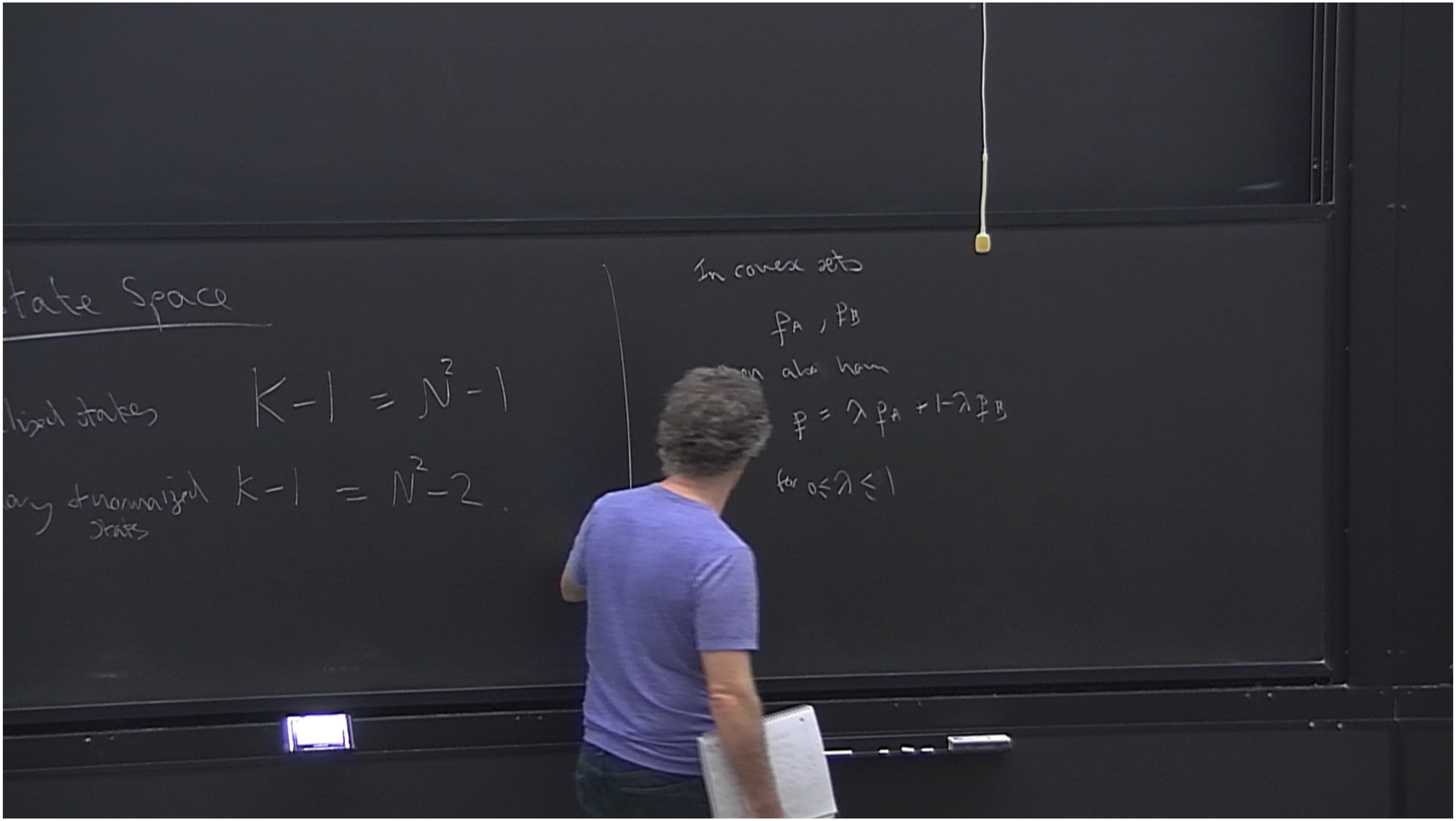
Title: Foundations of Quantum Mechanics-13

Date: Jan 20, 2015 11:30 AM

URL: http://pirsa.org/15010054

Abstract:


tate Space
$K-1 = N^2-1$
$K-1 = N^2-2$
In convex sets
ρ_A , ρ_B
$\rho = \lambda \rho_A + 1-\lambda \rho_B$
for $0 \leq \lambda \leq 1$

...e Space

...states $K-1 = N^2-1$

...t normalized states $K-1 = N^2-2$.

