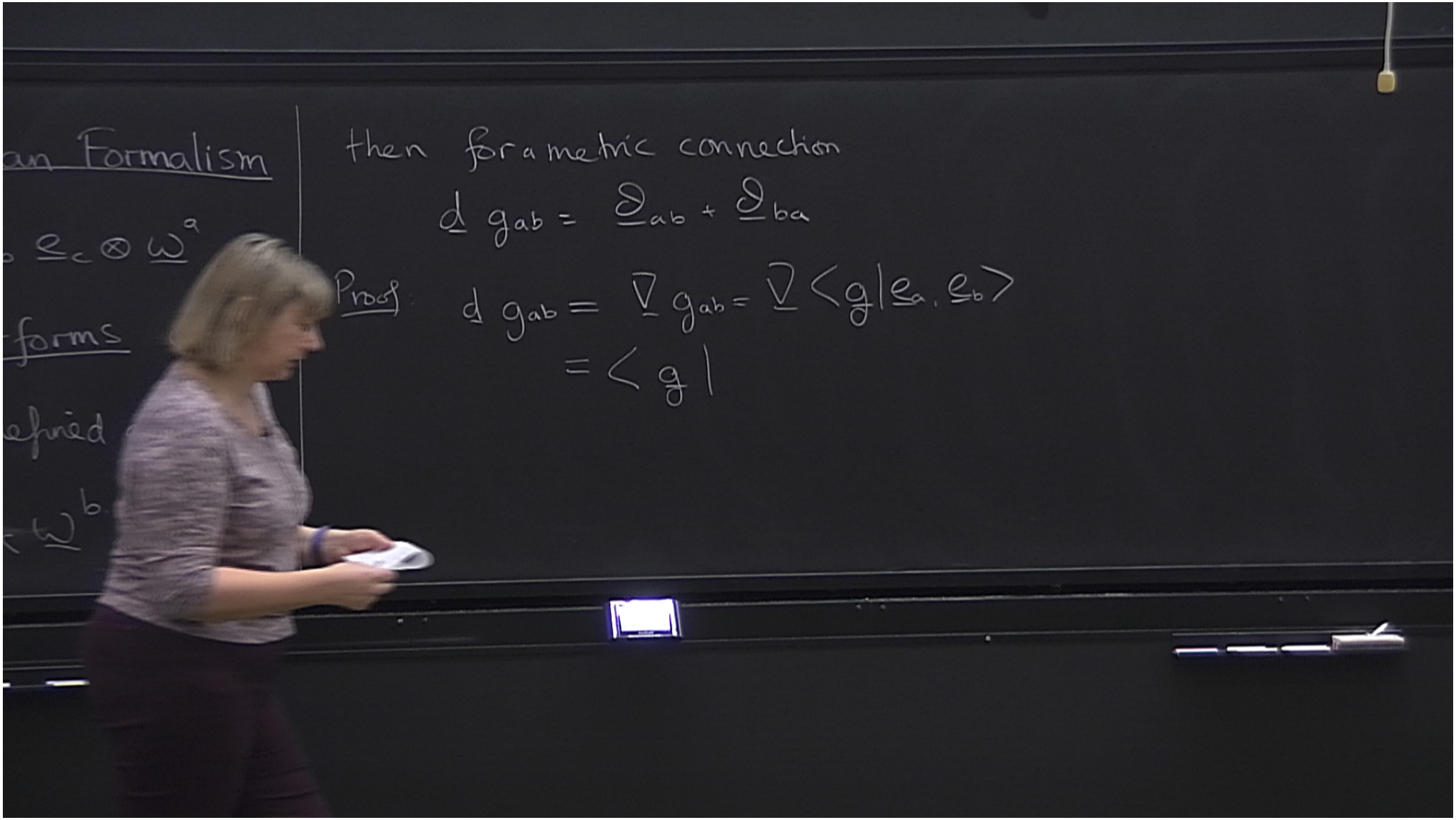
Title: Gravitational Physics-4

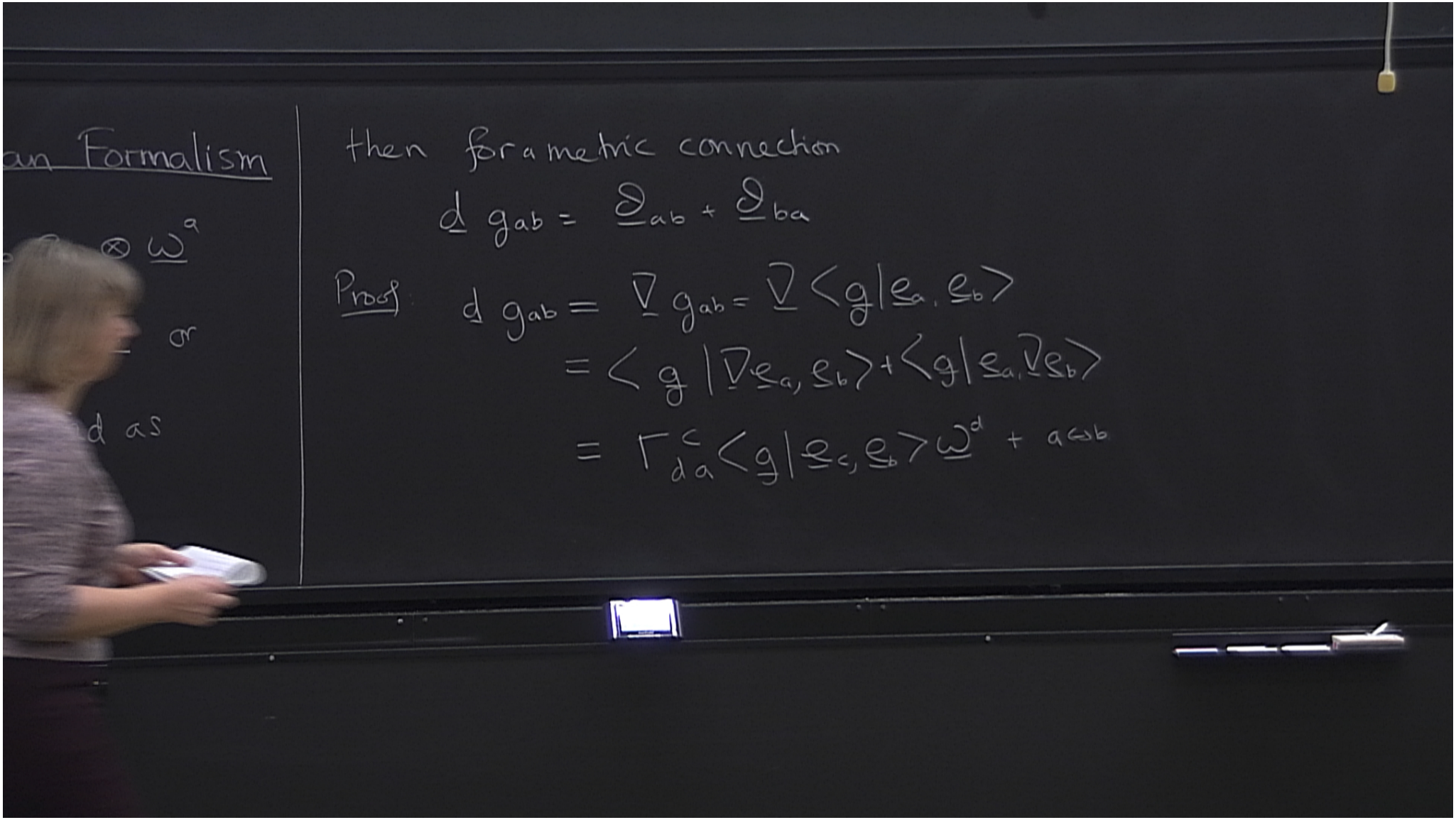
Date: Jan 06, 2015 10:15 AM

URL: http://pirsa.org/15010020

Abstract:




an Formalism
$e_c \otimes \omega^a$
forms
efined
ω^b
then for a metric connection
$\underline{d}\, g_{ab} = \underline{\partial}_{ab} + \underline{\partial}_{ba}$
Proof $\underline{d}\, g_{ab} = \underline{\nabla}\, g_{ab} = \underline{\nabla} \langle g | e_a, e_b \rangle$
$= \langle g |$


an Formalism
⊗ ω^a
or
d as
then for a metric connection
d g_ab = ∂_ab + ∂_ba
Proof d g_ab = ∇ g_ab = ∇ <g|e_a, e_b>
= <g|∇e_a, e_b> + <g|e_a, ∇e_b>
= Γ^c_da <g|e_c, e_b> ω^d + a<>b

