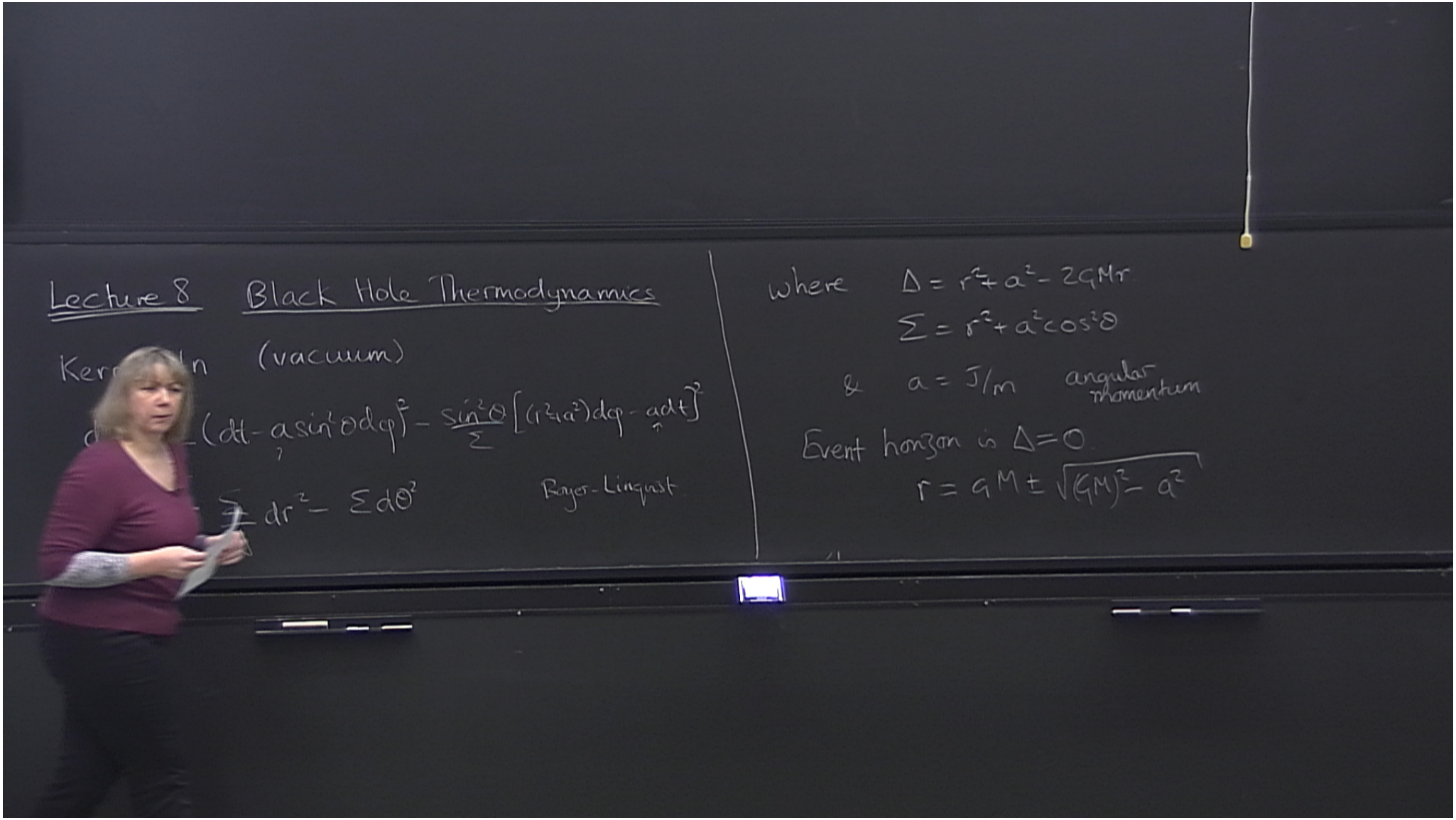
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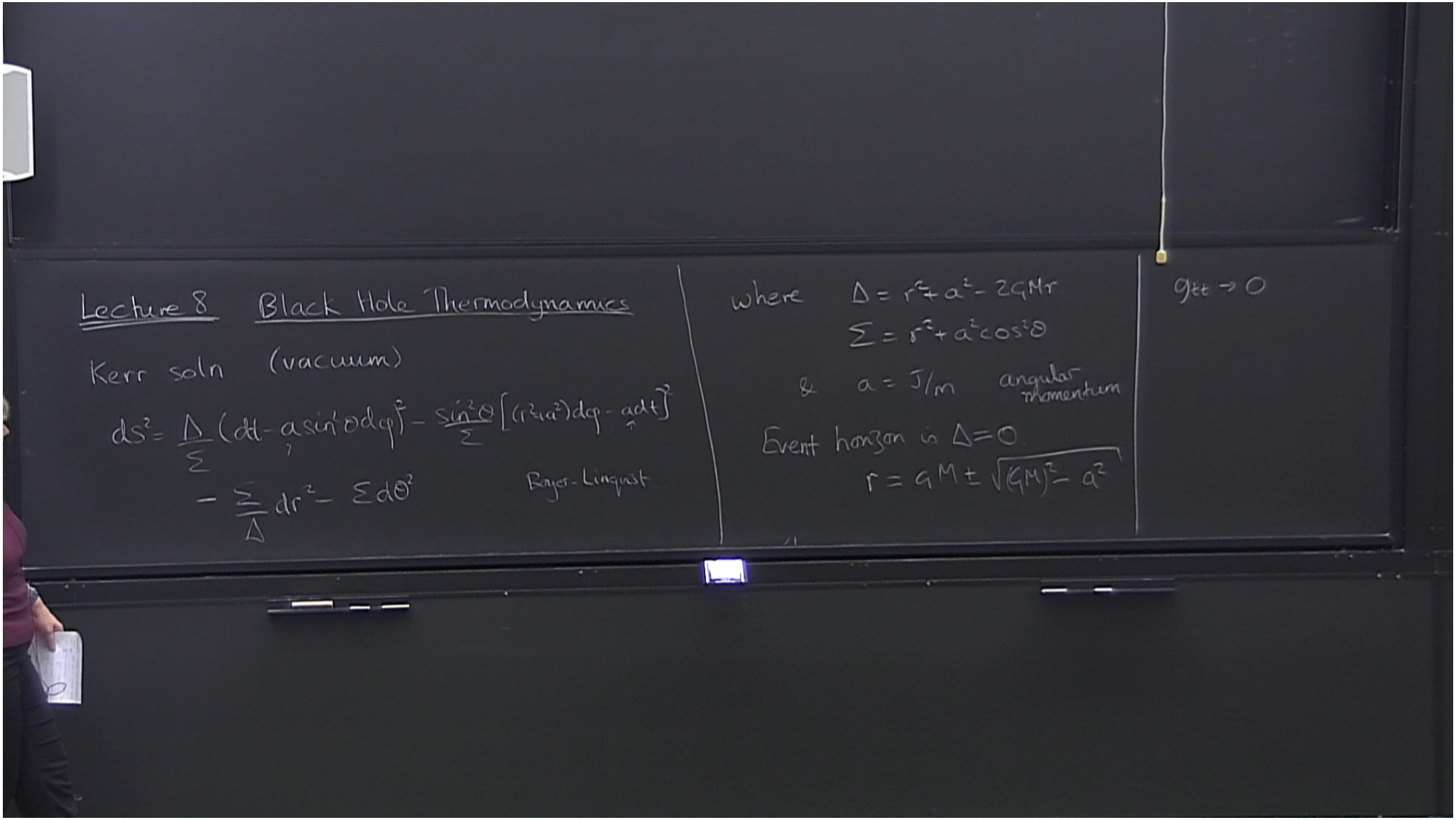
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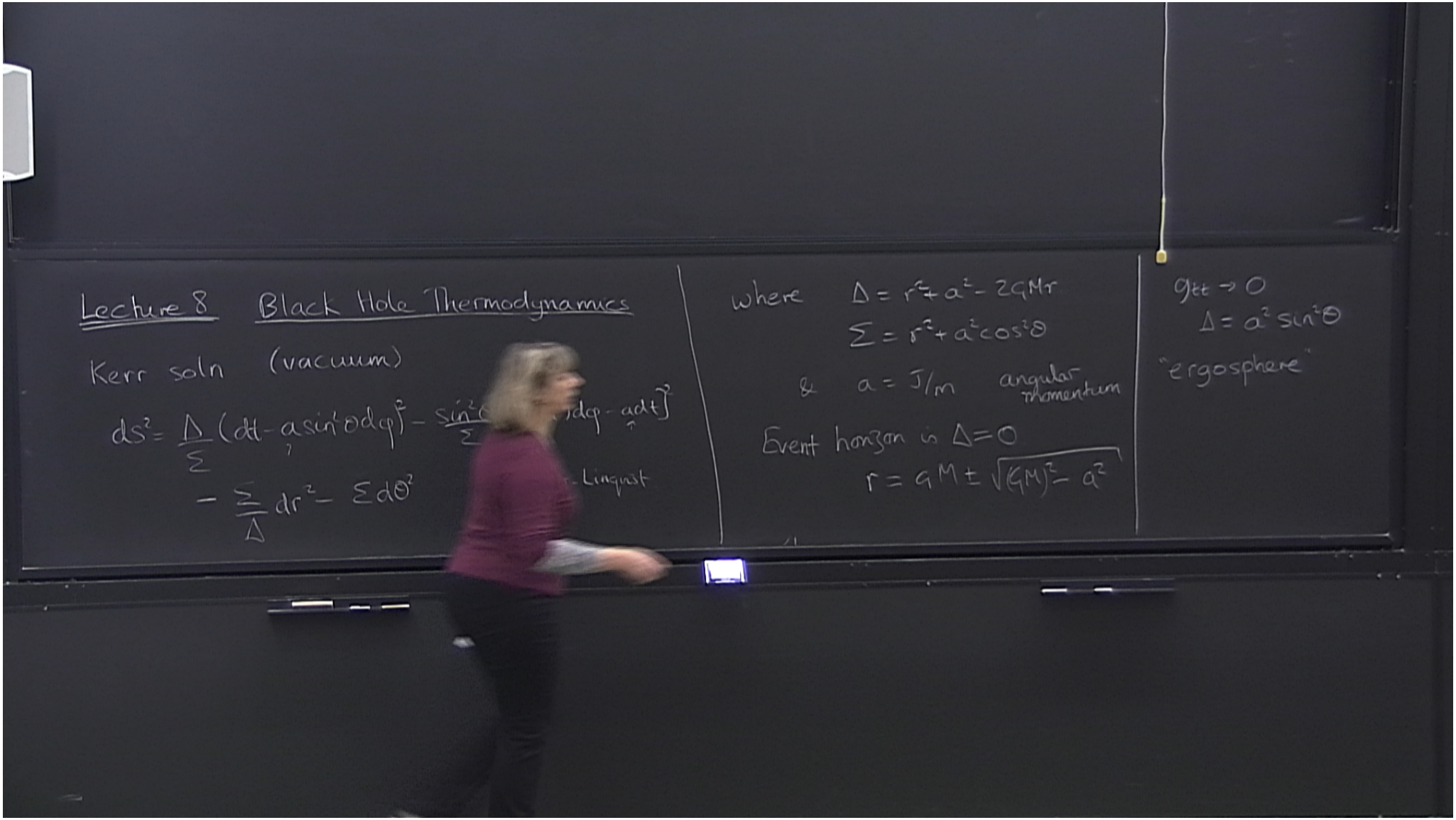
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Abstract:

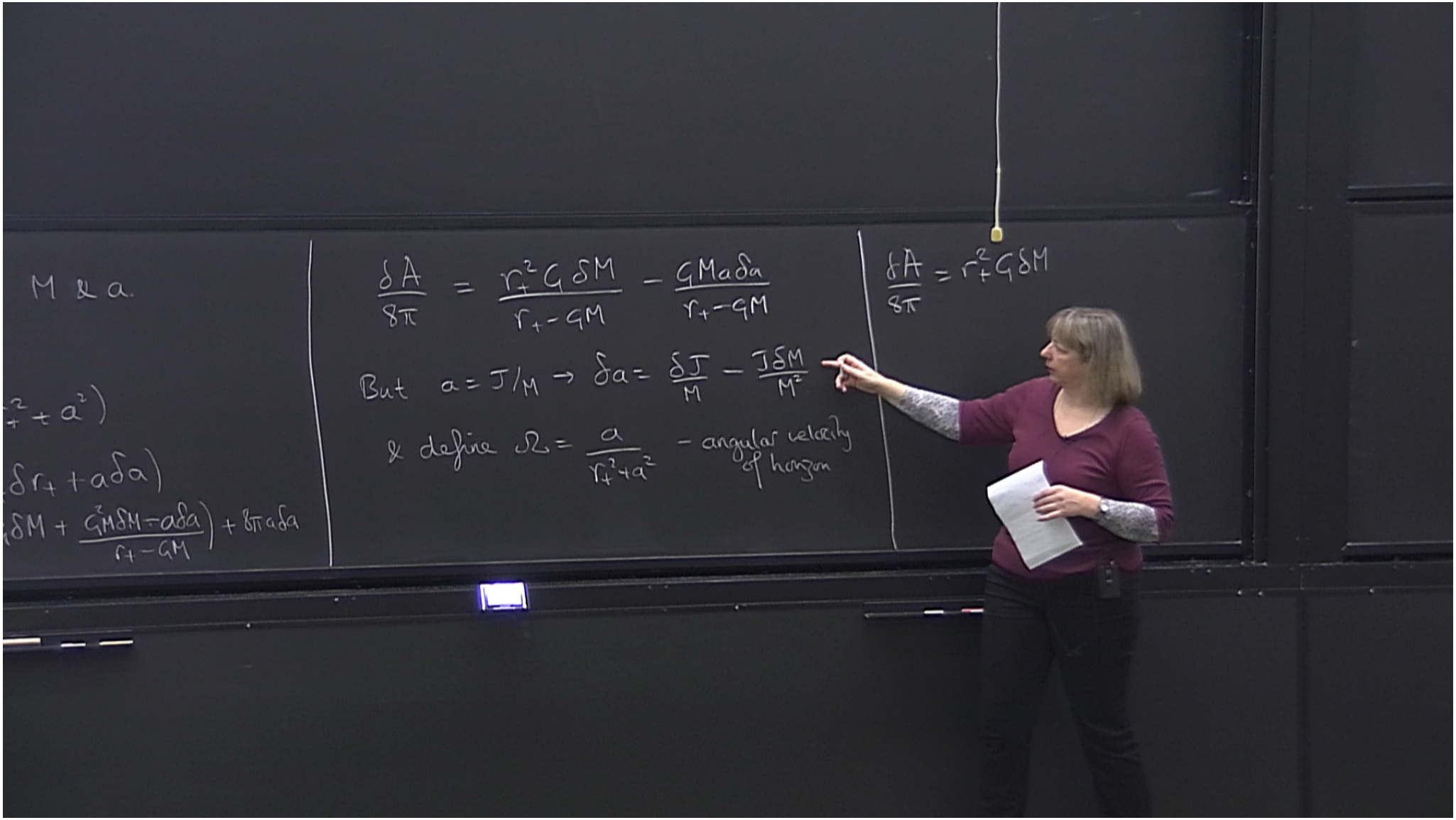


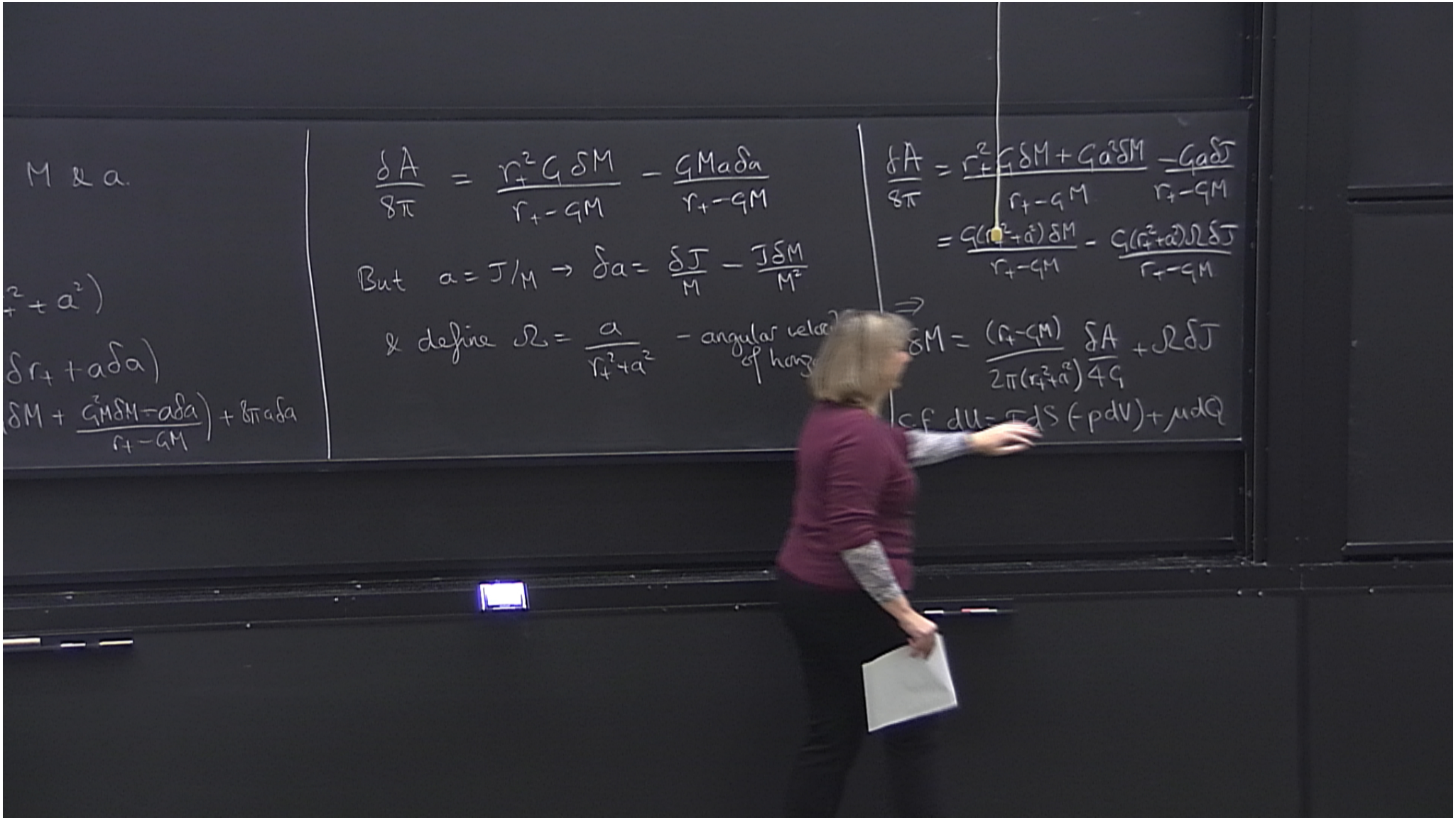

Lecture 8 Black Hole Thermodynamics
(vacuum)
Boyer-Lindquist
where $\Delta = r^2 + a^2 - 2GMr$
$\Sigma = r^2 + a^2\cos^2\theta$
& $a = J/m$ angular momentum
Event horizon is $\Delta = 0$
$r = GM \pm \sqrt{(GM)^2 - a^2}$


Lecture 8 Black Hole Thermodynamics
Kerr soln (vacuum)
$ds^2 = \frac{\Delta}{\Sigma}(dt - a\sin^2\theta\, d\varphi)^2 - \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\varphi - a\,dt\right]^2$
$- \frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2$
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$\Delta = a^2\sin^2\theta$
"ergosphere"