In convex sets

ρ_A, ρ_B

then also have

$\rho = \lambda \rho_A + 1-\lambda \rho_B$

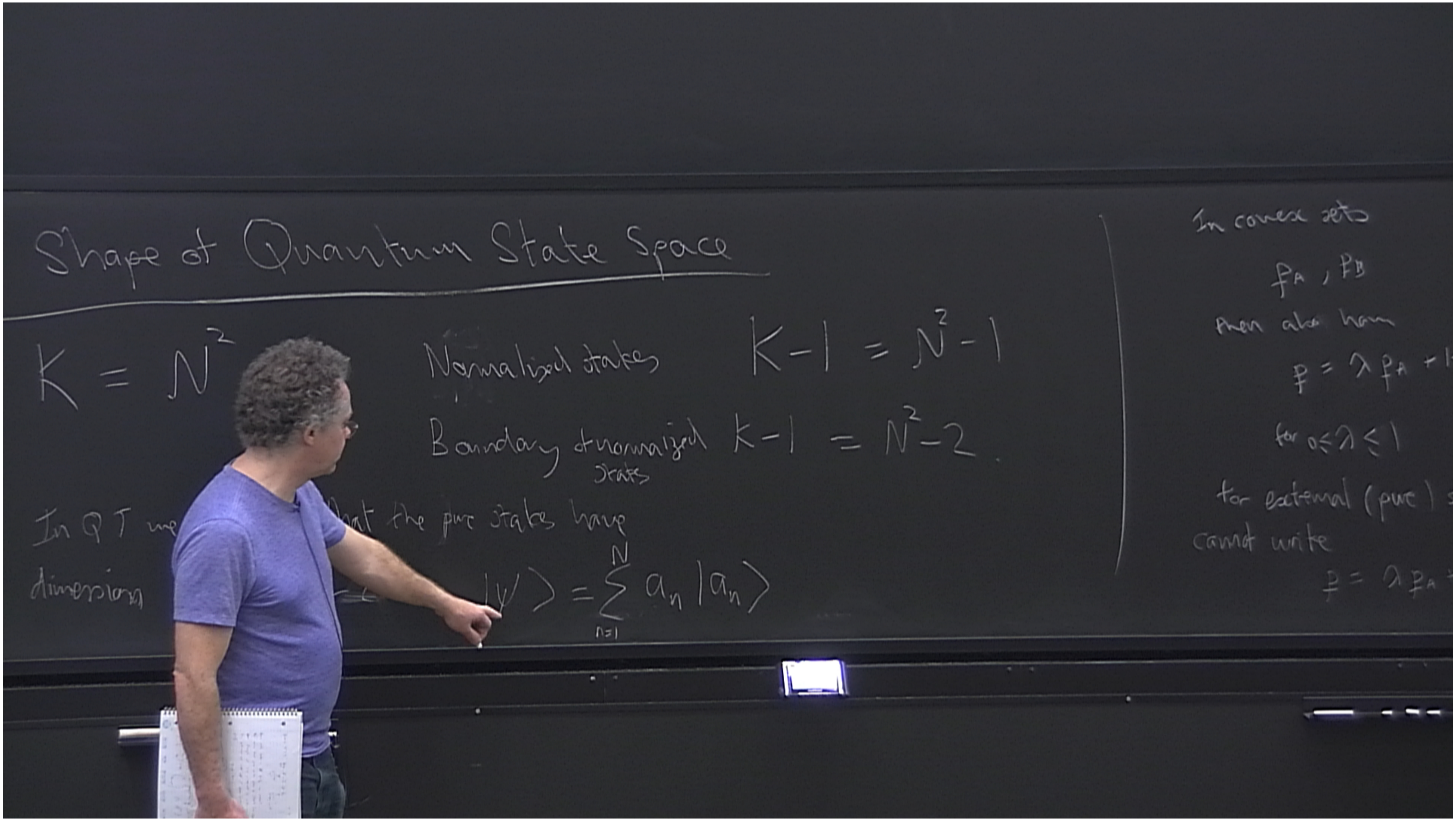
for $0 \leq \lambda \leq 1$

for extremal (pure) states ρ

cannot write

$\rho = \lambda \rho_A + (1-\lambda)\rho_B$ for distinct ρ_A, ρ_B

and $0 < \lambda < 1$


Shape of Quantum State Space
K = N^2
Normalised states K-1 = N^2-1
Boundary of normalized states K-1 = N^2-2
the pure states have
dimension
|ψ> = Σ a_n |a_n>
In convex sets
ρA , ρB
then also have
ρ = λ ρA +
for 0 ≤ λ ≤ 1
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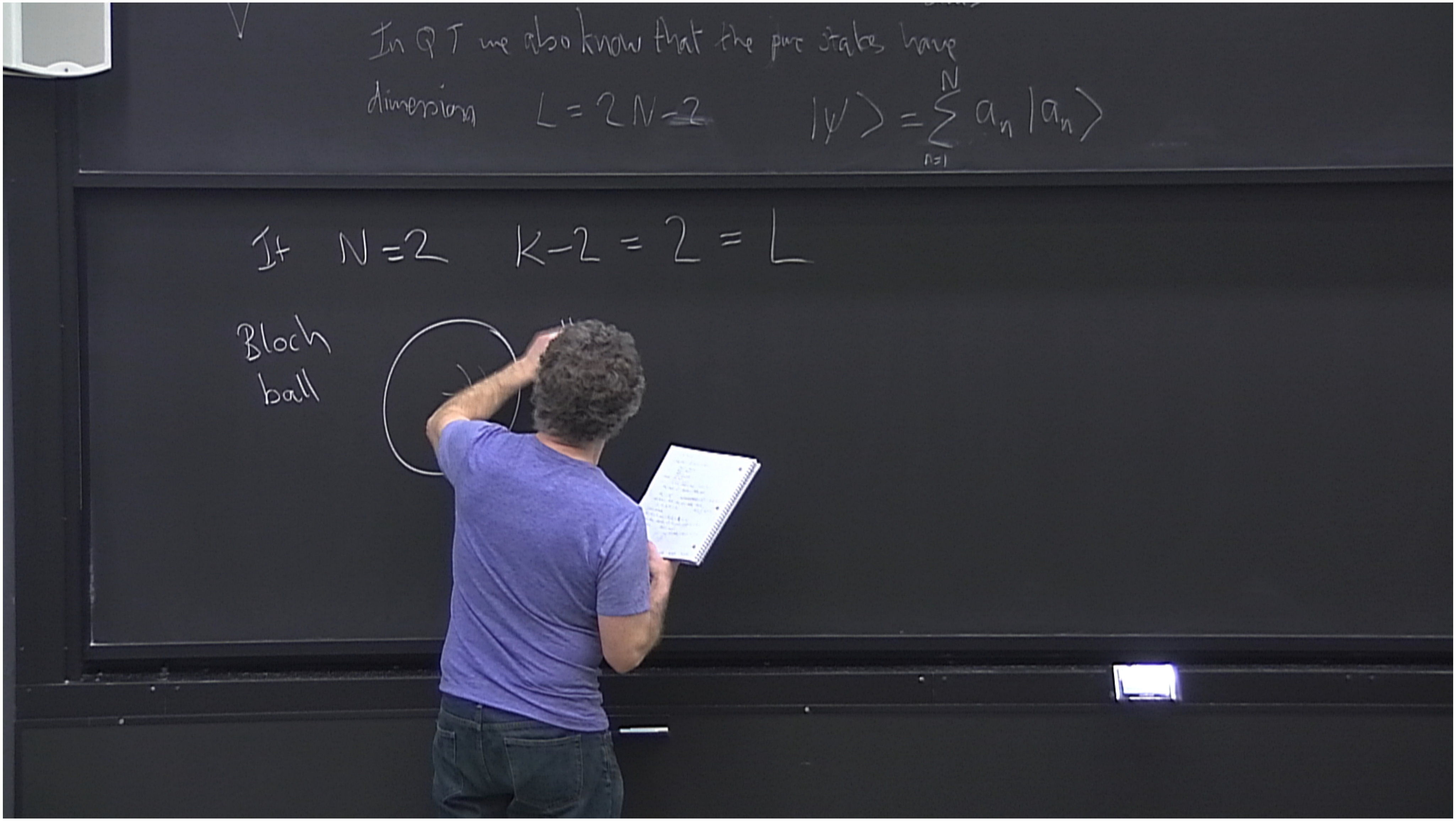
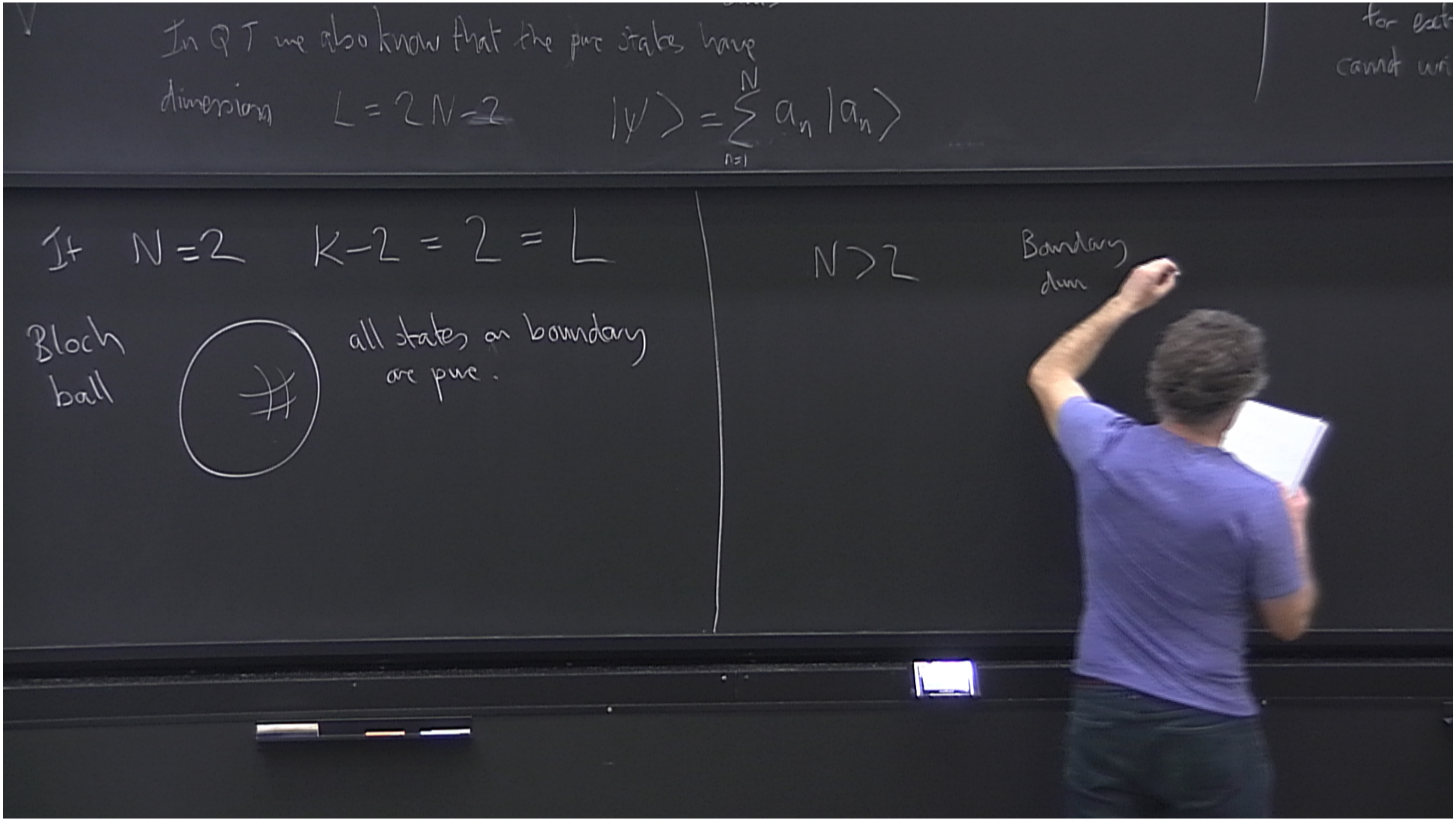
ρ = λ ρA


In QT we also know that the pure states have
dimension L = 2N-2
|ψ> = Σ a_n |a_n>
If N=2 K-2 = 2 = L
Bloch ball


