Title: Gravitational Physics-11

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URL: http://pirsa.org/15010033

Abstract:

Lecture 11 Kaluza-Klein theory

Extracts 4D gravity as a low-energy limit of a higher dim^{l} theory. The extra dim(s) are small, and geometry is not dependent on them.

Star

Start with 5D Einstein gravity

$$ds^2 = \tilde{g}_{\mu\nu} dx^\mu dx^\nu - e^{2\sigma(x)} [d\psi + A_\mu dx^\mu]^2$$

Note $\sqrt{g_5} = \sqrt{\tilde{g}}\, e^{\sigma}$

'small' circle, $\frac{\partial}{\partial \psi}$ killing vector

Let $\underline{\omega}^a = e^a{}_\mu dx^\mu$ be an o/n basis of $\tilde{g}$

$\underline{\omega}^4 = e^{\sigma}[d\psi + A_\mu dx^\mu]$ or $e^{\sigma}[d\psi + \underline{A}]$

$$\underline{d\omega}^a = -\underline{\Theta}_o{}^a{}_b \wedge \underline{\omega}^b = -\underline{\theta}^a{}_b \wedge \underline{\omega}^b - \underline{\Theta}^a{}_\psi \wedge \underline{\omega}^\psi$$

$$\underline{d\omega}^\psi = \sigma_{,a}\, \underline{\omega}^a \wedge \underline{\omega}^\psi + e^\sigma \underline{F} = -\underline{\Theta}^\psi{}_a \wedge \underline{\omega}^a$$

$\frac{1}{2} e^\sigma F_{ab}\, \underline{\omega}^a \wedge \underline{\omega}^b$

$$\underline{\Theta}^\psi{}_a = \sigma_{,a}\, \underline{\omega}^\psi + \frac{1}{2} e^\sigma F_{ab}\, \underline{\omega}^b$$

$$\underline{\theta}^a{}_b = \underline{\Theta}_o{}^a{}_b + \frac{1}{2} e^\sigma F^a{}_b\, \underline{\omega}^\psi$$

$$\underline{R}^a{}_b = \underline{d}\,\underline{\Theta}^a{}_b + \tfrac{1}{2}\underline{d}\left(e^\sigma F^a{}_{b\sigma}\underline{\omega}^\psi\right) + \left(\underline{\Theta}^a{}_c + \tfrac{1}{2}e^\sigma F^a{}_{c\sigma}\underline{\omega}^\psi\right)\wedge\left(\underline{\Theta}^c{}_b + \tfrac{1}{2}e^\sigma F^c{}_{b\sigma}\underline{\omega}^\psi\right)$$

$$+ \left(\sigma^{,a}\underline{\omega}^\tau + \tfrac{1}{2}e^\sigma F^a{}_{c\sigma}\underline{\omega}^c\right)\wedge\left(\sigma_{,b}\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F_{bd\sigma}\underline{\omega}^d\right)$$

$$= \underline{d}\,\underline{\Theta}^a{}_b + \tfrac{1}{2}e^\sigma \sigma_{,c} F^a{}_{b\sigma}\underline{\omega}^c\wedge\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F^a{}_{b,c\sigma}\underline{\omega}^c\wedge\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F^a{}_{b\sigma}\left(\sigma_{,c}\underline{\omega}^c\wedge\underline{\omega}^\psi + e^\sigma\underline{F}\right)$$

$$+ \underline{\Theta}^a{}_c\wedge\underline{\Theta}^c{}_b + \tfrac{1}{2}e^\sigma\left(F^a{}_{c\sigma}\underline{\omega}^\psi\wedge\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c\wedge F^c{}_{b\sigma}\underline{\omega}^\psi\right)$$

$$+ \tfrac{1}{4}e^{2\sigma}F^a{}_{c\sigma}F_{bd\sigma}\underline{\omega}^c\wedge\underline{\omega}^d + \tfrac{1}{2}e^\sigma\left(F^a{}_{c\sigma}\underline{\omega}^c\wedge\sigma_{,b}\underline{\omega}^\psi + F_{bd\sigma}\sigma^{,a}\underline{\omega}^\psi\wedge\underline{\omega}^d\right)$$

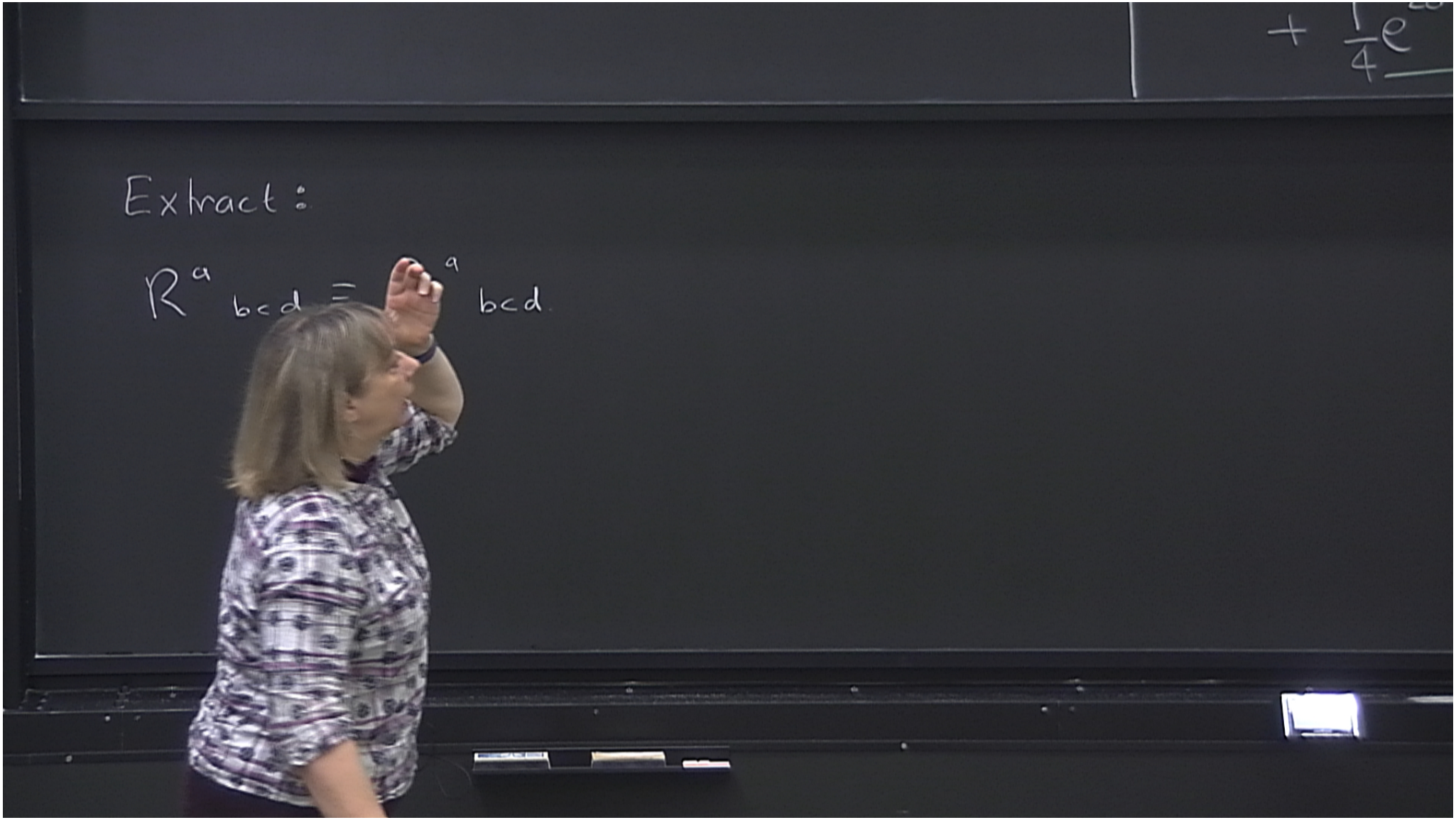
$$\underline{R}^a{}_b = \underline{d}\,\underline{\Theta}^a_{\ 0\,b} + \tfrac{1}{2}\underline{d}\left(e^\sigma F^a{}_{b\,\sigma\psi}\underline{\omega}^\psi\right) + \left(\underline{\Theta}^a_{\ 0\,c} + \tfrac{1}{2}e^\sigma F^a{}_{c\,\sigma\psi}\underline{\omega}^\psi\right)\wedge\left(\underline{\Theta}^c_{\ 0\,b} + \tfrac{1}{2}e^\sigma F^c{}_{b\,\sigma\psi}\underline{\omega}^\psi\right)$$

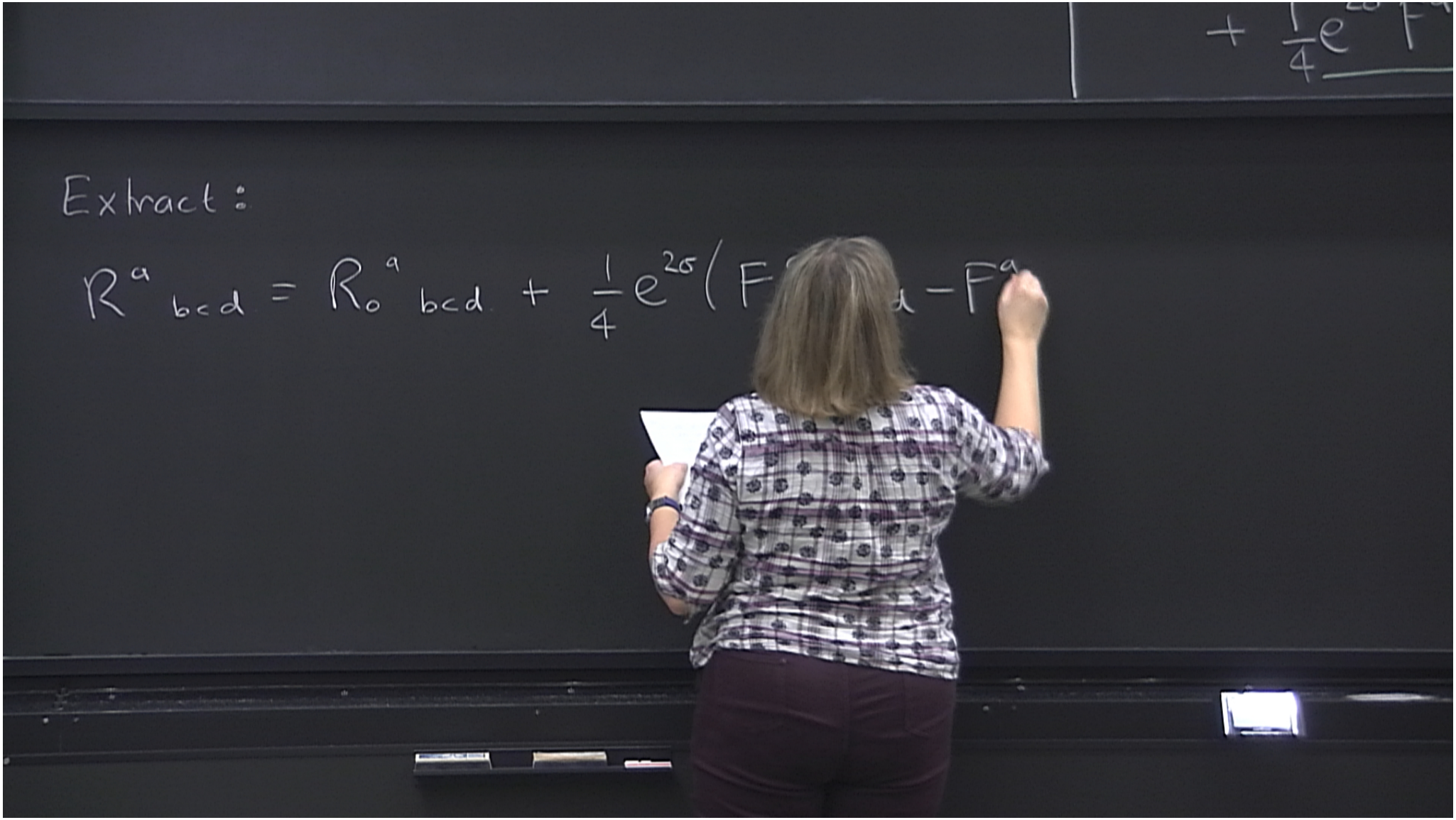
$$+ \left(\sigma^{,a}\underline{\omega}^\tau + \tfrac{1}{2}e^\sigma F^a{}_{c\,\sigma\tau}\underline{\omega}^c\right)\wedge\left(\sigma_{,b}\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F_{bd\,\sigma\psi}\underline{\omega}^d\right)$$

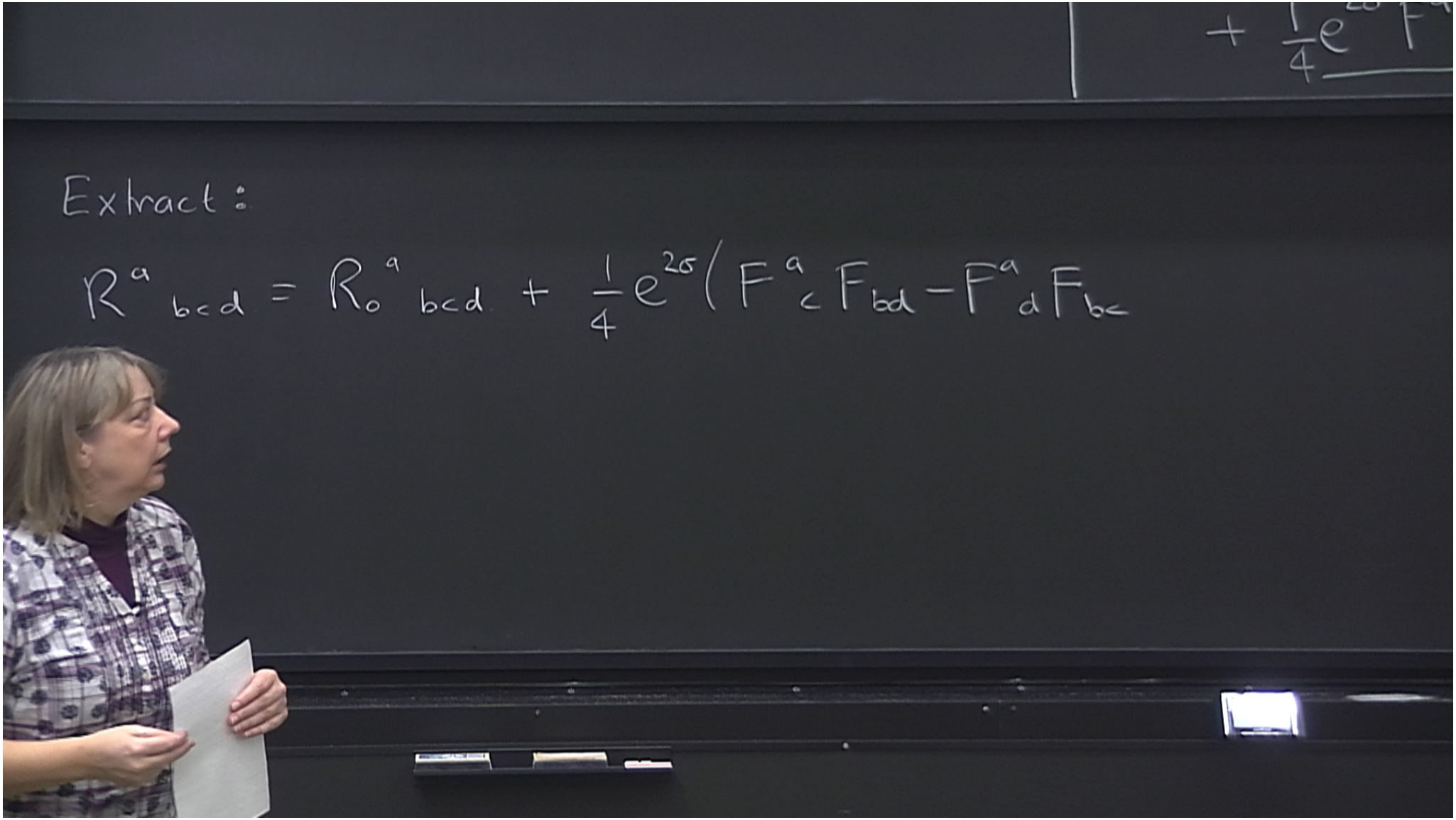
$$= \underline{d\,\Theta^a_{\ 0\,b}} + \tfrac{1}{2}e^\sigma\sigma_{,c}F^a{}_{b\,\sigma\psi}\underline{\omega}^c\wedge\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F^a{}_{b,c\,\sigma\psi}\underline{\omega}^c\wedge\underline{\omega}^\psi + \tfrac{1}{2}e^\sigma F^a{}_{b\,\sigma\psi}\left(\sigma_{,c}\underline{\omega}^c\wedge\underline{\omega}^\psi + e^\sigma\underline{F}\right)$$

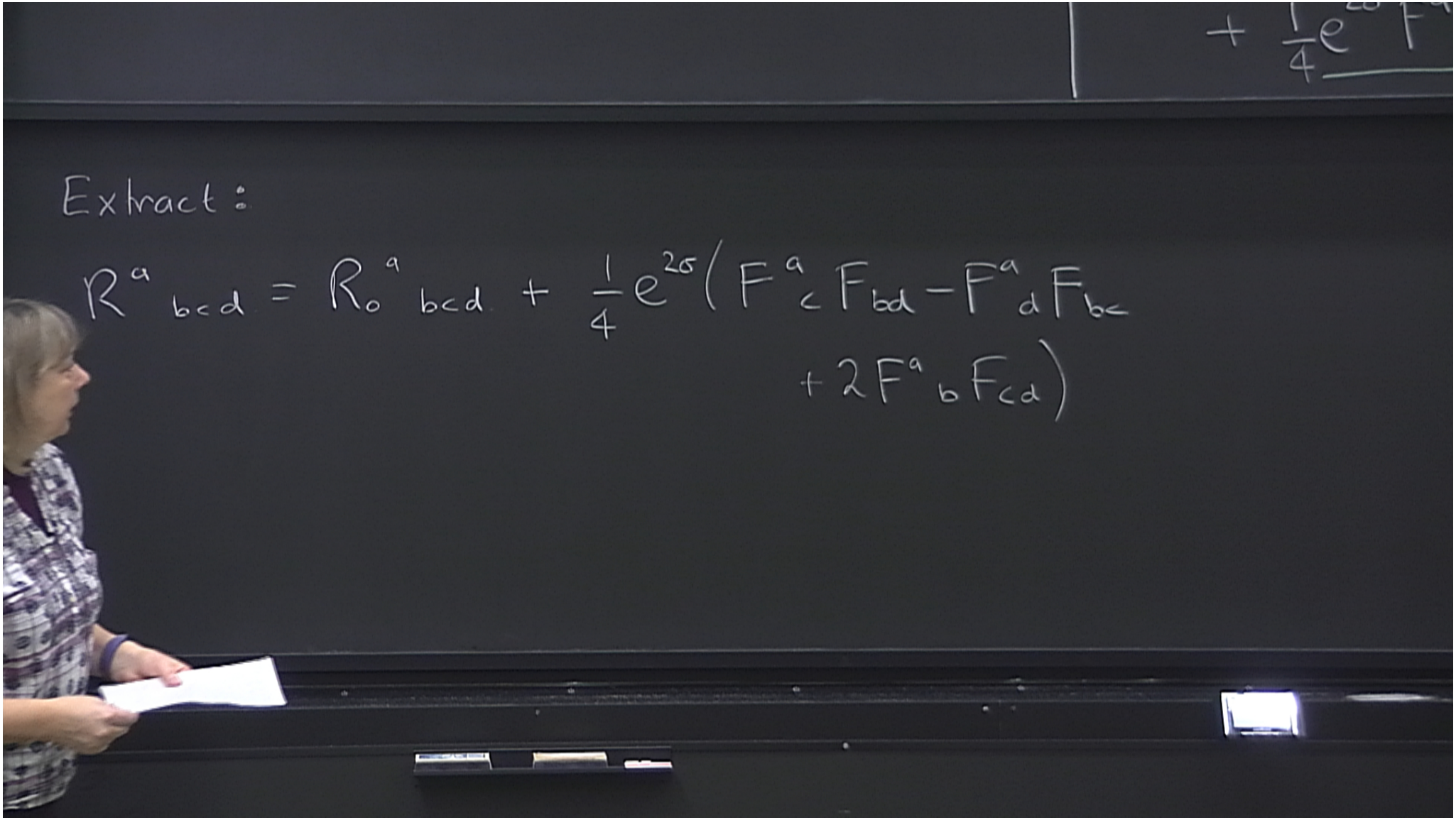
$$+ \underline{\Theta^a_{\ 0\,c}\wedge\Theta^c_{\ 0\,b}} + \tfrac{1}{2}e^\sigma\left(F^a{}_{c\,\sigma\psi}\underline{\omega}^\psi\wedge\underline{\Theta}^a_{\ 0\,b} + \underline{\Theta}^a_{\ 0\,c}\wedge F^c{}_{b\,\sigma\psi}\underline{\omega}^\psi\right)$$

