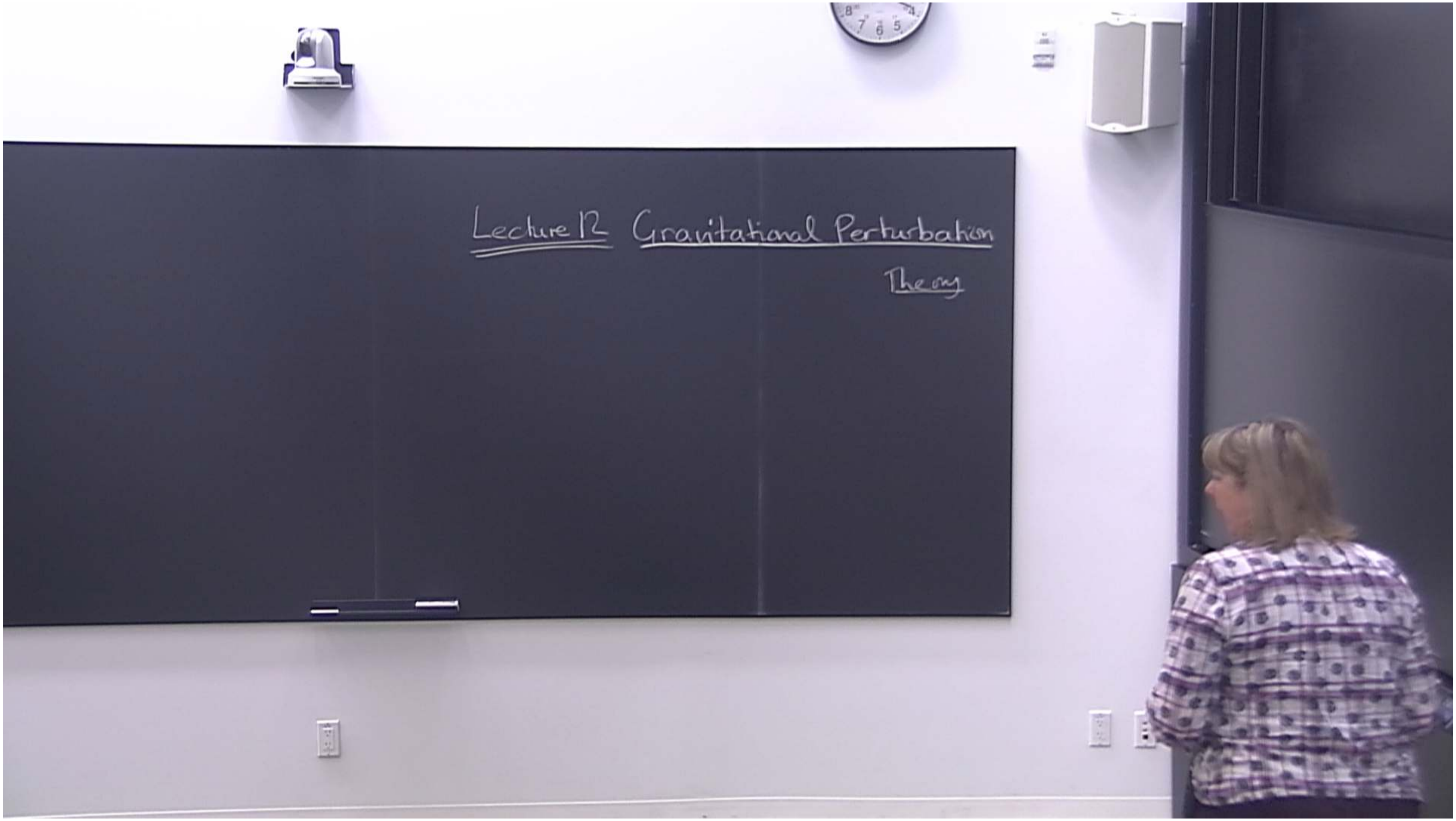
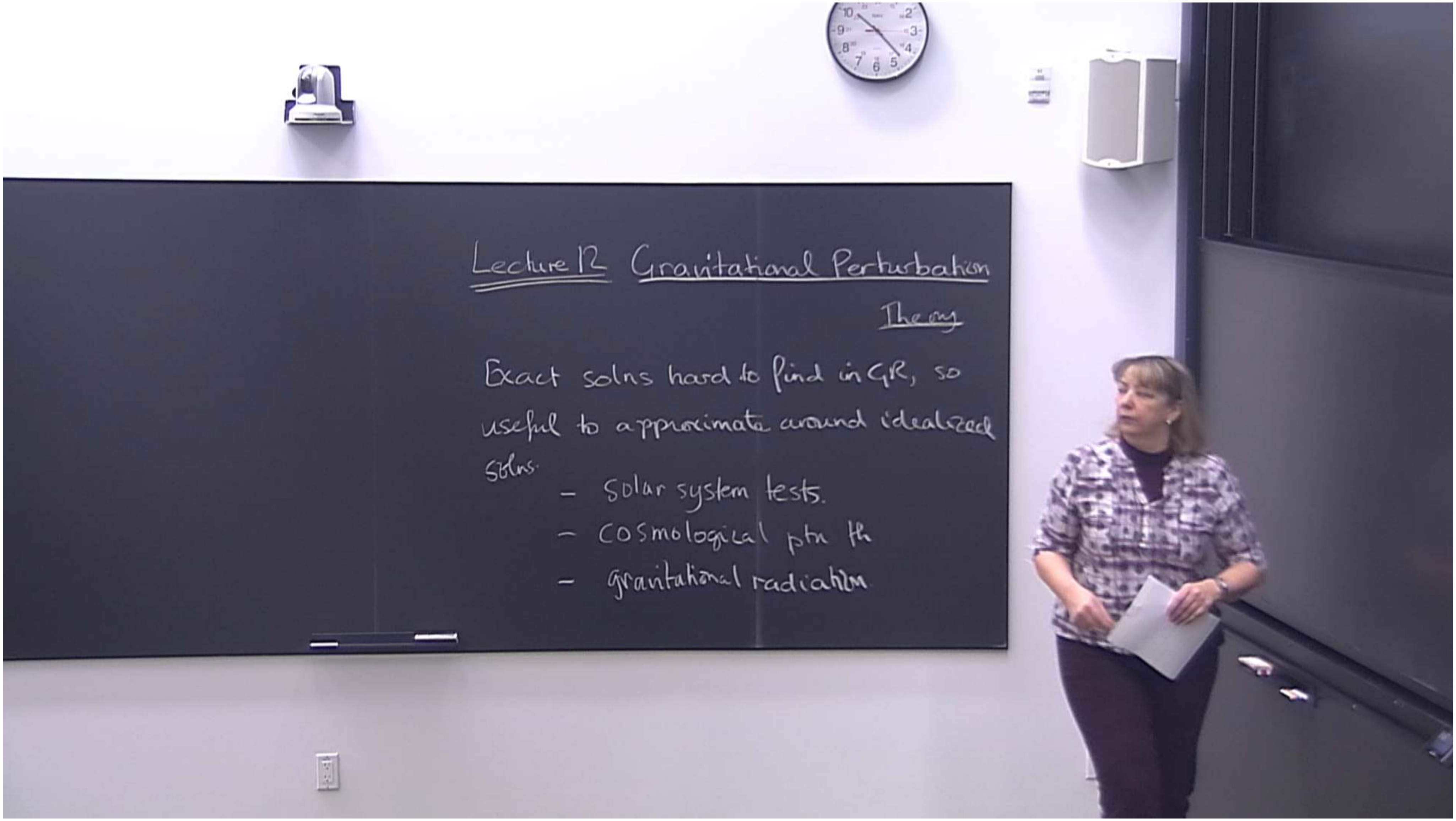
Title: Gravitational Physics-12

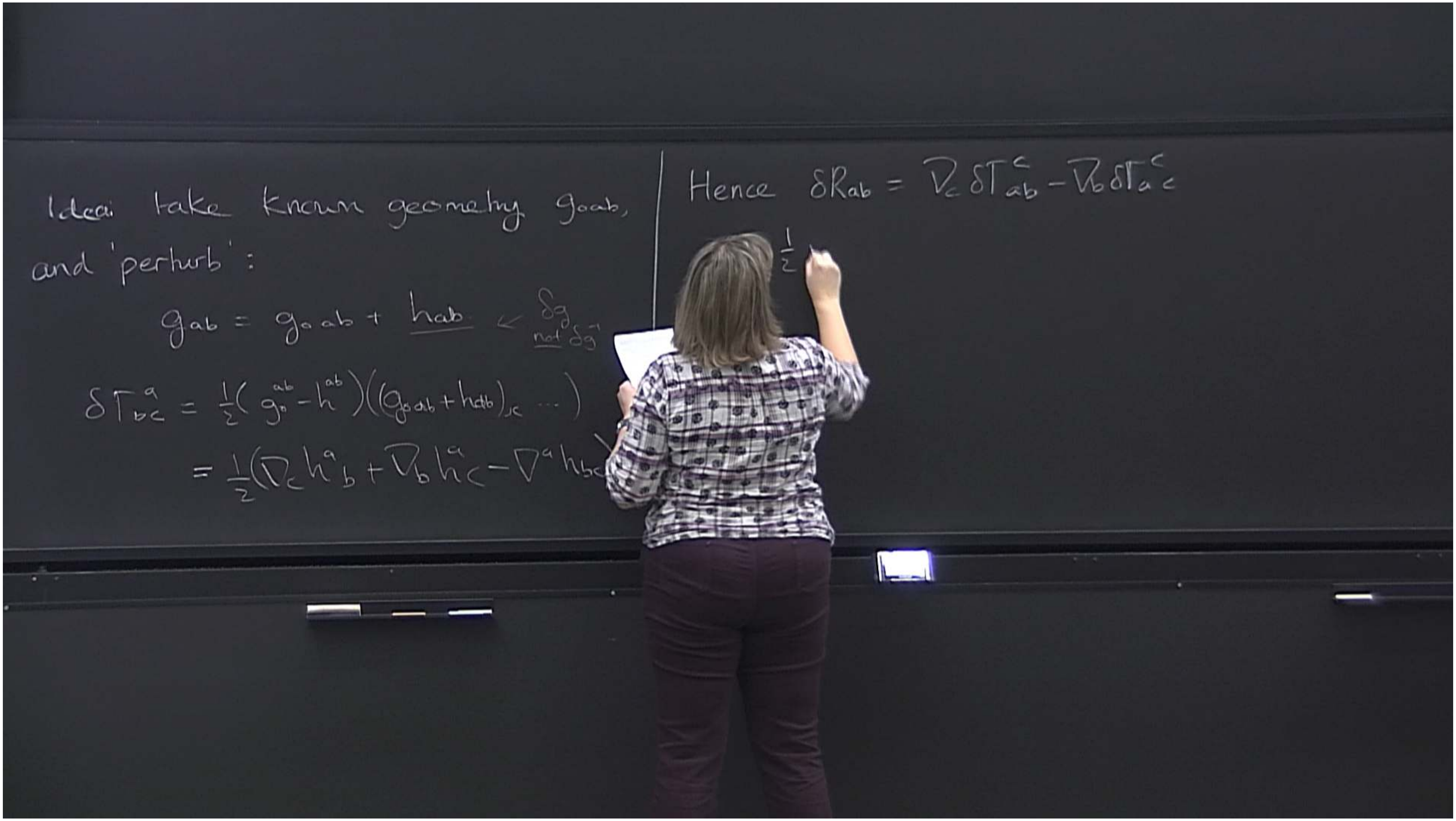
Date: Jan 20, 2015 10:15 AM

URL: http://pirsa.org/15010034

Abstract:


Lecture 12 Gravitational Perturbation Theory


Lecture 12 Gravitational Perturbation Theory
Exact solns hard to find in GR, so useful to approximate around idealized solns.
- solar system tests.
- cosmological ptn th
- gravitational radiation


Idea: take known geometry g_{0ab},
and 'perturb':
$g_{ab} = g_{0ab} + h_{ab} \leftarrow \delta g$ not δg^{-1}
$\delta\Gamma^a_{bc} = \frac{1}{2}(g_0^{ab} - h^{ab})((g_{0ab} + h_{ab})_{,c} \ldots)$
$= \frac{1}{2}(\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc})$
Hence $\delta R_{ab} = \nabla_c \delta\Gamma^c_{ab} - \nabla_b \delta\Gamma^c_{ac}$
$\frac{1}{2}$

ke known geometry g_{0ab},
rb':

$$g_{ab} = g_{0ab} + h_{ab} \leftarrow \delta g \text{ not } \delta g^{-1}$$

$$= \frac{1}{2}\left(g_0^{ab} - h^{ab}\right)\left((g_{0ab} + h_{ab})_{,c} \cdots\right)$$

$$= \frac{1}{2}\left(\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc}\right)$$