In QT we also know that the pure states have
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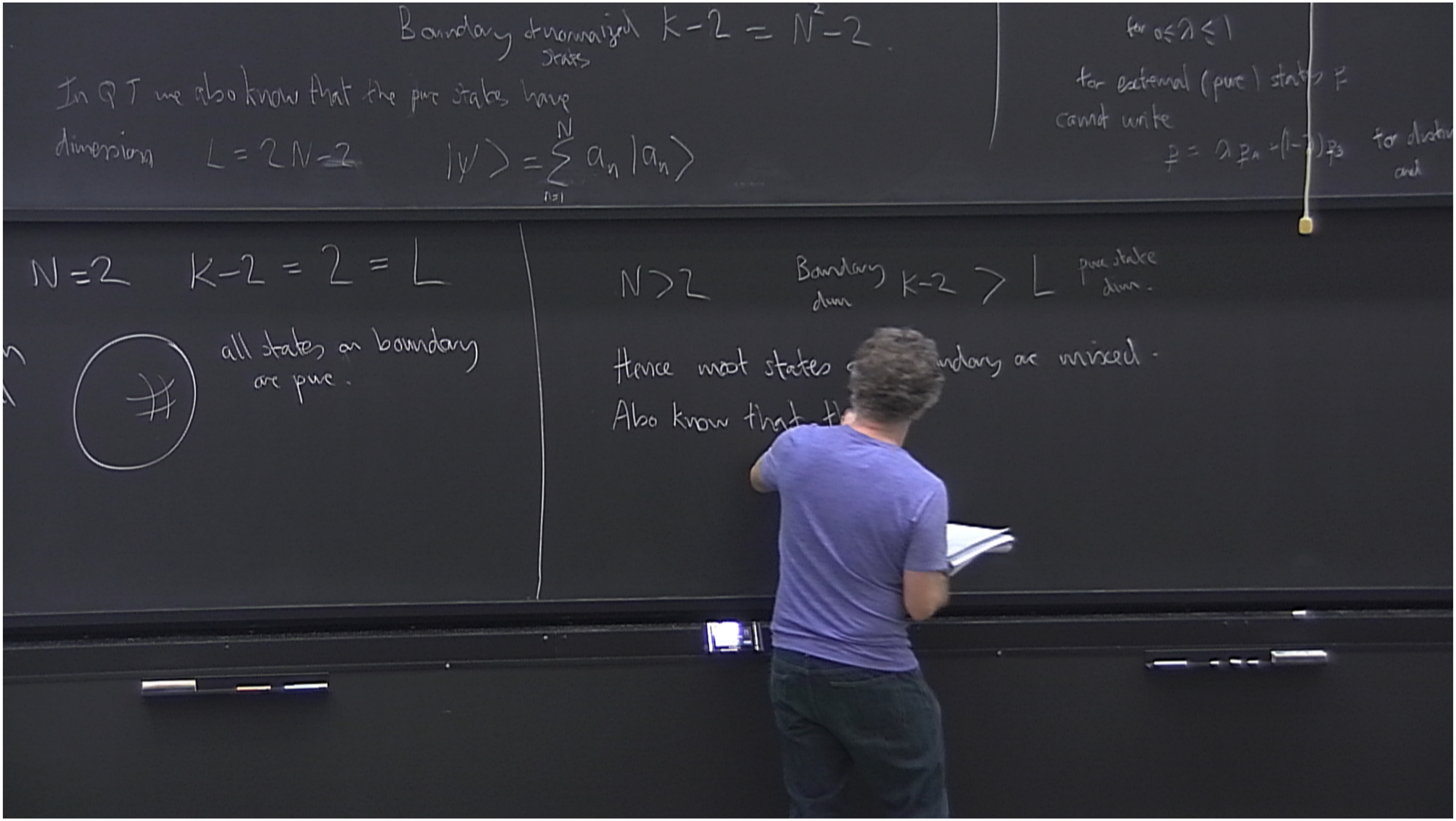
all states on boundary are pure.
N>2
Boundary dim


Boundary of normalized states $K-2 = N^2-2$.
In QT we also know that the pure states have dimension $L = 2N-2$
$|\psi\rangle = \sum_{n=1}^{N} a_n |a_n\rangle$
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$\rho = \lambda \rho_A + (1-\lambda)\rho_B$
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$N=2$ $K-2 = 2 = L$
all states on boundary are pure.
$N>2$ Boundary dim $K-2 > L$ pure state dim.
Hence most states
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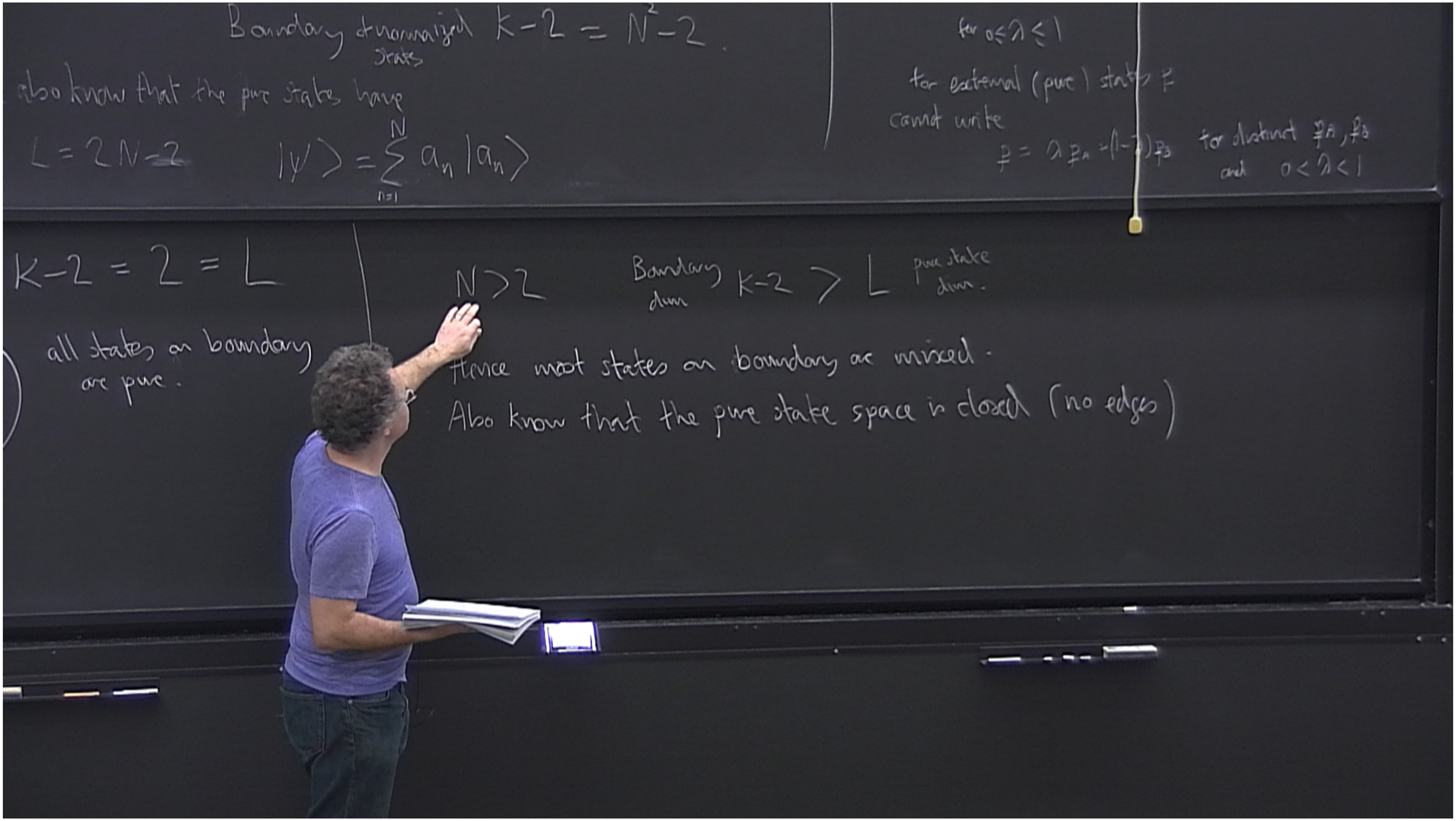
Also know that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for external (pure) states ρ
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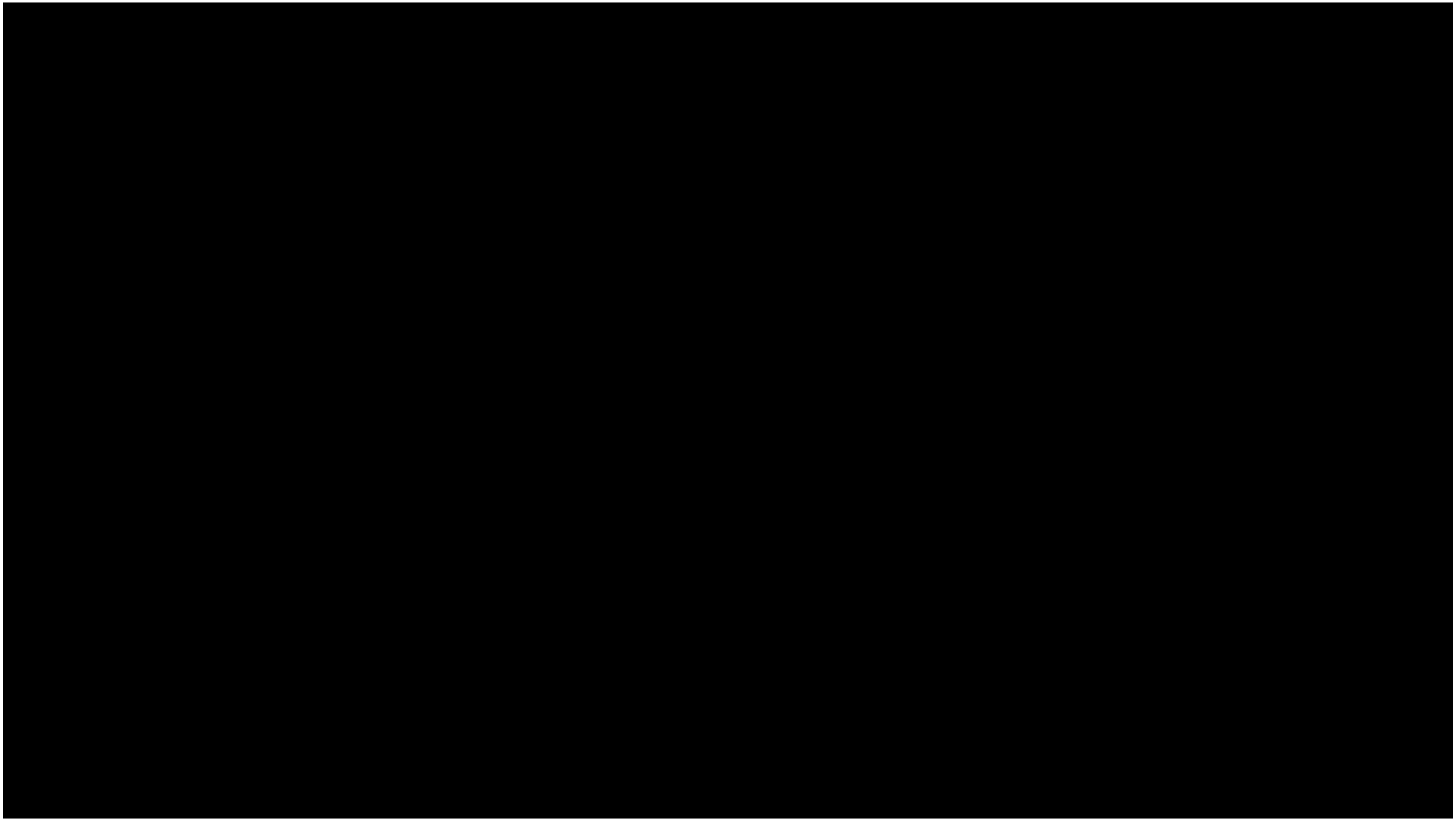
Also know that the pure state space is closed (no edges)