sm

then for a metric connection

$$\underline{d}\, g_{ab} = \underline{\theta}_{ab} + \underline{\theta}_{ba}$$

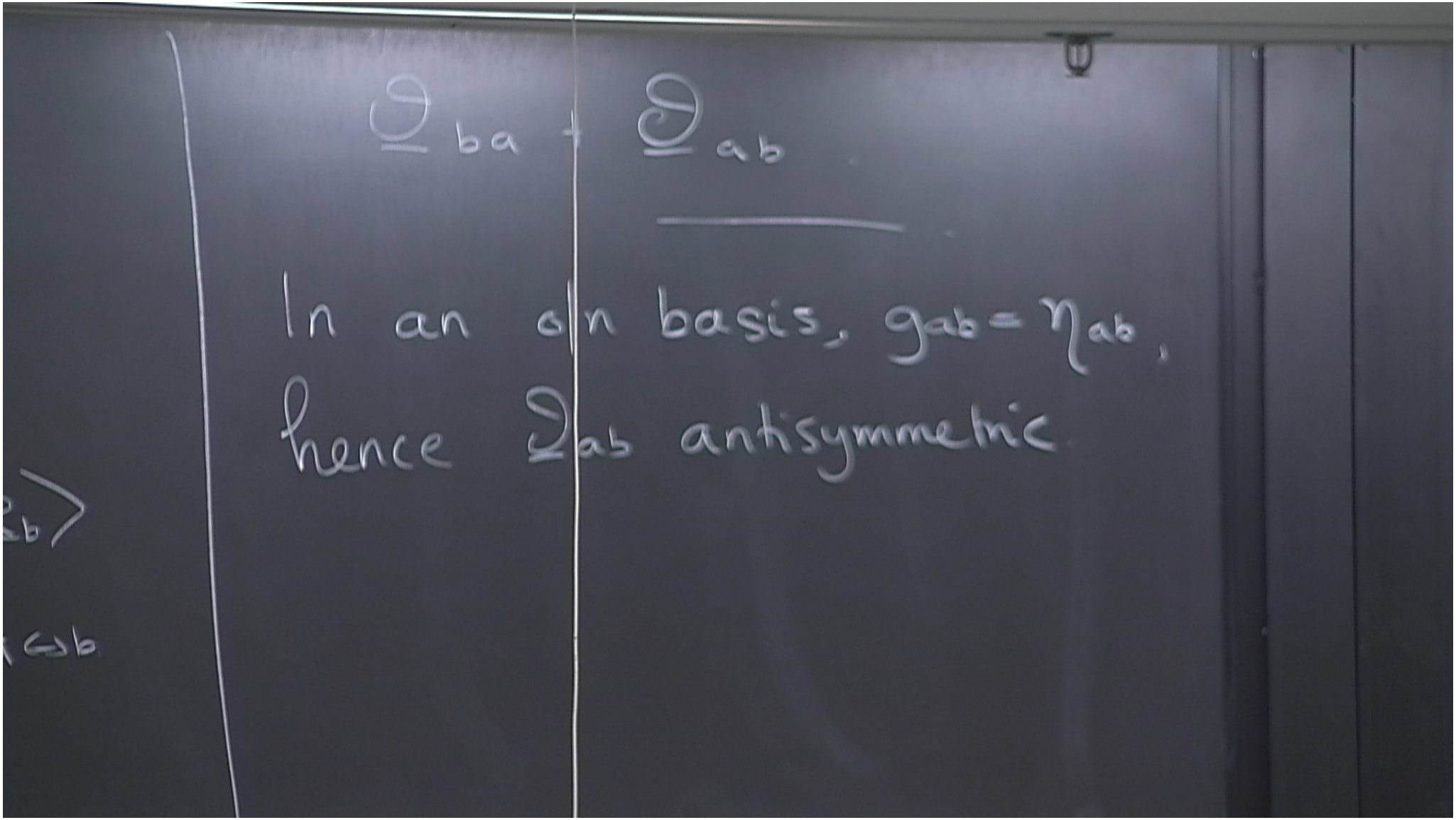
Proof

$$\underline{d}\, g_{ab} = \underline{\nabla}\, g_{ab} = \underline{\nabla} \langle g | \underline{e}_a, \underline{e}_b \rangle$$

$$= \langle g | \underline{\nabla} \underline{e}_a, \underline{e}_b \rangle + \langle g | \underline{e}_a, \underline{\nabla} \underline{e}_b \rangle$$

$$= \Gamma^{c}_{da} \langle g | \underline{e}_c, \underline{e}_b \rangle \underline{\omega}^d + a \leftrightarrow b$$

$$= g_{cb}\, \underline{\theta}^{c}_{a} + a \leftrightarrow b.$$


$\Sigma_{ba} + \Sigma_{ab}$
In an o|n basis, $g_{ab} = \eta_{ab}$,
hence Σ_{ab} antisymmetric