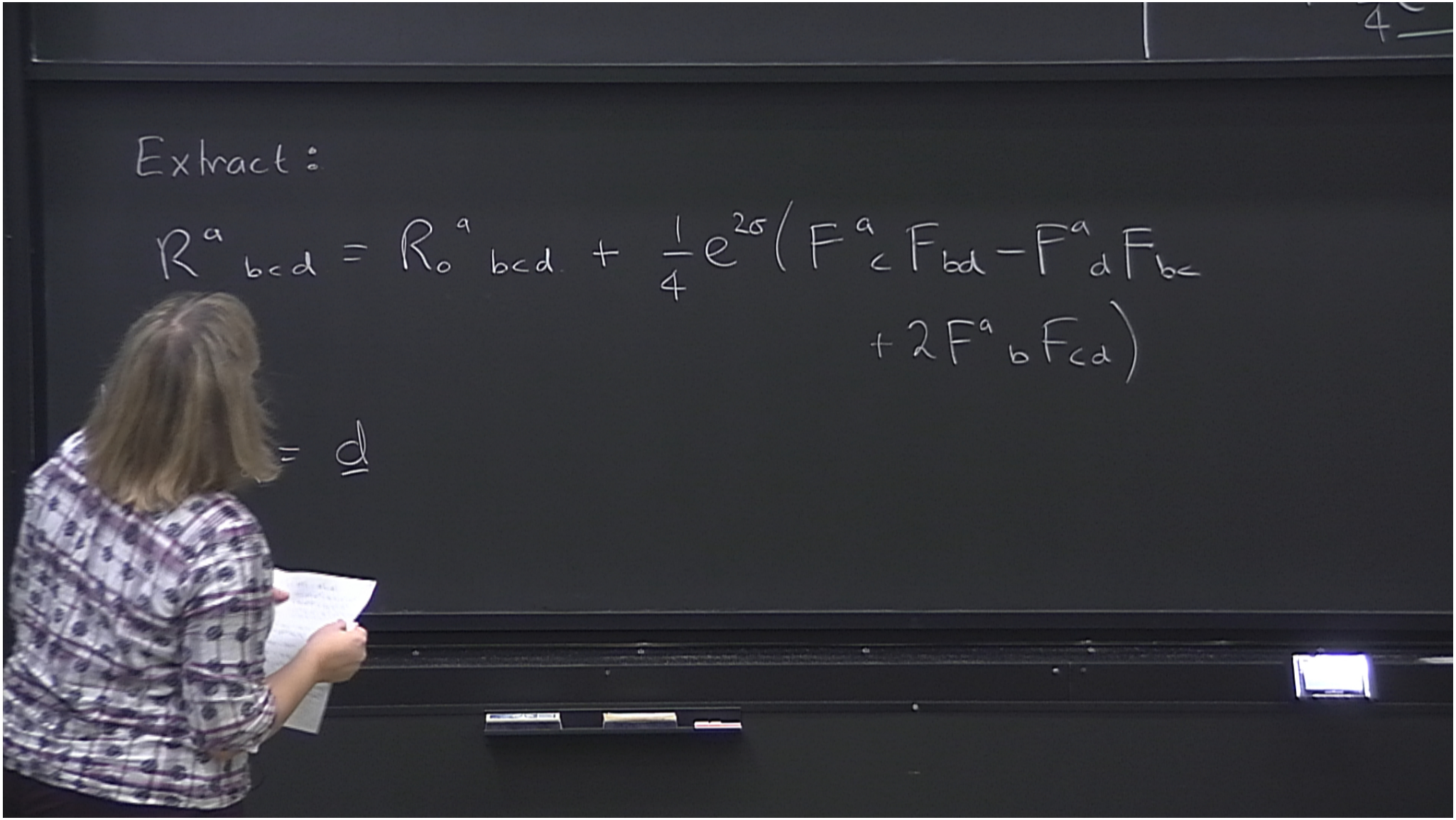
$$+ \underline{\tfrac{1}{4}e^{2\sigma}F^a{}_{c\,\sigma\psi}F_{bd\,\sigma\tau}\underline{\omega}^c\wedge\underline{\omega}^d} + \tfrac{1}{2}e^\sigma\left(F^a{}_{c\,\sigma\psi}\underline{\omega}^c\wedge\sigma_{,b}\underline{\omega}^\psi + F_{bd\,\sigma\tau}\sigma^{,a}\underline{\omega}^\psi\wedge\underline{\omega}^\tau\right)$$

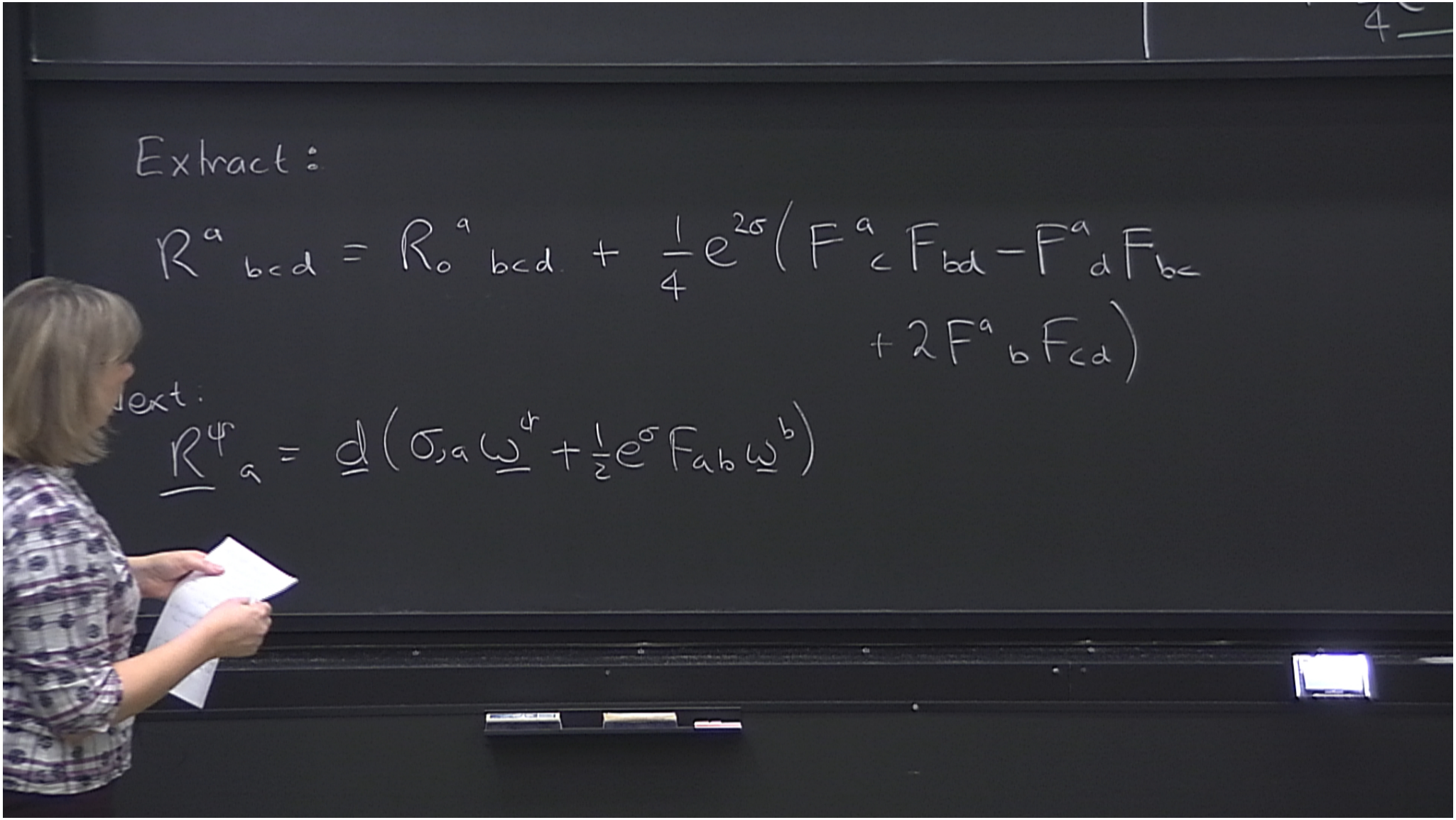

Extract:
$R^a{}_{bcd} = \quad {}^a{}_{bcd}$

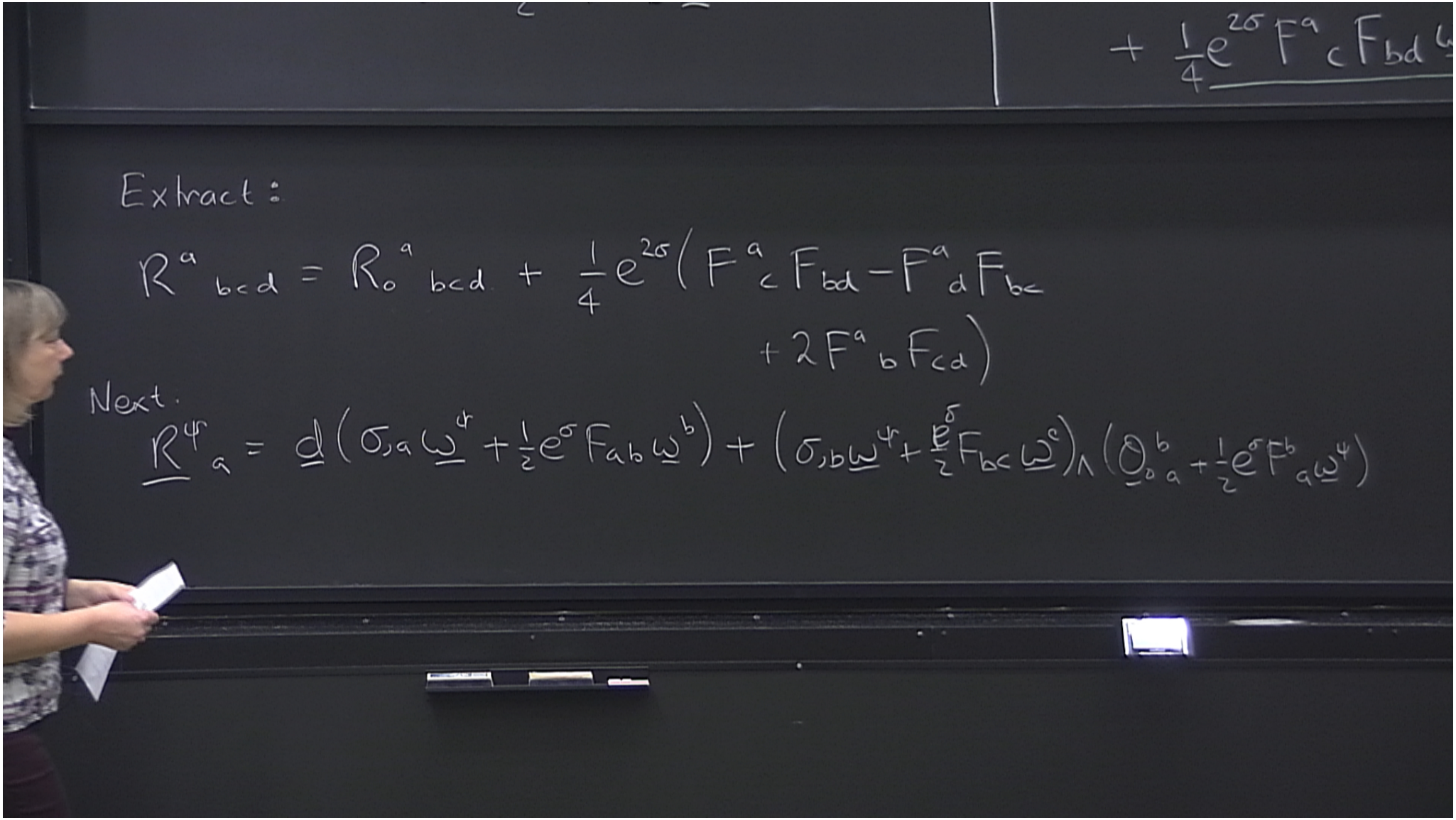

Extract:
$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}(F$
$+ \frac{1}{4}e^{2\sigma}F$


Extract:
$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}(F^a{}_c F_{bd} - F^a{}_d F_{bc}$


Extract:
$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}\left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd}\right)$


Extract:
$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd})$
$= d$


Extract:
$R^a{}_{bcd} = R_o{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd})$
Next:
$\underline{R}^{\psi}{}_a = \underline{d}(\sigma_{,a}\underline{\omega}^a + \frac{1}{2}e^{\sigma}F_{ab}\underline{\omega}^b)$


$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}$
Extract:
$R^{a}{}_{bcd} = R_{o}{}^{a}{}_{bcd} + \frac{1}{4}e^{2\sigma}\left(F^{a}{}_{c}F_{bd} - F^{a}{}_{d}F_{bc} + 2F^{a}{}_{b}F_{cd}\right)$
Next.
$\underline{R}^{\psi}{}_{a} = \underline{d}\left(\sigma_{,a}\underline{\omega}^{\psi} + \frac{1}{2}e^{\sigma}F_{ab}\underline{\omega}^{b}\right) + \left(\sigma_{,b}\underline{\omega}^{\psi} + \frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\right)\wedge\left(\underline{\theta}_{o}{}^{b}{}_{a} + \frac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{\psi}\right)$

$$+ \frac{1}{4} e^{2\sigma} F^a{}_c F_{bd}$$

Extract:

$$R^a{}_{bcd} = R_o{}^a{}_{bcd} + \frac{1}{4} e^{2\sigma} \left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2 F^a{}_b F_{cd} \right)$$

Next:

$$\underline{R}^{\psi}{}_{a} = \underline{d}\left(\sigma_{,a} \underline{\omega}^{\psi} + \tfrac{1}{2} e^{\sigma} F_{ab} \underline{\omega}^{b} \right) + \left(\sigma_{,b} \underline{\omega}^{\psi} + \tfrac{e^{\sigma}}{2} F_{bc} \underline{\omega}^{c} \right) \wedge \left(\underline{\theta}_o{}^{b}{}_{a} + \tfrac{1}{2} e^{\sigma} F^{b}{}_{a} \underline{\omega}^{\psi} \right)$$

$$= \sigma_{,ab} \underline{\omega}^{b} \wedge \underline{\omega}^{\psi}$$

$$\underline{\Theta}^a{}_b = \underline{\Theta}_0{}^a{}_b + \frac{1}{2}e^\sigma F^a{}_b \underline{\omega}^\psi$$

$$+ \underline{\Theta_0{}^a{}_c \wedge \Theta_0{}_b} + \frac{1}{2}e^{\sigma}(F_c \underline{\omega} \wedge \Theta_0{}_b \underline{\quad}$$

$$+ \frac{1}{4}e^{2\sigma} F^a{}_c F_{bd} \underline{\omega^c \wedge \omega^d} + \frac{1}{2}e^\sigma (F^a{}_c$$

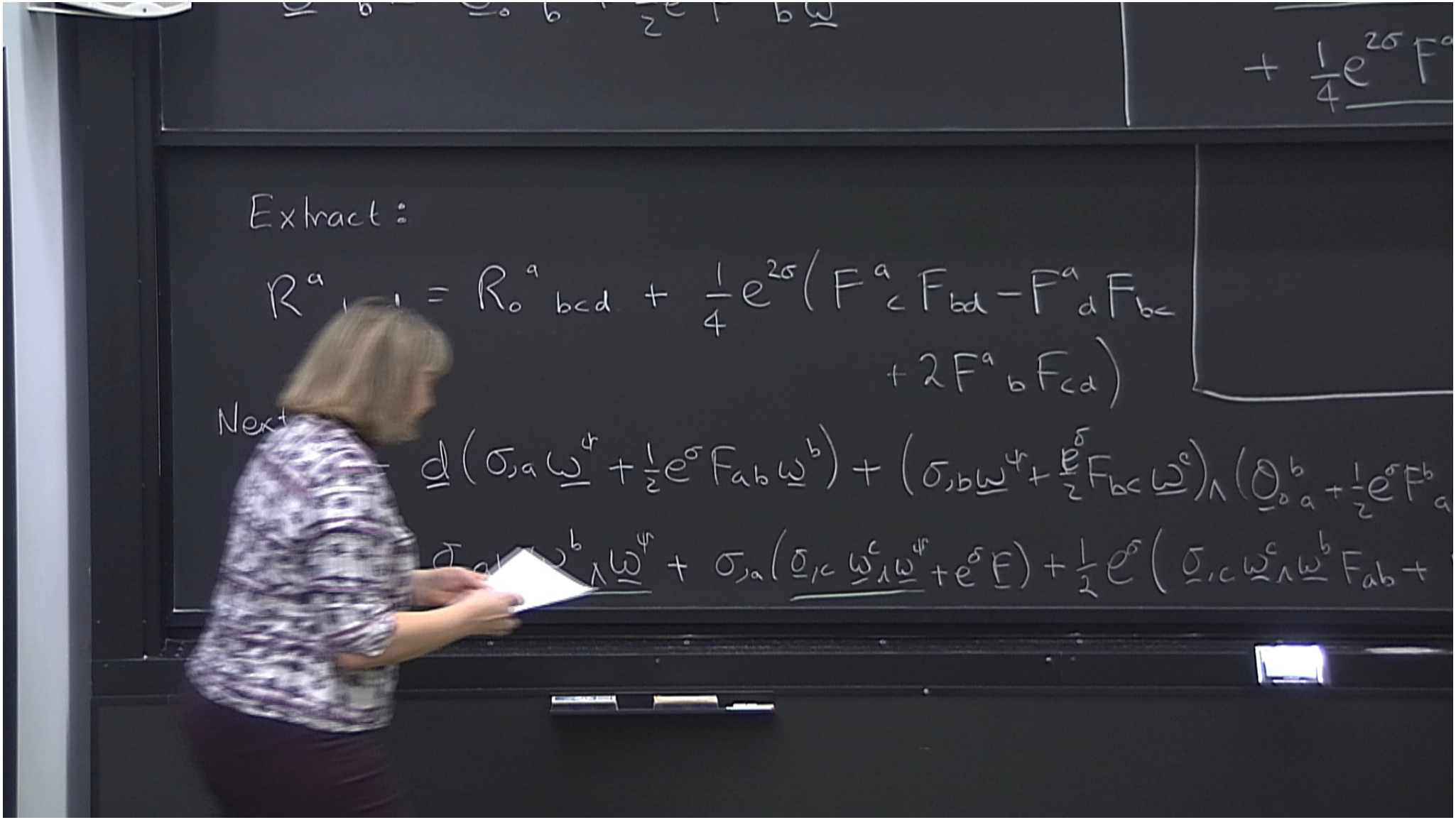
Extract:

$$R^a{}_{bcd} = R_0{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}\left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd}\right)$$

Next.

$$\underline{R}^\psi{}_a = \underline{d}\left(\sigma_{,a}\underline{\omega}^\psi + \frac{1}{2}e^\sigma F_{ab}\underline{\omega}^b\right) + \left(\sigma_{,b}\underline{\omega}^\psi + \frac{e^\sigma}{2}F_{bc}\underline{\omega}^c\right) \wedge \left(\underline{\Theta}_0{}^b{}_a + \frac{1}{2}e^\sigma F^b{}_a \underline{\omega}^\psi\right)$$

$$= \sigma_{,ab}\,\underline{\omega}^b \wedge \underline{\omega}^\psi + \sigma_{,a}\left(\underline{\Theta}_{,c}\,\underline{\omega}^c \wedge \underline{\omega}^\psi + e^\sigma \underline{F}\right) + \frac{1}{2}e^\sigma\left(\underline{\Theta}_{,c}\,\underline{\omega}^c \wedge \underline{\omega}^b F_{ab} + F_{ab,c}\underline{\omega}^c \wedge \underline{\omega}^b - \frac{1}{2}F_{ab}\Theta_0{}^b{}_c \wedge \underline{\omega}^c\right)$$


Extract:
$R^a{}_{bcd} = R_o{}^a{}_{bcd} + \frac{1}{4}e^{2\sigma}\left(F^a{}_c F_{bd} - F^a{}_d F_{bc} + 2F^a{}_b F_{cd}\right)$
Next
$d\left(\sigma_{,a}\omega^a + \frac{1}{2}e^\sigma F_{ab}\omega^b\right) + \left(\sigma_{,b}\omega^a + \frac{1}{2}e^\sigma F_{bc}\omega^c\right)\wedge\left(\omega_o{}^b{}_a + \frac{1}{2}e^\sigma F^b{}_a\right.$
$+ \sigma_{,a}\left(\omega_{,c}\omega^c\wedge\omega^a + e^\sigma F\right) + \frac{1}{2}e^\sigma\left(\omega_{,c}\omega^c\wedge\omega^b F_{ab} +$