M & a.
$\frac{\delta A}{8\pi} = \frac{r_+^2 G \delta M}{r_+ - GM} - \frac{GMa\delta a}{r_+ - GM}$
But $a = J/M \rightarrow \delta a = \frac{\delta J}{M} - \frac{J\delta M}{M^2}$
& define $\Omega_+ = \frac{a}{r_+^2 + a^2}$ - angular velocity of horizon
$\frac{\delta A}{8\pi} = r_+^2 G \delta M$


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of hor
$\frac{\delta A}{8\pi} = \frac{r_+^2 G\delta M + Ga^2\delta M}{r_+ - GM} - \frac{Ga\delta J}{r_+ - GM}$
$= \frac{G(r_+^2+a^2)\delta M}{r_+ - GM} - \frac{G(r_+^2+a^2)\Omega\delta J}{r_+ - GM}$
$\Rightarrow$
$\delta M = \frac{(r_+ - GM)}{2\pi(r_+^2+a^2)} \frac{\delta A}{4G} + \Omega\delta J$
c.f $dU = TdS (+pdV) + \mu dQ$

Area of b.h.

$$A = 4\pi(r_+^2 + a^2)$$

$$\Rightarrow \quad \delta A = 8\pi(r_+ \delta r_+ + a\delta a)$$

$$= 8\pi G\left(G\delta M + \frac{G^2 M\delta M - a\delta a}{r_+ - GM}\right) + 8\pi a\delta a$$

But $a = J/M \rightarrow \delta a = \frac{\delta J}{M} - \frac{J\delta M}{M^2}$

& define $\Omega = \frac{a}{r_+^2 + a^2}$ – angular velocity of horizon

$$= \frac{G(r_+^2+a^2)\delta M}{r_+ - GM} -$$

$\Rightarrow$

$$\delta M = \left(\frac{r_+ - GM}{2\pi(r_+^2+a^2)}\right)\delta$$

cf $dU = TdS + P$

Suggests thermodynamic interpretation

$$T = \frac{r_+ - GM}{2\pi(r_+^2+a^2)} \xrightarrow{a\to 0} \frac{1}{8\pi GM}$$

$$S = \frac{A}{4G}$$

$4\pi(r_+^2 + a^2)$

$8\pi(r_+\delta r_+ + a\delta a)$

$8\pi G\left(G\delta M + \frac{G^2 M\delta M - a\delta a}{r_+ - GM}\right) + 8\pi a\delta a$

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$\Rightarrow$

$\boxed{\delta M = \frac{(r_+ - GM)}{2\pi(r_+^2+a^2)}\frac{\delta A}{4G} + \Omega\delta J}$

cf $\boxed{dU = TdS} + (-pdV) + \mu dQ$

thermodynamic interpretation

$\frac{M}{r_+^2+a^2} \xrightarrow{a\to 0} \frac{1}{8\pi GM}$

Back up by Euclidean path integral calculation (with caution)

$Z \sim \mathrm{tr}\, e^{-\beta H}$ $\quad \beta = 1/T$ $(k_B = 1)$

$H \sim \int d^3x\, \mathcal{H} = \frac{1}{\beta}\int d^3x\, d\tau\, \mathcal{H}$

