Title: Gravitational Physics-10

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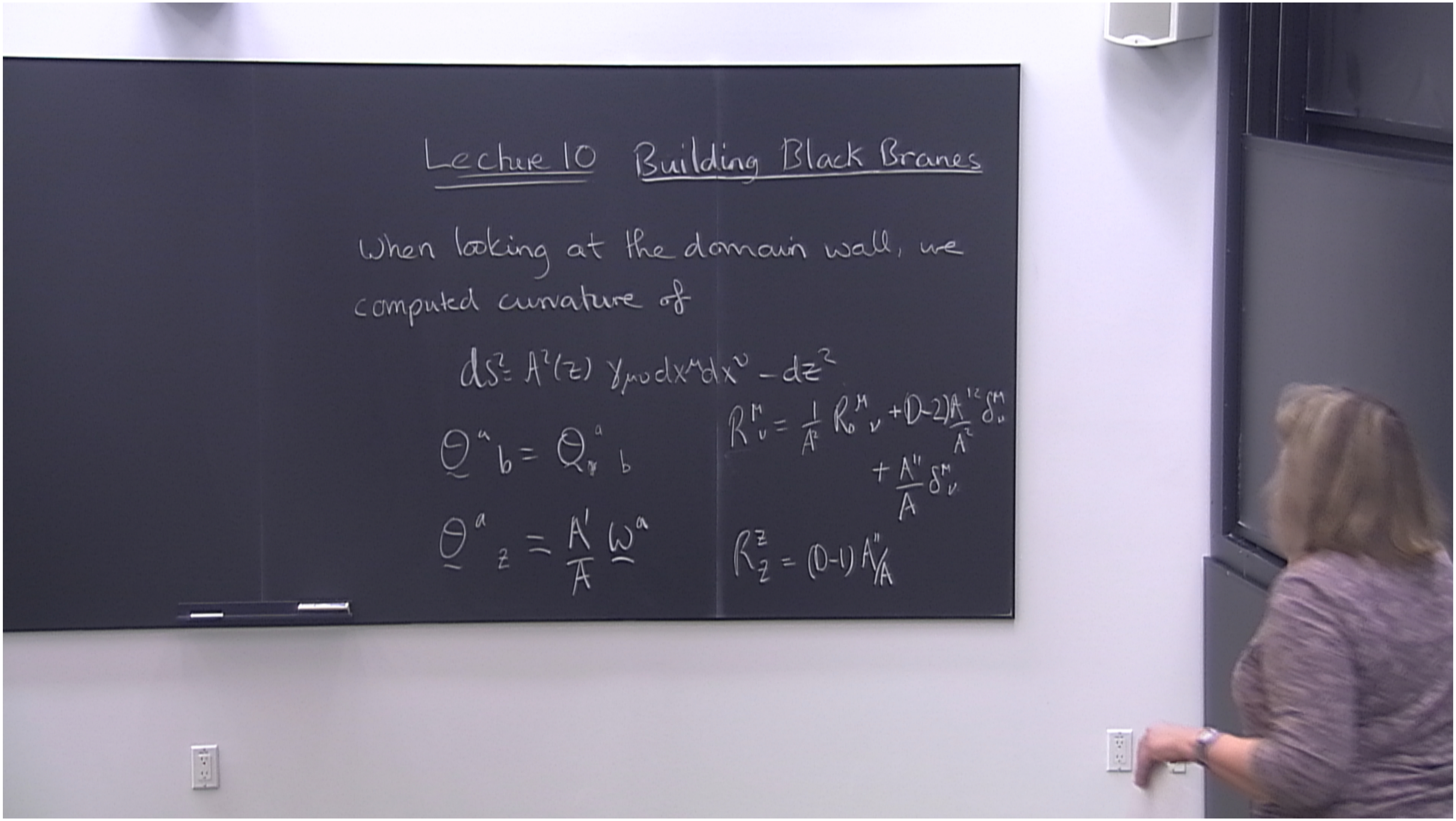
Abstract:


Lecture 10 Building Black Branes
When looking at the domain wall, we computed curvature of
$ds^2 = A^2(z)\, \gamma_{\mu\nu} dx^\mu dx^\nu - dz^2$
$\underset{\sim}{\Theta}^a{}_b = \underset{\sim}{\Theta}_\gamma{}^a{}_b$
$\underset{\sim}{\Theta}^a{}_z = \frac{A'}{A} \underset{\sim}{\omega}^a$
$R^\mu{}_\nu = \frac{1}{A^2} R_0{}^\mu{}_\nu + (D-2)\frac{A'^2}{A^2}\delta^\mu_\nu + \frac{A''}{A}\delta^\mu_\nu$
$R^z_z = (D-1) A''/A$

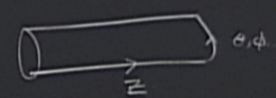
e.g. if $A \equiv 1$

$$R^{z}{}_{z} = 0 \qquad R^{\mu}{}_{\nu} = \frac{1}{A^2} R_0{}^{\mu}{}_{\nu}$$

If $\gamma_{\mu\nu}$ a soln to vacuum Einstein eqns then so is higher dim^{l} metric

<u>Black String</u> SCH $\times \mathbb{R}$

$$ds^2 = \left(1 - \frac{2GM}{r}\right)dt^2 - \frac{dr^2}{1 - \frac{2GM}{r}} - r^2 d\Omega_2^2 - dz^2$$



then have a <u>black brane</u>

let $dz^2 \to d\vec{z}^{\,2} \; (\mathbb{R}^n)$

$\frac{A'}{A}\,\omega^{\hat{\mu}}$ $\quad \Theta^{\hat{\lambda}}{}_{\hat{\tau}} = \frac{C'}{C}\,\underline{\omega}^{\hat{\lambda}}$

rms are same/similar

$\underline{R}_{\gamma}{}^{\hat{\lambda}}{}_{\hat{\beta}} + \left(\frac{C'}{C}\right)^2 \underline{\omega}^{\hat{\lambda}} \wedge \underline{\omega}_{\hat{\beta}}$ $+ \hat{\mu},\hat{\beta}$

