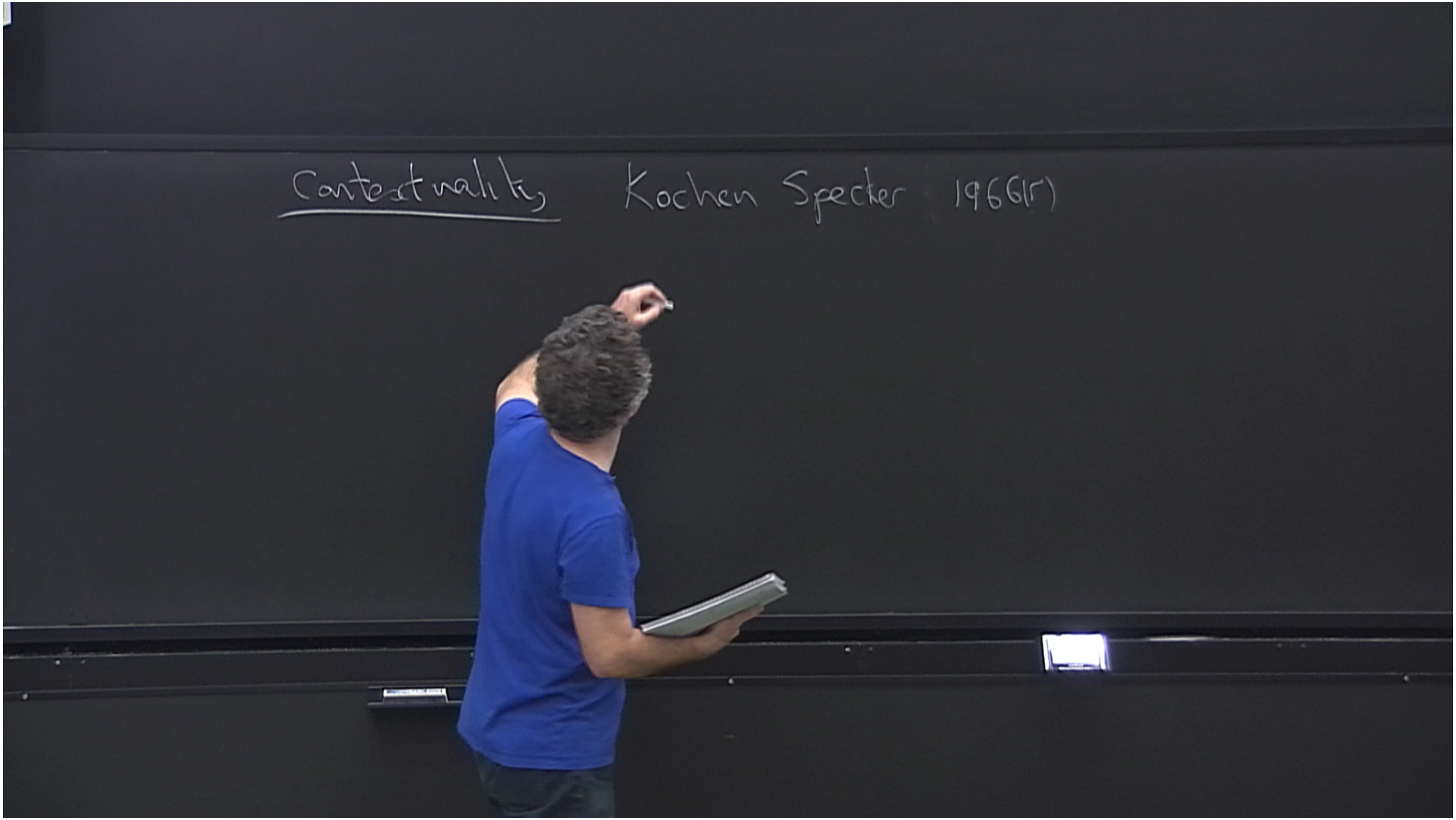
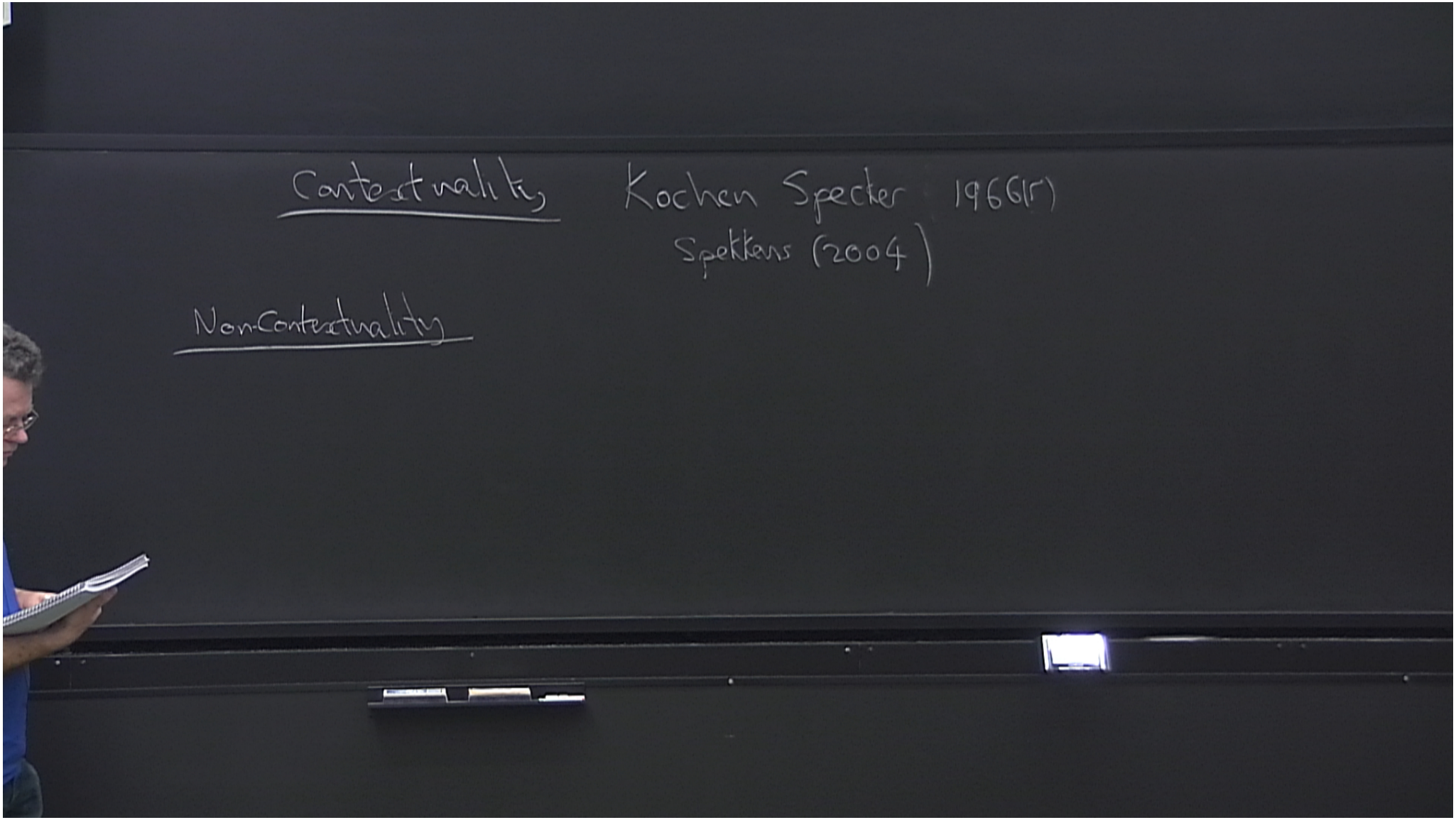
Title: Foundations of Quantum Mechanics-11

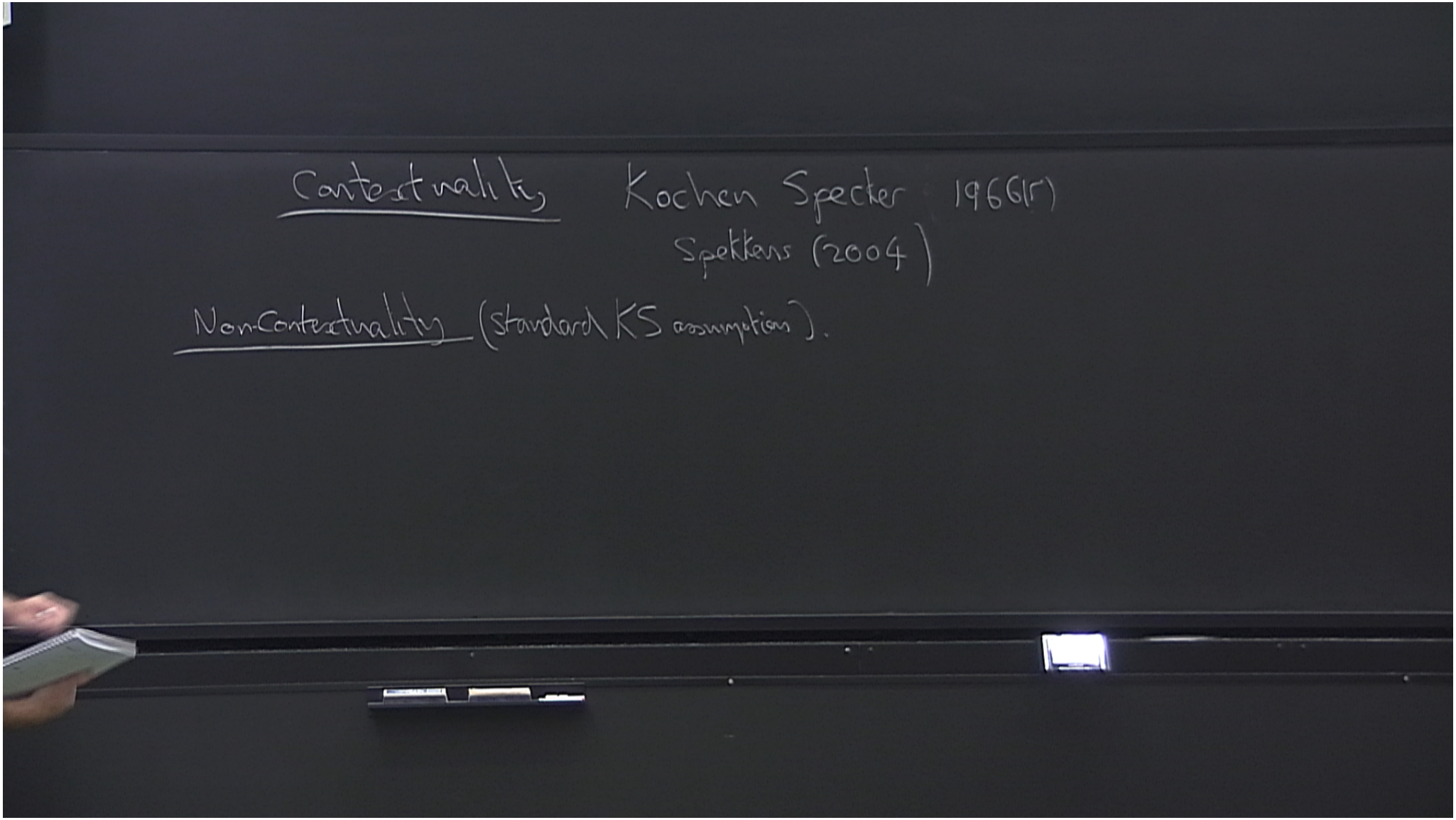
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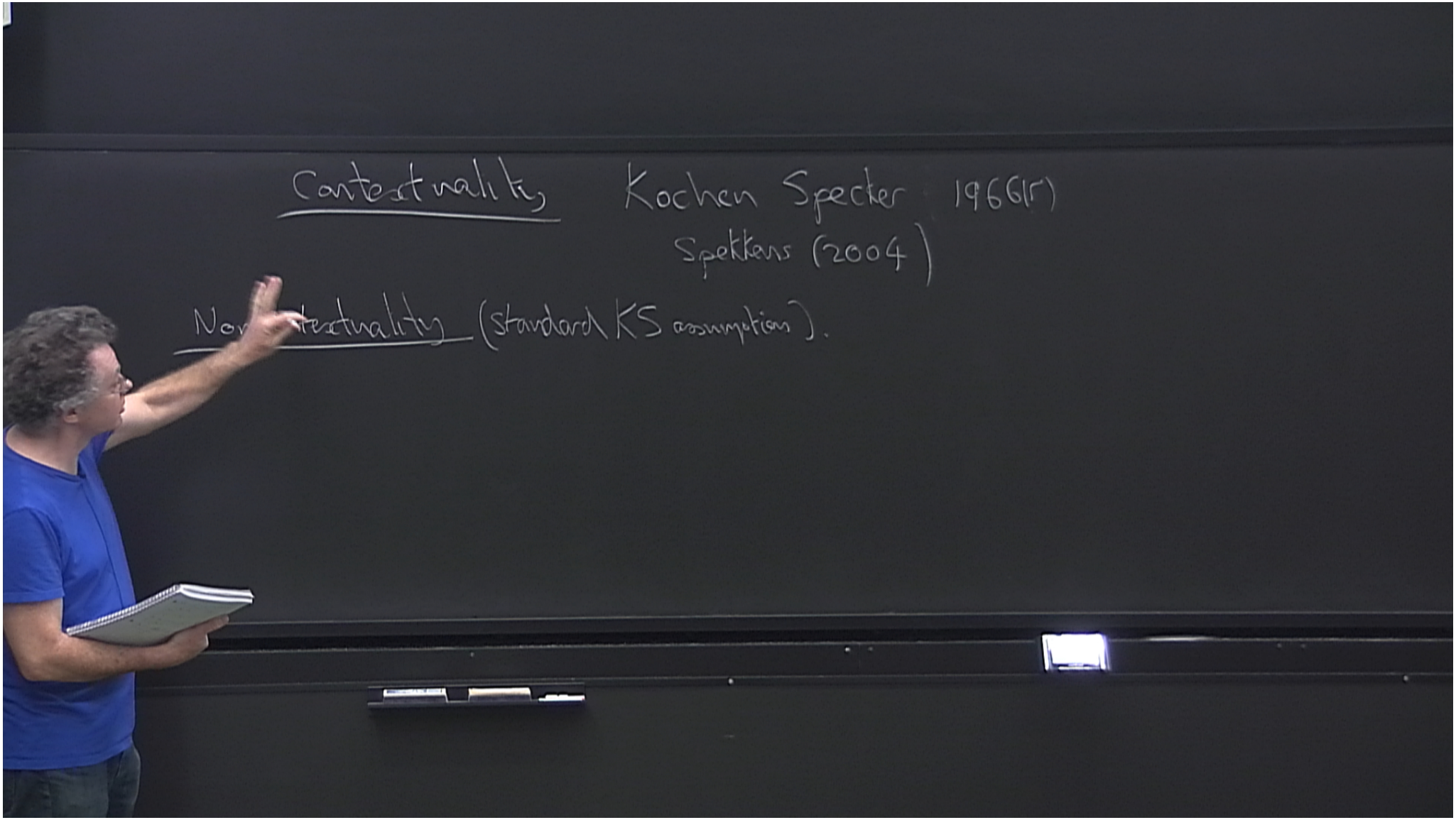
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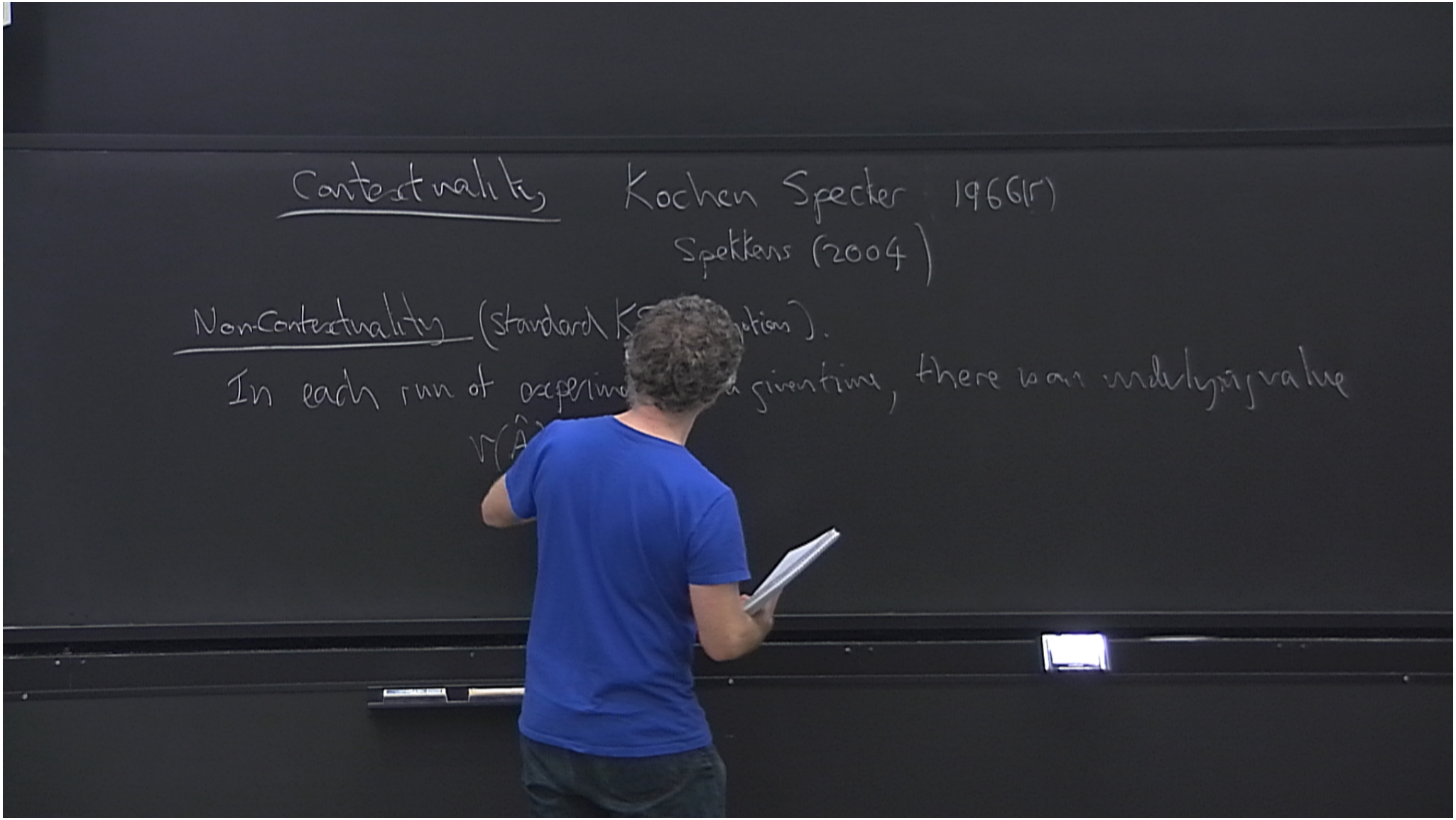
Abstract:

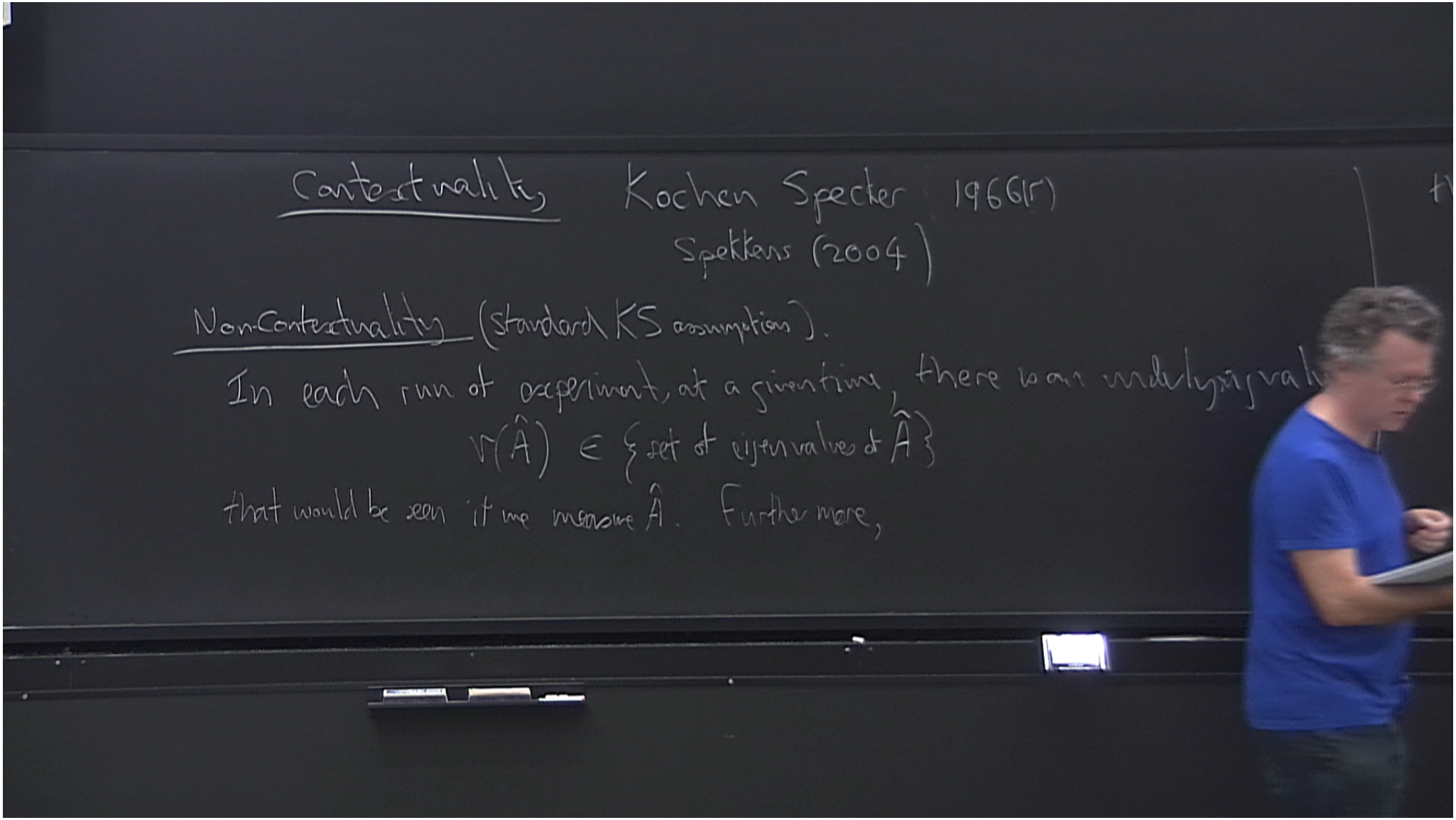

Contextuality
Kochen Specker 1967

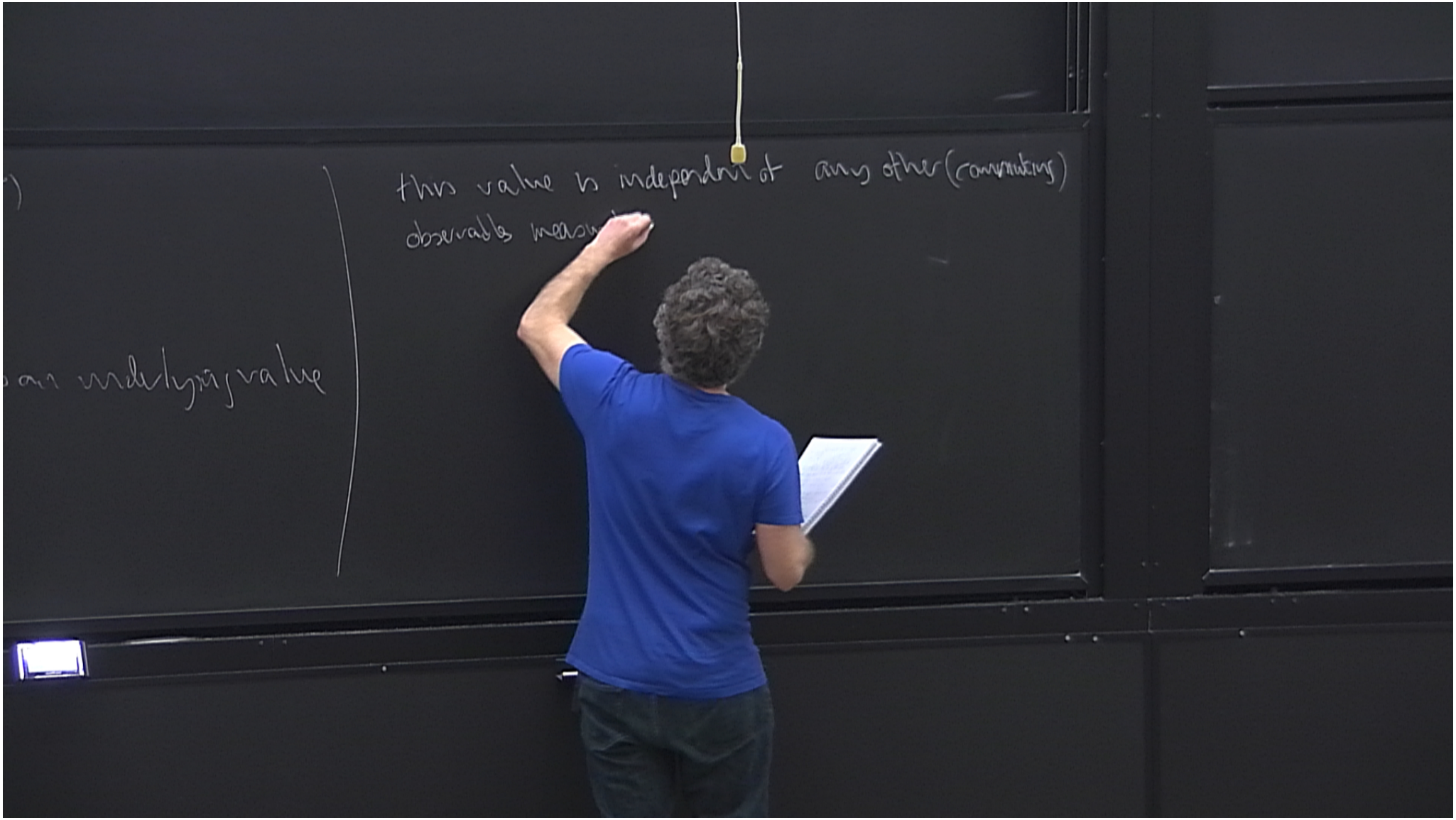

Contextuality
Kochen Specker 1966(7)
Spekkens (2004)
Non-Contextuality

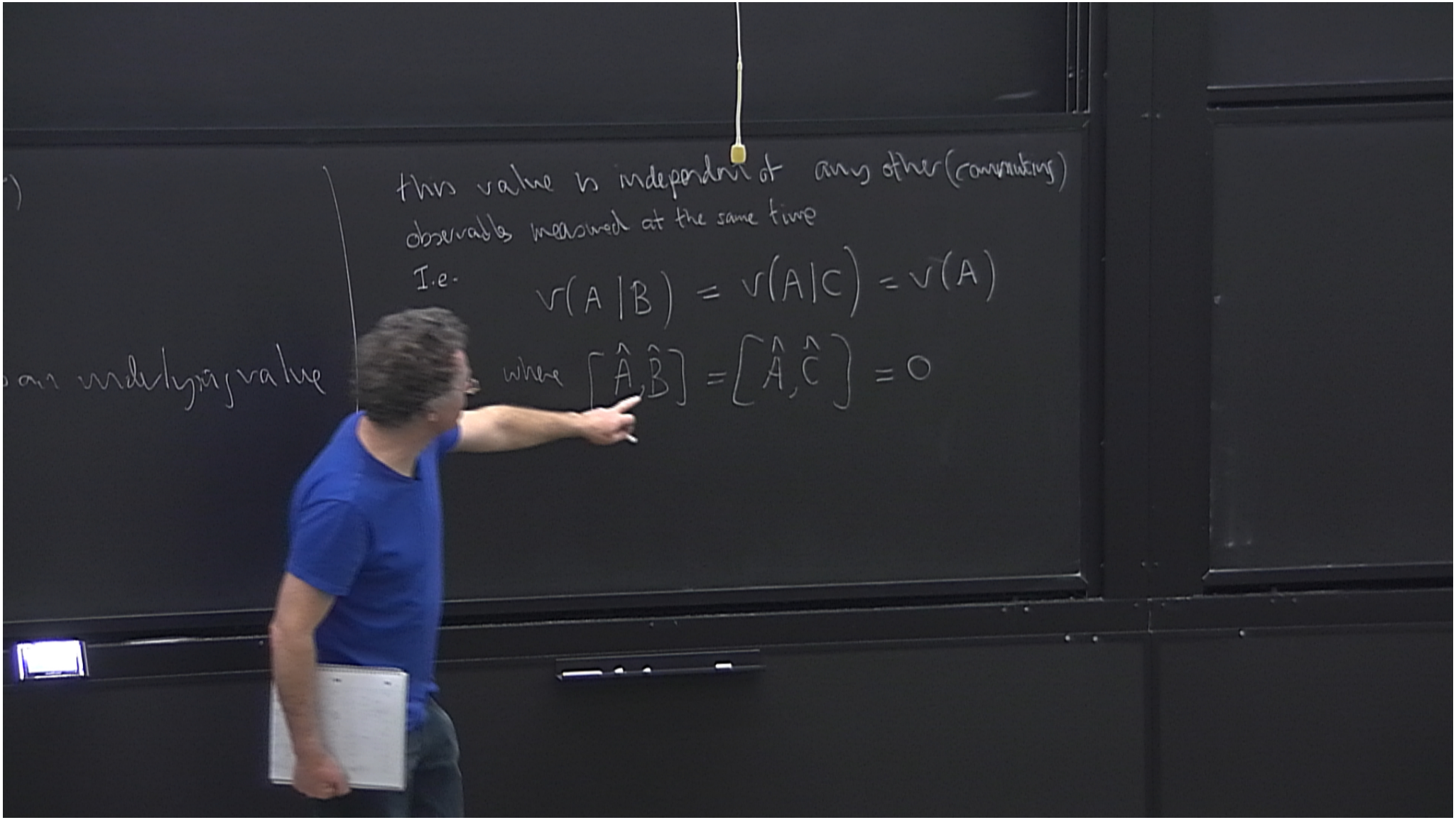

Contextuality Kochen Specker 1966(r)
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Non-Contextuality (standard KS assumption).

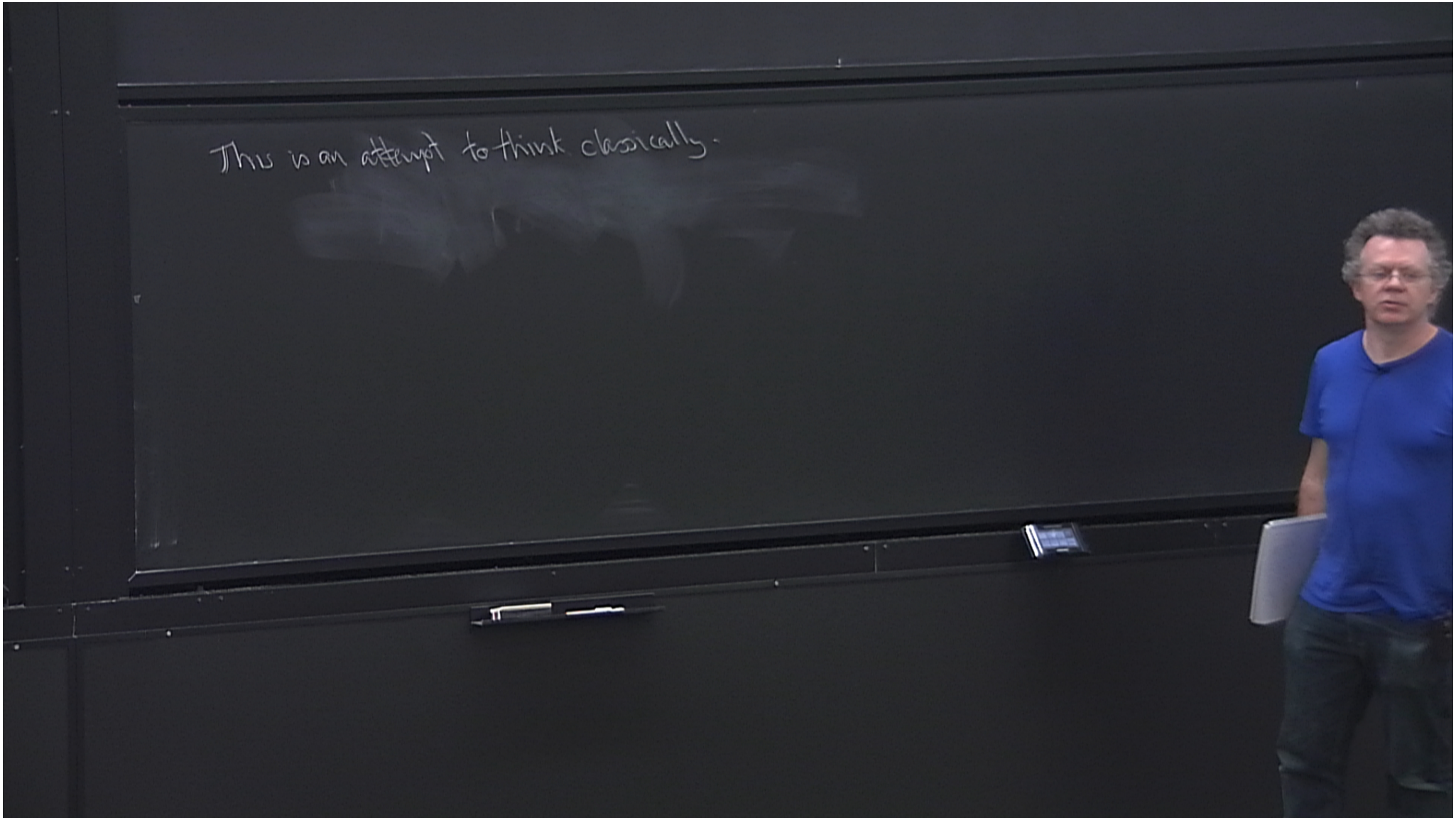

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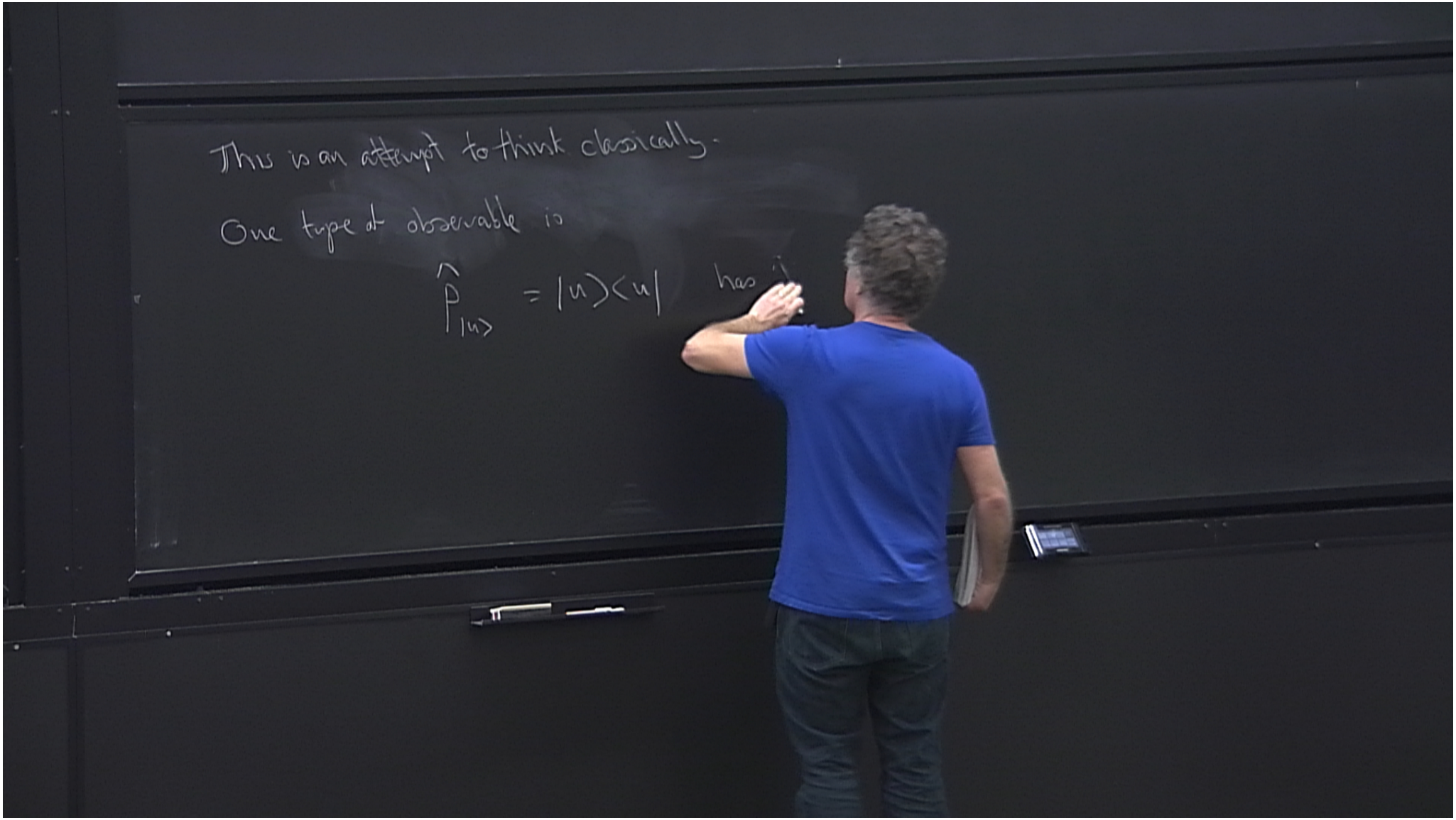

Contextuality Kochen Specker 1966(7)
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In each run of
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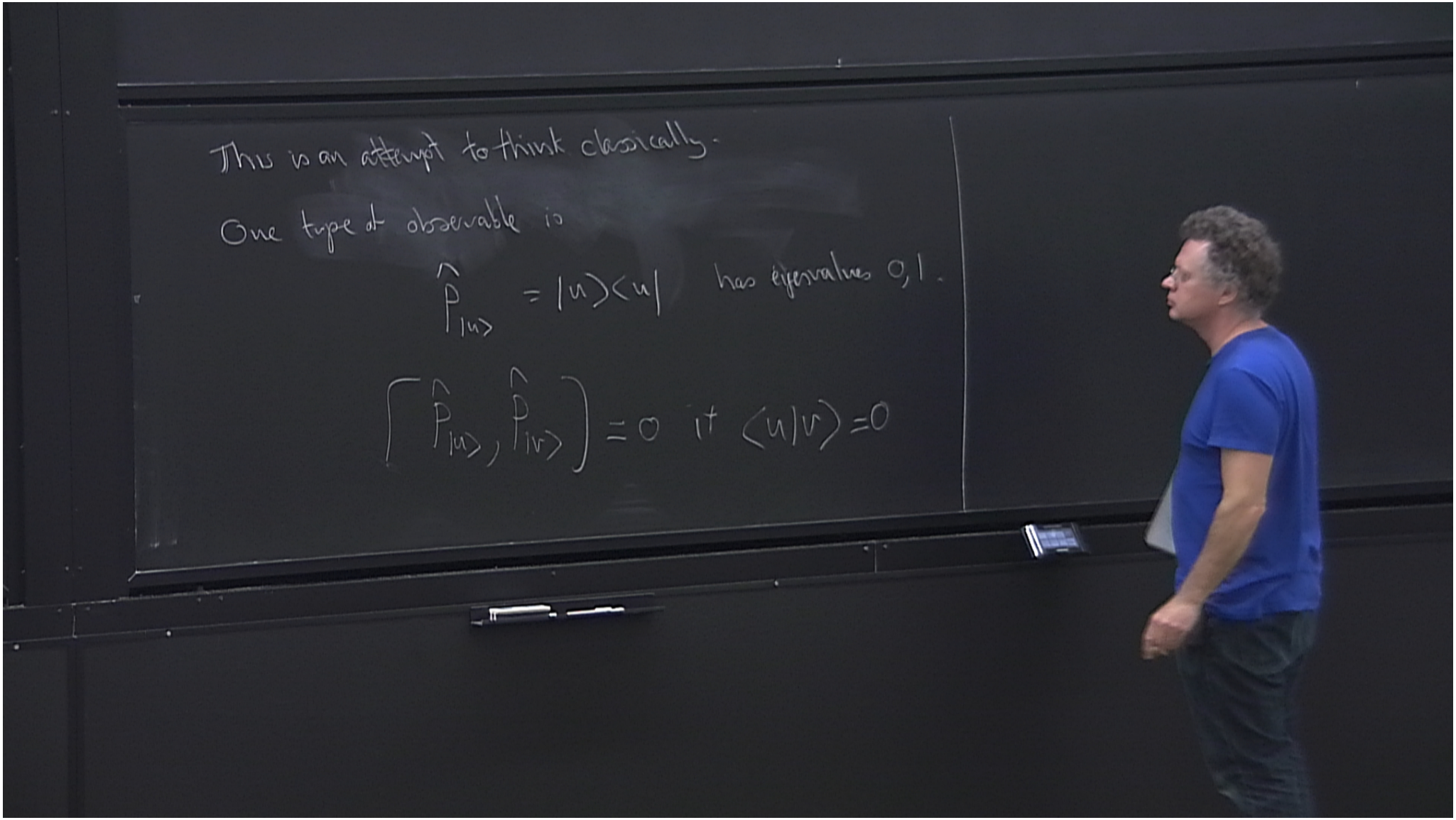

Contextuality Kochen Specker 1966(67)
Spekkens (2004)
Non-Contextuality (standard KS assumption).
In each run of experiment, at a given time, there is an underlying
$v(\hat{A}) \in \{$ set of eigenvalues of $\hat{A}\}$
that would be seen if we measure $\hat{A}$. Furthermore,

