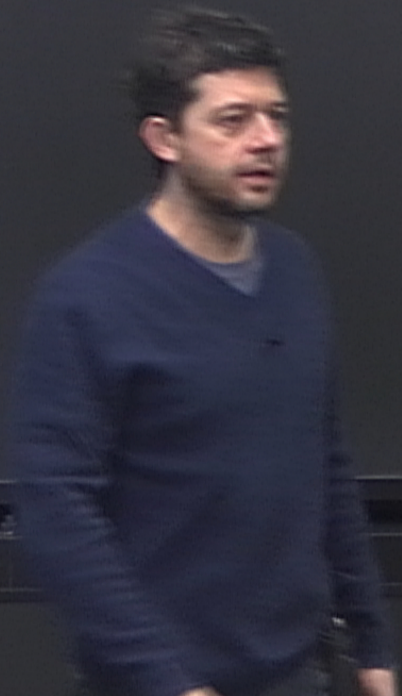
$$X \subset \Lambda$$

$$\text{vol } X = |X|$$

$$\text{diam } X = \max_{x, y} \dots$$

$$\text{dist}(c, g) =$$

R-Local Hamiltonian



R-Local Hamiltonian

Sum of R-local Herm. operators

$X \longrightarrow \Phi_X$ hermitean

$$H = \sum_x \Phi_x$$

R-Local Hamiltonian

Sum of R-local Herm. operators

$X \longrightarrow \bar{\Phi}_X$ hermitean

$$H = \sum_x \bar{\Phi}_x$$

$$[\bar{\Phi}_x, \bar{\Phi}_y] = 0$$

$$x \cap y = \emptyset$$

R-Local Hamiltonian

Sum of R-local Herm. operators

$X \rightarrow \bar{\Phi}_X$ hermitian

$$H = \sum_X \bar{\Phi}_X$$

$$[\bar{\Phi}_X, \bar{\Phi}_Y] = 0 \\ X \cap Y = \emptyset$$

Quantum Ising Model

$$H = -g \sum_i \hat{\sigma}_i^x - J \sum_{(i,j)} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

M all edges are lines

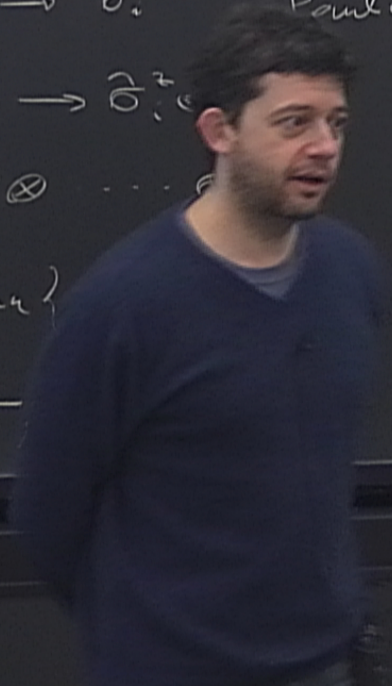
$X \begin{cases} i \rightarrow \hat{\sigma}_i^x \\ (i,j) \rightarrow \hat{\sigma}_i^z \hat{\sigma}_j^z \end{cases}$ Pauli matrices

$$\mathcal{H} = \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2 \\ \mathbb{C}^2 = \text{span}\{|0\rangle, |1\rangle\}$$

$\text{path}(x,y) = \dots$ $L(\text{path}(z,y))$
 $\Lambda = X U \bar{X}$ R -local if diam S

Ising Model
 $-g \sum_i \hat{\sigma}_i^x - J \sum_{(i,j)} \hat{\sigma}_i^z \hat{\sigma}_j^z$
 edges are links
 $i \rightarrow \hat{\sigma}_i^x$ Pauli matrices
 $(i,j) \rightarrow \hat{\sigma}_i^z \hat{\sigma}_j^z$
 $= \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2$
 span