$\frac{C''}{C}\,\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\lambda}}$

plus $\underline{R}^{\hat{\mu}}{}_{\hat{\lambda}} = \Theta^{\hat{\mu}}{}_{\hat{r}} \wedge \underline{\Theta}^{\hat{r}}{}_{\hat{\lambda}}$

$= \frac{A'C'}{AC}\,\underline{\omega}^{\hat{\mu}} \wedge \underline{\omega}_{\hat{\lambda}}$

Riemann cpts

$$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g{}^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2\left(\delta^\mu{}_\lambda \delta^\nu{}_\tau - \delta^\nu{}_\lambda \delta^\mu{}_\tau\right)$$

$$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_\gamma{}^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2\left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right)$$

$R^{\mu r}{}_{\nu r} = \frac{A''}{A}\,\delta^\mu{}_\nu$

$R^{\alpha r}{}_{\beta r} = \frac{C''}{C}\,\delta^\alpha_\beta$

Plus $R^{\mu\alpha}{}_{\nu\beta} = \frac{A'C'}{AC}\,\delta^\mu{}_\nu\,\delta^\alpha_\beta$

$$\underline{R}^{\hat\alpha}{}_{\hat\beta} = \underline{R}_{\tilde g}{}^{\hat\alpha}{}_{\hat\beta} + \left(\frac{C'}{C}\right)^2 \underline{\omega}^{\hat\alpha} \wedge \underline{\omega}_{\hat\beta}$$

$$\underline{R}^{\hat a}{}_{\hat r} = \frac{C''}{C}\,\underline{\omega}^{\hat r} \wedge \underline{\omega}^{\hat a}$$

$+\hat\mu, \hat\rho$

$$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g{}^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left(\delta^\mu{}_\lambda \delta^\nu{}_\tau - \delta^\nu{}_\lambda \delta^\mu{}_\tau\right)$$

$$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_{\tilde g}{}^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 \left(\delta^\alpha{}_\gamma \delta^\beta{}_\delta - \delta^\alpha{}_\delta \delta^\beta{}_\gamma\right)$$

Hence

$$R^\mu{}_\nu = A^{-2} R_g{}^\mu{}_\nu + \left(\frac{A''}{A} + (p-1)\frac{A'^2}{A^2} + n\frac{A'C'}{AC}\right)\delta^\mu_\nu$$

$$R^\alpha{}_\beta = C^{-2} R_{\tilde g}{}^\alpha{}_\beta + \left(\frac{C''}{C} + (n-1)\frac{C'^2}{C^2} + p\frac{A'C'}{AC}\right)\delta^\alpha_\beta$$

$$R^r{}_r = p\frac{A''}{A} + n\frac{C''}{C}$$

$$\underline{R}^{\hat{\alpha}}{}_{\hat{r}} = \frac{C''}{C}\,\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\alpha}}$$

$$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left(\delta^\mu{}_\lambda \delta^\nu{}_\tau - \delta^\nu_\lambda \delta^\mu_\tau\right)$$

$$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_\gamma^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 \left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right)$$

