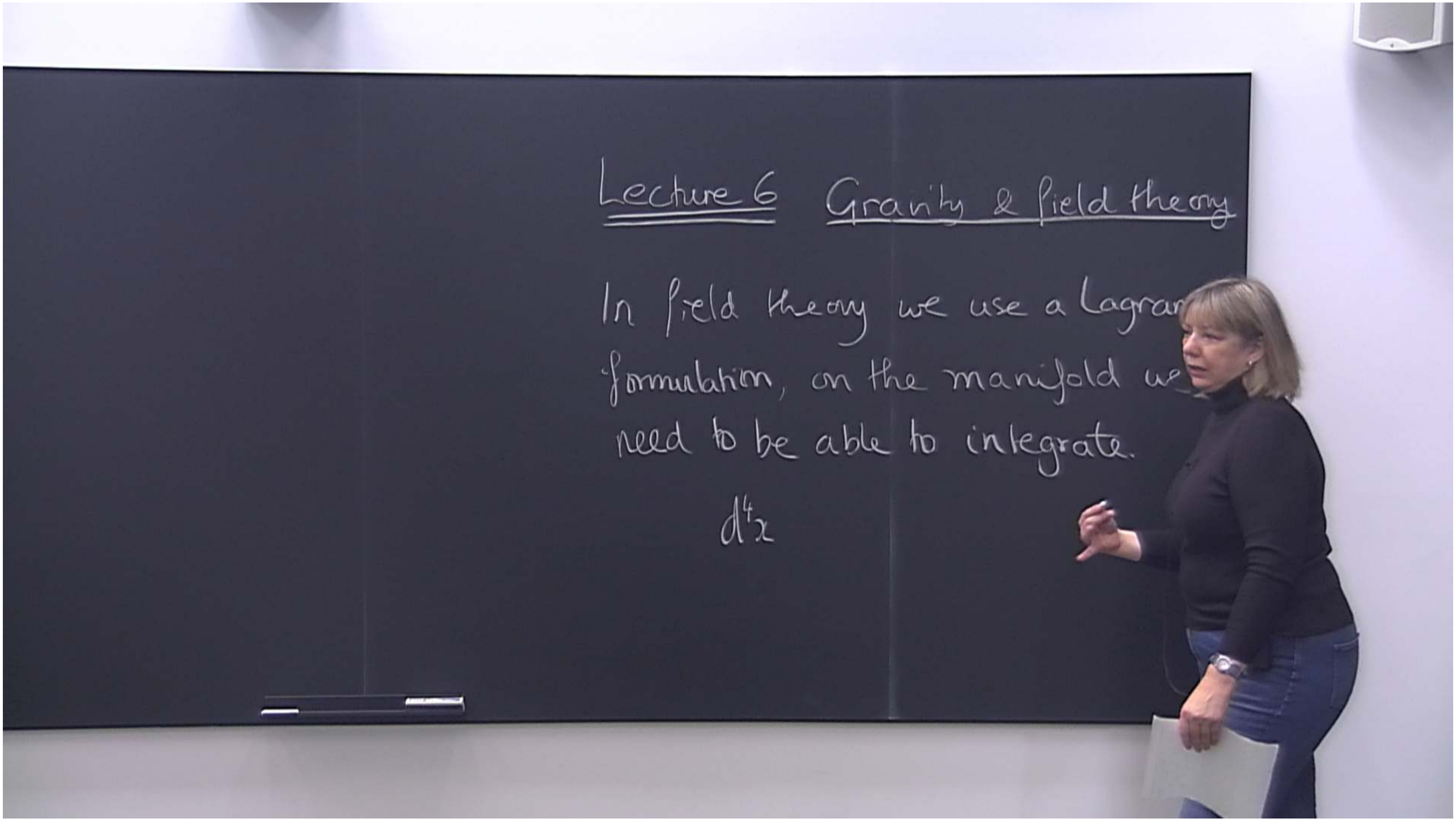
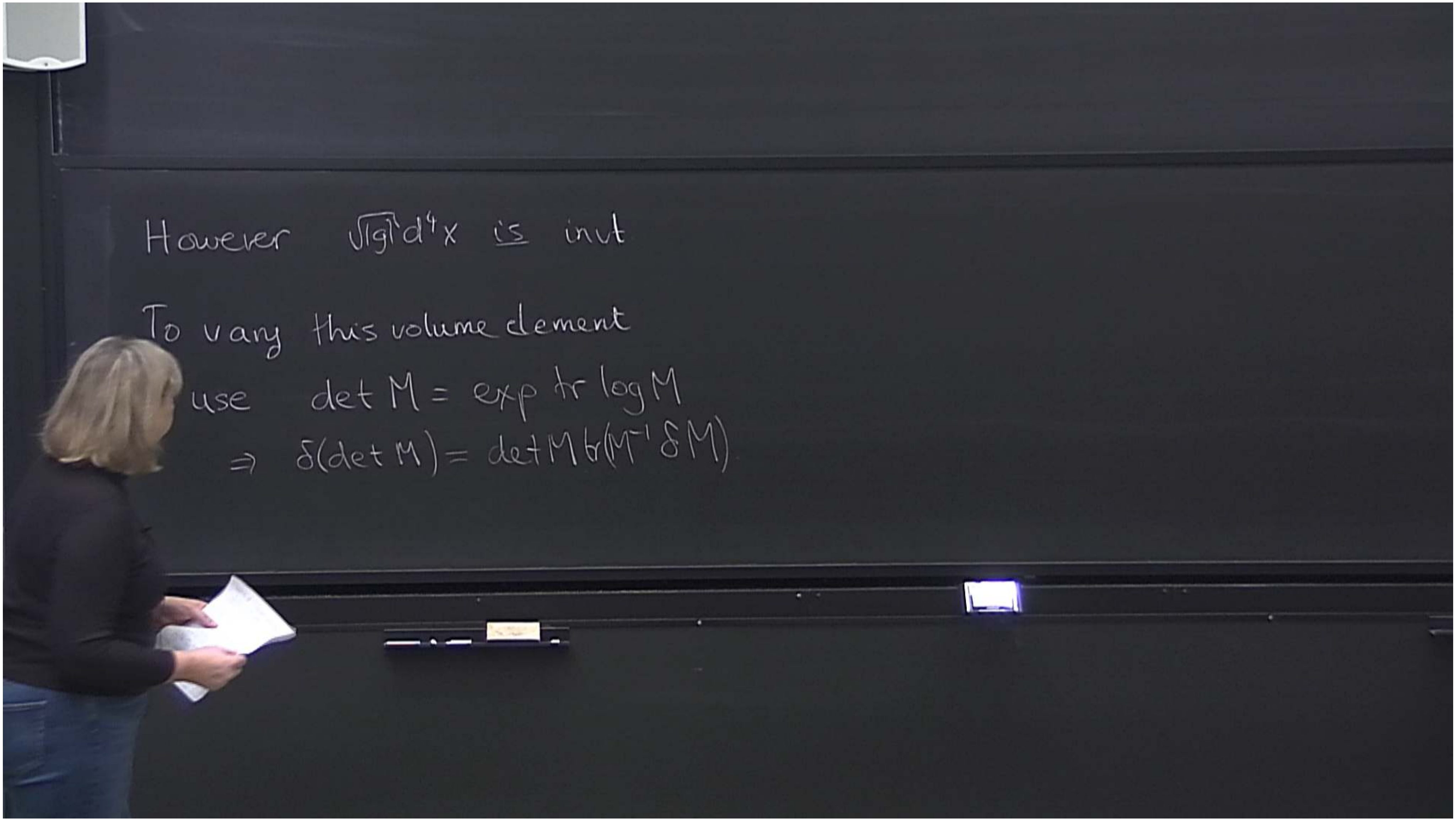
Title: Gravitational Physics-6

Date: Jan 08, 2015 10:15 AM

URL: http://pirsa.org/15010022

Abstract:


Lecture 6 Gravity & field theory
In field theory we use a Lagran
formulation, on the manifold we
need to be able to integrate.
d^4x


However $\sqrt{|g|}\,d^4x$ is invt
To vary this volume element
use $\det M = \exp \operatorname{tr} \log M$
$\Rightarrow \delta(\det M) = \det M \operatorname{tr}(M^{-1}\delta M)$