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}{}_{c}\underline{\omega}^{c}\wedge\underline{\omega}^{d})$$

Extract:

$$R^{ar}{}_{a\psi b}=-\sigma_{,ab}$$

$$\lambda-F^{a}{}_{d}F_{bc}$$

$$^{a}{}_{b}F_{cd})$$

$$\underline{\omega}^{\psi}+\tfrac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c})\wedge(\underline{\theta}^{b}_{\ 0\,a}+\tfrac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{\psi})$$

$$+\frac{1}{4}e^{2\sigma}F^{b}{}_{a}F_{bc}\underline{\omega}^{c}\wedge\underline{\omega}^{\psi}$$

$$\underline{F})+\tfrac{1}{2}e^{\sigma}(\underline{\sigma}_{,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}F_{ab}+F_{ab,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}-\tfrac{1}{2}F_{ab}\underline{\theta}^{b}_{\ 0\,c}\wedge\underline{\omega}^{c})+\underline{\sigma_{,b}\underline{\omega}^{\psi}\wedge\underline{\theta}^{b}_{\ 0\,a}}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\wedge\underline{\theta}^{b}_{\ 0\,a}$$

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}{}_{c}\underline{\omega}^{c}\wedge\underline{\omega}^{d})$$

Extract:

$$R^{ar}{}_{a\psi b}=-\sigma_{,ab}-\sigma_{,a}\sigma_{,b}$$

$$\lambda-F^{a}{}_{d}F_{bc}$$

$$^{a}{}_{b}F_{cd})$$

$$\underline{\omega}^{\psi}+\frac{\beta^{\sigma}}{2}F_{bc}\underline{\omega}^{c})\wedge(\underline{\theta}^{b}{}_{a}+\frac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{\psi})$$

$$+\frac{1}{4}e^{2\sigma}F^{b}{}_{a}F_{bc}\underline{\omega}^{c}\wedge\underline{\omega}^{\psi}$$

$$^{\sigma}\underline{F})+\frac{1}{2}e^{\sigma}(\underline{\sigma}_{,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}F_{ab}+F_{ab,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}-\frac{1}{2}F_{ab}\underline{\theta}^{b}{}_{c}\wedge\underline{\omega}^{c})+\sigma_{,b}\underline{\omega}^{\psi}\wedge\underline{\theta}^{b}{}_{a}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\wedge\underline{\theta}^{b}{}_{a}$$

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}\underline{\omega}^{c}\wedge\underline{\omega}^{d})$$

Extract:

$$R^{ar}{}_{a\,rb}=-\sigma_{;ab}-\sigma_{;a}\sigma_{;b}+\sigma_{;c}\Gamma^{c}_{ba}$$

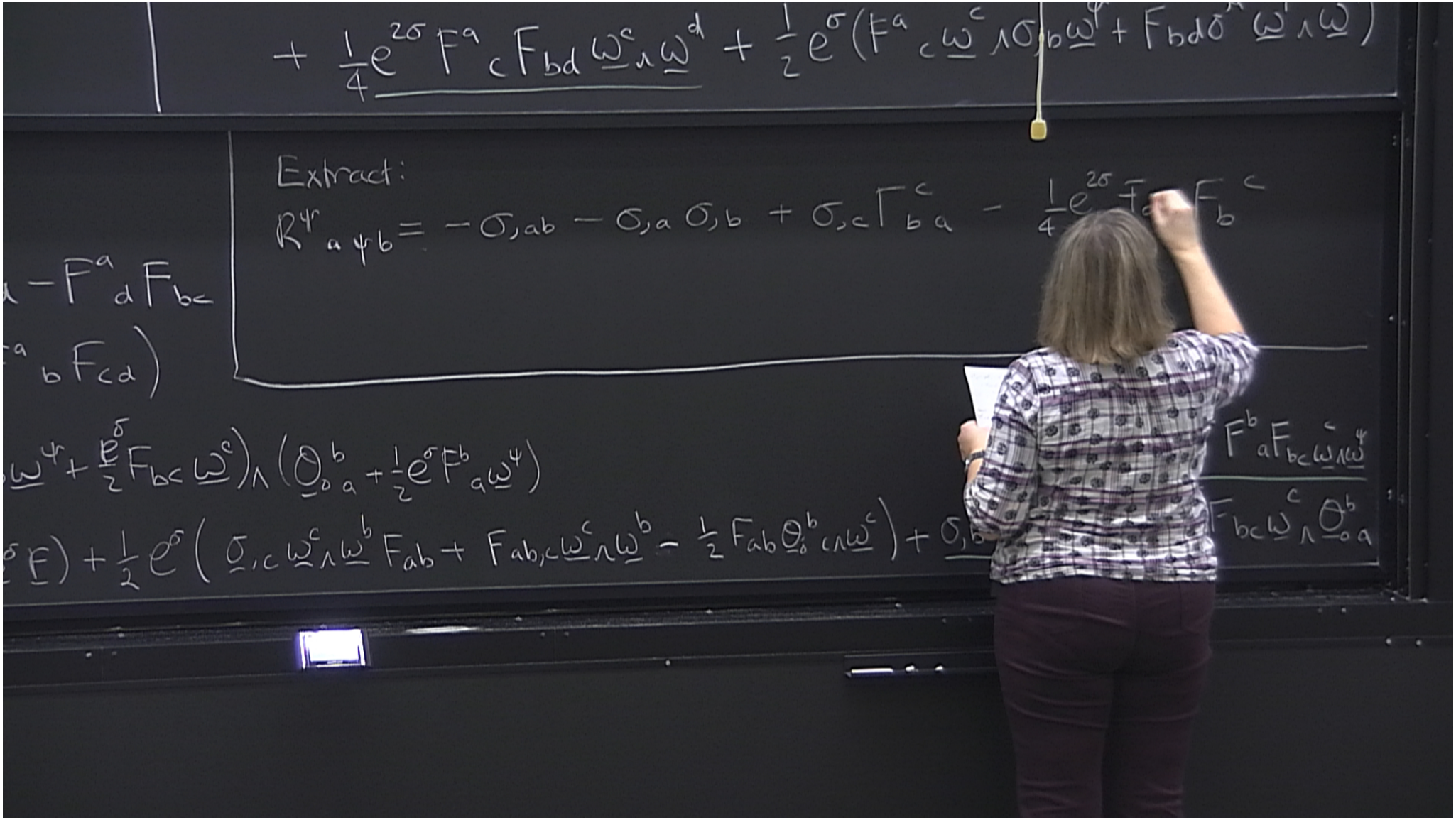
$$\lambda-F^{a}{}_{d}F_{bc}$$

$$\Gamma^{a}{}_{b}F_{cd})$$

$$\underline{\omega}^{\psi}+\frac{e^{\delta}}{2}F_{bc}\underline{\omega}^{c})\wedge(\underline{\theta}^{b}_{\ 0\,a}+\frac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{\psi})$$

$$+\frac{1}{4}e^{2\sigma}F^{b}{}_{a}F_{bc}\underline{\omega}^{c}\wedge\underline{\omega}^{\psi}$$

$$\underline{F})+\frac{1}{2}e^{\sigma}(\underline{\sigma}_{,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}F_{ab}+F_{ab,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}-\frac{1}{2}F_{ab}\underline{\theta}^{b}_{\ 0\,c}\wedge\underline{\omega}^{c})+\underline{\sigma_{,b}\underline{\omega}^{\psi}\wedge\underline{\theta}^{b}_{\ 0\,a}}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\wedge\underline{\theta}^{b}_{\ 0\,a}$$


Extract:

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}\left(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}{}_{c}\underline{\omega}^{c}\wedge\underline{\omega}^{d}\right)$$

Extract:

$$R^{ab}{}_{cd}=-\sigma_{,ab}-\sigma_{,a}\sigma_{,b}+\sigma_{,c}\Gamma^{c}_{ba}-\frac{1}{4}e^{2\sigma}F_{ac}F_{b}{}^{c}$$

$$\wedge-F^{a}{}_{d}F_{bc}$$

$$\left.{}^{a}{}_{b}F_{cd}\right)$$

$$\underline{\omega}^{b}+\tfrac{1}{2}e^{\sigma}F_{bc}\underline{\omega}^{c})\wedge(\underline{\theta}^{b}_{0\,a}+\tfrac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{d})$$

$$+\frac{1}{4}e^{2\sigma}F^{b}{}_{a}F_{bc}\underline{\omega}^{c}\wedge\underline{\omega}^{d}$$

$$\left.\underline{F}\right)+\frac{1}{2}e^{\sigma}\left(\underline{\sigma}_{,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}F_{ab}+F_{ab,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}-\frac{1}{2}F_{ab}\underline{\theta}^{b}_{0\,c}\wedge\underline{\omega}^{c}\right)+\sigma_{,b}\underline{\omega}^{b}\wedge\underline{\theta}^{b}_{0\,a}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\wedge\underline{\theta}^{b}_{0\,a}$$

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}{}_{c}\underline{\omega}^{c}\wedge\underline{\omega}^{d})$$

Extract:

$$R^{ar}{}_{a\,rb}=-\sigma_{,ab}-\sigma_{,a}\sigma_{,b}+\sigma_{,c}\Gamma^{c}_{ba}-\frac{1}{4}e^{2\sigma}F_{ac}F_{b}{}^{c}$$

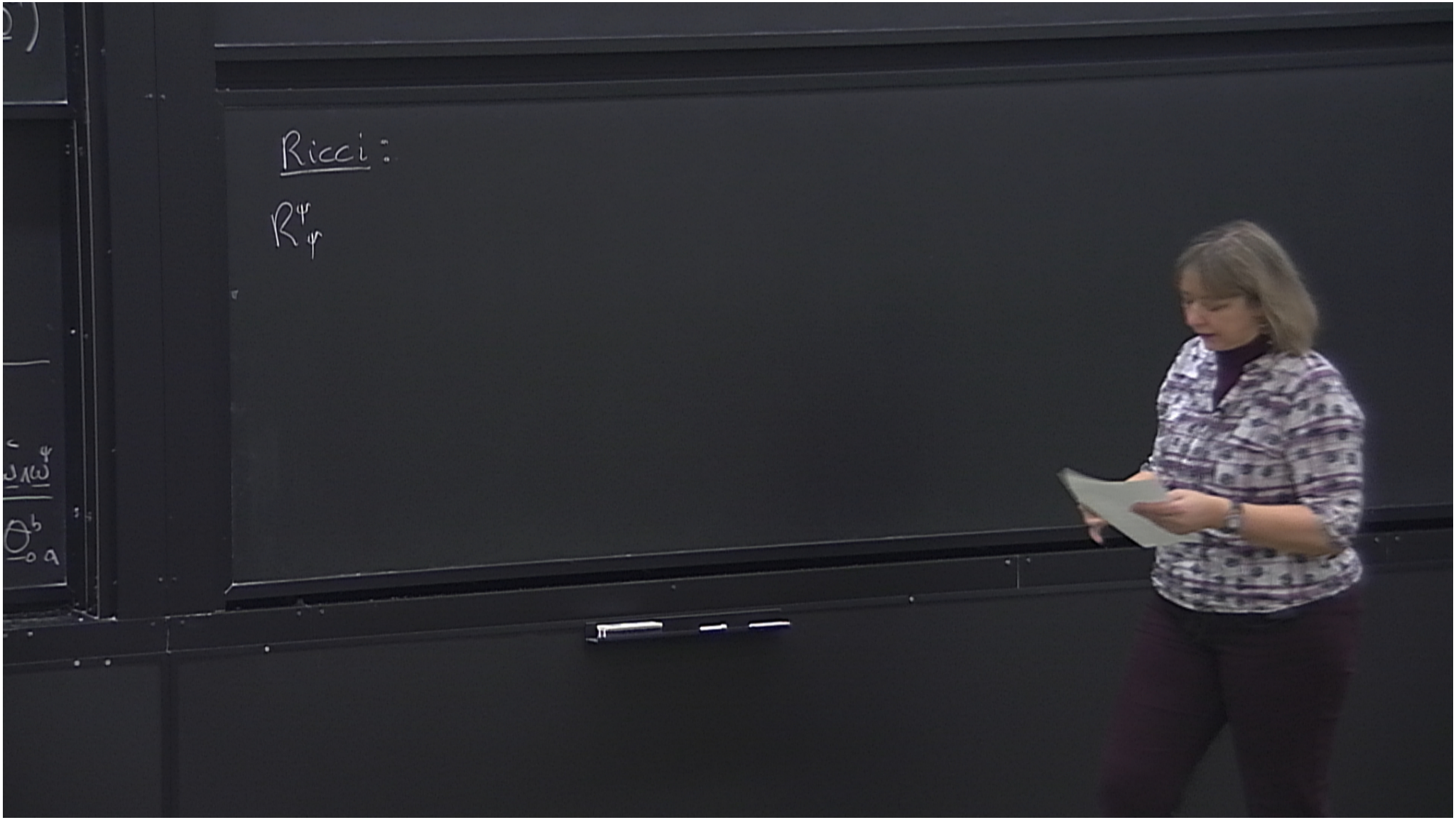
$$\wedge-F^{a}{}_{d}F_{bc}$$

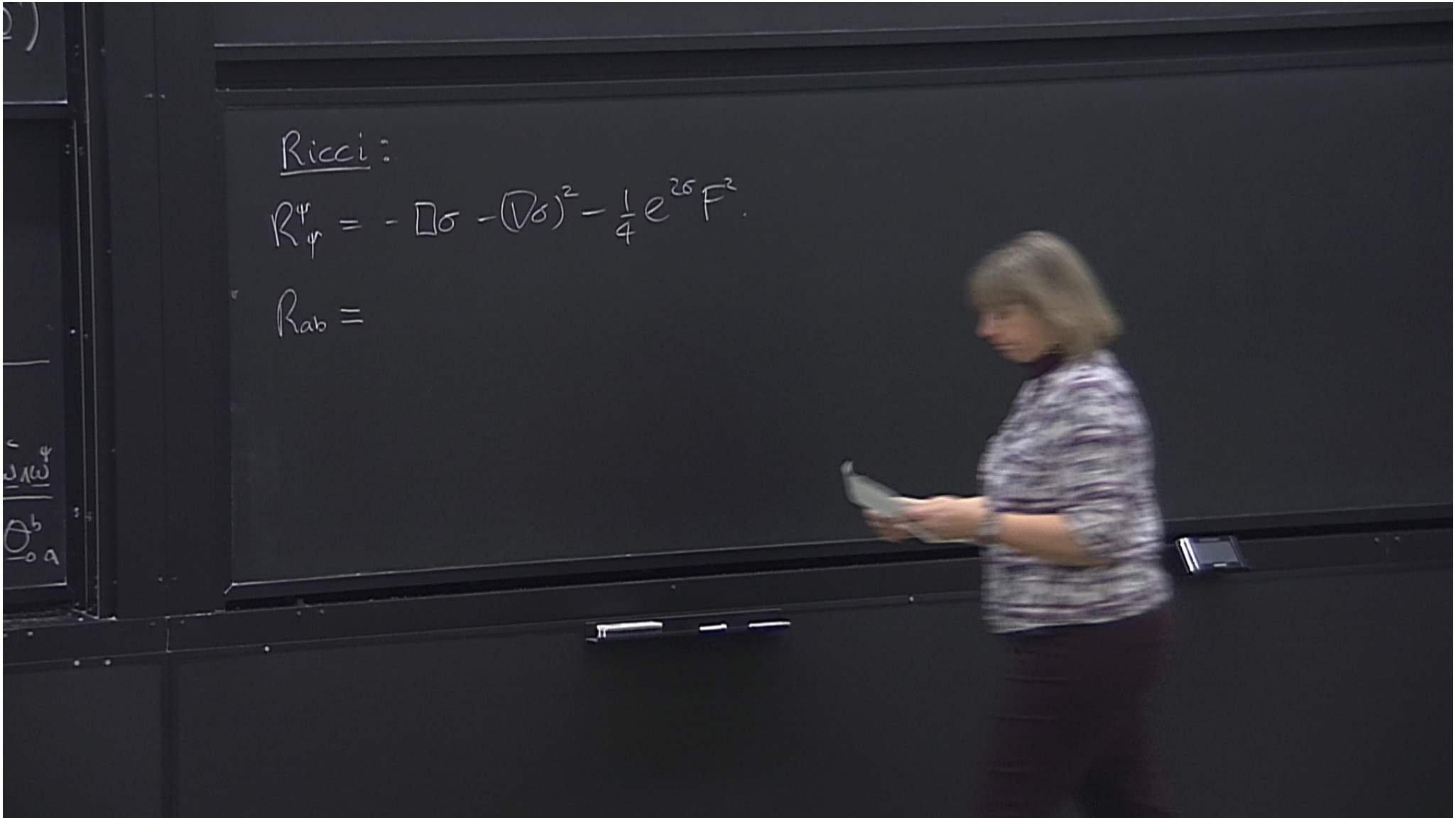
$$\Gamma^{a}{}_{b}F_{cd})$$

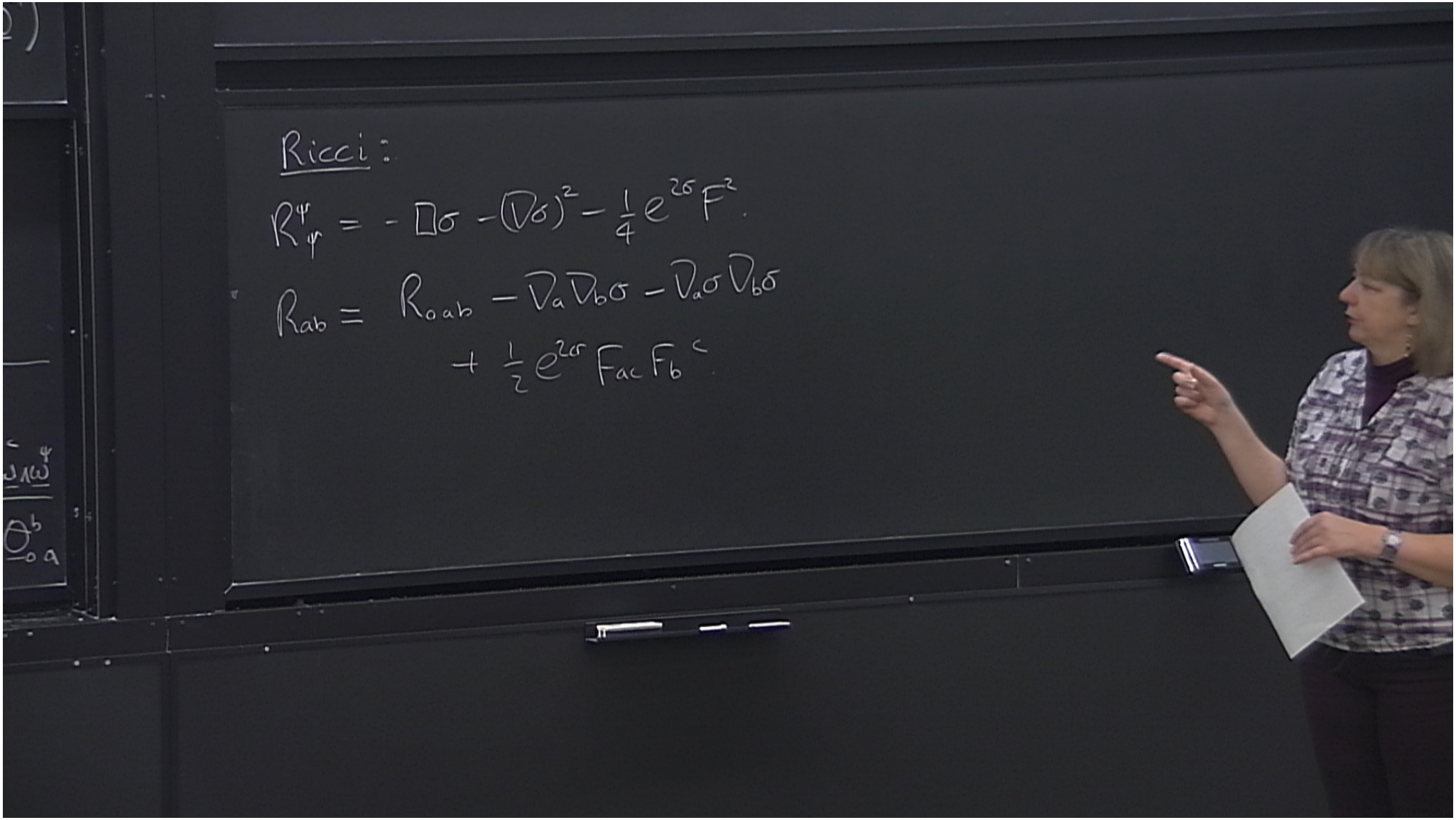
$$\underline{\omega}^{r}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c})\wedge(\underline{\theta}^{b}_{\ 0\,a}+\frac{1}{2}e^{\sigma}F^{b}{}_{a}\underline{\omega}^{r})$$