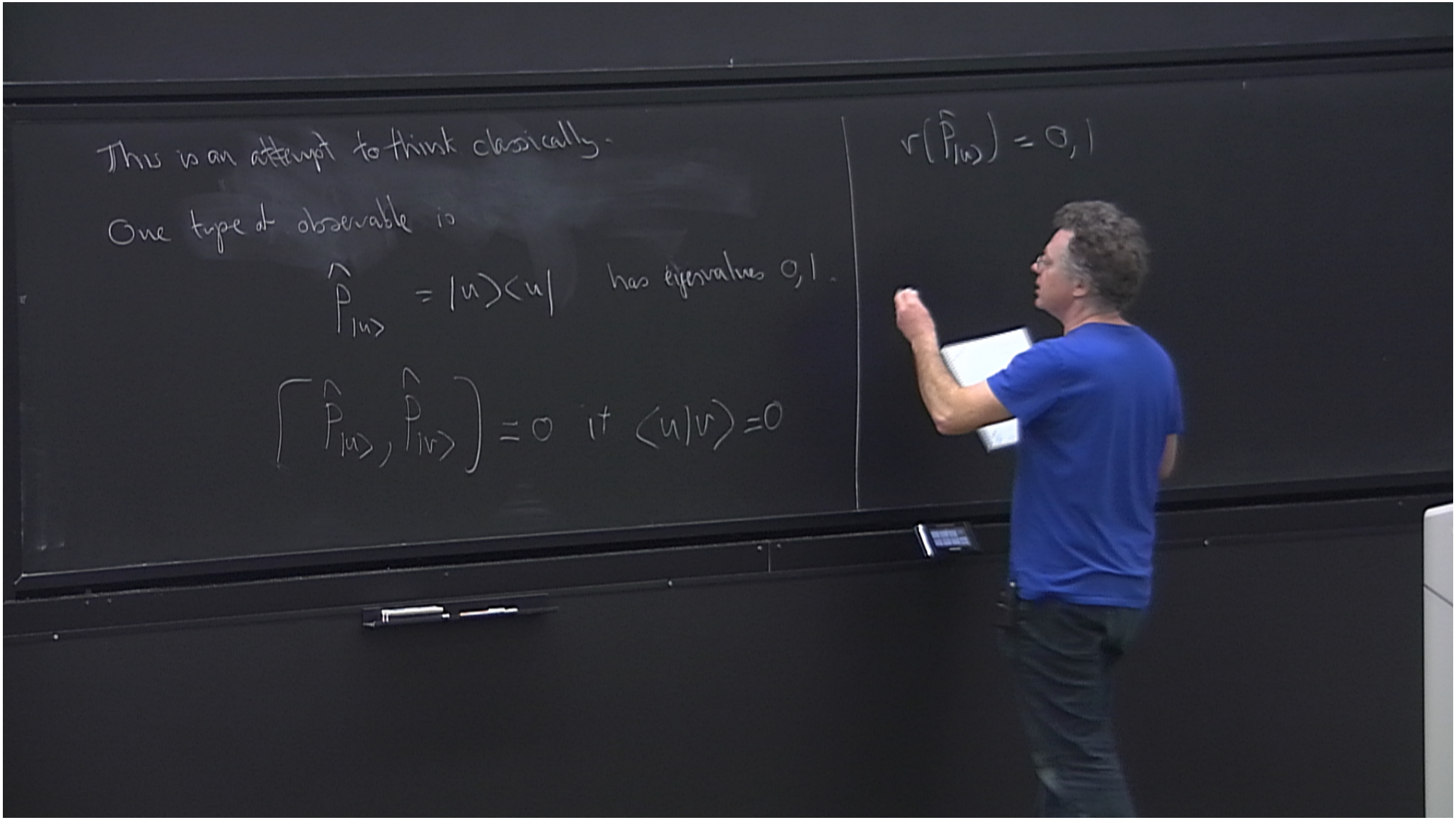

this value is independent of any other (commuting)
observable

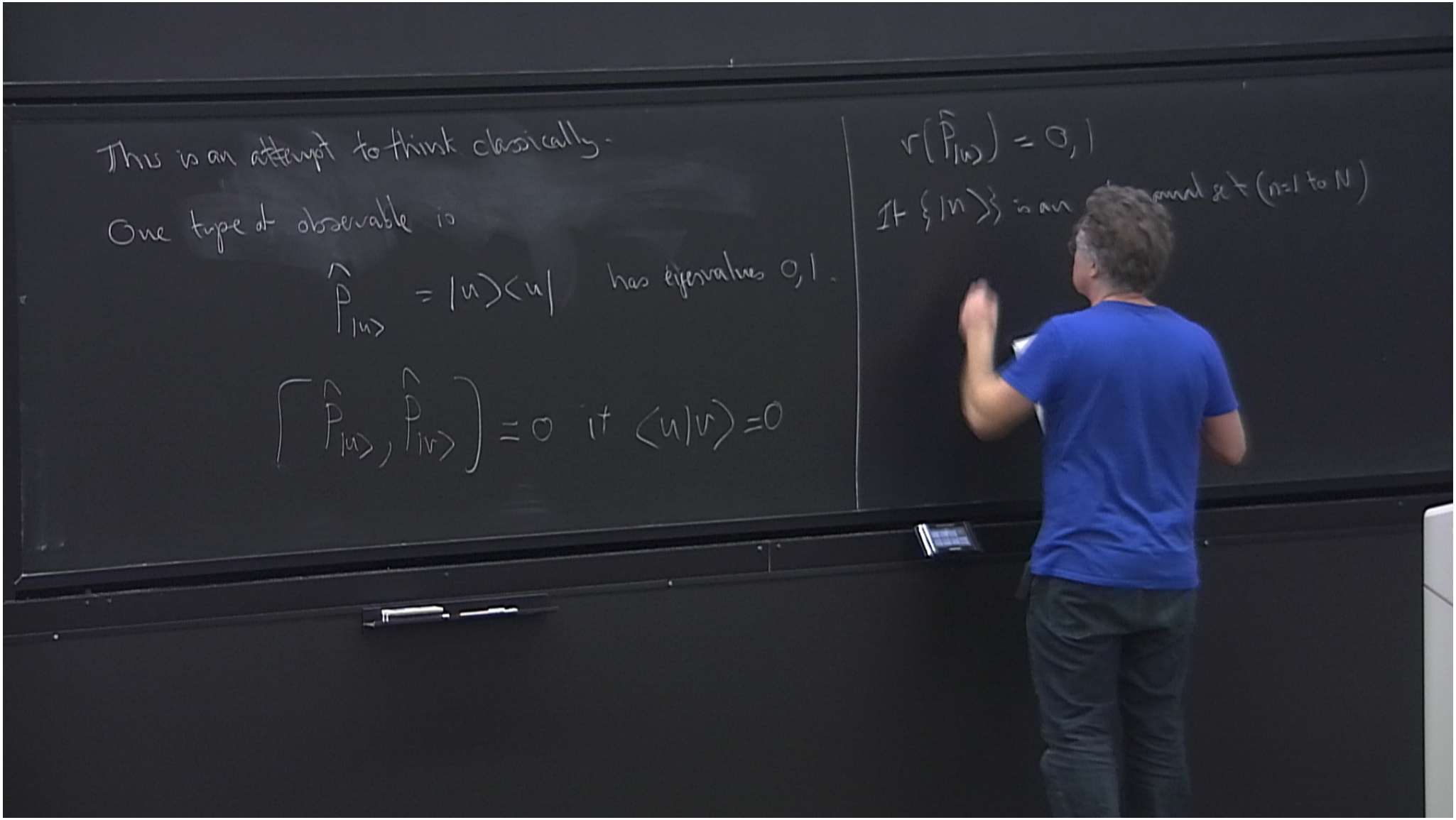

this value is independent of any other (commuting) observable measured at the same time
I.e.
$v(A|B) = v(A|C) = v(A)$
where $[\hat{A},\hat{B}] = [\hat{A},\hat{C}] = 0$

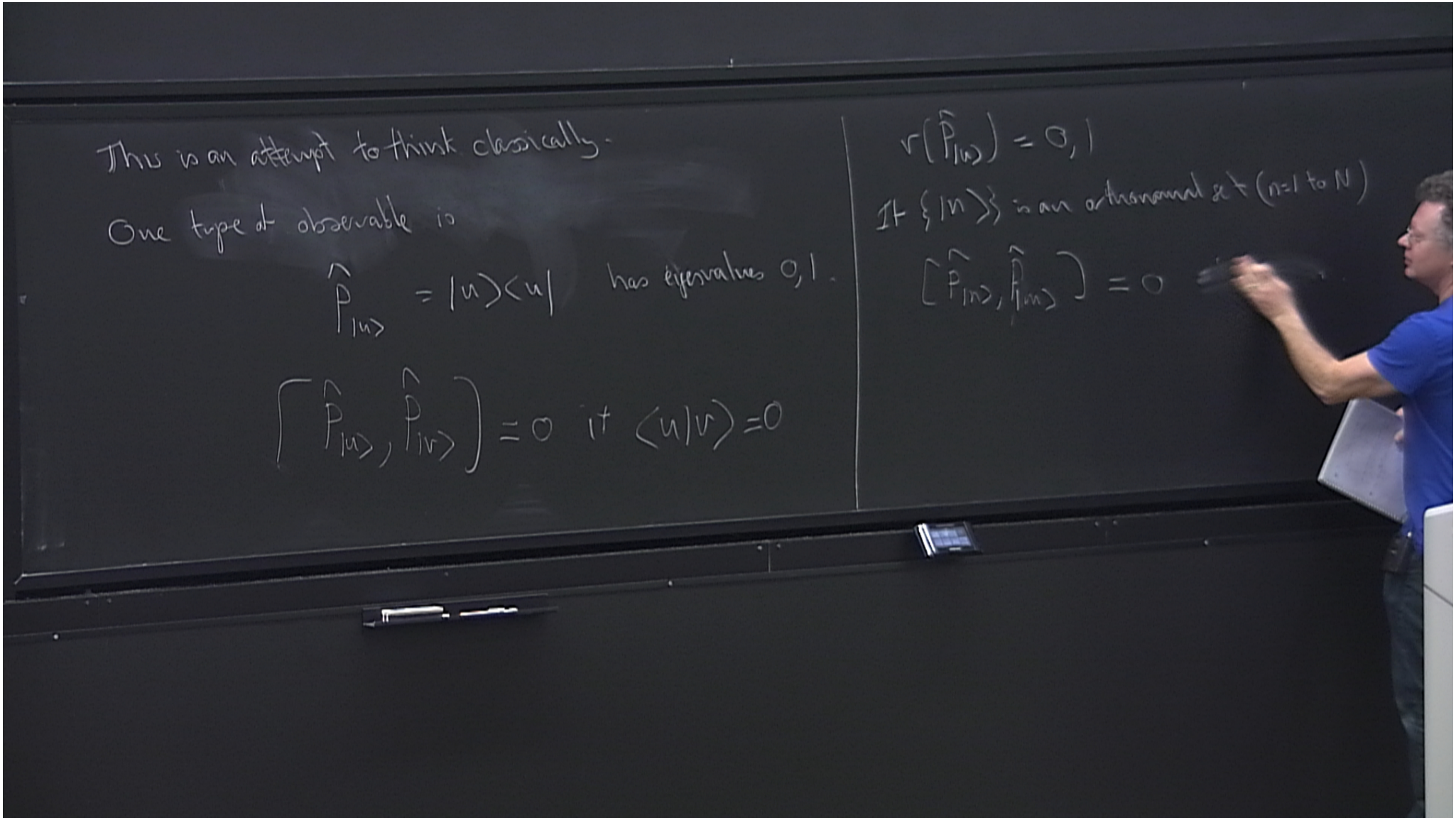

This is an attempt to think classically.

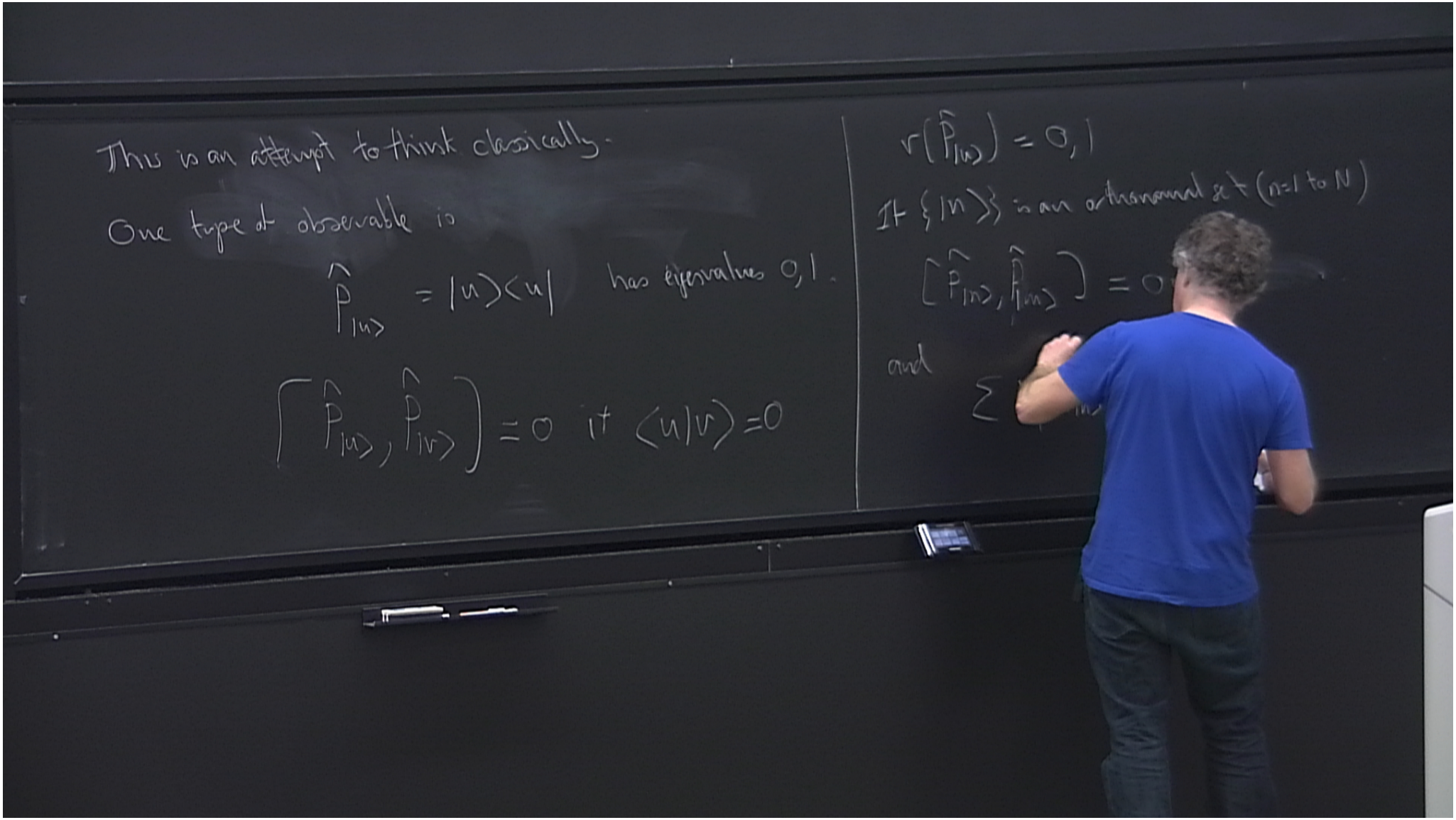

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One type of observable is
$\hat{P}_{|u\rangle} = |u\rangle\langle u|$ has

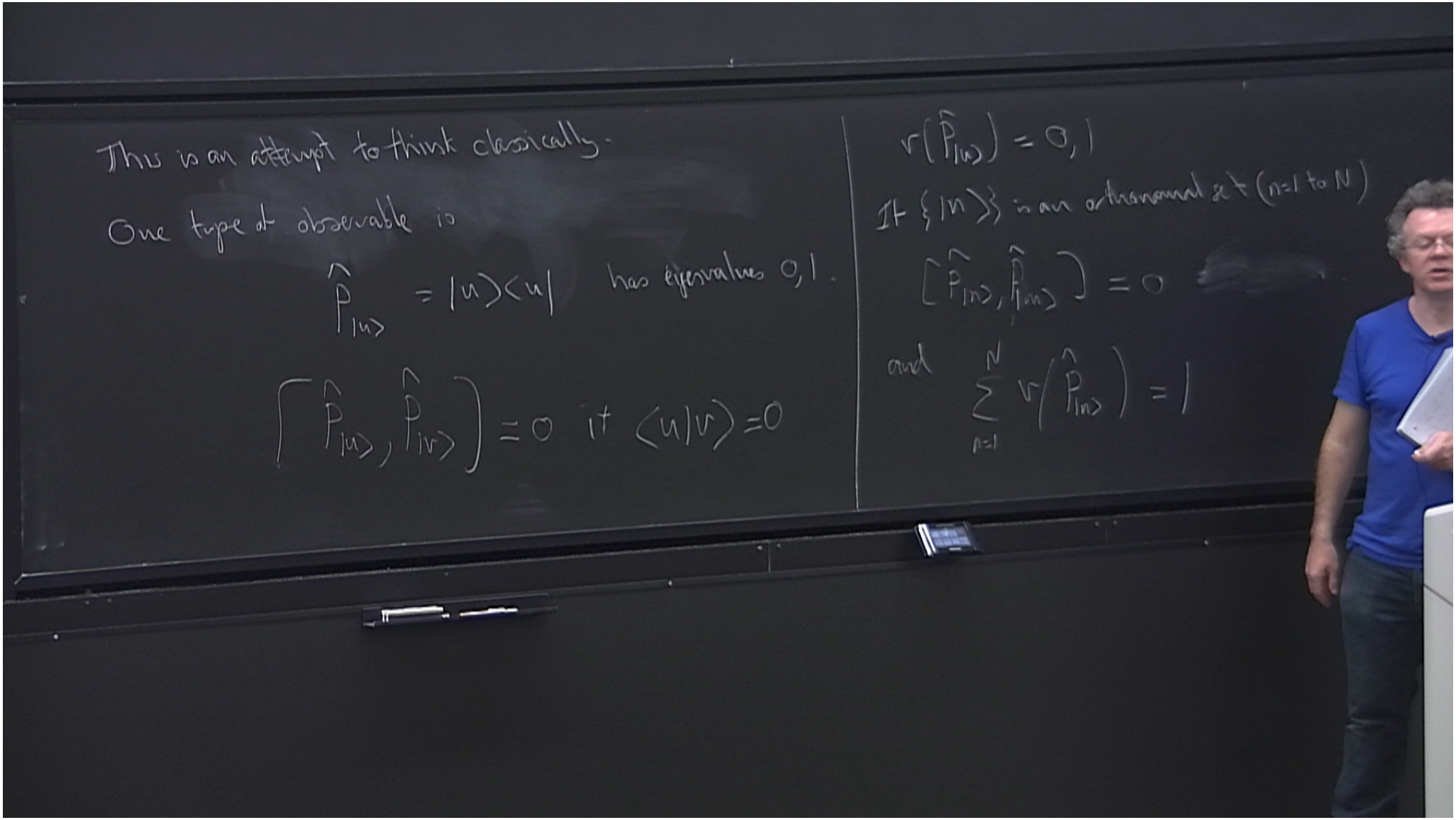

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One type of observable is
$\hat{P}_{|u\rangle} = |u\rangle\langle u|$ has eigenvalues 0, 1.
$\left[\hat{P}_{|u\rangle}, \hat{P}_{|v\rangle}\right] = 0$ if $\langle u|v\rangle = 0$

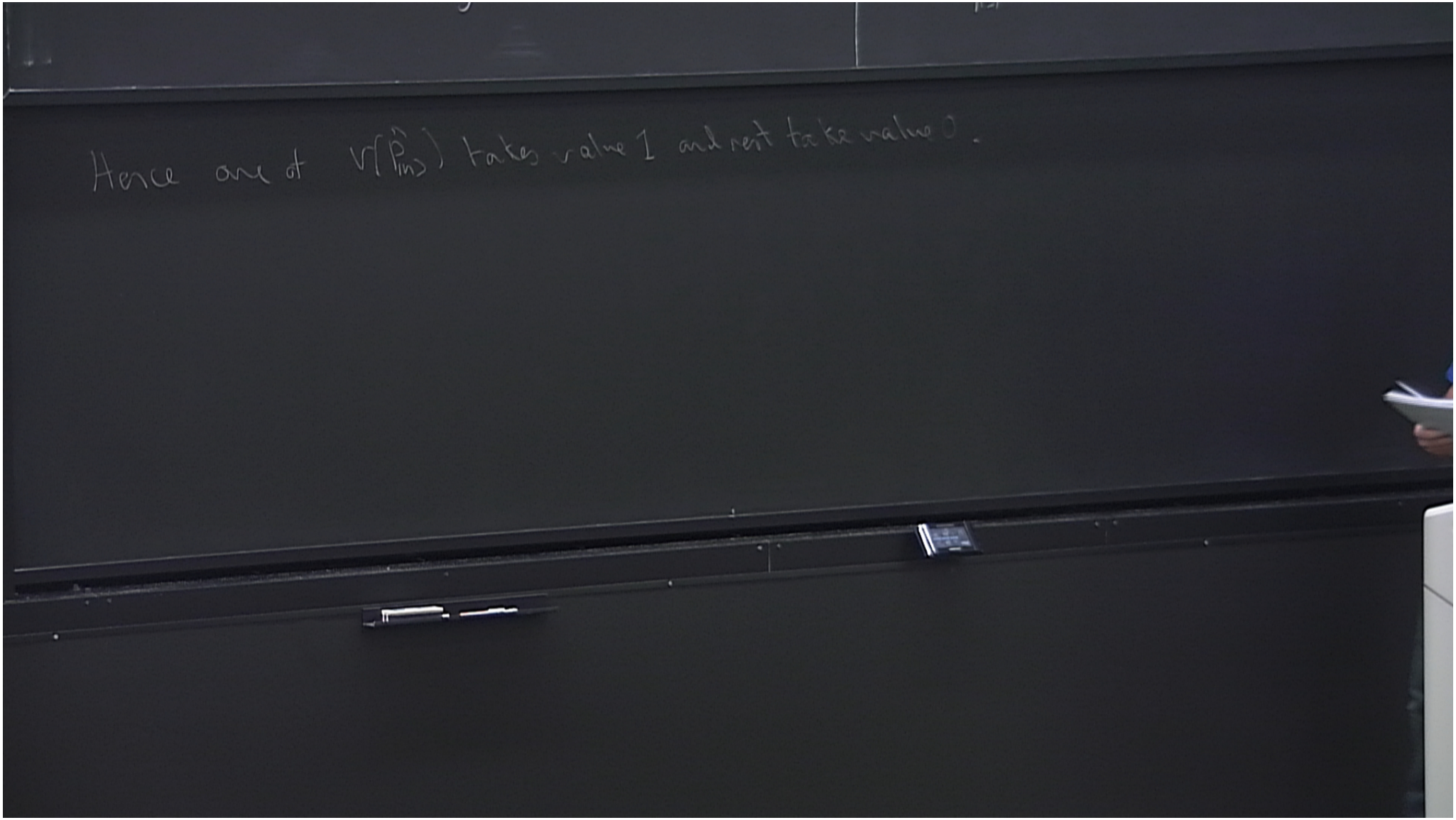

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$r(\hat{P}_{|u\rangle}) = 0, 1$