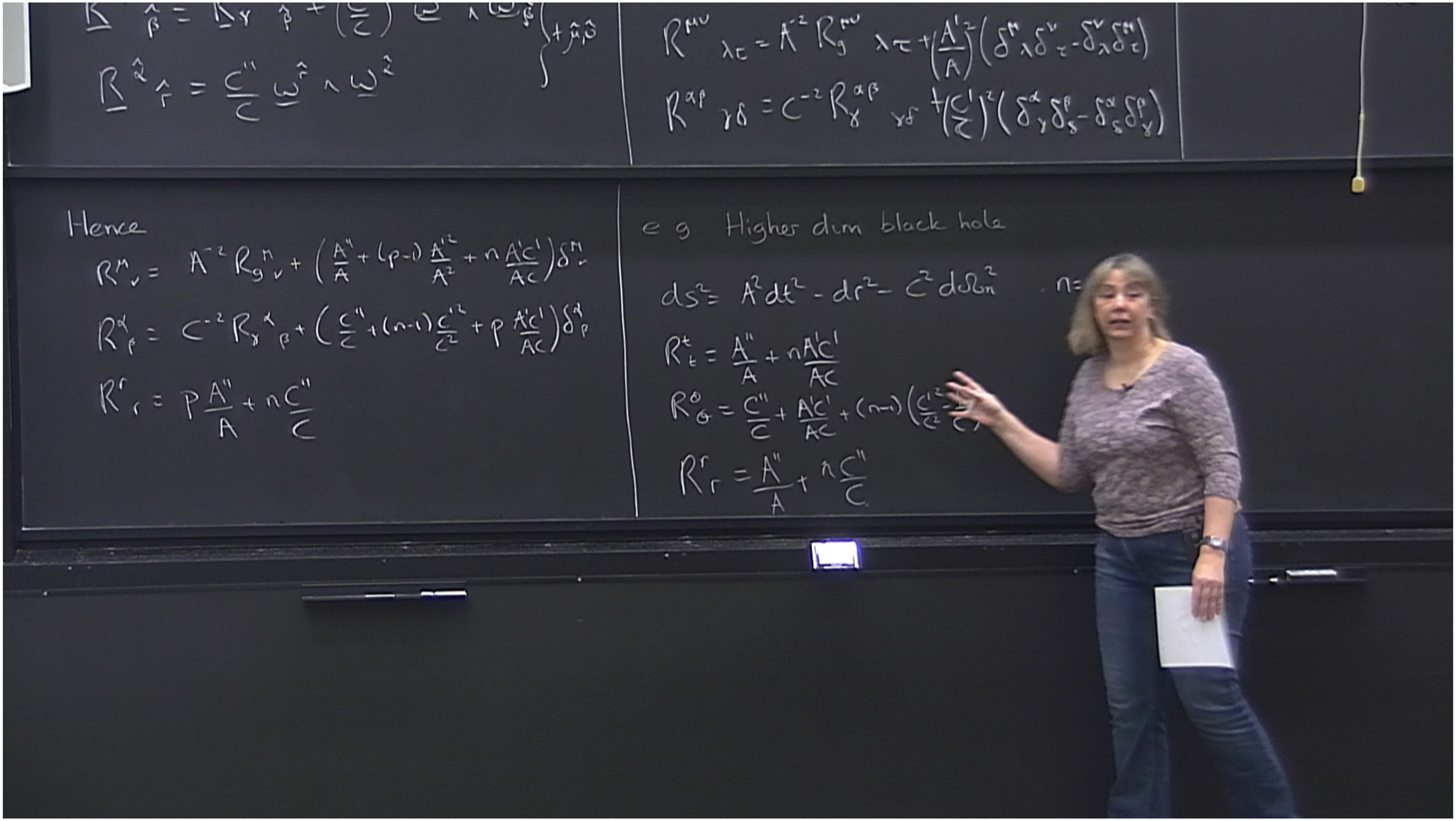
Hence

$$R^\mu{}_\nu = A^{-2} R_g{}^\mu{}_\nu + \left(\frac{A''}{A} + (p-1)\frac{A'^2}{A^2} + n\frac{A'C'}{AC}\right)\delta^\mu_\nu$$

$$R^\alpha{}_\beta = C^{-2} R_\gamma{}^\alpha{}_\beta + \left(\frac{C''}{C} + (n-1)\frac{C'^2}{C^2} + p\frac{A'C'}{AC}\right)\delta^\alpha_\beta$$

$$R^r{}_r = p\frac{A''}{A} + n\frac{C''}{C}$$

e g Higher dim black hole


$R^{\hat\alpha}{}_{\hat r} = \frac{C''}{C}\, \omega^{\hat r} \wedge \omega^{\hat\alpha}$
$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g{}^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left(\delta^\mu{}_\lambda \delta^\nu{}_\tau - \delta^\nu{}_\lambda \delta^\mu{}_\tau\right)$
$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_\gamma{}^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 \left(\delta^\alpha{}_\gamma \delta^\beta{}_\delta - \delta^\alpha{}_\delta \delta^\beta{}_\gamma\right)$
Hence
$R^\mu{}_\nu = A^{-2} R_g{}^\mu{}_\nu + \left(\frac{A''}{A} + (p-1)\frac{A'^2}{A^2} + n\frac{A'C'}{AC}\right)\delta^\mu_\nu$
$R^\alpha{}_\beta = C^{-2} R_\gamma{}^\alpha{}_\beta + \left(\frac{C''}{C} + (n-1)\frac{C'^2}{C^2} + p\frac{A'C'}{AC}\right)\delta^\alpha_\beta$
$R^r{}_r = p\frac{A''}{A} + n\frac{C''}{C}$
e g Higher dim black hole
$ds^2 = A^2 dt^2 - dr^2 - C^2 d\Omega_n^2$. n=
$R^t_t = \frac{A''}{A} + n\frac{A'C'}{AC}$
$R^\theta_\theta = \frac{C''}{C} + \frac{A'C'}{AC} + (n-1)\left(\frac{C'^2}{C^2}\right.$
$R^r_r = \frac{A''}{A} + n\frac{C''}{C}$

$$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left(\delta^\mu_\lambda \delta^\nu_\tau - \delta^\nu_\lambda \delta^\mu_\tau\right)$$

$$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_g^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 \left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right)$$

$$p-1)\frac{A'^2}{A^2} + n\frac{A'C'}{AC}\Big)\delta^\mu_\nu$$

$$n-1)\frac{C'^2}{C^2} + p\frac{A'C'}{AC}\Big)\delta^\alpha_\beta$$

e.g. Higher dim black hole

$$ds^2 = A^2 dt^2 - dr^2 - C^2 d\Omega_n^2 \qquad n = D-2$$

$$R^t{}_t = \frac{A''}{A} + n\frac{A'C'}{AC}$$

$$R^\theta{}_\theta = \frac{C''}{C} + \frac{A'C'}{AC} + (n-1)\left(\frac{C'^2}{C^2} - \frac{1}{C^2}\right)$$