Hence $\delta R_{ab} = \nabla_c \delta\Gamma^c_{ab} - \nabla_b \delta\Gamma^c_{ac}$

$$= \frac{1}{2}\nabla_c\nabla_a h^c{}_b + \frac{1}{2}\nabla_c\nabla_b h^c{}_a - \frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

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rb':

$$g_{ab} = g_{0ab} + h_{ab} \leftarrow \delta g \text{ not } \delta g^{-1}$$

$$= \frac{1}{2}(g_0^{ab} - h^{ab})((g_{0ab} + h_{ab})_{,c} \ldots)$$

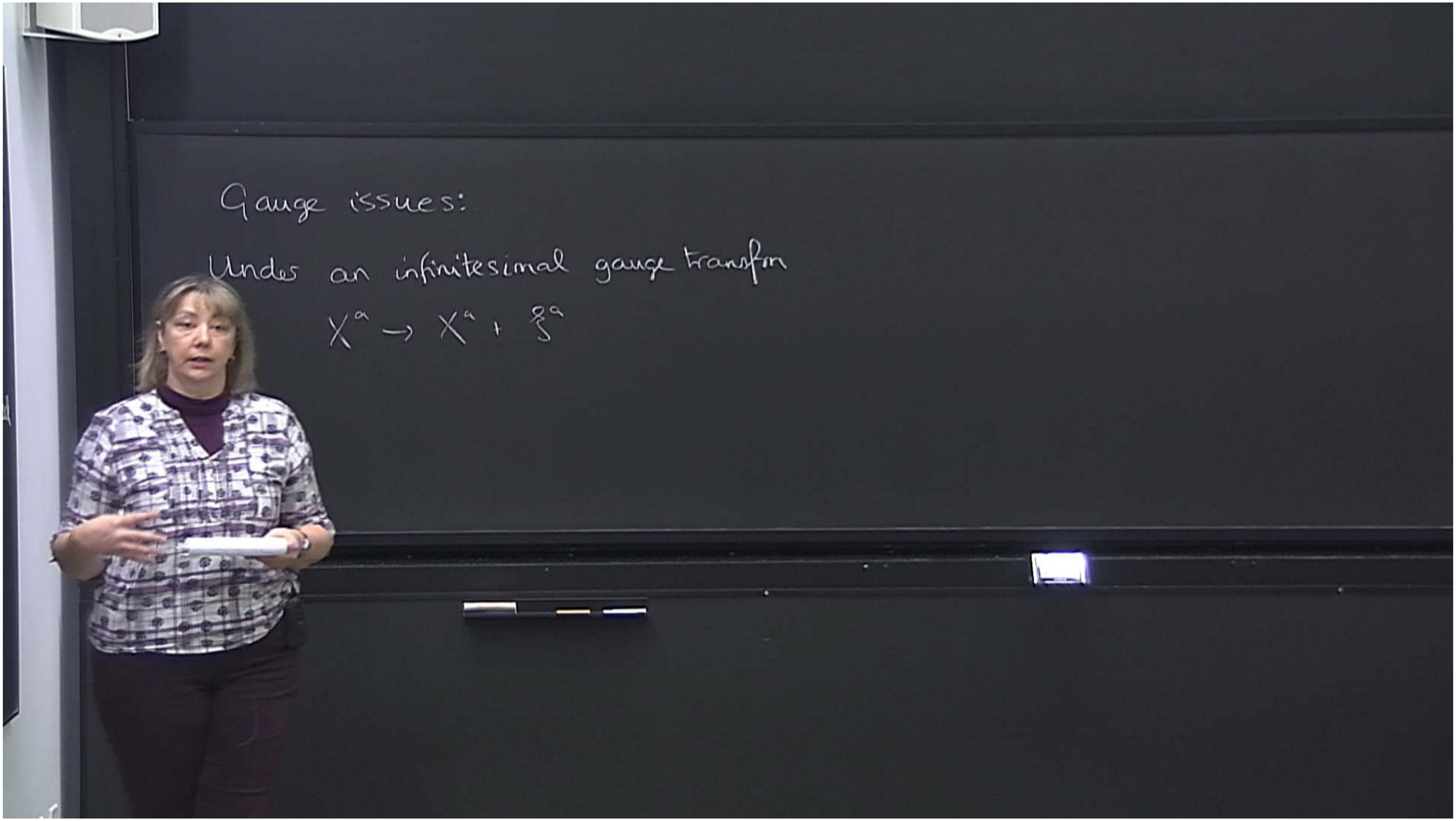
$$= \frac{1}{2}(\nabla_c h^a{}_b + \nabla_b h^a{}_c - \nabla^a h_{bc})$$

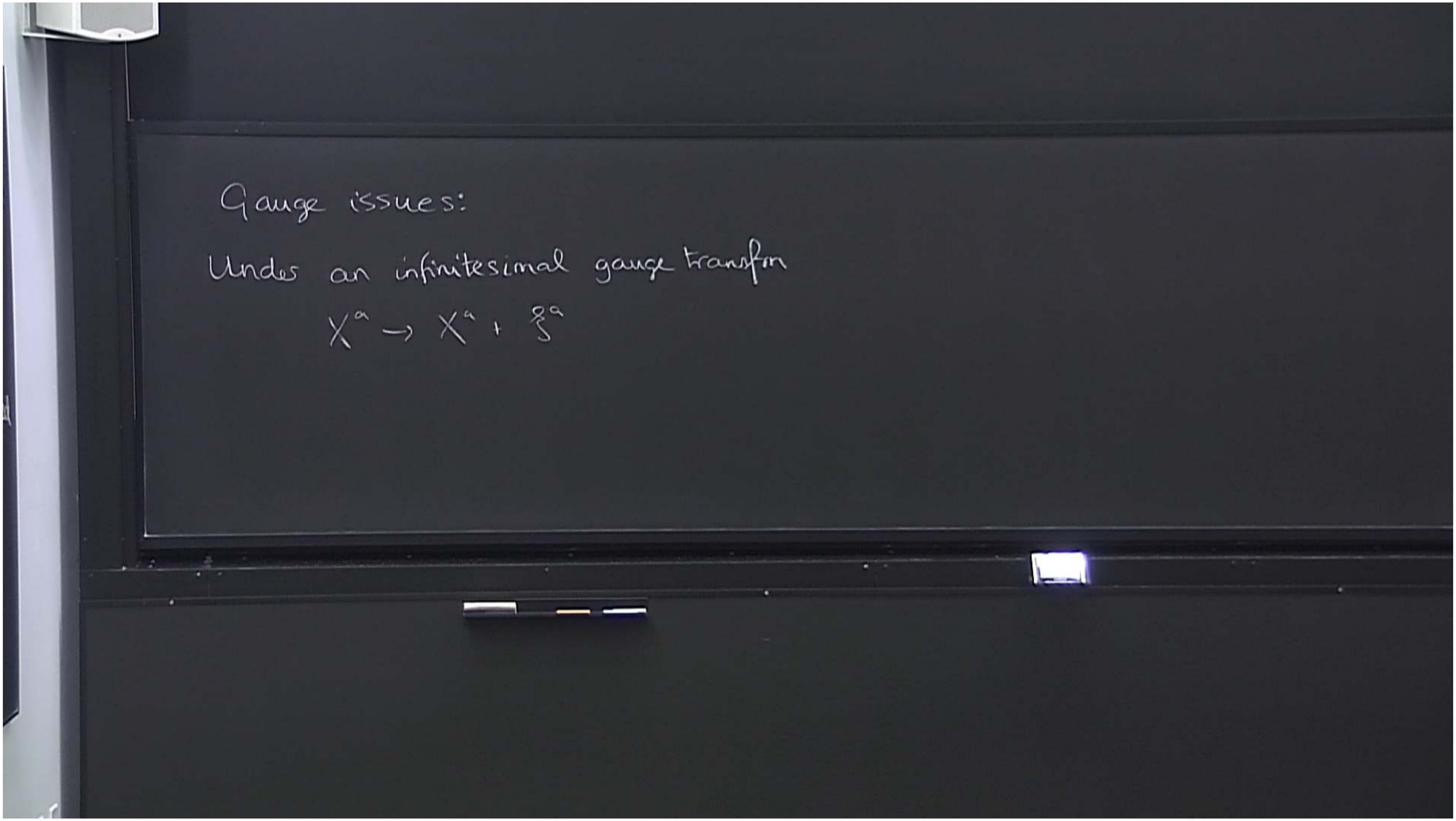
Hence $\delta R_{ab} = \nabla_c \delta\Gamma^c_{ab} - \nabla_b \delta\Gamma^c_{ac}$

$$= \frac{1}{2}\nabla_c\nabla_a h^c{}_b + \frac{1}{2}\nabla_c\nabla_b h^c{}_a - \frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

$$= \frac{1}{2}\nabla_a\nabla_c h^c{}_b + \frac{1}{2}\nabla_b\nabla_c h^c{}_a - \frac{1}{2}\Box h_{ab} - \frac{1}{2}\nabla_a\nabla_b h$$

$$+ \frac{1}{2}R^c{}_{dca} h^d{}_b + \frac{1}{2}R_{bdca} h^{cd} + \frac{1}{2}R^c{}_{dcb} h^d{}_a + \frac{1}{2}R_{adcb} h^{c\ldots}$$