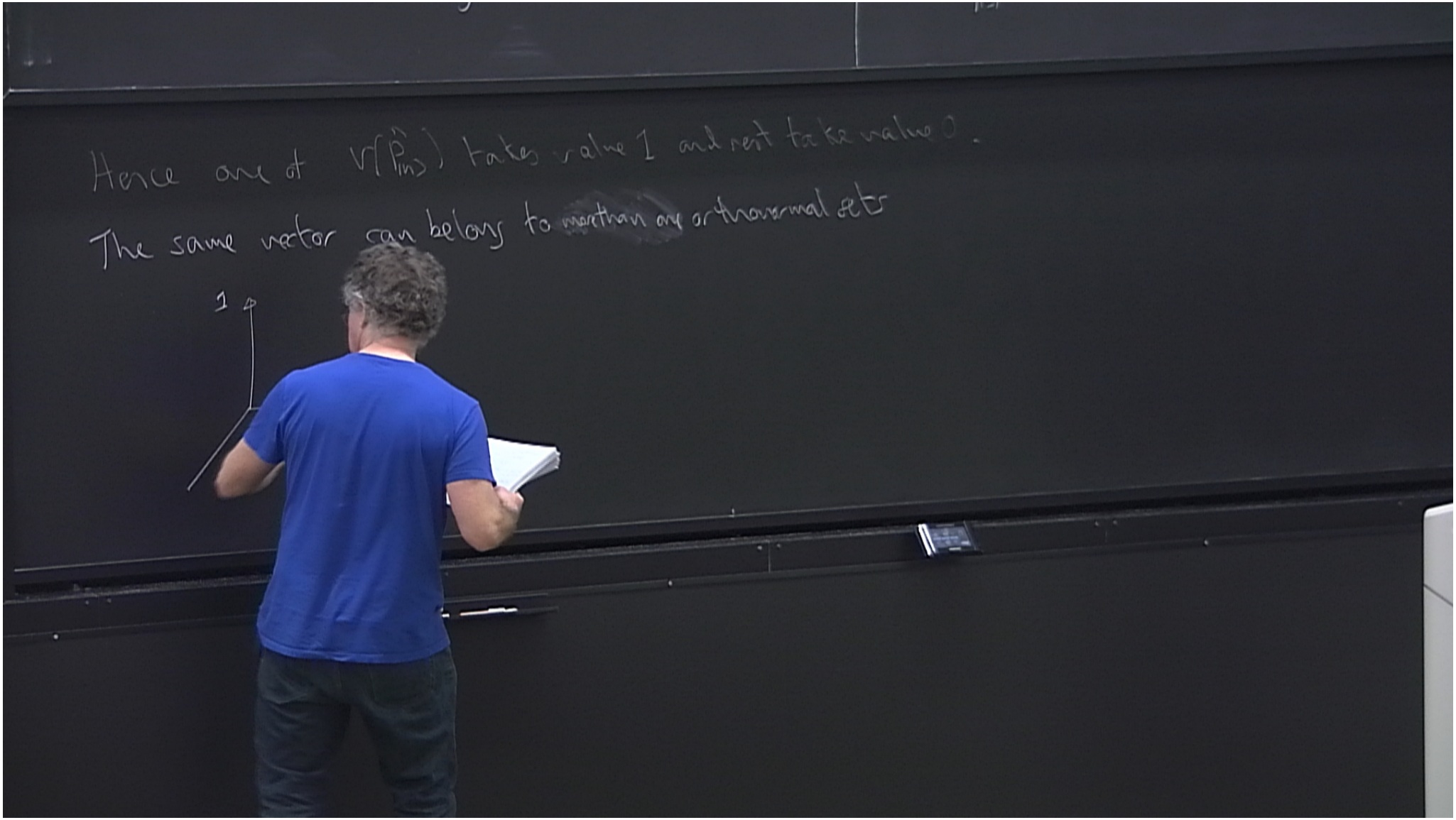

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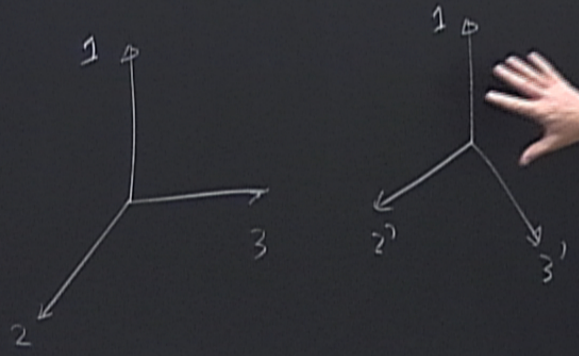

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$r(\hat{P}_{|u\rangle}) = 0, 1$
If $\{|n\rangle\}$ is an orthonormal set (n=1 to N)
$\left[\hat{P}_{|n\rangle}, \hat{P}_{|m\rangle}\right] = 0$
and $\sum_{n=1}^{N} r(\hat{P}_{|n\rangle}) = 1$

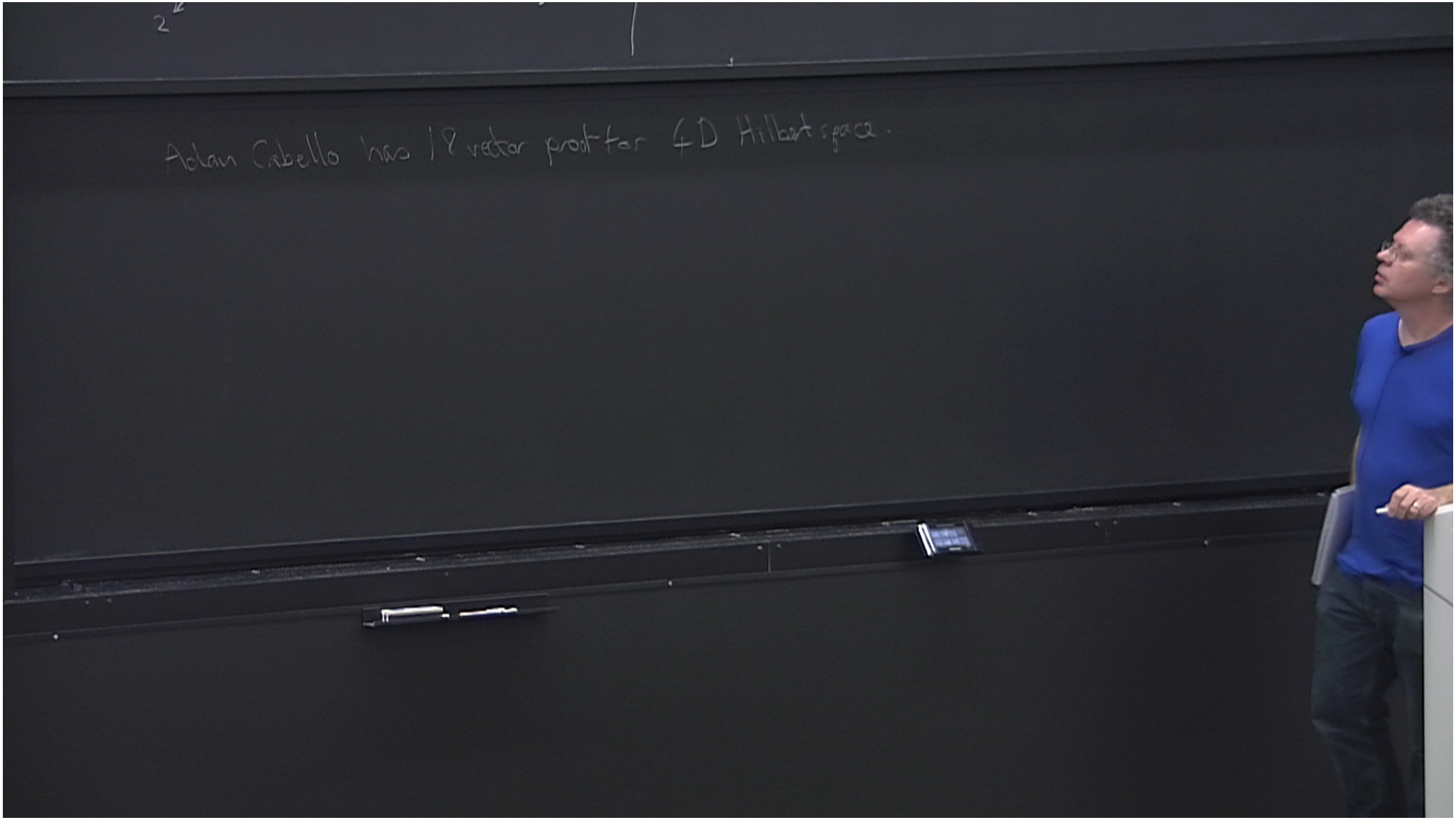

Hence one of $V(\hat{P}_{ins})$ takes value 1 and rest take value 0.

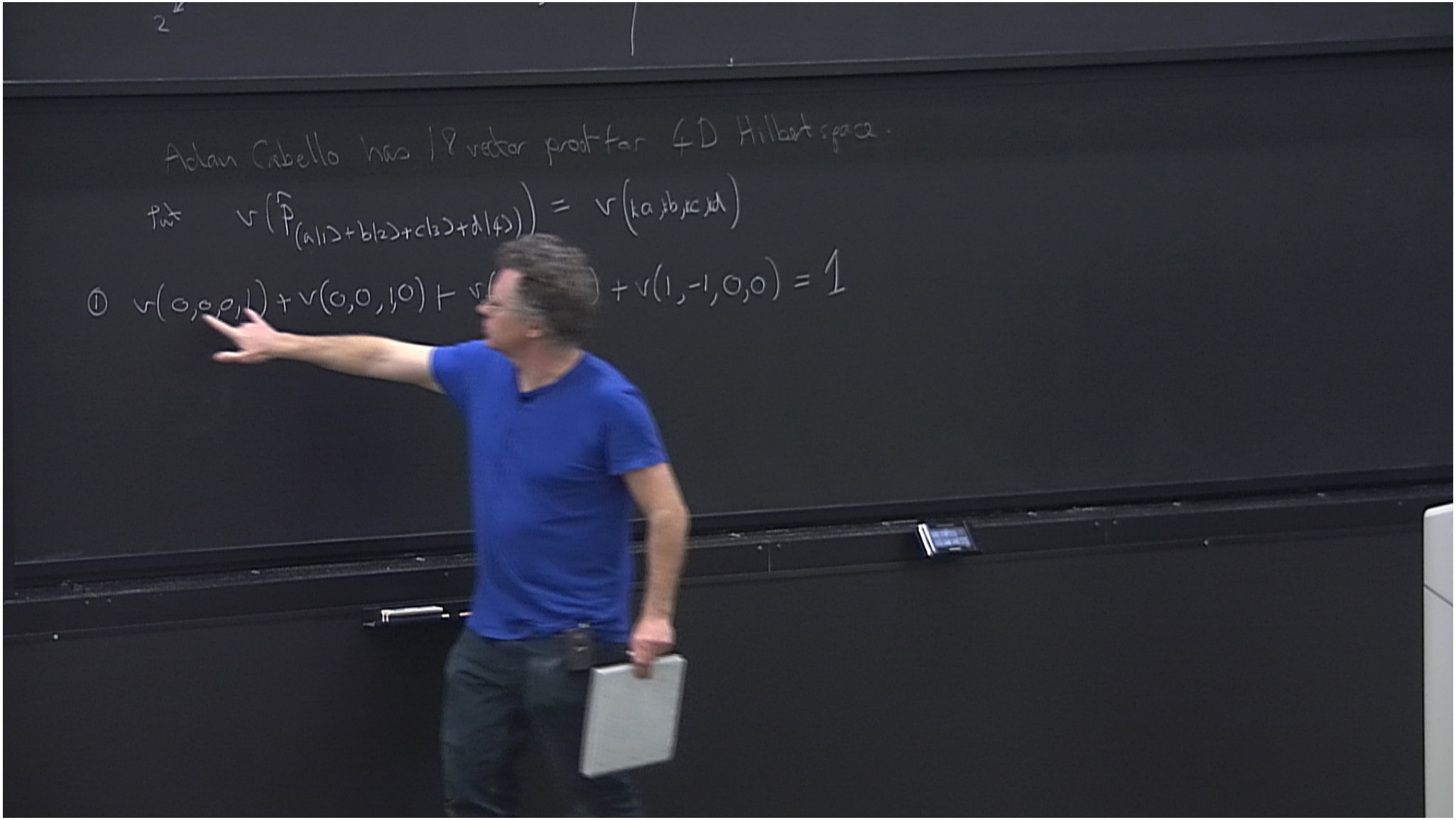

Hence one of $V(\hat{P}_{|n\rangle})$ takes value 1 and rest take value 0.
The same vector can belong to more than one orthonormal sets
1

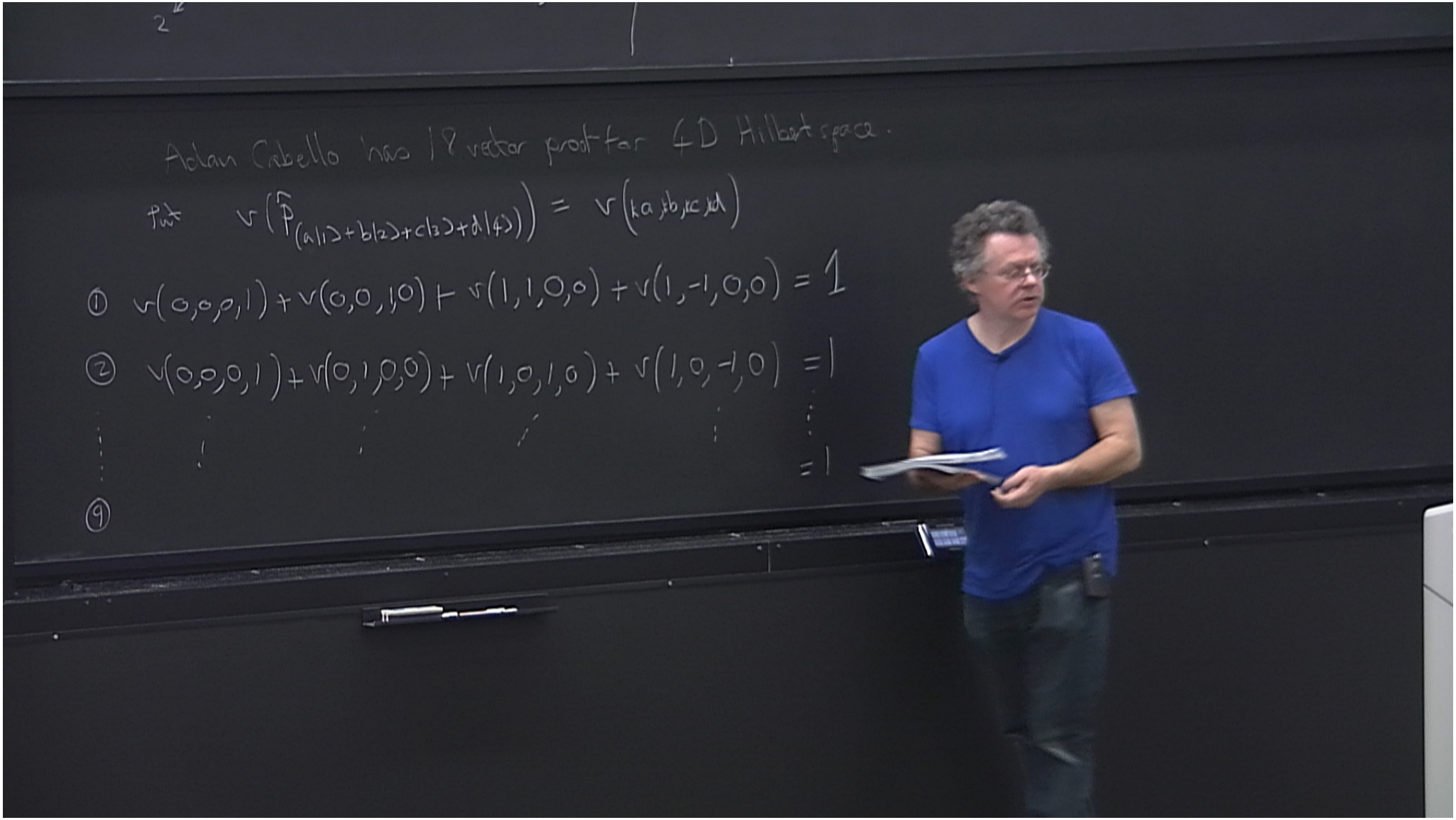
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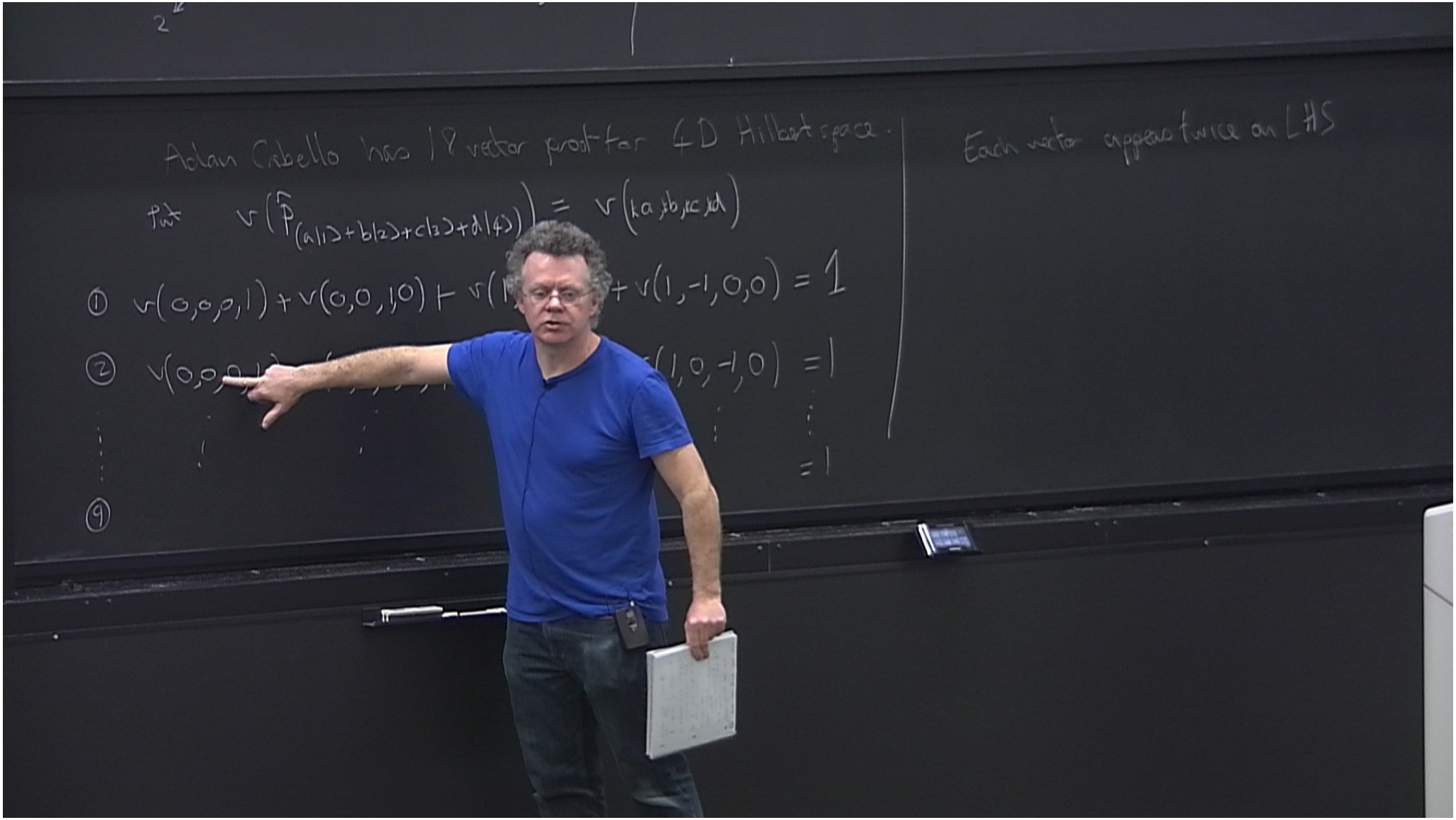
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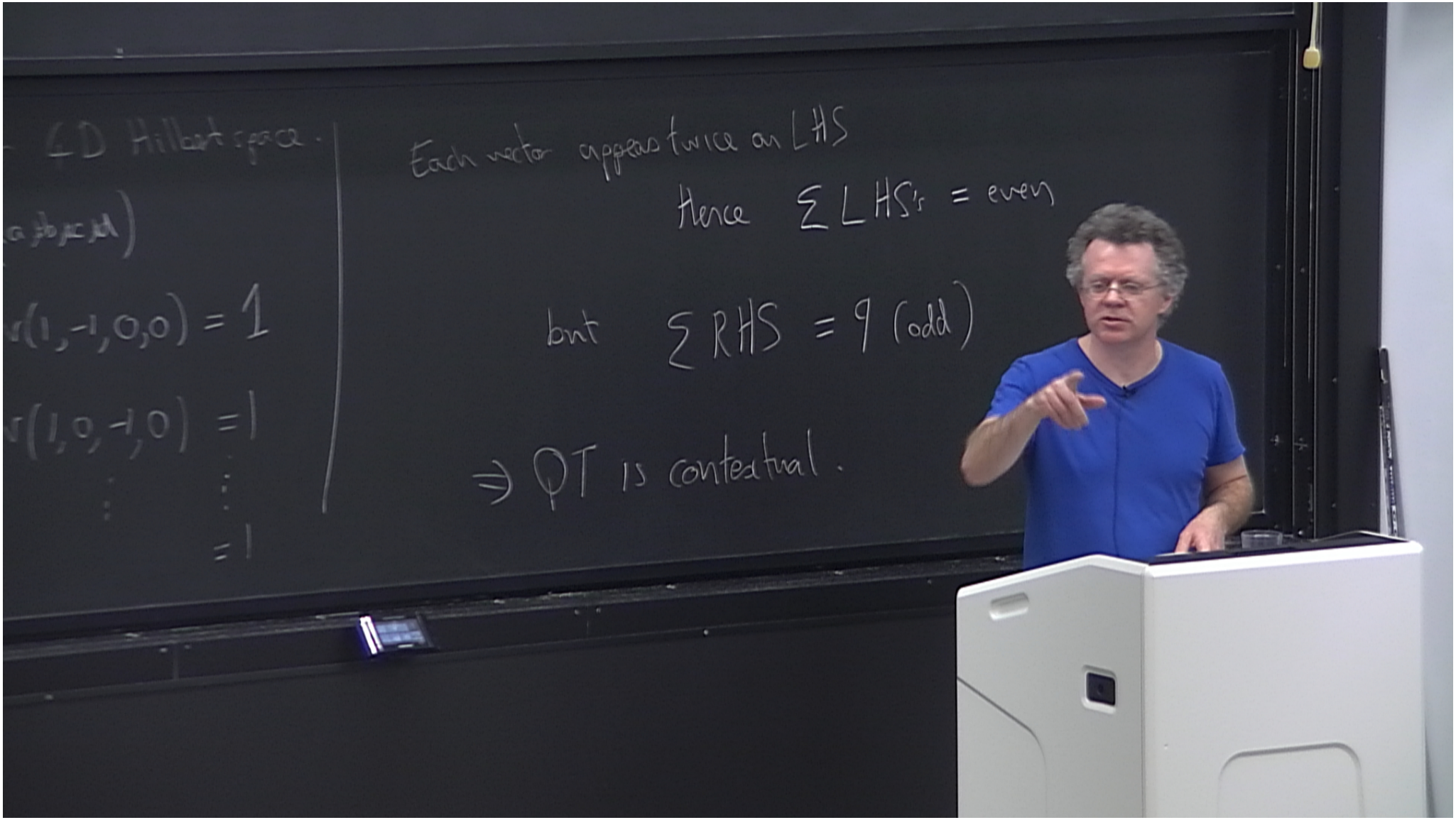