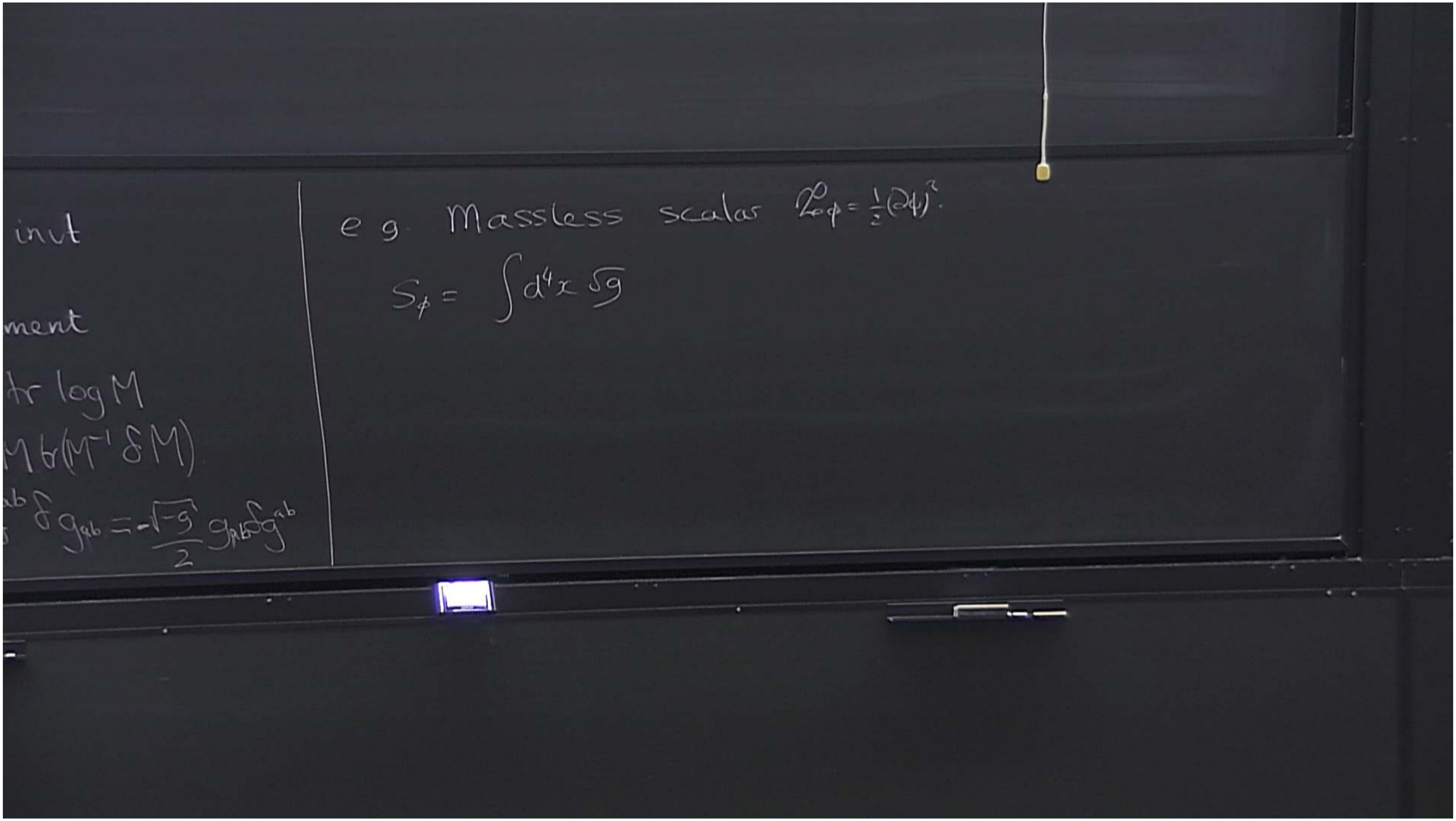
However $\sqrt{|g|}\,d^4x$ is invt

ary this volume element

use $\det M = \exp \operatorname{tr} \log M$

$\Rightarrow \delta(\det M) = \det M \, \operatorname{tr}(M^{-1}\delta M)$

$\Rightarrow \delta\sqrt{-g} = \frac{1}{2}\frac{-1}{\sqrt{-g}}\, g\, g^{ab}\,\delta g_{ab}$



e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$.
$S_\phi = \int d^4x \sqrt{g}$
init
ment
tr log M
M tr(M^{-1} δM)
δg_{ab} = $-\frac{\sqrt{-g}}{2} g_{ab}\delta g^{ab}$

init

ment

$\mathrm{tr}\log M$

$M\,\mathrm{tr}(M^{-1}\delta M)$

${}^{ab}\,\delta g_{ab} = -\frac{\sqrt{-g}}{2}\, g_{ab}\,\delta g^{ab}$

e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$.

$$S_\phi = \int d^4x \sqrt{g}\, \frac{1}{2}(\partial\phi)^2$$

$$\delta S_\phi = \int d^4x \sqrt{g}\, \partial_\mu\phi\, \partial_\nu\delta\phi\, g^{\mu\nu}.$$

e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$.

$$S_\phi = \int d^4x \sqrt{g}\, \tfrac{1}{2}(\partial\phi)^2 = \int d^4x \tfrac{\sqrt{g}}{2}\, \phi_{,\mu}\phi_{,\nu}\, g^{\mu\nu}$$

$$\delta S_\phi = \int d^4x \sqrt{g}\, \partial_\mu\phi\, \partial_\nu\delta\phi\, g^{\mu\nu}$$

$$+ \tfrac{1}{2}\sqrt{g}\left(\phi_{,\mu}\phi_{,\nu} - \tfrac{1}{2}(\partial\phi)^2 g_{\mu\nu}\right)\delta g^{\mu\nu}$$

