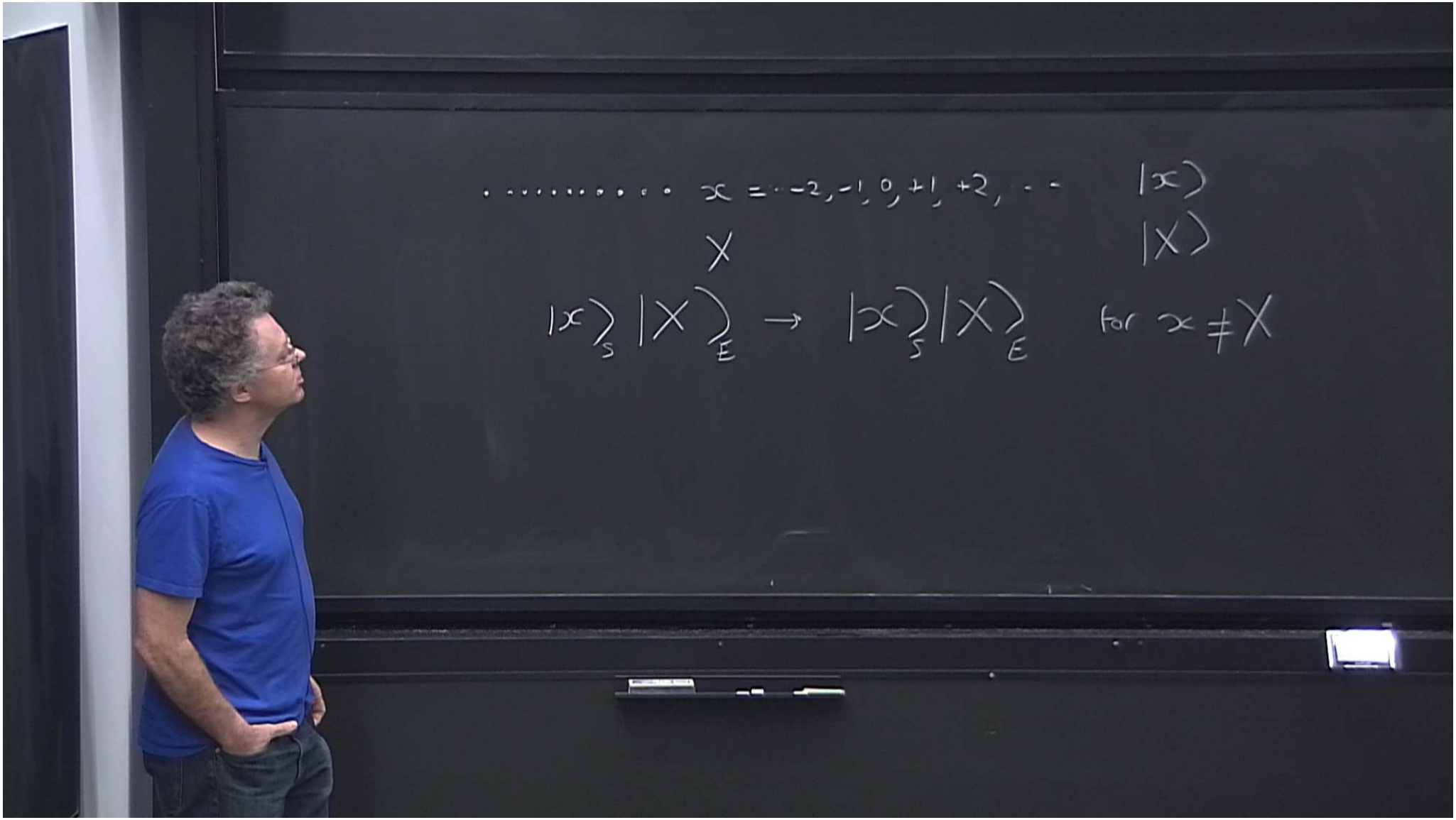
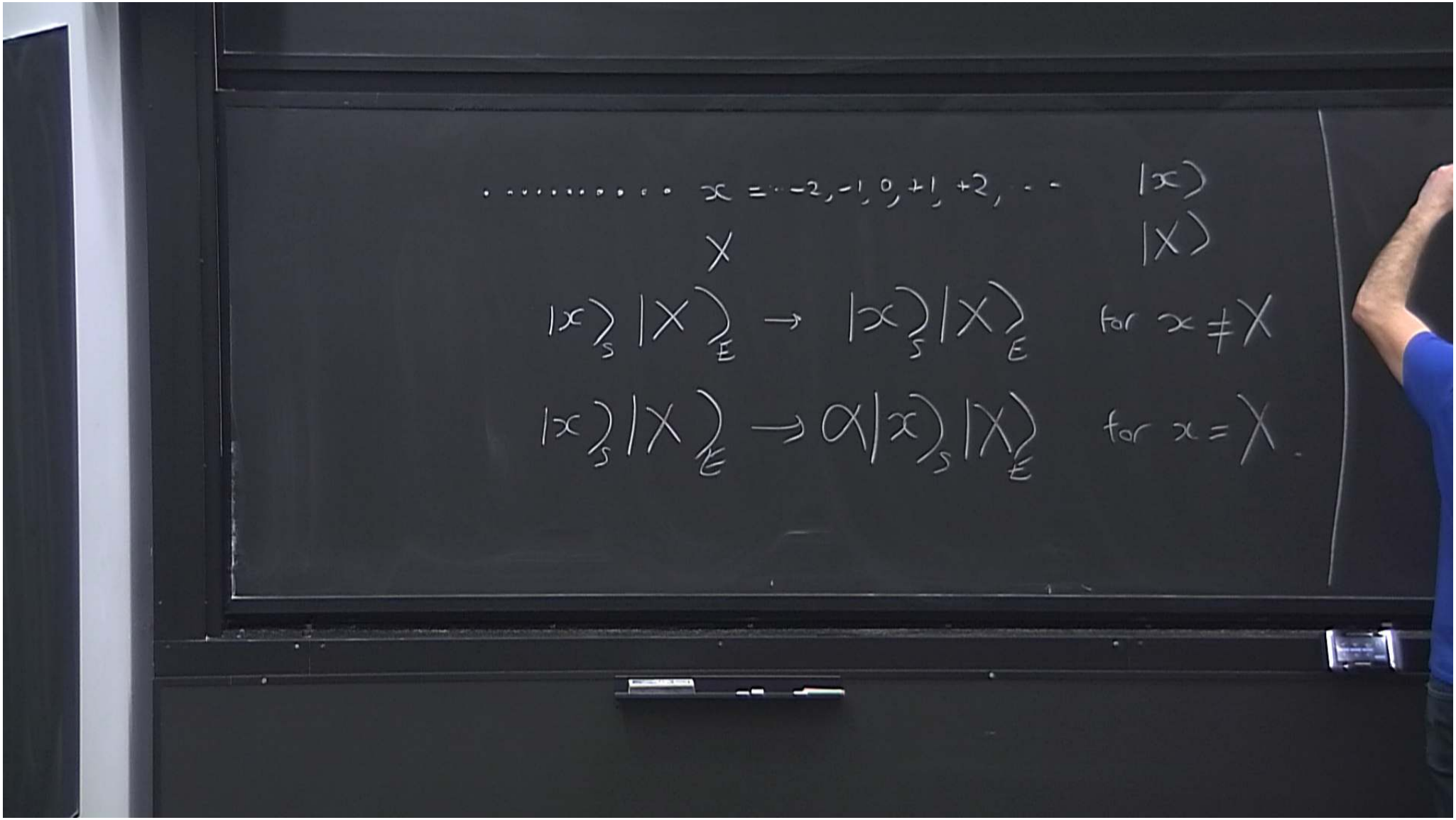
Title: Foundations of Quantum Mechanics-3

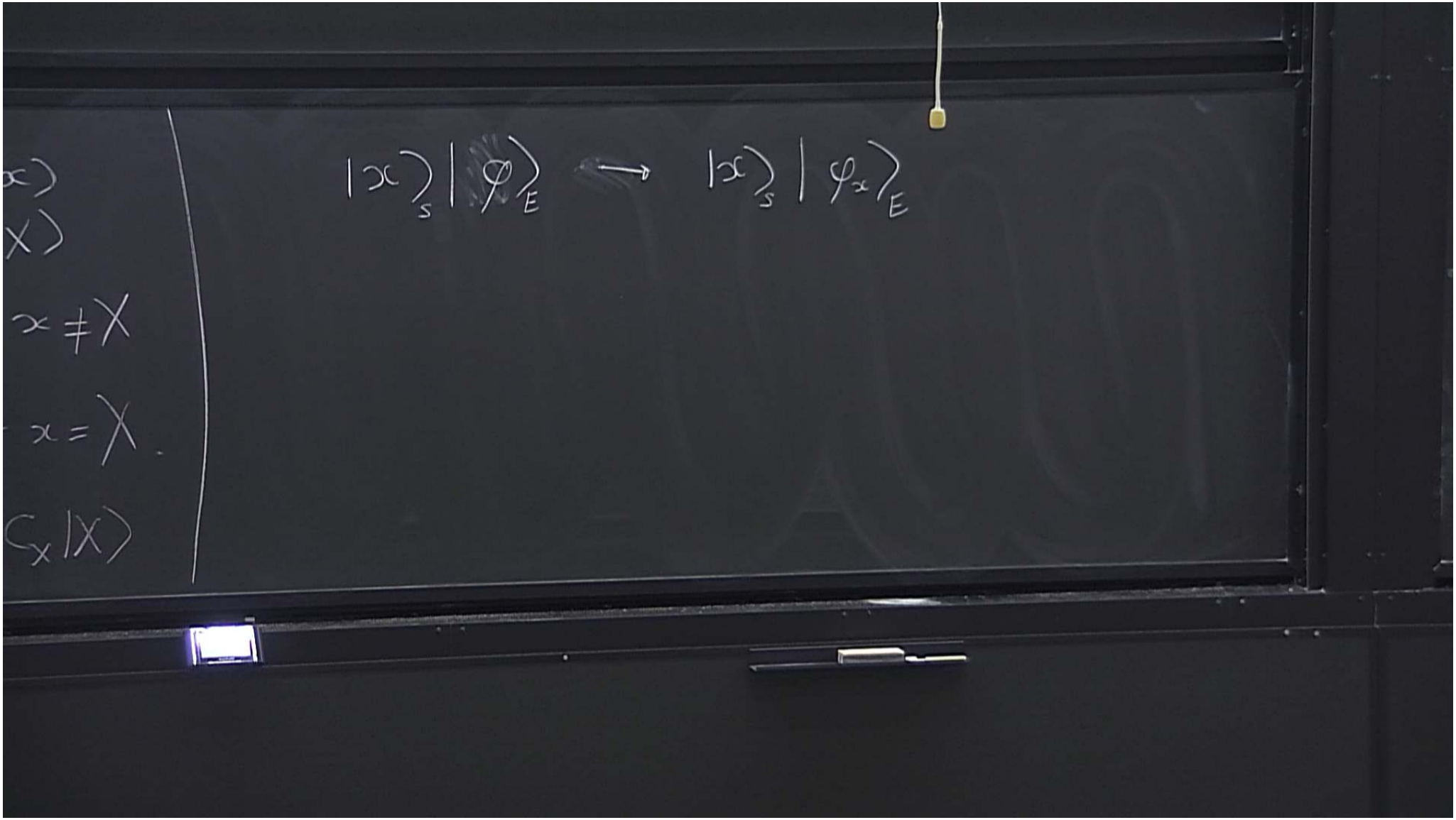
Date: Jan 07, 2015 11:30 AM

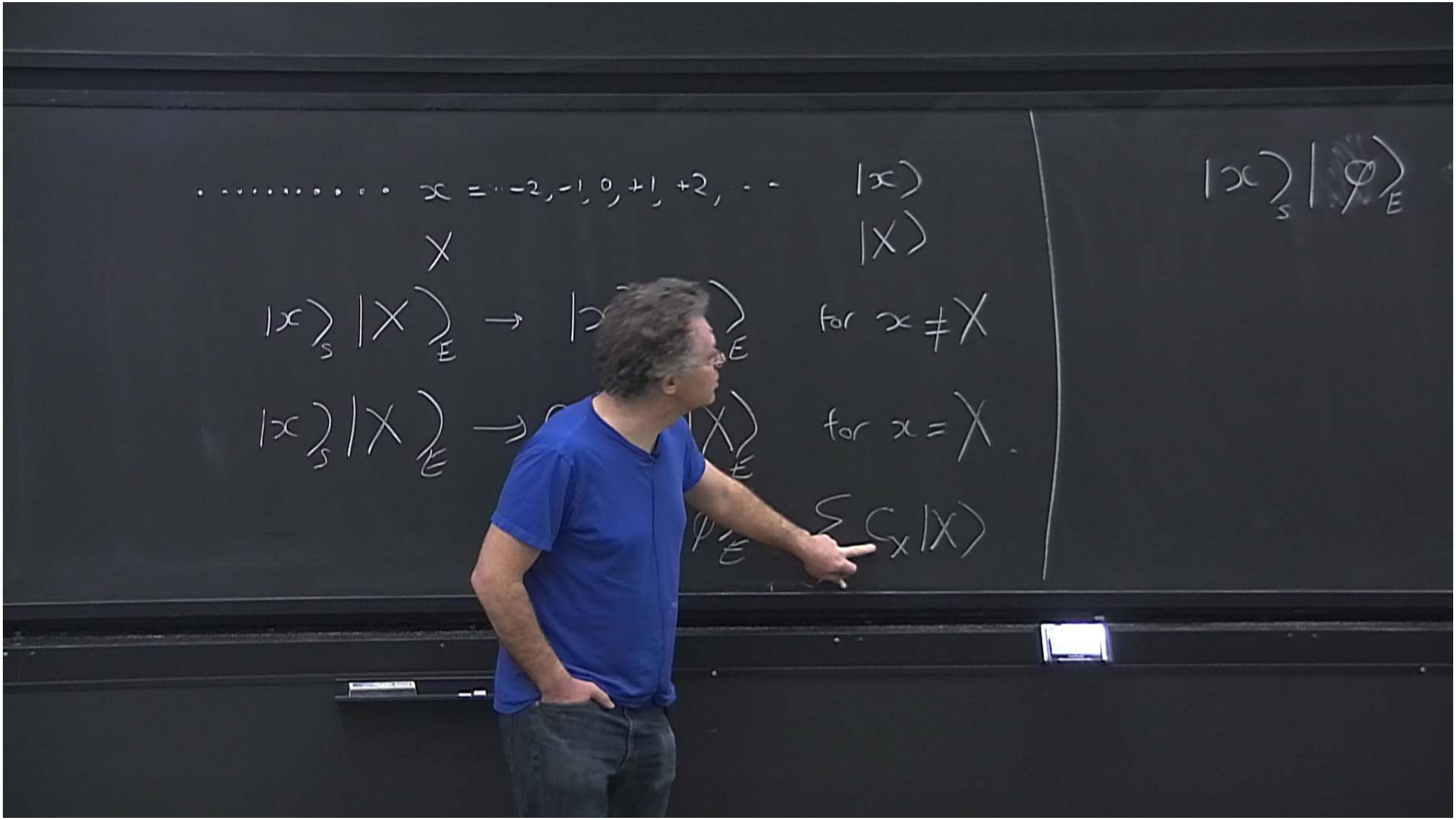
URL: http://pirsa.org/15010040

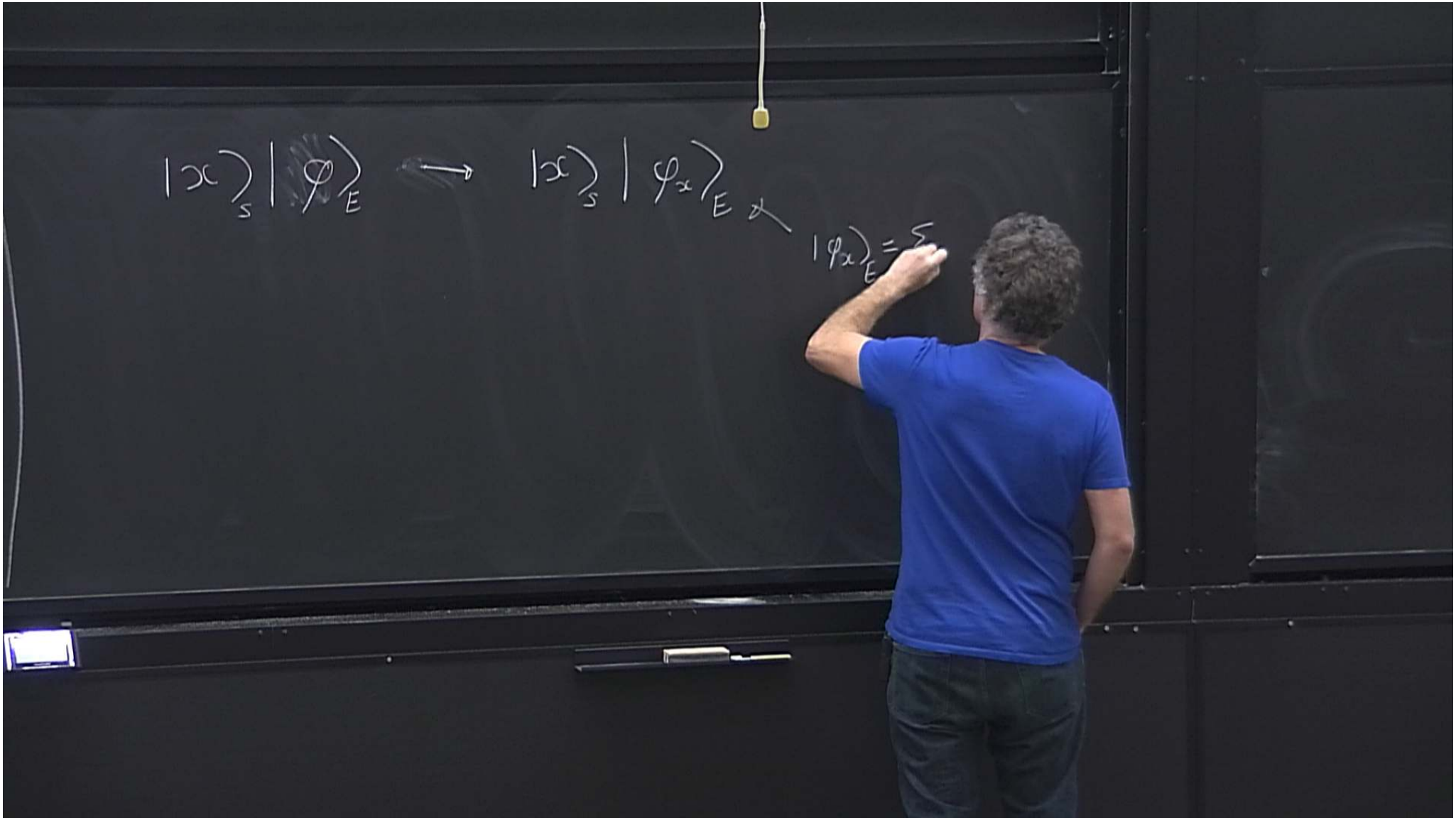
Abstract:

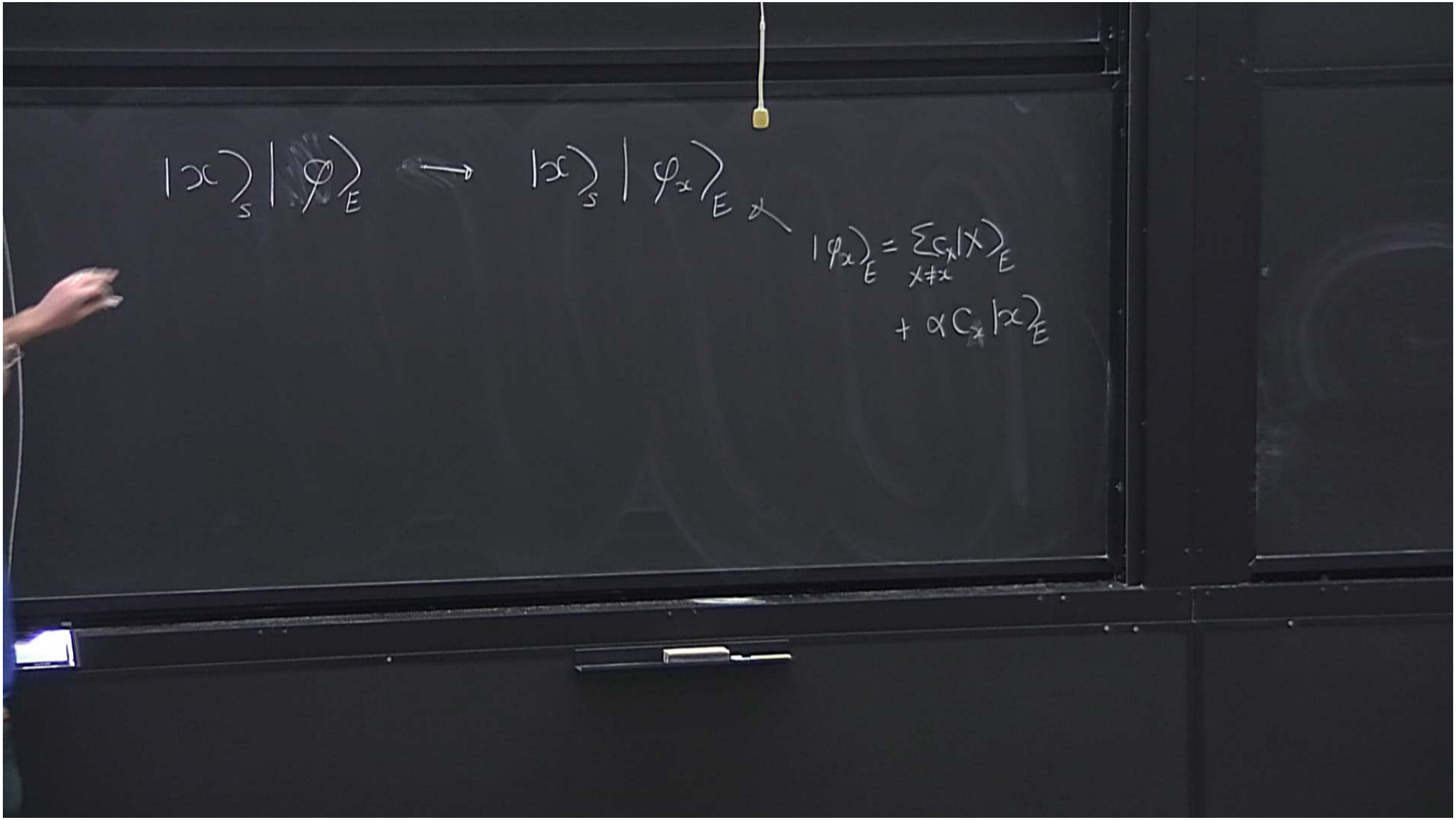

x = -2, -1, 0, +1, +2, ...
|x⟩
X
|X⟩
|x⟩_S |X⟩_E → |x⟩_S |X⟩_E for x ≠ X

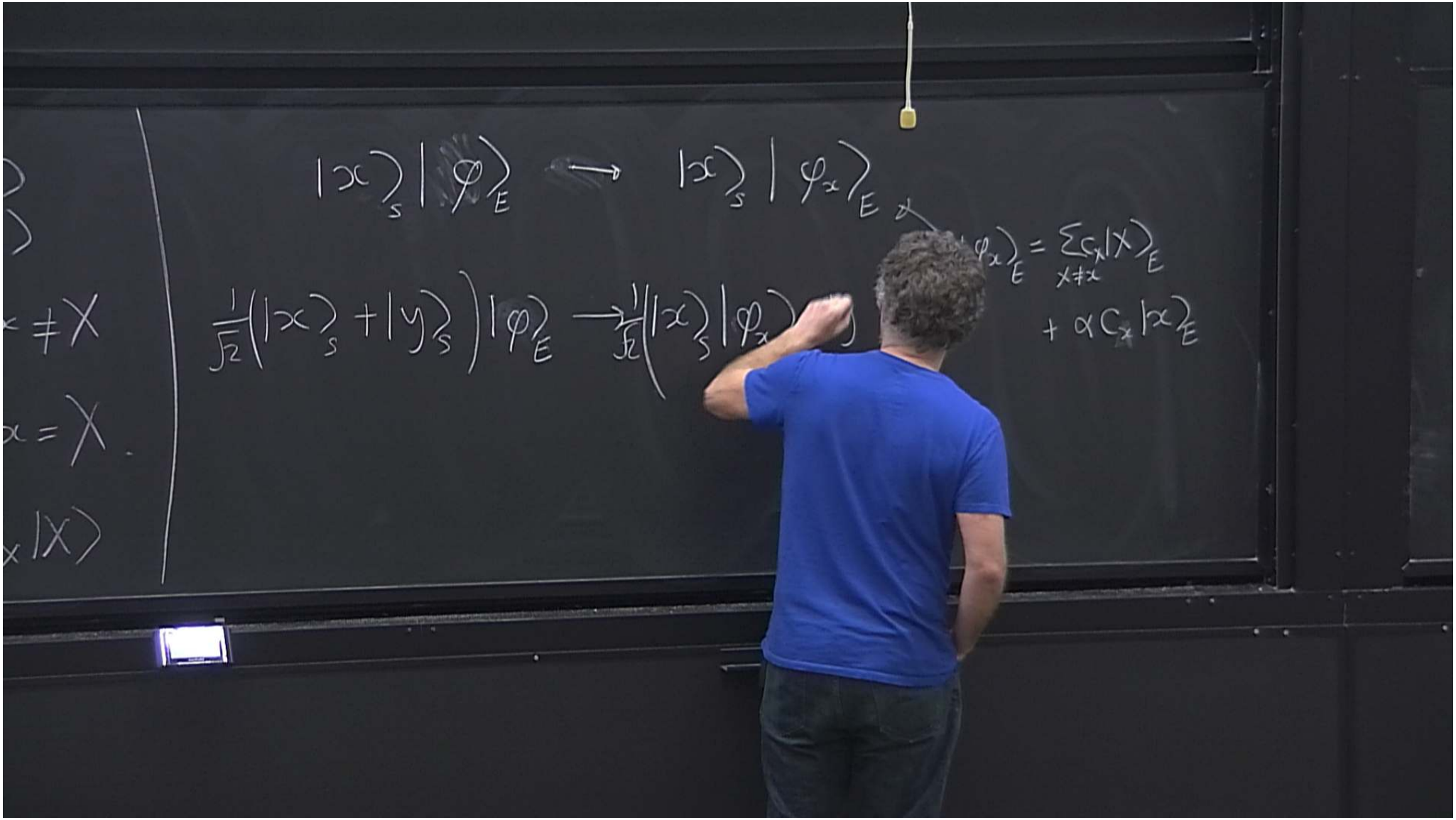

$x = \cdots -2, -1, 0, +1, +2, \cdots$ $|x\rangle$
X $|X\rangle$
$|x\rangle_S |X\rangle_E \rightarrow |x\rangle_S |X\rangle_E$ for $x \neq X$
$|x\rangle_S |X\rangle_E \rightarrow \alpha |x\rangle_S |X\rangle_E$ for $x = X$.

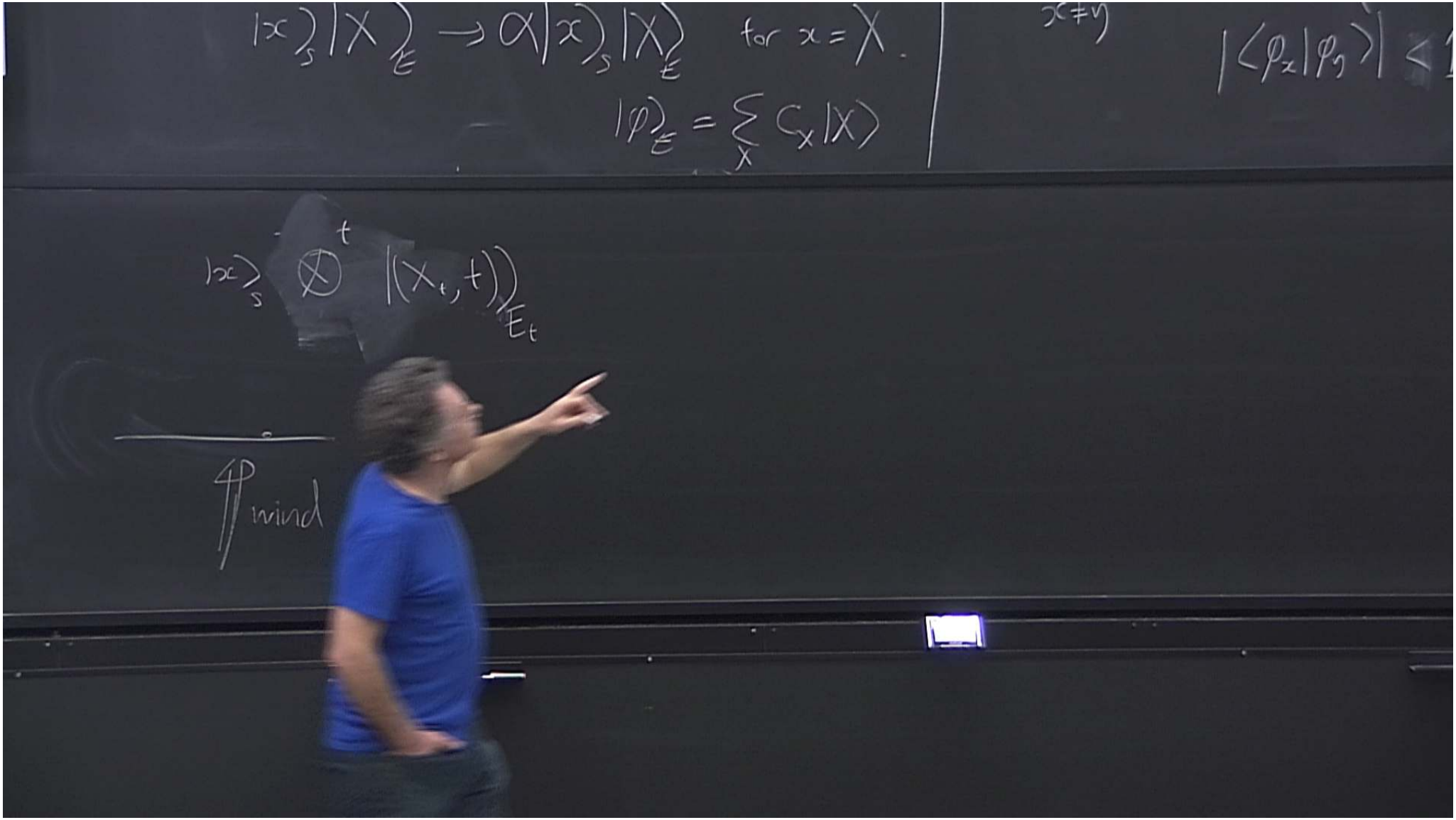

$|x\rangle_S |\varphi\rangle_E \longrightarrow |x\rangle_S |\varphi_x\rangle_E$
$x \neq X$
$x = X$
$C_x |X\rangle$

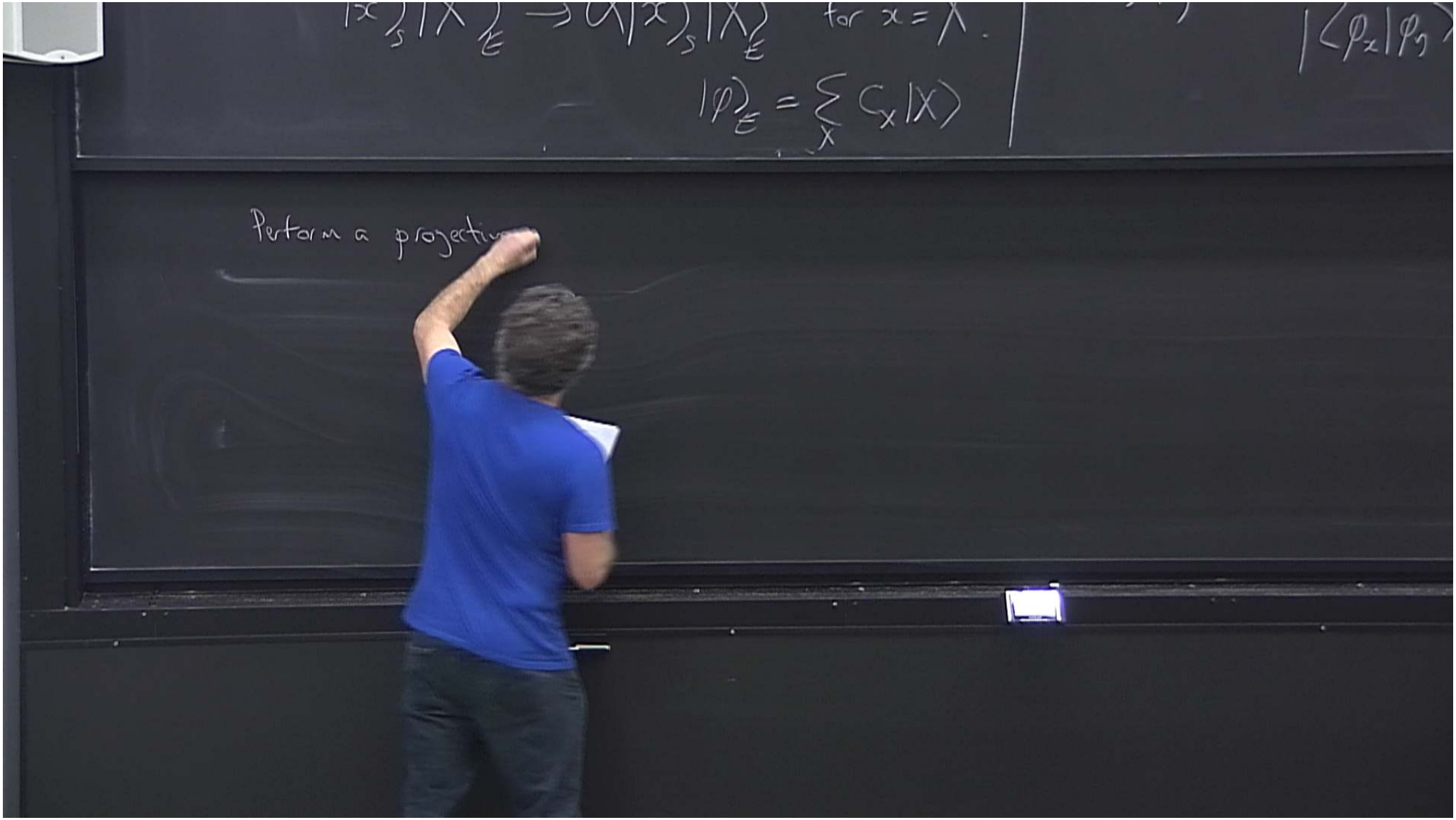

$x = \ldots -2, -1, 0, +1, +2, \ldots$
$|x\rangle$
X
$|X\rangle$
$|x\rangle_S |X\rangle_E \rightarrow$
for $x \neq X$
$|x\rangle_S |X\rangle_E \rightarrow$
for $x = X$
$\sum c_x |X\rangle$
$|x\rangle_S |\varphi\rangle_E$

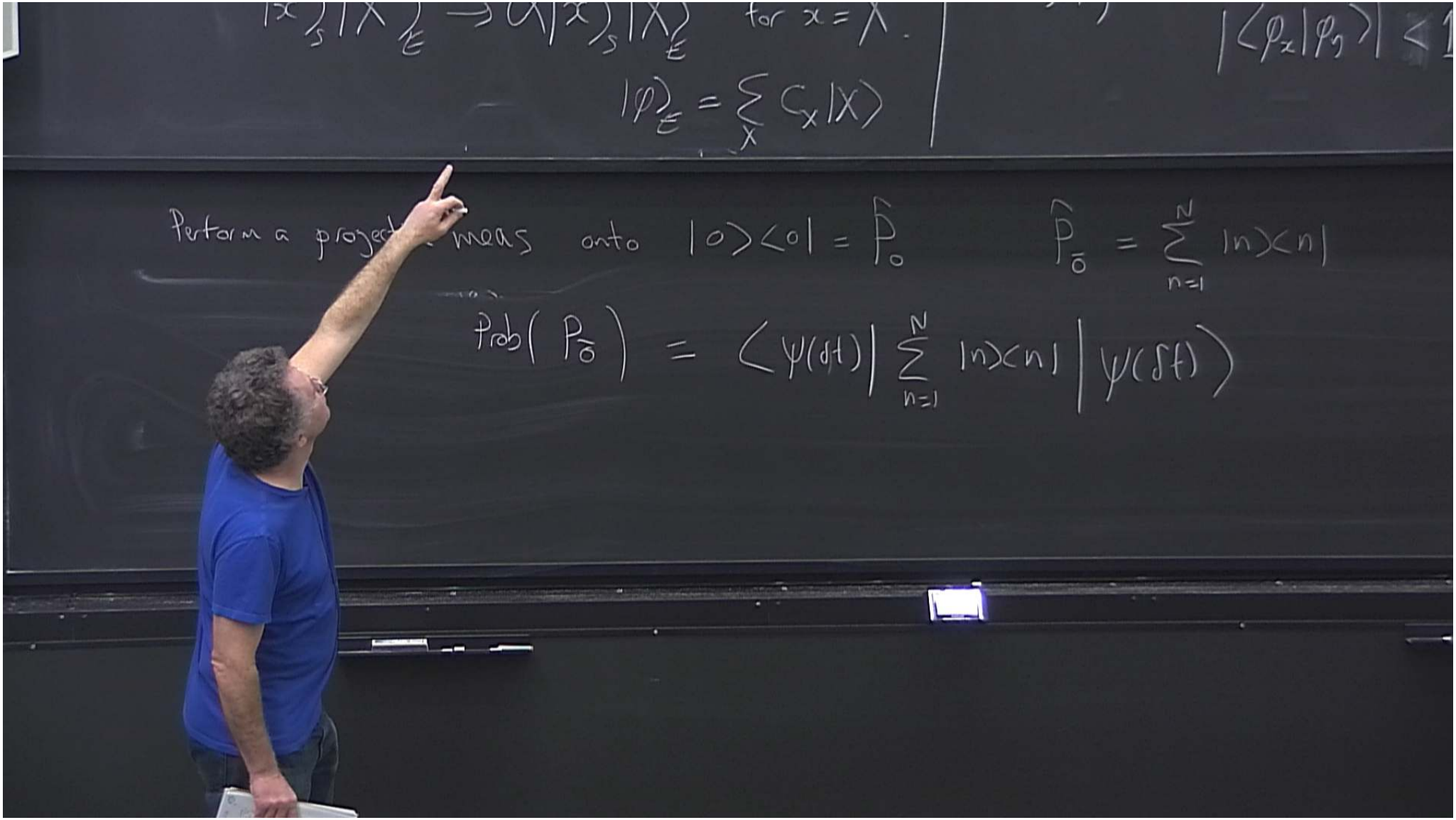

$|x\rangle_S |\varphi\rangle_E \longrightarrow |x\rangle_S |\varphi_x\rangle_E$
$|\varphi_x\rangle_E =$