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Pringle (Ben)

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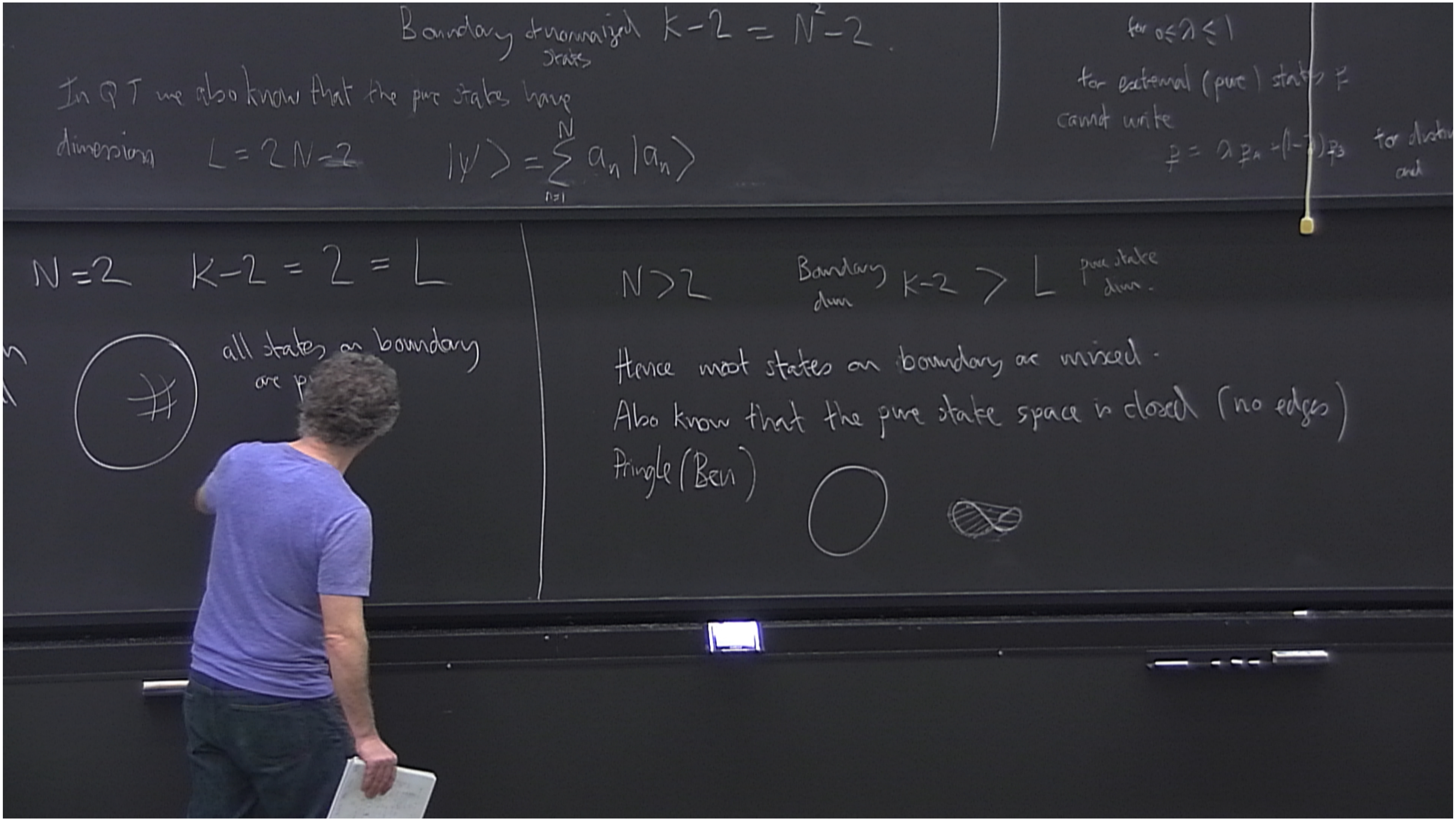
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