Gauge issues:
Under an infinitesimal gauge transfm
$X^a \to X^a + \xi^a$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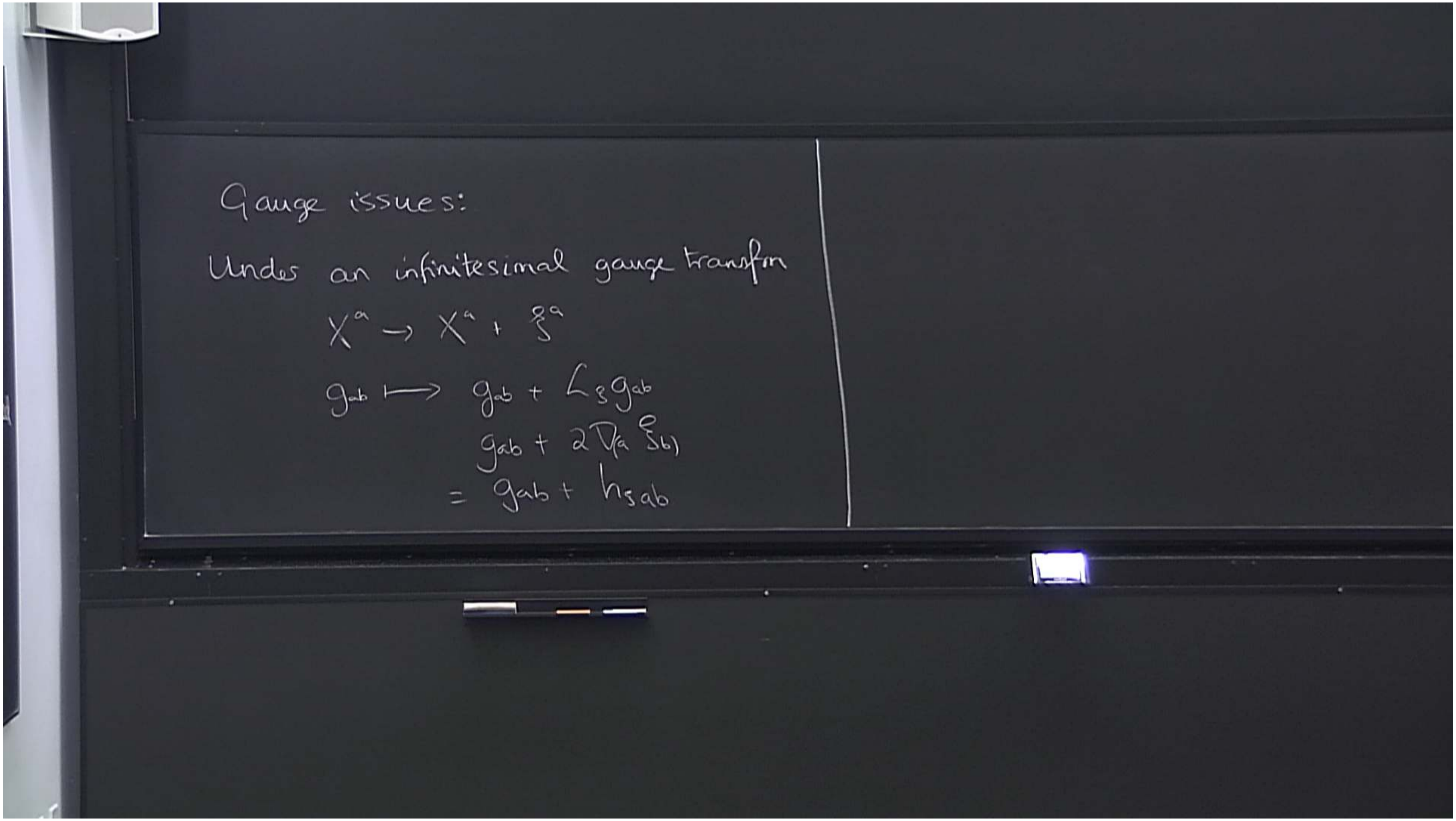
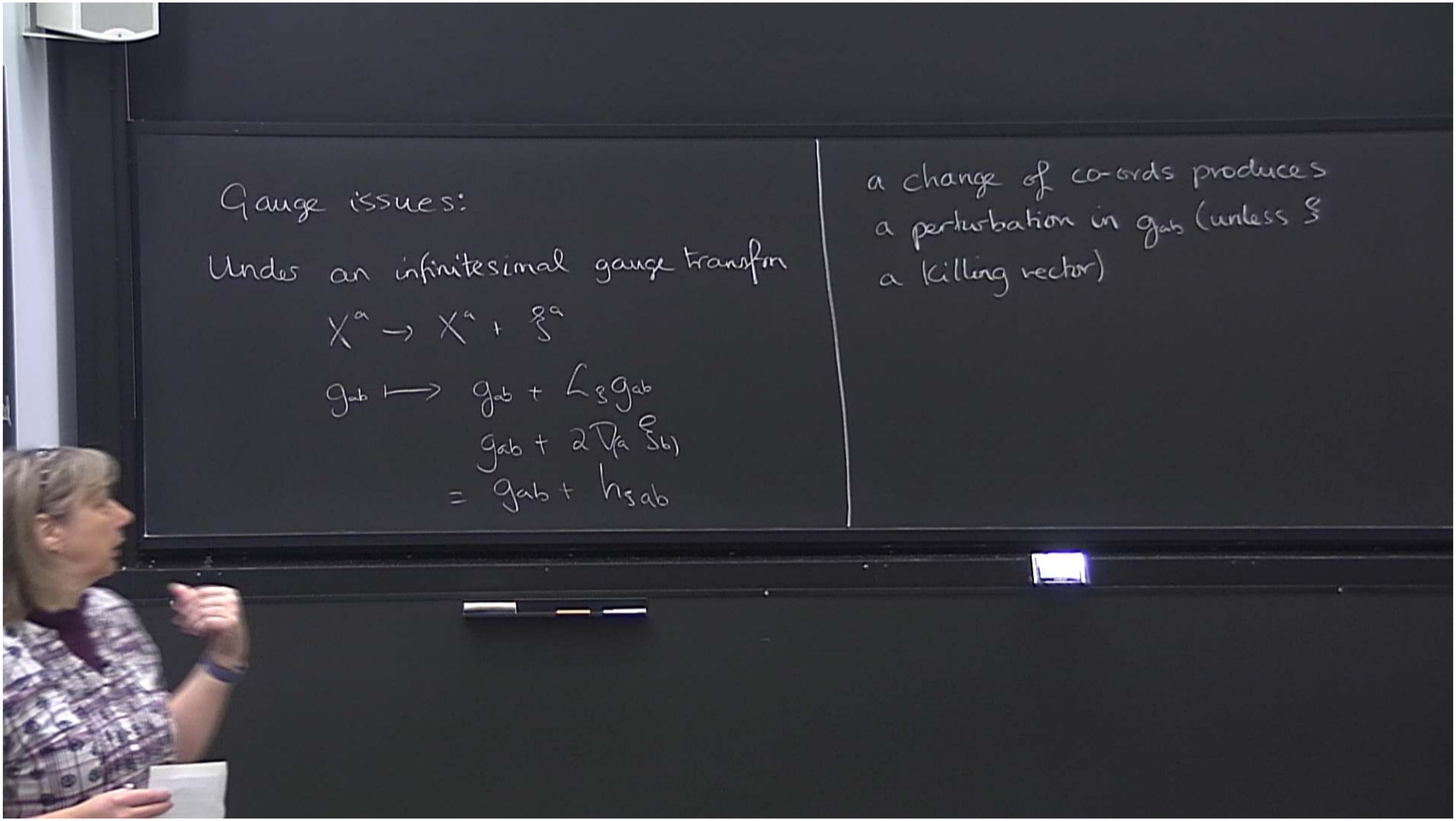

Gauge issues:
Under an infinitesimal gauge transfm
$x^a \to x^a + \xi^a$
$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$
$g_{ab} + 2\nabla_{(a}\xi_{b)}$
$= g_{ab} + h_{\xi ab}$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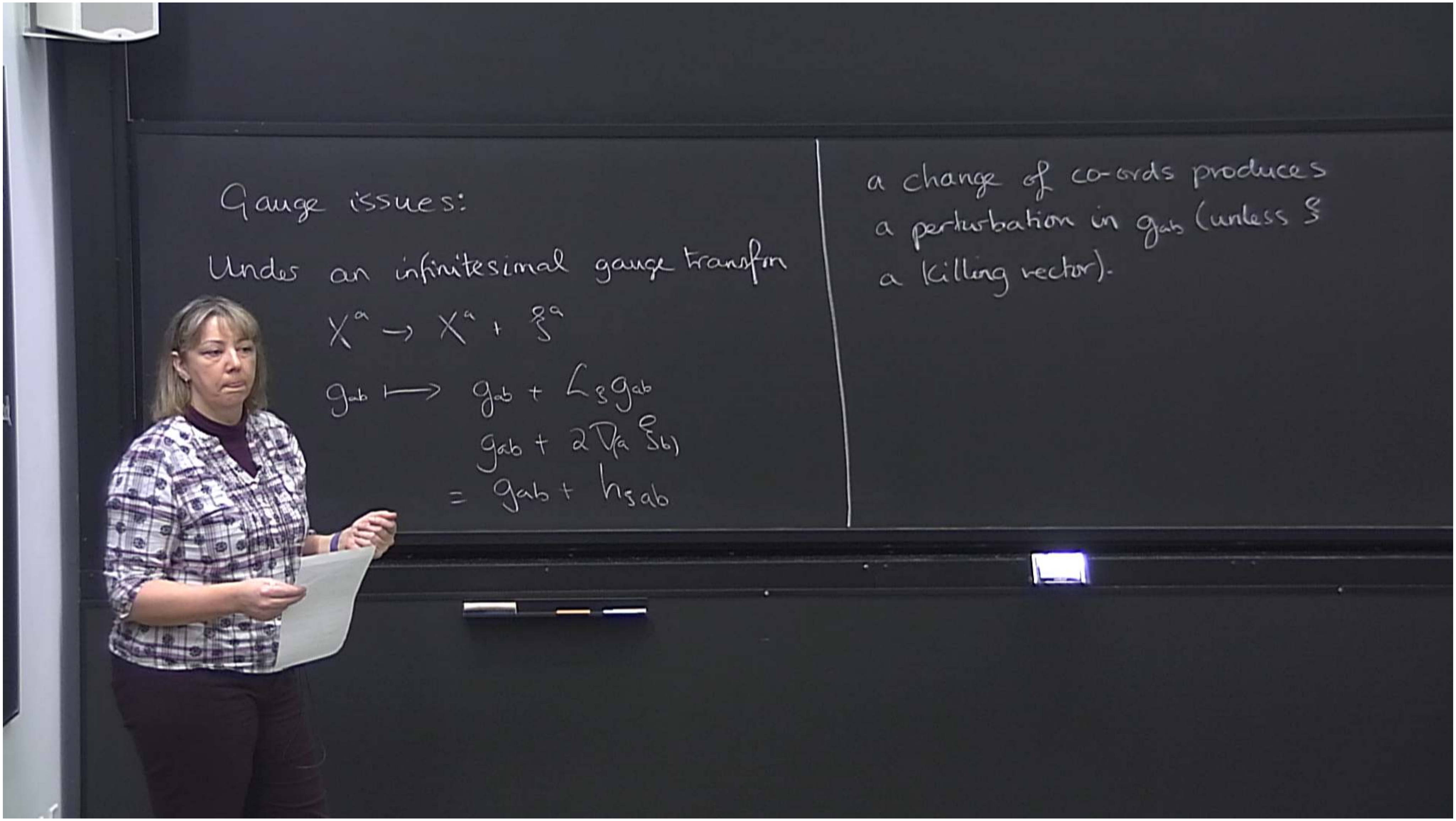
a change of co-ords produces
a perturbation in g_{ab} (unless ξ
a Killing vector)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Gauge issues:

Under an infinitesimal gauge transfm

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$$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$$

$$g_{ab} + 2\nabla_{(a}\xi_{b)}$$

$$= g_{ab} + h_{\xi ab}$$

a change of co-ords produces a perturbation in g_{ab} (unless ξ a Killing vector).

$$\bar{h}_{\xi ab} = 2\nabla_{(a}\xi_{b)} - (\nabla\cdot\xi)g_{ab}$$

$$\nabla^a \bar{h}_{\xi ab} = \Box\xi_b + \nabla^a\nabla_b\xi_a - \nabla_b\nabla^a\xi_a$$