$\to Z = \mathrm{tr}\, e^{-I_E}$

I_E = (4D) Euclidean action

$4\pi(r_+^2 + a^2)$

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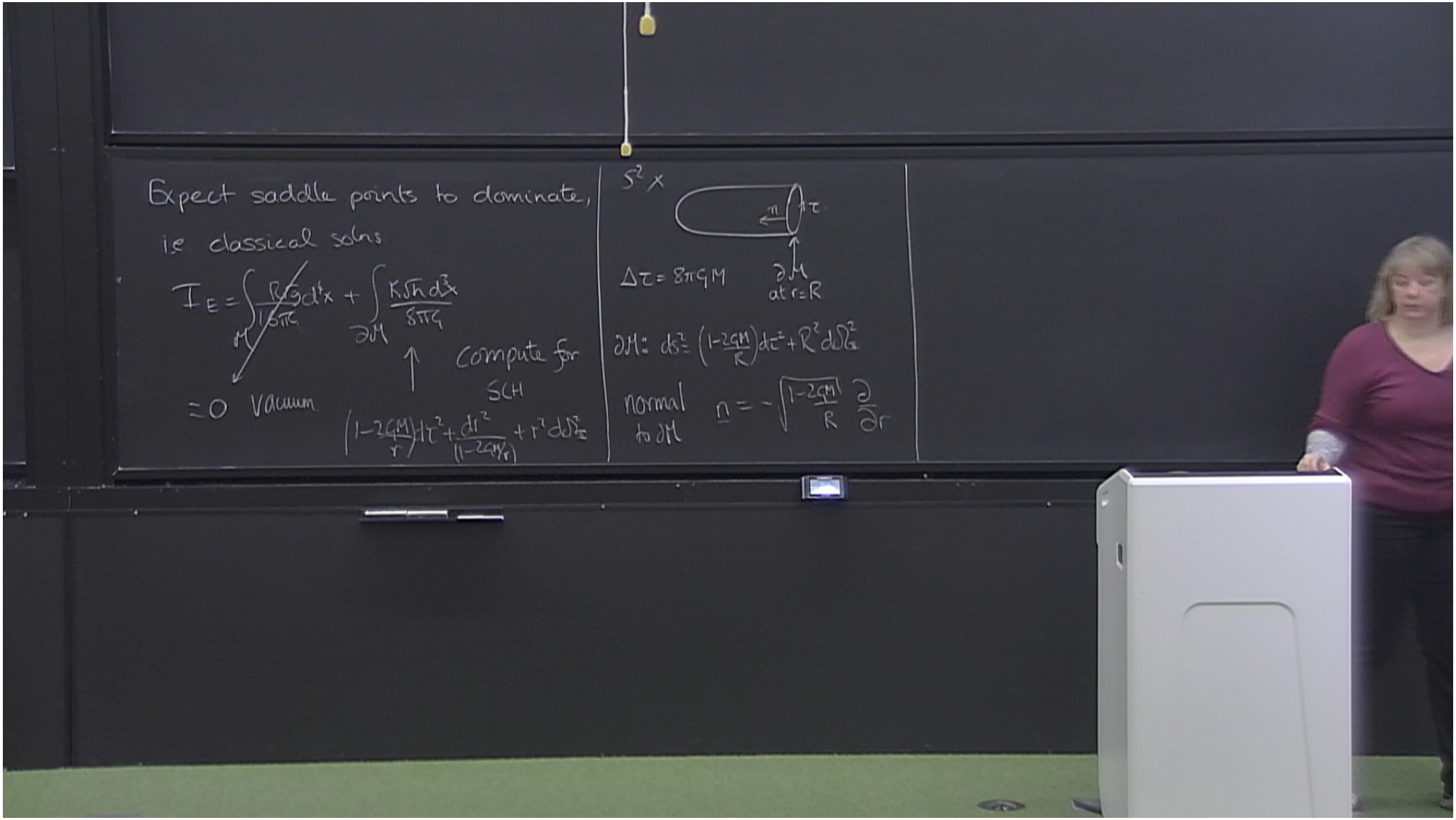
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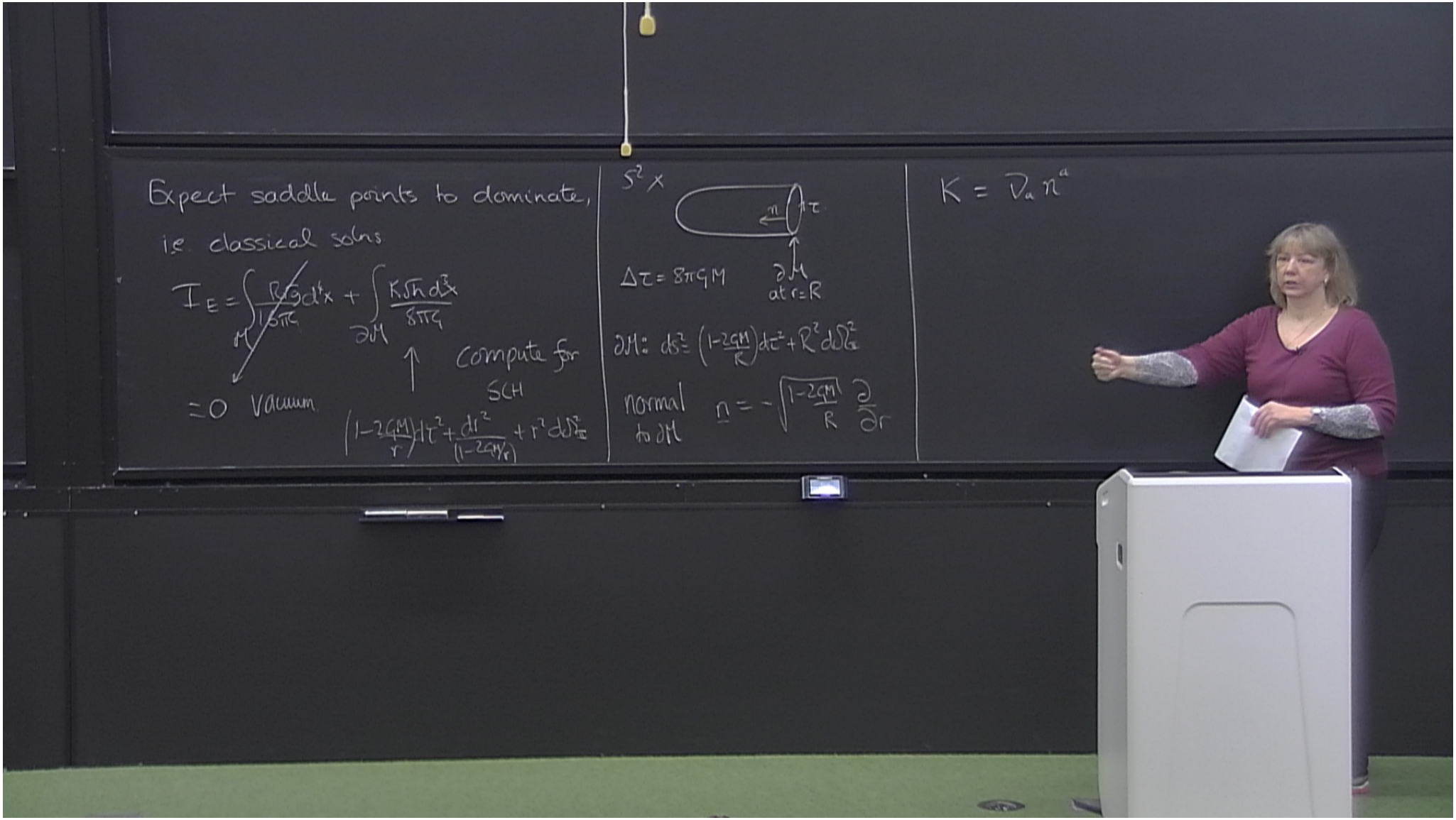
$Z \sim \mathrm{tr}\, e^{-\beta H}$ $\quad \beta = 1/T$ $(k_B = 1)$