$\hat{T} \psi = \psi$
 $[H, \hat{T}] = 0$



$$L(x,y) = \sum_{\text{path}(x,y)} L(\text{path}(x,y))$$

$$\Lambda = X U \bar{X}$$

R-local if diam S

1D Ising Model

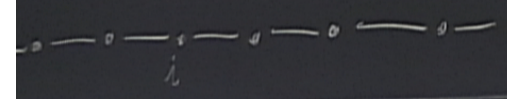
$$-g \sum_i \hat{\sigma}_i^x - J \sum_{(i,j)} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

edges are links
 $i \rightarrow \hat{\sigma}_i^x$ Pauli matrices

$$(i,j) \rightarrow \hat{\sigma}_i^z \hat{\sigma}_j^z$$

$$= \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2$$

span $\{|0\rangle, |1\rangle\}$



$$\hat{T} \psi = \psi'$$

Symmetry

$$[H, \hat{T}] = 0$$

$$L(x,y) = \sum_{\text{path}(x,y)} L(\text{path}(x,y))$$

$$\Lambda = X U \bar{X}$$

R-local if diam S

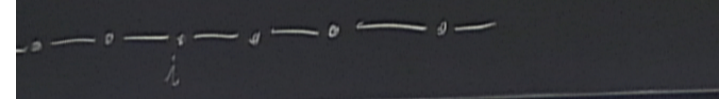
1D Ising Model
 $-g \sum_i \hat{\sigma}_i^x - J \sum_{(i,j)} \hat{\sigma}_i^z \hat{\sigma}_j^z$

edges are links
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$$(i,j) \rightarrow \hat{\sigma}_i^z \otimes \hat{\sigma}_j^z$$

$$= \mathbb{C}^2 \otimes \dots \otimes \mathbb{C}^2$$

\wedge
 $\text{span} \{ |0\rangle, |1\rangle \}$



$$\hat{T} \psi = \psi'$$

Symmetry

$$[H, \hat{T}] = 0$$

$\text{path}(x, y) \subset \text{path}(z, y)$

$$\Lambda = X U \bar{X}$$

R-local if $\dim \text{Supp}(\hat{\theta}) < R$

$$\sum_{(i,j)} \hat{\sigma}_i^2 \otimes \hat{\sigma}_j^2$$

multi matrices

$$\hat{T} \psi = \psi' \quad \text{Symmetry}$$

$$[H, \hat{T}] = 0$$

$$\hat{T} = \hat{\sigma}_1^x \otimes \dots \otimes \hat{\sigma}_N^x$$

$\text{path}_1(x, y) \subset \text{path}_2(x, y)$

$$\Lambda = X U \bar{X}$$

R-local if $\dim \text{Supp}(\hat{\theta}) < R$

$$\sum_{(i,j)} \hat{\sigma}_i^2 \otimes \hat{\sigma}_j^2$$

multi matrices

$$\hat{T} \psi = \psi' \quad \mathbb{Z}_2 \text{ Symmetry}$$

$$[H, \hat{T}] = 0$$

$$\hat{T} = \hat{\sigma}_1^x \otimes \dots \otimes \hat{\sigma}_N^x$$

$$\{\hat{T}, H\} = \mathbb{Z}_2$$

$\text{path}(x, y) \subset \text{path}(z, y)$

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R-local if $\dim \text{Supp}(\hat{\theta}) < R$

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multi matrices

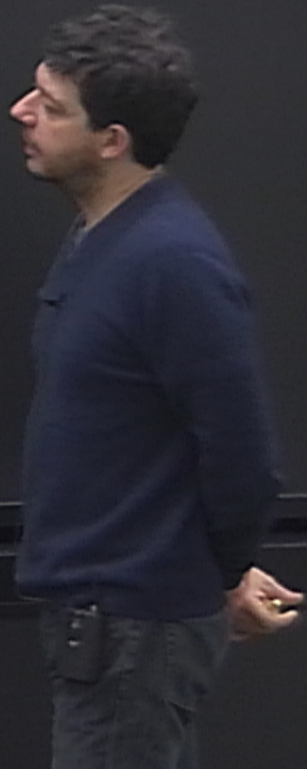
$$\hat{T} \psi = \psi \quad \mathbb{Z}_2 \text{ Symmetry}$$

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1. $g \rightarrow \infty$



path(x,y) \subset (path(z,y))

$$\Lambda = X U \bar{X}$$

R-local if $\dim \text{Supp}(\bar{\theta}) < R$

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$$\hat{T} \psi = \psi \quad \mathbb{Z}_2 \text{ Symmetry}$$

$$[H, \hat{T}] = 0$$

$$\hat{T} = \hat{\sigma}_1^x \otimes \dots \otimes \hat{\sigma}_N^x$$

$$\{\hat{T}, H\} = \mathbb{Z}_2$$

$$\begin{aligned} |\uparrow\rangle &= \hat{\sigma}^z |\uparrow\rangle & \hat{\sigma}^x |\pm\rangle &= \pm |\pm\rangle \\ |\downarrow\rangle &= -\hat{\sigma}^z |\downarrow\rangle \\ \hat{\sigma}^x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \hat{\sigma}^z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