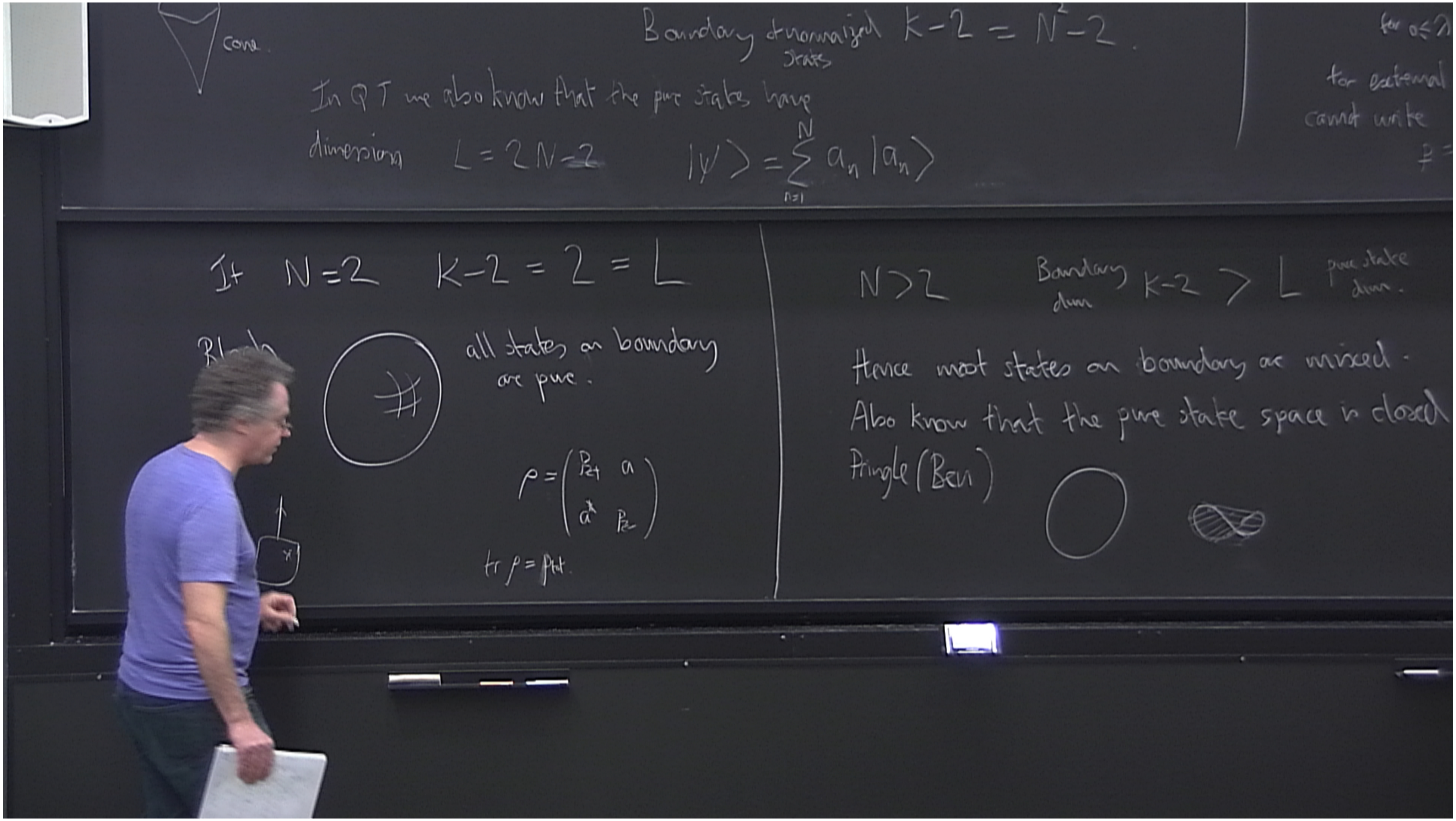
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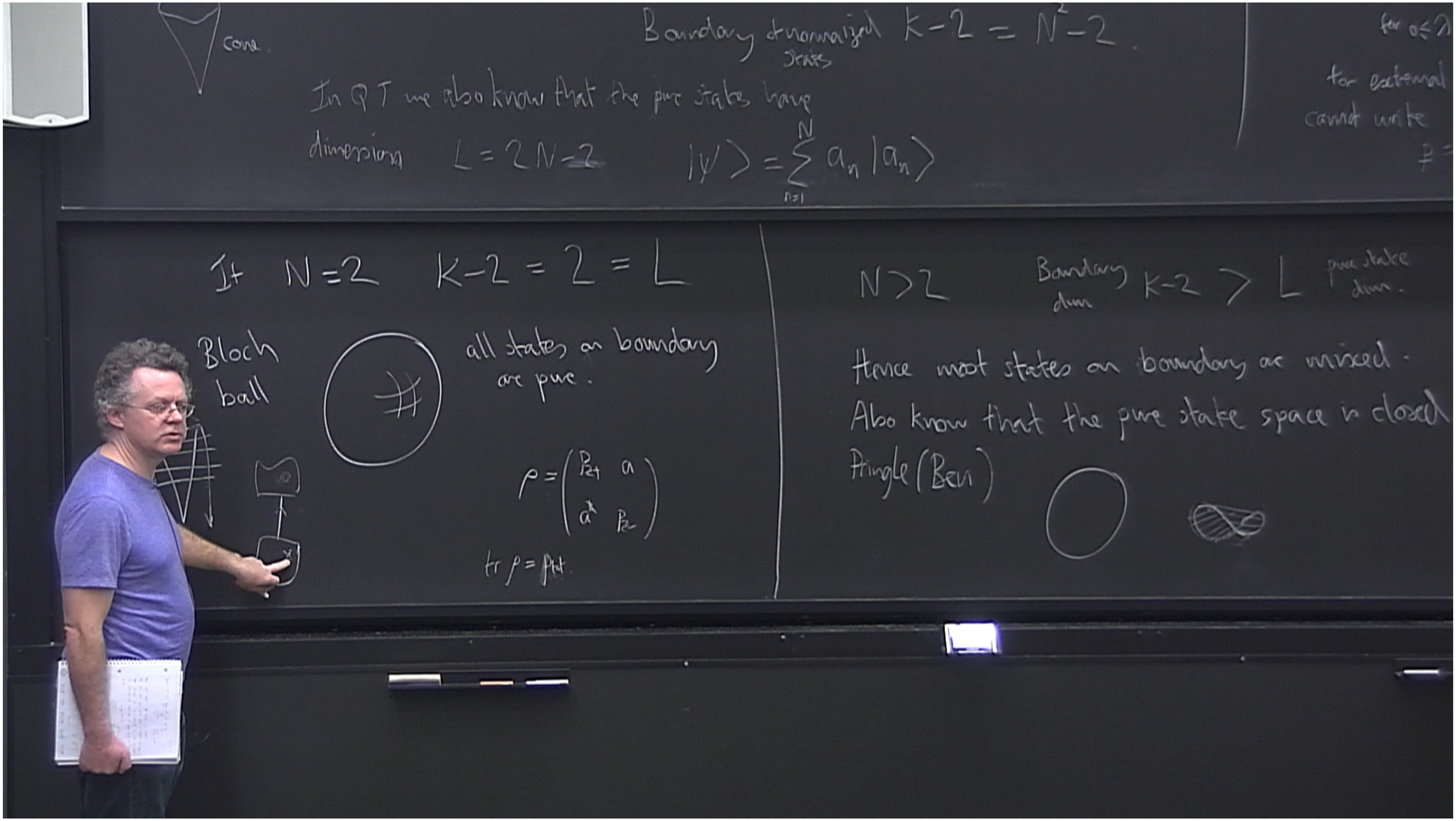
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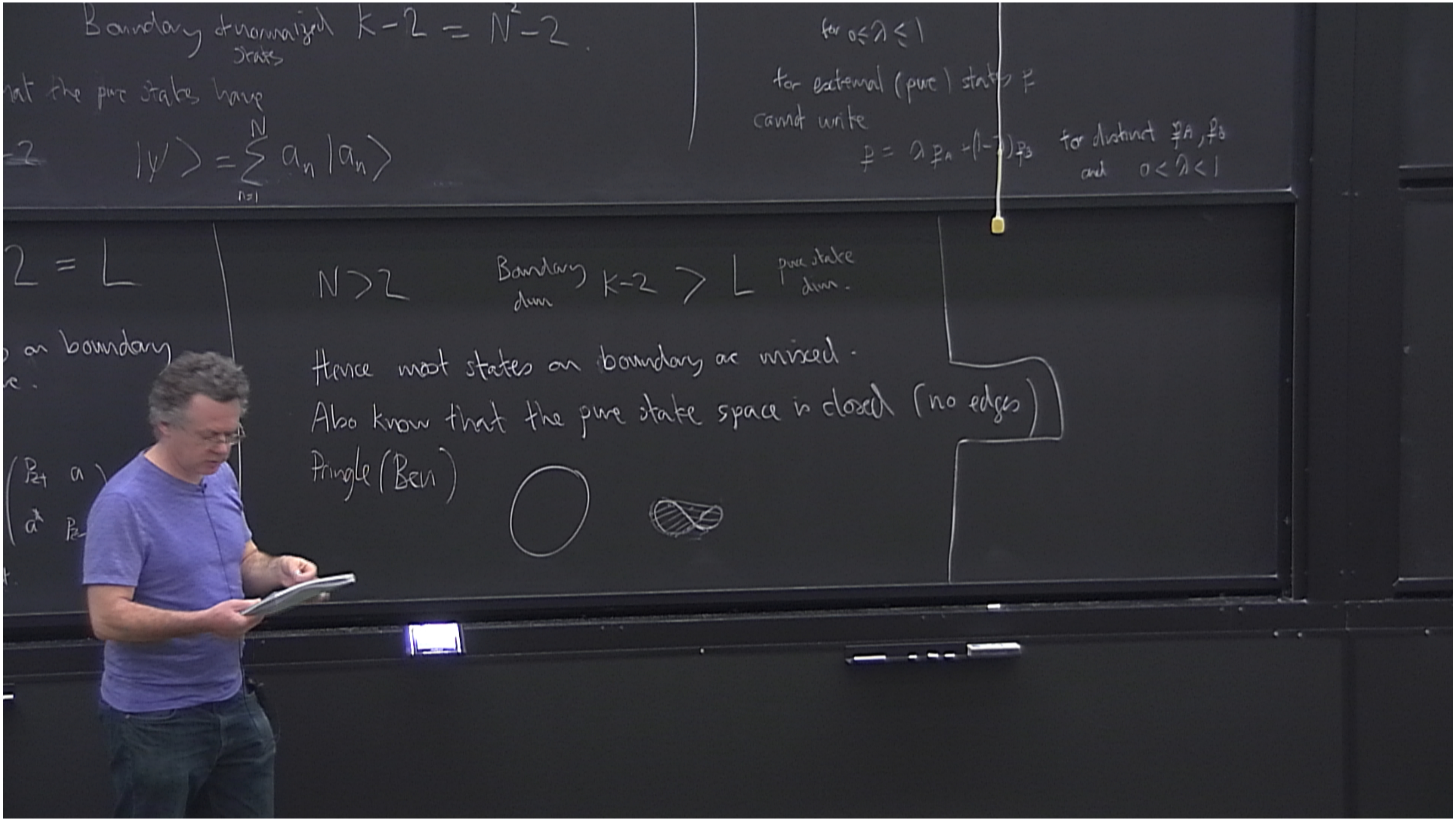