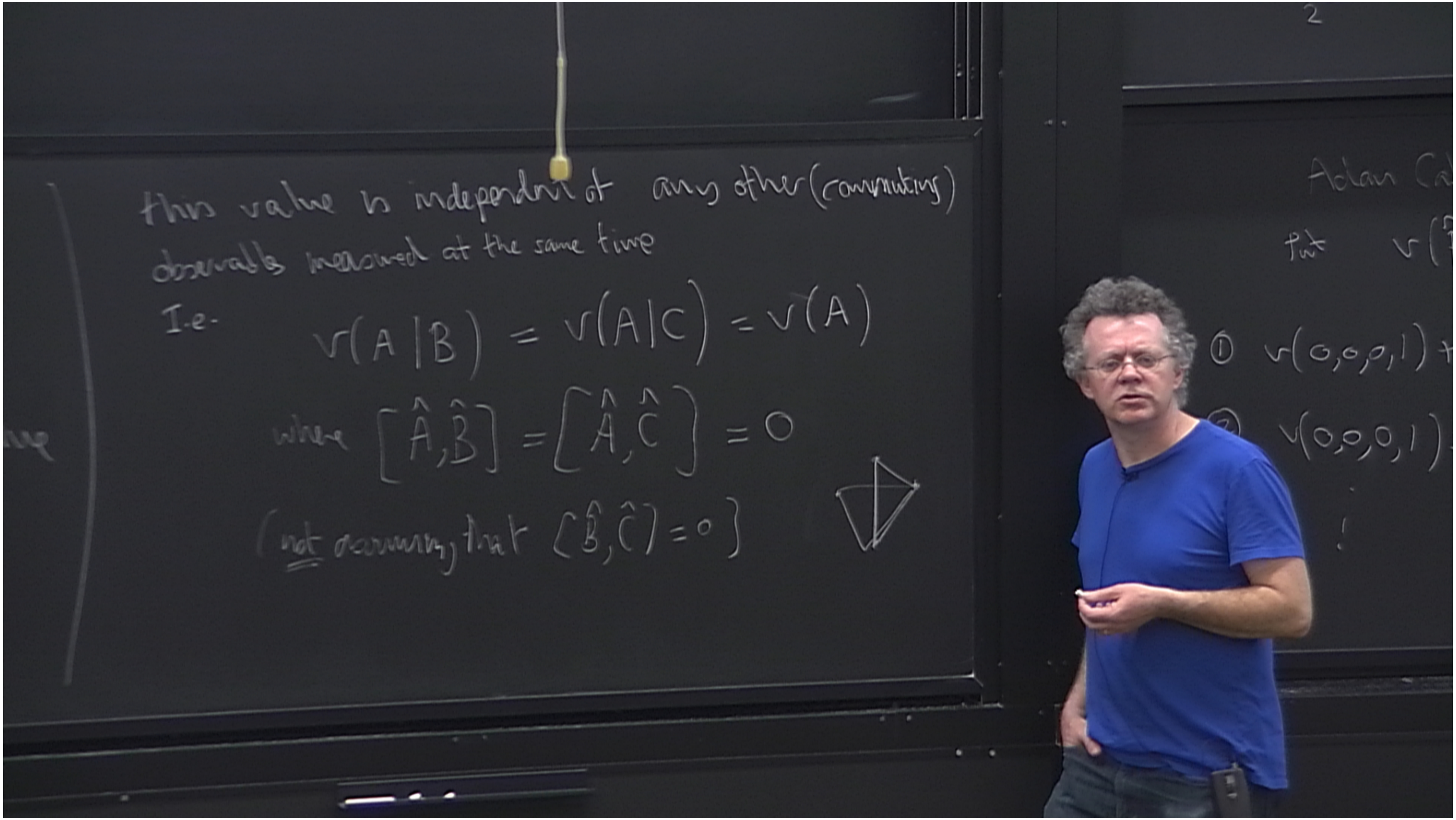

Adan Cabello has 18 vector proof for 4D Hilbert space.

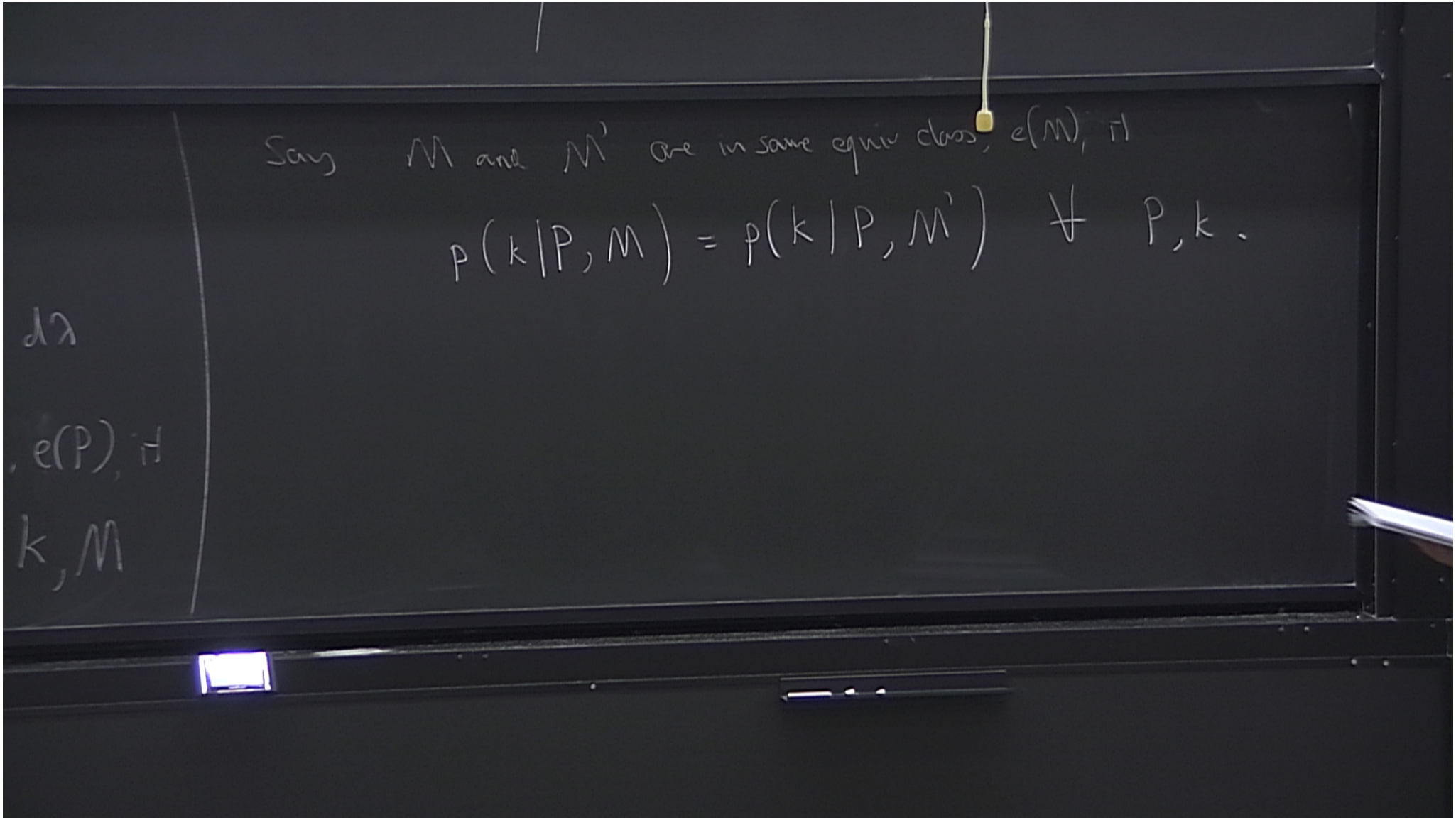

Adan Cabello has 18 vector proof for 4D Hilbert space.
but $v\left(\hat{P}_{(a|1\rangle+b|2\rangle+c|3\rangle+d|4\rangle)}\right) = v(ka, kb, kc, kd)$
① $v(0,0,0,1) + v(0,0,1,0) + v(\quad) + v(1,-1,0,0) = 1$

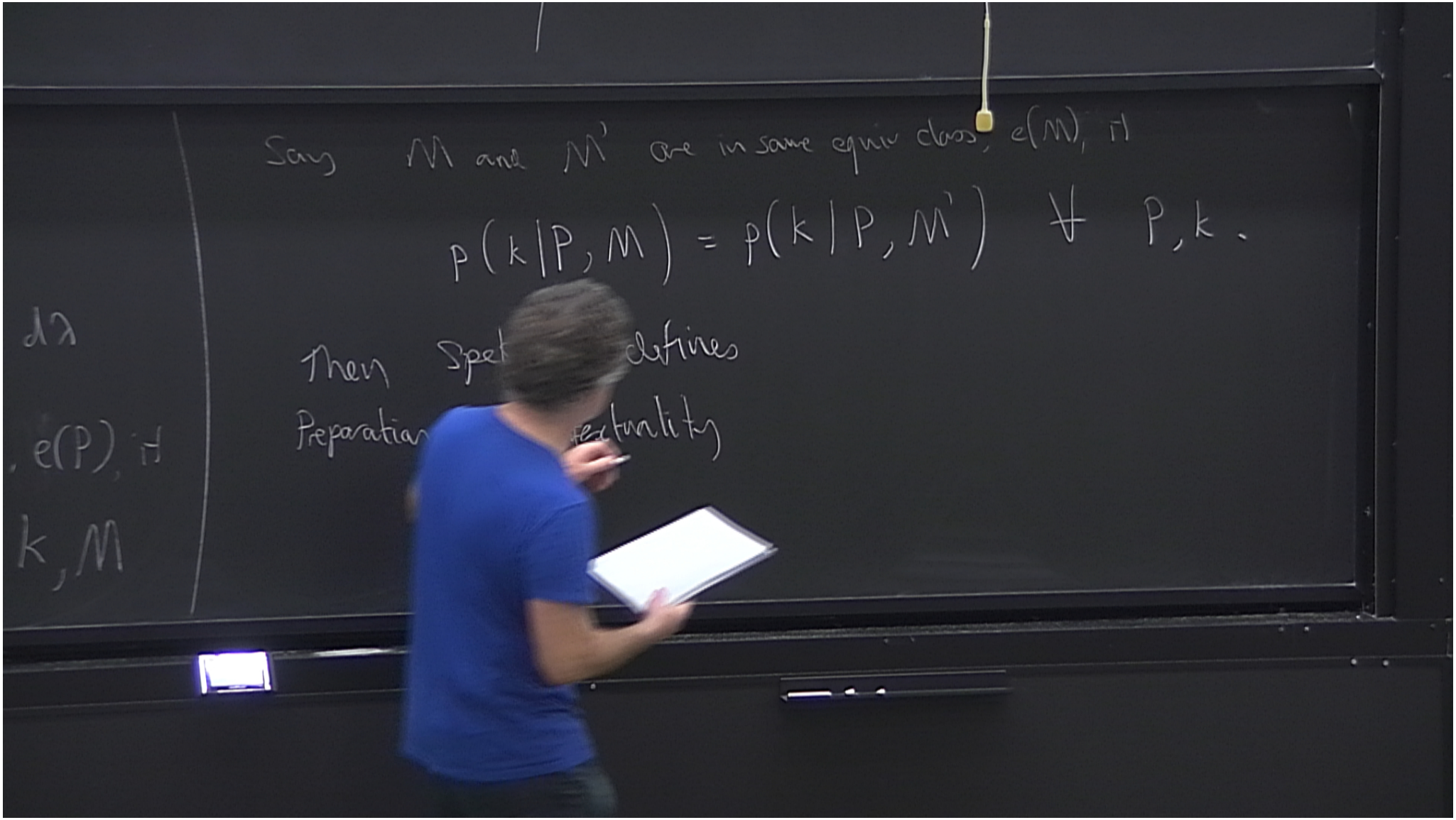

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$= 1$
⑨

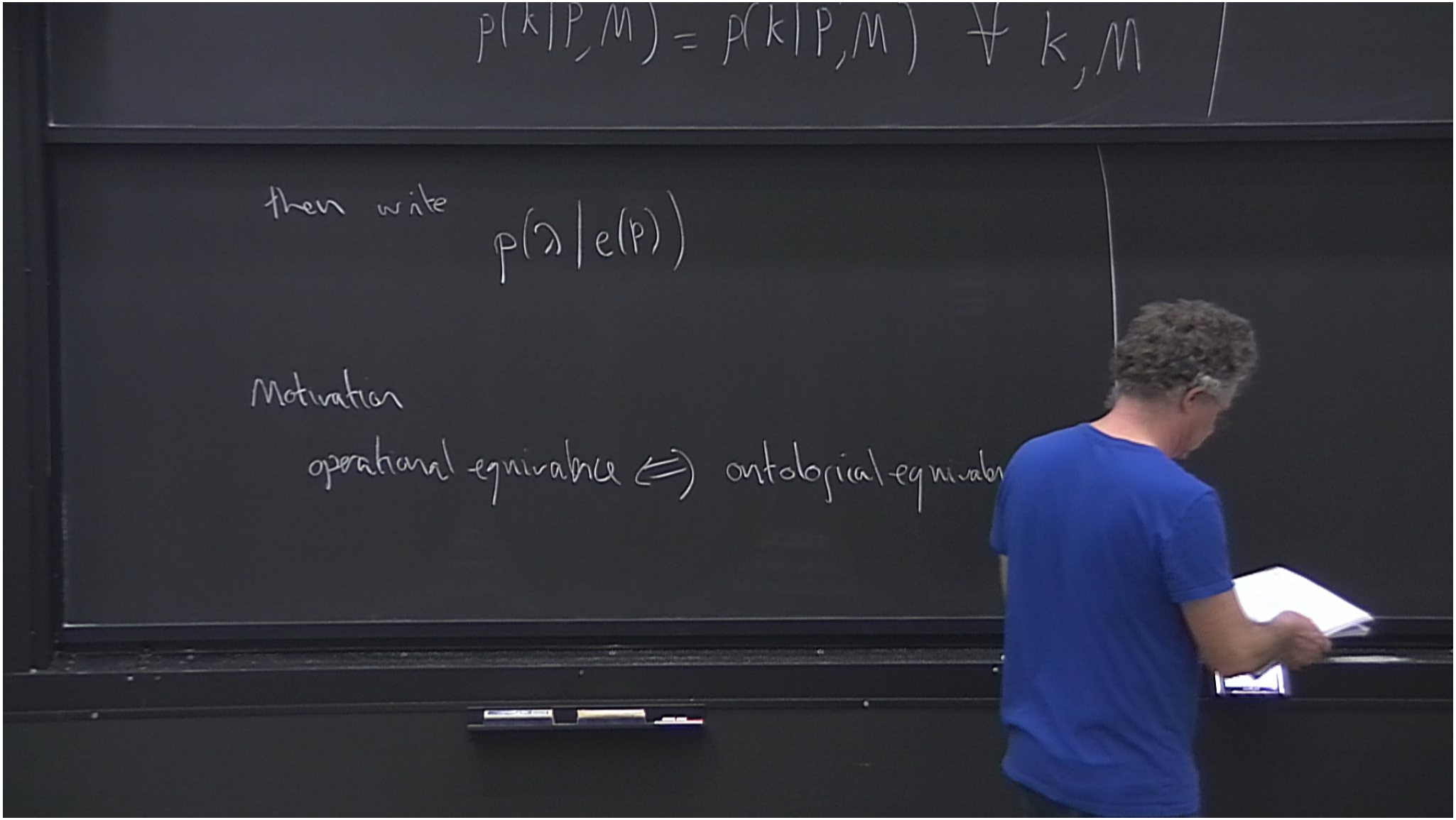

Adan Cabello has 18 vector proof for 4D Hilbert space.
Each vector appears twice on LHS

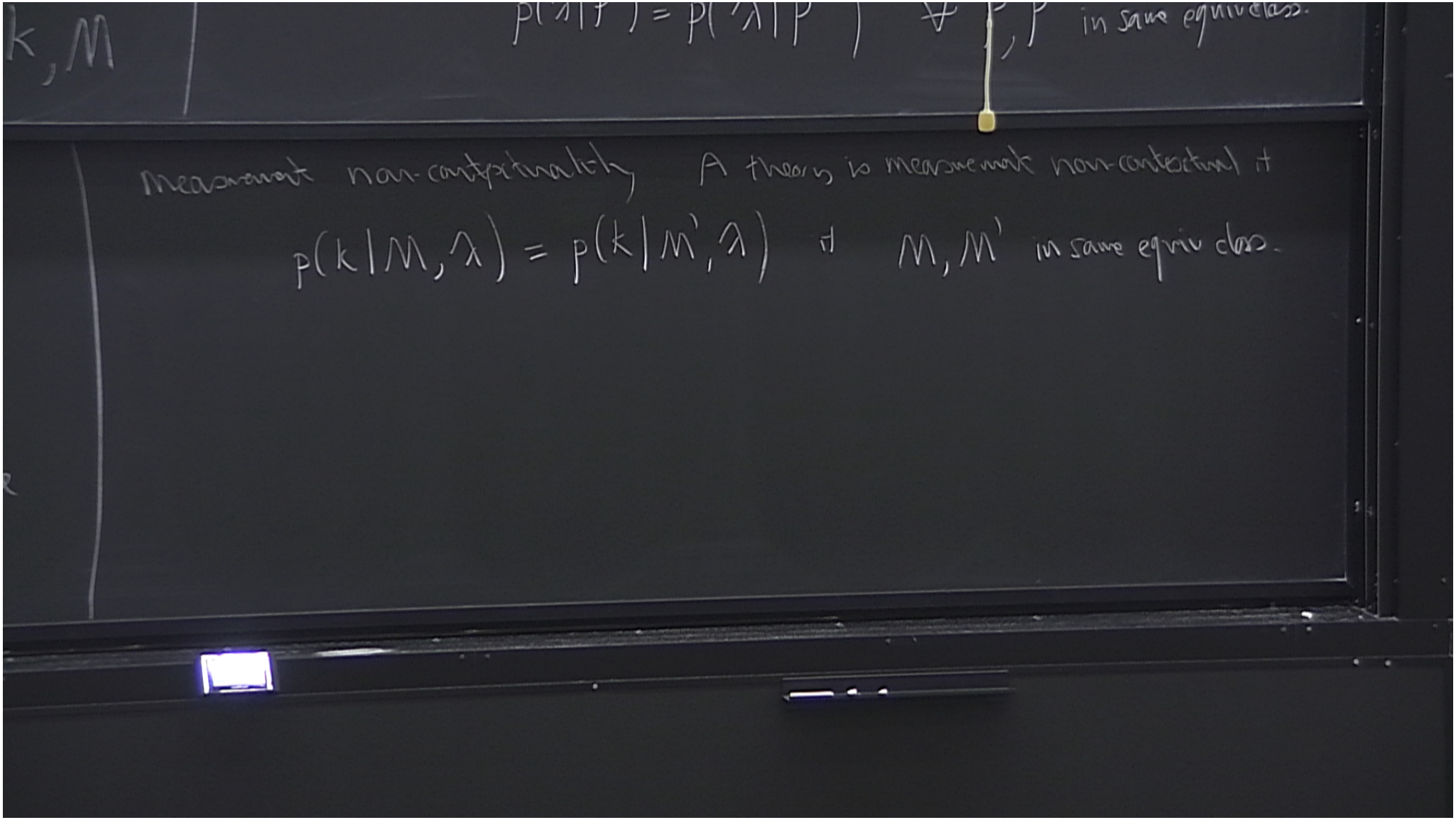

4D Hilbert space.
Each vector appears twice on LHS
Hence Σ LHS's = even
but Σ RHS = 9 (odd)
⇒ QT is contextual.