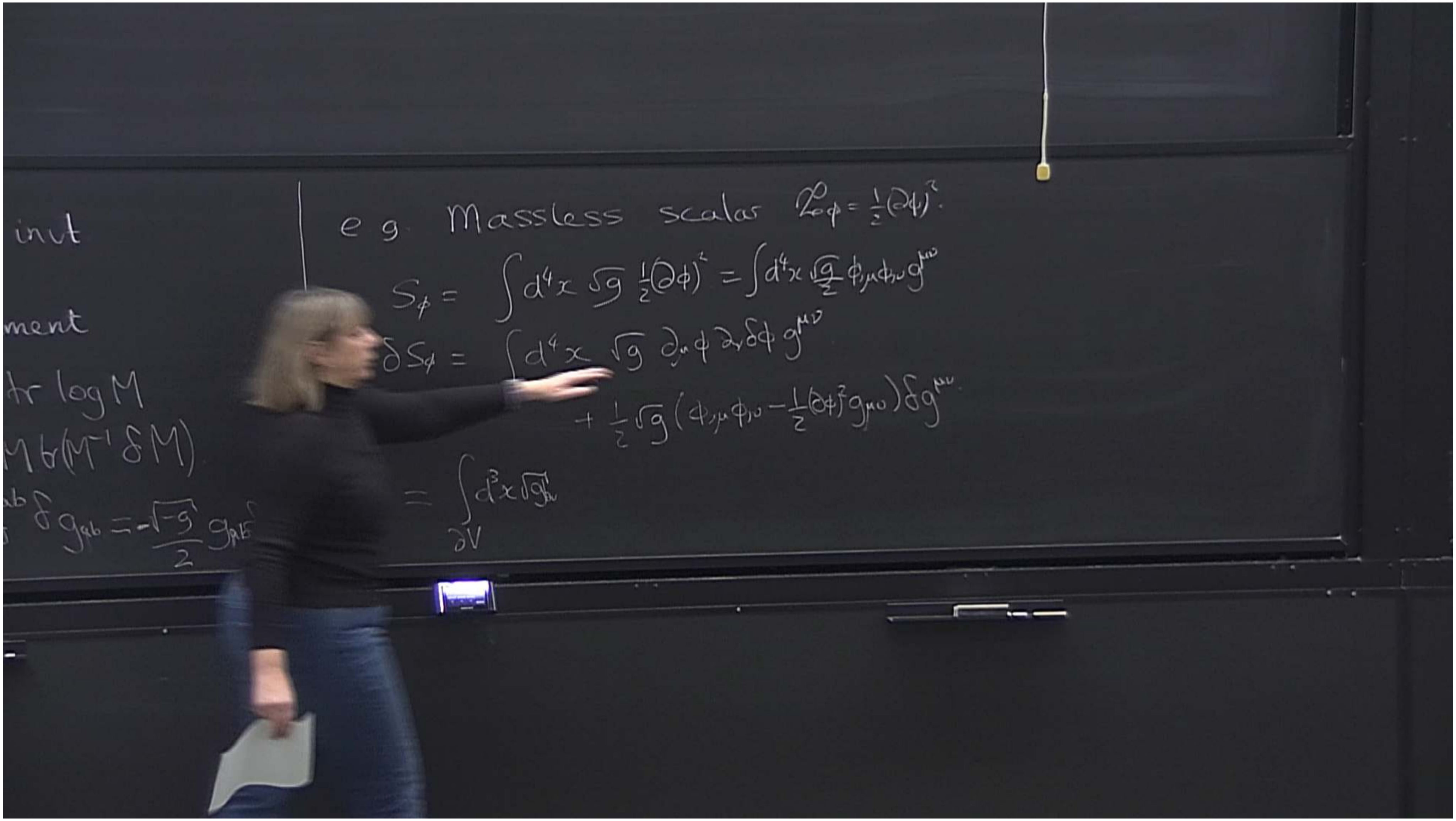
int

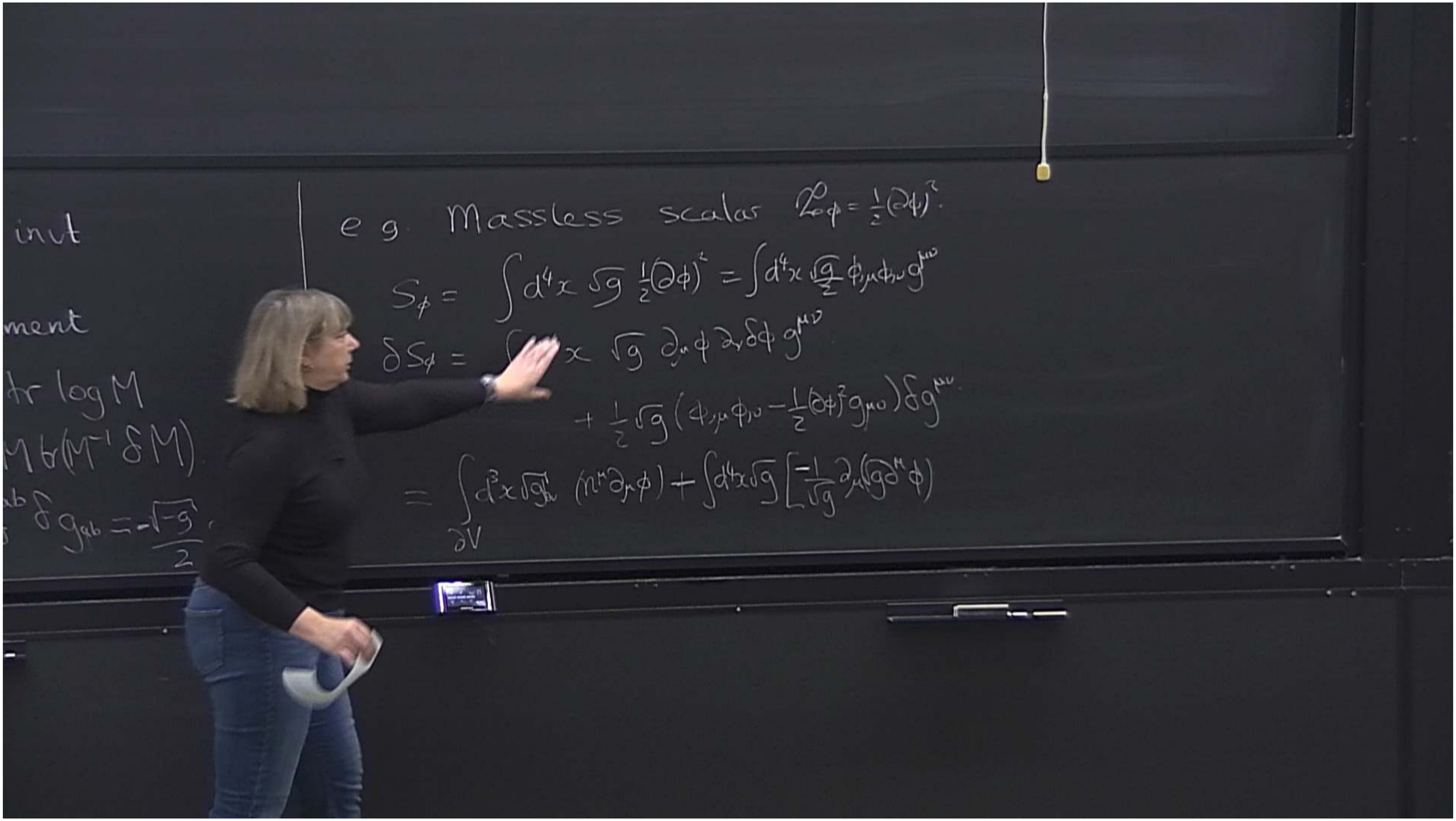
ent

$\log M$

$M\,\mathrm{tr}(M^{-1}\delta M)$

$\delta g_{ab} = -\frac{\sqrt{-g}}{2} g_{ab}\delta g^{ab}$


init
ment
tr log M
M tr(M⁻¹ δM)
δ g_ab = -√-g/2 g_ab
e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$.
$S_\phi = \int d^4x \sqrt{g}\, \frac{1}{2}(\partial\phi)^2 = \int d^4x \frac{\sqrt{g}}{2} \phi_{,\mu}\phi_{,\nu} g^{\mu\nu}$
$\delta S_\phi = \int d^4x \sqrt{g}\, \partial_\mu\phi\, \partial_\nu\delta\phi\, g^{\mu\nu}$
$+ \frac{1}{2}\sqrt{g}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}(\partial\phi)^2 g_{\mu\nu}\right)\delta g^{\mu\nu}$
$= \int_{\partial V} d^3x \sqrt{g_h}$



inut
ment
tr log M
M tr(M⁻¹ δM)
δ g_ab = -√-g / 2
e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$
$S_\phi = \int d^4x \sqrt{g}\,\frac{1}{2}(\partial\phi)^2 = \int d^4x \frac{\sqrt{g}}{2}\phi_{,\mu}\phi_{,\nu}g^{\mu\nu}$
$\delta S_\phi = \int d^4x \sqrt{g}\,\partial_\mu\phi\,\partial_\nu\delta\phi\,g^{\mu\nu}$
$+ \frac{1}{2}\sqrt{g}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}(\partial\phi)^2 g_{\mu\nu}\right)\delta g^{\mu\nu}$
$= \int_{\partial V} d^3x \sqrt{g_h}\,(n^\mu\partial_\mu\phi) + \int d^4x \sqrt{g}\left[\frac{-1}{\sqrt{g}}\partial_\mu(\sqrt{g}\,\partial^\mu\phi)\right]$

inut

ment

tr log M

$M\,\mathrm{tr}(M^{-1}\delta M)$

$^{ab}\delta g_{ab} = -\frac{\sqrt{-g}}{2} g_{ab}\delta g^{ab}$

e.g. Massless scalar $\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2$.

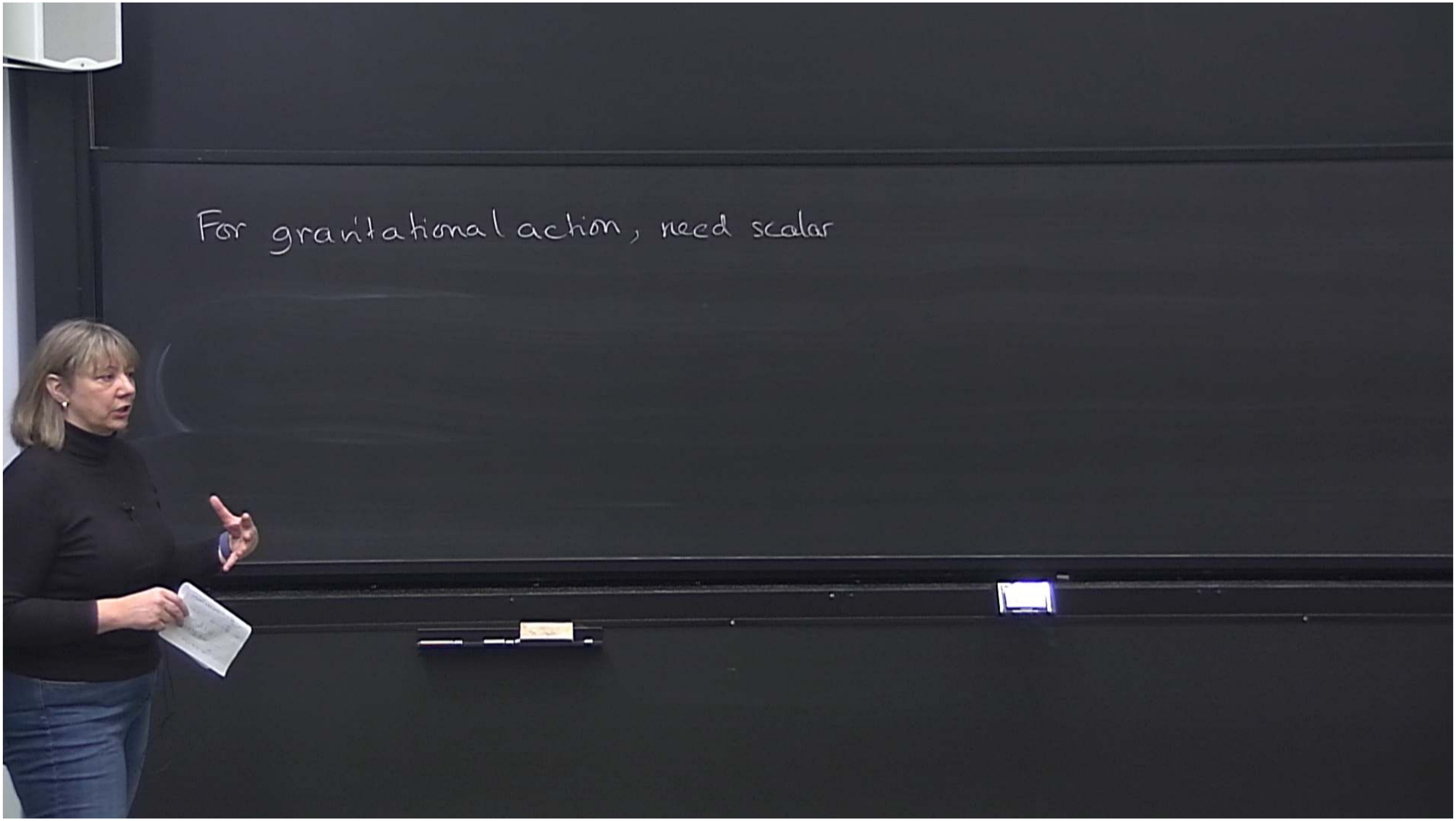
$$S_\phi = \int d^4x \sqrt{g}\, \frac{1}{2}(\partial\phi)^2 = \int d^4x \frac{\sqrt{g}}{2} \phi_{,\mu}\phi_{,\nu} g^{\mu\nu}$$

$$\delta S_\phi = \int d^4x \sqrt{g}\, \partial_\mu\phi\, \partial_\nu\delta\phi\, g^{\mu\nu}$$