Cartan's 1st eqn

Consider $\underline{\Theta}^a{}_c \wedge \underline{\omega}^c$

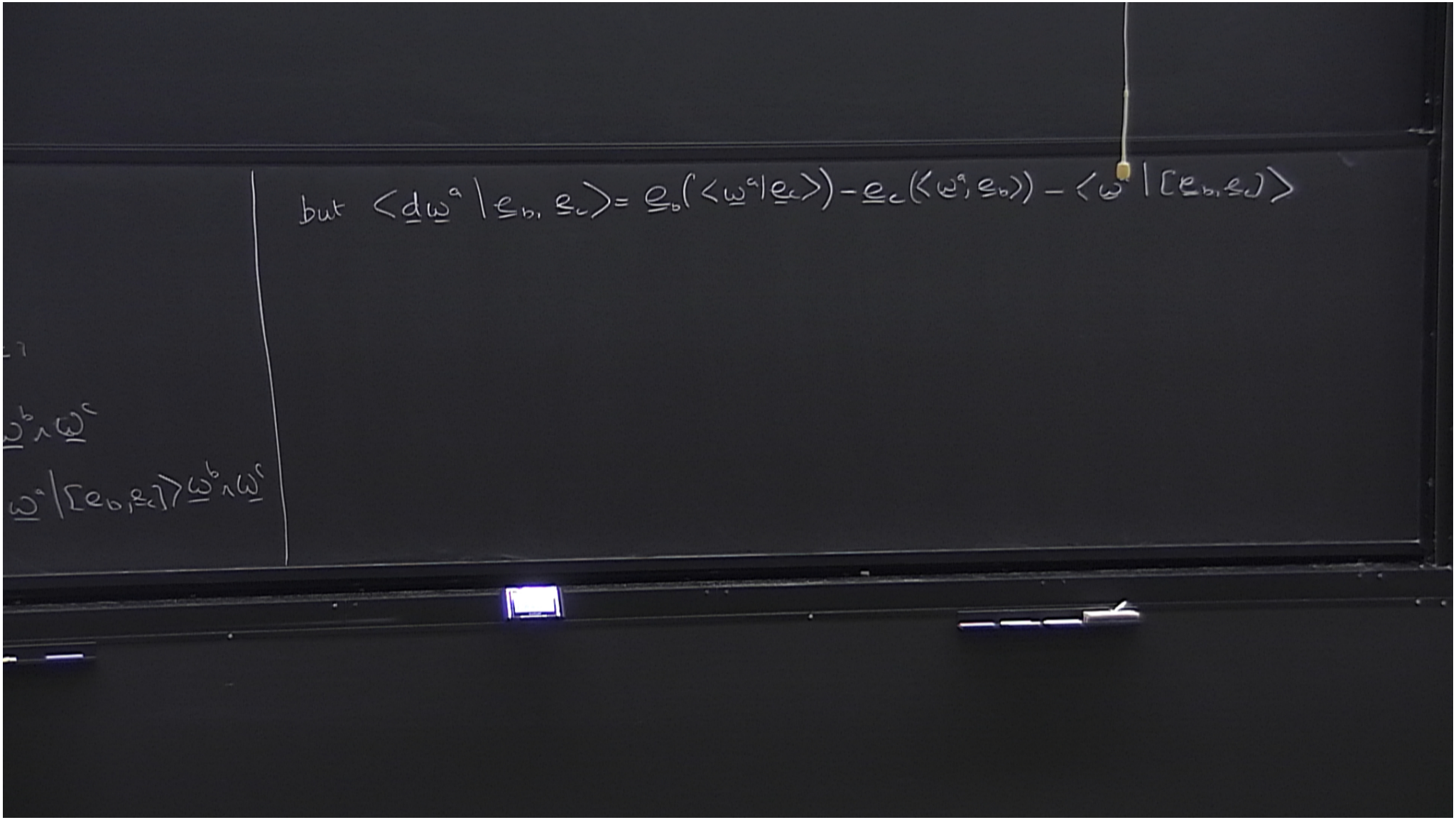
$$= \Gamma_{[bc]}{}^a \underline{\omega}^b \wedge \underline{\omega}^c$$

$$= \frac{1}{2}\left(T^a_{bc} + C^a_{bc}\right)\underline{\omega}^b \wedge \underline{\omega}^c$$

$$= \underline{T}^a + \frac{1}{2}\langle \underline{\omega}^a \,|\, [e_b, e_c] \rangle \underline{\omega}^b \wedge \underline{\omega}^c$$

$\underline{T}^a = \frac{1}{2} T^a_{bc} \underline{\omega}^b \wedge \underline{\omega}^c$ ← torsion 2-form

but


but $\langle \underline{d}\underline{\omega}^a \mid \underline{e}_b, \underline{e}_c \rangle = \underline{e}_b(\langle \underline{\omega}^a \mid \underline{e}_c \rangle) - \underline{e}_c(\langle \omega^a, \underline{e}_b \rangle) - \langle \omega^a \mid [\underline{e}_b, \underline{e}_c] \rangle$
$\underline{\omega}^b \wedge \underline{\omega}^c$
$\underline{\omega}^a \mid [\underline{e}_b, \underline{e}_c] \rangle \underline{\omega}^b \wedge \underline{\omega}^c$

but $\langle \underline{d}\underline{\omega}^a \,|\, \underline{e}_b, \underline{e}_c \rangle = \underline{e}_b(\langle \underline{\omega}^a | \underline{e}_c \rangle) - \underline{e}_c(\langle \underline{\omega}^a | \underline{e}_b \rangle) - \langle \underline{\omega}^a \,|\, [\underline{e}_b, \underline{e}_c] \rangle$