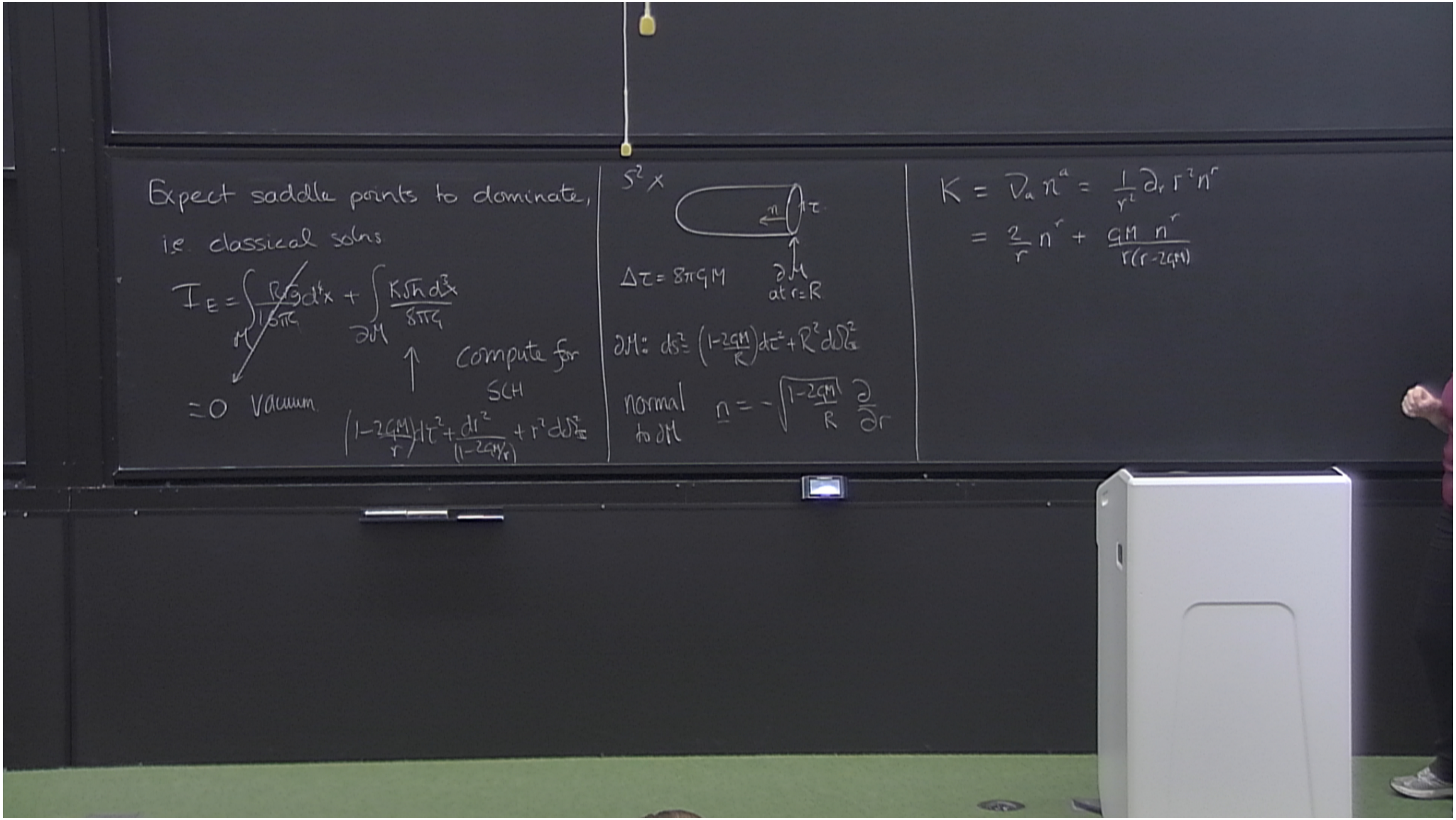
$H \sim \int d^3x\, \mathcal{H} = \frac{1}{\beta}\int d^3x\, d\tau\, \mathcal{H}$