$$= \Box\xi_b + R_{ac}{}^a{}_b\,\xi^c$$

$$= \Box\xi_b + R^c{}_b\,\xi_c$$

...uge transfm

g_{ab}

$\xi_{b)}$

$_{\xi ab}$

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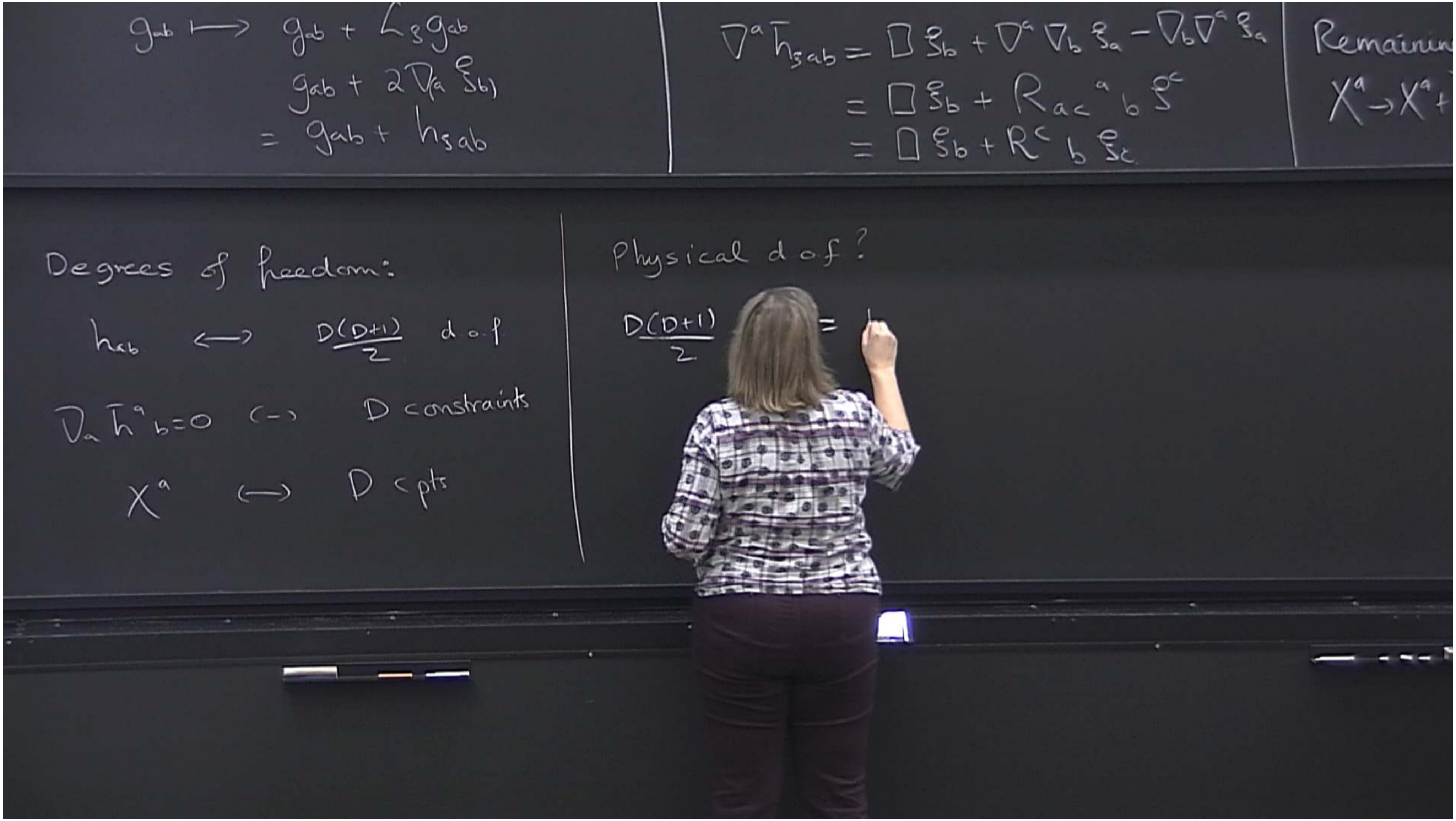
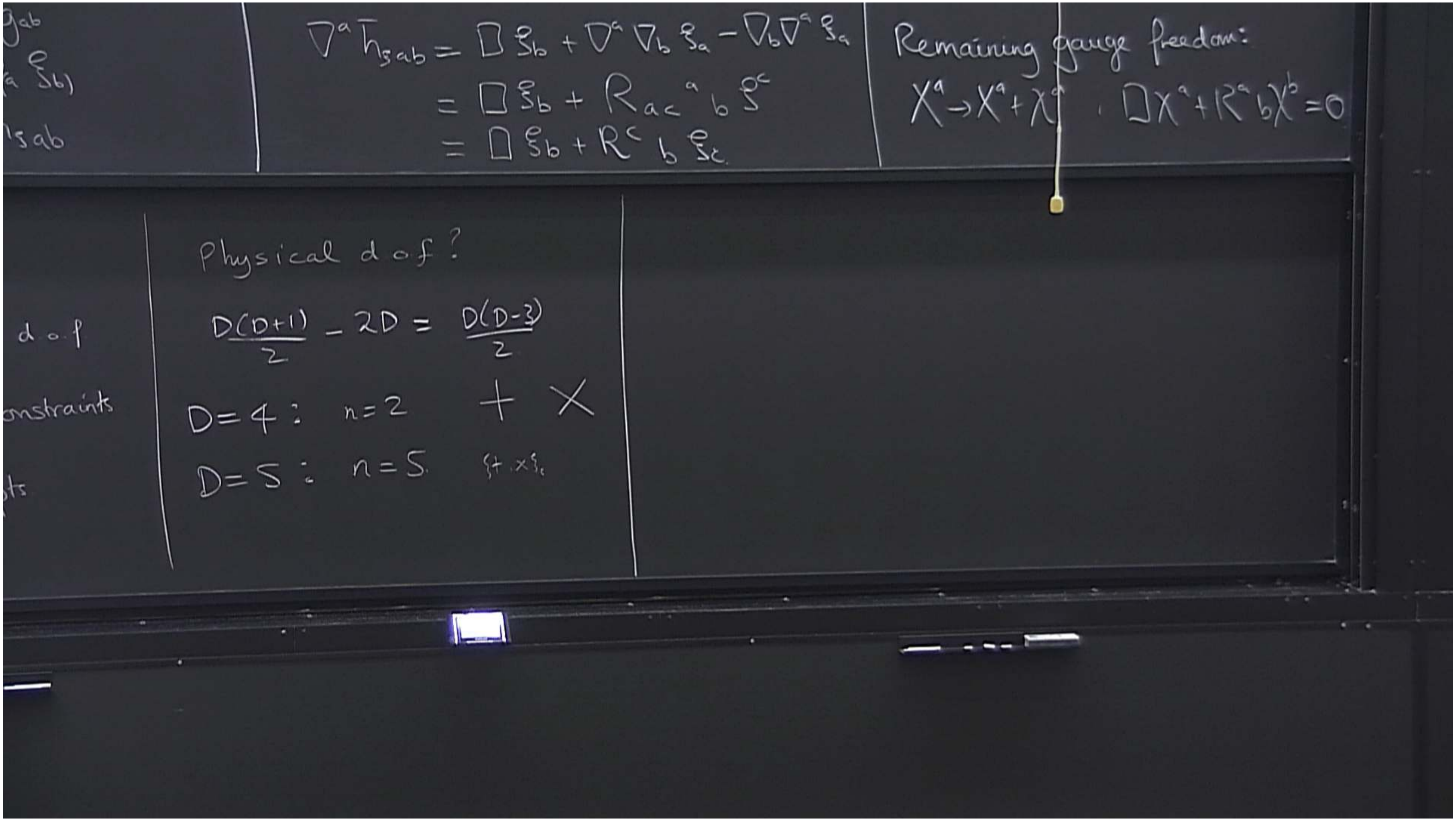
– a well posed diff eqn, so can solve to set $\nabla_a \bar{h}^a{}_b = 0$.



$g_{ab} \longmapsto g_{ab} + \mathcal{L}_\xi g_{ab}$
$g_{ab} + 2\nabla_{(a}\xi_{b)}$
$= g_{ab} + h_{\xi ab}$
$\nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla^a \nabla_b \xi_a - \nabla_b \nabla^a \xi_a$
$= \Box \xi_b + R_{ac}{}^a{}_b \xi^c$
$= \Box \xi_b + R^c{}_b \xi_c$
Remainin
$X^a \to X^a +$
Degrees of freedom:
$h_{ab} \longleftrightarrow \frac{D(D+1)}{2}$ d.o.f
$\nabla_a \bar{h}^a{}_b = 0 \longleftrightarrow$ D constraints
$X^a \longleftrightarrow$ D cpts
Physical d.o.f?
$\frac{D(D+1)}{2}$ =



$\nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla^a \nabla_b \xi_a - \nabla_b \nabla^a \xi_a$
$= \Box \xi_b + R_{ac}{}^a{}_b \xi^c$
$= \Box \xi_b + R^c{}_b \xi_c$
Remaining gauge freedom:
$X^a \to X^a + \chi^a$, $\Box \chi^a + R^a{}_b \chi^b = 0$
Physical d.o.f?
$\frac{D(D+1)}{2} - 2D = \frac{D(D-3)}{2}$
$D=4$: $n=2$ + ×
$D=5$: $n=5$

g_{ab}

ξ

$(a \xi_b)$

$h_{\xi ab}$

$$\nabla^a \bar{h}_{\xi ab} = \Box \xi_b + \nabla^a \nabla_b \xi_a - \nabla_b \nabla^a \xi_a$$
$$= \Box \xi_b + R_{ac}{}^a{}_b \xi^c$$
$$= \Box \xi_b + R^c{}_b \xi_c$$

Remaining gauge freedom:

$X^a \to X^a + \chi^a \;,\; \Box \chi^a + R^a{}_b \chi^b = 0$

d.o.f

onstraints

ts

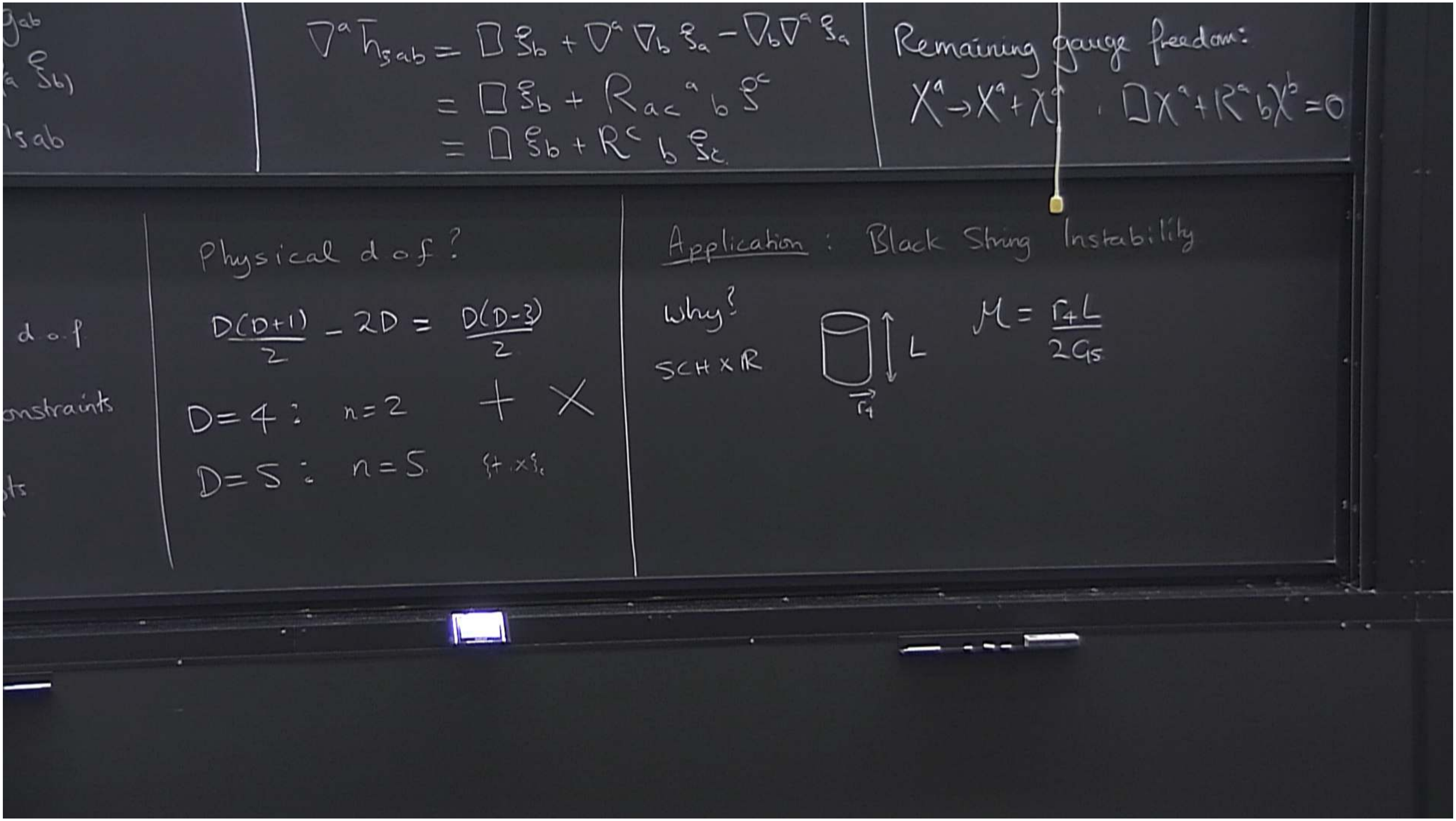
Physical d.o.f?