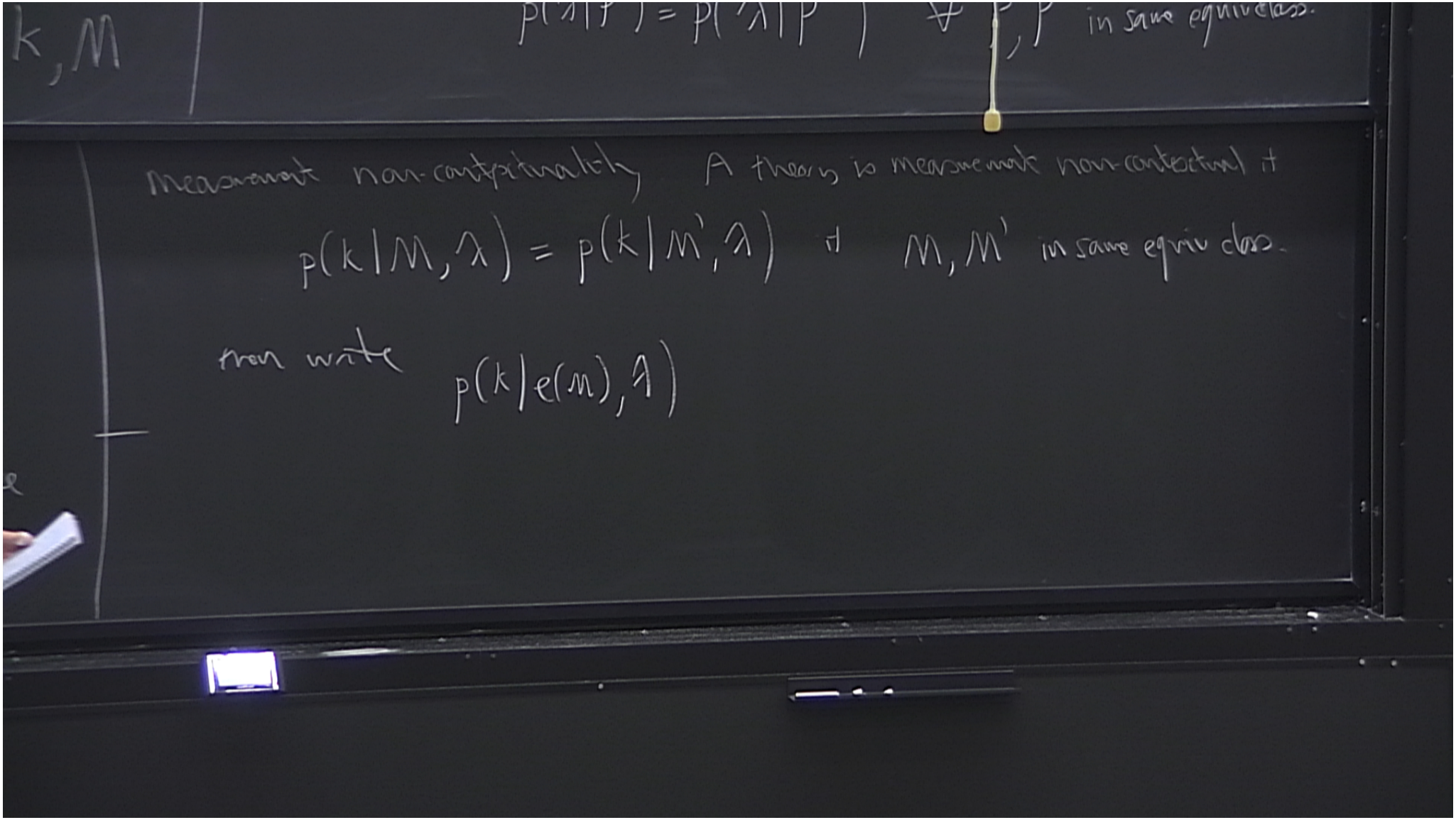

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(not assuming that $[\hat{B},\hat{C}] = 0$)
2
Adan Ca
Put v(
① v(0,0,0,1)
② v(0,0,0,1)

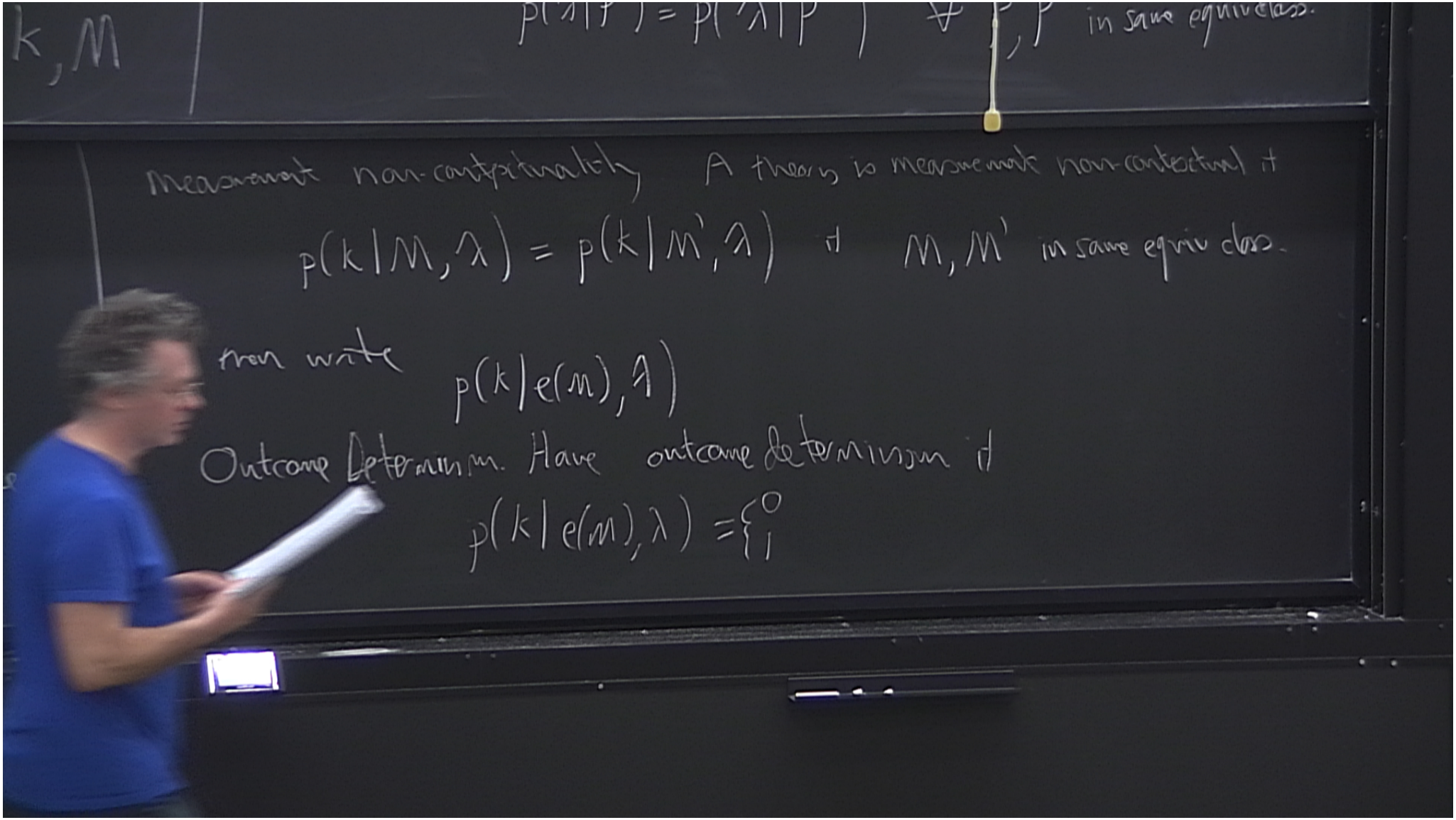

Say M and M' are in same equiv class: e(M), ii
p(k|P, M) = p(k|P, M') ∀ P, k.
dλ
, e(P), ii
k, M


Say M and M' are in same equiv class: e(M), i
$p(k|P,M) = p(k|P,M') \quad \forall \quad P,k.$
Then Spek defines
Preparation
dλ
e(P), i
k, M


$p(k|P,M) = p(k|P,M) \quad \forall\ k, M$
then write
$p(\lambda|e(P))$
Motivation
operational-equivalence $\Leftrightarrow$ ontological-equivalence


k, M
in same equiv class.
Measurement non-contextuality A theory is measurement non-contextual if
$p(k|M,\lambda) = p(k|M',\lambda)$ if M, M' in same equiv class.


k, M
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Outcome determinism. Have outcome determinism if
$p(k|e(M),\lambda) = \{0, 1$

k, M | $p(\lambda|P) = p(\lambda|P')$ $\forall$ P, P' in same equiv class.

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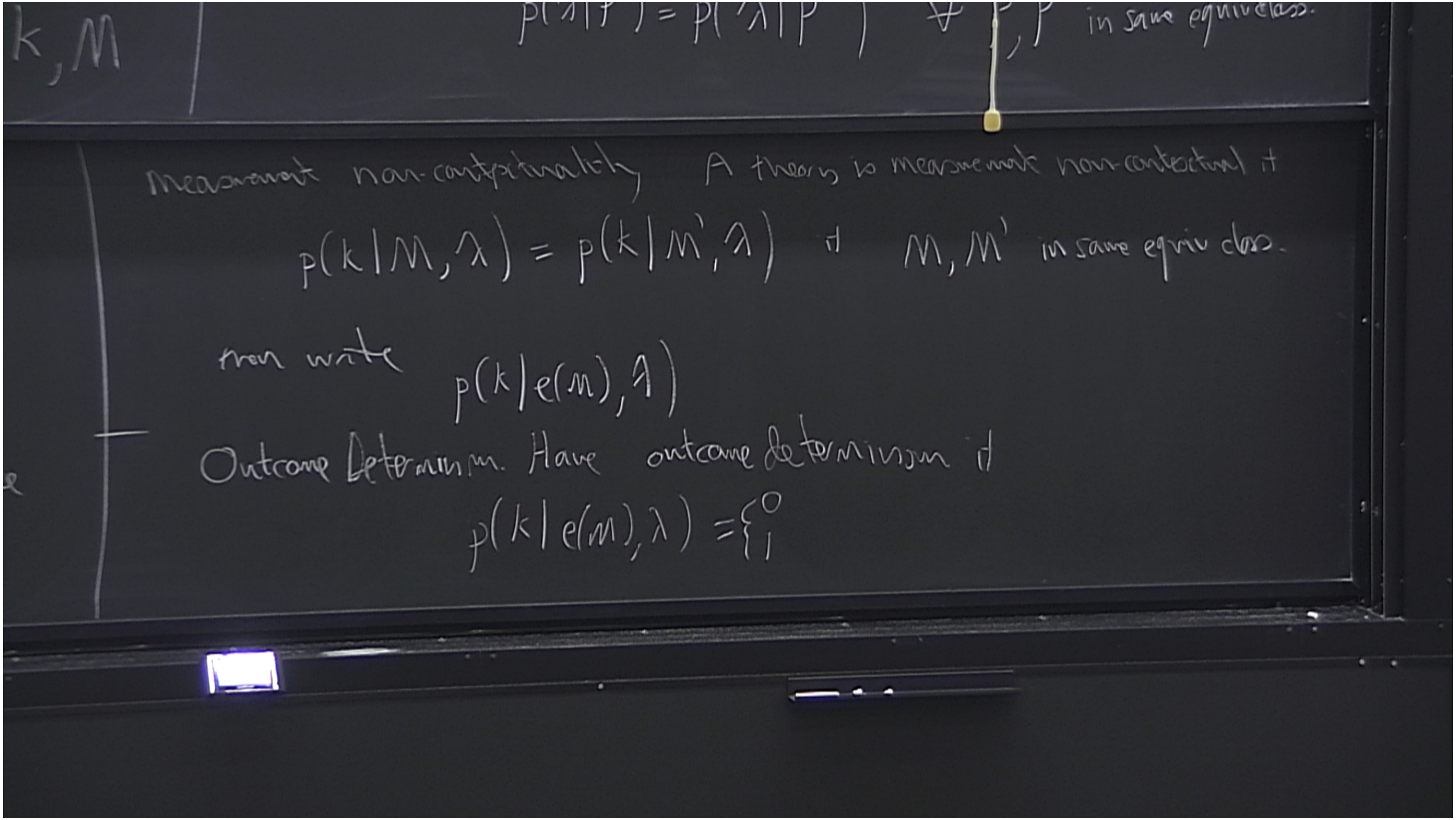
$$p(k|M,\lambda) = p(k|M',\lambda) \quad \text{if} \quad M, M' \text{ in same equiv class.}$$