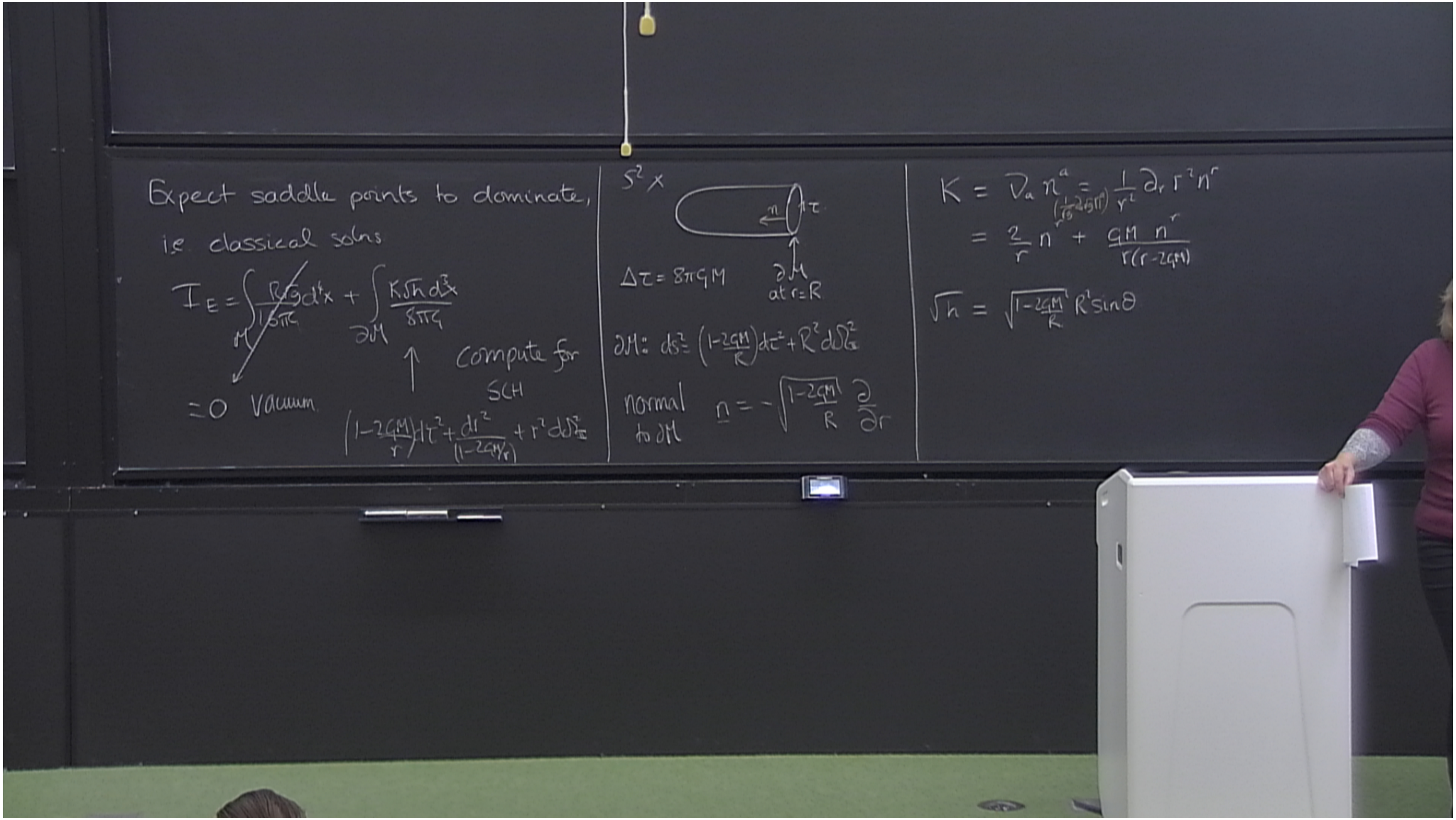
$\to Z = \mathrm{tr}\, e^{-I_E}$

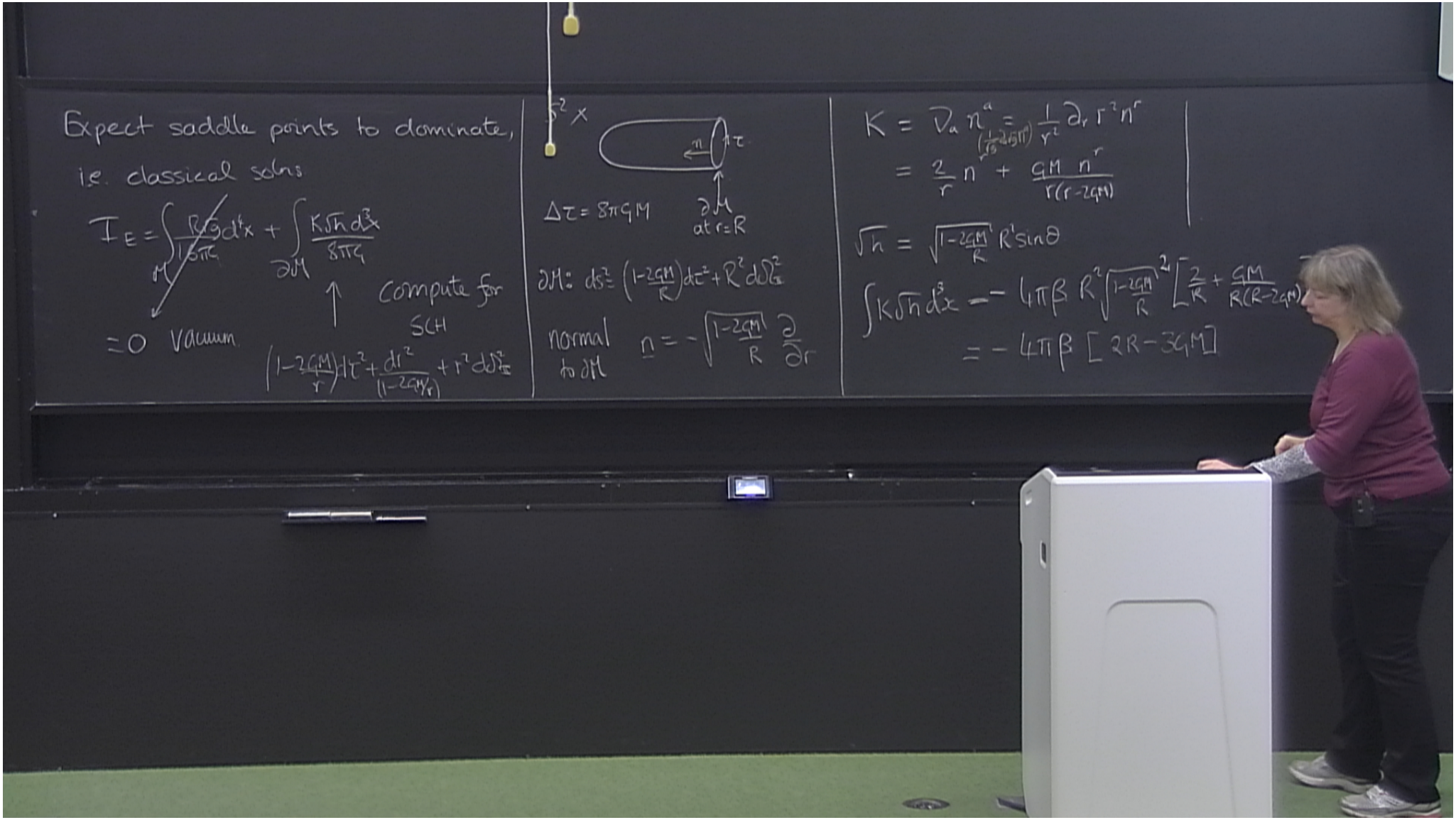