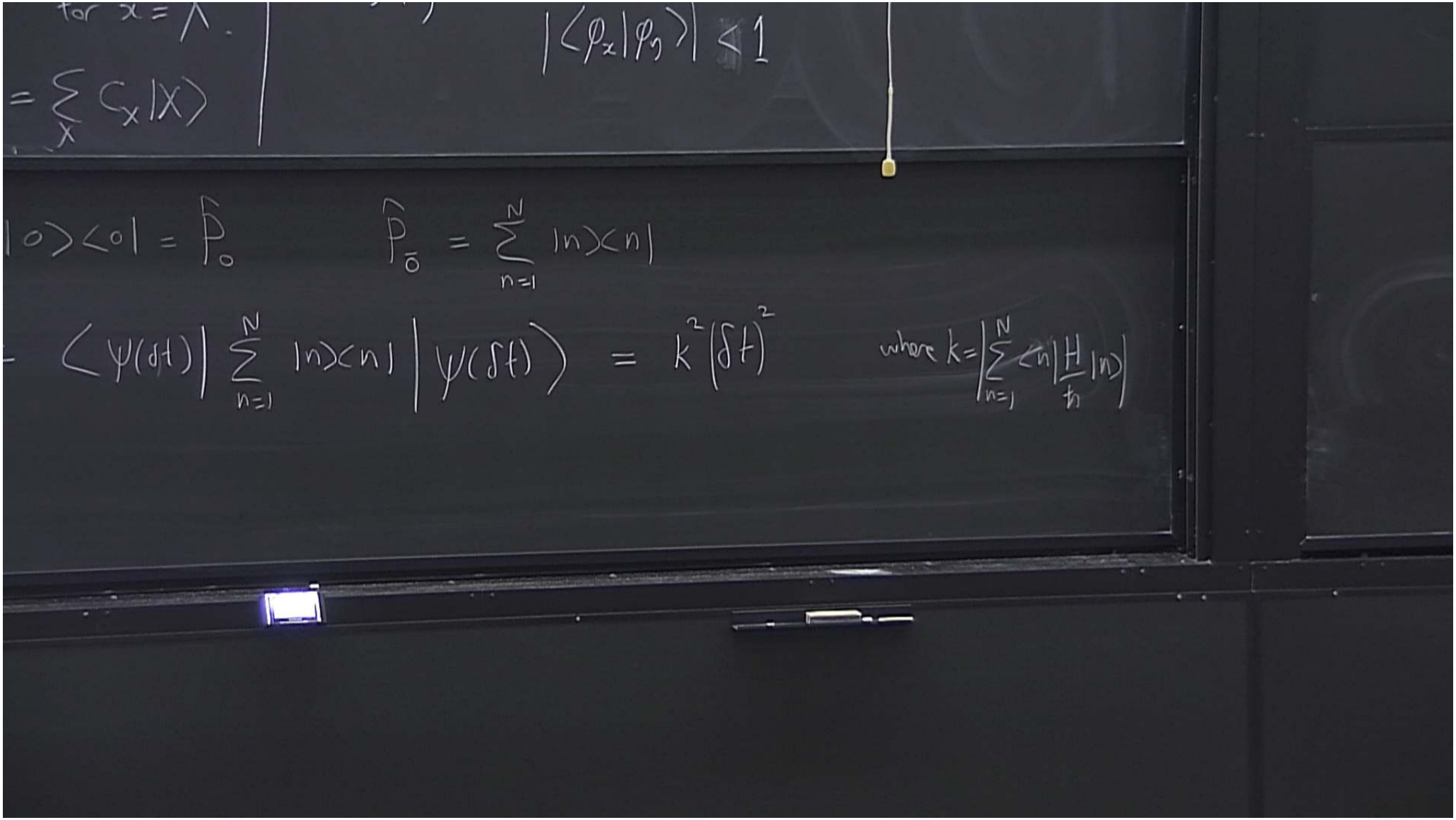

$|x\rangle_S |\varphi\rangle_E \longrightarrow |x\rangle_S |\varphi_x\rangle_E$
$|\varphi_x\rangle_E = \sum_{x' \neq x} c_{x'} |x'\rangle_E$
$+ \alpha c_x |x\rangle_E$

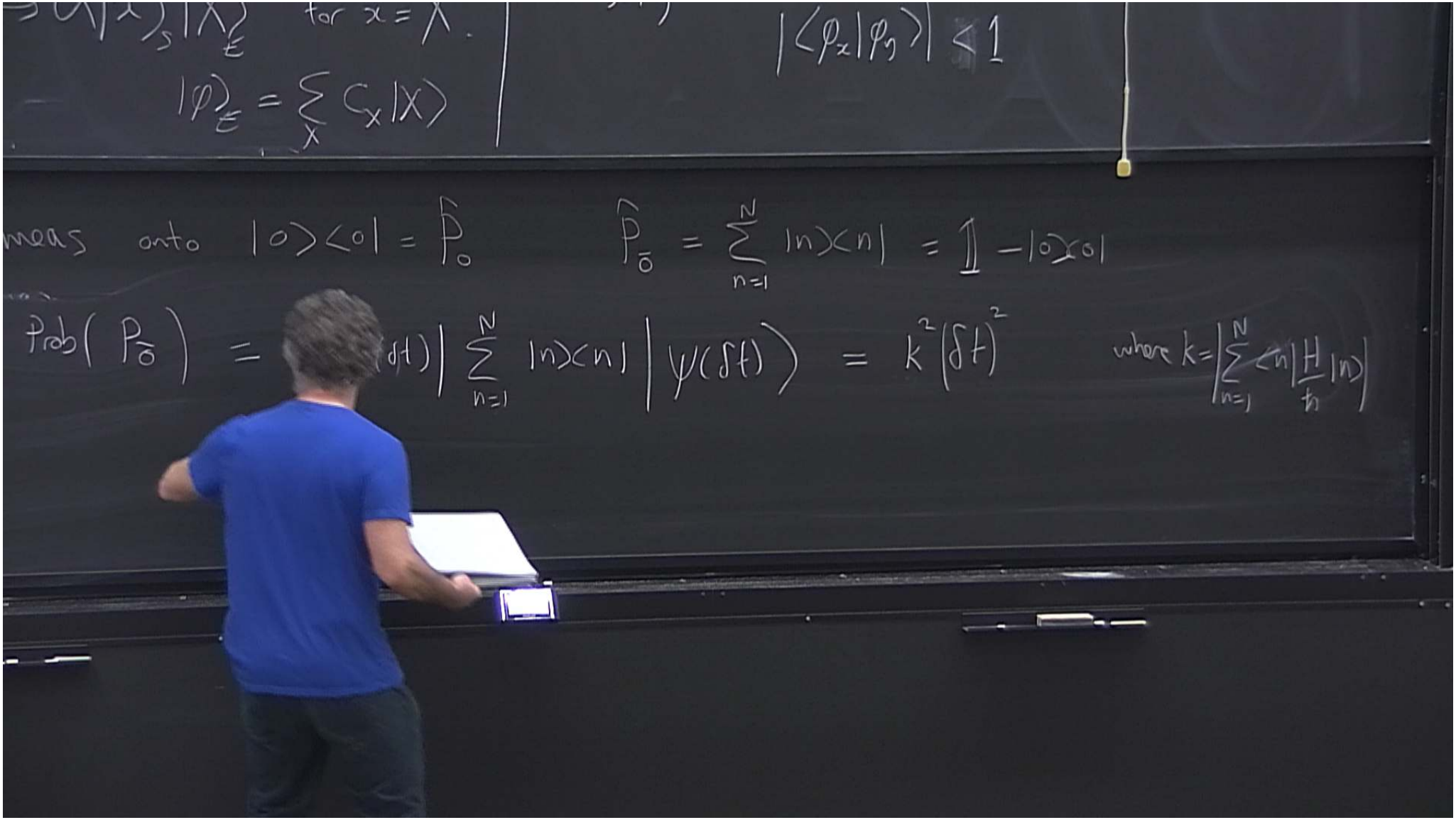

$|x\rangle_S|\varphi\rangle_E \longrightarrow |x\rangle_S|\varphi_x\rangle_E$
$\frac{1}{\sqrt{2}}(|x\rangle_S + |y\rangle_S)|\varphi\rangle_E \longrightarrow \frac{1}{\sqrt{2}}(|x\rangle_S|\varphi_x\rangle_E$
$|\varphi_x\rangle_E = \sum_{x' \neq x} c_{x'}|x'\rangle_E$
$+ \alpha c_x |x\rangle_E$

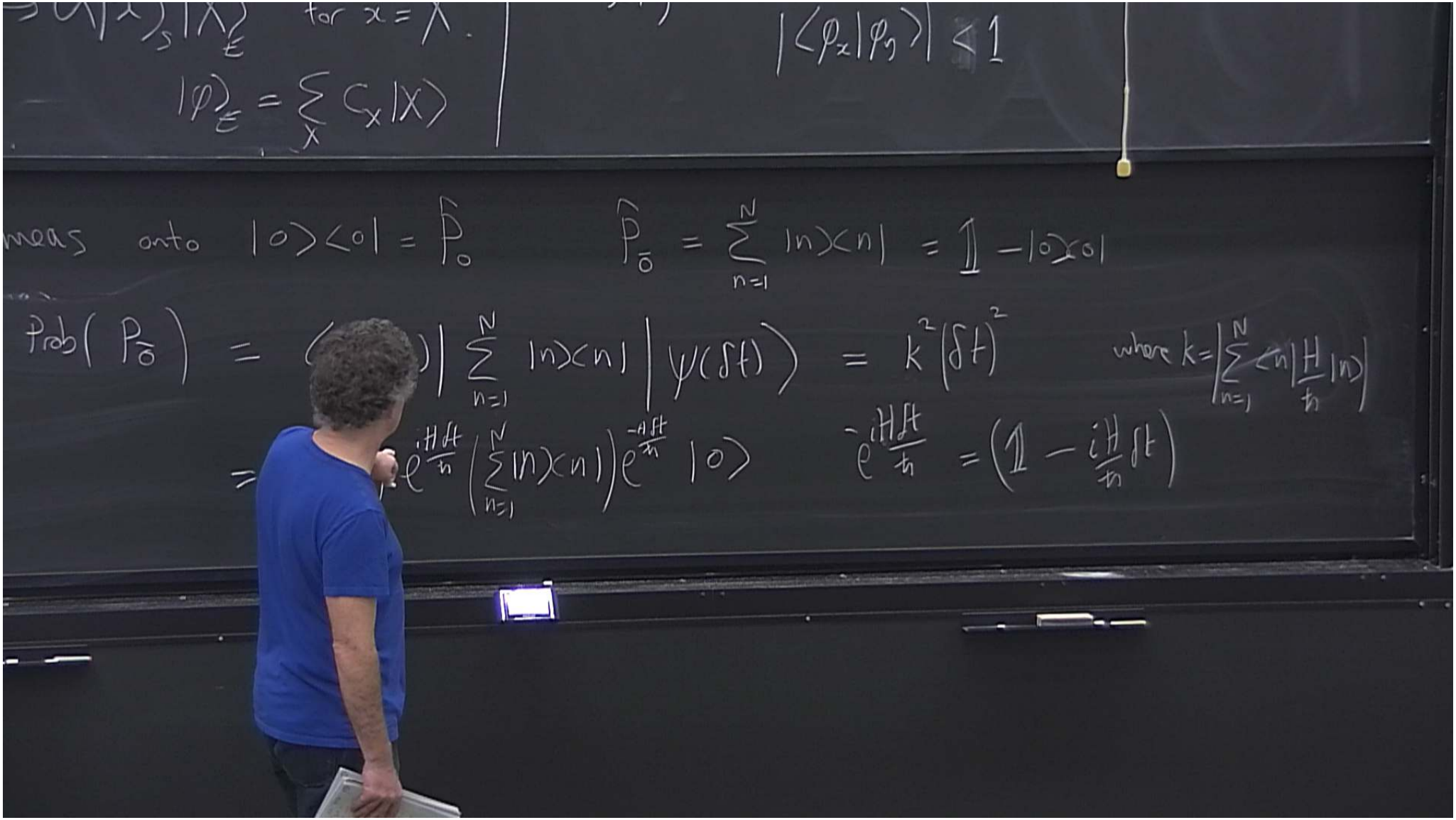

$|x\rangle_S|X\rangle_E \to \alpha|x\rangle_S|X\rangle_E$ for $x = X$.
$|\rho\rangle_E = \sum_X c_X|X\rangle$
$x \neq y$
$|\langle\rho_x|\rho_y\rangle| <$
$|x\rangle_S \bigotimes^t |(X_t, t)\rangle_{E_t}$
Ψ_{mind}


for $x = x'$.
$|\varphi\rangle_E = \sum_x c_x |x\rangle$
$|\langle \varphi_x | \varphi_y \rangle$
Perform a projecti


$|\varphi\rangle_E = \sum_x c_x |x\rangle$
$|\langle \varphi_x | \varphi_y \rangle| < 1$
Perform a projective meas onto $|0\rangle\langle 0| = \hat{P}_0$
$\hat{P}_{\bar{0}} = \sum_{n=1}^{N} |n\rangle\langle n|$
$\text{Prob}(P_{\bar{0}}) = \langle \psi(dt) | \sum_{n=1}^{N} |n\rangle\langle n| | \psi(\delta t) \rangle$


for $x = \lambda$.
$= \sum_x C_x |x\rangle$
$|\langle \rho_x | \rho_y \rangle| \leq 1$
$|0\rangle\langle 0| = \hat{P}_0$
$\hat{P}_0 = \sum_{n=1}^{N} |n\rangle\langle n|$
$\langle \psi(\delta t) | \sum_{n=1}^{N} |n\rangle\langle n| \psi(\delta t) \rangle = k^2 (\delta t)^2$
where $k = |\sum_{n=1}^{N} \langle n | \frac{H}{\hbar} | n \rangle|$


for $x = \lambda$.
$|\psi\rangle_E = \sum_x c_x |x\rangle$
$|\langle \rho_x | \rho_y \rangle| \leq 1$
meas onto $|0\rangle\langle 0| = \hat{P}_0$
$\hat{P}_{\bar{0}} = \sum_{n=1}^{N} |n\rangle\langle n| = \mathbb{1} - |0\rangle\langle 0|$
$\text{Prob}(P_{\bar{0}}) = \langle \psi(\delta t) | \sum_{n=1}^{N} |n\rangle\langle n| | \psi(\delta t) \rangle = k^2 (\delta t)^2$
where $k = |\sum_{n=1}^{N} \langle n | \frac{H}{\hbar} | n \rangle|$


for $x = \lambda$
$|\varphi\rangle_E = \sum_x c_x |x\rangle$
$|\langle \varphi_x | \varphi_y \rangle| \leq 1$
meas onto $|0\rangle\langle 0| = \hat{P}_0$
$\hat{P}_{\bar{0}} = \sum_{n=1}^{N} |n\rangle\langle n| = \mathbb{1} - |0\rangle\langle 0|$
$\text{Prob}(P_{\bar{0}}) = \langle \psi(\delta t) | \sum_{n=1}^{N} |n\rangle\langle n| \psi(\delta t) \rangle = k^2 (\delta t)^2$
where $k = |\sum_{n=1}^{N} \langle n | \frac{H}{\hbar} | n \rangle|$
$= \langle 0 | e^{\frac{iH\delta t}{\hbar}} \left(\sum_{n=1}^{N} |n\rangle\langle n| \right) e^{\frac{-iH\delta t}{\hbar}} |0\rangle$
$e^{\frac{iH\delta t}{\hbar}} = \left(\mathbb{1} - \frac{iH}{\hbar} \delta t \right)$

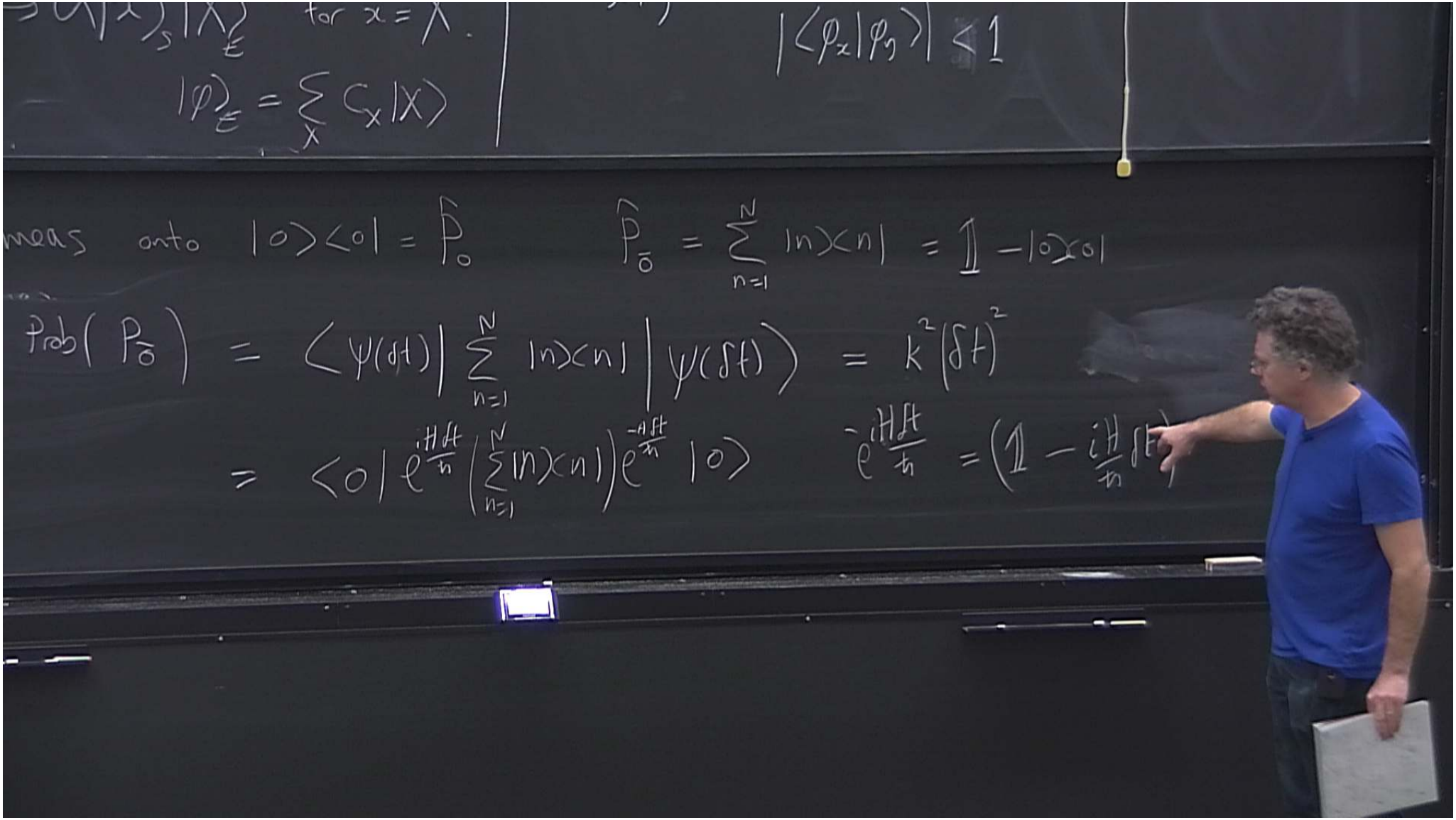
$|\varphi\rangle_E = \sum_x c_x |x\rangle$ for $x = \lambda$.