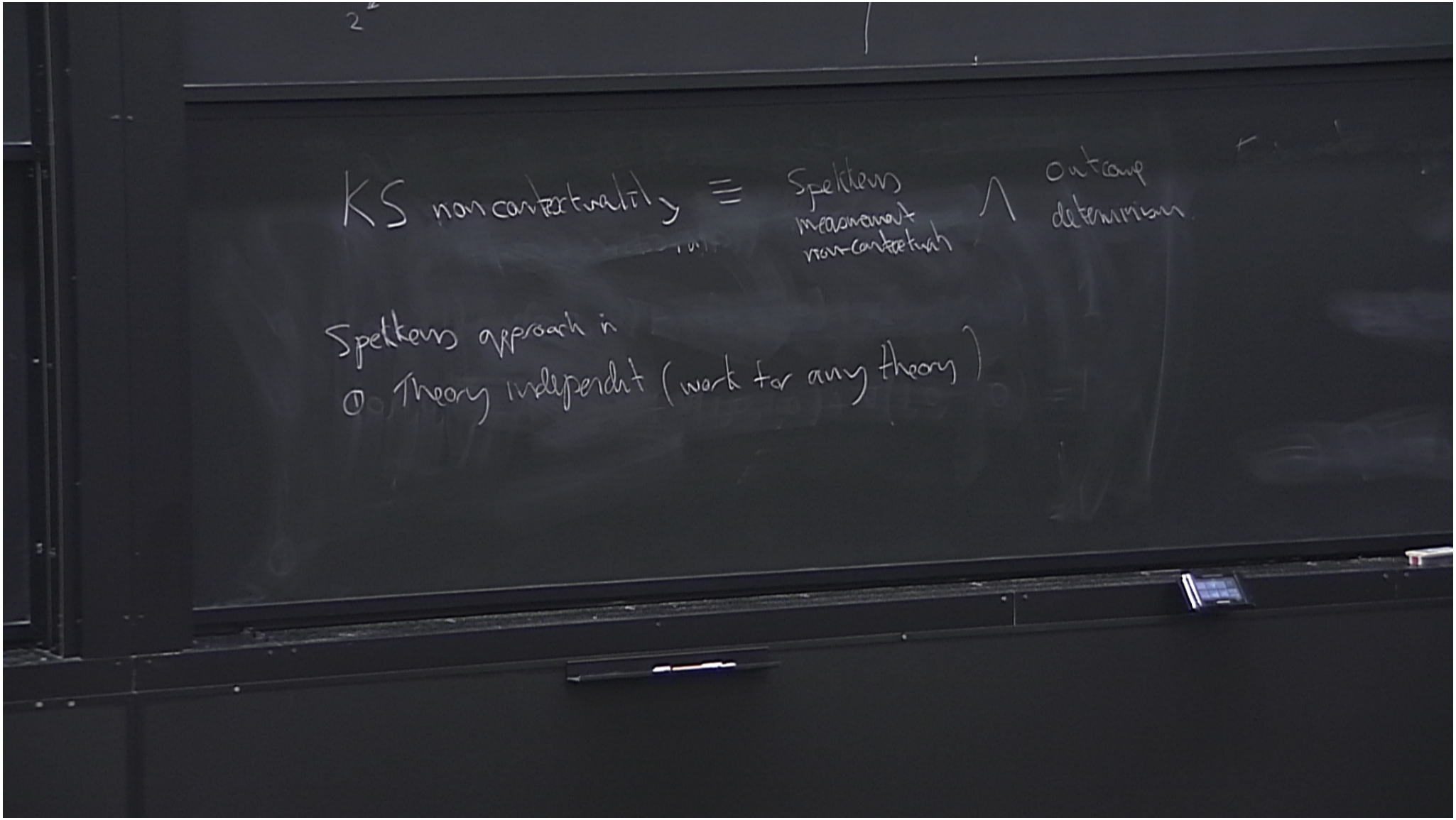
then write

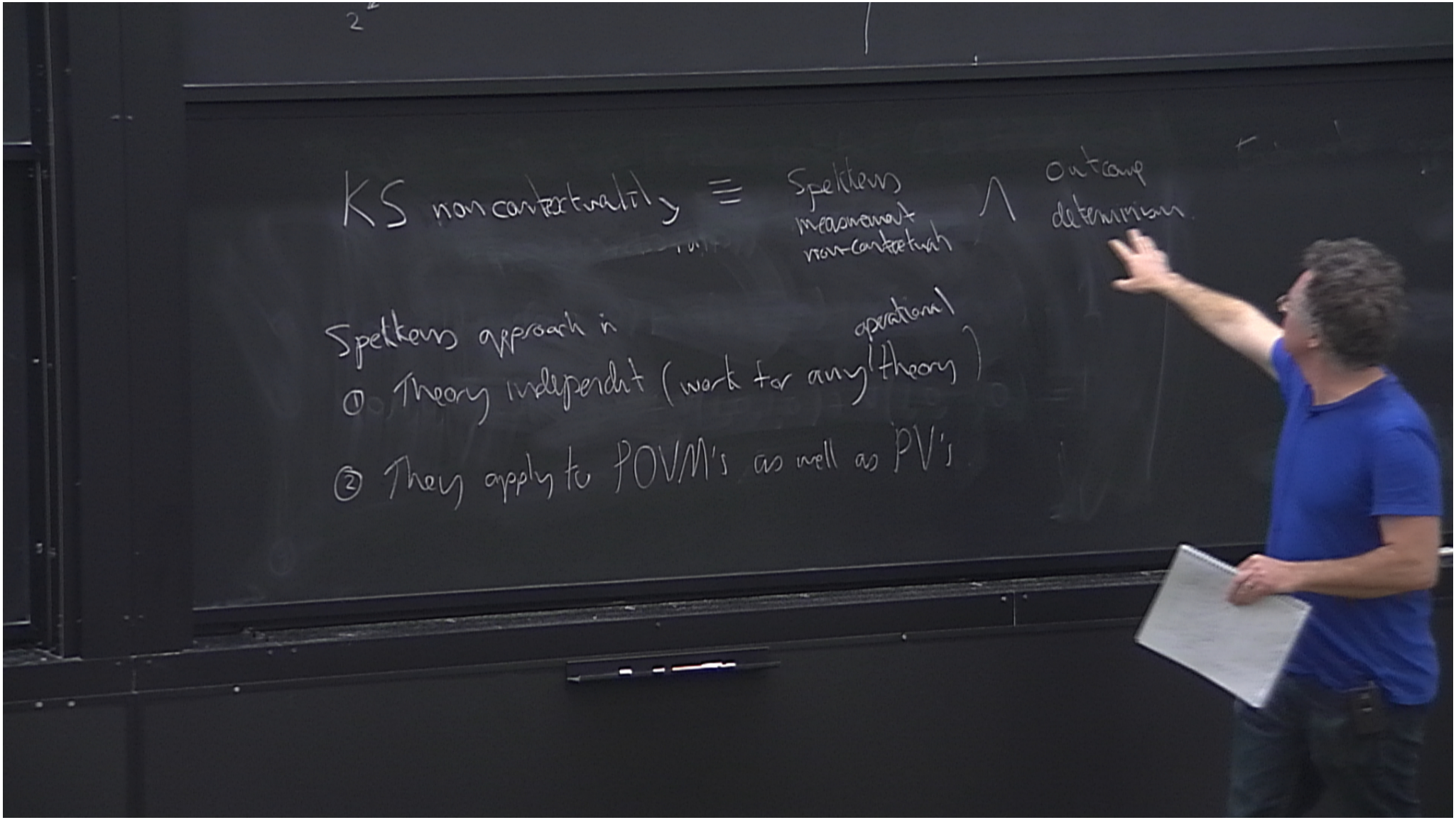
$$p(k|e(m),\lambda)$$

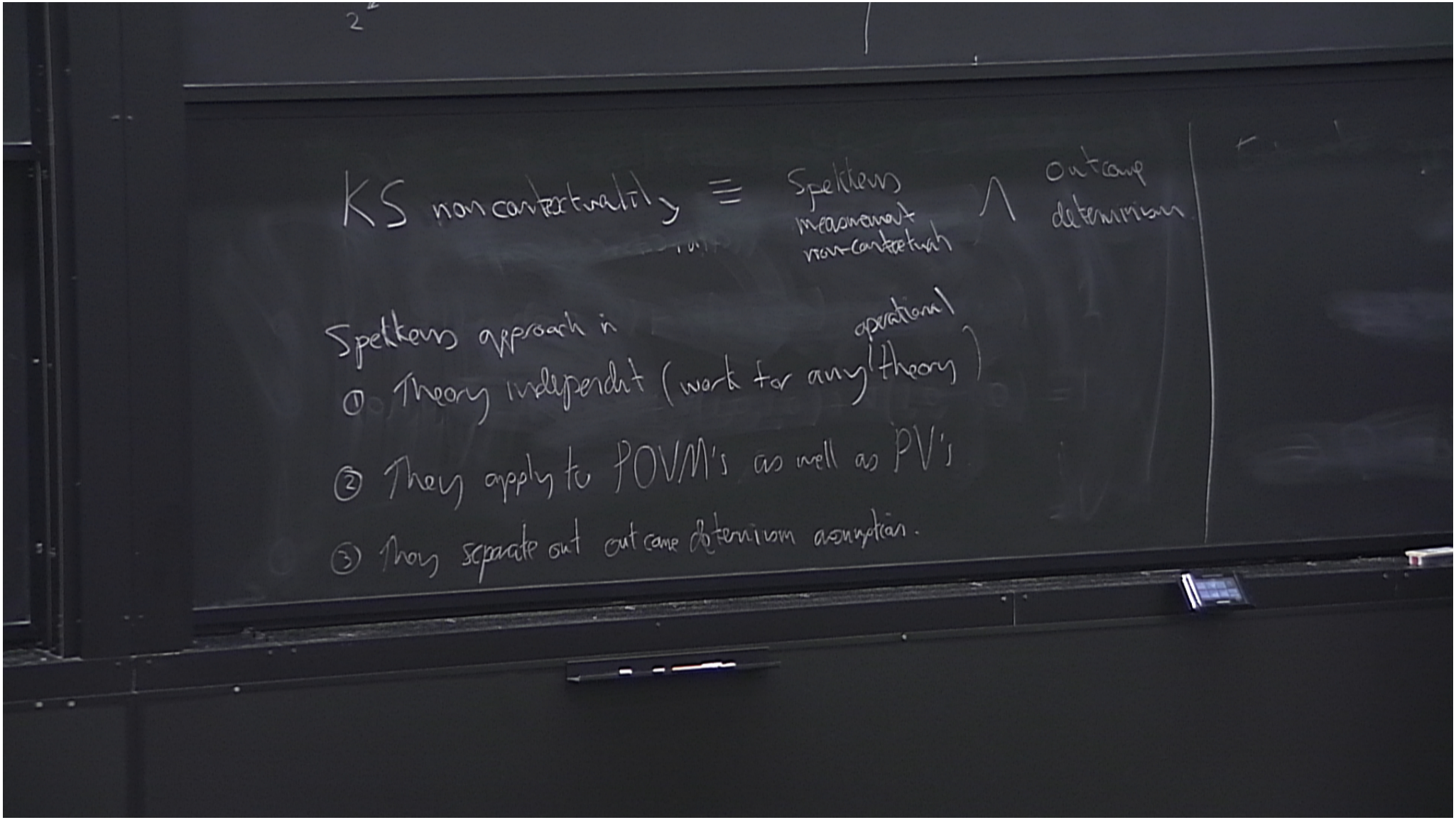
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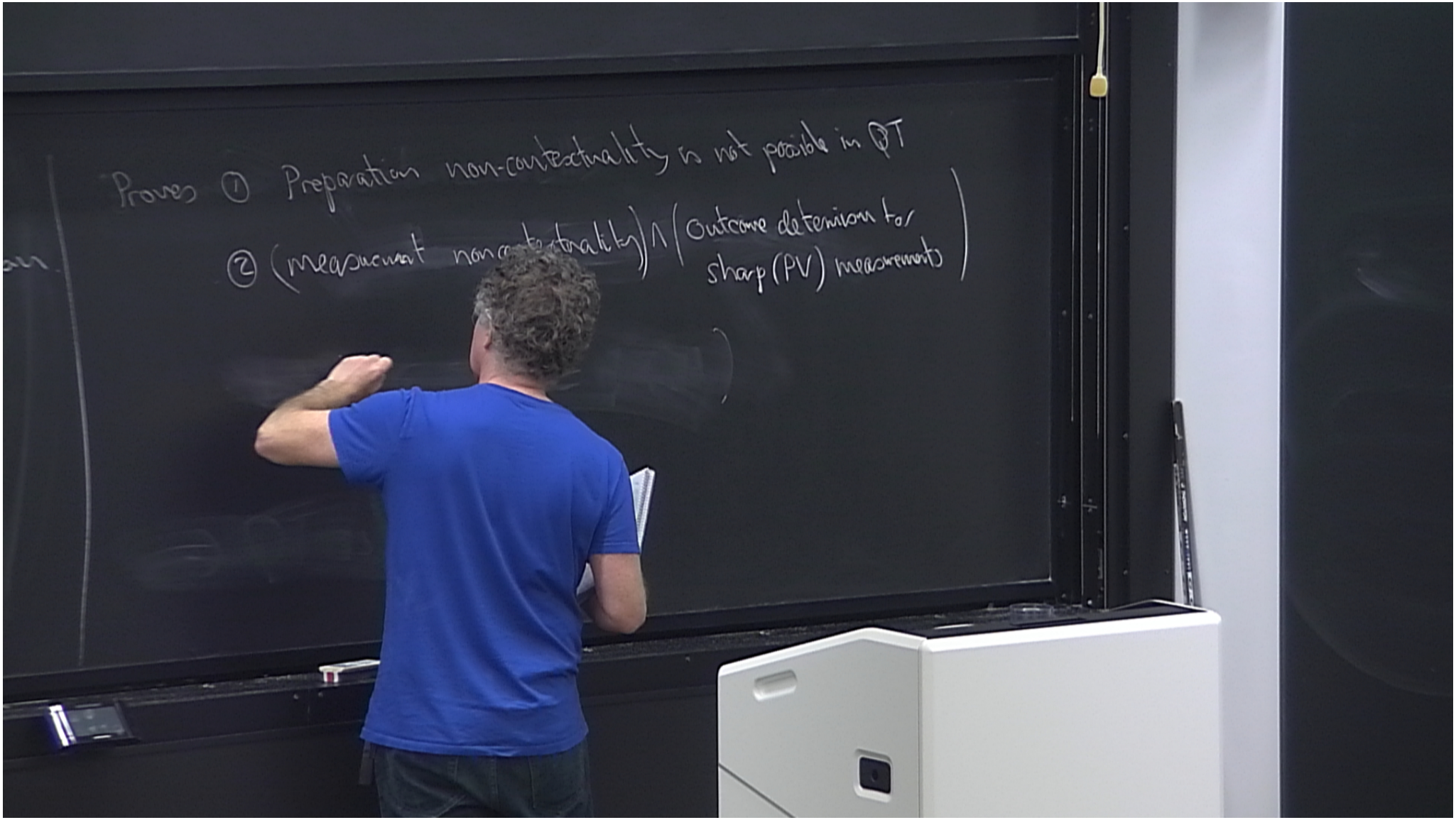
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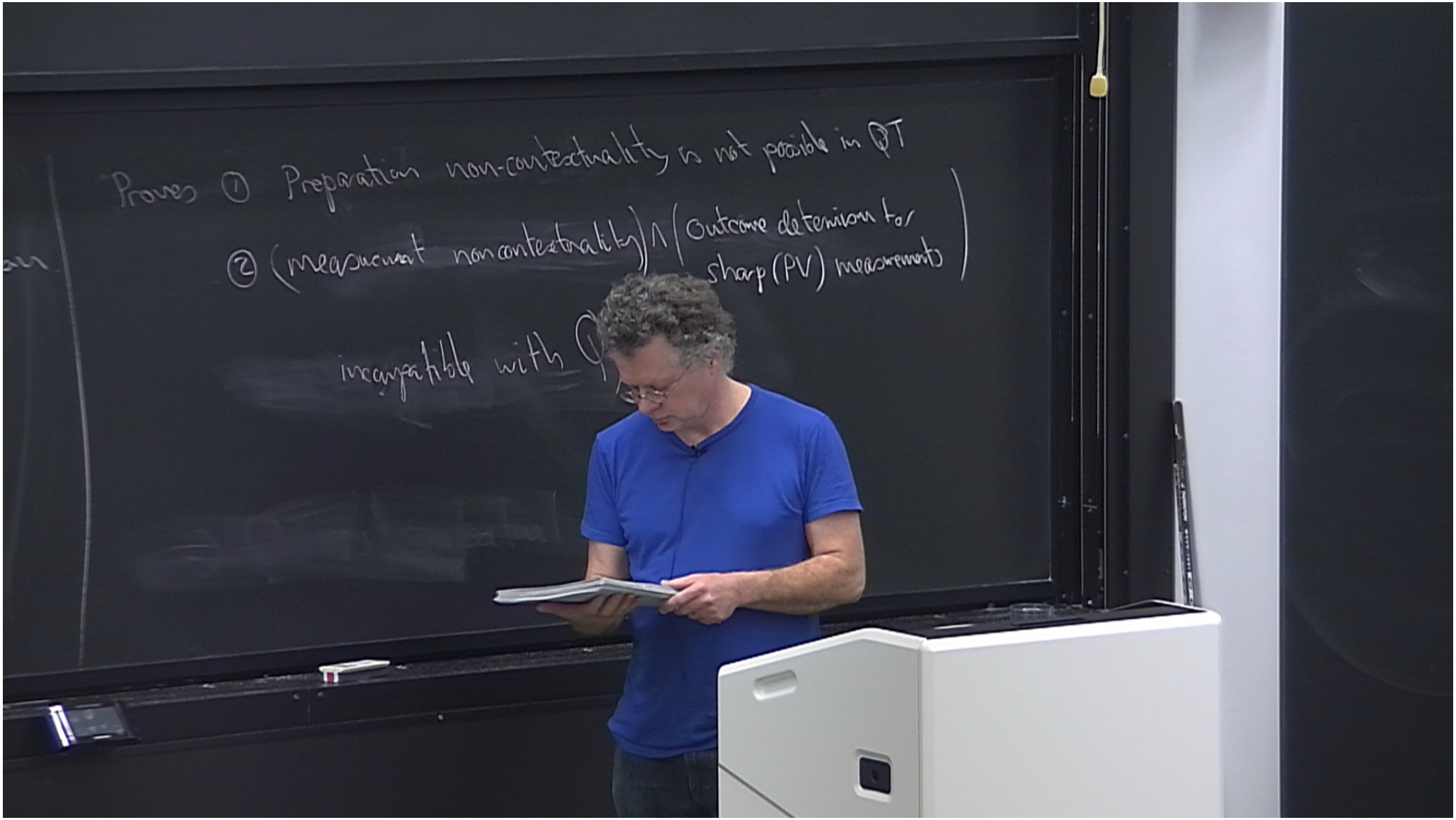

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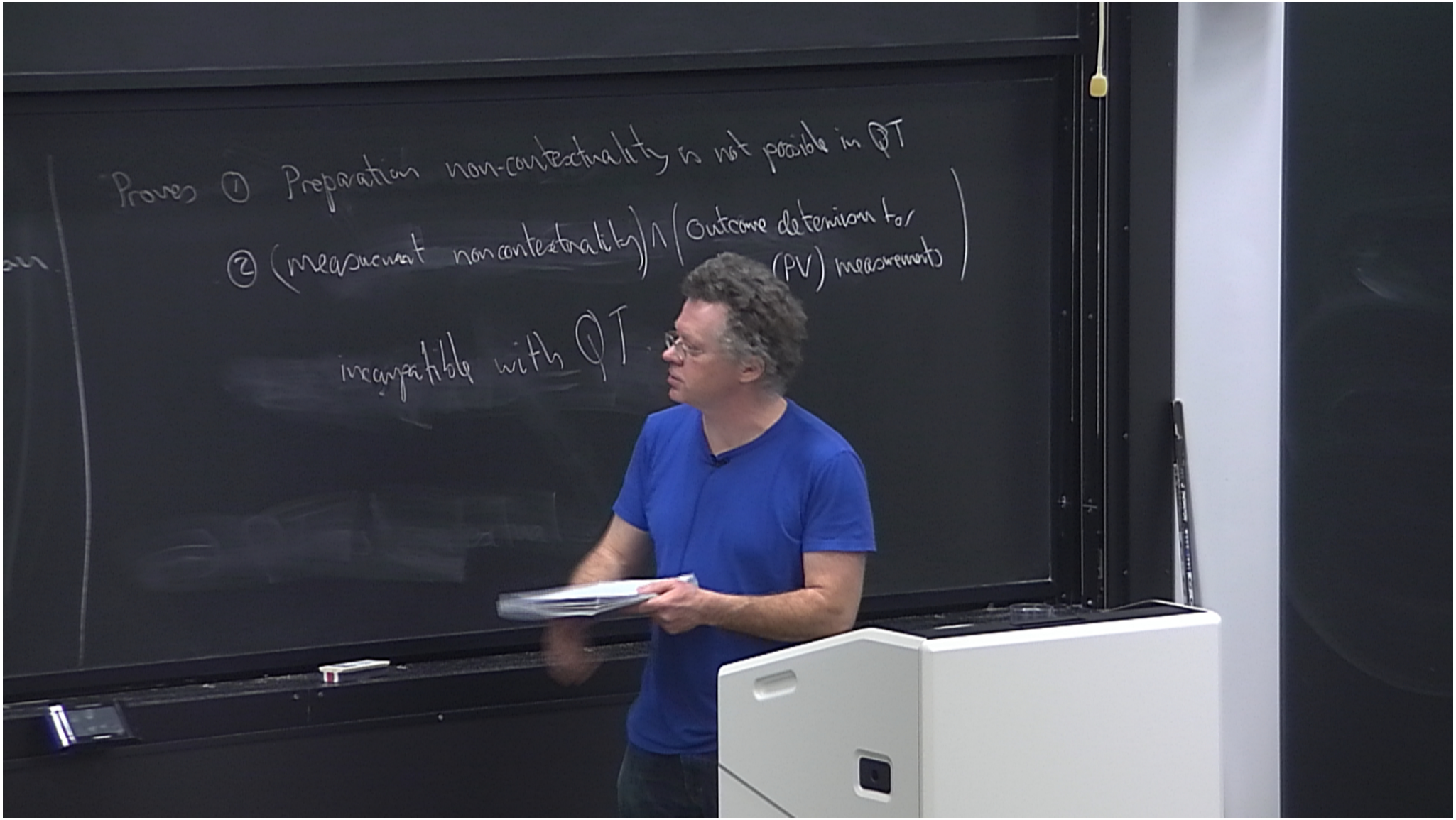

KS non contextuality ≡ Spekkens measurement non-contextuality ∧ Outcome determinism
Spekkens approach is
① Theory independent (work for any theory)

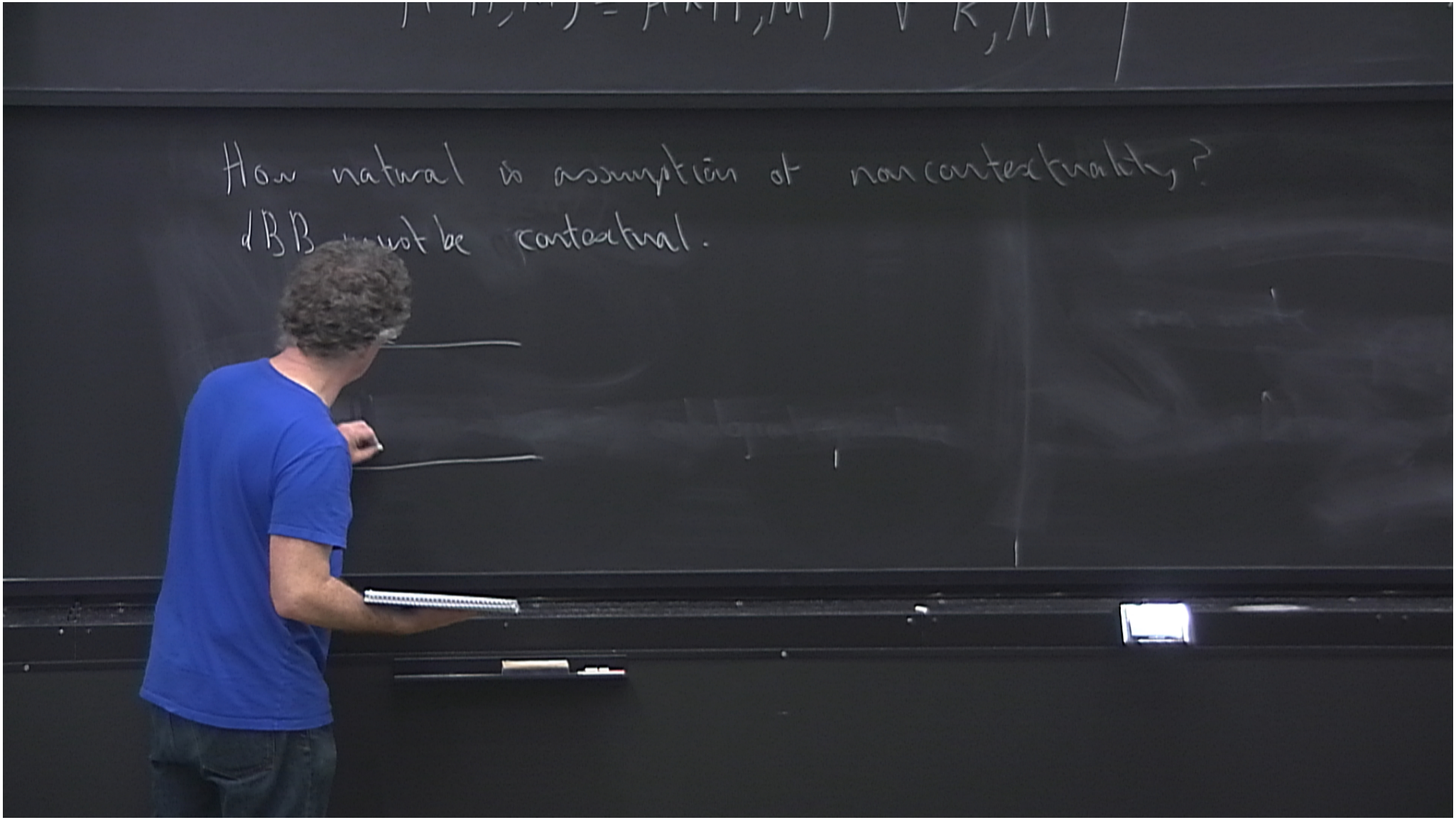

KS non contextuality ≡ Spekkens measurement non-contextuality
Outcome determinism
Spekkens approach is
① Theory independent (work for any operational theory)
② They apply to POVM's as well as PV's

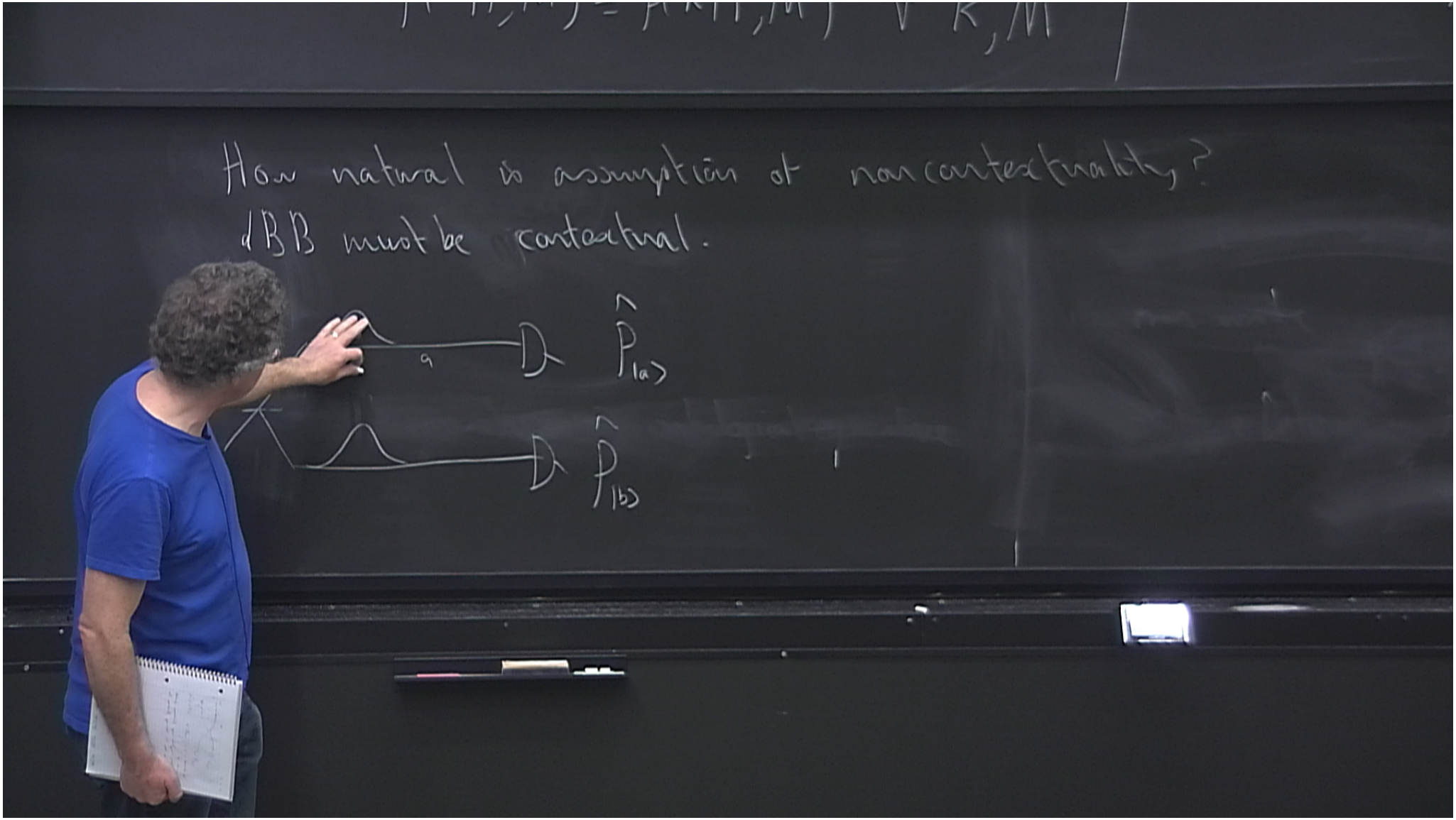

KS non contextuality ≡ Spekkens measurement non-contextuality + 1 outcome determinism
Spekkens approach is
① Theory independent (work for any operational theory)
② They apply to POVM's as well as PV's
③ They separate out outcome determinism assumption.