($\langle \underline{\omega}^a | \underline{e}_c \rangle = \delta^a_c$)

hence $\underline{T}^a = \underline{\theta}^a{}_{c\wedge} \underline{\omega}^c + \frac{1}{2} \langle \underline{d}\underline{\omega}^a \,|\, \underline{e}_b, \underline{e}_c \rangle \, \underline{\omega}^b \wedge \underline{\omega}^c$

$= \underline{\theta}^a{}_{c\wedge} \underline{\omega}^c + \underline{d}\underline{\omega}^a$

$\underline{\omega}^b \wedge \underline{\omega}^c$

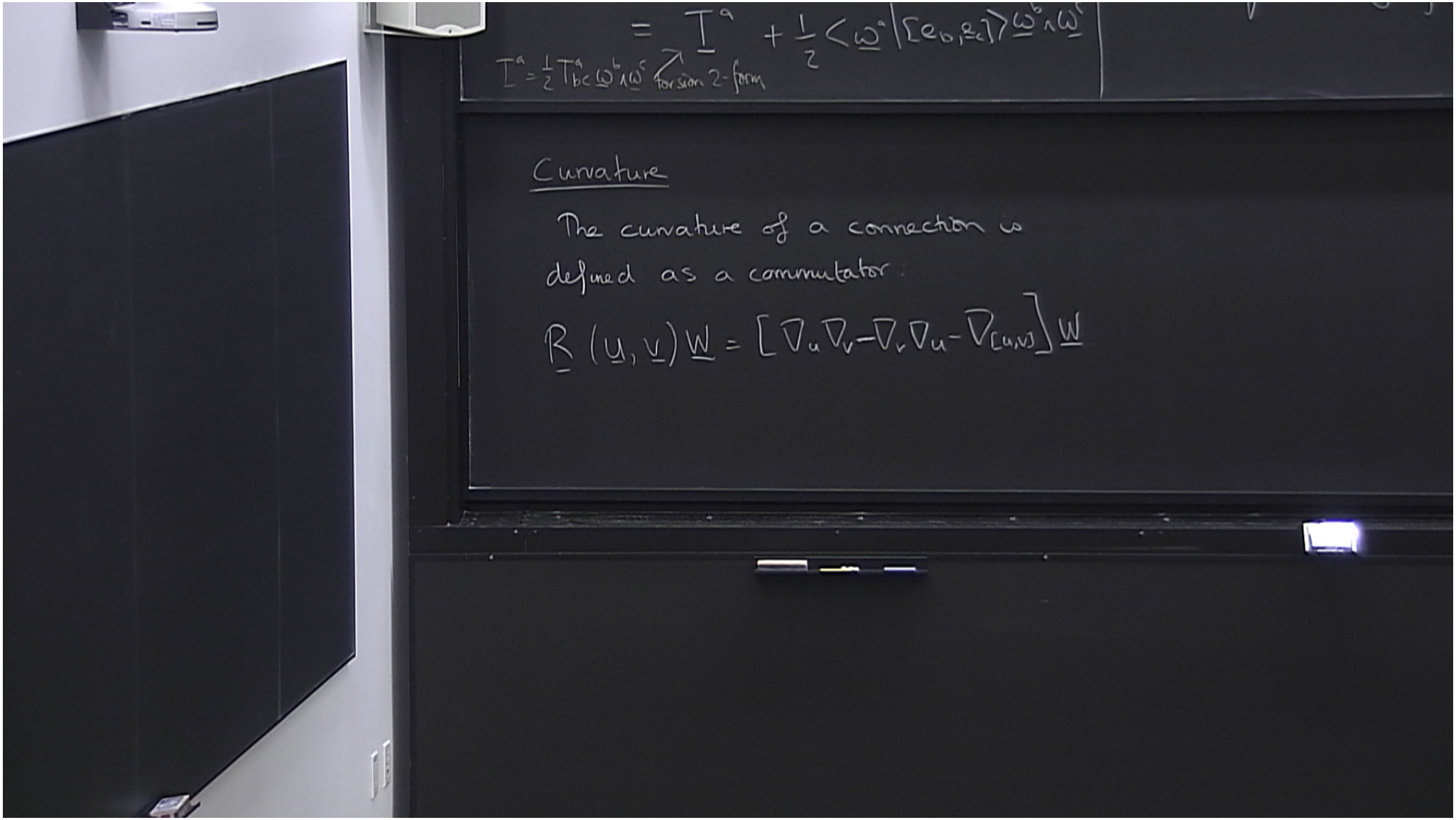
$\underline{\omega}^a | [\underline{e}_b, \underline{e}_c] \rangle \underline{\omega}^b \wedge \underline{\omega}^c$

but $\langle \underline{d}\underline{\omega}^a \,|\, \underline{e}_b, \underline{e}_c \rangle = \underline{e}_b(\langle \underline{\omega}^a \,|\, \underline{e}_c \rangle) - \underline{e}_c(\langle \underline{\omega}^a \,|\, \underline{e}_b \rangle) - \langle \underline{\omega}^a \,|\, [\underline{e}_b, \underline{e}_c] \rangle$