$$\frac{D(D+1)}{2} - 2D = \frac{D(D-3)}{2}$$

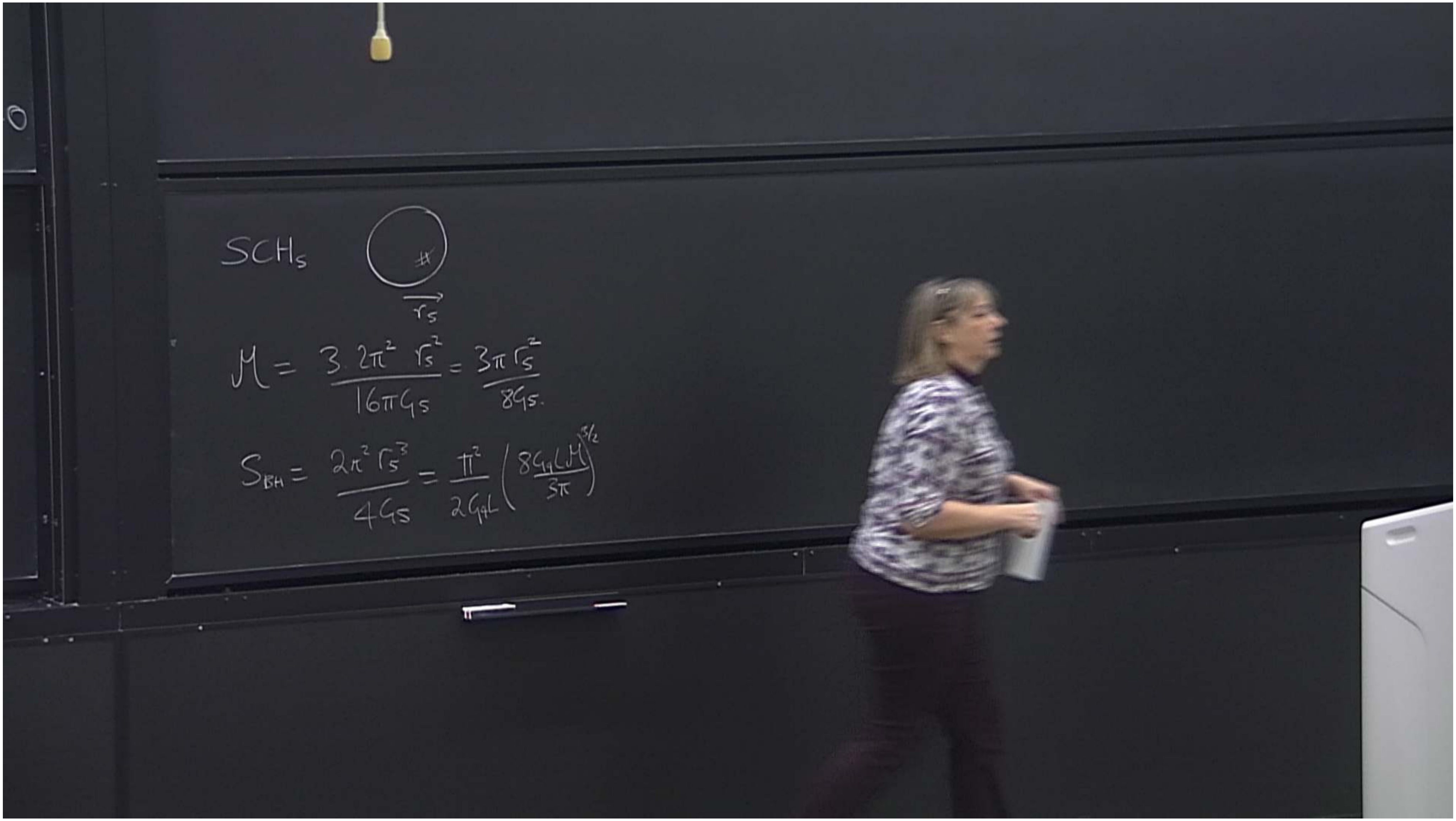
$D=4$: $n=2$ $\quad + \quad \times$

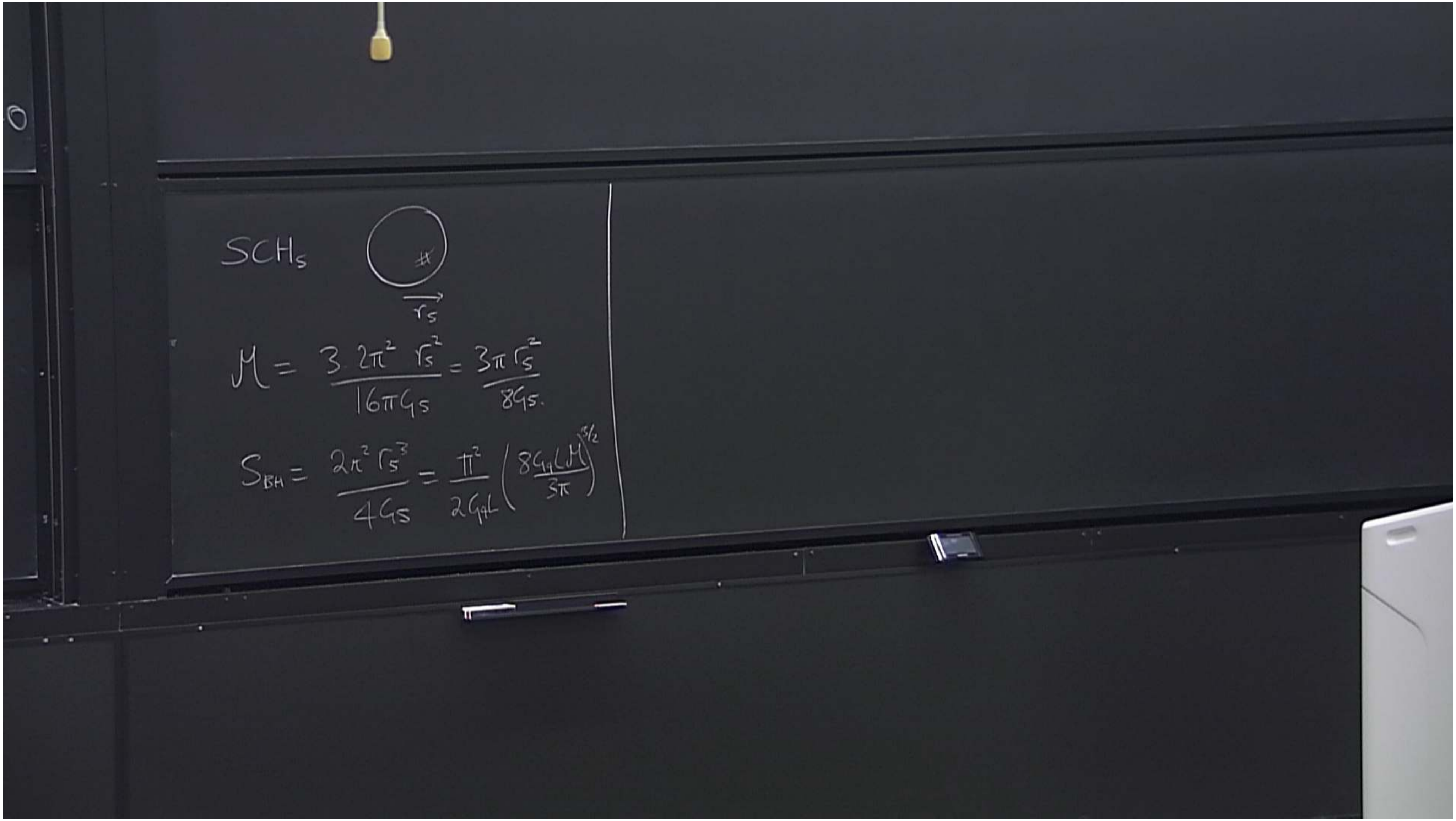
$D=5$: $n=5$ $\quad \{+, \times\},$

Application : Black String Instability



∇^a h̄_ξab = □ξ_b + ∇^a ∇_b ξ_a − ∇_b ∇^a ξ_a
= □ξ_b + R_ac^a_b ξ^c
= □ξ_b + R^c_b ξ_c
Remaining gauge freedom:
X^a → X^a + χ^a : □χ^a + R^a_b χ^b = 0
Physical d.o.f?
D(D+1)/2 − 2D = D(D−3)/2
D=4 : n=2 + ×
D=5 : n=5
d.o.f
onstraints
Application : Black String Instability
Why?
SCH × R
L
r4
M = r4 L / 2G5


SCH5
rs
M = 3.2π² rs² / 16πG5 = 3π rs² / 8G5.
SBH = 2π² rs³ / 4G5 = π² / 2G4L (8G4LM / 3π)^3/2


SCHs
r_s
$M = \frac{3 \cdot 2\pi^2 r_s^2}{16\pi G_5} = \frac{3\pi r_s^2}{8 G_5}$
$S_{BH} = \frac{2\pi^2 r_s^3}{4 G_5} = \frac{\pi^2}{2 G_4 L}\left(\frac{8 G_4 L M}{3\pi}\right)^{3/2}$

SCH_5

$$M = \frac{3 \cdot 2\pi^2 r_s^2}{16\pi G_5} = \frac{3\pi r_s^2}{8G_5}$$

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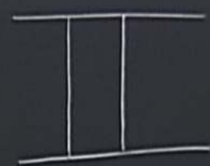
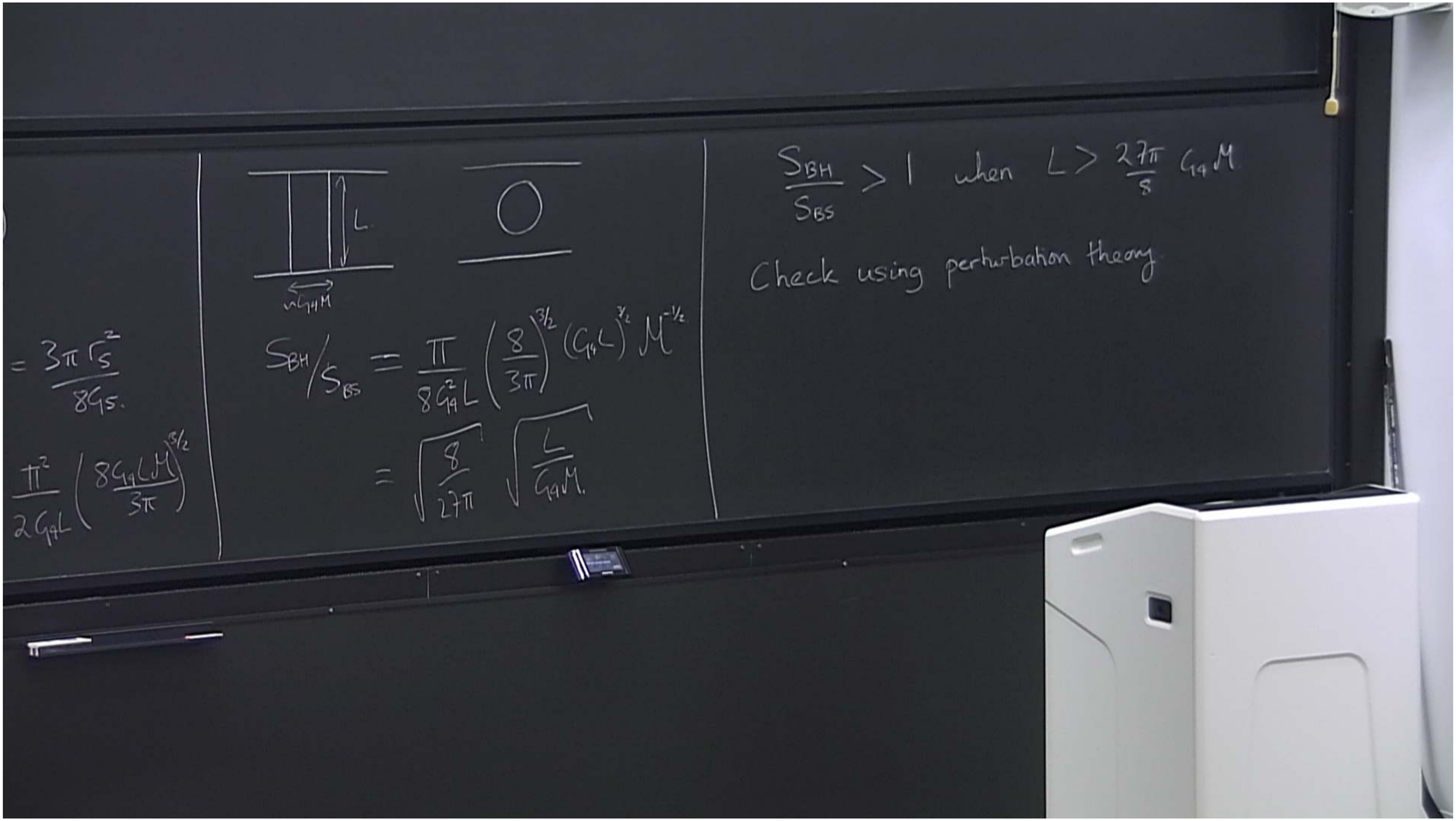
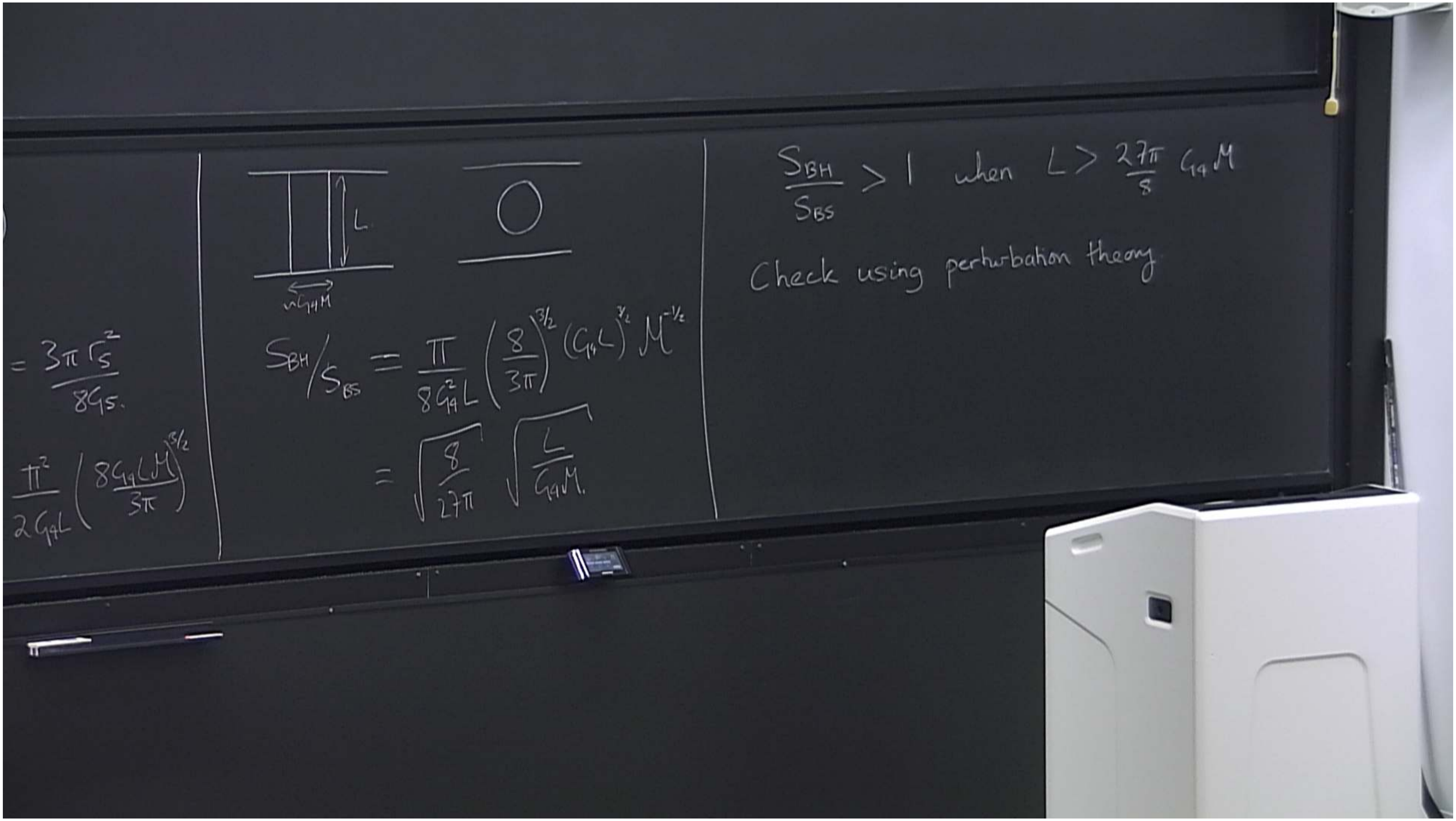