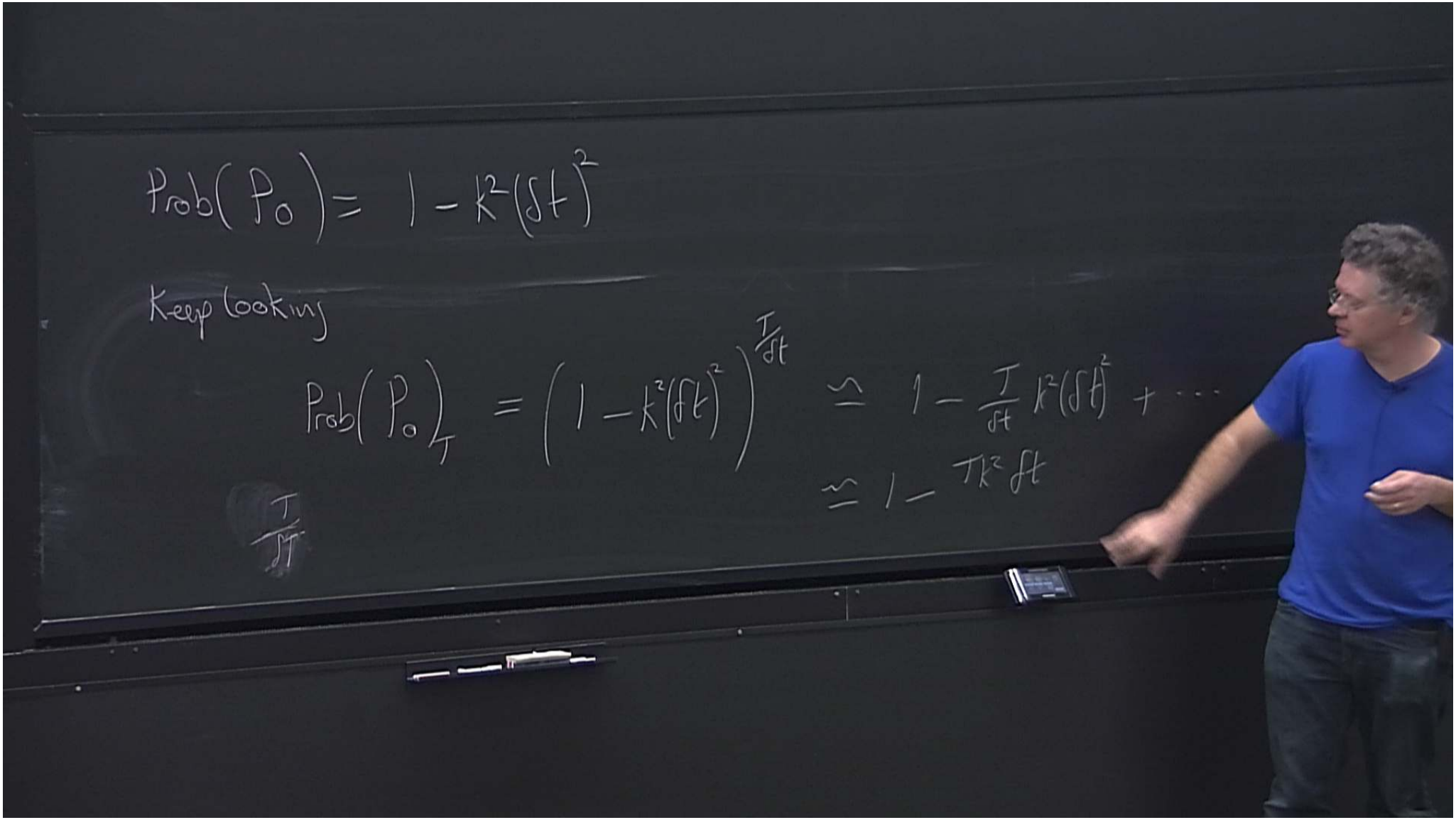
$|\langle \varphi_x | \varphi_y \rangle| \leq 1$

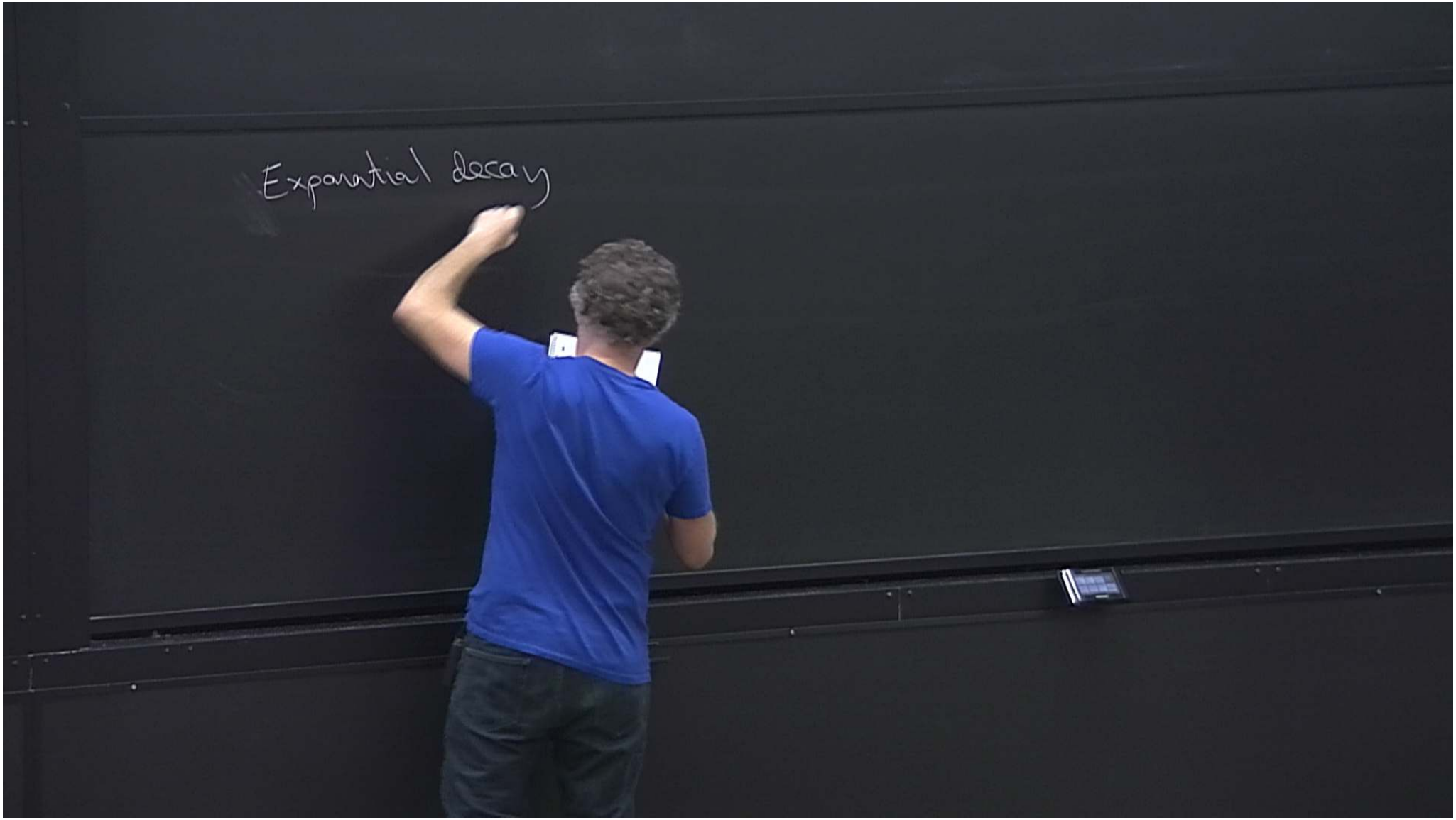
meas onto $|0\rangle\langle 0| = \hat{P}_0$ $\quad \hat{P}_{\bar{0}} = \sum_{n=1}^{N} |n\rangle\langle n| = \mathbb{1} - |0\rangle\langle 0|$

$$\text{Prob}(P_{\bar{0}}) = \langle \psi(\delta t) | \sum_{n=1}^{N} |n\rangle\langle n| \, | \psi(\delta t) \rangle = k^2 (\delta t)^2$$

$$= \langle 0 | e^{\frac{iH\delta t}{\hbar}} \left(\sum_{n=1}^{N} |n\rangle\langle n| \right) e^{\frac{-iH\delta t}{\hbar}} |0\rangle \qquad e^{\frac{iH\delta t}{\hbar}} = \left(\mathbb{1} - \frac{iH}{\hbar} \delta t \right)$$

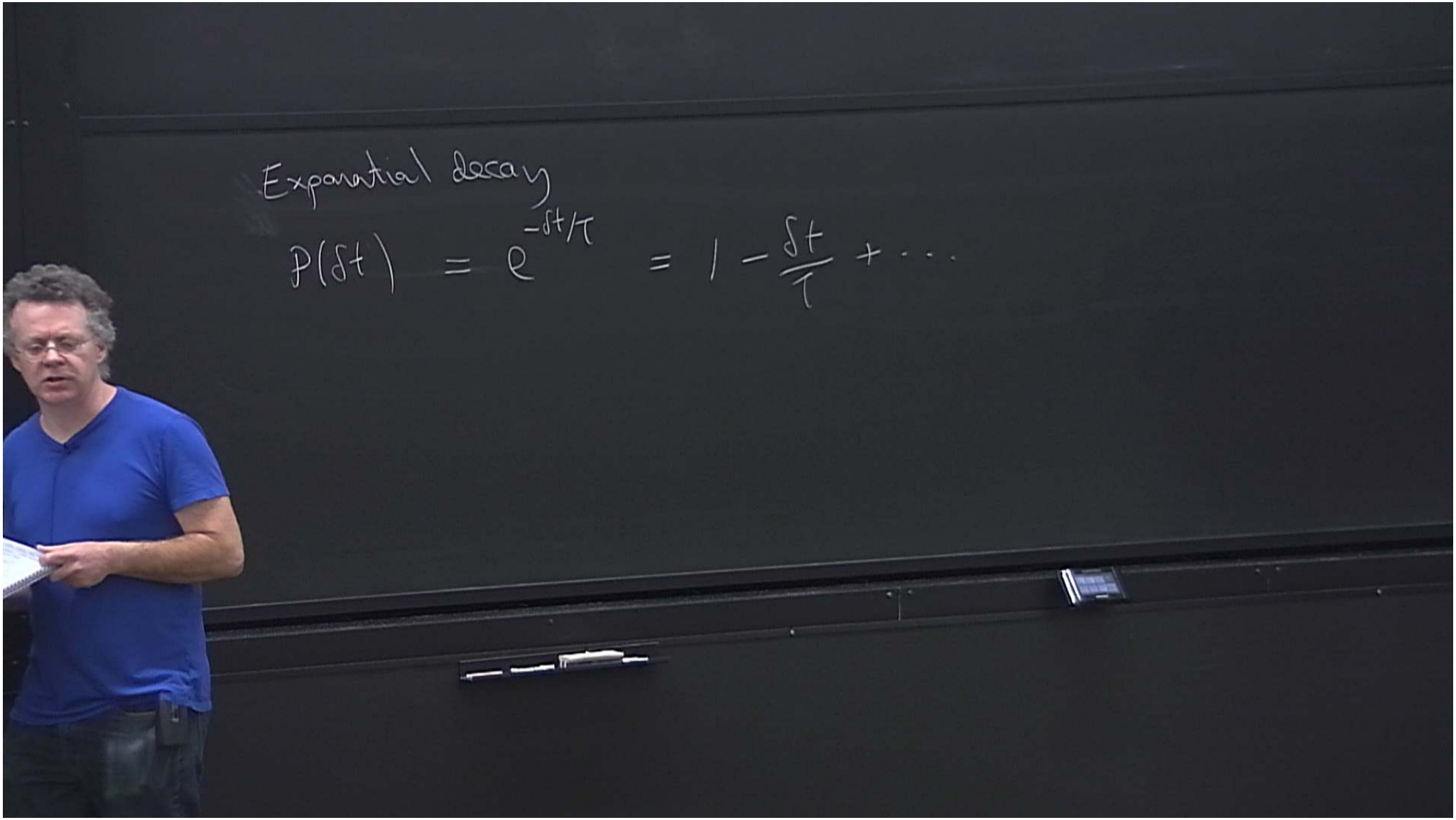



$\text{Prob}(P_0) = 1 - k^2(\delta t)^2$
Keep looking
$\text{Prob}(P_0)_T = \left(1 - k^2(\delta t)^2\right)^{\frac{T}{\delta t}} \simeq 1 - \frac{T}{\delta t} k^2(\delta t)^2 + \cdots$
$\simeq 1 - Tk^2 \delta t$
$\frac{T}{\delta T}$


Exponential decay

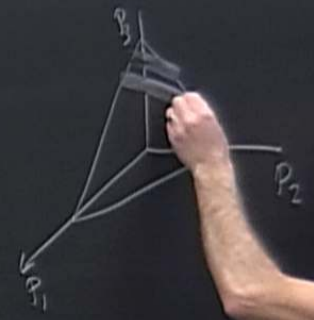
Exponential decay

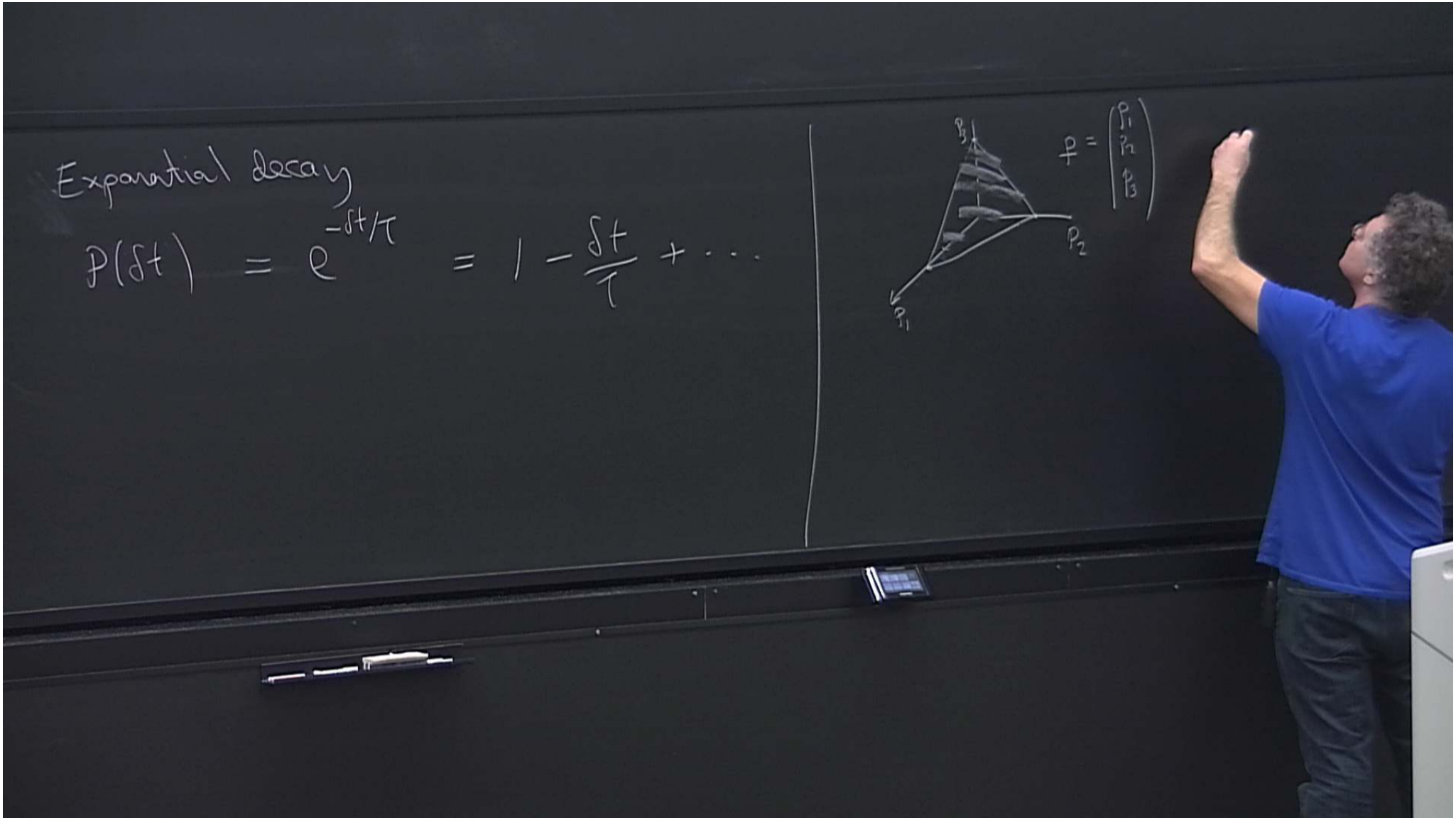
$$P(\delta t) = e^{-\delta t/\tau} = 1 - \frac{\delta t}{\tau} + \cdots$$