($\langle \underline{\omega}^a \,|\, \underline{e}_c \rangle = \delta^a_c$)

hence $\underline{T}^a = \underline{\theta}^a{}_{c\wedge}\, \underline{\omega}^c + \frac{1}{2} \langle \underline{d}\underline{\omega}^a \,|\, \underline{e}_b, \underline{e}_c \rangle\, \underline{\omega}^b \wedge \underline{\omega}^c$

$= \underline{\theta}^a{}_{c\wedge} \underline{\omega}^c + \underline{d}\underline{\omega}^a$


$= \underline{T}^a + \frac{1}{2} \langle \underline{\omega}^a | [e_b, e_c] \rangle \underline{\omega}^b \wedge \underline{\omega}^c$
$\underline{T}^a = \frac{1}{2} T^a_{bc} \underline{\omega}^b \wedge \underline{\omega}^c$ torsion 2-form
Curvature
The curvature of a connection is defined as a commutator:
$\underline{R}(\underline{u}, \underline{v}) \underline{W} = [\nabla_u \nabla_v - \nabla_v \nabla_u - \nabla_{[u,v]}] \underline{W}$

$L = \frac{1}{2} T^{a}_{bc} \underline{\omega}^{b} \wedge \underline{\omega}^{c}$ torsion 2-form

Curvature

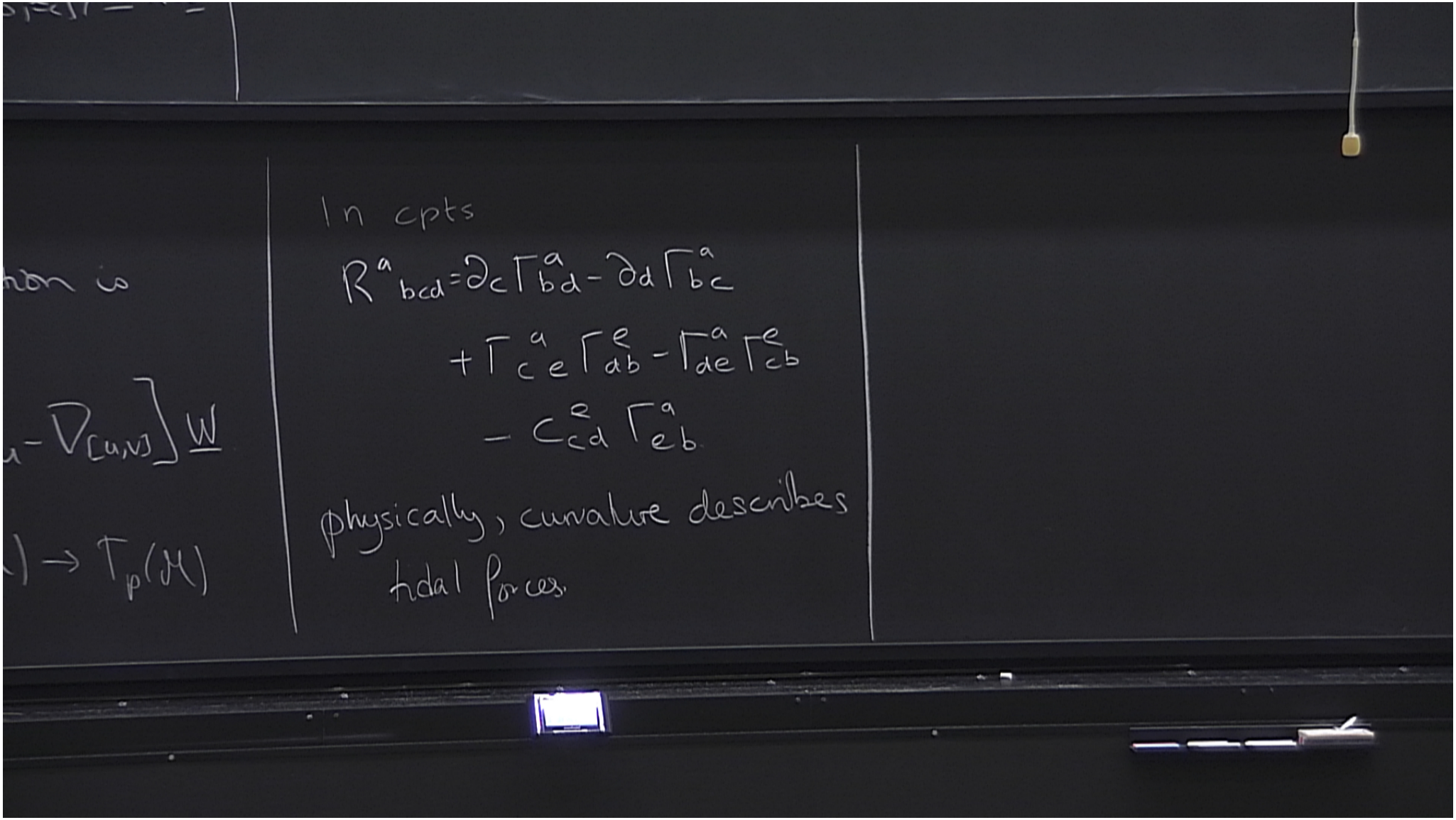
The curvature of a connection is defined as a commutator:

$$\underline{R}(\underline{u}, \underline{v})\underline{W} = [\nabla_u \nabla_v - \nabla_v \nabla_u - \nabla_{[u,v]}]\underline{W}$$

is a map $T_p(M) \times T_p(M) \times T_p(M) \to T_p(M)$

In

R^{a}


In cpts
$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc}$
$+ \Gamma^a_{ce} \Gamma^e_{ab} - \Gamma^a_{de} \Gamma^e_{cb}$
$- C^e_{cd} \Gamma^a_{eb}$
physically, curvature describes
tidal forces.

on is

$-\nabla_{[u,v]}]\underline{W}$

$\to T_p(\mathcal{M})$

In cpts

$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc}$$
$$+ \Gamma^a_{ce}\Gamma^e_{ab} - \Gamma^a_{de}\Gamma^e_{cb}$$
$$- C^e_{cd}\Gamma^a_{eb}$$

physically, curvature describes tidal forces.

Car

Defn:

Cartan's 2nd eqn

Defn: The curvature 2-form is defined as

$$\underline{R}^a{}_b = \frac{1}{2} R^a{}_{b c d}\, \underline{\omega}^c \wedge \underline{\omega}^d$$

Cartan's 2nd eqn

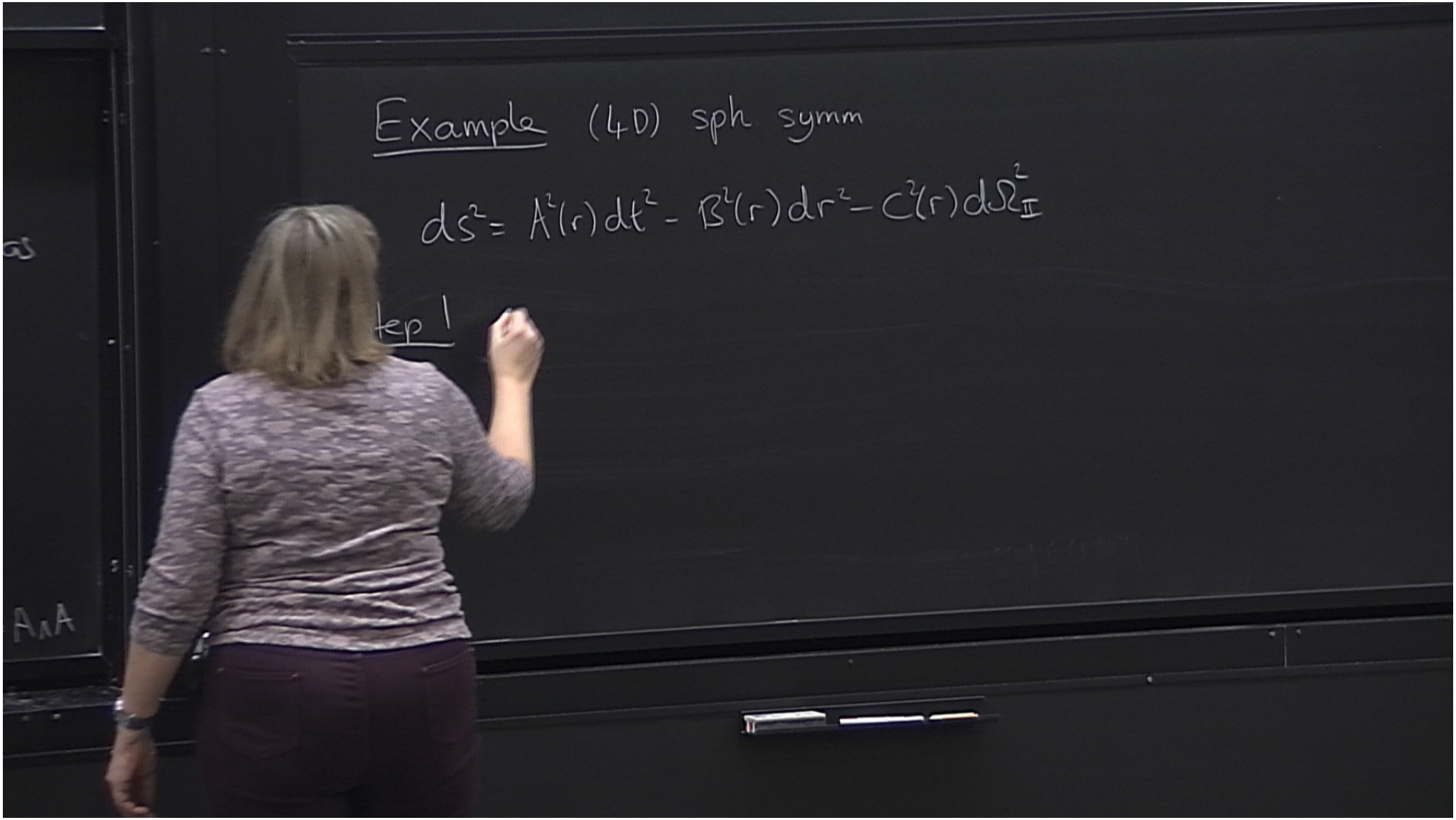
Defn: The curvature 2-form is defined as