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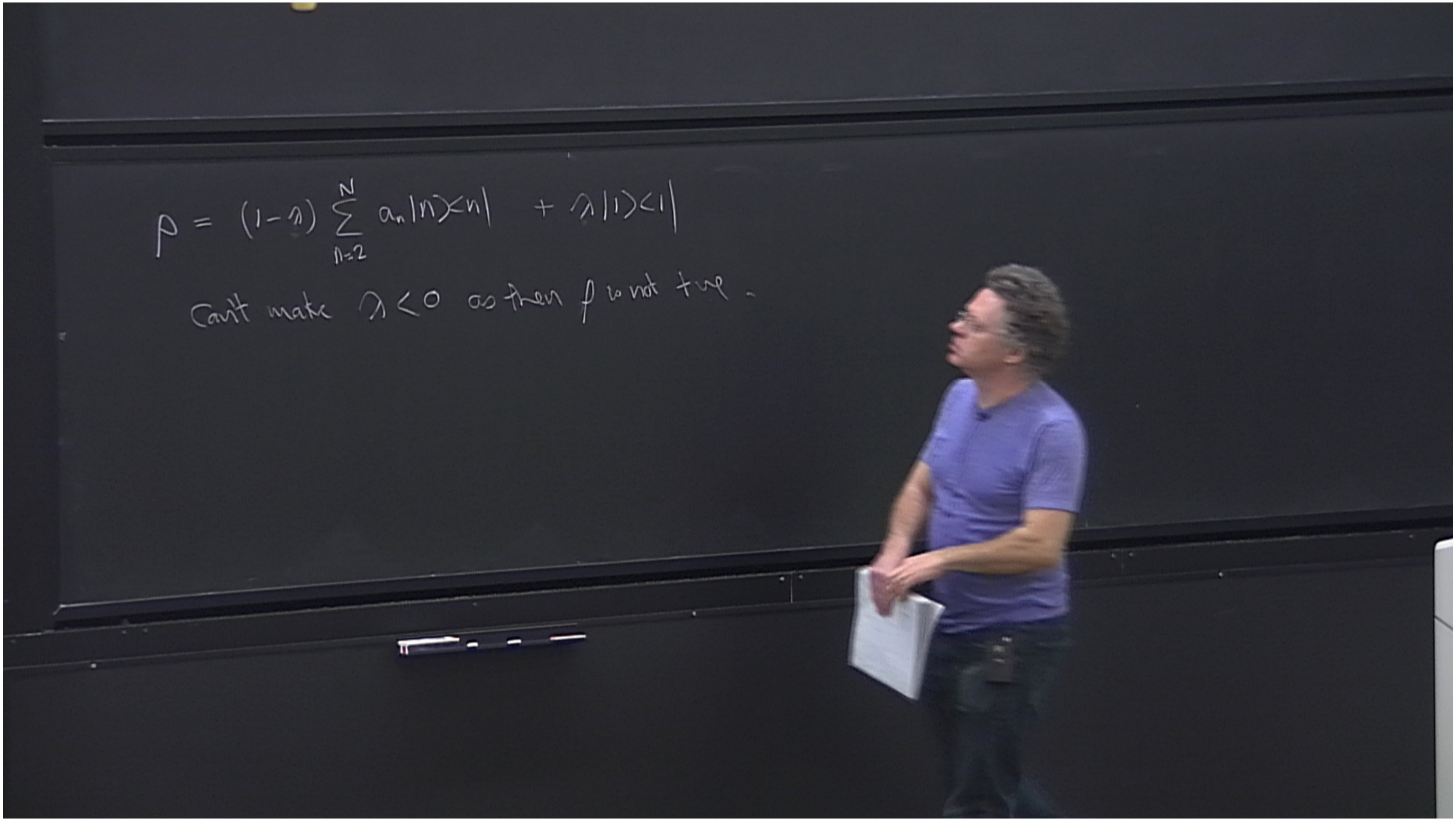
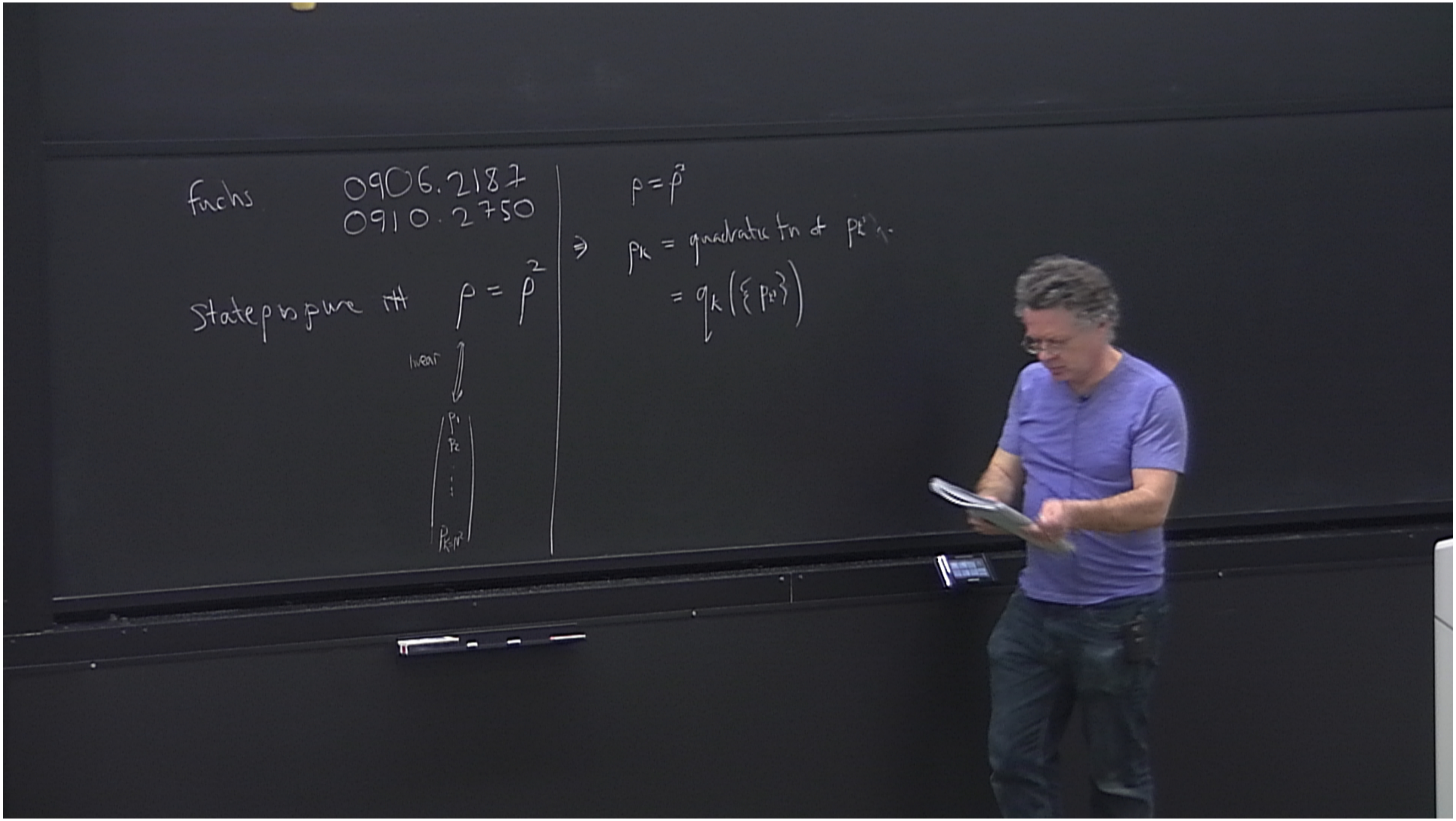
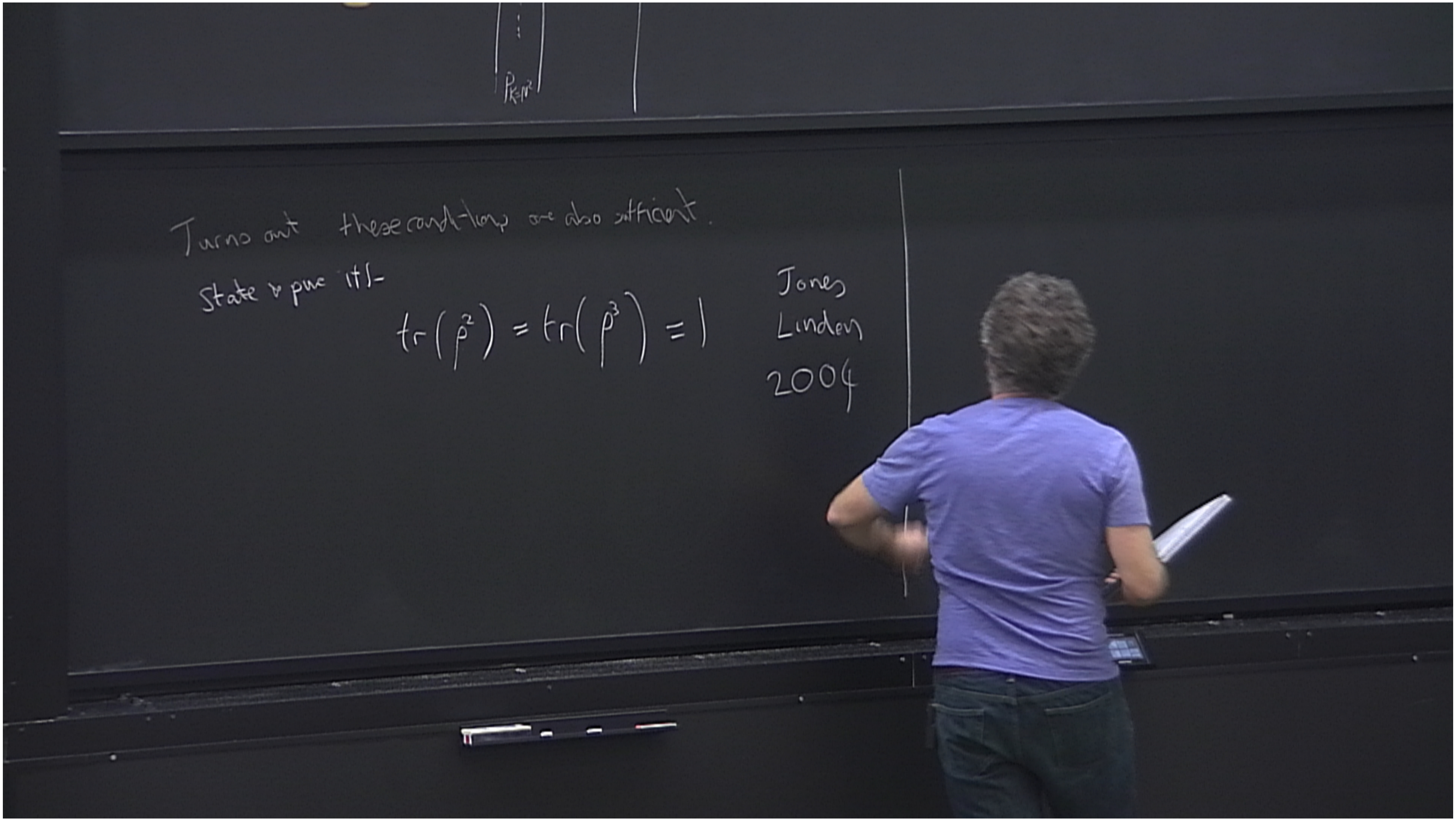
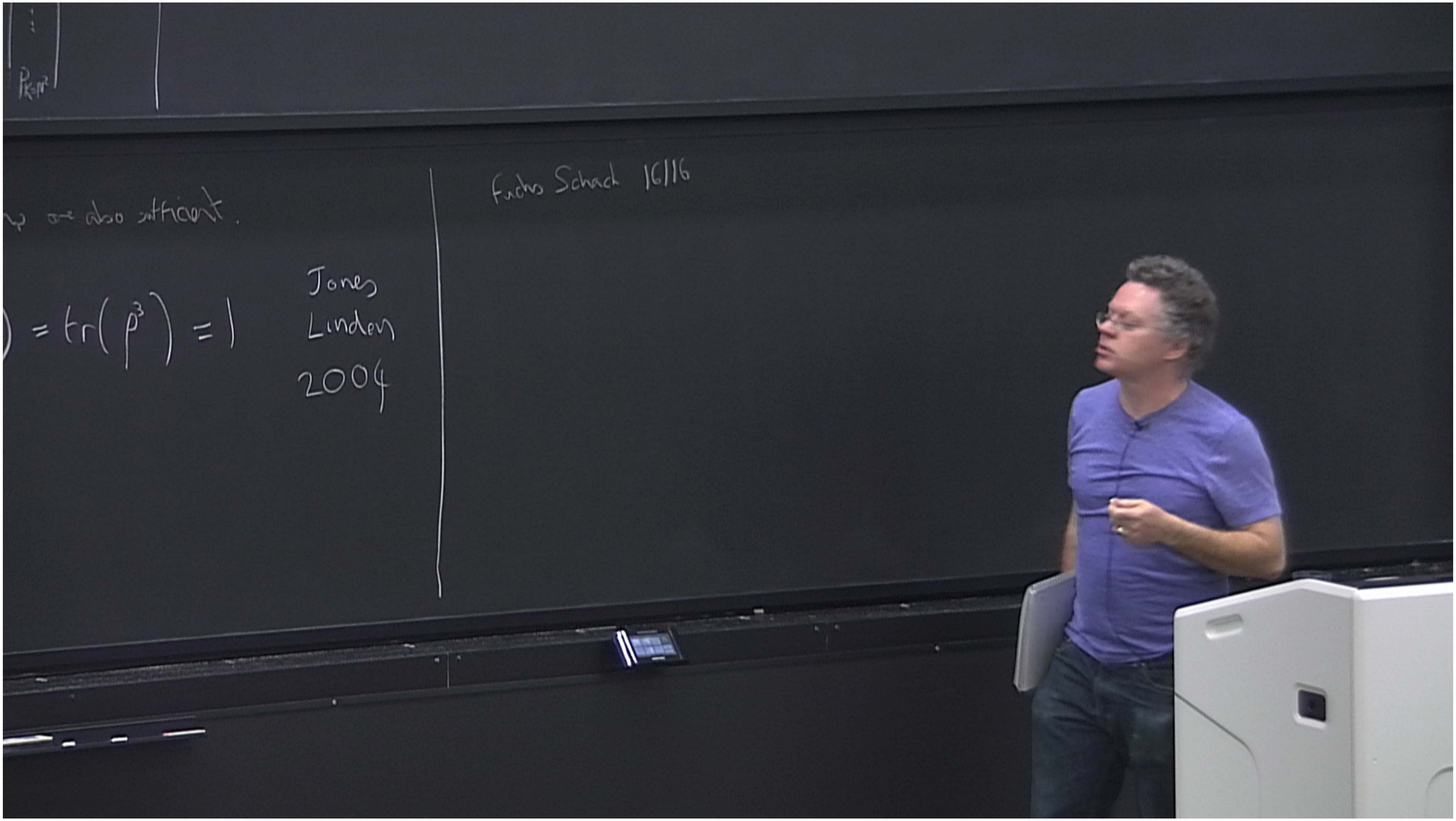