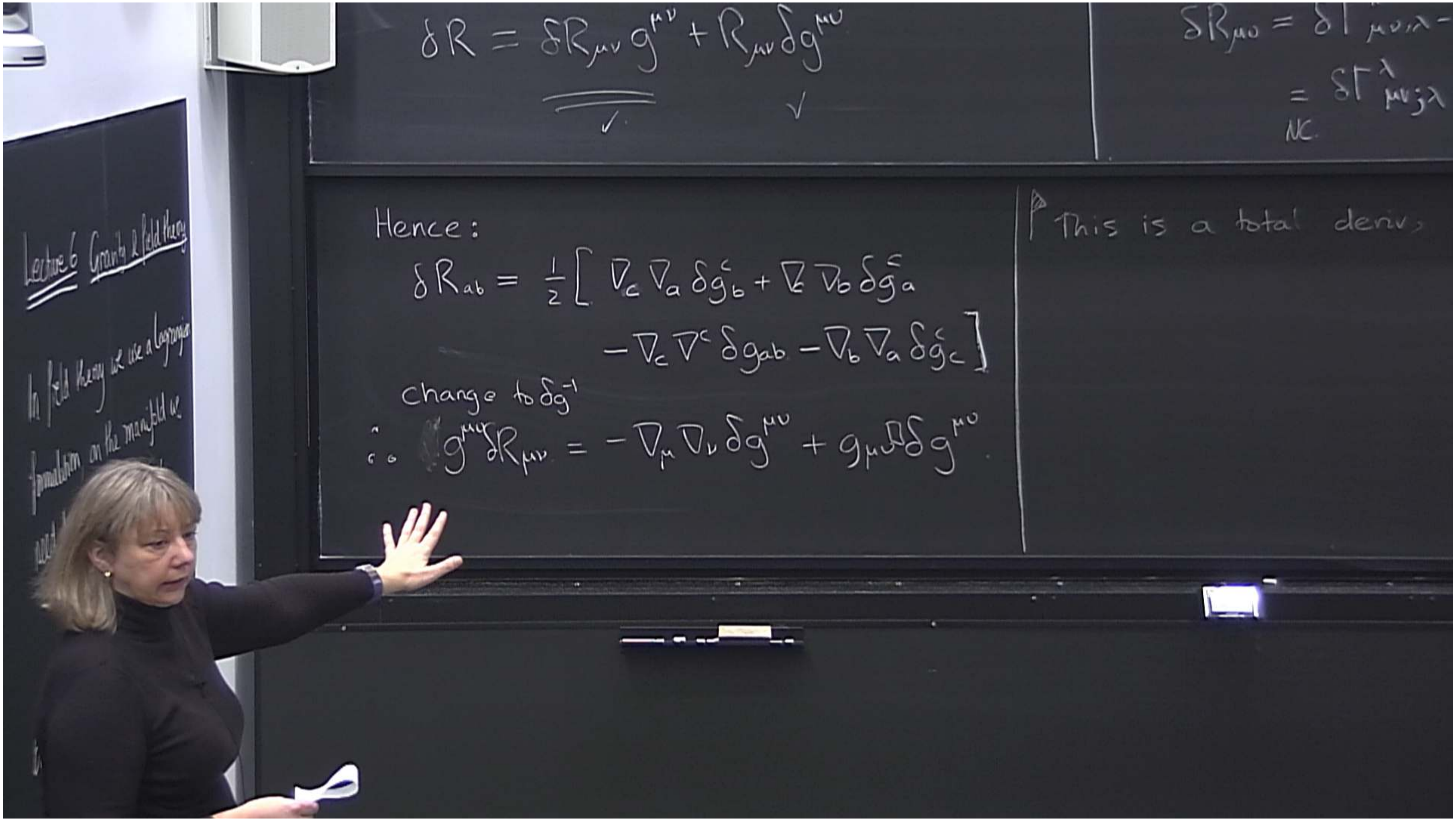
$$+ \frac{1}{2}\sqrt{g}\left(\phi_{,\mu}\phi_{,\nu} - \frac{1}{2}(\partial\phi)^2 g_{\mu\nu}\right)\delta g^{\mu\nu}$$

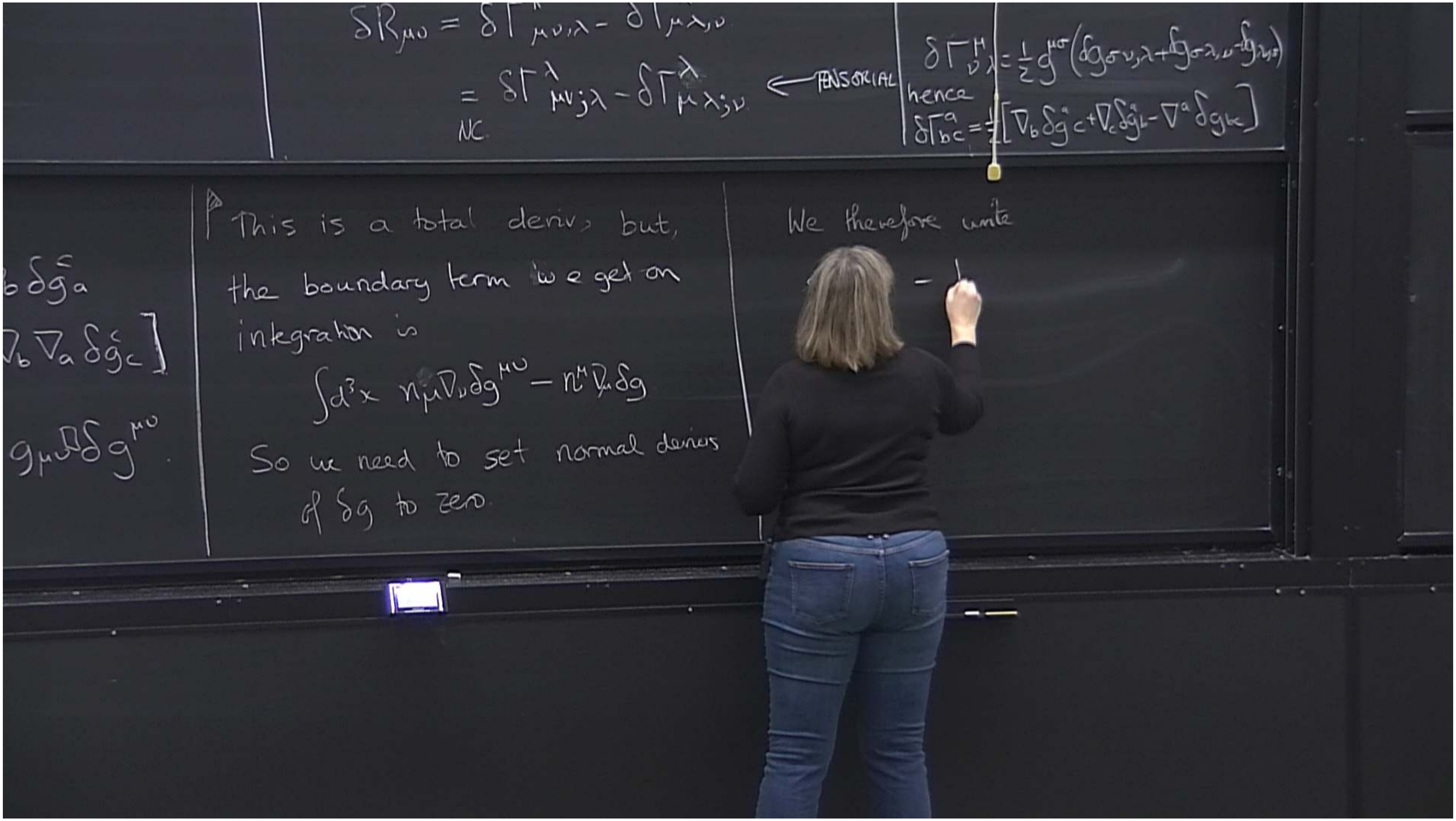
$$= \int_{\partial V} d^3x \sqrt{g_{bd}}\, (n^\mu \partial_\mu \phi) + \int d^4x \sqrt{g}\left[\frac{-1}{\sqrt{g}}\partial_\mu(\sqrt{g}\partial^\mu\phi)\delta\phi + \frac{1}{2}\left[\partial_\mu\phi\,\partial_\nu\phi - \frac{1}{2}(\partial\phi)^2 g_{\mu\nu}\right]\delta g^{\mu\nu}\right]$$