Proves ① Preparation non-contextuality is not possible in QT
② (measurement non-contextuality) ∧ (Outcome determinism for sharp (PV) measurements)


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incompatible with

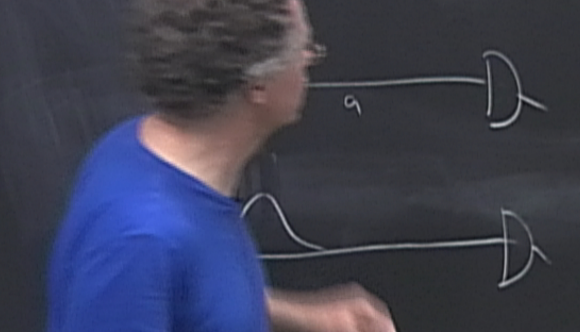

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② (measurement noncontextuality) ∧ (Outcome determination for
(PV) measurements)
incompatible with QT


How natural is assumption of noncontextuality?
dBB must be contextual.


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q
D
$\hat{P}_{|a\rangle}$
D
$\hat{P}_{|b\rangle}$

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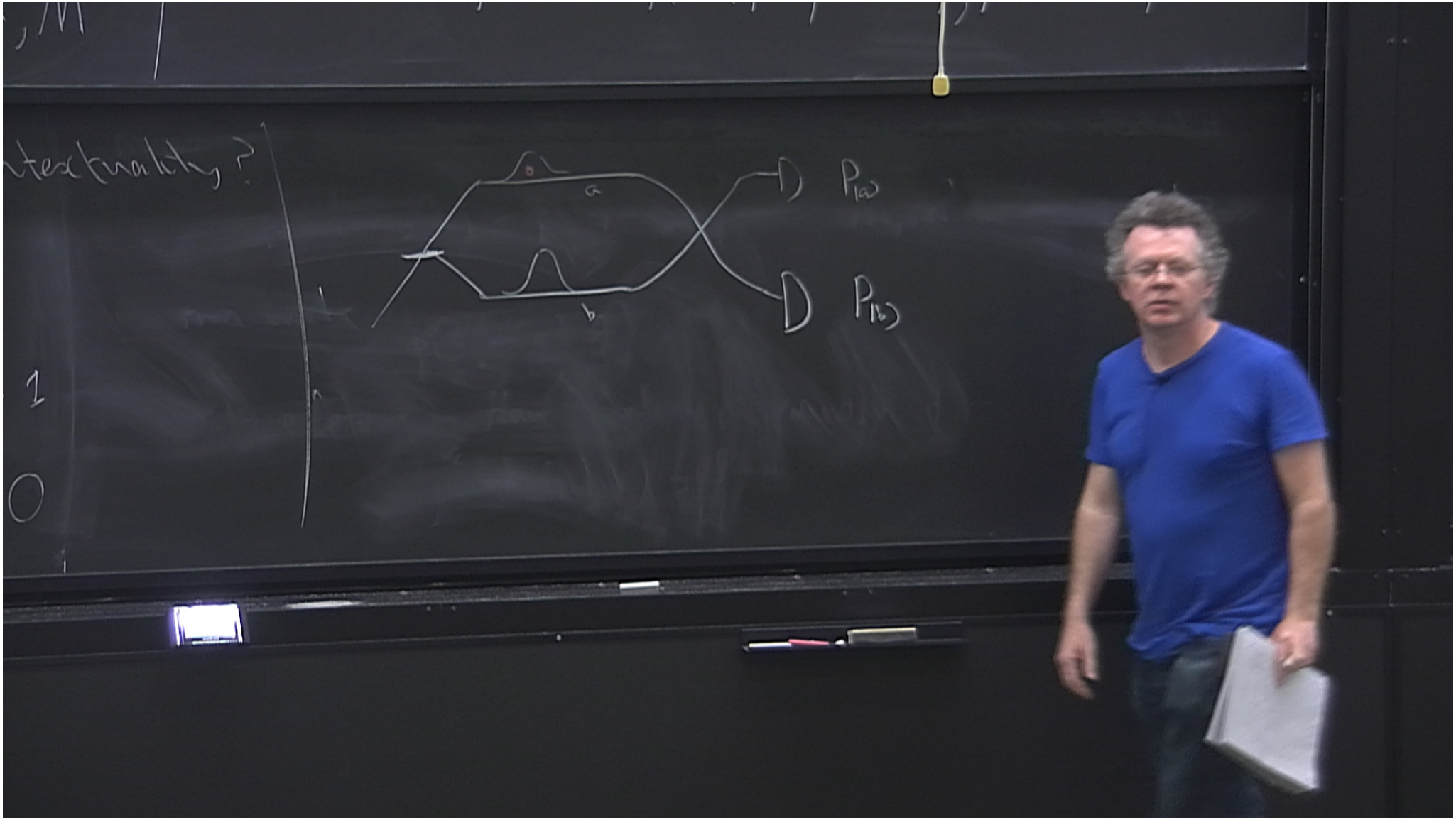


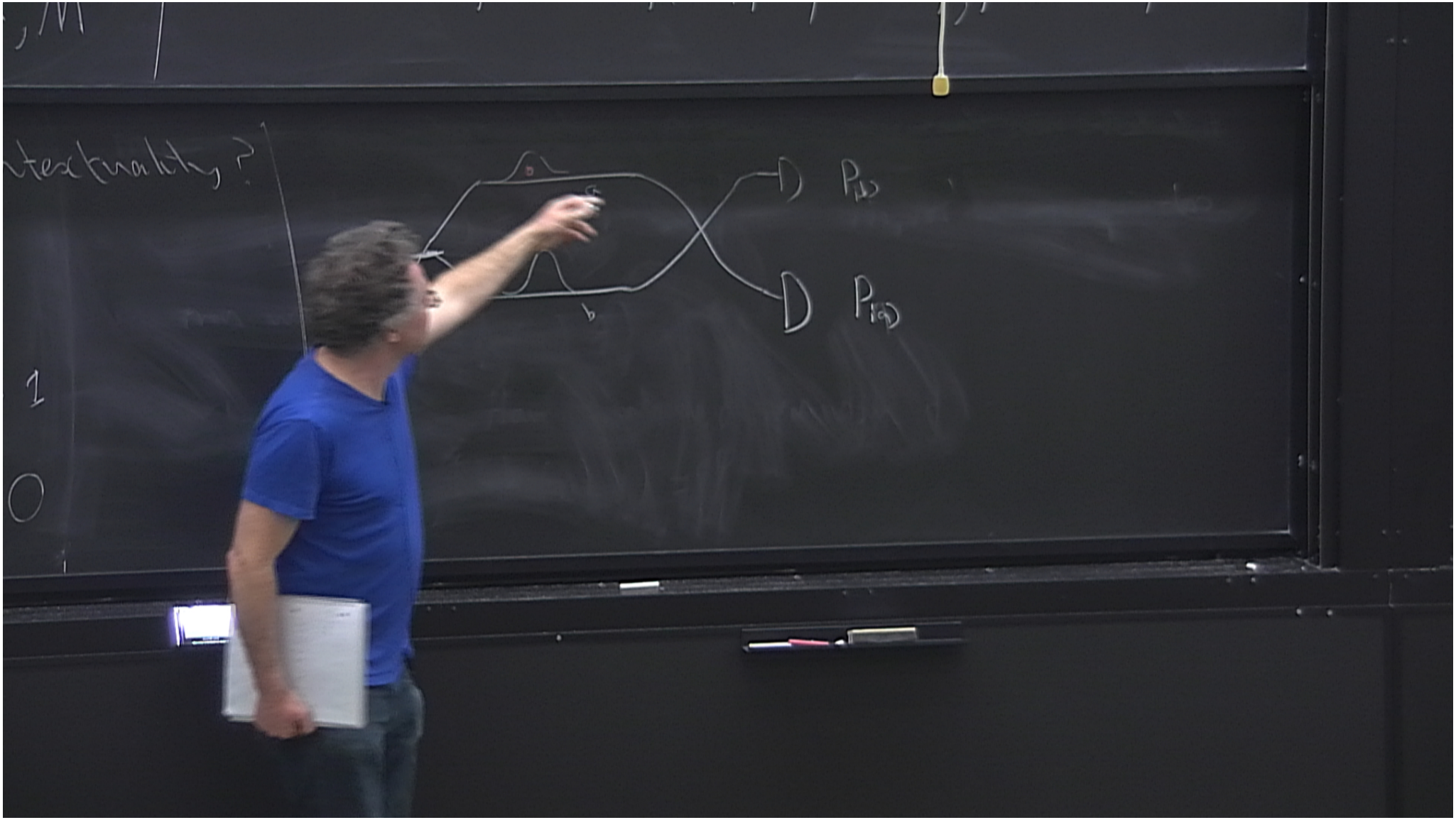
$\hat{P}_{|a\rangle}$ $\hat{P}_{|b\rangle}$

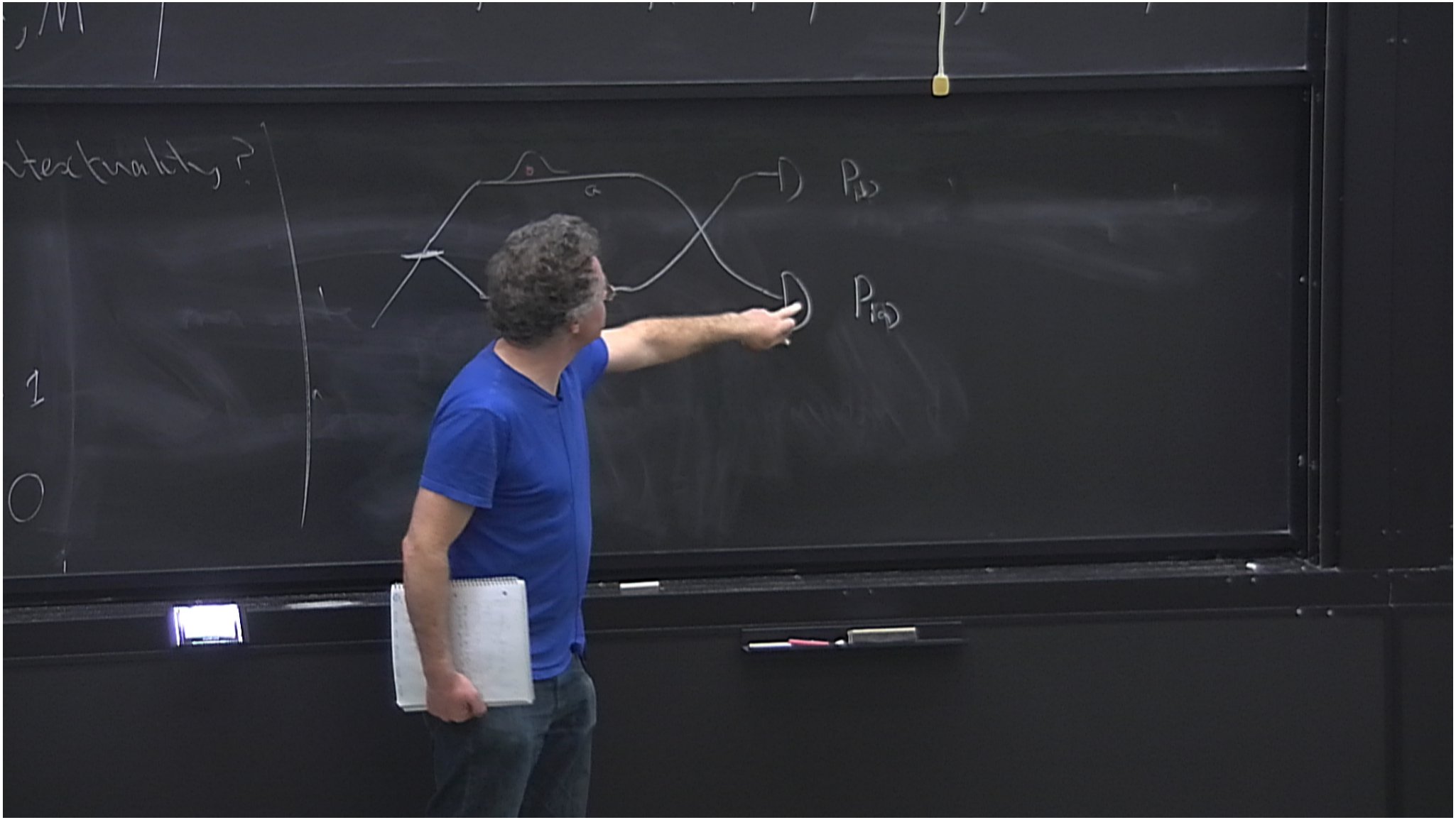
value

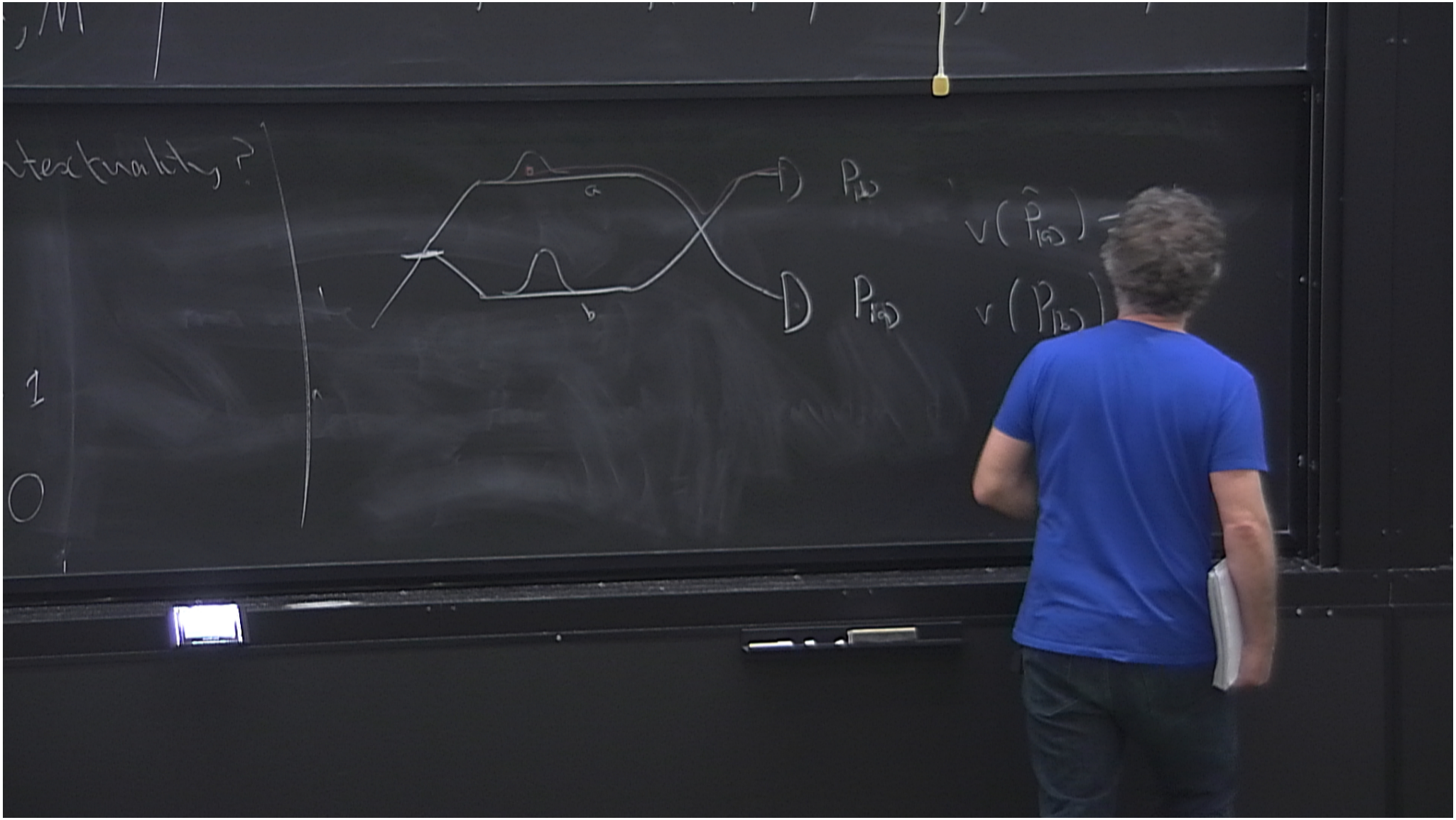
$v(\hat{P}_{|a\rangle}) = 1$

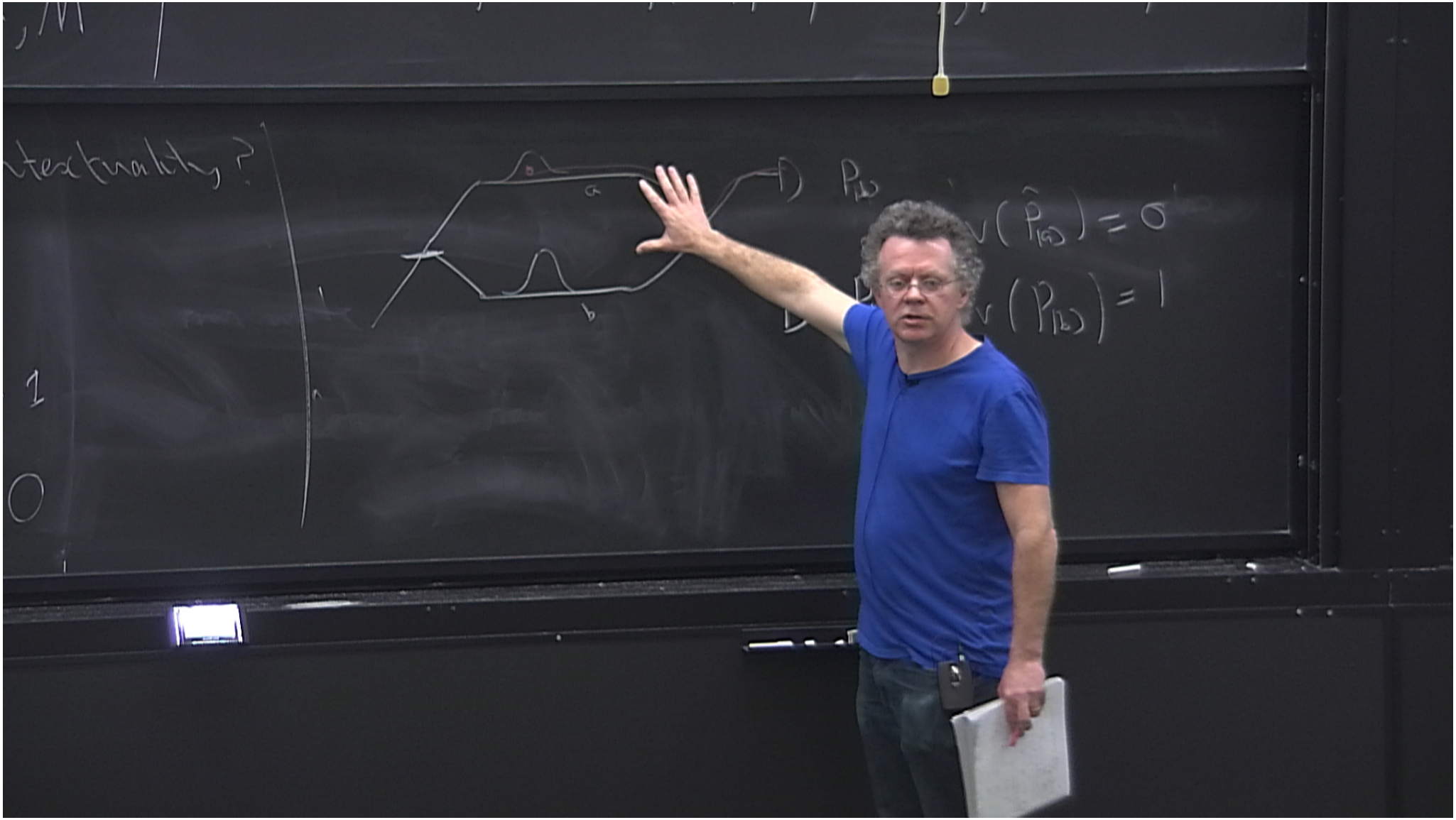
$v(\hat{P}_{|b\rangle}) = 0$


nterctuality?
a
b
D
D
1
0


nterctuality?
1
0
a
b
D
D


nterctuality?
1
0
a
b
$P_{(a)}$
$P_{(b)}$


a
b


a
b
1
0