$\rho = (1-\lambda) \sum_{n=2}^{N} a_n |n\rangle\langle n| + \lambda |1\rangle\langle 1|$
Can't make $\lambda < 0$ as then ρ is not +ve.


fuchs 0906.2187
0910.2750
State ρ is pure iff ρ = ρ²
linear
p₁
p₂
⇒ ρ = ρ²
p_k = quadratic fn of p_k's
= q_k({p_k})


Turns out these conditions are also sufficient.
State ρ pure iff
tr(ρ²) = tr(ρ³) = 1
Jones
Linden
2004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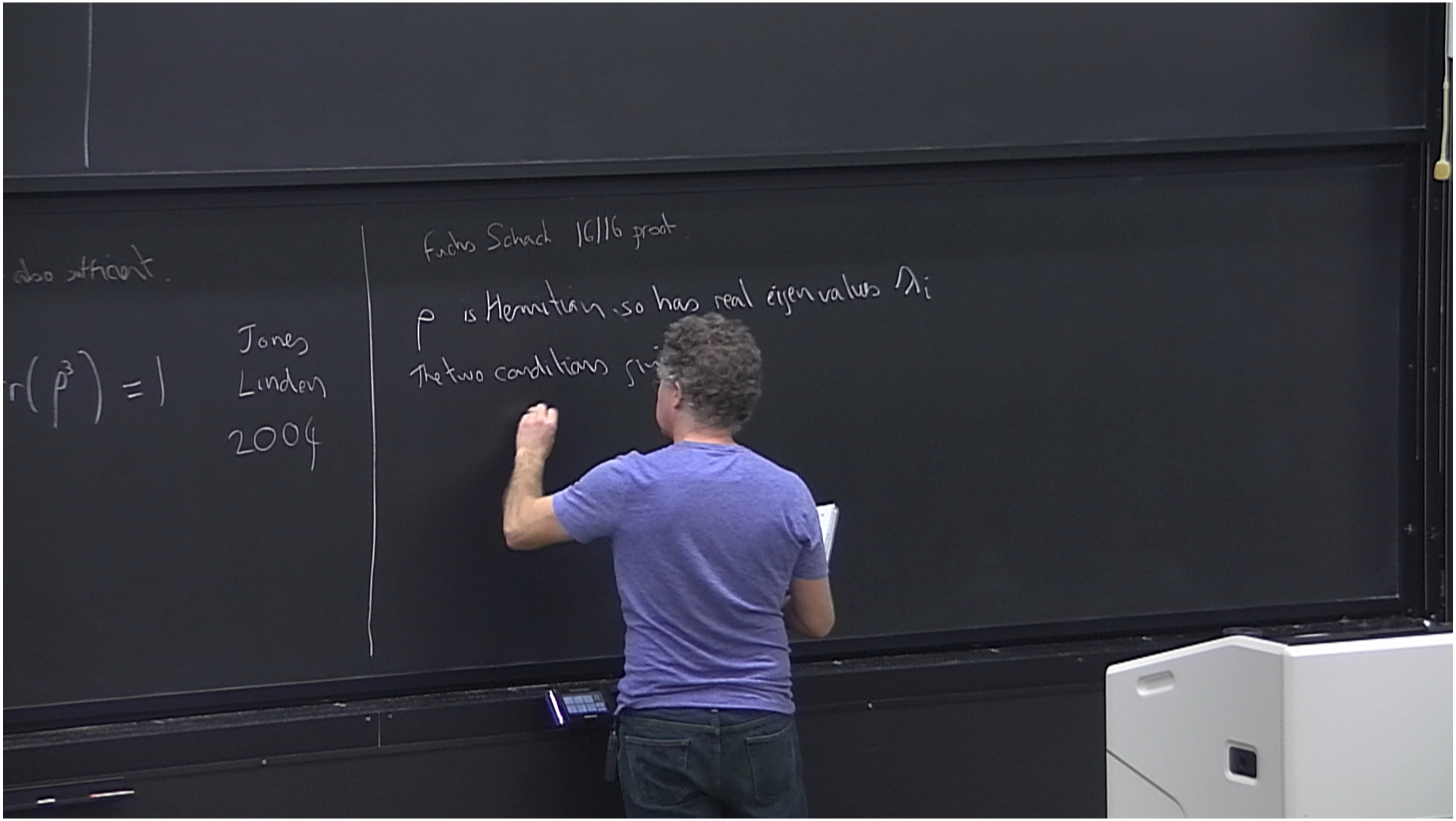
also sufficient.
r(ρ³) = 1
Jones
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2004
Fuchs Schack 16/16 proof
ρ is Hermitian so has real eigenvalues λi
The two conditions

$$\begin{pmatrix} p_1 \\ p_2 \\ \vdots \\ p_{K-N^2} \end{pmatrix}$$

(1) $|\lambda_i| \leq 1 \quad \forall i$

$\Rightarrow \quad 1 - \lambda_i \geq 0 \quad \forall i$.