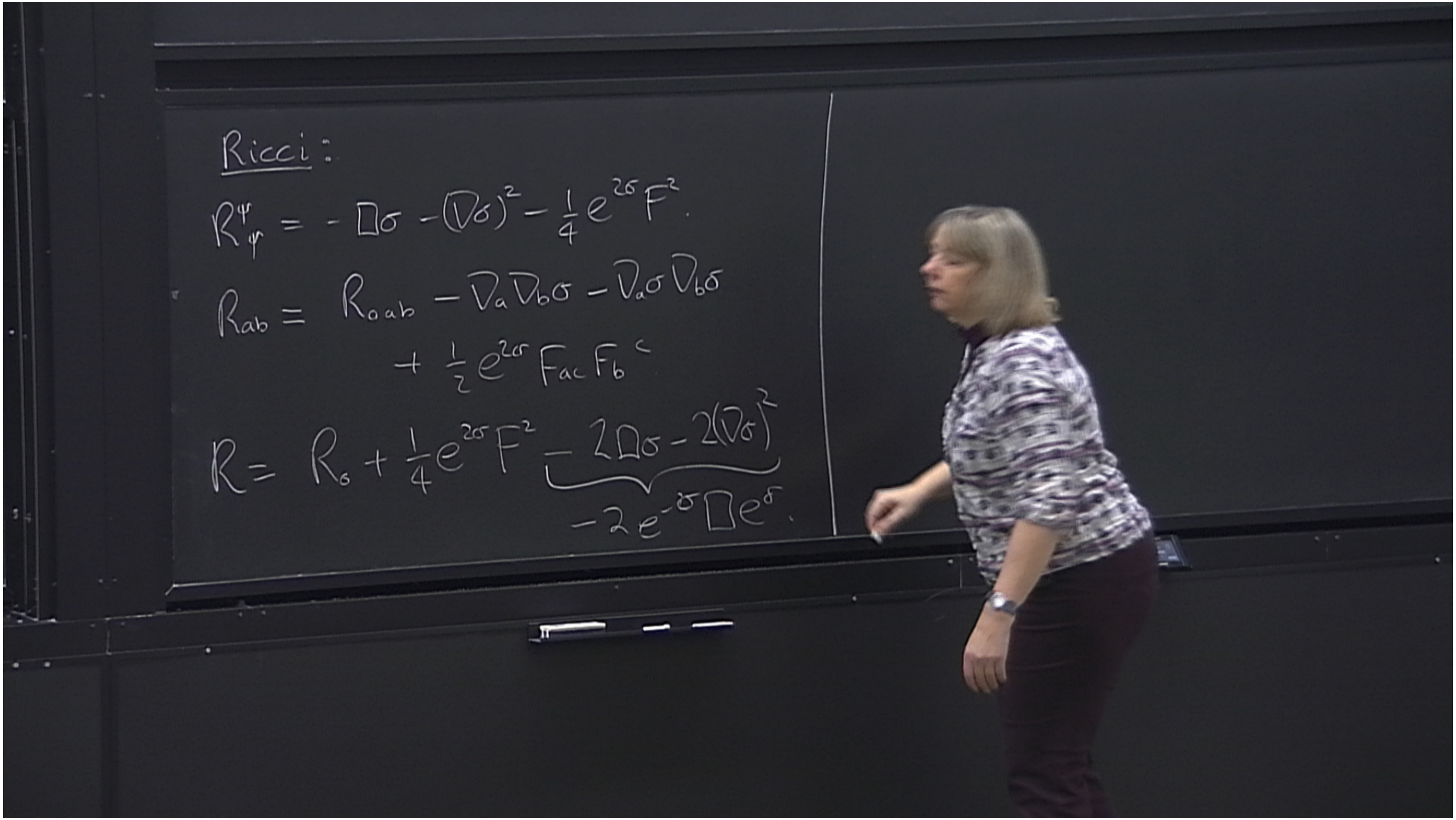
$$+\frac{1}{4}e^{2\sigma}F^{b}{}_{a}F_{bc}\underline{\omega}^{c}\wedge\underline{\omega}^{r}$$

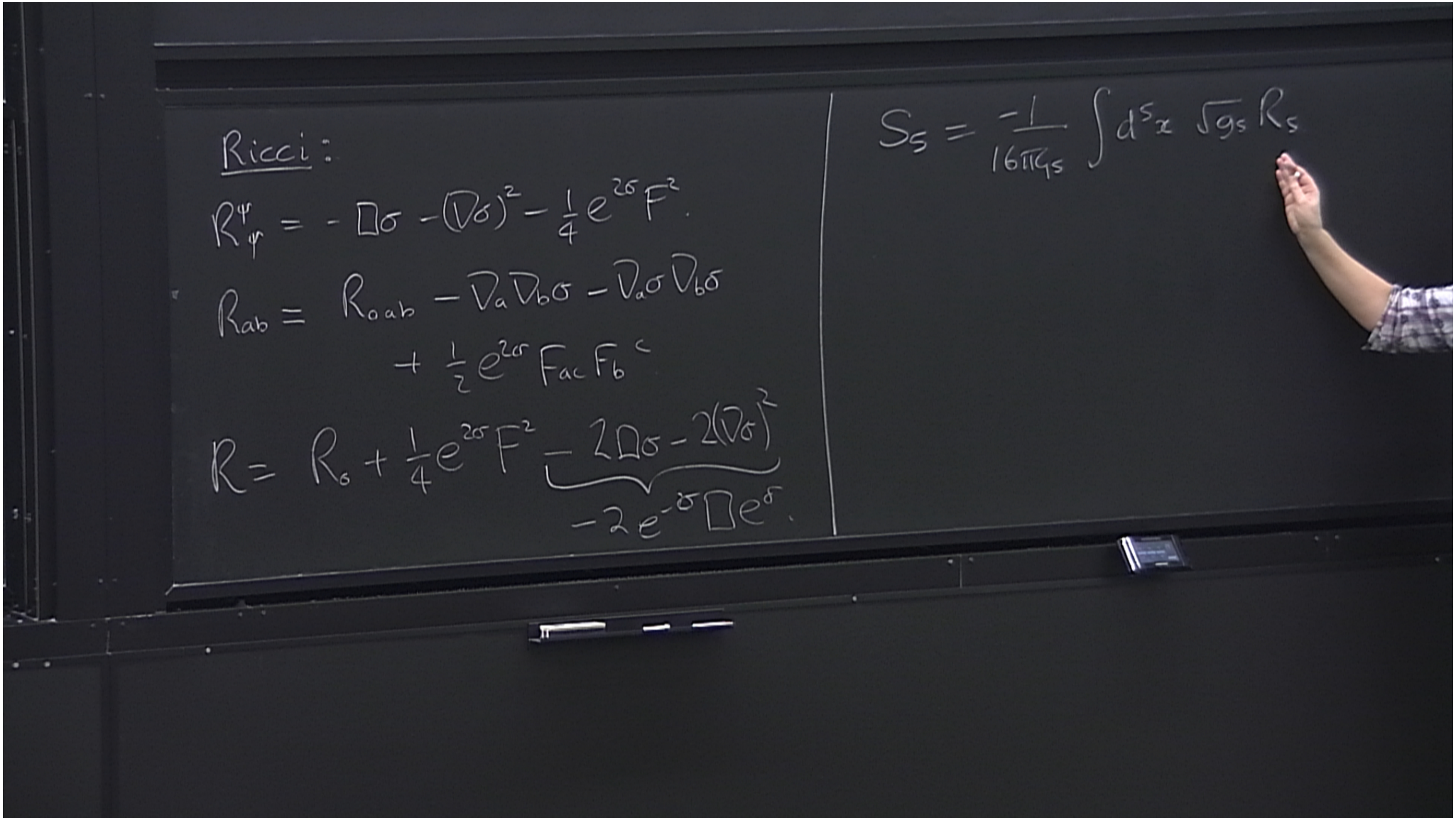
$$\underline{F})+\frac{1}{2}e^{\sigma}(\underline{\sigma}_{,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}F_{ab}+F_{ab,c}\underline{\omega}^{c}\wedge\underline{\omega}^{b}-\frac{1}{2}F_{ab}\underline{\theta}^{b}_{\ 0\,c}\wedge\underline{\omega}^{c})+\underline{\sigma_{,b}\underline{\omega}^{r}\wedge\underline{\theta}^{b}_{\ 0\,a}}+\frac{e^{\sigma}}{2}F_{bc}\underline{\omega}^{c}\wedge\underline{\theta}^{b}_{\ 0\,a}$$

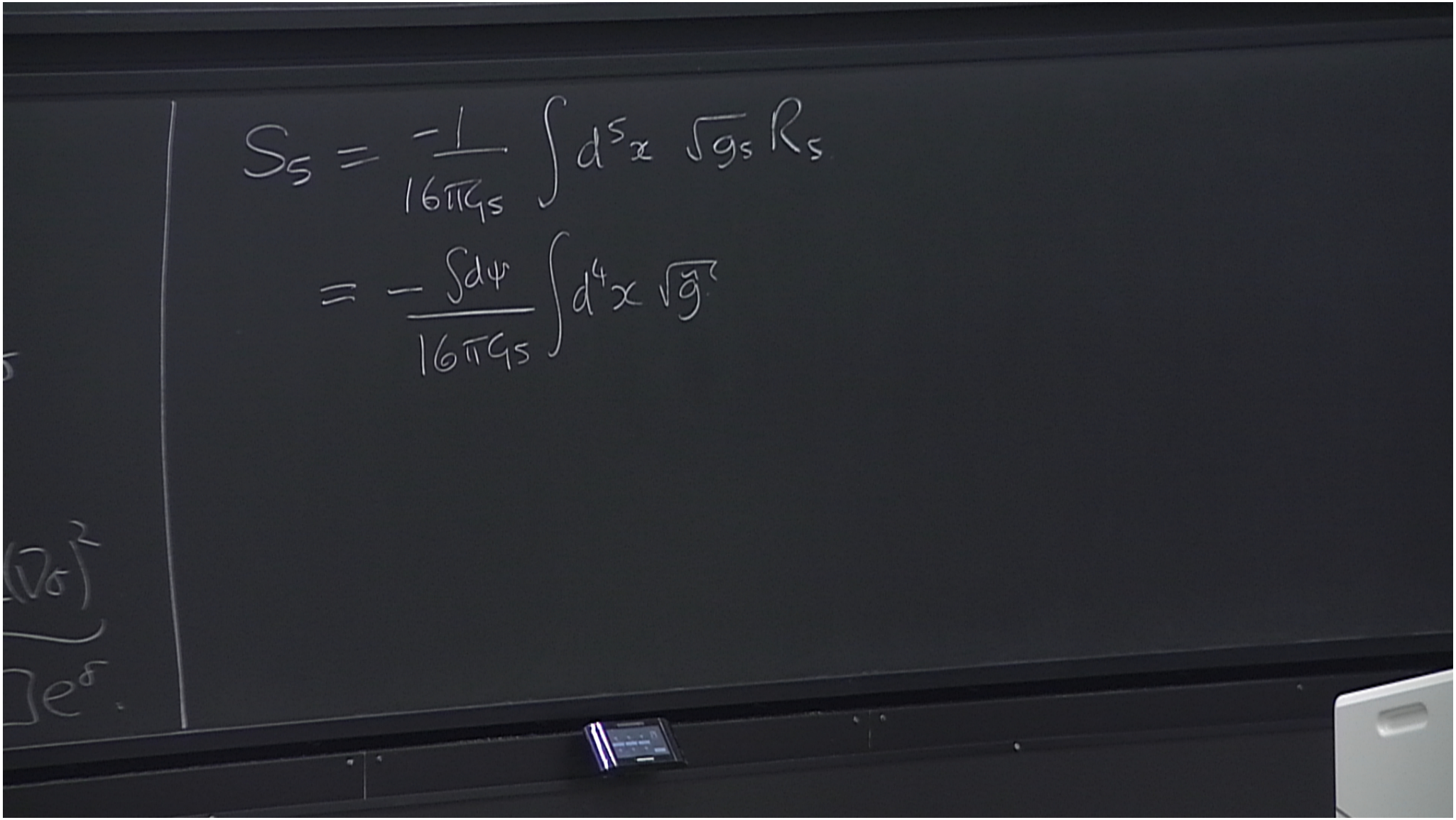

Ricci:
R^{ψ}_{ψ}

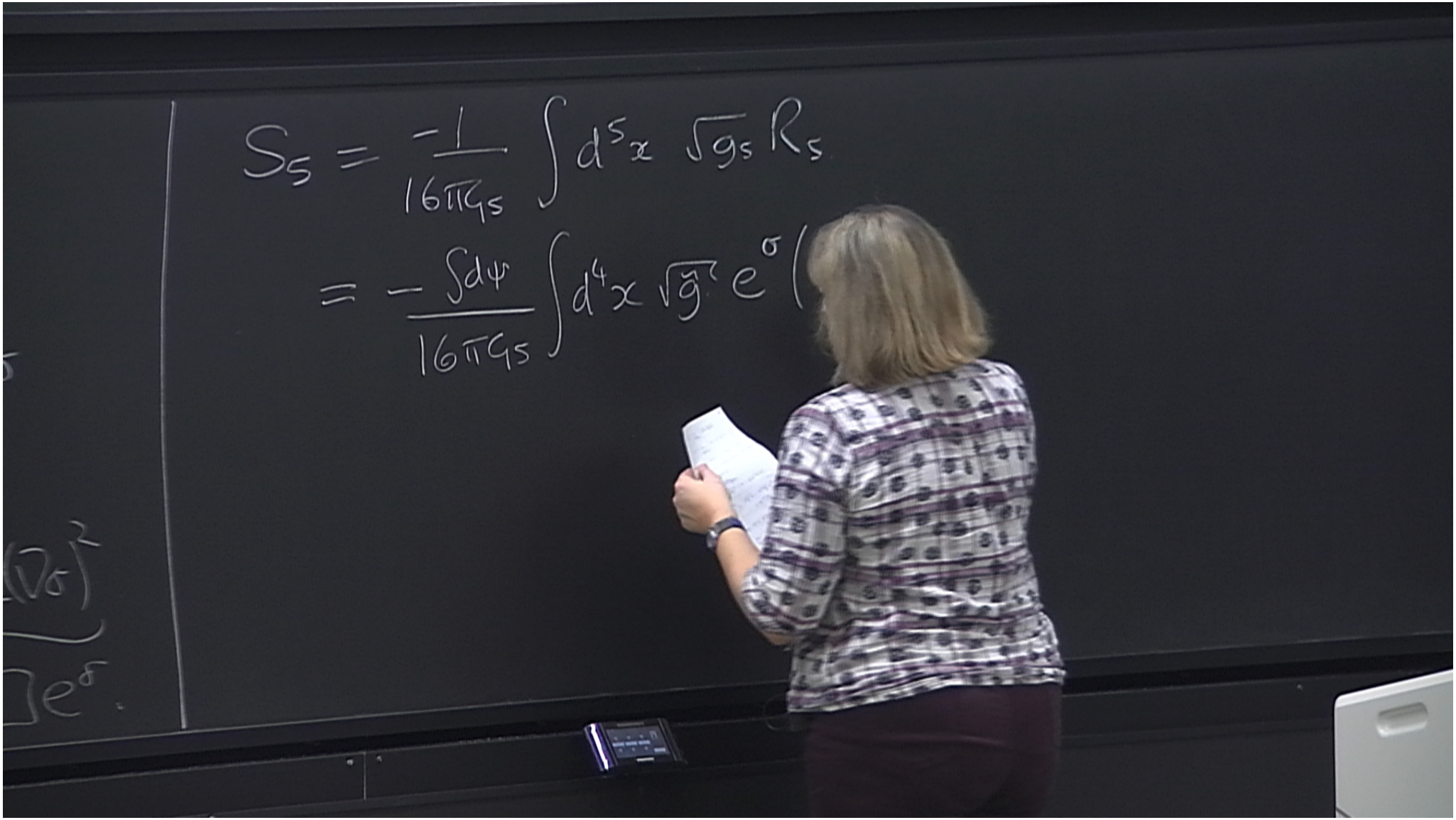

Ricci:
$R^{\psi}_{\psi} = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4}e^{2\sigma}F^2$.
$R_{ab} =$

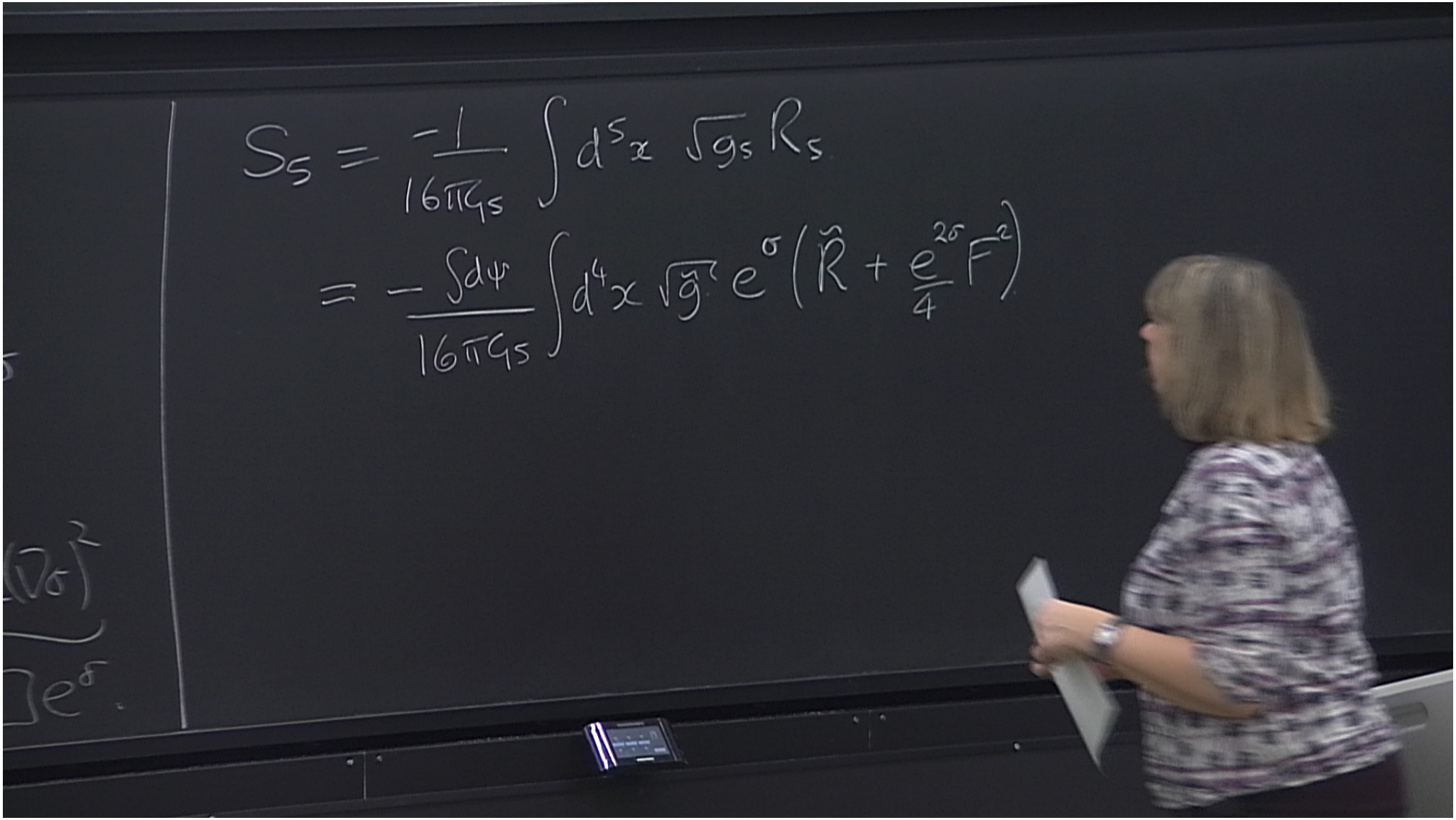

Ricci:
$R^{\psi}_{\psi} = -\Box\sigma - (\nabla\sigma)^2 - \frac{1}{4}e^{2\sigma}F^2$
$R_{ab} = R_{0ab} - \nabla_a\nabla_b\sigma - \nabla_a\sigma\nabla_b\sigma + \frac{1}{2}e^{2\sigma}F_{ac}F_b{}^c$