$$R^r{}_r = \frac{A''}{A} + n\frac{C''}{C}$$

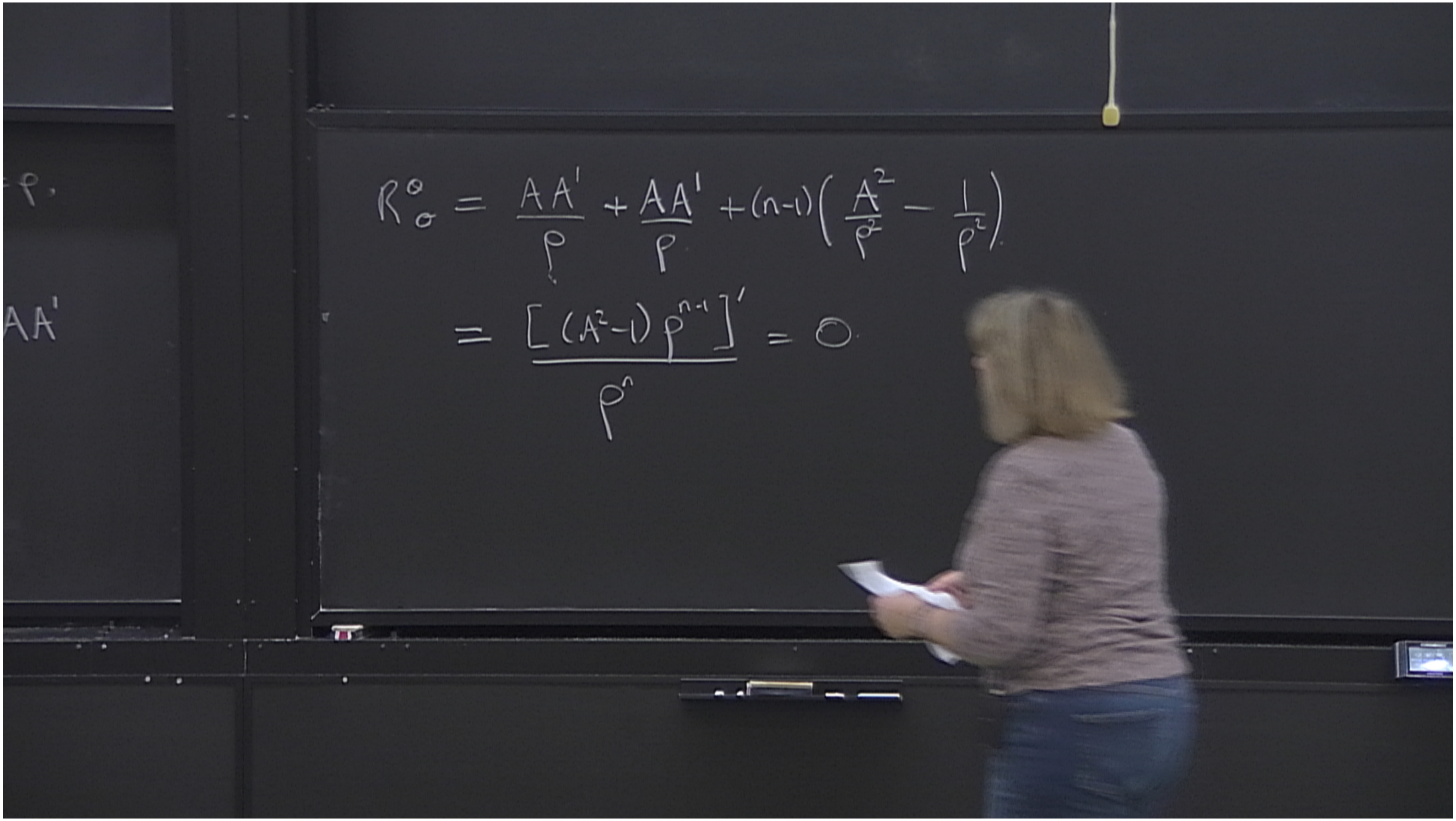
Look for a soln with $C = \rho$.

$$\frac{d\rho}{A} = dr$$

Then $\dfrac{dC}{dr} = A$, $\dfrac{d^2C}{dr^2} = AA'$

$$\frac{d}{dr} = A\frac{d}{d\rho}$$

Now let $' = \dfrac{d}{d\rho}$


$R^{\theta}_{\theta} = \frac{AA'}{\rho} + \frac{AA'}{\rho} + (n-1)\left(\frac{A^2}{\rho^2} - \frac{1}{\rho^2}\right)$
$= \frac{[(A^2-1)\rho^{n-1}]'}{\rho^n} = 0$

$$R^{\sigma}{}_{\sigma} = \frac{AA'}{\rho} + \frac{AA'}{\rho} + (n-1)\left(\frac{A^2}{\rho^2} - \frac{1}{\rho^2}\right)$$