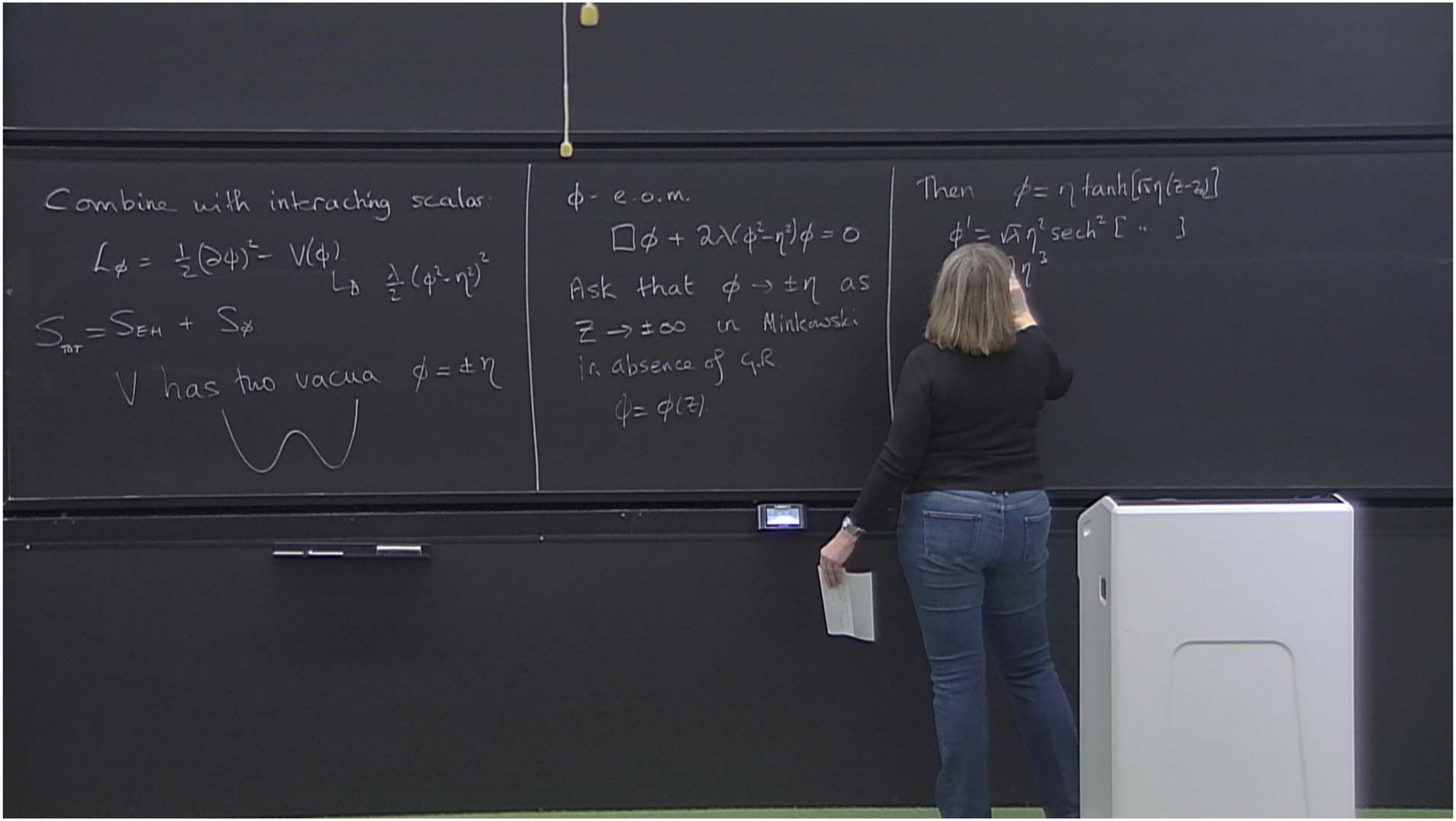
$$\frac{\delta S_\phi}{\delta\phi} =$$

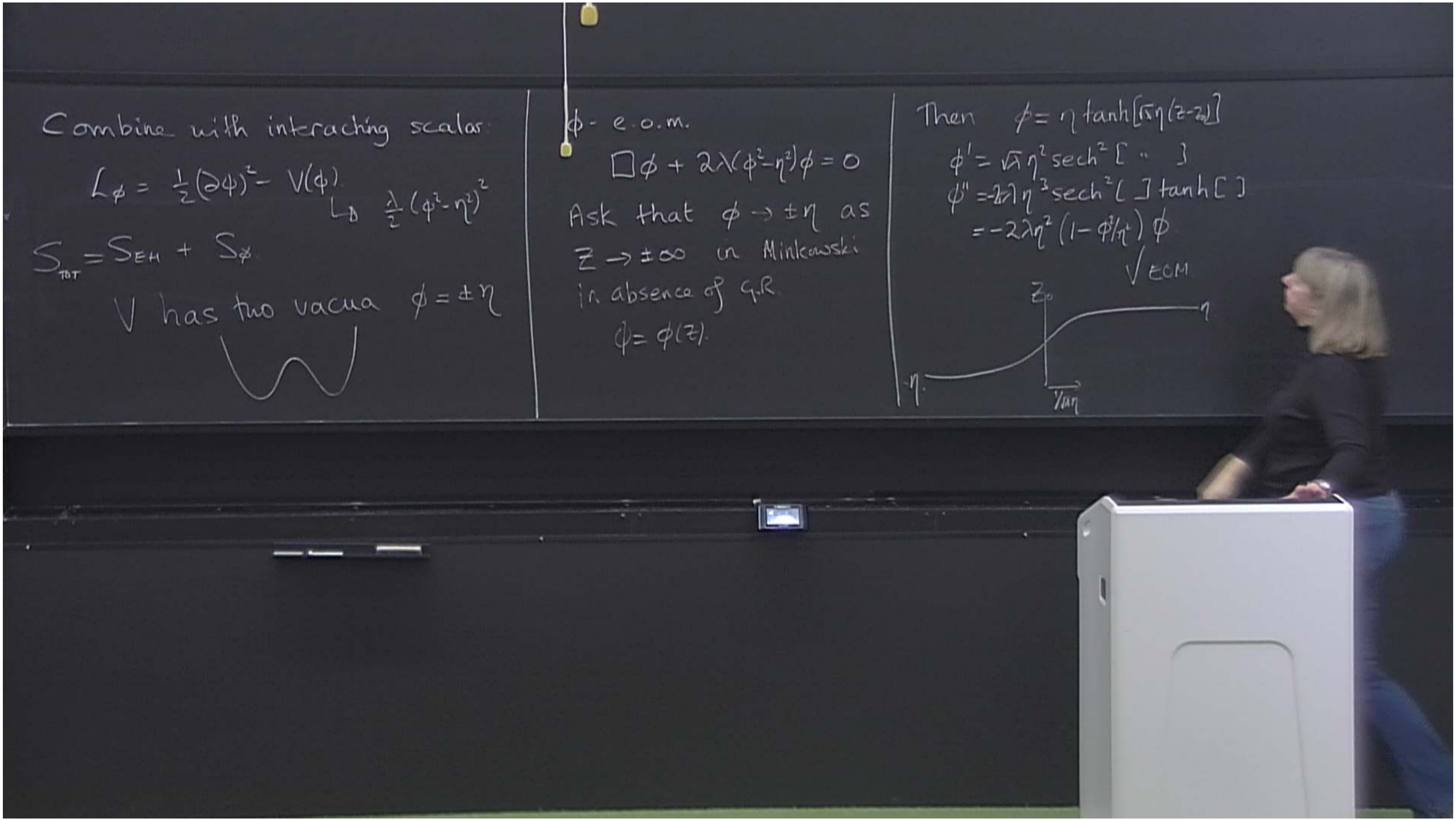

For gravitational action, need scalar

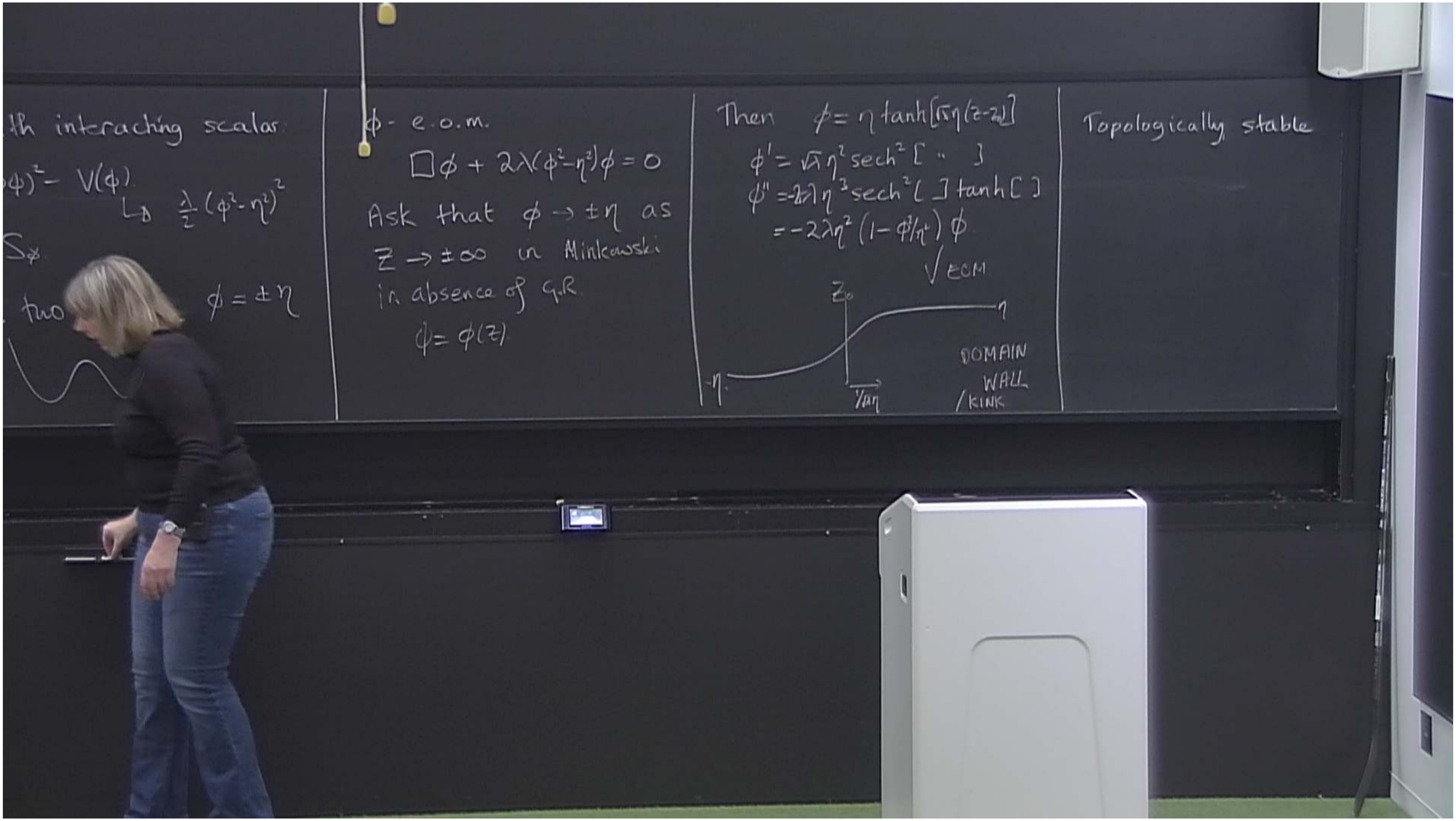
For gravitational action, need scalar

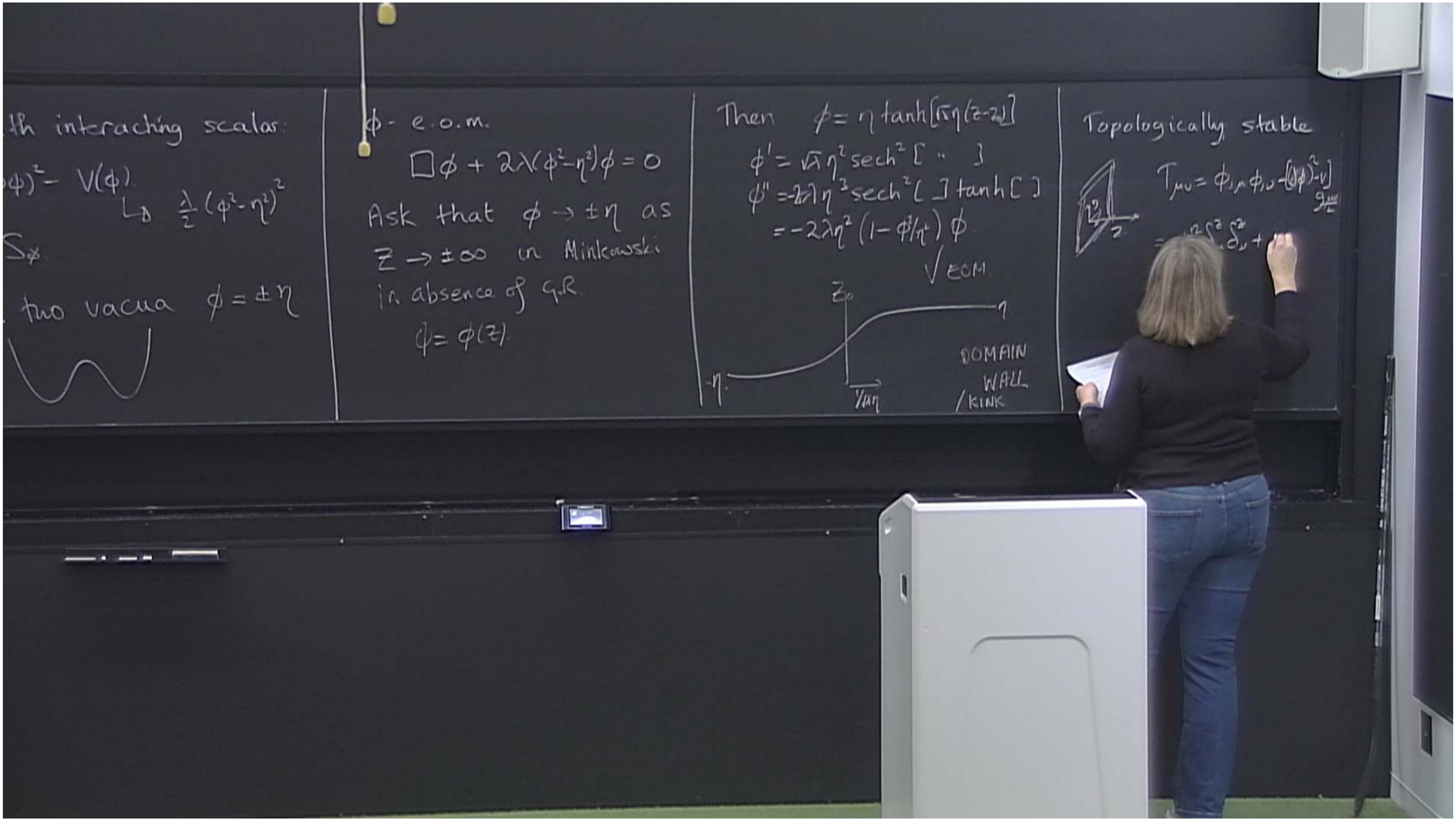

Lecture 6 Gravity & Field Theory
$\delta R = \delta R_{\mu\nu} g^{\mu\nu} + R_{\mu\nu}\delta g^{\mu\nu}$
$\delta R_{\mu\nu} = \delta\Gamma^{\lambda}_{\mu\nu;\lambda} -$
$\underset{NC}{=} \delta\Gamma^{\lambda}_{\mu\nu;\lambda}$
Hence:
$\delta R_{ab} = \frac{1}{2}\left[\nabla_c \nabla_a \delta g^c_b + \nabla_c \nabla_b \delta g^c_a - \nabla_c \nabla^c \delta g_{ab} - \nabla_b \nabla_a \delta g^c_c \right]$
Change to δg^{-1}
$\therefore g^{\mu\nu}\delta R_{\mu\nu} = -\nabla_\mu \nabla_\nu \delta g^{\mu\nu} + g_{\mu\nu}\Box\delta g^{\mu\nu}$
This is a total deriv.