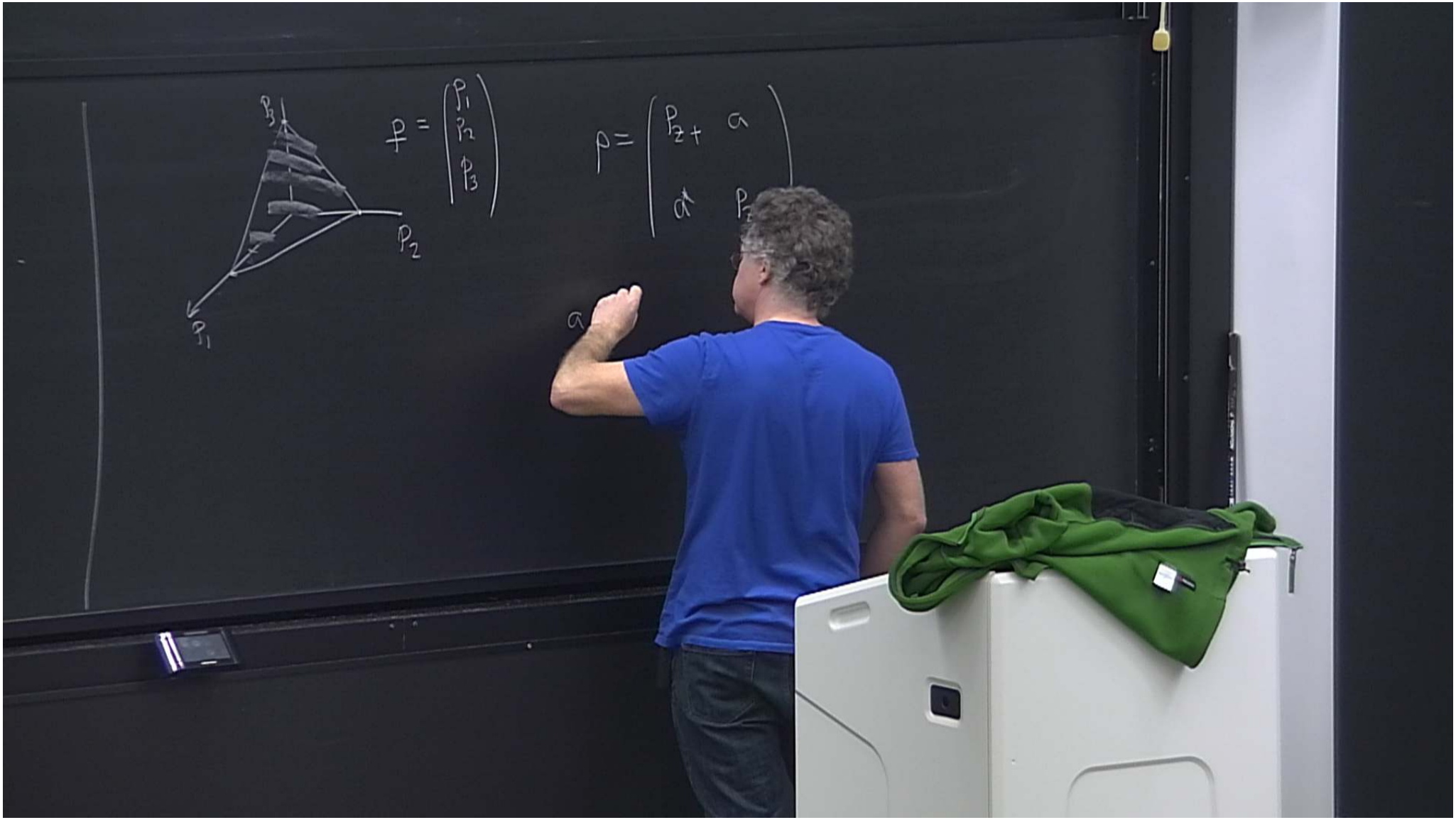

Exponential decay
$P(\delta t) = e^{-\delta t/\tau} = 1 - \frac{\delta t}{\tau} + \cdots$

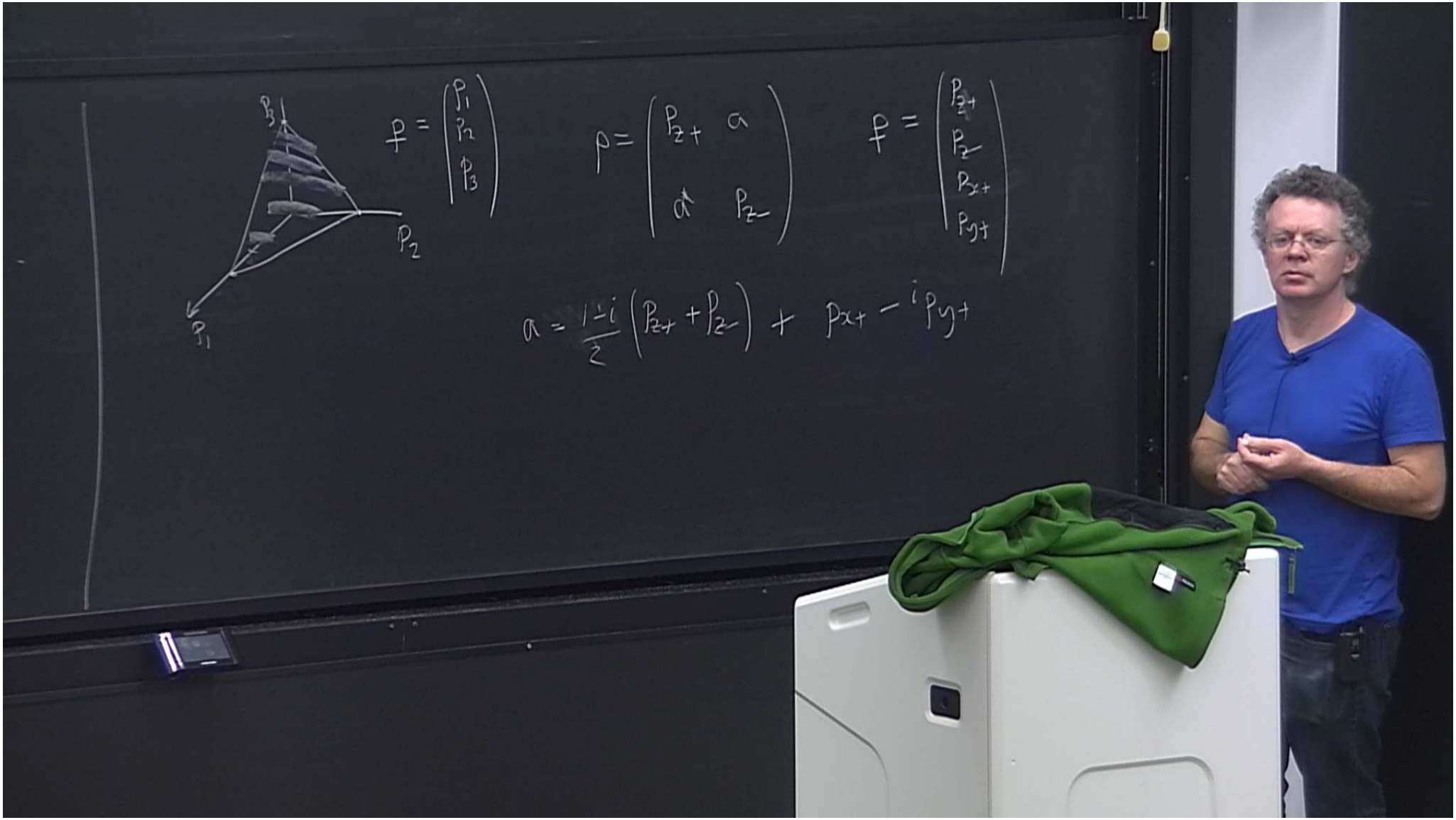
Exponential decay

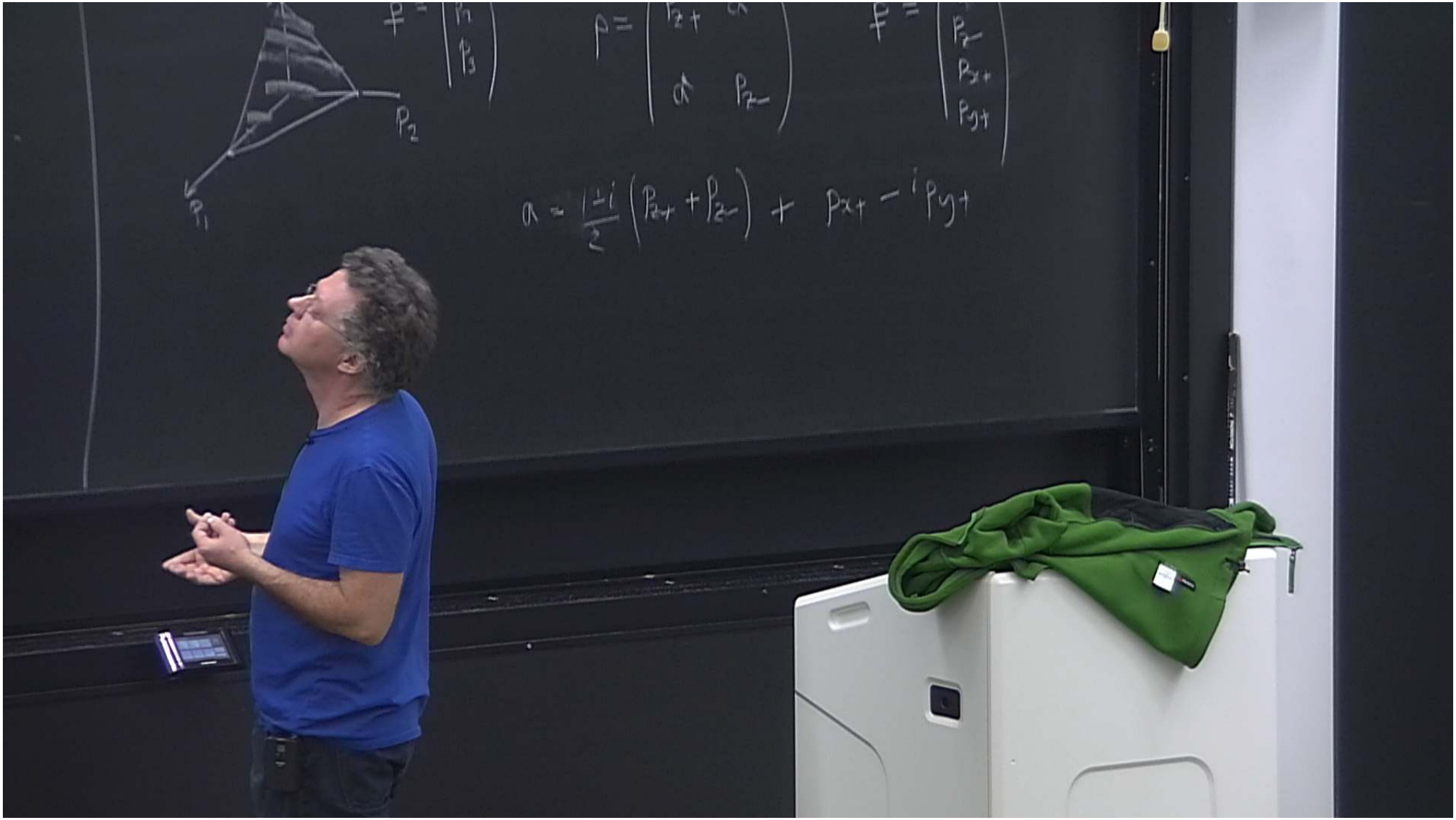
$$P(\delta t) = e^{-\delta t/\tau} = 1 - \frac{\delta t}{\tau} + \cdots$$

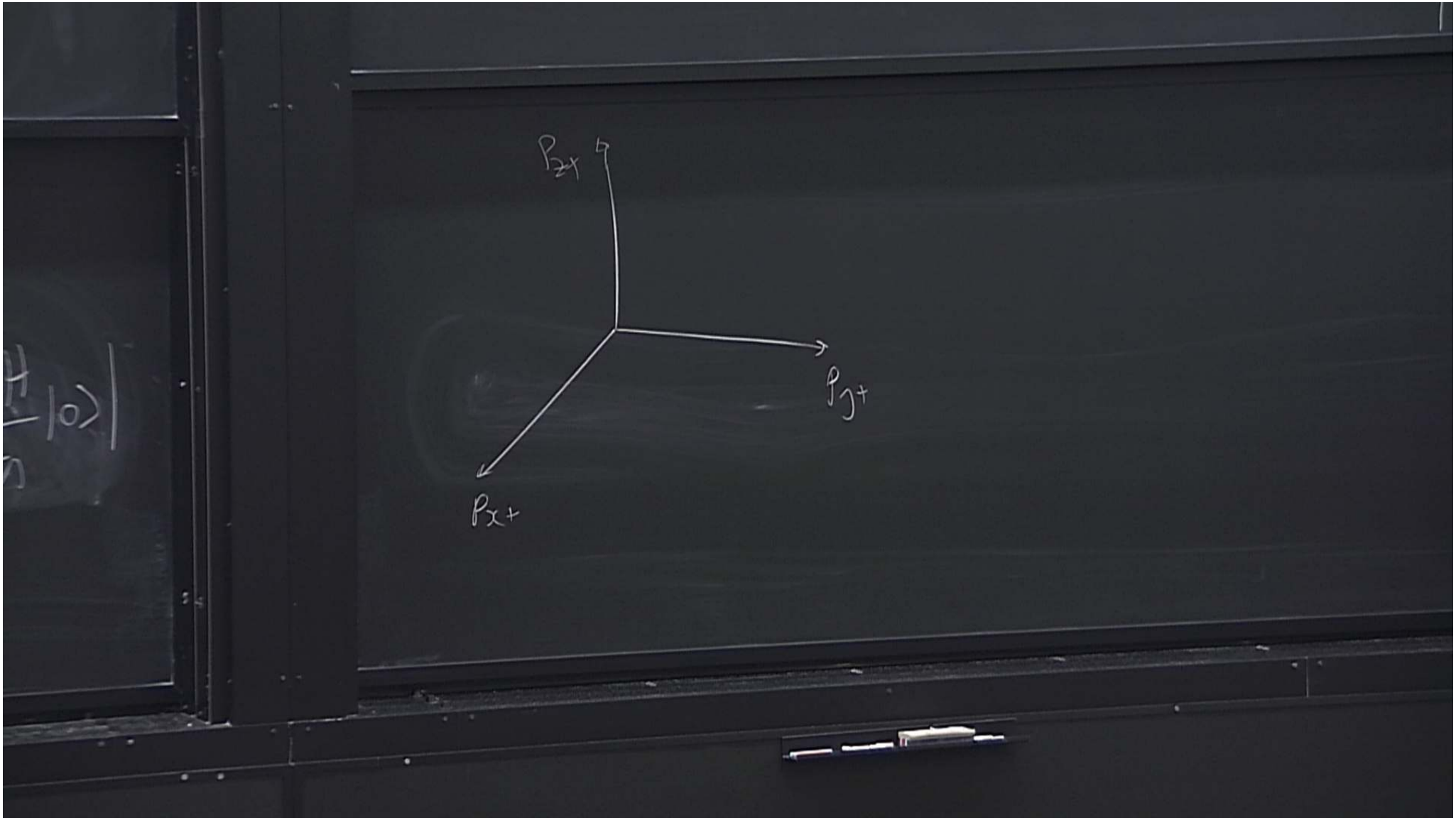


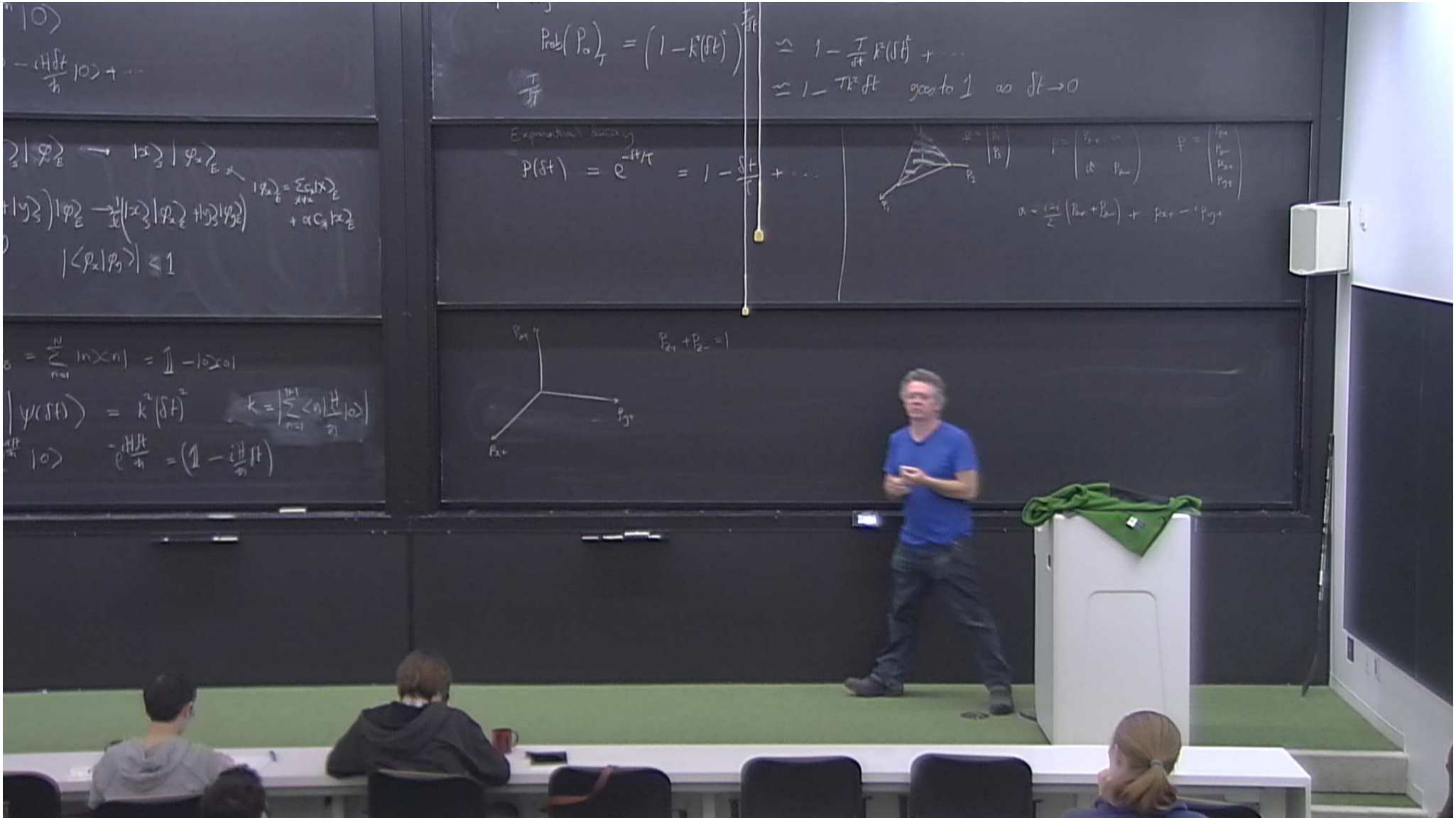

Exponential decay
$P(\delta t) = e^{-\delta t/\tau} = 1 - \frac{\delta t}{\tau} + \dots$
p_3
p_2
p_1
$p = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$