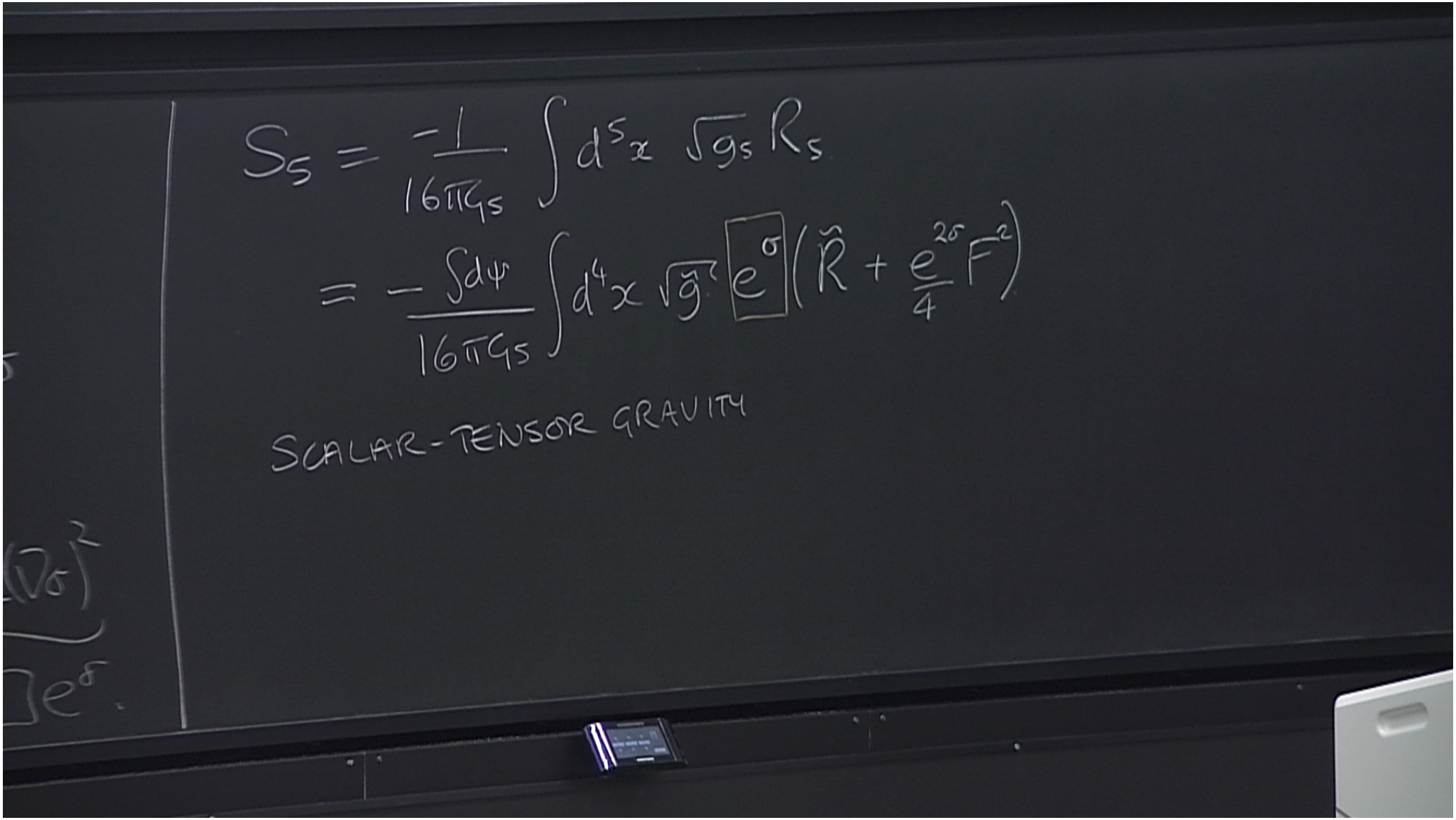

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$S_5 = \frac{-1}{16\pi G_5}\int d^5x\,\sqrt{g_5}\,R_5$


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SCALAR-TENSOR GRAVITY

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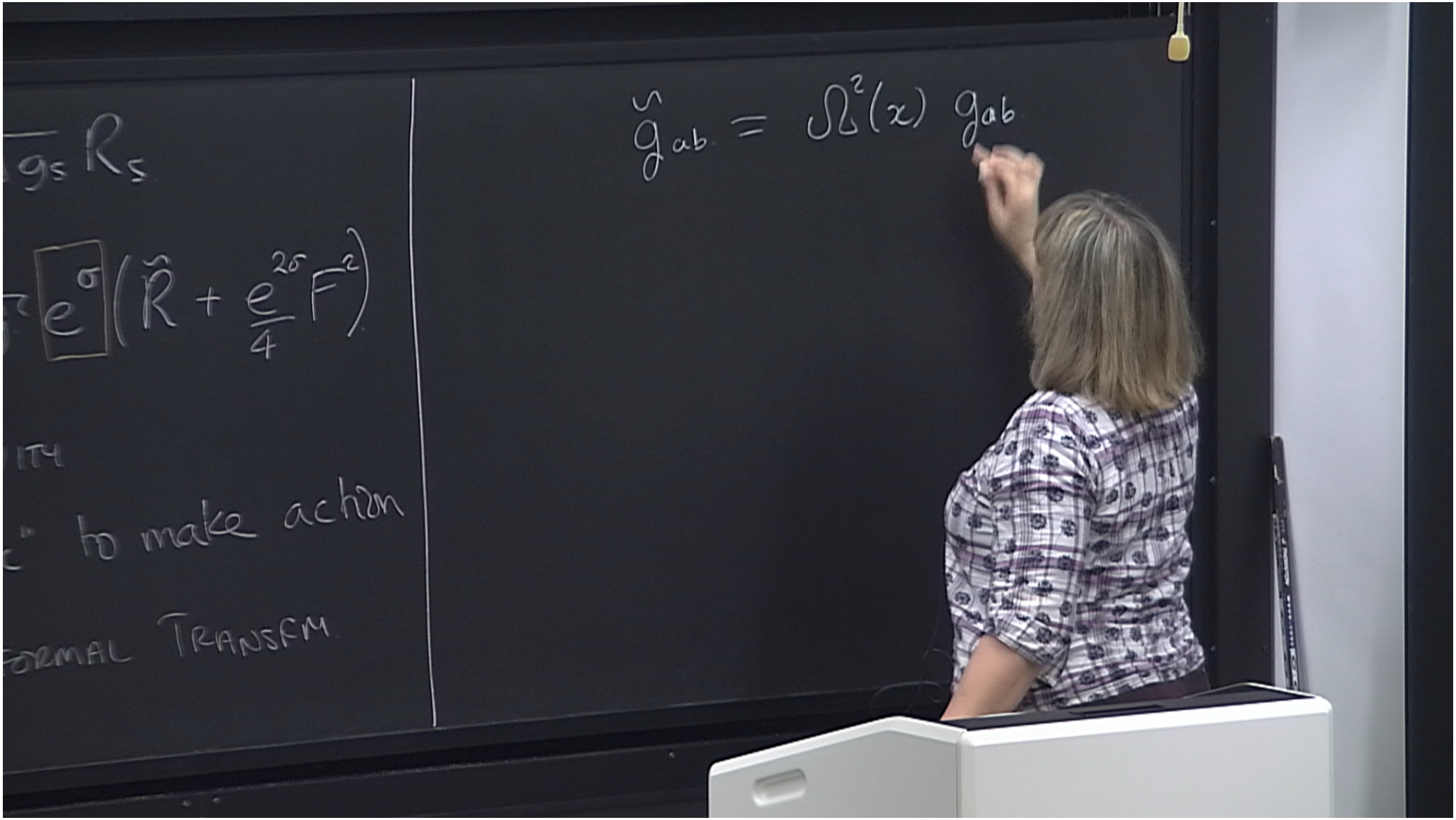
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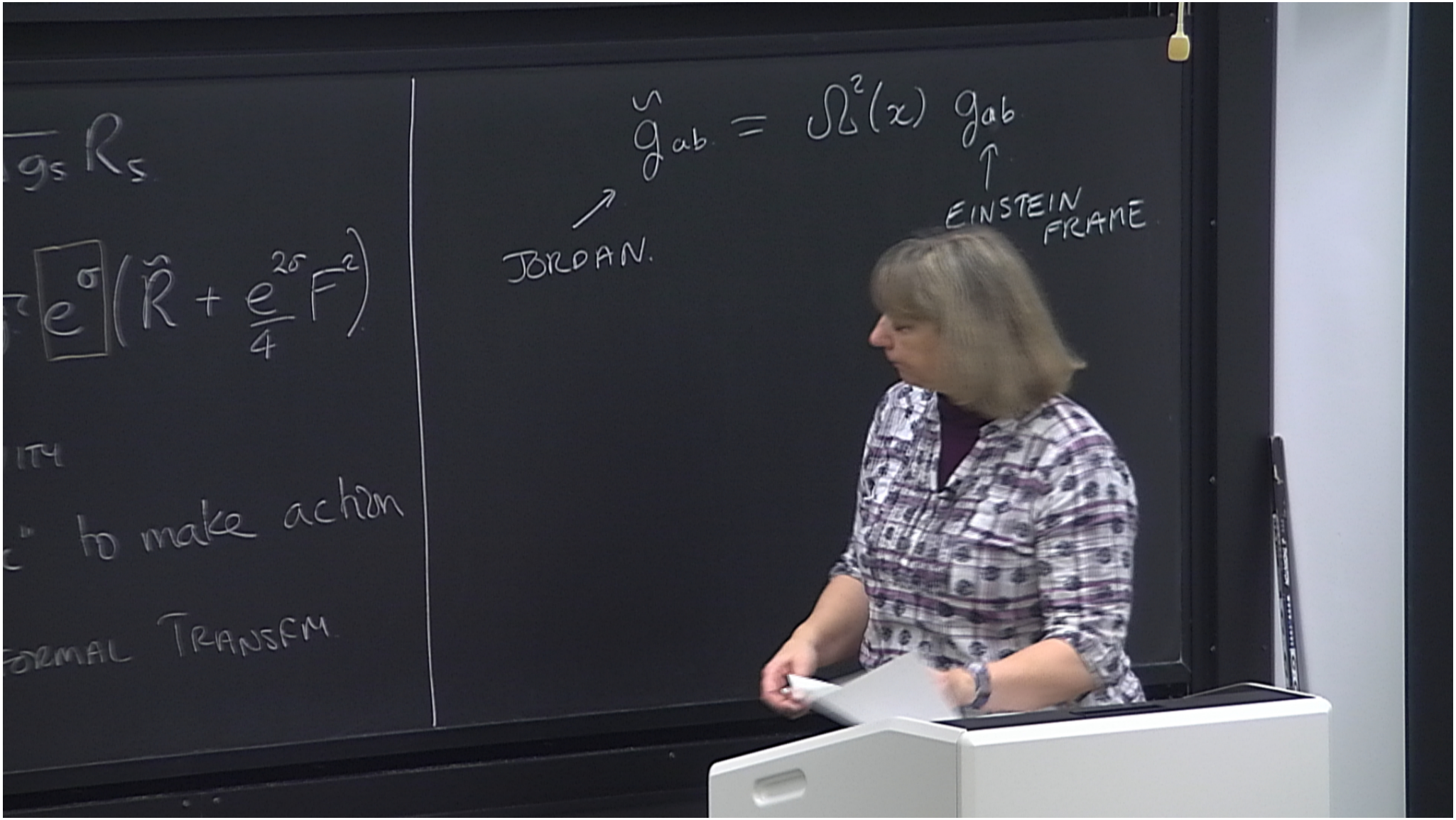
Can change our "metric" to make action look like S_{EH} - CONFORMAL TRANSFM