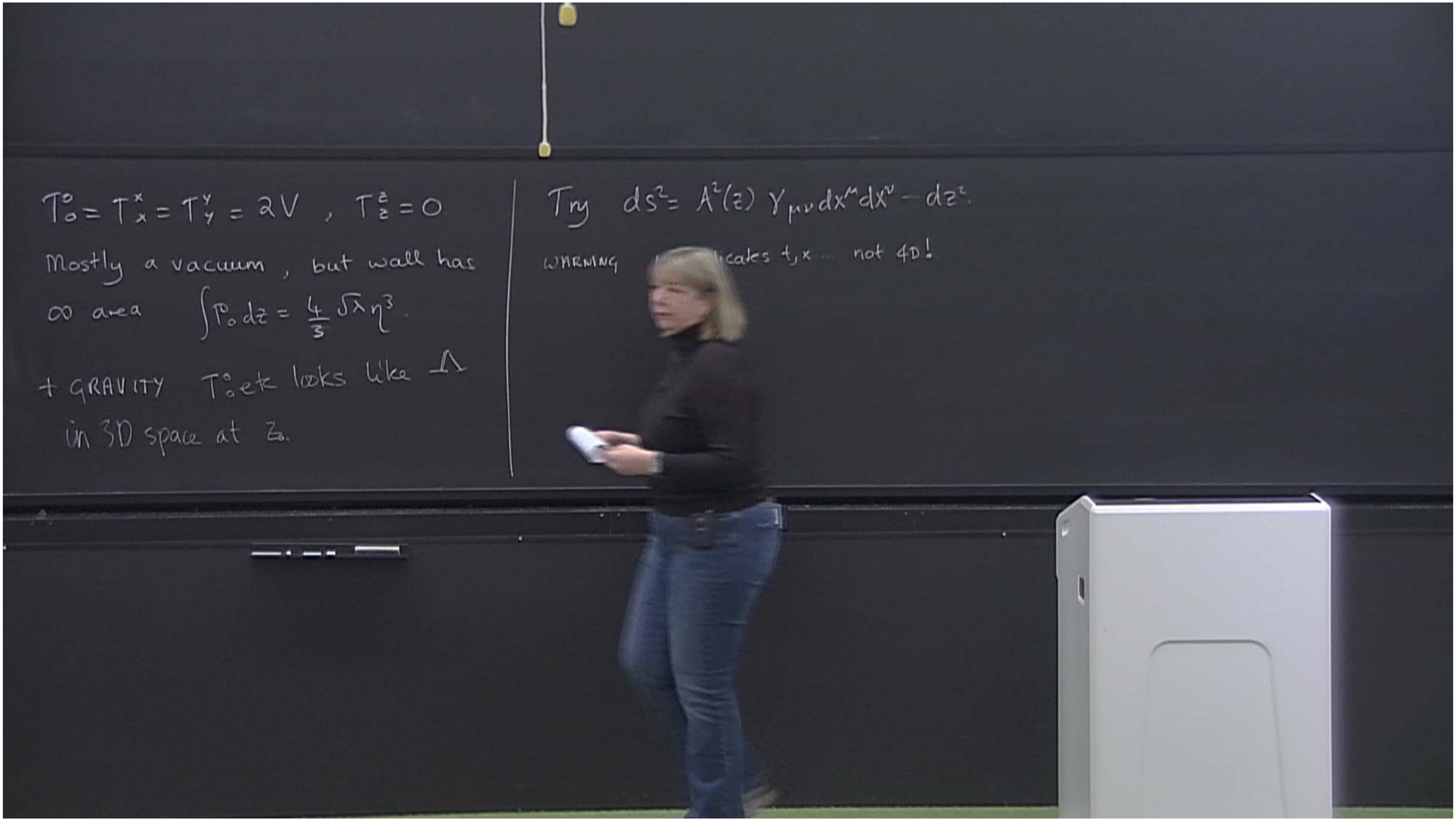

$\delta R_{\mu\nu} = \delta\Gamma^\lambda_{\mu\nu,\lambda} - \delta\Gamma^\lambda_{\mu\lambda,\nu}$
$\underset{NC}{=} \delta\Gamma^\lambda_{\mu\nu;\lambda} - \delta\Gamma^\lambda_{\mu\lambda;\nu} \longleftarrow$ TENSORIAL
$\delta\Gamma^\mu_{\nu\lambda} = \frac{1}{2} g^{\mu\sigma}(\delta g_{\sigma\nu,\lambda} + \delta g_{\sigma\lambda,\nu} - \delta g_{\nu\lambda,\sigma})$
hence
$\delta\Gamma^a_{bc} = \frac{1}{2}[\nabla_b \delta g^a{}_c + \nabla_c \delta g^a{}_b - \nabla^a \delta g_{bc}]$
$\delta g^c{}_a$
$\nabla_b \nabla_a \delta g^c{}_c]$
$g_{\mu\nu}\Box\delta g^{\mu\nu}$
This is a total deriv, but,
the boundary term we get on
integration is
$\int d^3x \; n_\mu \nabla_\nu \delta g^{\mu\nu} - n^\mu \nabla_\mu \delta g$
So we need to set normal derivs
of δg to zero.
We therefore write

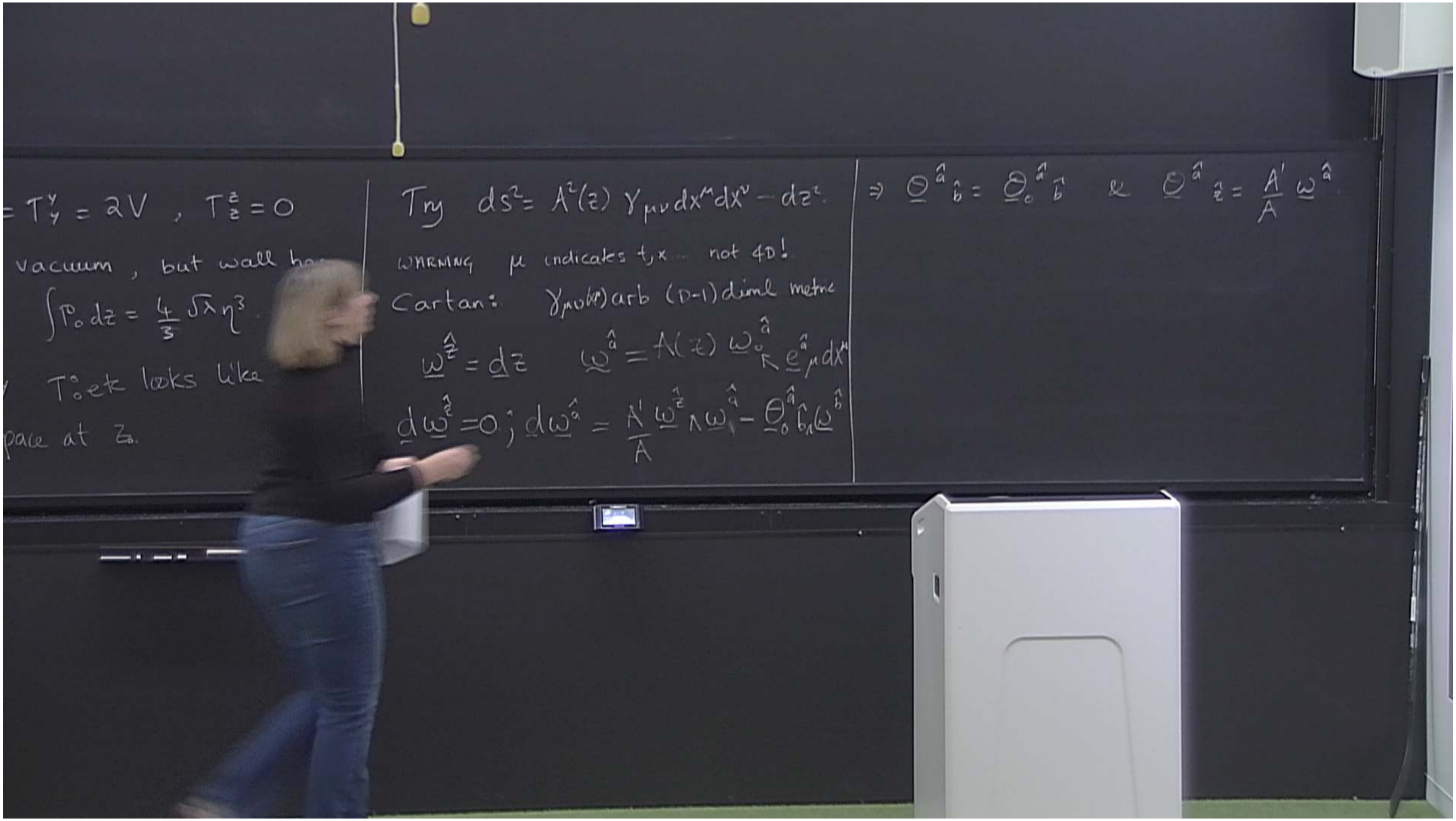

Combine with interacting scalar:
$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 - V(\phi)$
$\frac{\lambda}{2}(\phi^2-\eta^2)^2$
$S_{TOT} = S_{EH} + S_\phi$
V has two vacua $\phi = \pm\eta$
ϕ - e.o.m.
$\Box\phi + 2\lambda(\phi^2-\eta^2)\phi = 0$
Ask that $\phi \to \pm\eta$ as
$z \to \pm\infty$ in Minkowski
in absence of G.R.
$\phi = \phi(z)$
Then $\phi = \eta \tanh[\sqrt{\lambda}\eta(z-z_0)]$
$\phi' = \sqrt{\lambda}\eta^2 \operatorname{sech}^2[\ \cdot\cdot\ ]$
η^3