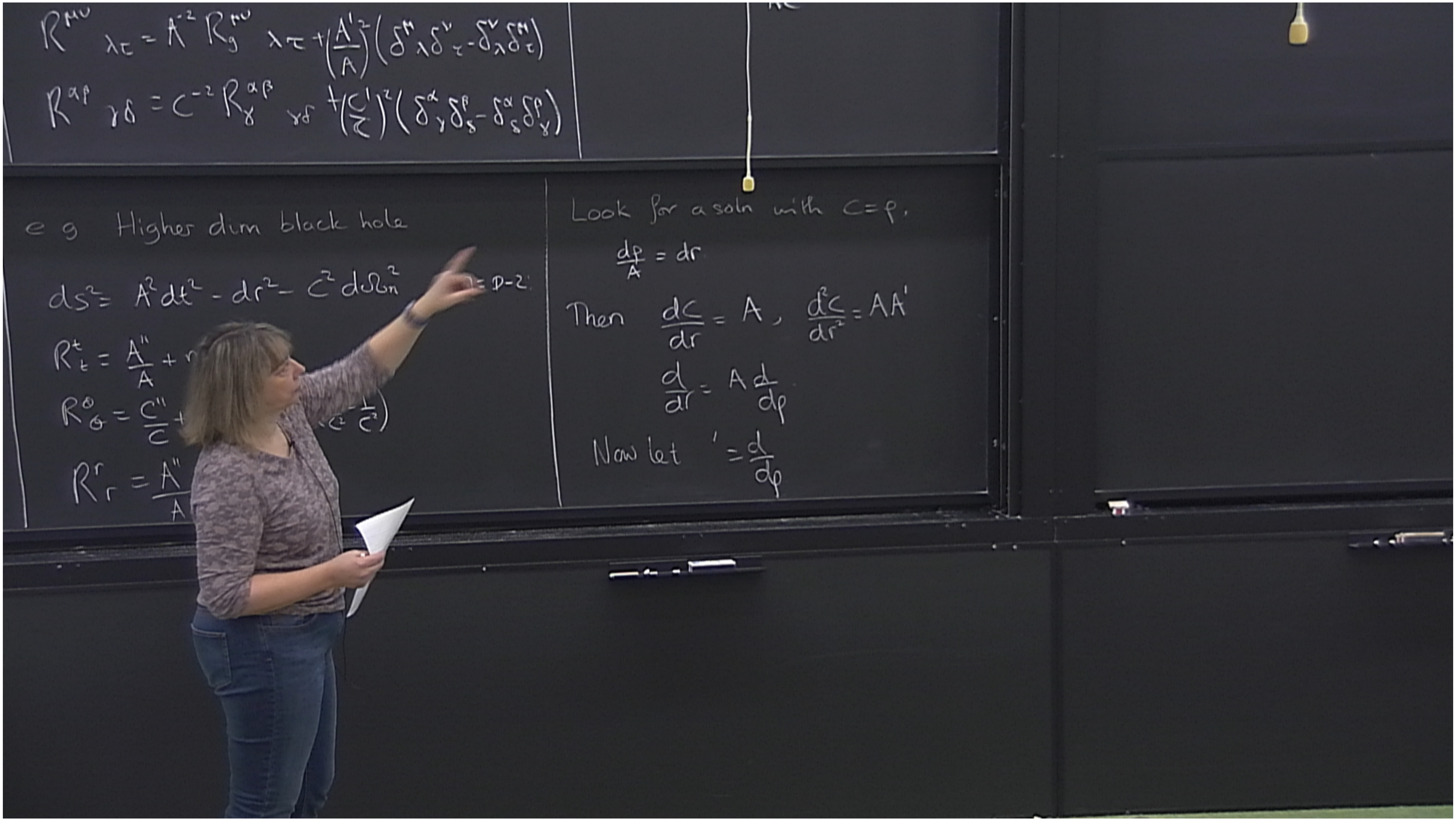
$$= \frac{\left[(A^2-1)\rho^{n-1}\right]'}{\rho^n} = 0$$

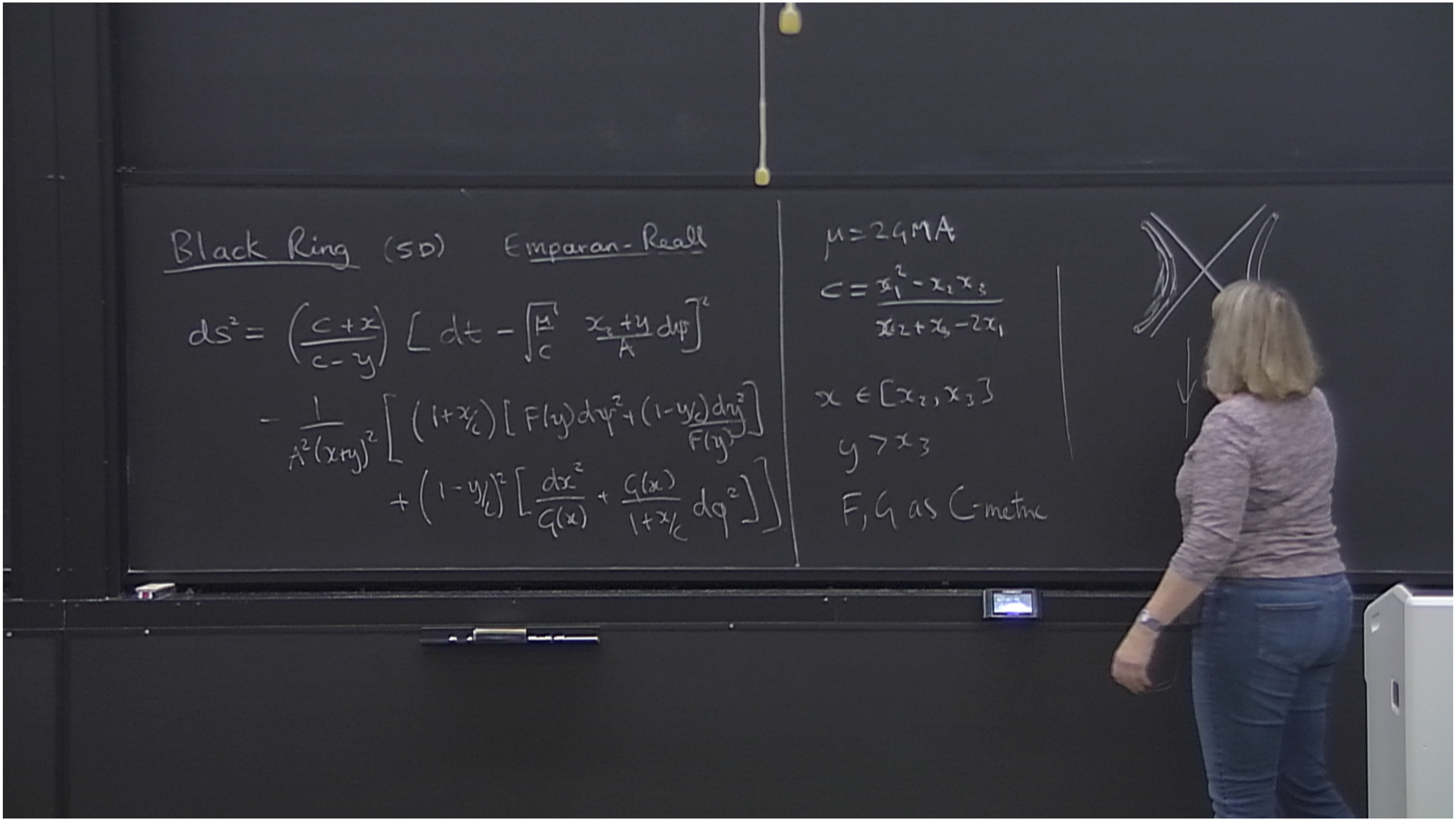
$$\Rightarrow A^2 = 1 + \frac{C}{\rho^{n-1}}$$