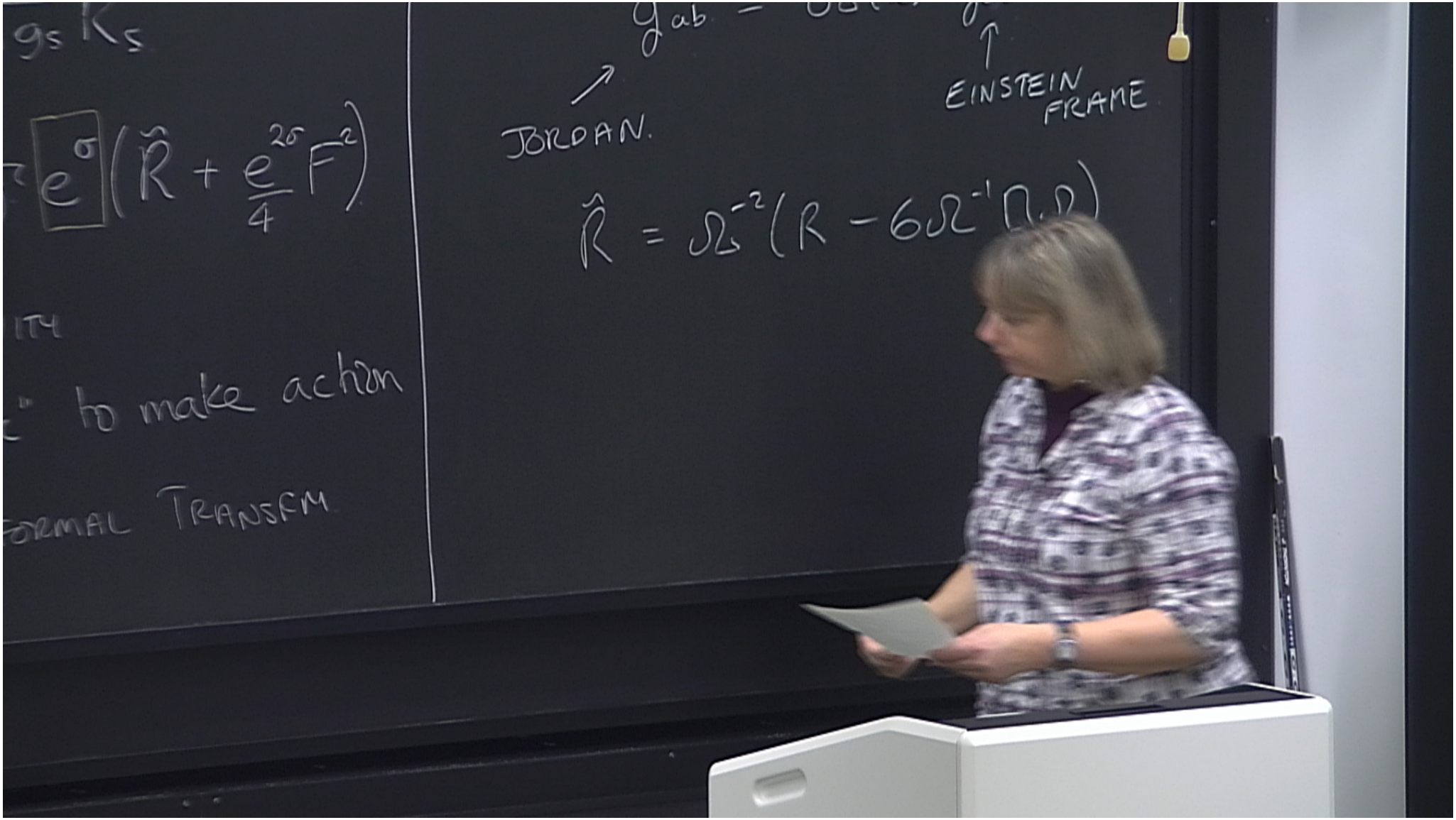
$\tilde{g}_{ab} =$

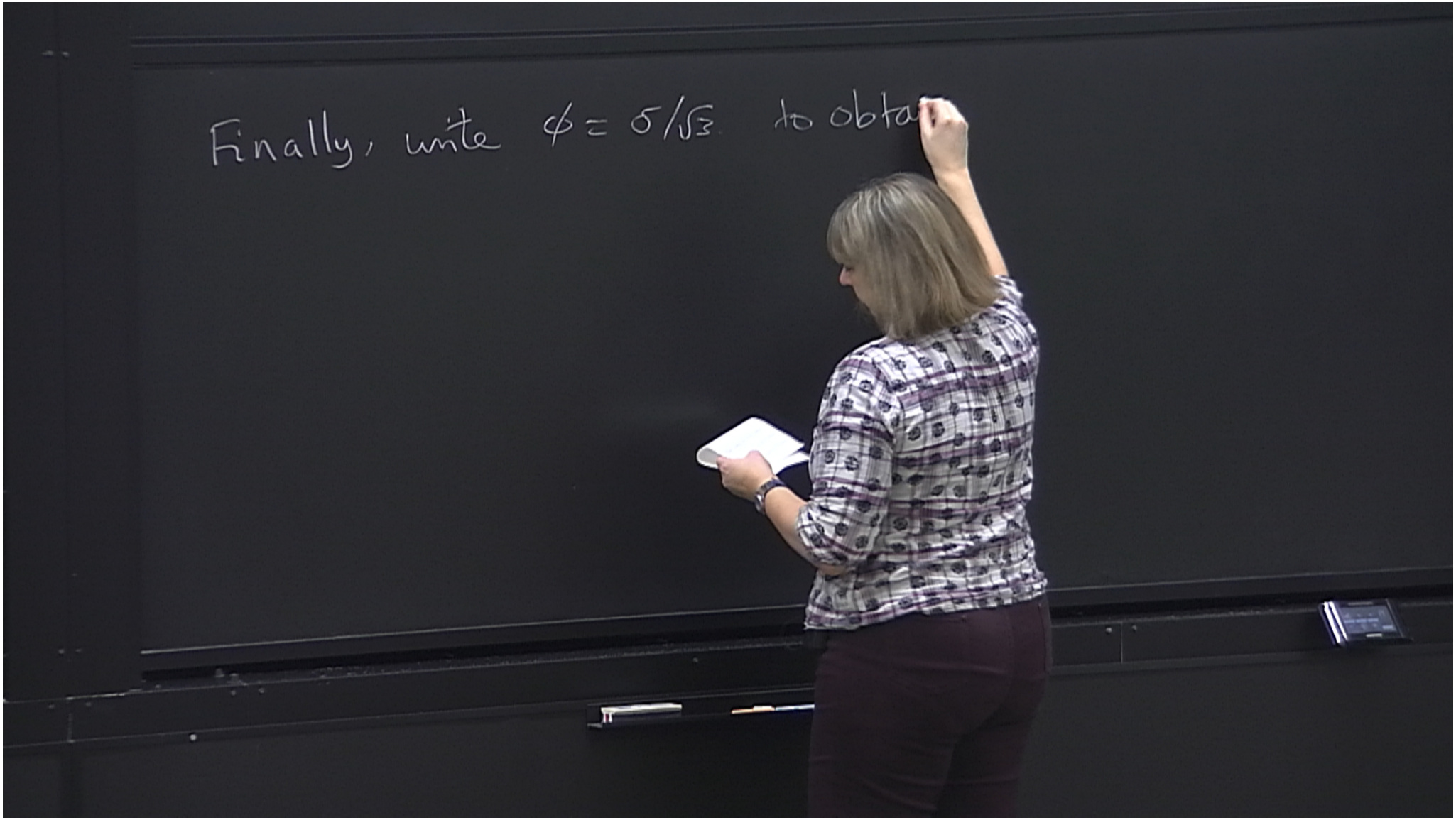
$(\partial\sigma)^2$

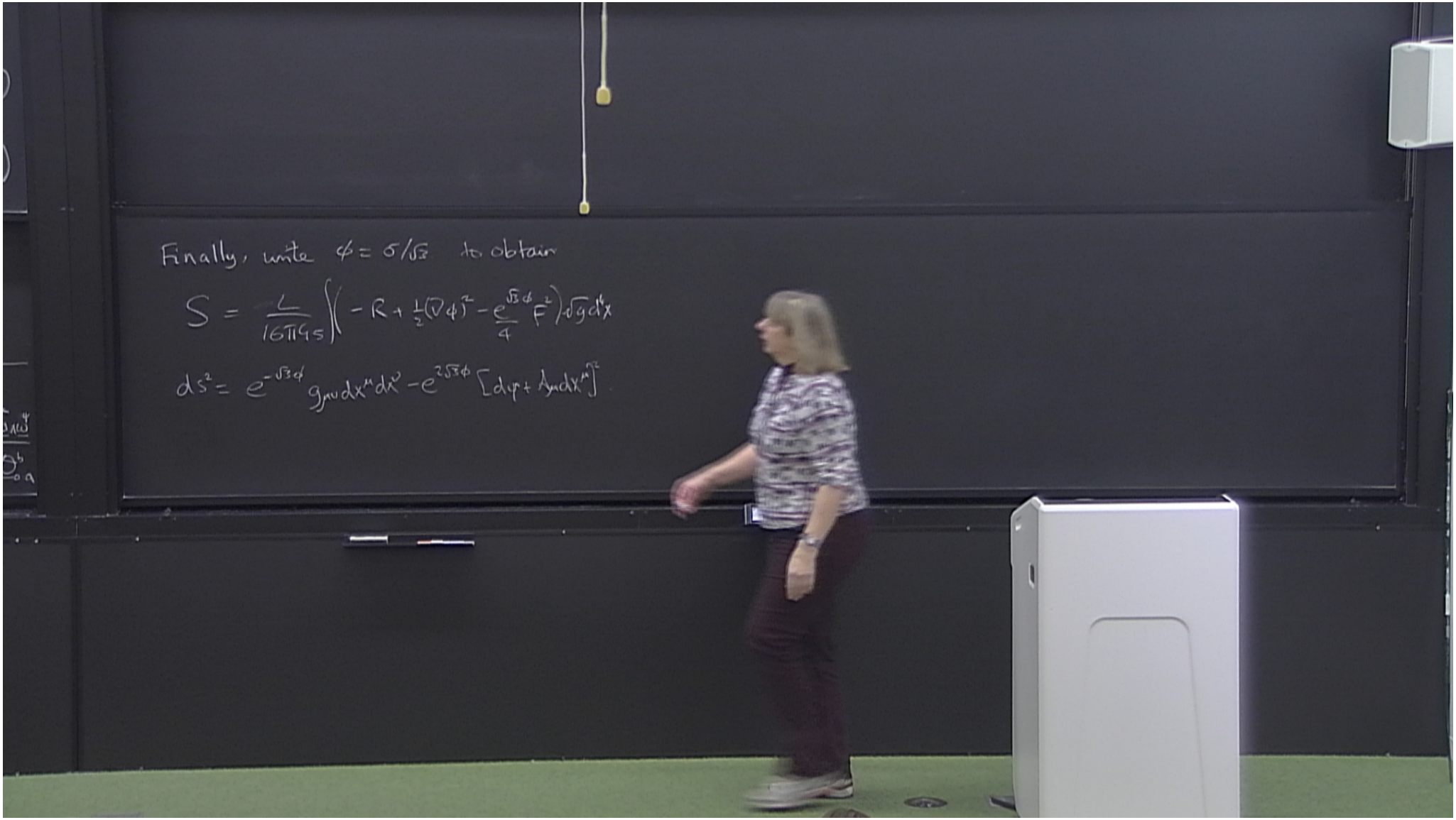
$\Box e^{\sigma}$

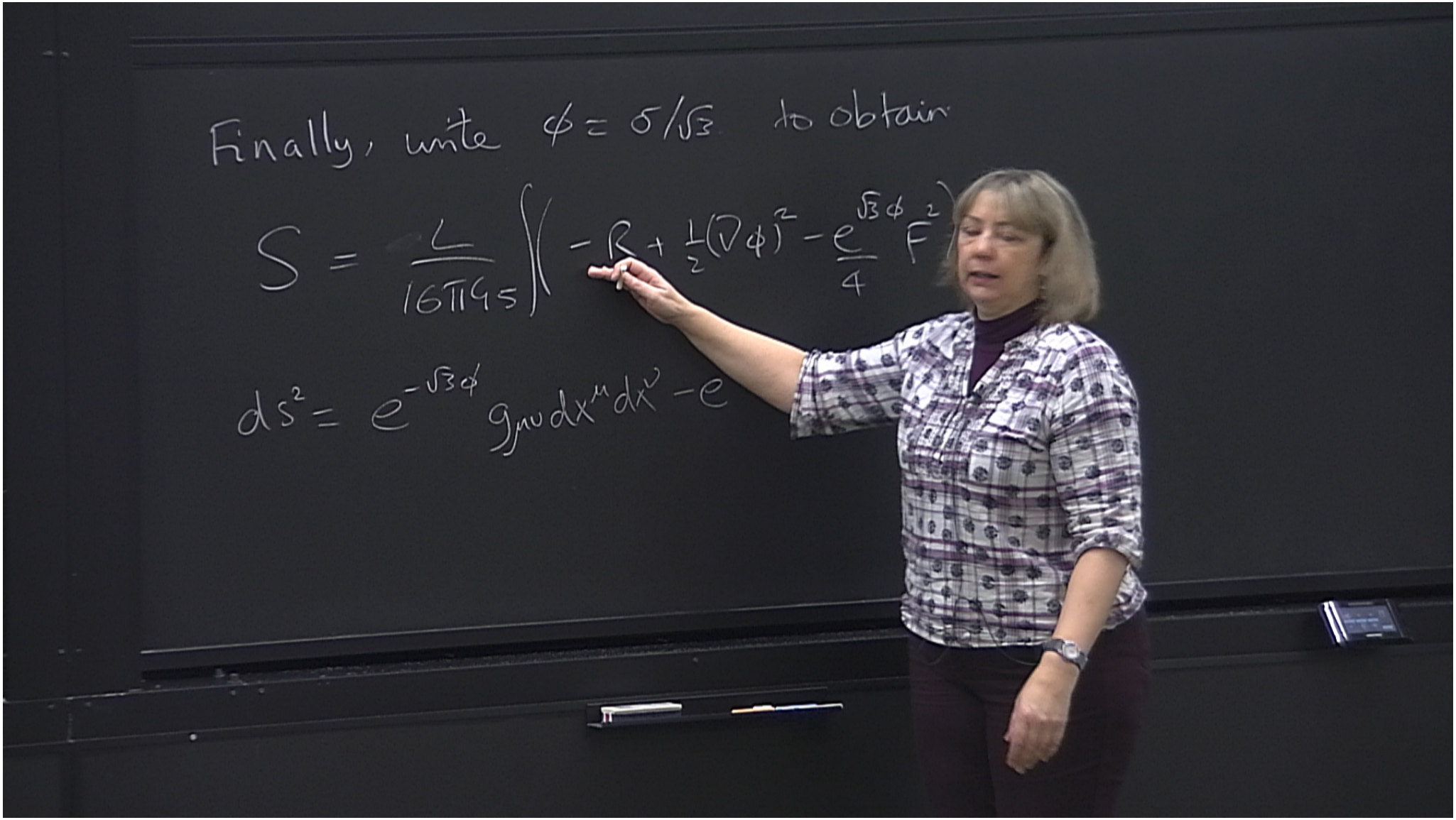

$\hat{g}_{ab} = \Omega^2(x)\, g_{ab}$
$g_5 R_5$
$e^{\sigma}\left(\tilde{R} + \frac{e^{2\sigma}}{4} F^2\right)$
ITY
to make action
FORMAL TRANSFM


$\tilde{g}_{ab} = \Omega^2(x)\, g_{ab}$
JORDAN.
EINSTEIN FRAME
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Finally, write $\phi = \sigma/\sqrt{3}$ to obta


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$S = \frac{1}{16\pi G_5}\int\left(-R + \frac{1}{2}(\nabla\phi)^2 - \frac{e^{\sqrt{3}\phi}}{4}F^2\right)\sqrt{g}\,d^5x$
$ds^2 = e^{-\sqrt{3}\phi}g_{\mu\nu}dx^\mu dx^\nu - e^{2\sqrt{3}\phi}\left[dy + A_\mu dx^\mu\right]^2$


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SPIN 2 MASSLESS · SPINO · SPIN 1

$$ds^2 = e^{-\sqrt{3}\phi}g_{\mu\nu}dx^\mu dx^\nu - e^{2\sqrt{3}\phi}\left[d\varphi + A_\mu dx^\mu\right]^2$$

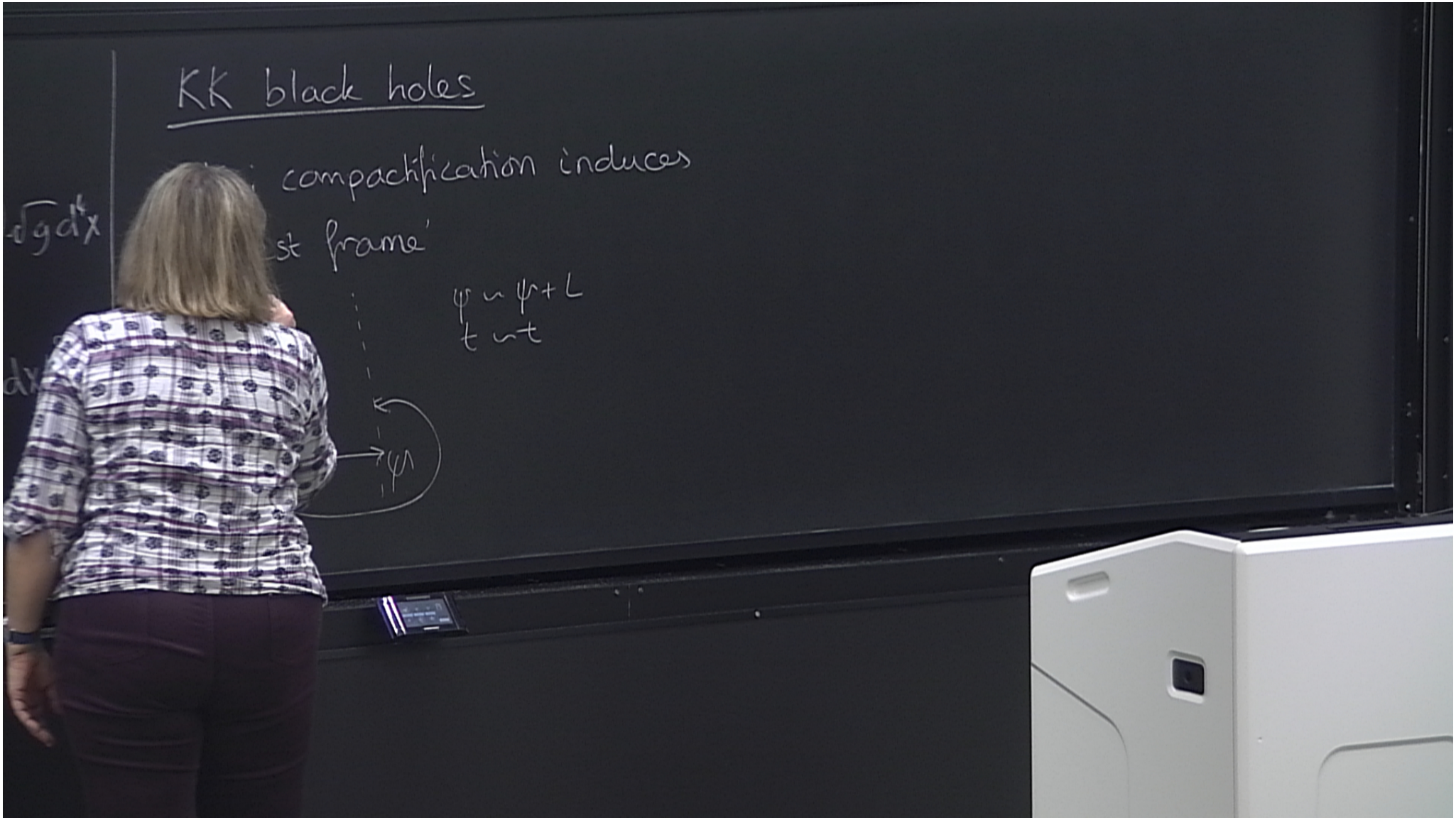
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SPIN 2 MASSLESS · SPIN 0 · SPIN 1

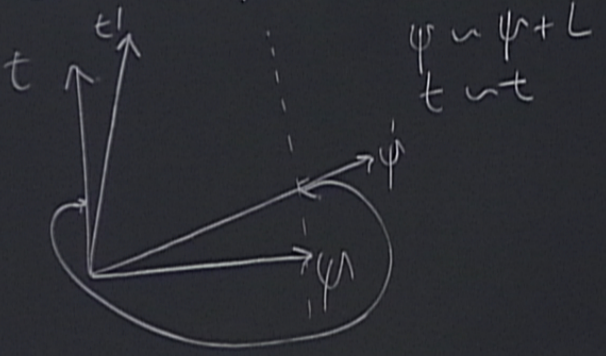
$$ds^2 = e^{-\sqrt{3}\phi}\,g_{\mu\nu}dx^\mu dx^\nu - e^{2\sqrt{3}\phi}\left[d\psi + A_\mu dx^\mu\right]^2$$

Note $L = \int d\psi$


KK black holes
compactification induces
st frame'
$\varphi \sim \varphi + L$
$t \sim t$
$\sqrt{g}d^4x$

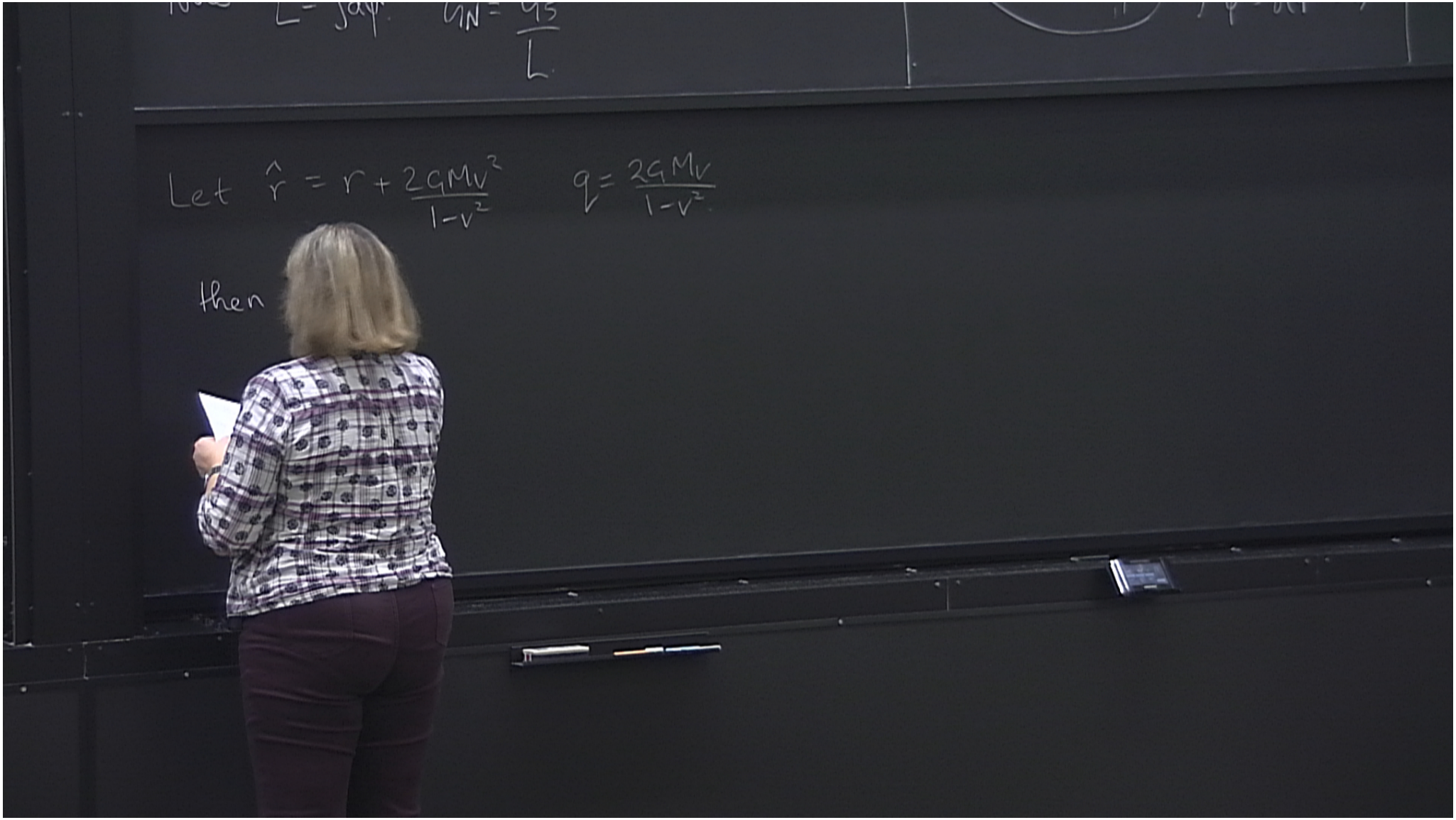
KK black holes

Note: compactification induces a 'rest frame'



$\psi \sim \psi + L$

$t \sim t$


Let $\hat{r} = r + \frac{2GMv^2}{1-v^2}$ $q = \frac{2GMv}{1-v^2}$
then

$L = \int a\varphi \qquad G_N = \frac{G_5}{L}$

Let $\hat{r} = r + \frac{2GMv^2}{1-v^2} \qquad q = \frac{2GMv}{1-v^2}$

then $\underline{A} = \frac{q}{r}\underline{dt} \qquad e^{2\sqrt{3}\phi} = \frac{\hat{r}}{\hat{r} - vq}$

$$ds_4^2 = \left(1 - \frac{2GM}{(1-v^2)\hat{r}}\right)\left(1 - \frac{vq}{\hat{r}}\right)^{-1/2} dt^2 - \left(1 - \frac{vq}{\hat{r}}\right)^{3/2} \hat{r}^2 \, d\Omega_{II}^2$$