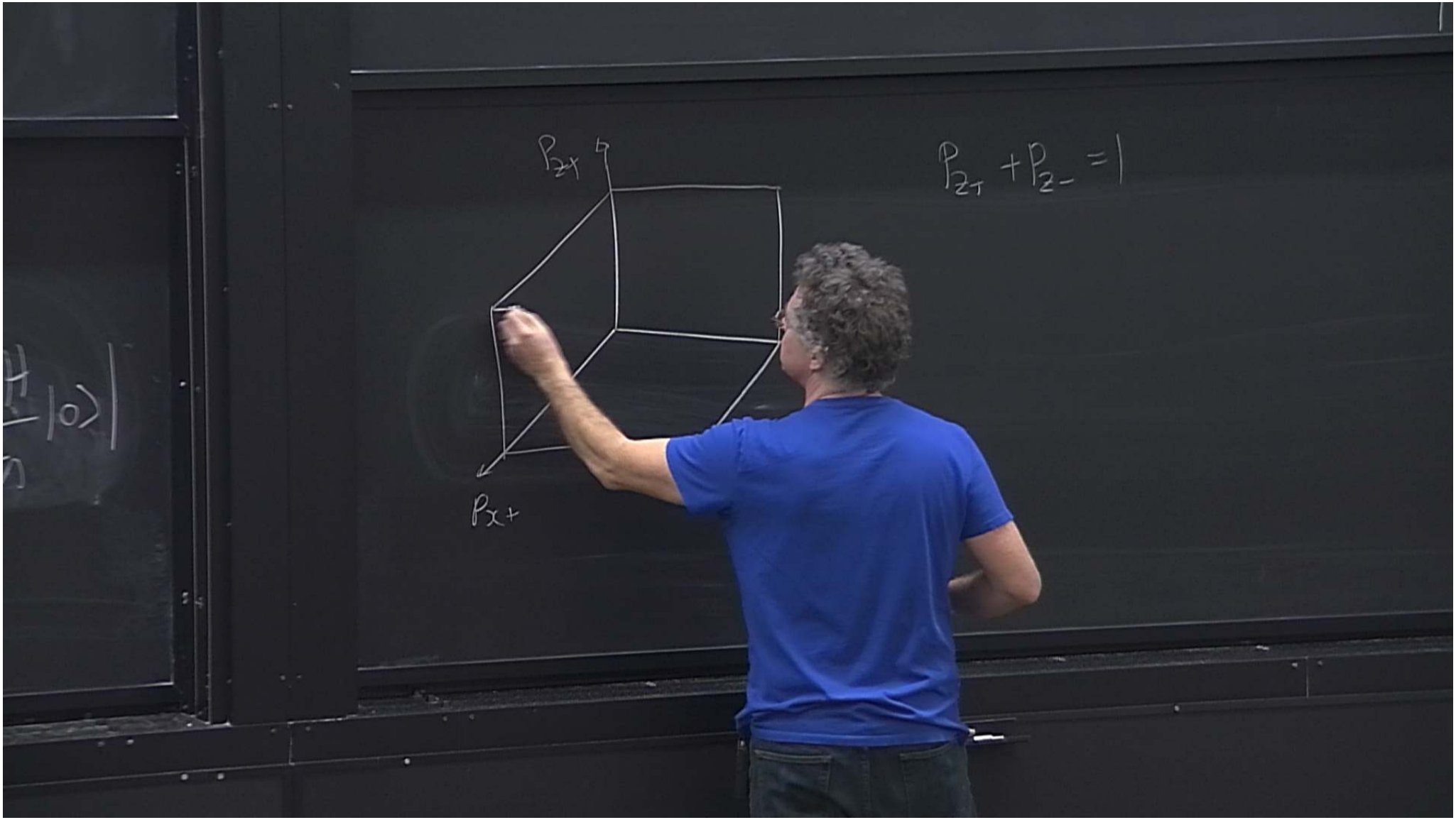

p_3
p_2
p_1
$p = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$
$\rho = \begin{pmatrix} p_2 + & \sigma \\ \sigma & p_2 \end{pmatrix}$
σ

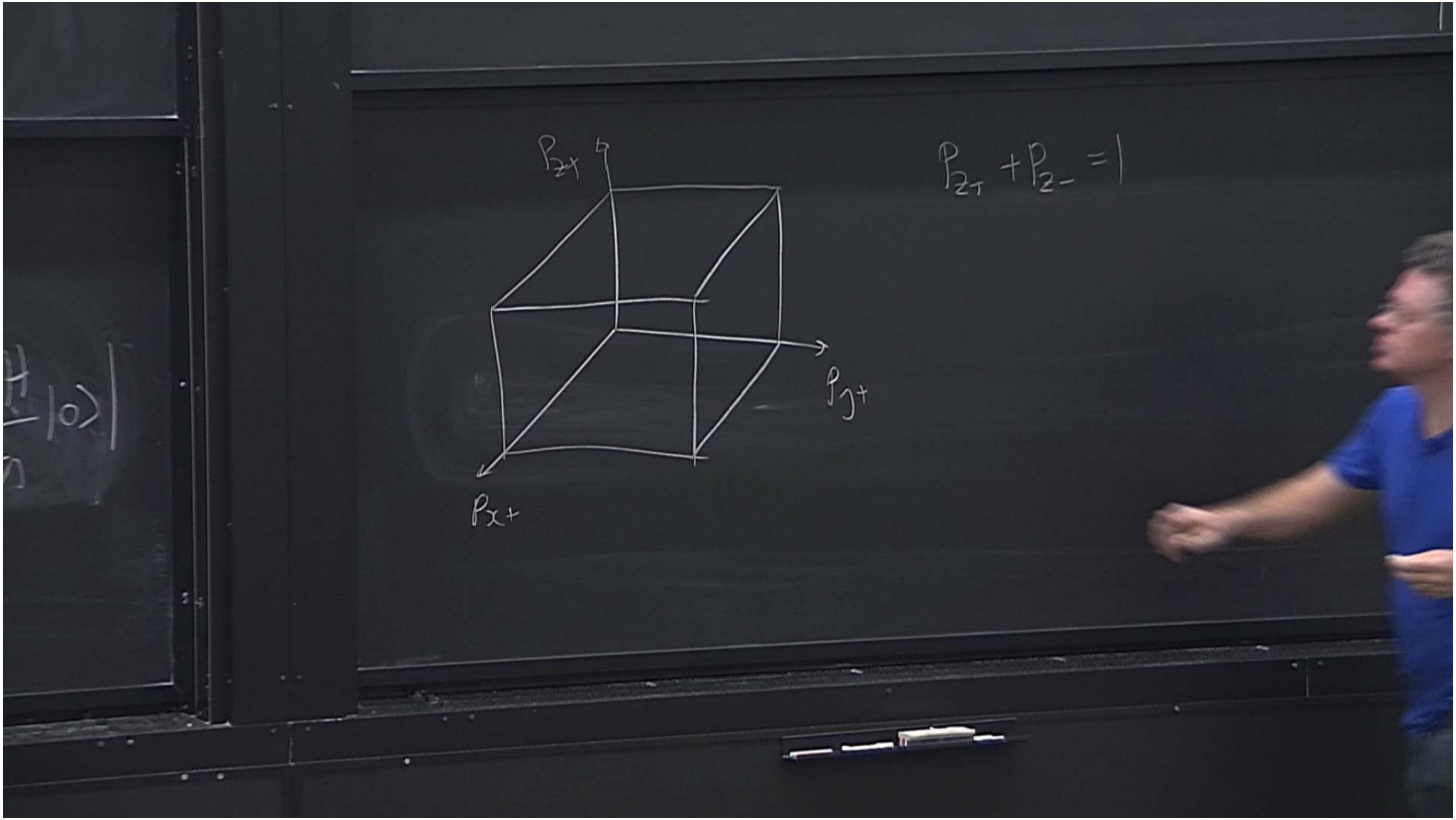

p_3
p_2
p_1
$\underline{p} = \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}$
$\rho = \begin{pmatrix} p_{z+} & \sigma \\ \sigma^* & p_{z-} \end{pmatrix}$
$\underline{p} = \begin{pmatrix} p_{z+} \\ p_{z-} \\ p_{x+} \\ p_{y+} \end{pmatrix}$
$\sigma = \frac{1-i}{2}\left(p_{z+} + p_{z-}\right) + p_{x+} - i\,p_{y+}$

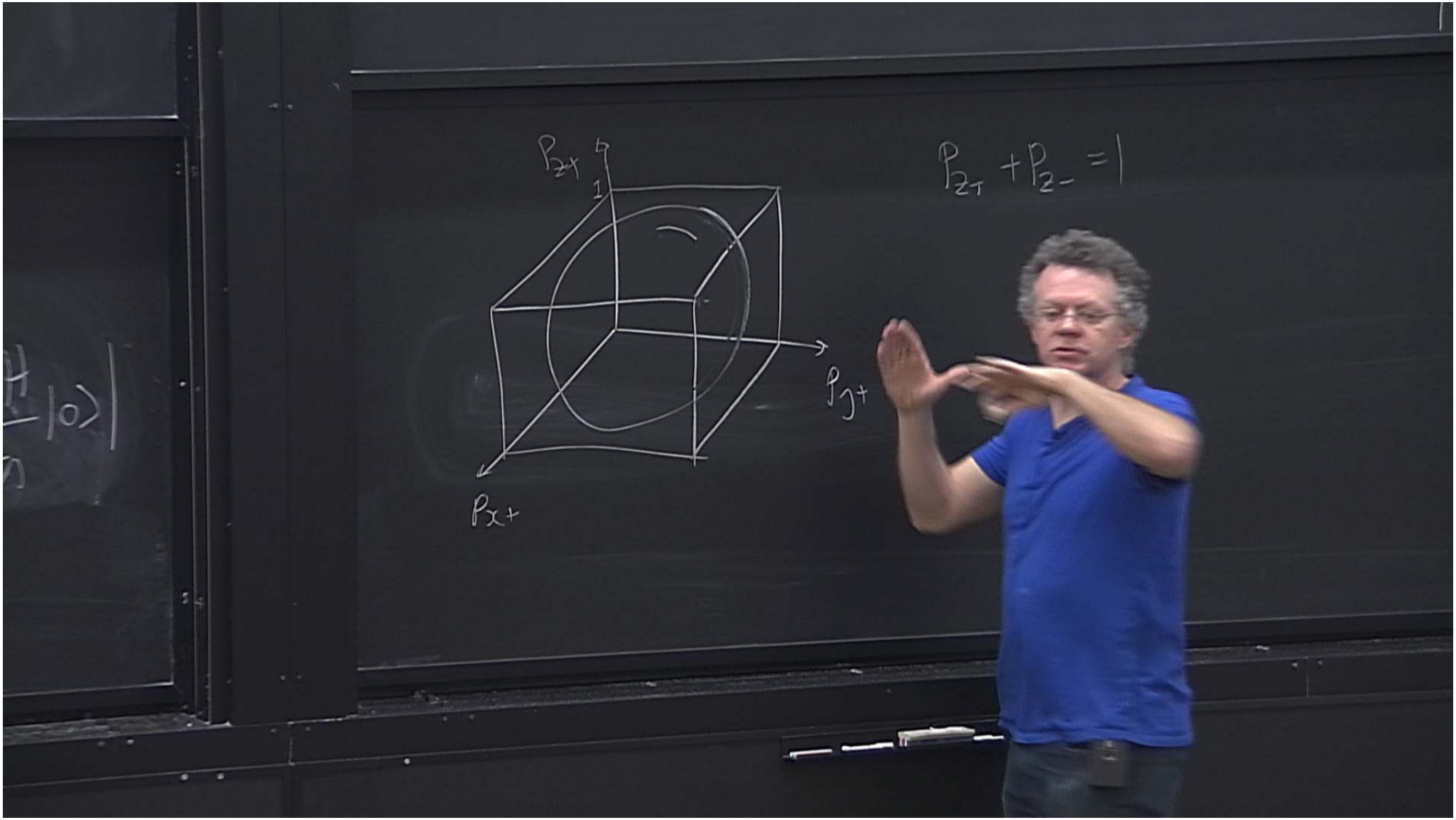

p_1
p_2
$a = \frac{1+i}{2}(p_{z+} + p_{z-}) + p_{x+} - i p_{y+}$

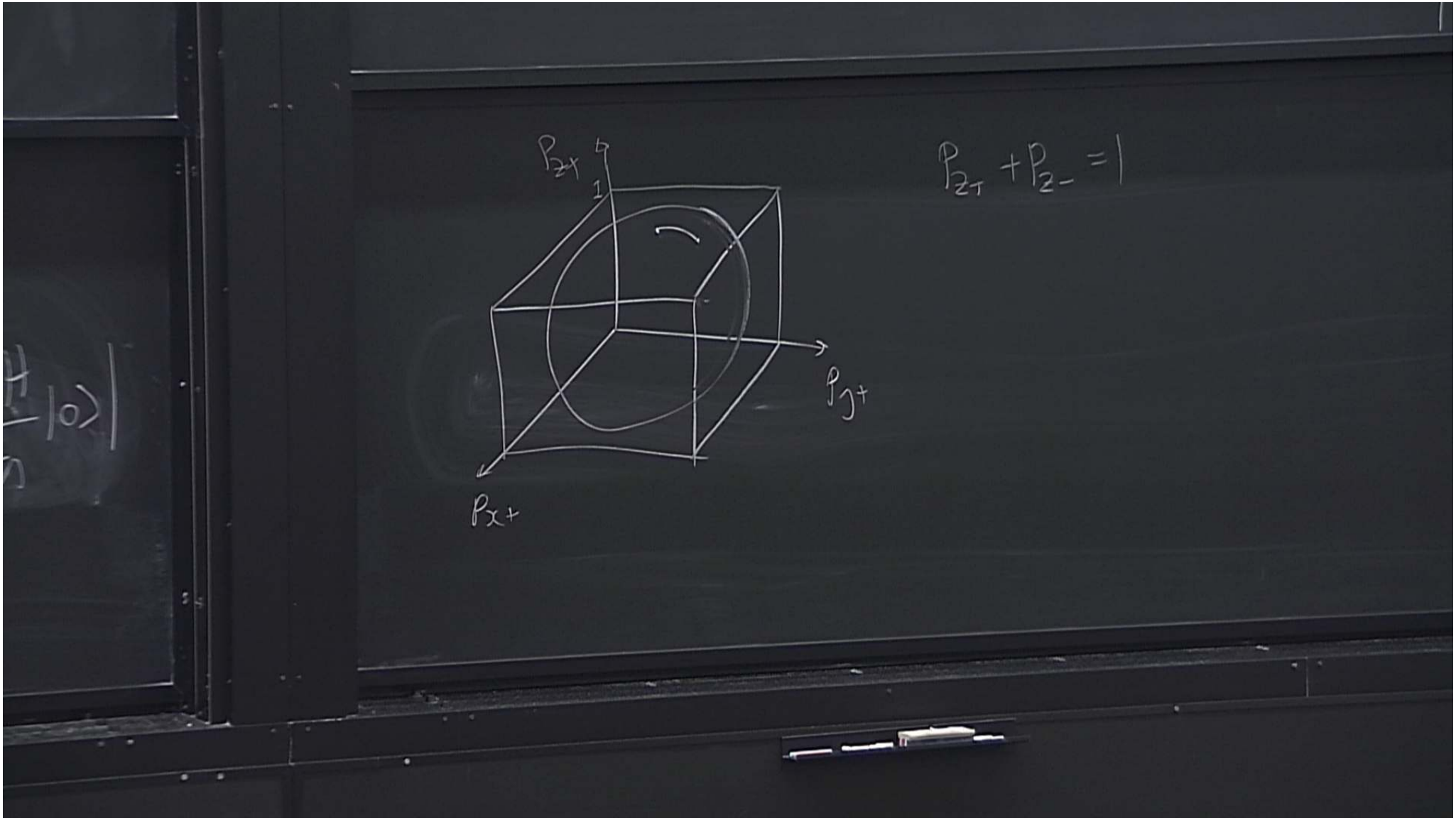

P_{z+}
P_{y+}
P_{x+}




P_{z+}
P_{x+}
$P_{z+} + P_{z-} = 1$

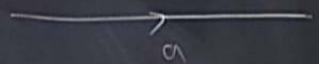

P_{z+}
P_{y+}
P_{x+}
$P_{z+} + P_{z-} = 1$


P_{z+}
1
P_{y+}
P_{x+}
$P_{z+} + P_{z-} = 1$


P_{z+}
1
P_{y+}
P_{x+}
$P_{z+} + P_{z-} = 1$

Quantum Optical Interferometry

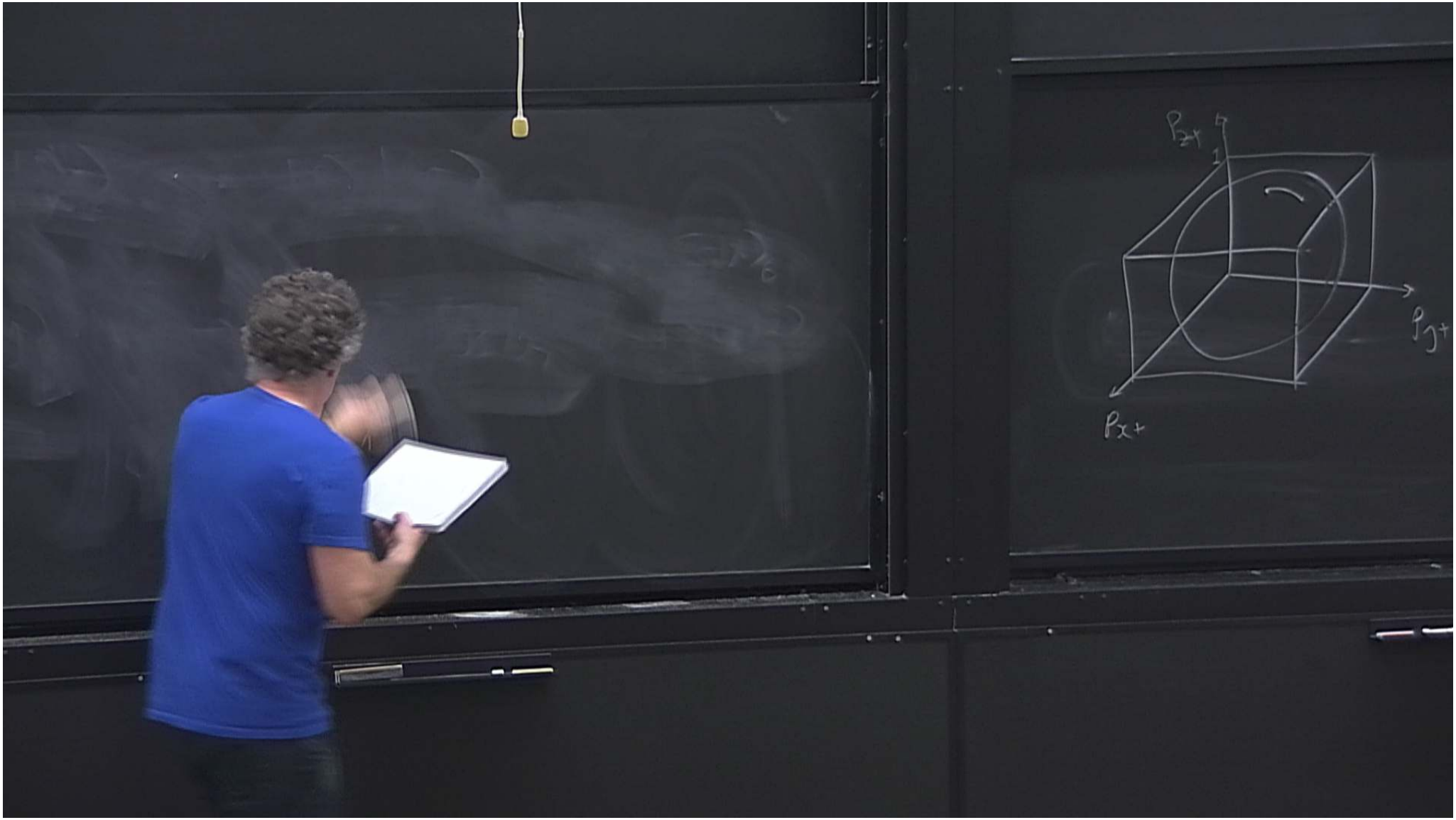
Single mode all polarizations the same.