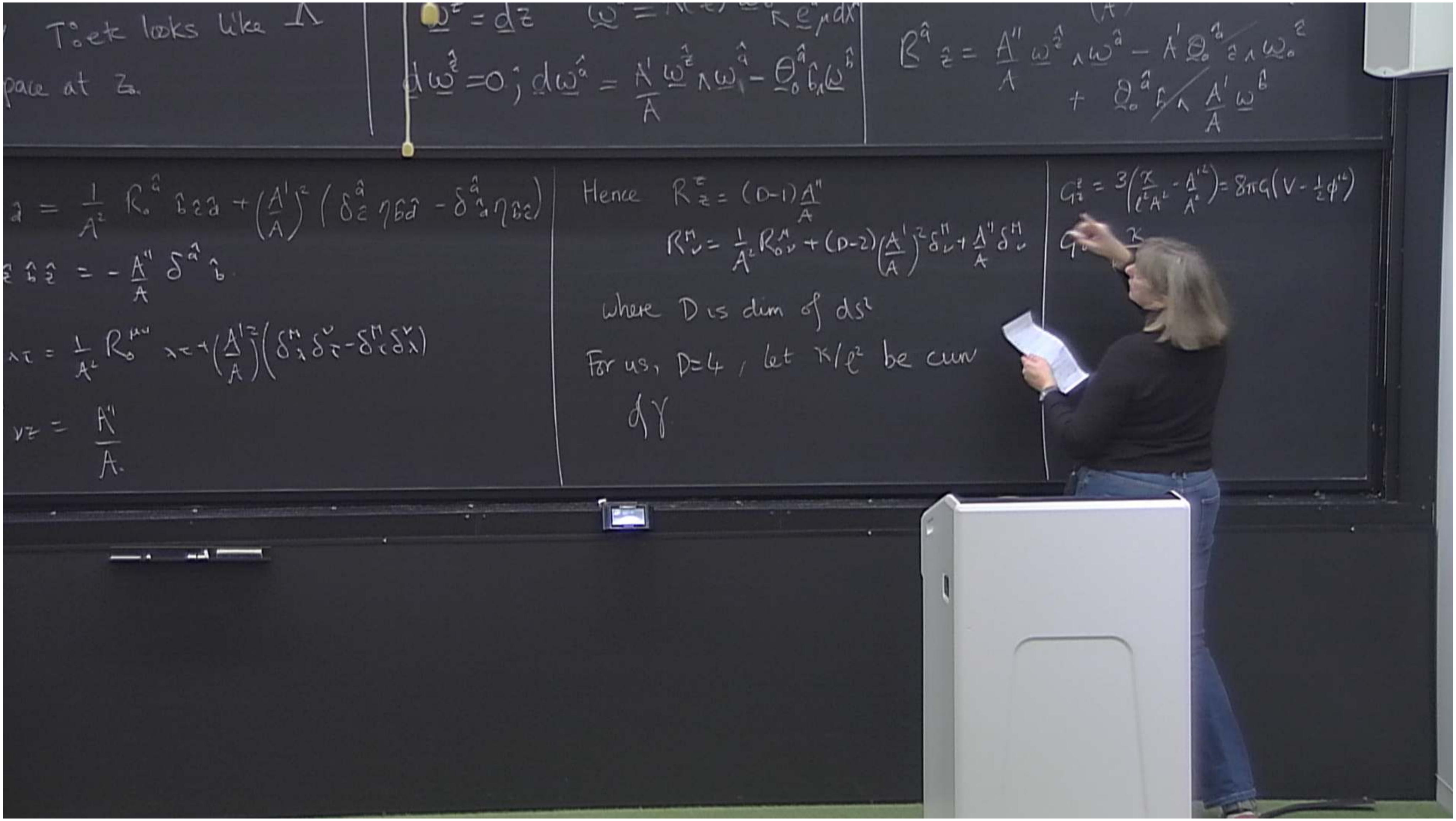

Combine with interacting scalar:
L_φ = ½(∂φ)² − V(φ)
L_φ λ/2 (φ² − η²)²
S_TOT = S_EH + S_φ
V has two vacua φ = ±η
φ - e.o.m.
□φ + 2λ(φ² − η²)φ = 0
Ask that φ → ±η as
z → ±∞ in Minkowski
in absence of G.R.
φ = φ(z).
Then φ = η tanh[√λ η(z − z₀)]
φ' = √λ η² sech²["]
φ'' = −2λ η³ sech²[ ] tanh[ ]
= −2λη² (1 − φ²/η²) φ
√ EOM
z₀
η
−η
1/√λη


th interacting scalar
$\phi)^2 - V(\phi)$
$\frac{\lambda}{2}(\phi^2-\eta^2)^2$
S_ϕ
two
$\phi = \pm\eta$
ϕ - e.o.m.
$\Box\phi + 2\lambda(\phi^2-\eta^2)\phi = 0$
Ask that $\phi \to \pm\eta$ as
$z \to \pm\infty$ in Minkowski
in absence of G.R.
$\phi = \phi(z)$
Then $\phi = \eta \tanh[\sqrt{\lambda}\eta(z-z_0)]$
$\phi' = \sqrt{\lambda}\eta^2 \operatorname{sech}^2[\ \ ]$
$\phi'' = -2\lambda\eta^3 \operatorname{sech}^2[\ ]\tanh[\ ]$
$= -2\lambda\eta^2(1-\phi^2/\eta^2)\phi$
$\surd$ EOM
z_0
η
$-\eta$
$1/\sqrt{\lambda}\eta$
DOMAIN
WALL
/KINK
Topologically stable


th interacting scalar:
ϕ - e.o.m.
□ϕ + 2λ(ϕ²-η²)ϕ = 0
Ask that ϕ → ±η as
Z → ±∞ in Minkowski
in absence of G.R.
ϕ = ϕ(z)
two vacua ϕ = ±η
Then ϕ = η tanh[√λ η(z-z₀)]
√EOM
DOMAIN
WALL
/KINK
Topologically stable


$T^0_0 = T^x_x = T^y_y = 2V$, $T^z_z = 0$
Mostly a vacuum, but wall has
∞ area $\int \rho_0 \, dz = \frac{4}{3} \sqrt{\lambda} \eta^3$.
+ GRAVITY T^0_0 etc looks like Λ
in 3D space at z_0.
Try $ds^2 = A^2(z) \gamma_{\mu\nu} dx^\mu dx^\nu - dz^2$.
WARNING
icates t, x ... not 4D!


$= T^y_y = 2V \; , \; T^z_z = 0$
vacuum , but wall ha
$\int \rho_0 \, dz = \frac{4}{3} \sqrt{\lambda} \eta^3$
T^0_0 etc looks like
pace at z_0.
Try $ds^2 = A^2(z) \gamma_{\mu\nu} dx^\mu dx^\nu - dz^2$.
WARNING μ indicates $t, x \ldots$ not 4D!
Cartan: $\gamma_{\mu\nu}(x^\mu)$ arb $(D-1)$ diml metric
$\underline{\omega}^{\hat{z}} = \underline{d}z \quad \underline{\omega}^{\hat{a}} = A(z) \underline{\omega}_0^{\hat{a}} \leftarrow \underline{e}^{\hat{a}}_\mu dx^\mu$
$\underline{d}\underline{\omega}^{\hat{z}} = 0 \; ; \; \underline{d}\underline{\omega}^{\hat{a}} = \frac{A'}{A} \underline{\omega}^{\hat{z}} \wedge \underline{\omega}^{\hat{a}} - \underline{\theta}_0{}^{\hat{a}}{}_{\hat{b}} \wedge \underline{\omega}^{\hat{b}}$
$\Rightarrow \underline{\theta}^{\hat{a}}{}_{\hat{b}} = \underline{\theta}_0{}^{\hat{a}}{}_{\hat{b}} \quad \& \quad \underline{\theta}^{\hat{a}}{}_{\hat{z}} = \frac{A'}{A} \underline{\omega}^{\hat{a}}$


T₀ looks like Λ
pace at z₀.
Hence
where D is dim of ds²
For us, D=4, let κ/ℓ² be curv

ϕ_{kink}

$$A = 1 - \epsilon\left(\frac{2}{3}\log\cosh z - \frac{1}{6}\operatorname{sech}^2 z + \frac{2}{3}\ln 2\right) \quad \text{in } 1 \mp z/\ell$$

This is a total deriv, but, the boundary term we get on integration is

$$\int d^3x \; n_\mu \nabla_\nu \delta g^{\mu\nu} - n^\mu \nabla_\mu \delta g$$

So we need to set normal derivs of δg to zero.

We therefore write

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x \sqrt{g}\, R$$

&

$$\frac{\delta S_{EH}}{\delta g^{\mu\nu}} = -\frac{1}{16\pi G}\left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}\right)$$