Identify with mass

$$16\pi G M = (D-2)\, A_{D-2}\,(-C)$$

area of unit S^{D-2}


$R^{\mu\nu}{}_{\lambda\tau} = A^{-2} R_g^{\mu\nu}{}_{\lambda\tau} + \left(\frac{A'}{A}\right)^2 \left(\delta^\mu{}_\lambda \delta^\nu{}_\tau - \delta^\nu{}_\lambda \delta^\mu{}_\tau\right)$
$R^{\alpha\beta}{}_{\gamma\delta} = C^{-2} R_\gamma^{\alpha\beta}{}_{\gamma\delta} + \left(\frac{C'}{C}\right)^2 \left(\delta^\alpha_\gamma \delta^\beta_\delta - \delta^\alpha_\delta \delta^\beta_\gamma\right)$
e g Higher dim black hole
$ds^2 = A^2 dt^2 - dr^2 - C^2 d\Omega_n^2$
$= D-2$
$R^t{}_t = \frac{A''}{A} +$
$R^\theta{}_\theta = \frac{C''}{C} +$
$R^r{}_r = \frac{A''}{A}$
Look for a soln with $C = \rho$.
$\frac{d\rho}{A} = dr$
Then $\frac{dC}{dr} = A$, $\frac{d^2C}{dr^2} = AA'$
$\frac{d}{dr} = A \frac{d}{d\rho}$
Now let $' = \frac{d}{d\rho}$


Black Ring (5D) Emparan-Reall
$ds^2 = \left(\frac{c+x}{c-y}\right)\left[dt - \sqrt{\frac{\mu}{c}}\,\frac{x_3+y}{A}\,d\psi\right]^2$
$-\frac{1}{A^2(x+y)^2}\left[\left(1+\frac{x}{c}\right)\left[F(y)\,d\psi^2 + \left(1-\frac{y}{c}\right)\frac{dy^2}{F(y)}\right]\right.$
$\left. + \left(1-\frac{y}{c}\right)^2\left[\frac{dx^2}{G(x)} + \frac{G(x)}{1+x/c}\,d\varphi^2\right]\right]$
$\mu = 2GMA$
$c = \frac{x_1^2 - x_2 x_3}{x_2 + x_3 - 2x_1}$
$x \in [x_2, x_3]$
$y > x_3$
F, G as C-metric

#6

Black Ring (5D) Emparan-Reall

$$ds^2 = \left(\frac{c+x}{c-y}\right)\left[dt - \sqrt{\frac{\mu}{c}}\,\frac{x_3+y}{A}\,d\psi\right]^2$$

$$-\frac{1}{A^2(x+y)^2}\left[\left(1+\tfrac{x}{c}\right)\left[F(y)\,d\psi^2 + \left(1-\tfrac{y}{c}\right)\frac{dy^2}{F(y)}\right] + \left(1-\tfrac{y}{c}\right)^2\left[\frac{dx^2}{G(x)} + \frac{G(x)}{1+x/c}\,d\phi^2\right]\right]$$