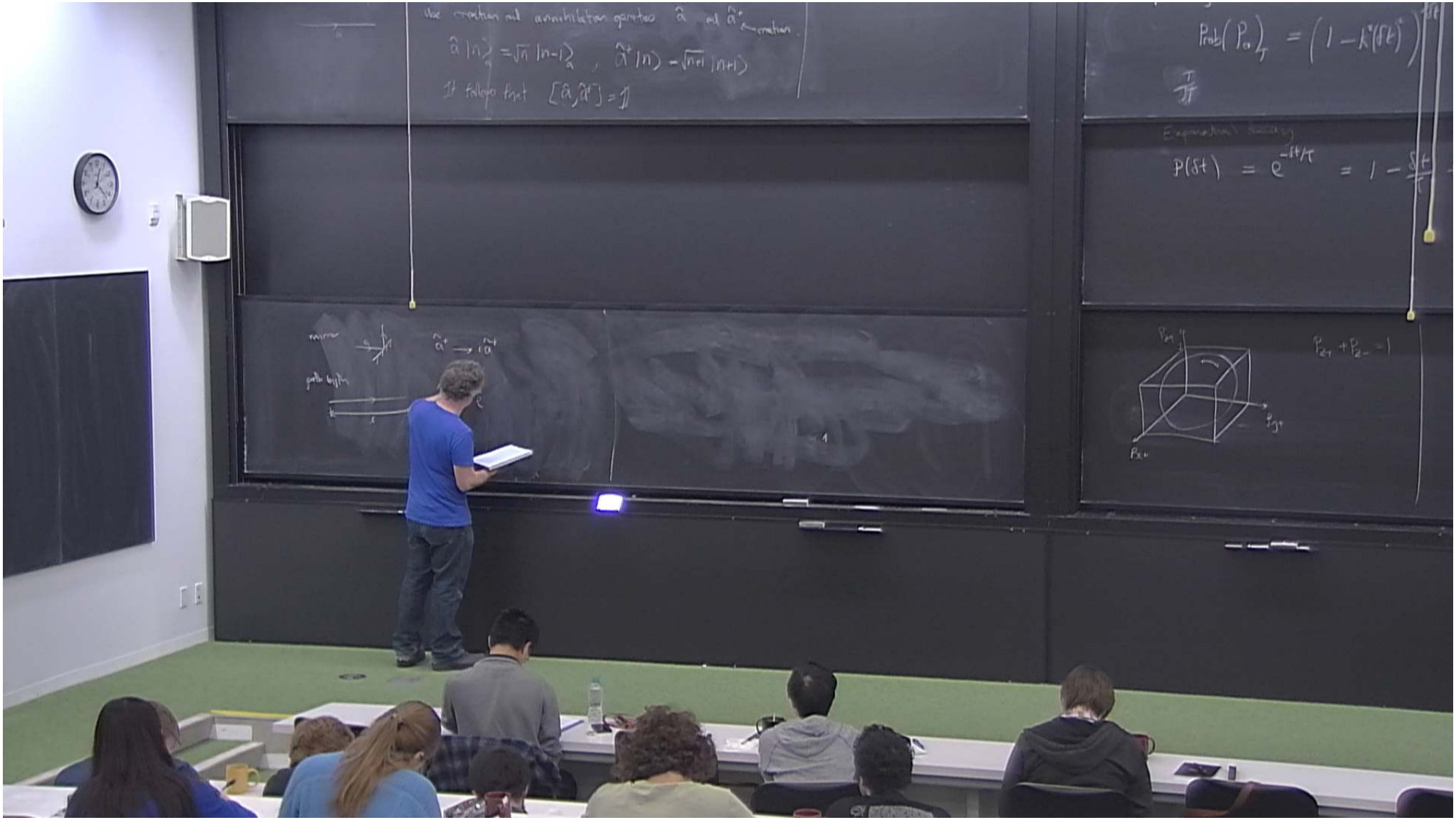


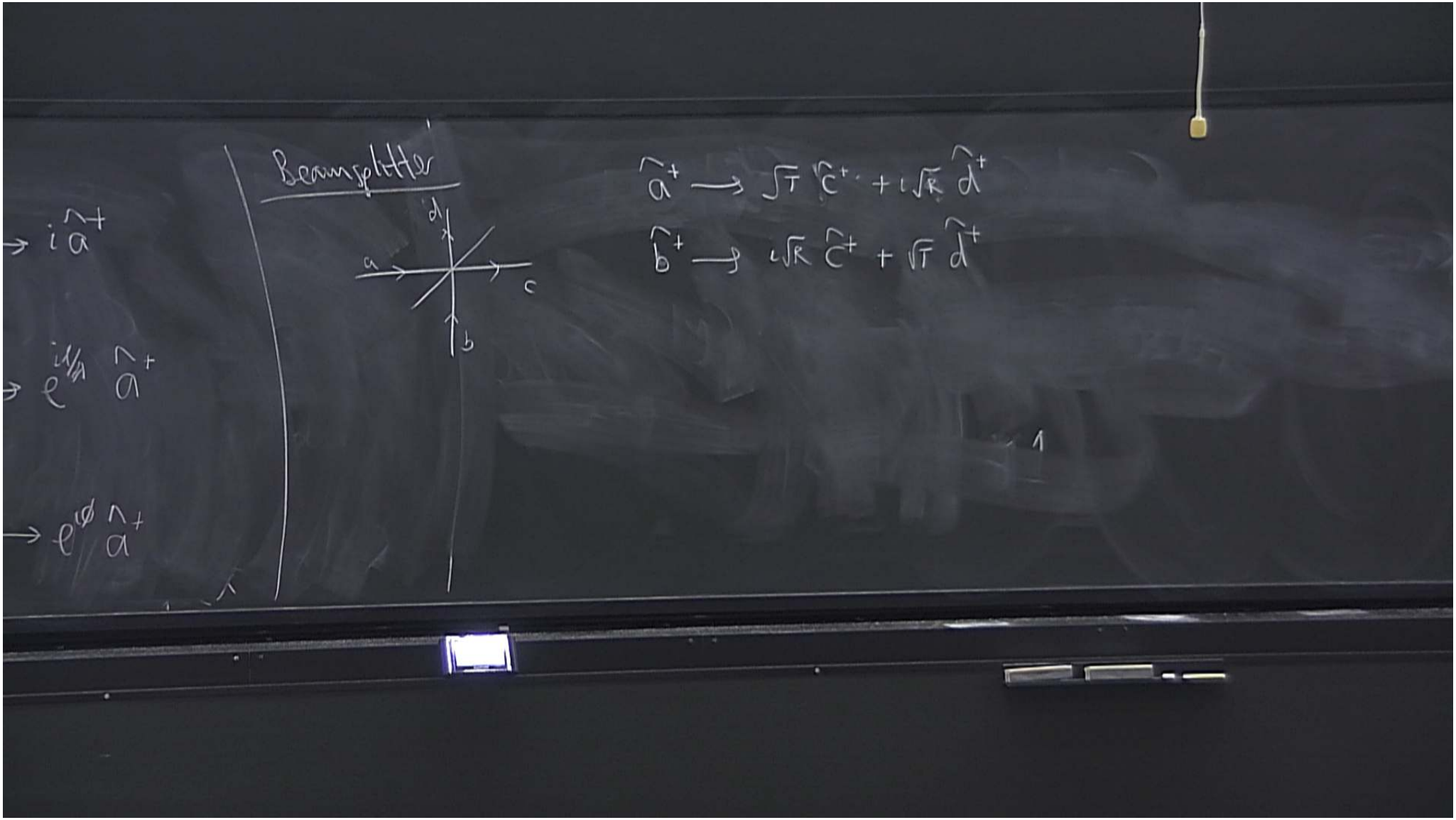
Use creation and annihilation operators $\hat{a}$ (annihilation) and $\hat{a}^+$ (creation).

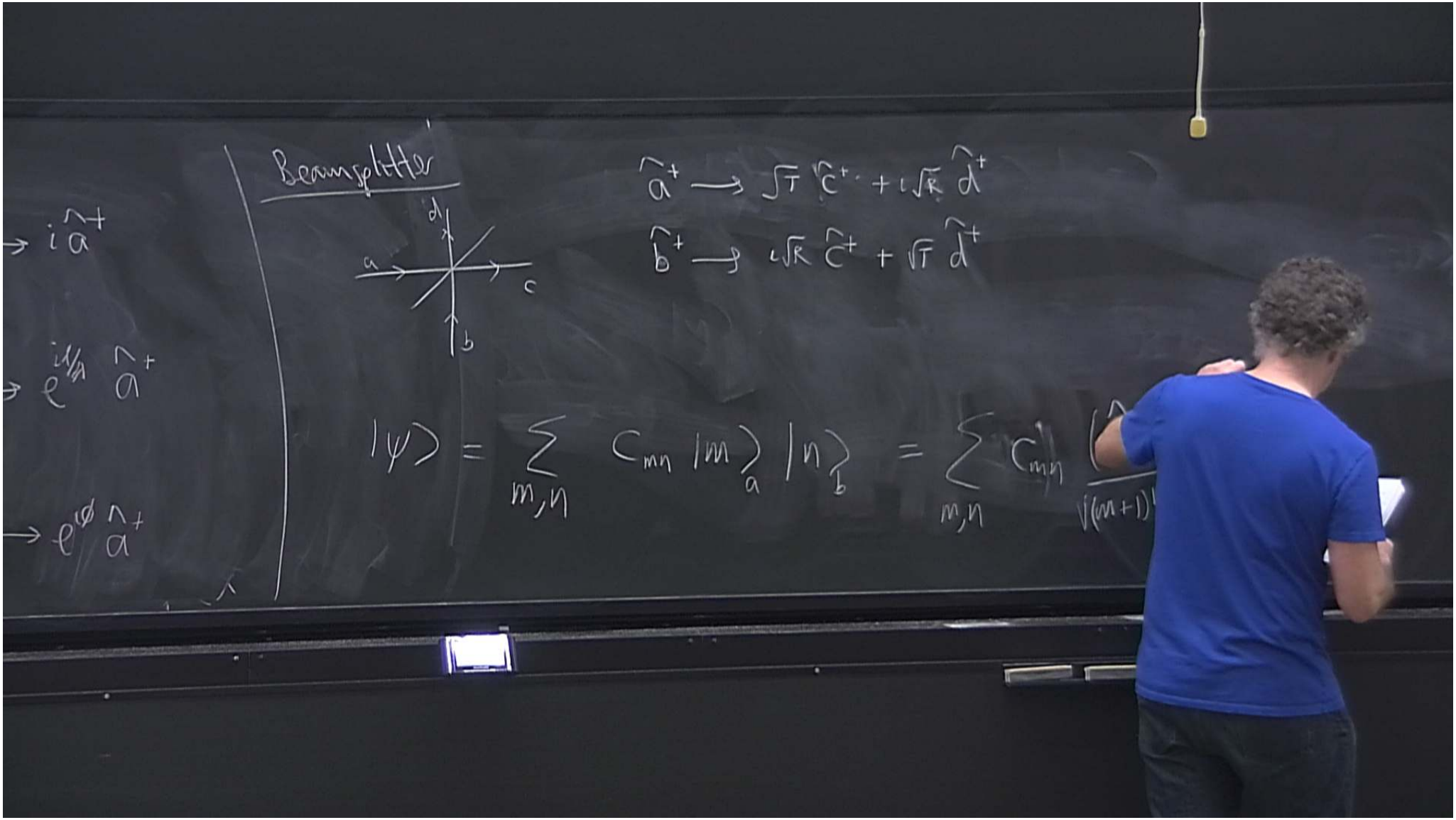
$$\hat{a}\,|n\rangle_a = \sqrt{n}\,|n-1\rangle_a \,, \quad \hat{a}^+|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

It follows that $[\hat{a}, \hat{a}^+] = \mathbb{1}$


1
p_{z+}
p_{y+}
p_{x+}


use creation and annihilation operators
It follows that
Exponential decay


Beamsplitter
d
a
c
b
$\hat{a}^+ \longrightarrow \sqrt{T}\,\hat{c}^+ + i\sqrt{R}\,\hat{d}^+$
$\hat{b}^+ \longrightarrow i\sqrt{R}\,\hat{c}^+ + \sqrt{T}\,\hat{d}^+$
$\rightarrow i\hat{a}^+$
$\rightarrow e^{i\varphi}\hat{a}^+$
$\rightarrow e^{i\phi}\hat{a}^+$

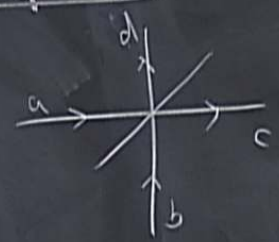

Beamsplitter
d
a
c
b
$\hat{a}^\dagger \rightarrow \sqrt{T}\,\hat{c}^\dagger + i\sqrt{R}\,\hat{d}^\dagger$
$\hat{b}^\dagger \rightarrow i\sqrt{R}\,\hat{c}^\dagger + \sqrt{T}\,\hat{d}^\dagger$
$|\psi\rangle = \sum_{m,n} c_{mn} |m\rangle_a |n\rangle_b = \sum_{m,n} c_{mn}$