$$S_{BH}/S_{BS} = \frac{\pi}{8G_4^2 L}\left(\frac{8}{3\pi}\right)^{3/2}(G_4 L)^{1/2} M^{-1/2}$$


$= \frac{3\pi r_s^2}{8G_5}$
$\frac{\pi^2}{2G_4L}\left(\frac{8G_4LM}{3\pi}\right)^{3/2}$
L
$\sim G_4M$
$S_{BH}/S_{BS} = \frac{\pi}{8G_4^2L}\left(\frac{8}{3\pi}\right)^{3/2}(G_4L)^{3/2}M^{-1/2}$
$= \sqrt{\frac{8}{27\pi}}\sqrt{\frac{L}{G_4M}}$
$\frac{S_{BH}}{S_{BS}} > 1$ when $L > \frac{27\pi}{8}G_4M$
Check using perturbation theory



L
$\sim G_4 M$
$= \frac{3\pi r_S^2}{8G_5}$
$\frac{\pi^2}{2G_4L}\left(\frac{8G_4LM}{3\pi}\right)^{3/2}$
$S_{BH}/S_{BS} = \frac{\pi}{8G_4^2L}\left(\frac{8}{3\pi}\right)^{3/2}(G_4L)^{3/2}M^{-1/2}$
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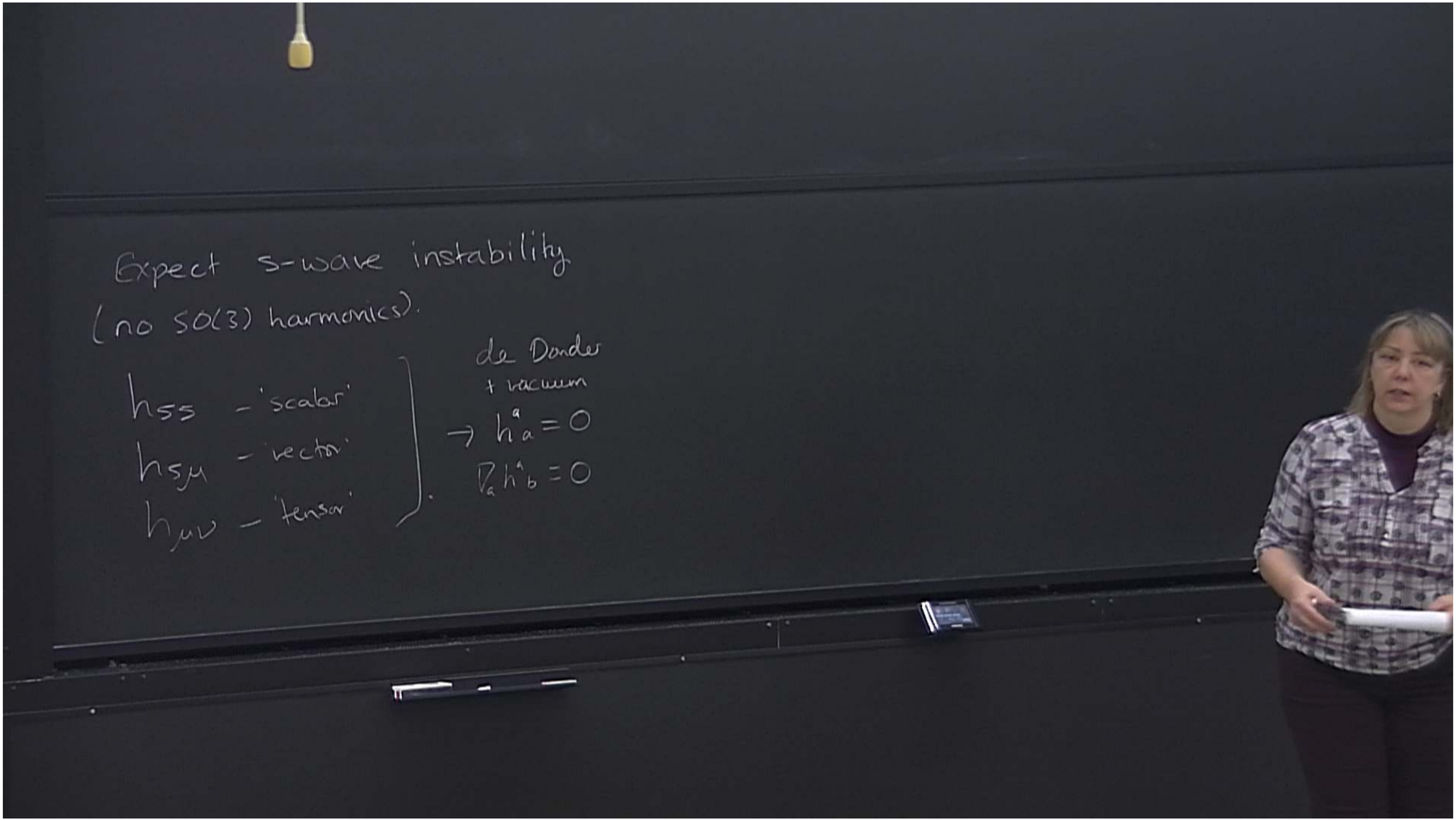
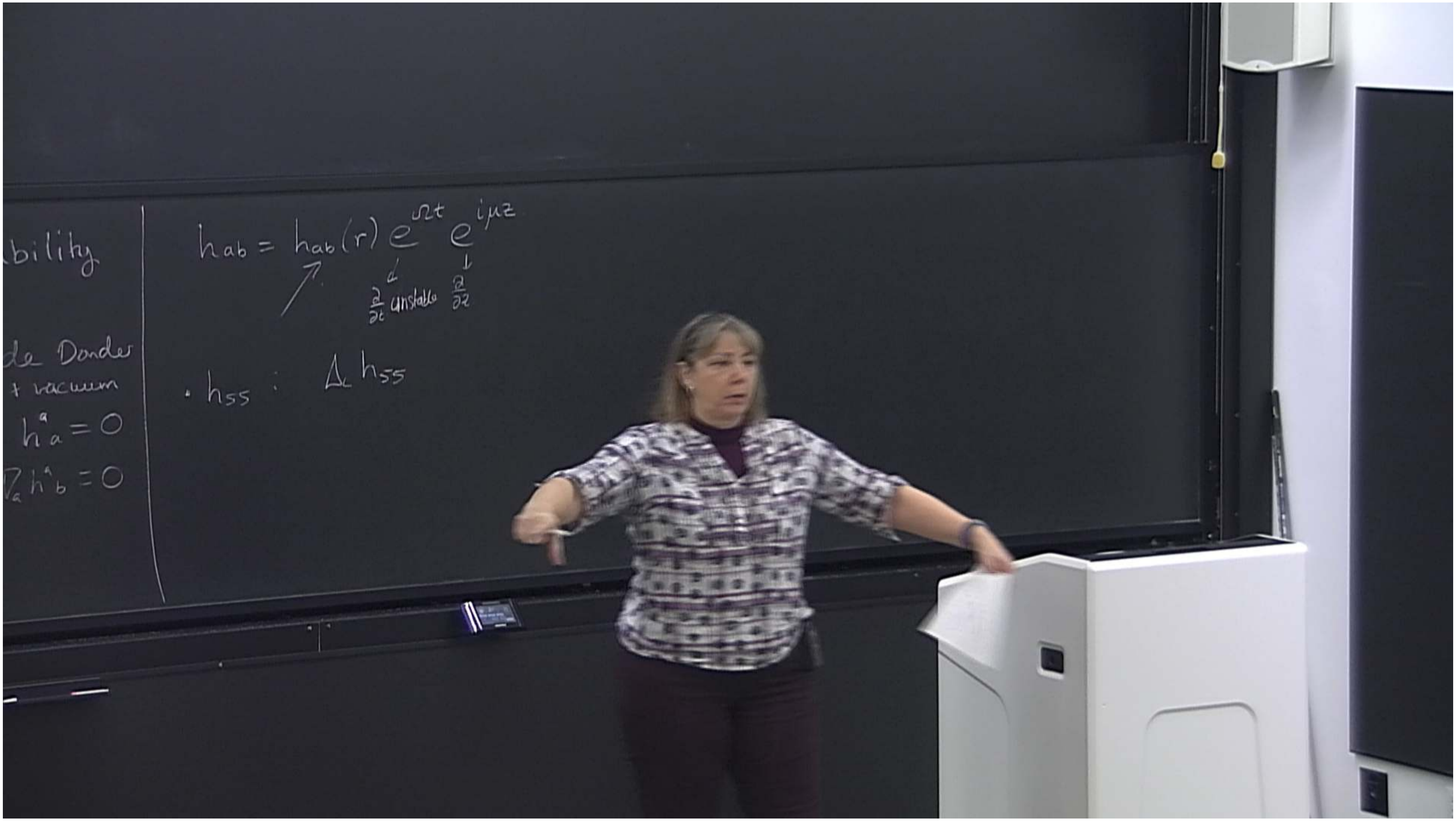
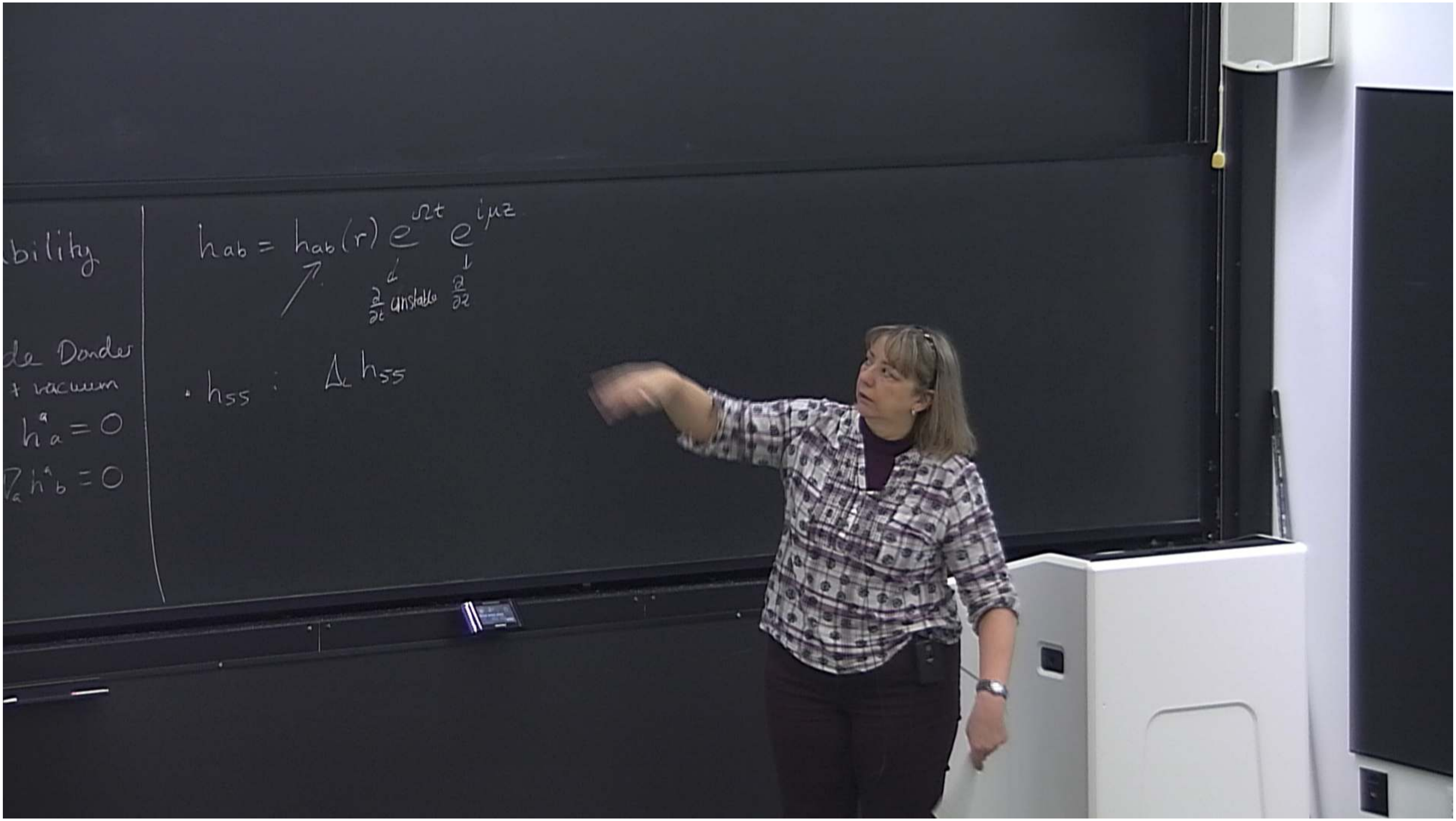
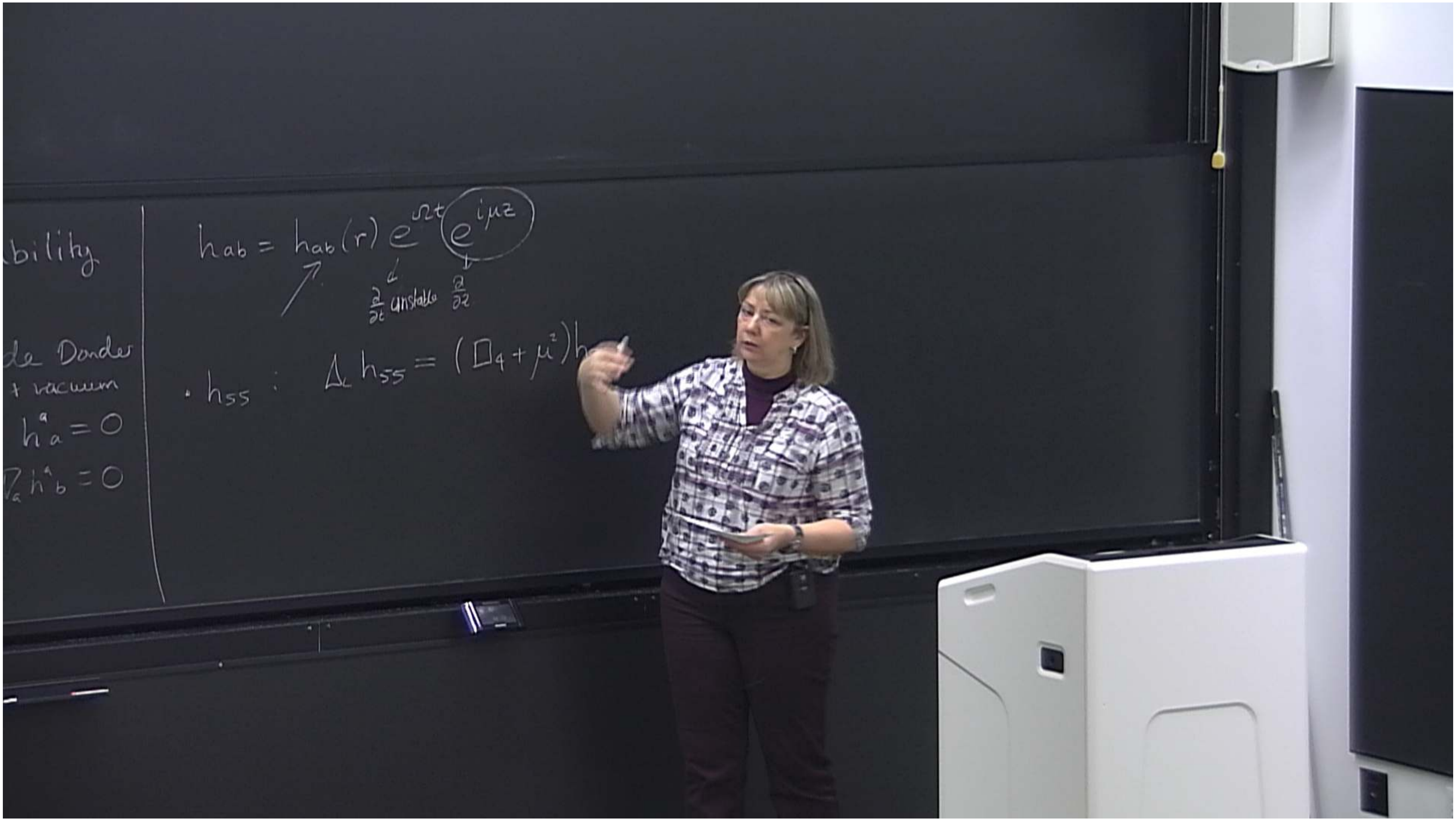
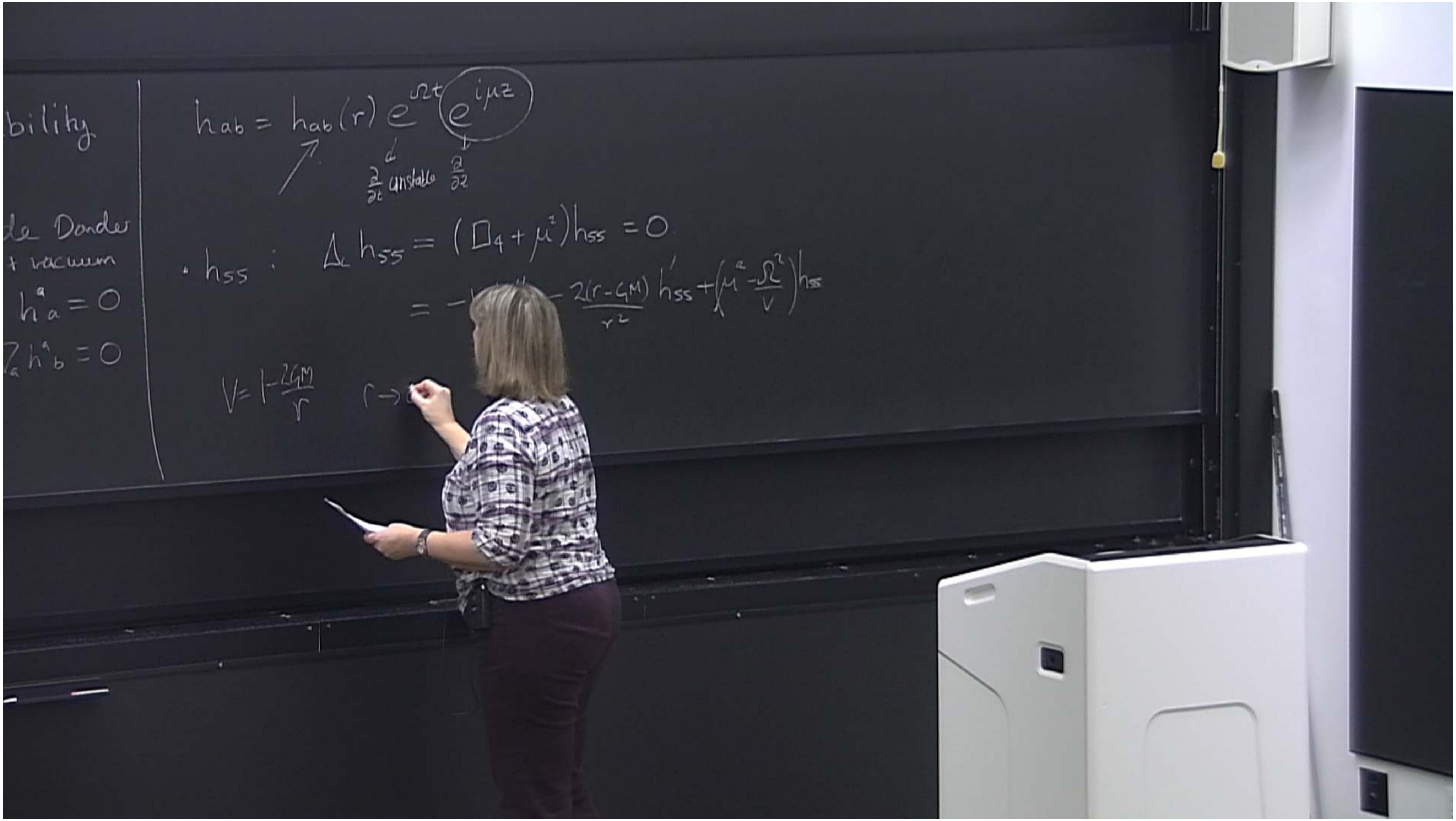
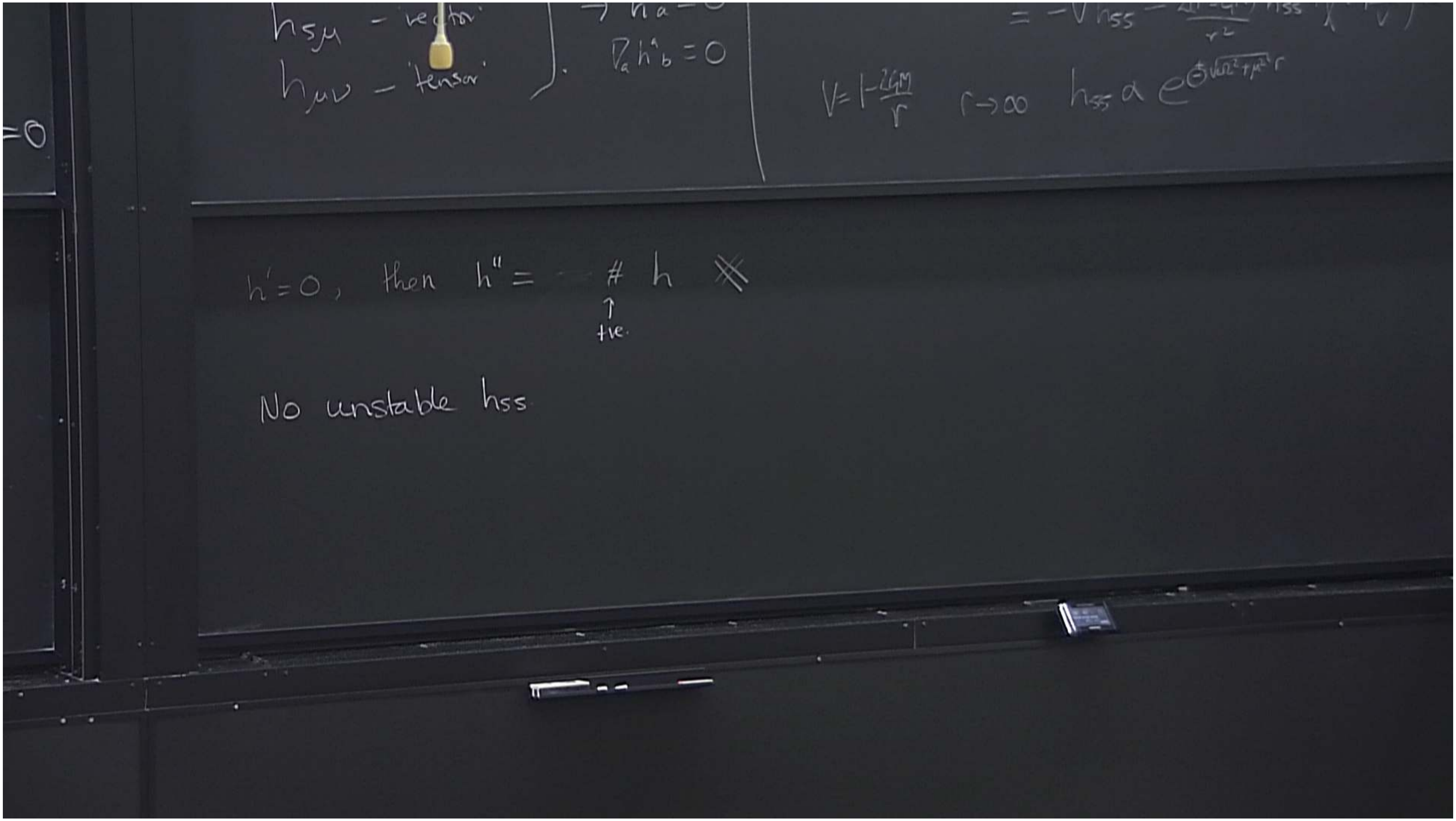