$\mu = 2GMA$

$$c = \frac{x_1^2 - x_2 x_3}{x_2 + x_3 - 2x_1}$$

$x \in [x_2, x_3]$

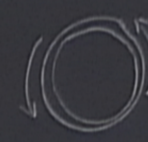
$y > x_3$

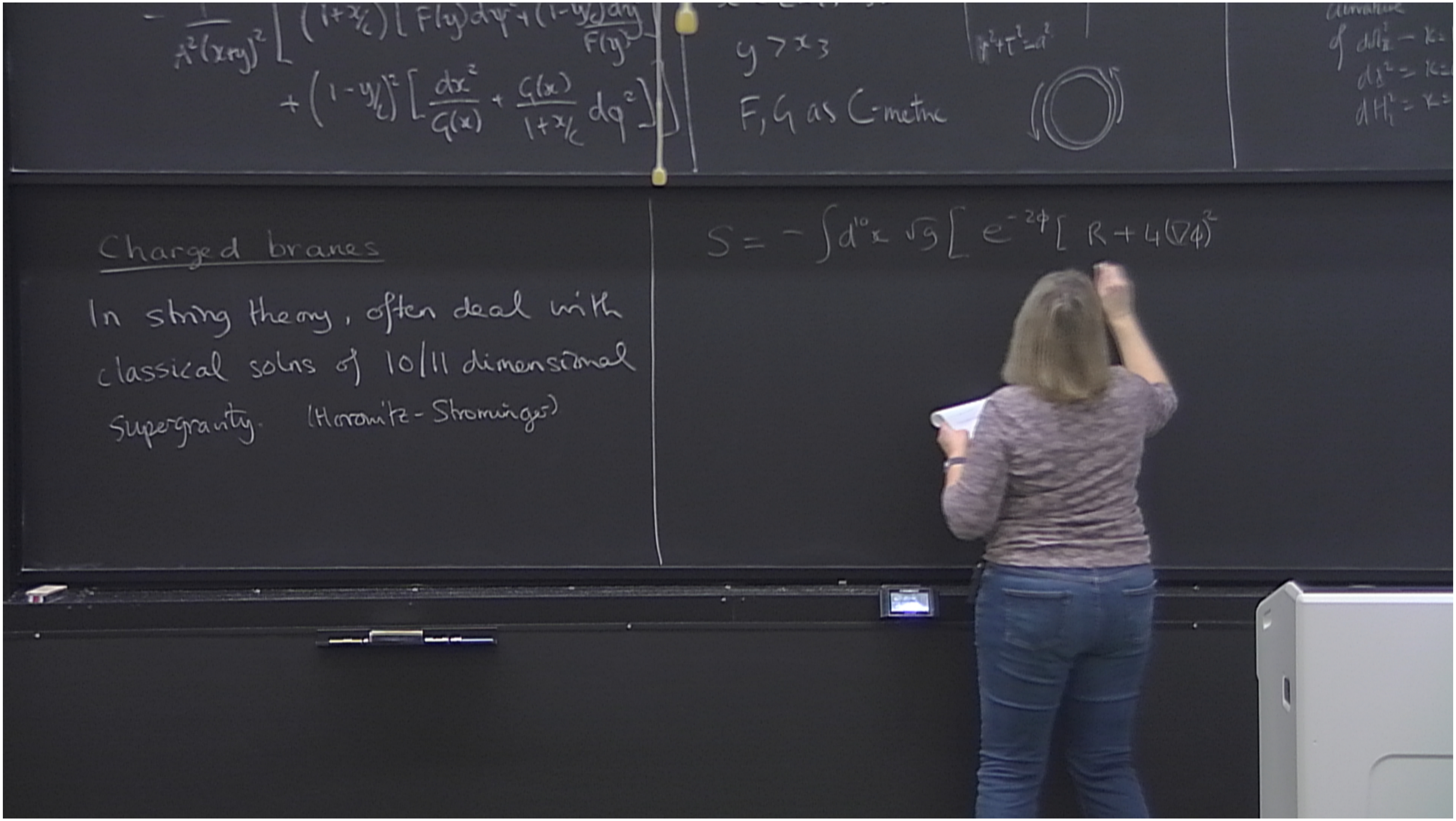
F, G as C-metric

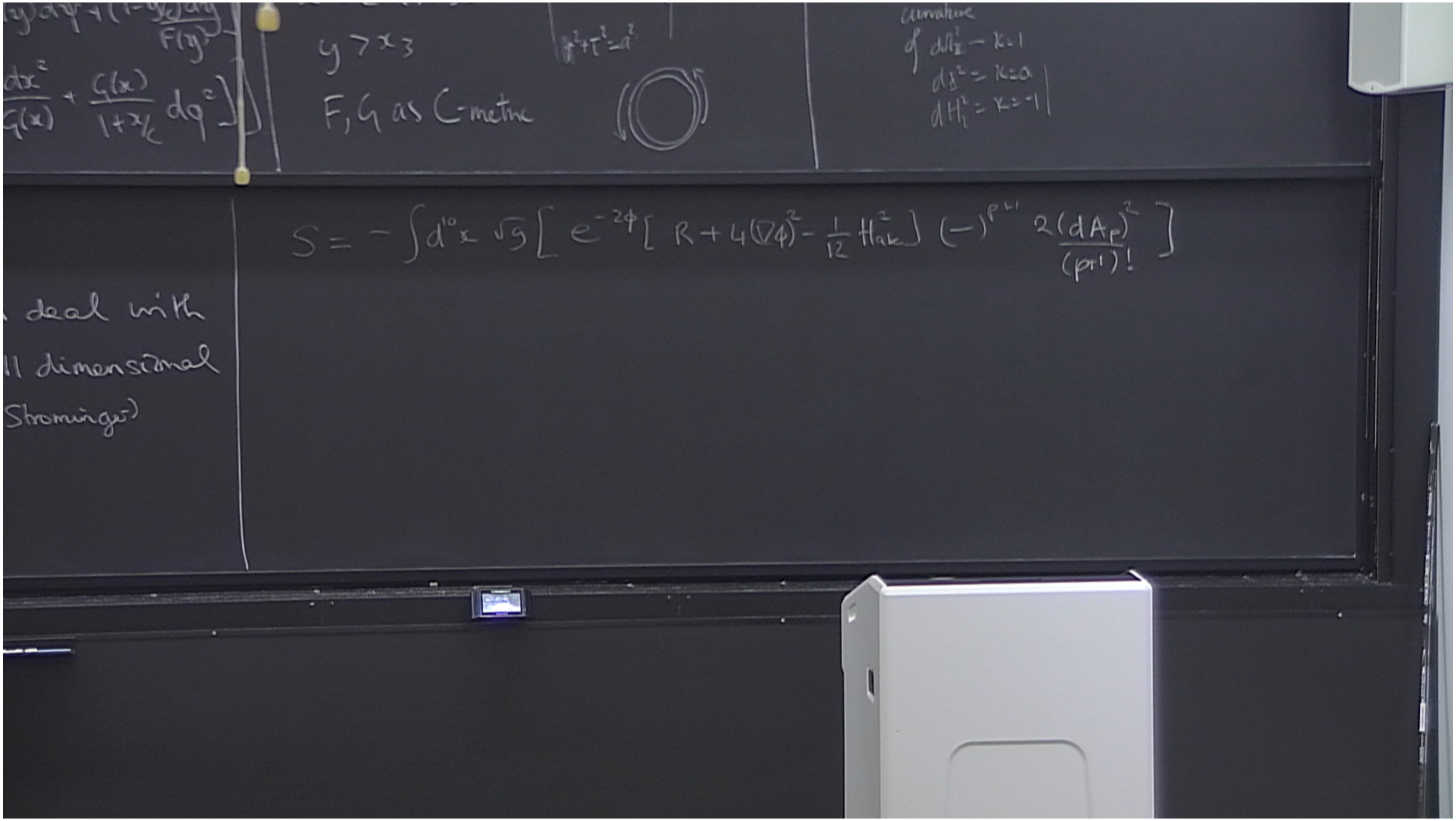
$r^2 - t^2 = a^2$

$t \to i\tau$

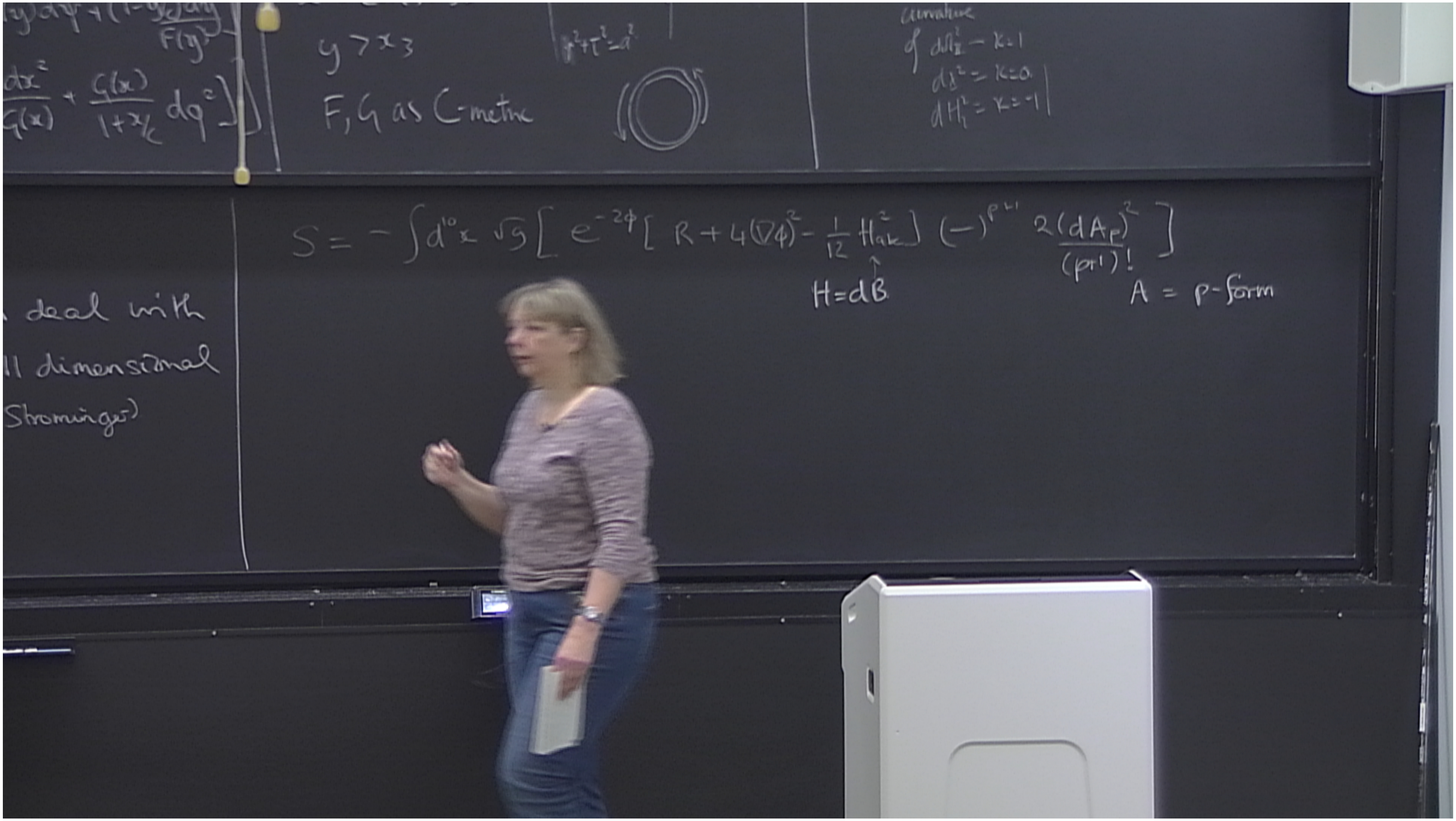
$r^2 + \tau^2 = a^2$




y > x3
F, G as C-metric
ψ²+τ²=a²
Charged branes
In string theory, often deal with
classical solns of 10/11 dimensional
supergravity. (Horowitz - Strominger)
S = − ∫d¹⁰x √g [e^−2φ [R + 4(∇φ)²



$y > x_3$
F, G as C-metric
$\rho^2+\tau^2=a^2$
curvature
$d\Omega^2_{\,} - k=1$
$dl^2 = k=0$
$dH^2_1 = k=-1$
$S = -\int d^{10}x \sqrt{g}\left[e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{12}H_{abc}^2\right] (-)^{p+1} \, 2\frac{(dA_p)^2}{(p+1)!} \right]$
deal with
11 dimensional
Strominger)


F, G as C-metric
S = − ∫d^10x √g [e^{−2φ} [R + 4(∇φ)^2 − 1/12 H^2] (−)^{p+1} 2(dA_p)^2/(p+1)!]
H=dB
A = p-form
deal with
dimensional
Strominger)