$\to i\hat{a}^+$

$\to e^{i\lambda/\hbar}\,\hat{a}^+$

$\to e^{i\phi}\,\hat{a}^+$

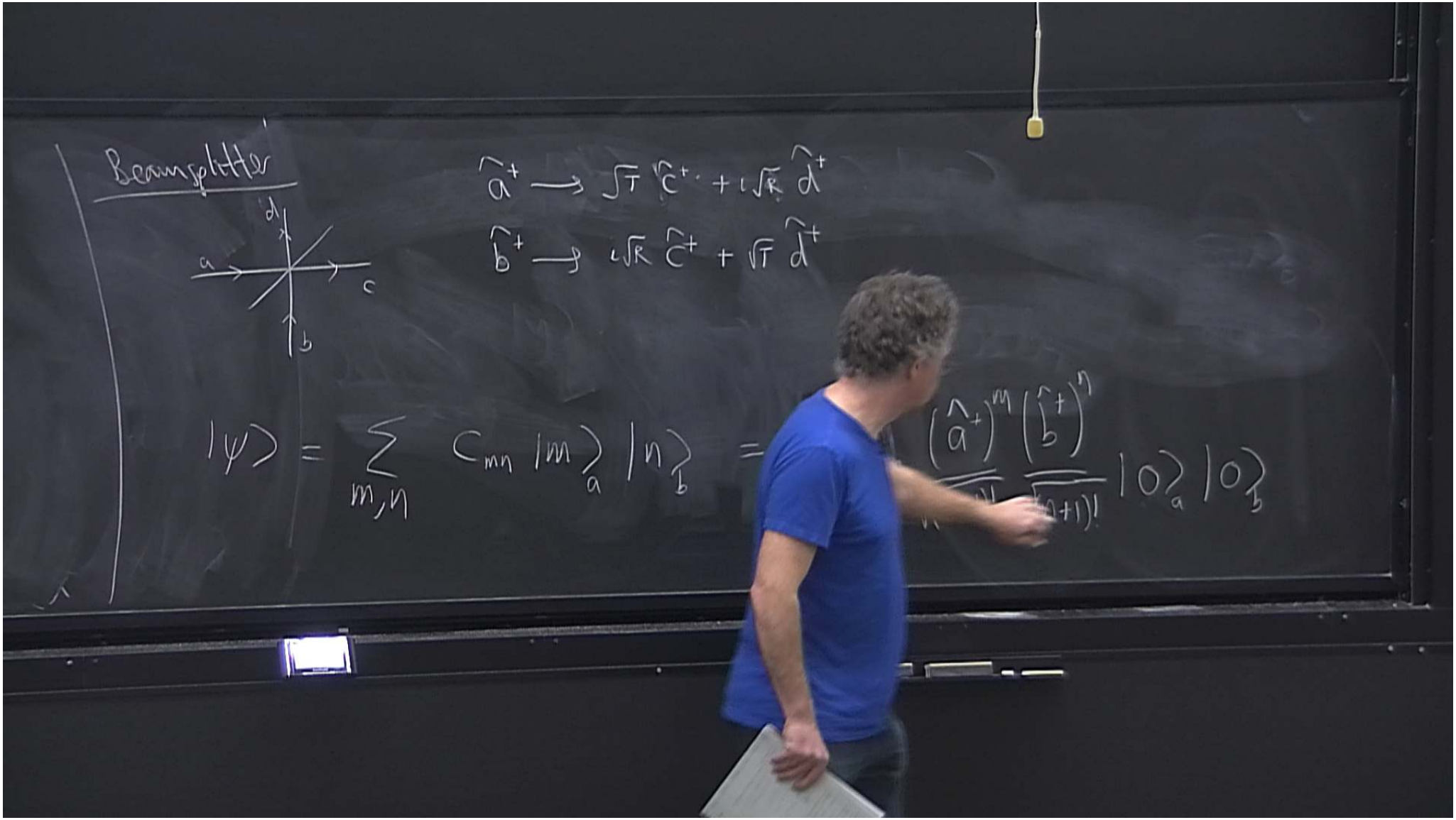
Beamsplitter

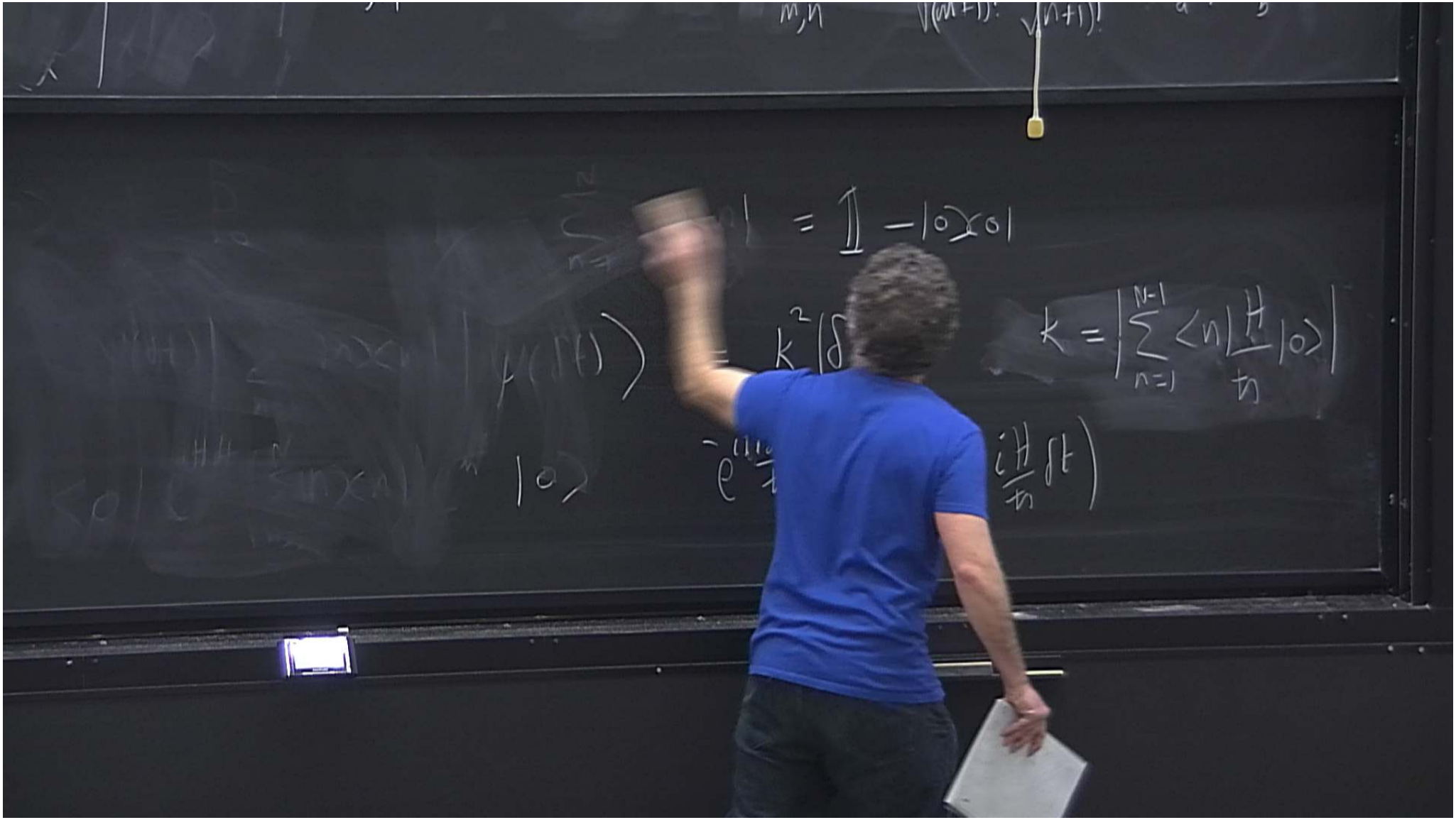


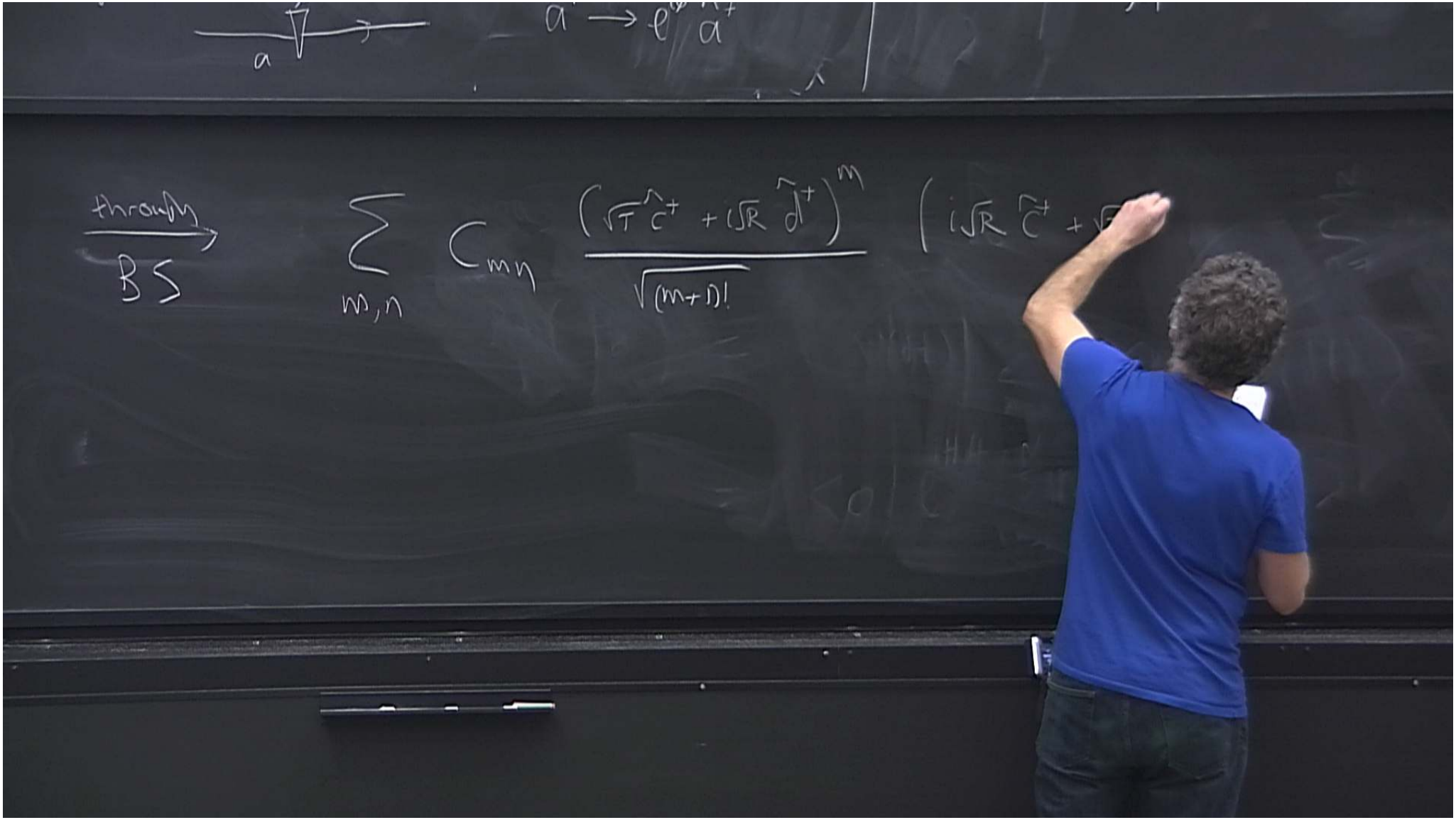
$$\hat{a}^+ \longrightarrow \sqrt{T}\,\hat{c}^+ + i\sqrt{R}\,\hat{d}^+$$

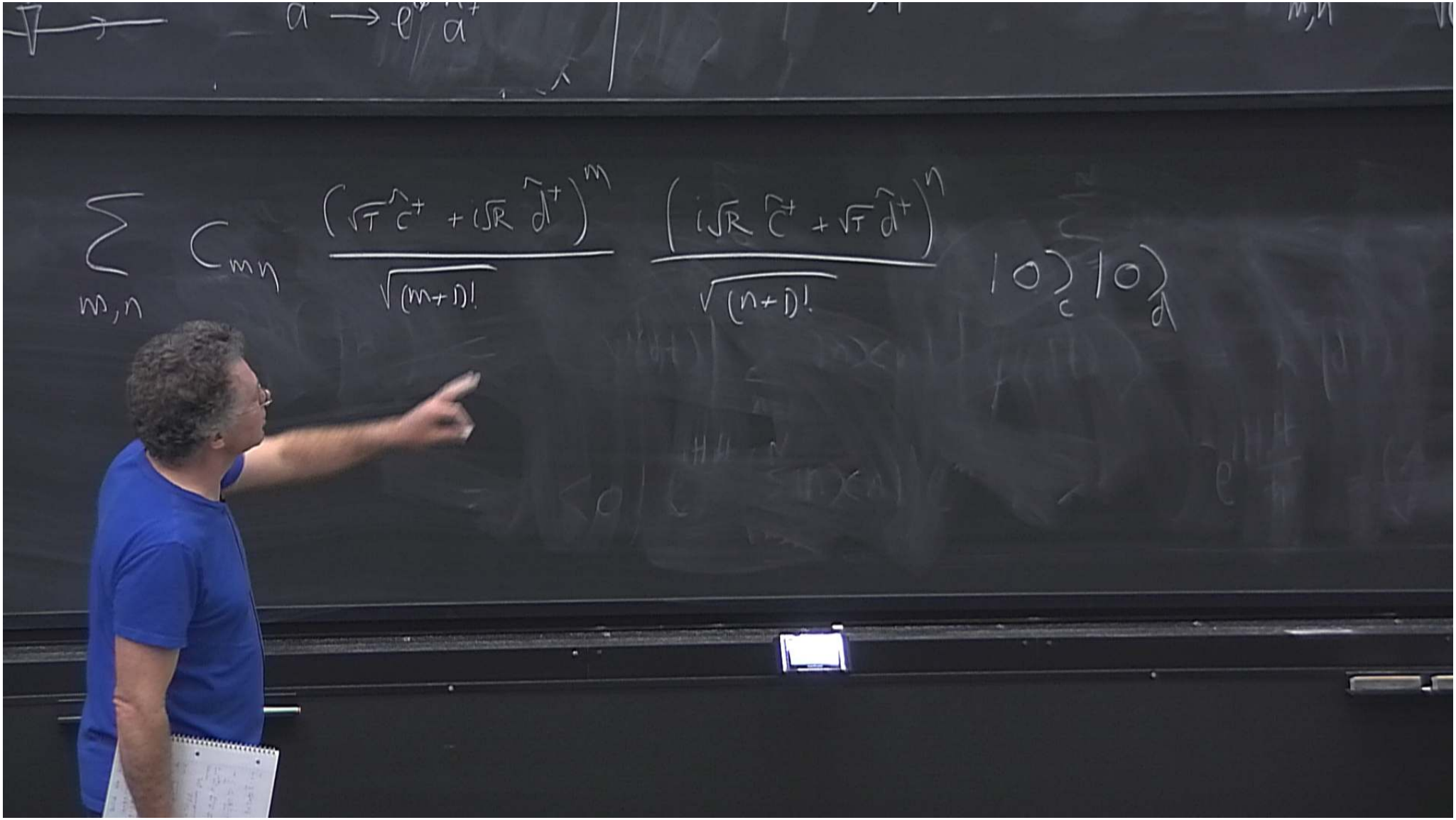
$$\hat{b}^+ \longrightarrow i\sqrt{R}\,\hat{c}^+ + \sqrt{T}\,\hat{d}^+$$

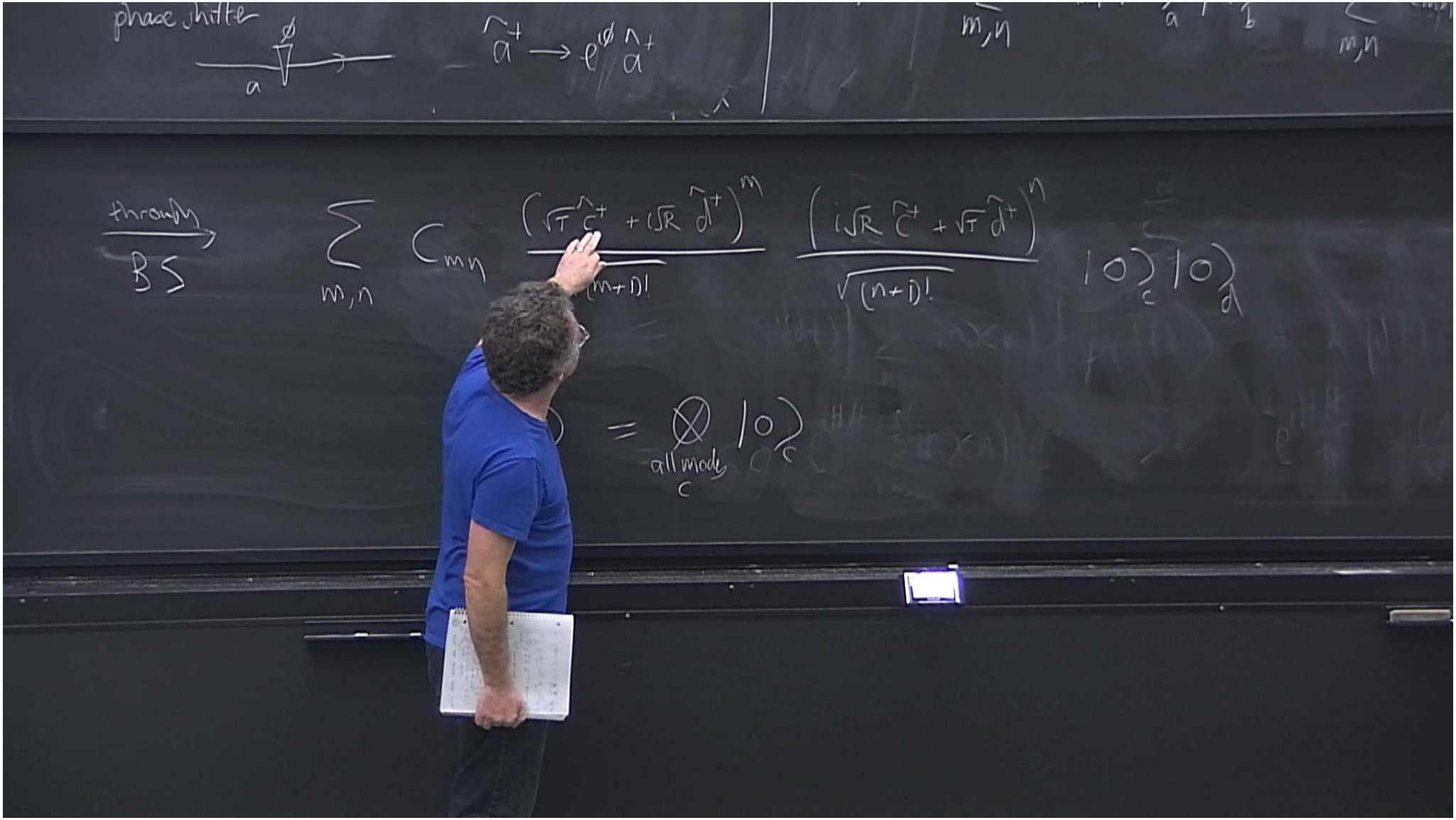
$$|\psi\rangle = \sum_{m,n} c_{mn}\,|m\rangle_a\,|n\rangle_b = \sum_{m,n} c_{mn}\,\frac{(\hat{a}^+)^m}{\sqrt{(m+1)!}}\,\frac{(\hat{b}^+)^n}{\sqrt{(n+1)!}}\,|0\rangle_a\,|0\rangle_b$$


Beamsplitter
d
a
c
b
$\hat{a}^+ \longrightarrow \sqrt{T}\,\hat{c}^+ + i\sqrt{R}\,\hat{d}^+$
$\hat{b}^+ \longrightarrow i\sqrt{R}\,\hat{c}^+ + \sqrt{T}\,\hat{d}^+$
$|\psi\rangle = \sum_{m,n} c_{mn} |m\rangle_a |n\rangle_b =$
$\frac{(\hat{a}^+)^m}{\sqrt{m!}} \frac{(\hat{b}^+)^n}{\sqrt{(n+1)!}} |0\rangle_a |0\rangle_b$


$= \mathbb{I} - |0\rangle\langle 0|$
$k = \left| \sum_{n=1}^{N-1} \langle n | \frac{H}{\hbar} | 0 \rangle \right|$


a
$a \rightarrow e^{\phi} a^{\dagger}$
through BS
$\sum_{m,n} C_{mn} \frac{\left(\sqrt{T}\hat{c}^{\dagger} + i\sqrt{R}\hat{d}^{\dagger}\right)^{m}}{\sqrt{(m+1)!}}$
$\left(i\sqrt{R}\hat{c}^{\dagger} + \sqrt{T}\right.$


$\hat{a} \rightarrow e^{i\theta} \hat{a}^\dagger$
$\sum_{m,n} C_{mn} \frac{(\sqrt{T}\hat{c}^\dagger + i\sqrt{R}\hat{d}^\dagger)^m}{\sqrt{(m+1)!}} \frac{(i\sqrt{R}\hat{c}^\dagger + \sqrt{T}\hat{d}^\dagger)^n}{\sqrt{(n+1)!}} |0\rangle_c |0\rangle_d$


phase shifter
a
$\hat{a}^\dagger \to e^{i\phi}\hat{a}^\dagger$
through BS
$\sum_{m,n} C_{mn} \frac{(\sqrt{T}\hat{c}^\dagger + i\sqrt{R}\hat{d}^\dagger)^m}{\sqrt{(m+1)!}} \frac{(i\sqrt{R}\hat{c}^\dagger + \sqrt{T}\hat{d}^\dagger)^n}{\sqrt{(n+1)!}} |0\rangle_c |0\rangle_d$
$= \bigotimes_c |0\rangle_c$
all modes