Expect s-wave instability
(no SO(3) harmonics).
de Donder
+ vacuum
h_{ss} - 'scalar'
$h_{s\mu}$ - 'vector'
$h_{\mu\nu}$ - 'tensor'
$\rightarrow h^a{}_a = 0$
$\nabla_a h^a{}_b = 0$

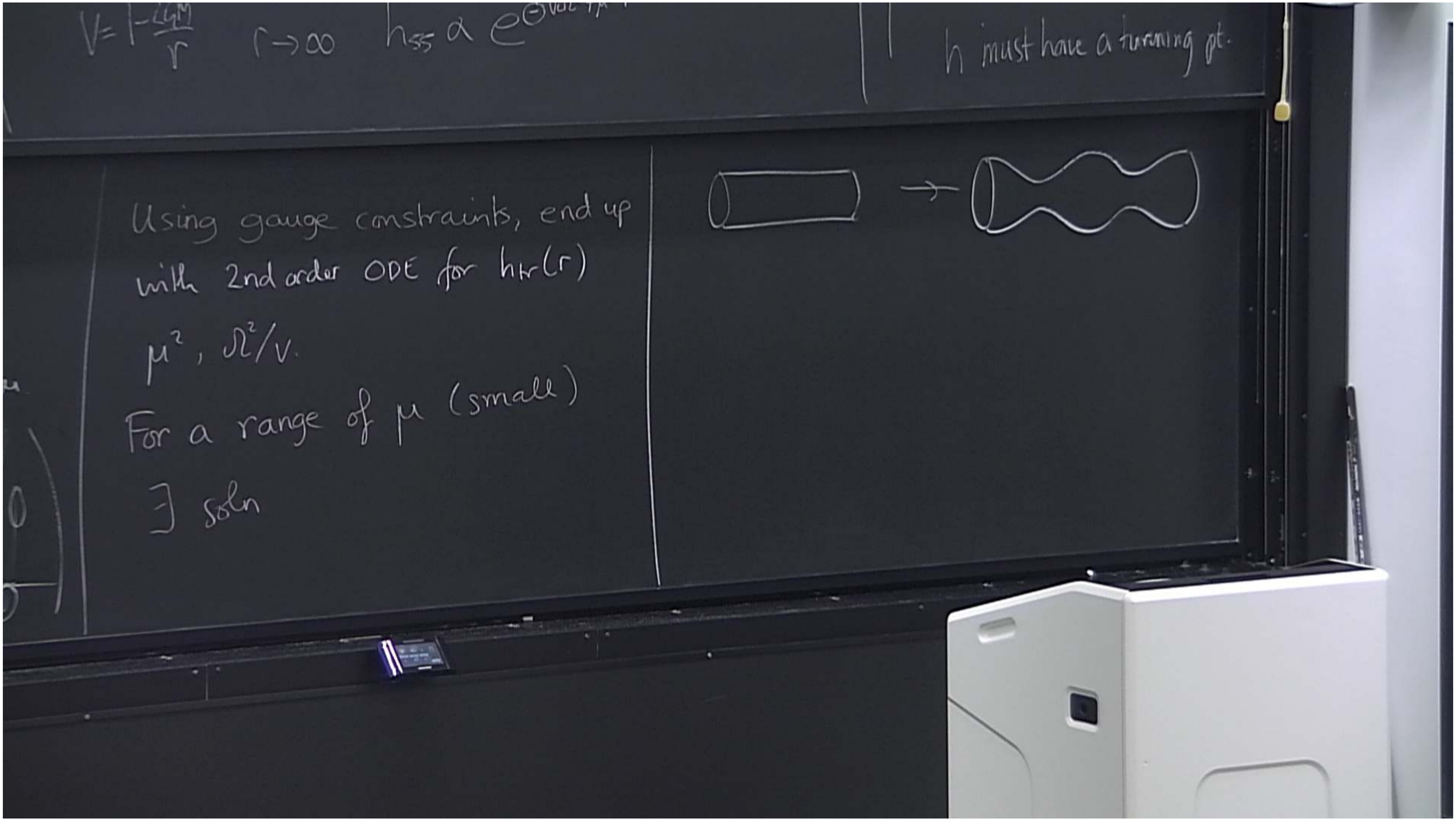

bility
de Donder
+ vacuum
$h^a_a = 0$
$\nabla_a h^a_b = 0$
$h_{ab} = h_{ab}(r)\, e^{\Omega t} e^{i\mu z}$
$\frac{\partial}{\partial t}$ unstable $\frac{\partial}{\partial z}$
$\cdot\ h_{55}$: $\Delta_L h_{55}$

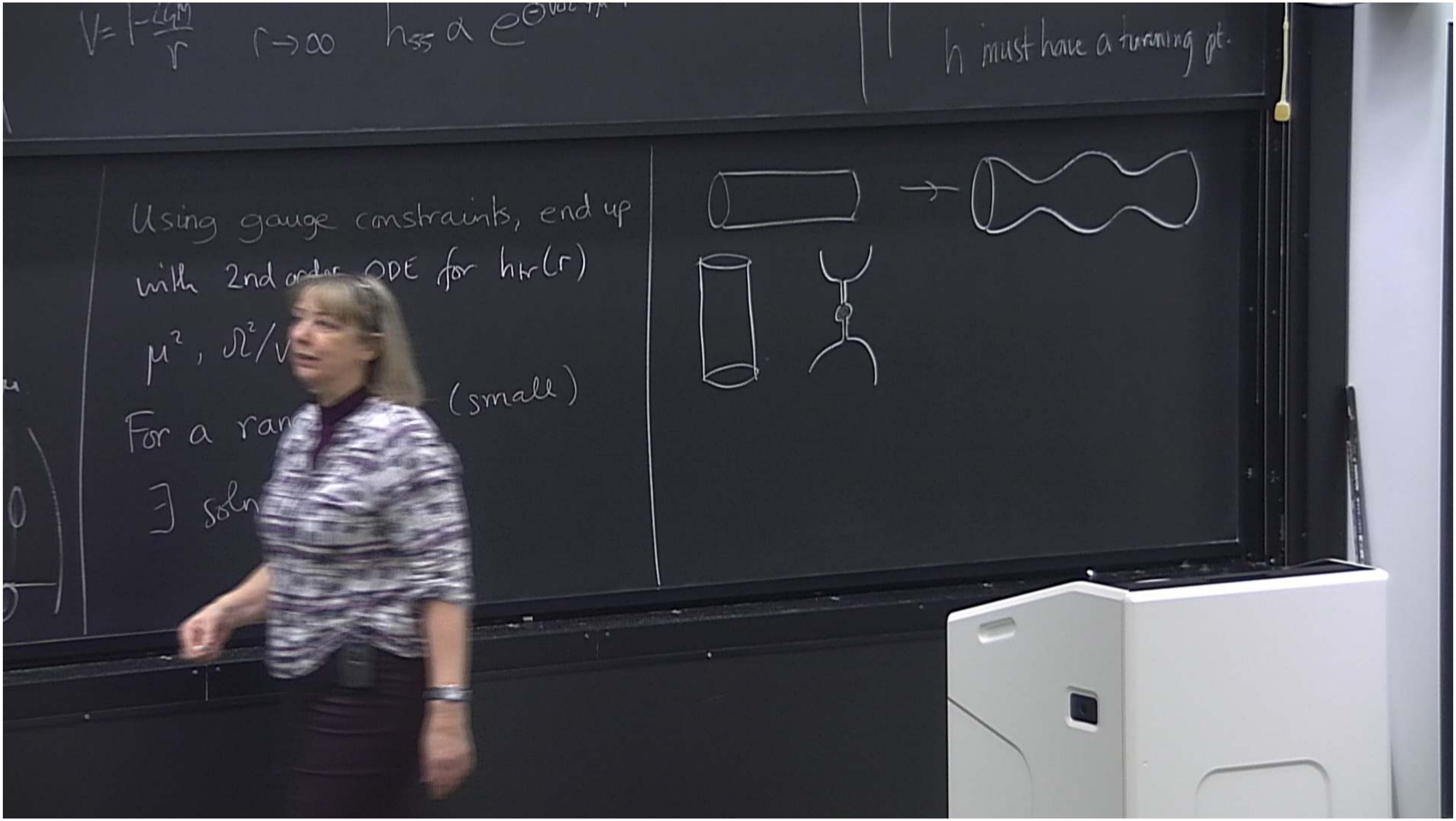

bility
de Donder
+ vacuum
h^a_a = 0
∇_a h^a_b = 0
h_ab = h_ab(r) e^Ωt e^iμz
∂/∂t unstable ∂/∂z
• h55 : Δ_L h55


bility
de Donder
+ vacuum
$h^a{}_a = 0$
$\nabla_a h^a{}_b = 0$
$h_{ab} = h_{ab}(r)\, e^{\Omega t}\, e^{i\mu z}$
$\frac{\partial}{\partial t}$ unstable $\frac{\partial}{\partial z}$
$\cdot\ h_{55}$: $\Delta_L h_{55} = (\Box_4 + \mu^2) h$


bility
de Donder
+ vacuum
h^a_a = 0
∇_a h^a_b = 0
h_ab = h_ab(r) e^{Ωt} e^{iμz}
∂/∂t unstable ∂/∂z
h_ss : Δ_L h_ss = (□_4 + μ²) h_ss = 0
V = 1 − 2GM/r


hsμ - 'vector'
hμν - 'tensor'
∇a h^a b = 0
V = 1 - 2GM/r
r → ∞
h' = 0, then h'' = # h
+ve.
No unstable hss.


r → ∞
h must have a turning pt.
Using gauge constraints, end up
with 2nd order ODE for h_tr(r)
μ², Ω²/V.
For a range of μ (small)
∃ soln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For a ran
∃ soln