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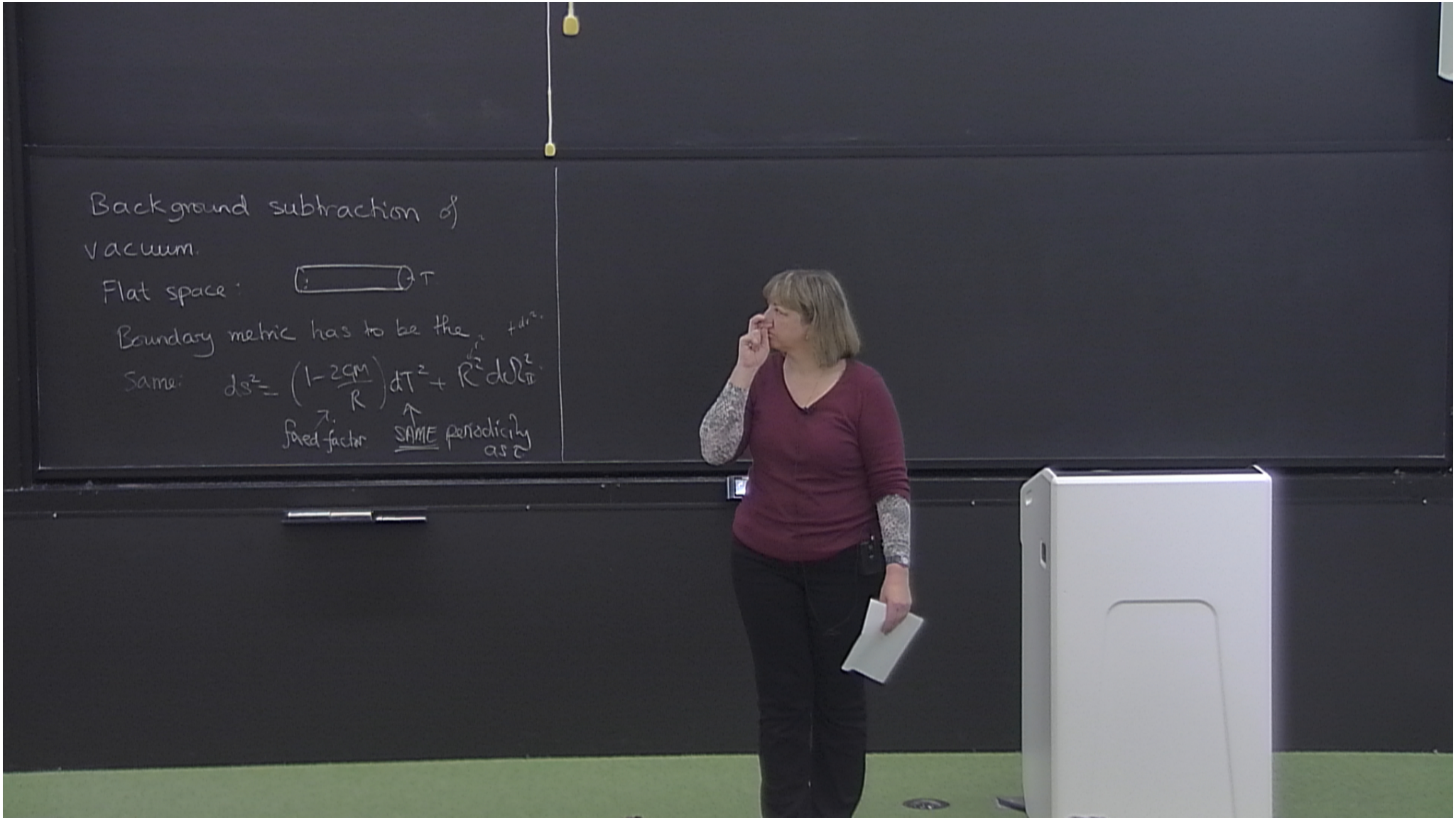