$$- \left(\quad " \quad \right)^{-1} \left(\quad " \quad \right)^{+1/2} dr^2$$

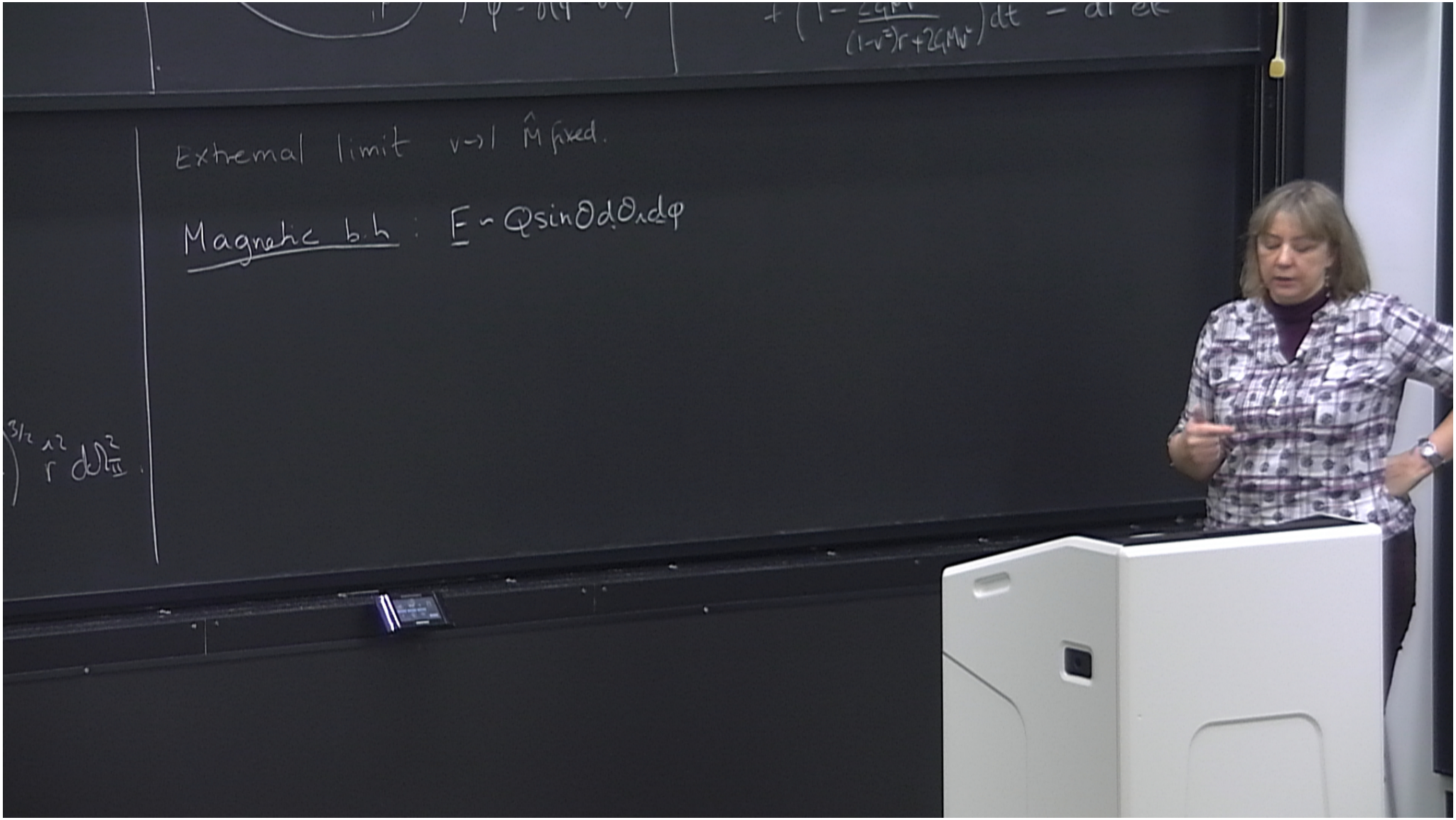
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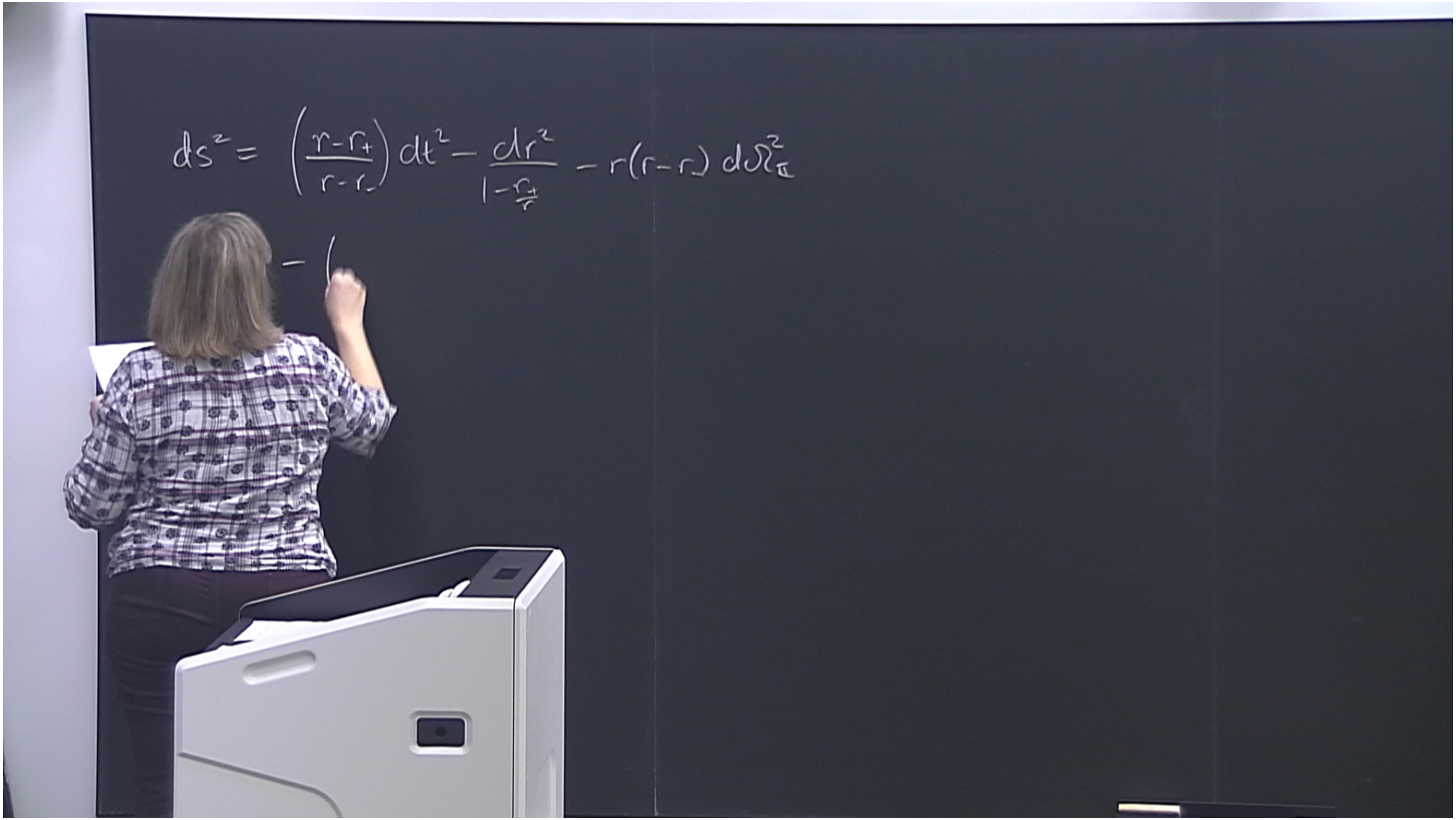
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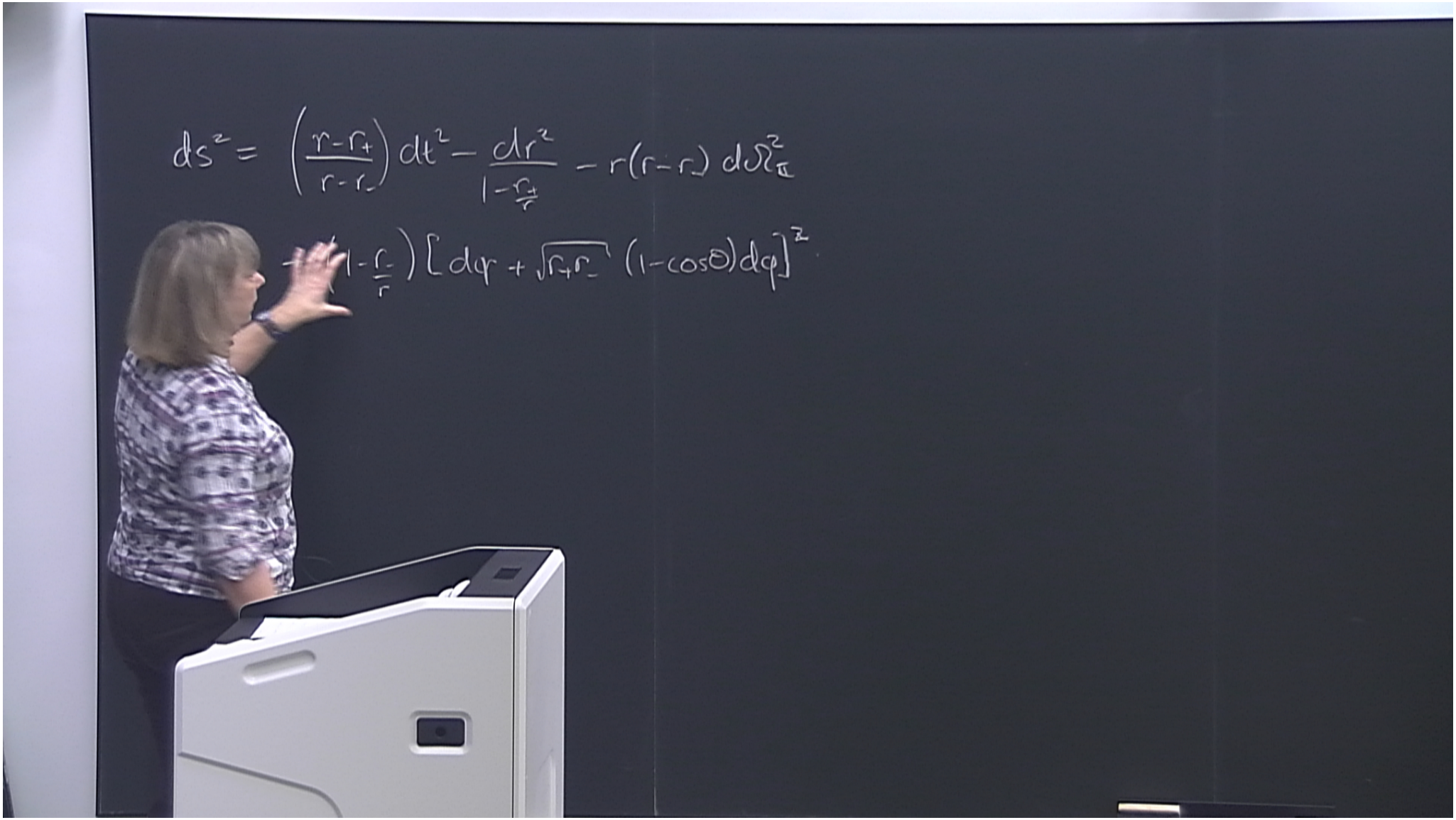
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Extremal limit $r \to l$ $\hat{M}$ fixed.
Magnetic b.h : $E \sim Q \sin\theta d\theta_1 d\varphi$

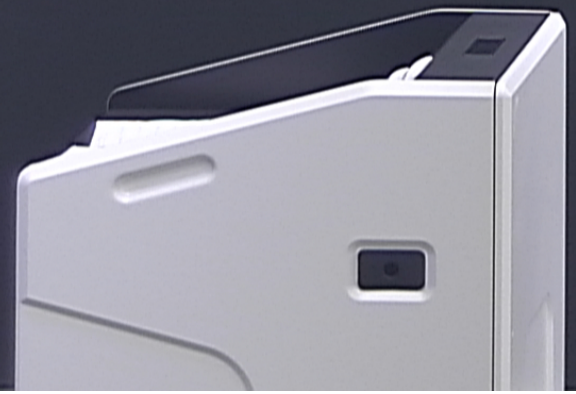

$ds^2 = \left(\frac{r-r_+}{r-r_-}\right) dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - r(r-r_-)\, d\Omega_2^2$
$-($

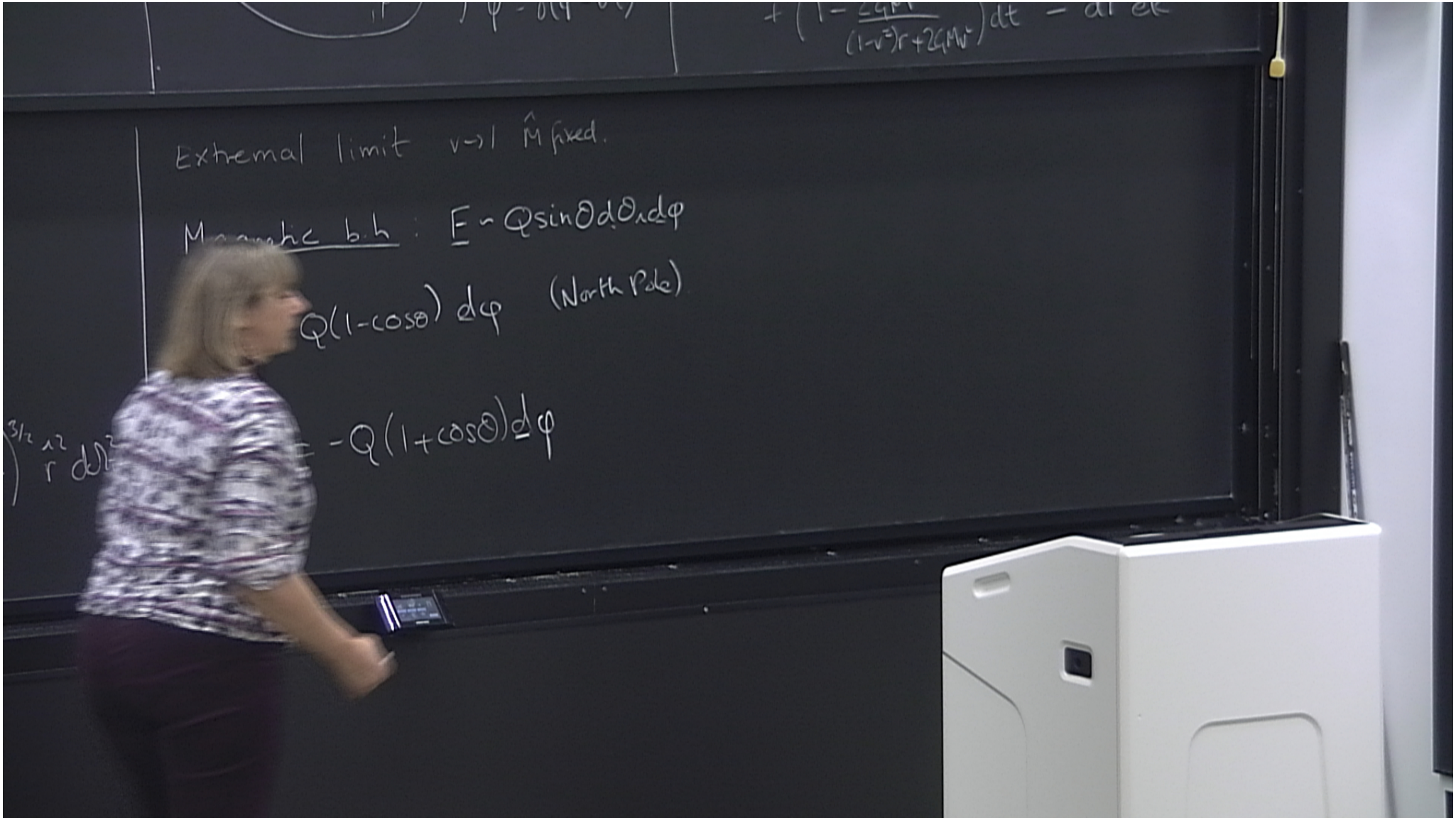

$ds^2 = \left(\frac{r-r_+}{r-r_-}\right) dt^2 - \frac{dr^2}{1-\frac{r_+}{r}} - r(r-r_-)\, d\Omega_{II}^2$
$\left(1-\frac{r_-}{r}\right)\left[dq + \sqrt{r_+ r_-}\,(1-\cos\theta)\, d\varphi\right]^2$

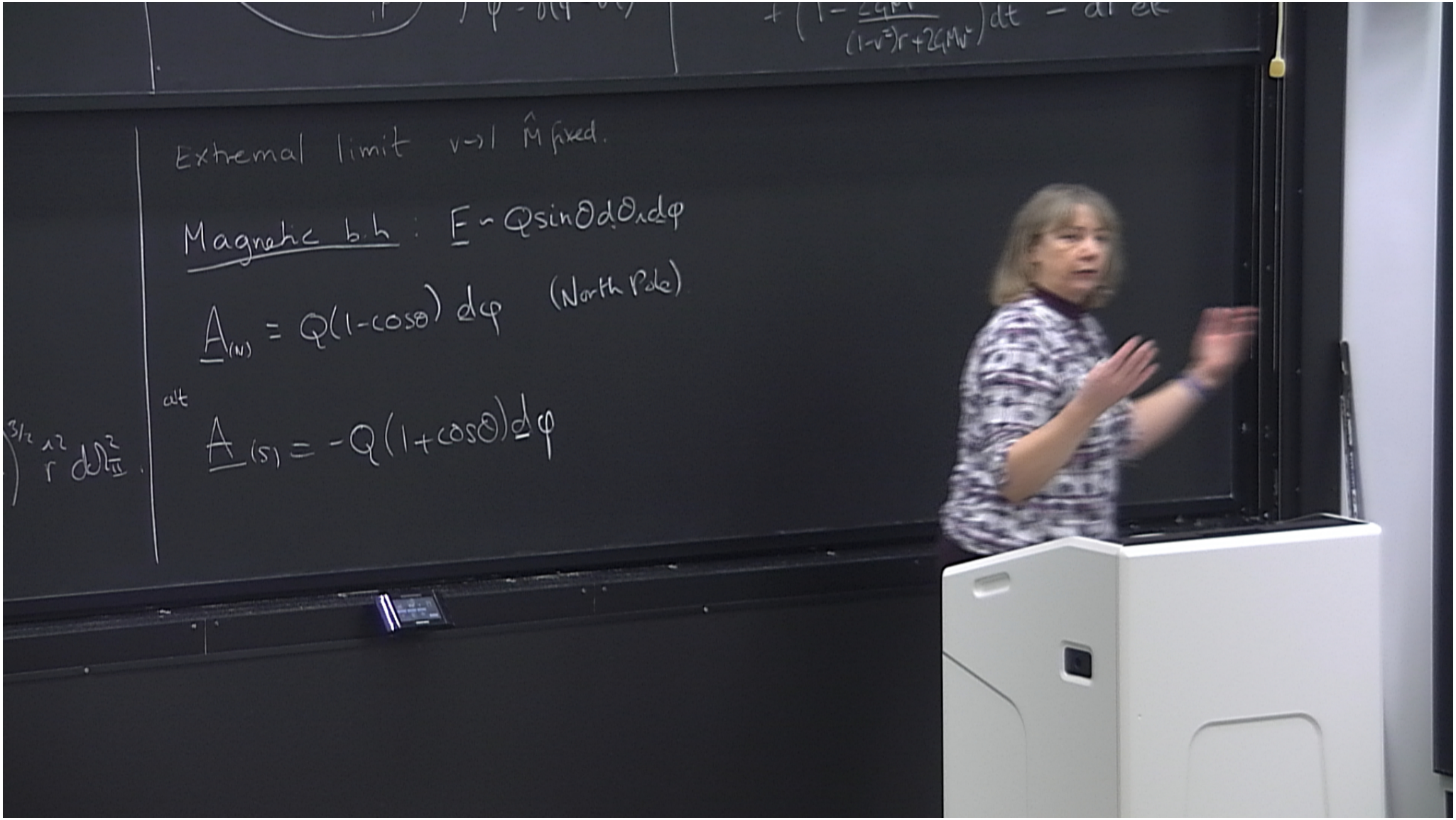
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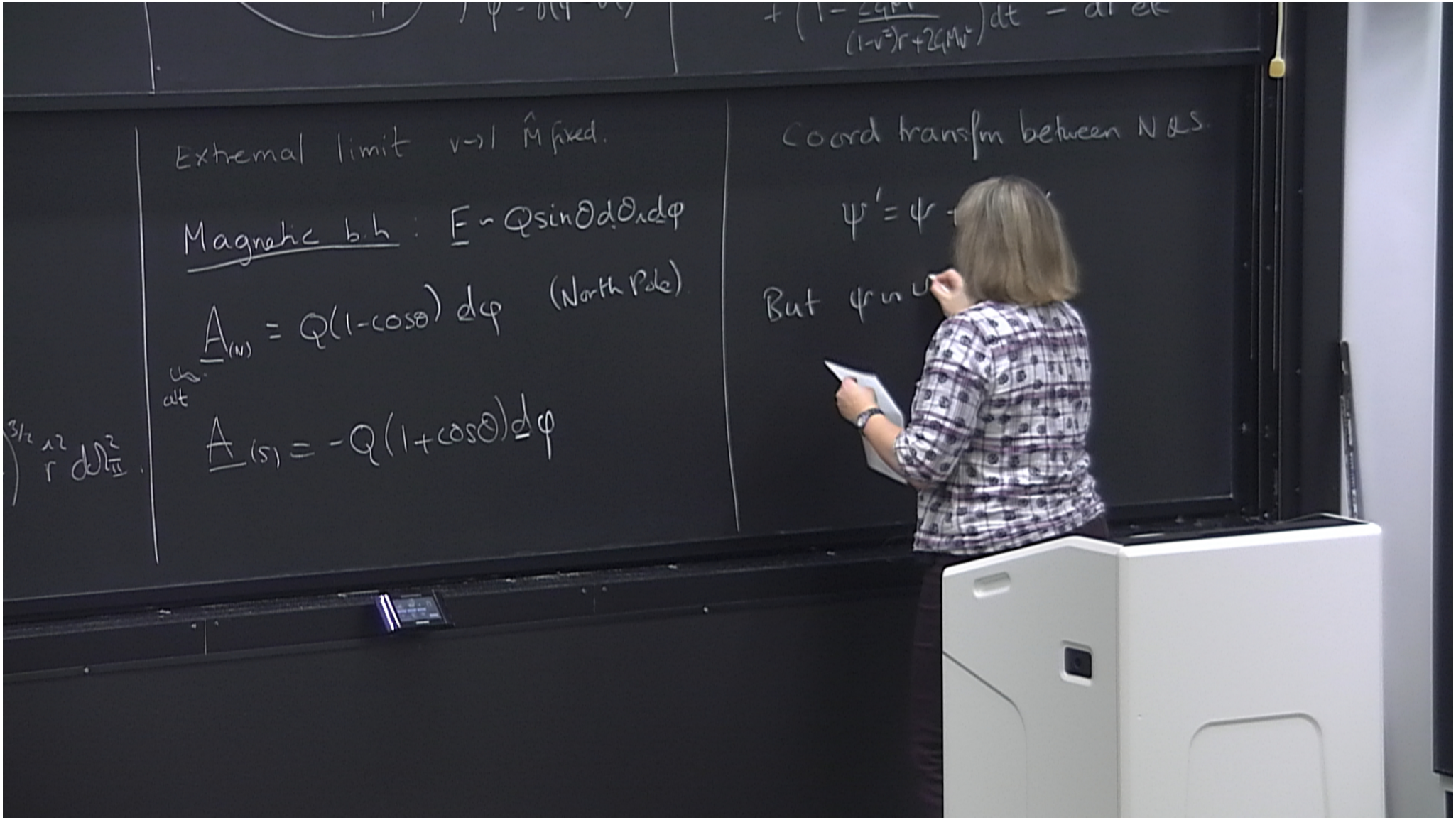
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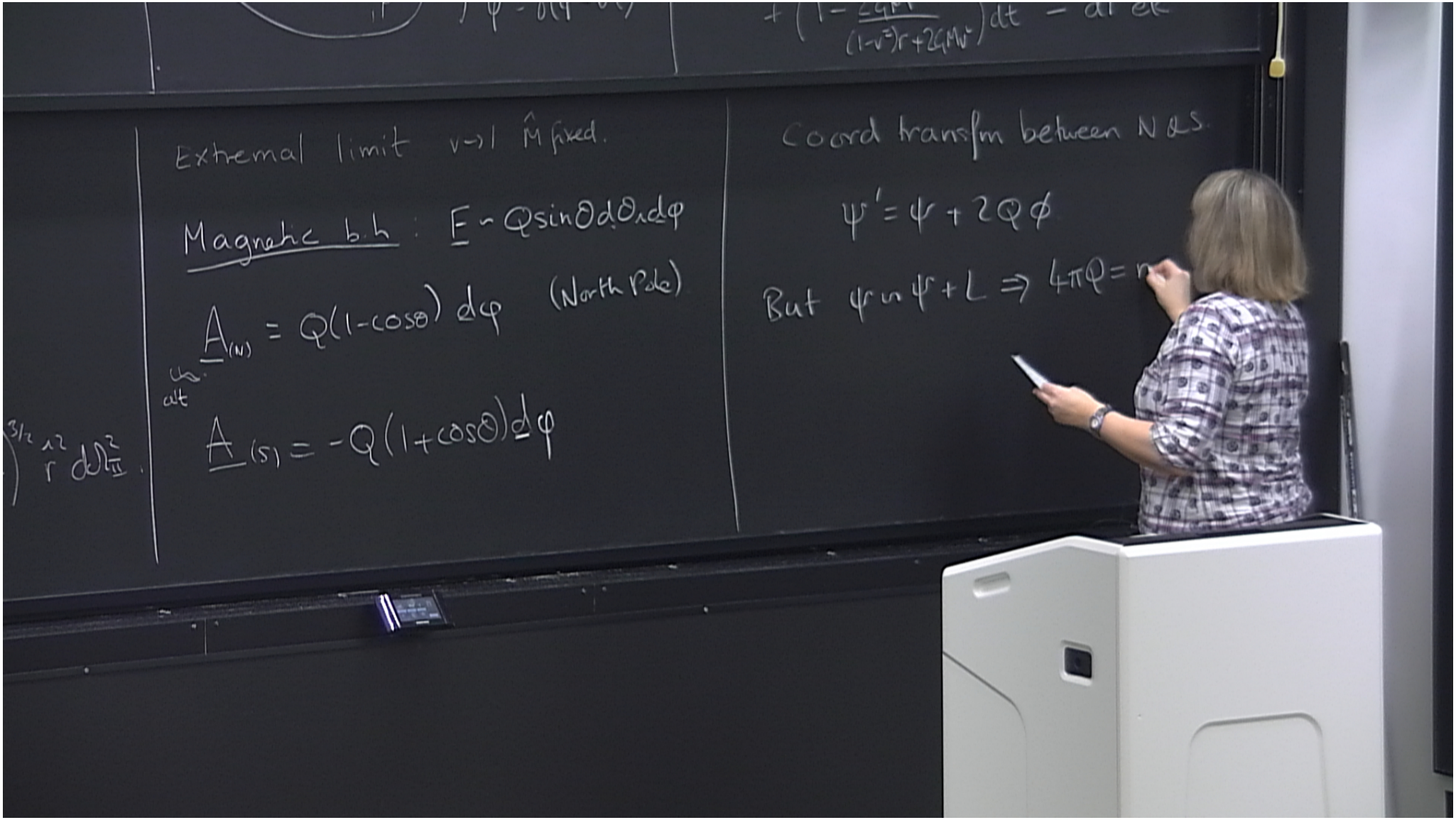
$\underline{A}$ is part of geometry, metric must be regular at poles