(1) − (2)

$$\sum_i \underbrace{\lambda_i^2}_{\geq 0} \underbrace{(1-\lambda_i)}_{\geq 0} = 0$$

$\Rightarrow \quad \lambda_i = 1$ or 0.

p_2

$\vdots$

$p_{k=n^2}$

$\rho =$

① $|\lambda_i| \leq 1 \quad \forall i$

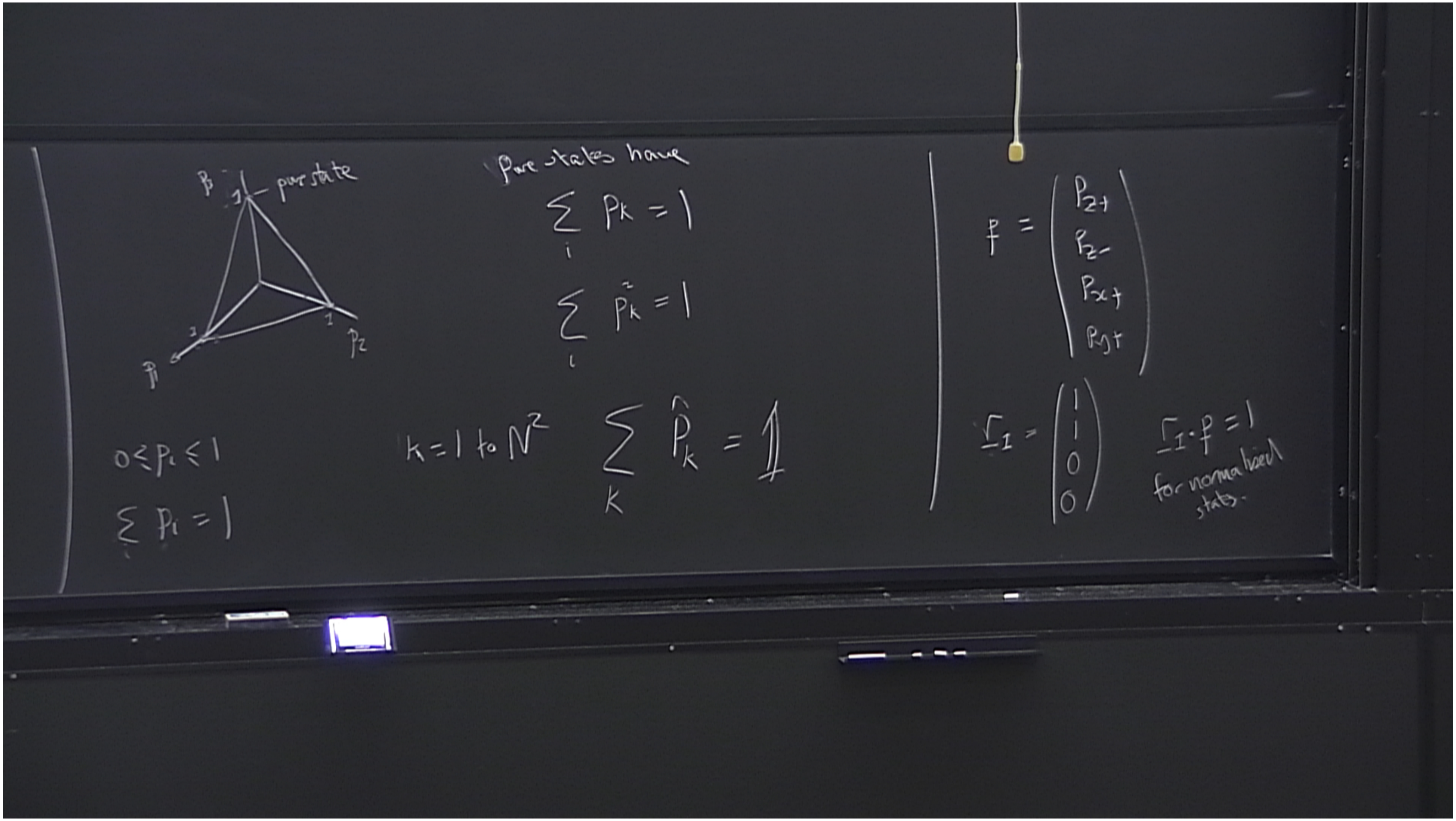
$\Rightarrow \quad 1 - \lambda_i \geq 0 \quad \forall i$

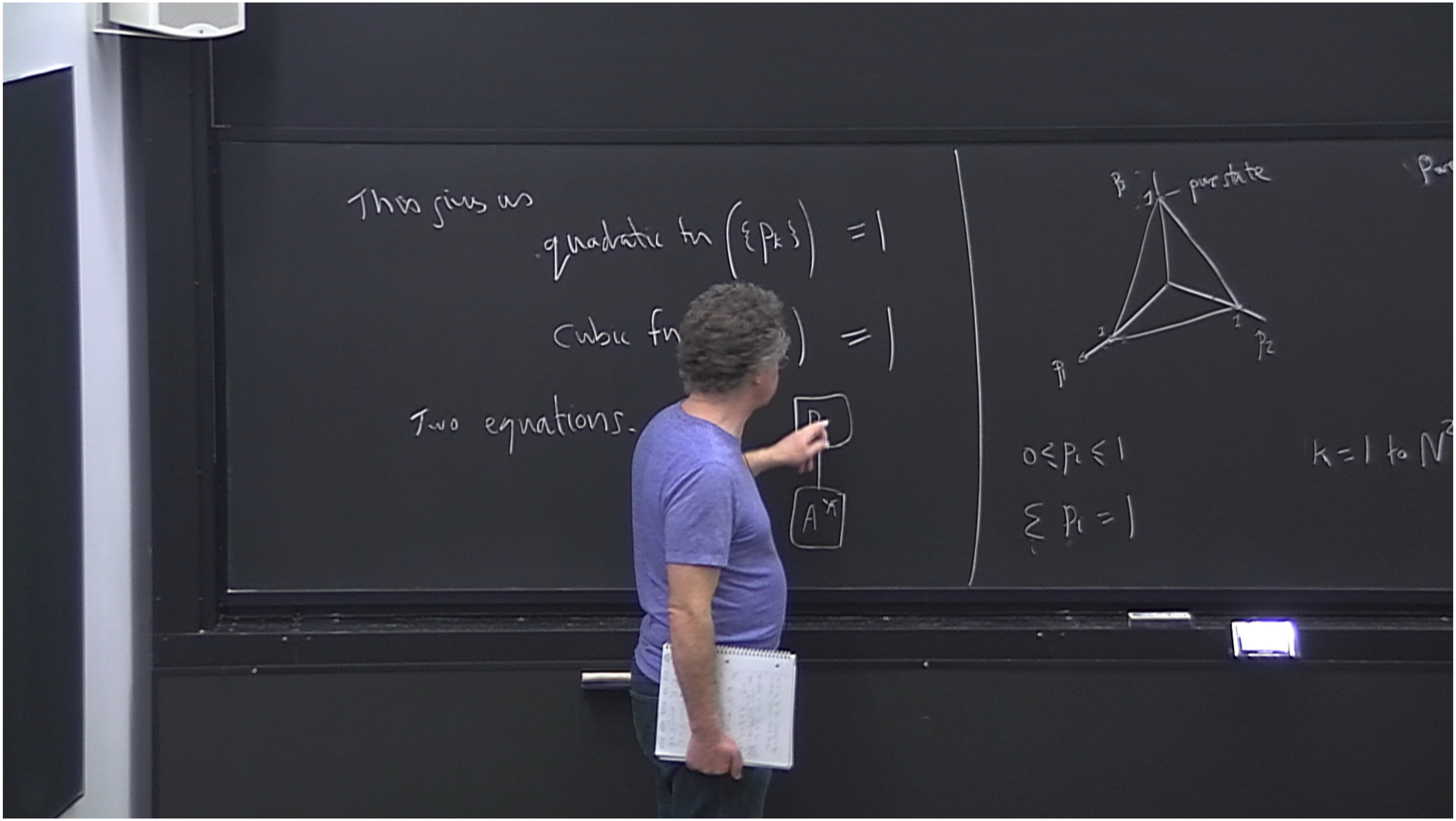
①−②

$$\sum_i \underbrace{\lambda_i^2}_{\geq 0} \underbrace{(1-\lambda_i)}_{\geq 0} = 0$$

$\Rightarrow \quad \lambda_i = 1$ or 0.

① only one λ_i can equal 1


pure state
p_3
p_1
p_2
$0 \leq p_i \leq 1$
$\sum p_i = 1$
Pure states have
$\sum_i p_k = 1$
$\sum_i p_k^2 = 1$
$k = 1$ to N^2
$\sum_k \hat{P}_k = \mathbb{1}$
$\underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$
$\underline{r}_I = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$
$\underline{r}_I \cdot \underline{p} = 1$
for normalised states.


This gives us
quadratic fn ({p_k}) = 1
cubic fn
= 1
Two equations.
pure state
0 ≤ p_i ≤ 1
Σ p_i = 1
k = 1 to N