$$1. g \rightarrow \infty$$

$$|4_0\rangle = \otimes_i |+\rangle_i$$

product state

PARAMAGNET

$$\frac{1}{N} \sum_i \langle 4_0 | \hat{\sigma}_i^z | 4_0 \rangle = 0$$

$$\langle 4_0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | 4_0 \rangle = 0 \sim e^{-|i-j|/3}$$

path(x,y) \subset (path(z,y))

$$\Lambda = X U \bar{X}$$

R-local if $\dim \text{Supp}(\bar{\theta}) < R$

$$\sum \hat{\sigma}_i^z \otimes \hat{\sigma}_i^z$$

multi matrices

$$\hat{T} \psi = \psi \quad \mathbb{Z}_2 \text{ Symmetry}$$

$$[H, \hat{T}] = 0$$

$$\hat{T} = \hat{\sigma}_1^x \otimes \dots \otimes \hat{\sigma}_N^x$$

$$\{\hat{T}, H\} = \mathbb{Z}_2$$

$$\begin{aligned}
 |\uparrow\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} & \hat{\sigma}^x |\pm\rangle &= \pm |\pm\rangle \\
 |\downarrow\rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\
 \hat{\sigma}^y &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} & \hat{\sigma}^z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}
 \end{aligned}$$

1. $g \rightarrow \infty$

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PARAMAGNET

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$$H_L(\vec{r}, \gamma)$$

$$\Lambda = X U X \quad R\text{-local if } \text{diam } \text{Supp}(\hat{\sigma}) < R$$

ψ Z_2 Symmetry

$$[\psi] = 0$$

$$\otimes \dots \otimes \hat{\sigma}_N^x$$

$$G = Z_2$$

$$\begin{matrix} \uparrow \\ \downarrow \end{matrix} \hat{\sigma}^x |\pm\rangle = \pm |\pm\rangle$$

$$\hat{\sigma}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

1. $g \rightarrow \infty$ $T|\psi_0\rangle = |\psi_0\rangle$
 $|\psi_0\rangle = \otimes_i |+\rangle_i$ $[T, \psi_0] = 0$ product state
 PARAMAGNET
 $\frac{1}{N} \sum_i \langle \psi_0 | \hat{\sigma}_i^z | \psi_0 \rangle = 0$
 $\langle \psi_0 | \hat{\sigma}_i^z \hat{\sigma}_j^z | \psi_0 \rangle = 0 \quad -11 \dots 11$
 $\sim e^{-1/3}$

2. $g \rightarrow 0$ $|\psi_0\rangle = \begin{cases} \otimes_i |1 \dots 1\rangle \\ \otimes_i |1 \dots \downarrow\rangle \end{cases}$ $[T, \psi_0] \neq 0$



$H_2(x, y)$

$\Lambda = X U \bar{X}$ R-local if $\text{diam Supp}(\bar{\theta}) < R$

ψ Z_2 Symmetry

$] = 0$

\dots

$\{ =$

$\pm | \pm \rangle$

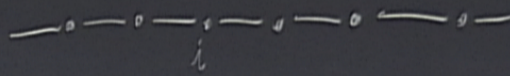
1. $g \rightarrow \infty$ $T|\psi_0\rangle = |\psi_0\rangle$
 $|\psi_0\rangle = \otimes_i |+\rangle_i$ $[T, \psi_0] = 0$ product state
 $\frac{1}{N} \sum_i \langle \psi_0 | \sigma_i^z | \psi_0 \rangle = 0$ PARAMAGNET
 $\langle \psi_0 | \sigma_i^z \sigma_j^z | \psi_0 \rangle = 0 \quad -11-1/3$
 $\sim e$

2. $g \rightarrow 0$ $|\psi_0\rangle = \begin{cases} \otimes_i | \uparrow \dots \uparrow \rangle \\ \otimes_i | \downarrow \dots \downarrow \rangle \end{cases}$ $[T, \psi_0] \neq 0$

$\psi^{\text{Sym}} = \frac{1}{\sqrt{2}} (\psi^\uparrow + \psi^\downarrow)$

$$x \uparrow \uparrow = \phi$$

$$\mathbb{C}^2 = \text{span}\{|0\rangle, |1\rangle\}$$



$$\begin{aligned} |1\rangle &= \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \\ \hat{\sigma}_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \end{aligned}$$

$$\hat{T} | \uparrow \uparrow \rangle = | \downarrow \downarrow \rangle \quad e^{-N} \rightarrow t \sim e^N$$

$$\hat{\sigma}_z | \text{sym} \rangle = | \text{antisym} \rangle$$