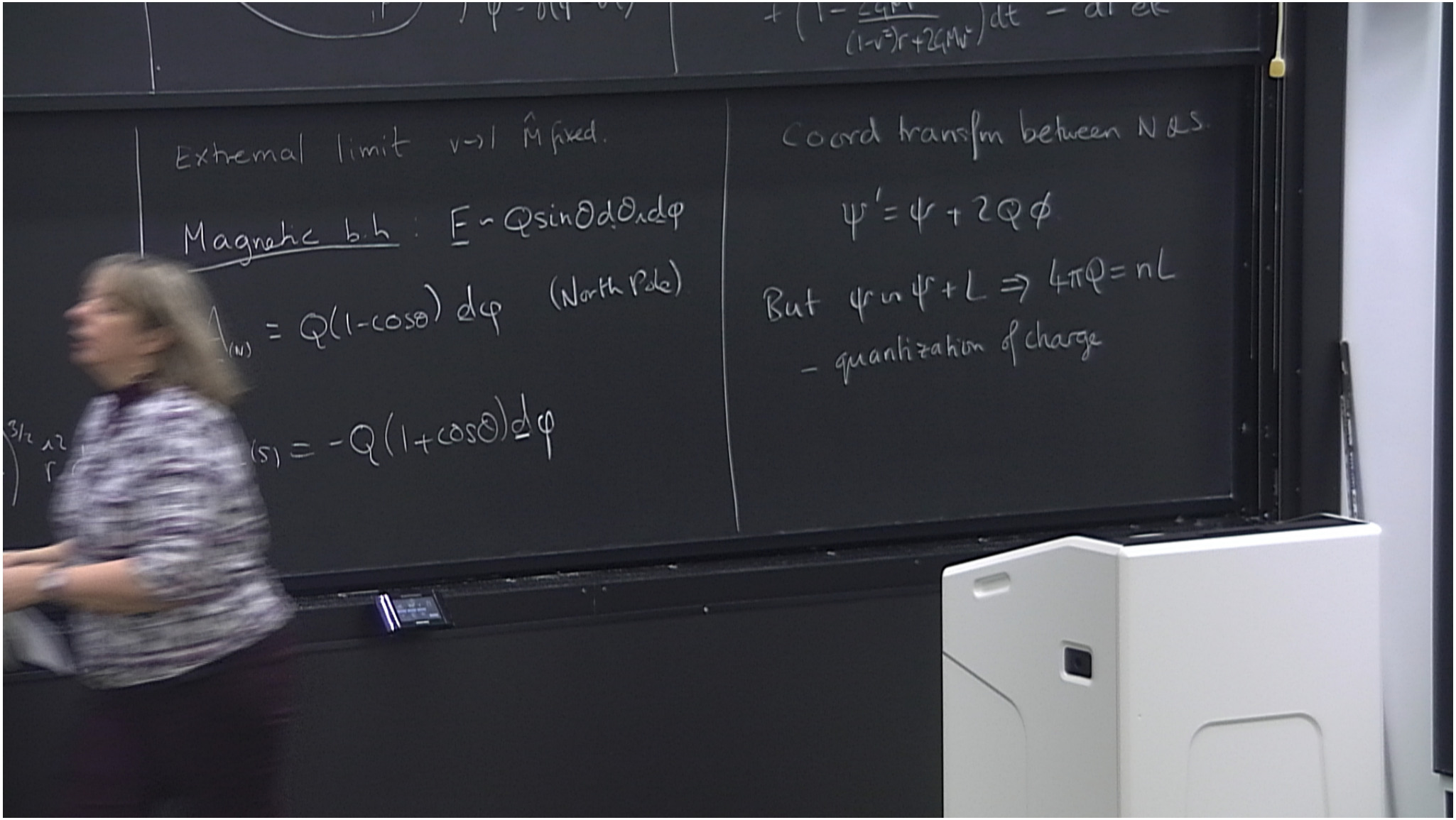



Extremal limit v->1 $\hat{M}$ fixed.
Magnetic b.h : $E \sim Q \sin\theta d\theta \wedge d\varphi$
$Q(1-\cos\theta)\, d\varphi$ (North Pole)
$-Q(1+\cos\theta) d\varphi$

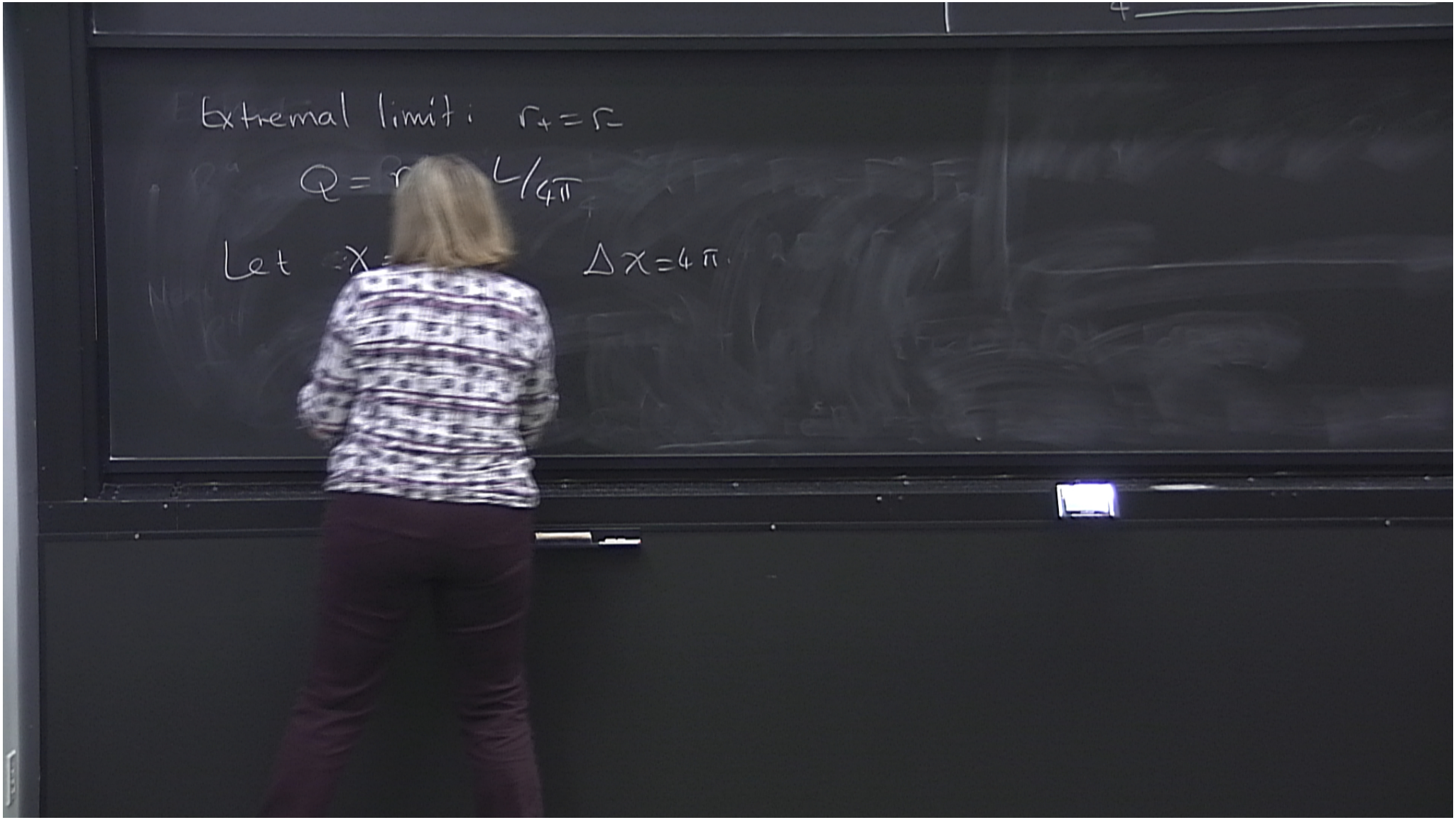

Extremal limit $v \to 1$ $\hat{M}$ fixed.
Magnetic b.h : $E \sim Q \sin\theta d\theta \wedge d\varphi$
$A_{(N)} = Q(1-\cos\theta)\, d\varphi$ (North Pole)
at
$A_{(S)} = -Q(1+\cos\theta) d\varphi$

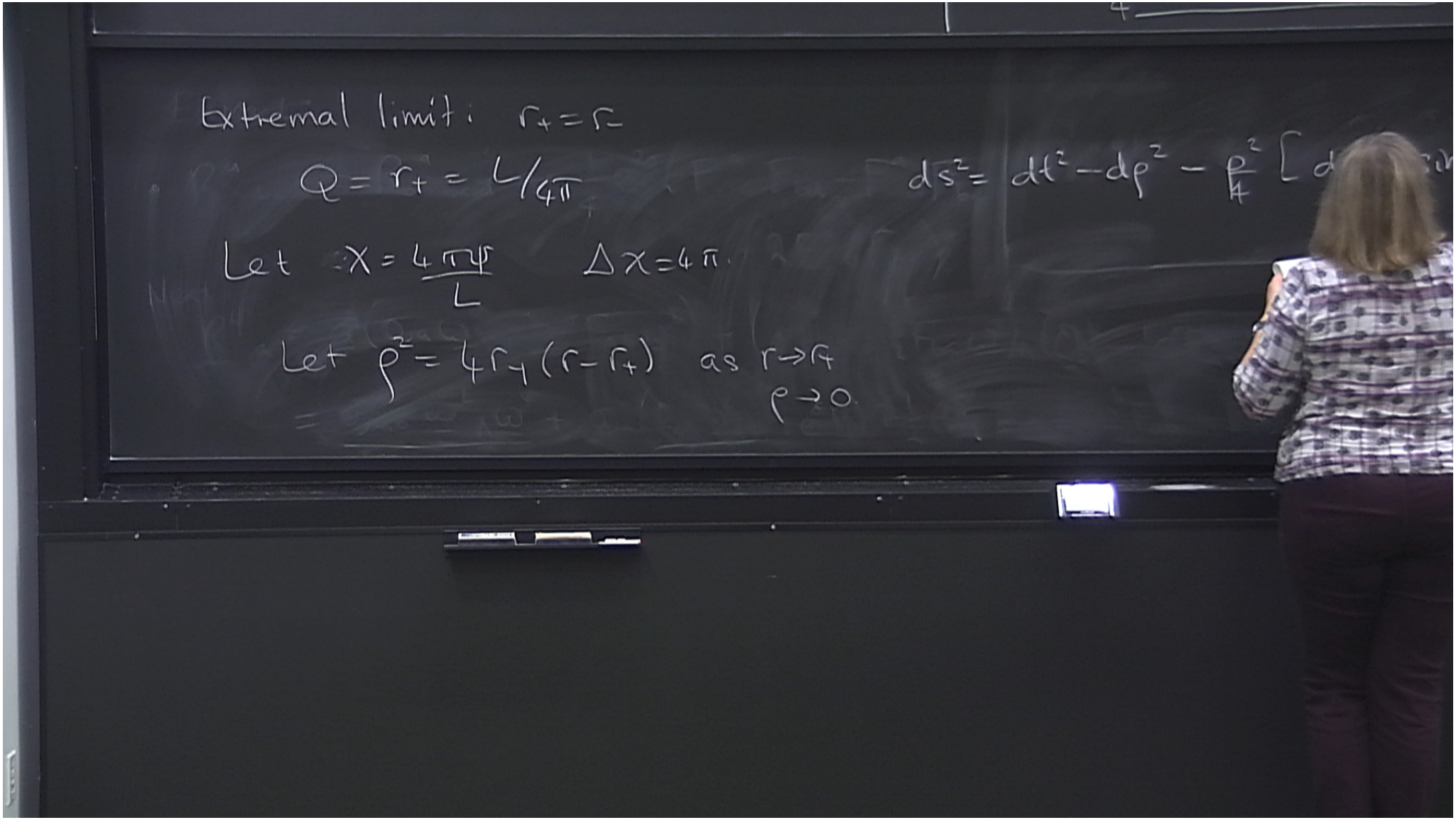

Extremal limit $r \to l$ $\hat{M}$ fixed.
Magnetic b.h : $\underline{F} \sim Q \sin\theta \, d\theta \wedge d\varphi$
$\underline{A}_{(N)} = Q(1-\cos\theta)\, d\varphi$ (North Pole)
$\underline{A}_{(S)} = -Q(1+\cos\theta)\, d\varphi$
Coord transf/m between N & S
$\psi' = \psi$
But ψ

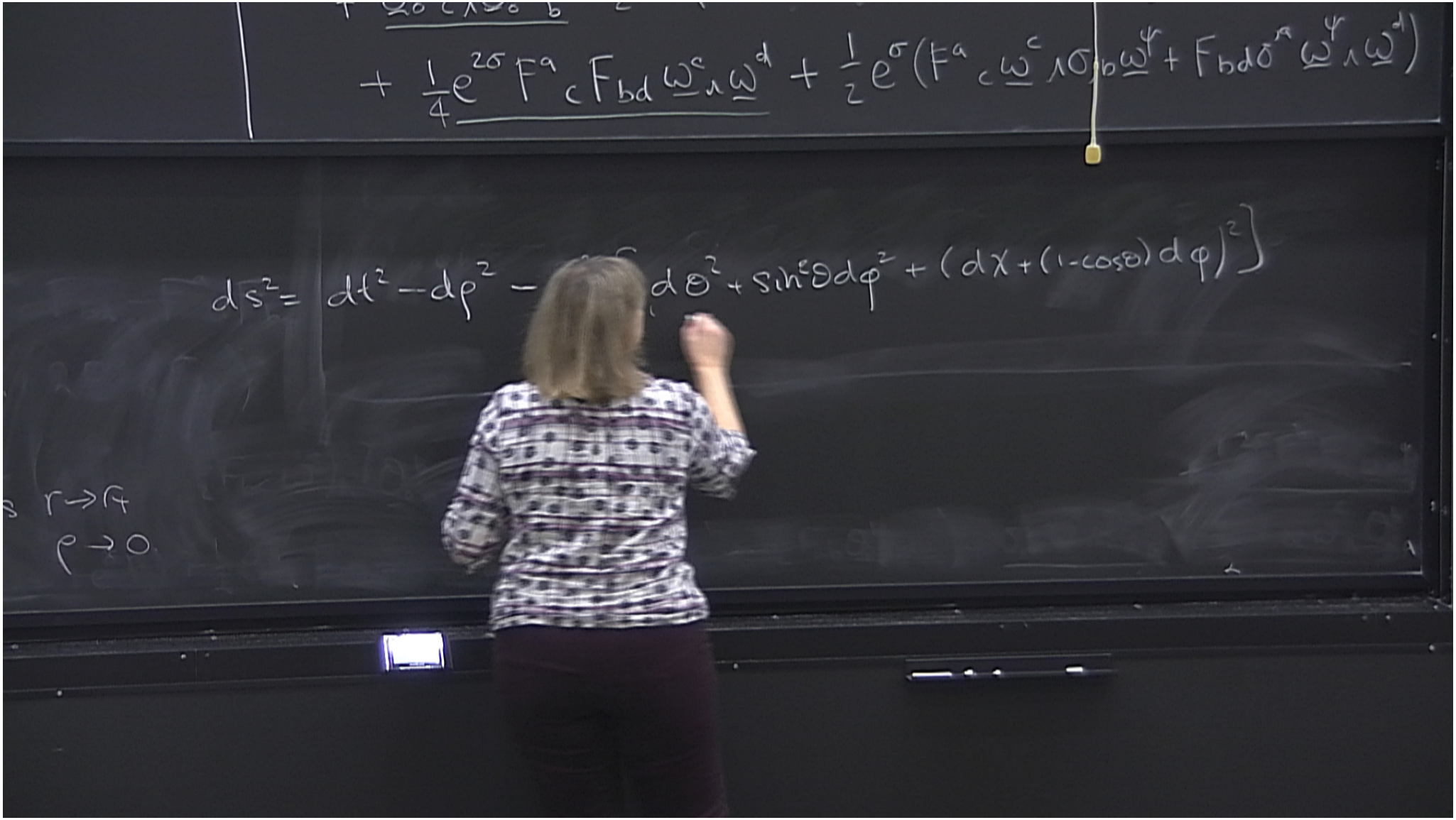

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Coord transf/m between N & S.
$\psi' = \psi + 2Q\phi$
But $\psi \sim \psi + L \Rightarrow 4\pi Q =$


Extremal limit $r \to l$ $\hat{M}$ fixed.
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Coord transf/m between N & S.
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But $\psi \sim \psi + L \Rightarrow 4\pi Q = nL$
- quantization of charge


Extremal limit: $r_+ = r_-$
$Q = r_+ = L/4\pi$
Let $\chi = \frac{4\pi\varphi}{L}$
$\chi = 4\pi$


Extremal limit: $r_+ = r_-$
$Q = r$ $L/4\pi$
Let χ
$\Delta\chi = 4\pi$


Extremal limit: $r_+ = r_-$
$Q = r_+ = L/4\pi$
$ds^2 = dt^2 - d\rho^2 - \frac{\rho^2}{4}[d$
Let $\chi = \frac{4\pi y}{L}$ $\Delta\chi = 4\pi$
Let $\rho^2 = 4r_+(r - r_+)$ as $r \to r_+$
$\rho \to 0$


$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}(F^{a}{}_{c}\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{a}\underline{\omega}^{b}\wedge\underline{\omega}^{d})$
$ds^2=dt^2-d\rho^2-\ldots d\theta^2+\sin^2\theta d\varphi^2+(d\chi+(1-\cos\theta)d\varphi)^2\}$
$r\to r_+$
$\rho\to 0$

$$+\frac{1}{4}e^{2\sigma}F^{a}{}_{c}F_{bd}\,\underline{\omega}^{c}\wedge\underline{\omega}^{d}+\frac{1}{2}e^{\sigma}\left(F^{a}{}_{c}\,\underline{\omega}^{c}\wedge\sigma_{,b}\underline{\omega}^{b}+F_{bd}\delta^{ac}\,\underline{\omega}^{b}\wedge\underline{\omega}^{d}\right)$$

$$ds^2=dt^2-d\rho^2-\frac{\rho^2}{4}\Big[\underbrace{d\theta^2+\sin^2\theta\,d\varphi^2}_{S^2}+\underbrace{(d\chi+(1-\cos\theta)\,d\varphi)^2}_{S^1}\Big]$$

S^3 in Euler angles

$r\to r_+$

$\rho\to 0$