Expect saddle points to dominate,
ie classical solns
$I_E = \int_M \frac{R\sqrt{g}}{16\pi G} d^4x + \int_{\partial M} \frac{K\sqrt{h}\,d^3x}{8\pi G}$
$= 0$ Vacuum
↑ Compute for SCH
$\left(1-\frac{2GM}{r}\right)dt^2 + \frac{dr^2}{\left(1-\frac{2GM}{r}\right)} + r^2 d\Omega^2$
$S^2 \times$
π
τ
$\Delta\tau = 8\pi GM$
∂M at $r=R$
∂M: $ds^2 = \left(1-\frac{2GM}{R}\right)dt^2 + R^2 d\Omega_2^2$
normal to ∂M
$n = -\sqrt{1-\frac{2GM}{R}}\,\frac{\partial}{\partial r}$

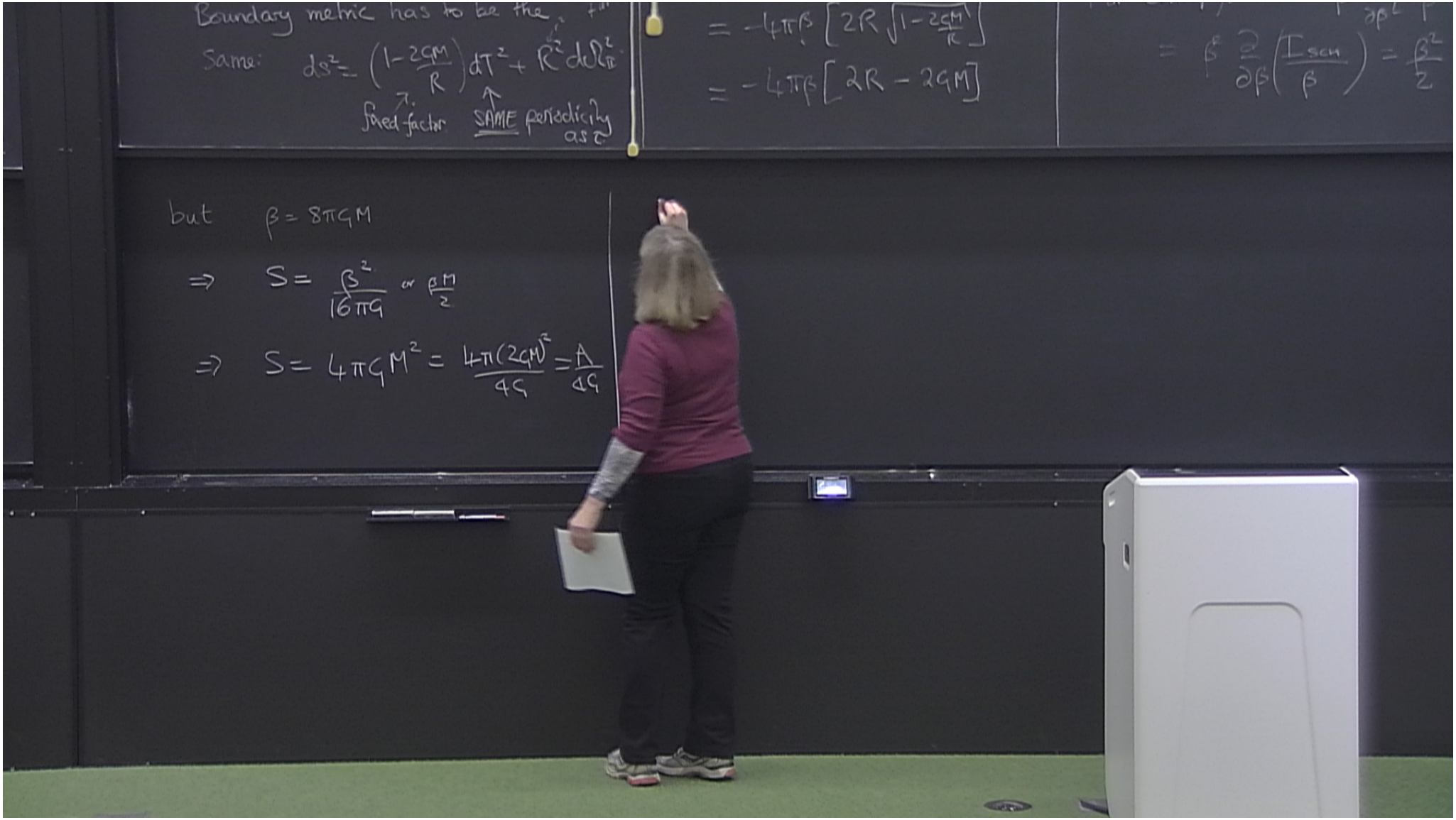

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$= \frac{2}{r} n^r + \frac{GM\, n^r}{r(r-2GM)}$


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τ
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$\sqrt{h} = \sqrt{1-\frac{2GM}{R}}\,R^2 \sin\theta$
$\int K\sqrt{h}\,d^3x = -4\pi\beta R^2\sqrt{1-\frac{2GM}{R}}\left[\frac{2}{R} + \frac{GM}{R(R-2GM)}\right]$
$= -4\pi\beta\,[2R - 3GM]$


Background subtraction of
vacuum.
Flat space:
T
Boundary metric has to be the same:
$ds^2 = \left(1 - \frac{2GM}{R}\right) dT^2 + R^2 d\Omega_{II}^2$
fixed factor
SAME periodicity as


Boundary metric has to be the
Same: $ds^2 = \left(1-\frac{2GM}{R}\right)dT^2 + R^2 d\Omega_2^2$
fixed factor
SAME periodicity as τ
$= -4\pi\beta\left[2R\sqrt{1-\frac{2GM}{R}}\right]$
$= -4\pi\beta\left[2R - 2GM\right]$
$= \beta^2 \frac{\partial}{\partial\beta}\left(\frac{I_{sch}}{\beta}\right) = \frac{\beta^2}{2}$
but $\beta = 8\pi GM$
$\Rightarrow S = \frac{\beta^2}{16\pi G}$ or $\frac{\beta M}{2}$
$\Rightarrow S = 4\pi G M^2 = \frac{4\pi(2GM)^2}{4G} = \frac{A}{4G}$