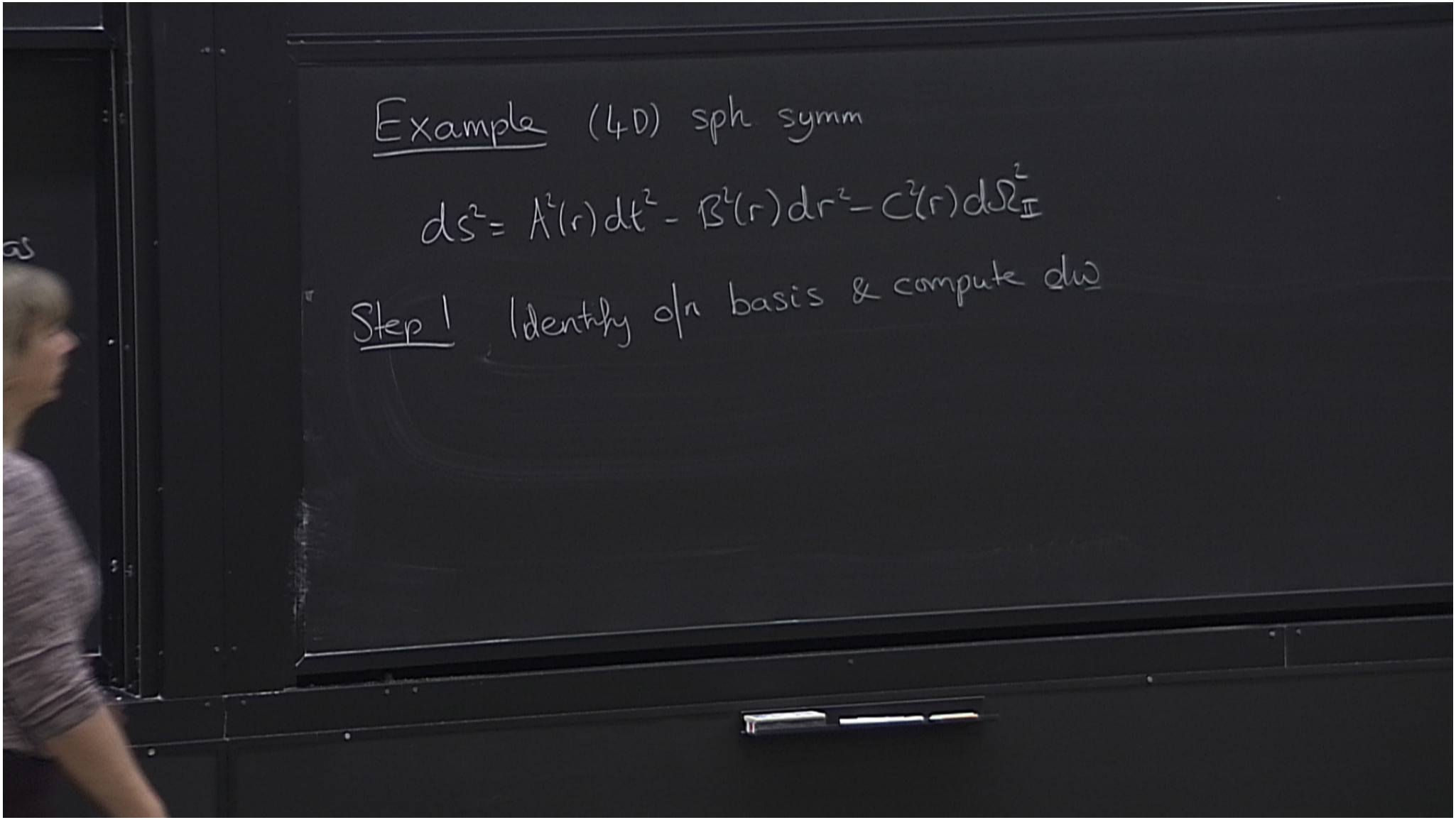
$$\underline{R}^a{}_b = \frac{1}{2} R^a{}_{bcd}\, \underline{\omega}^c \wedge \underline{\omega}^d$$

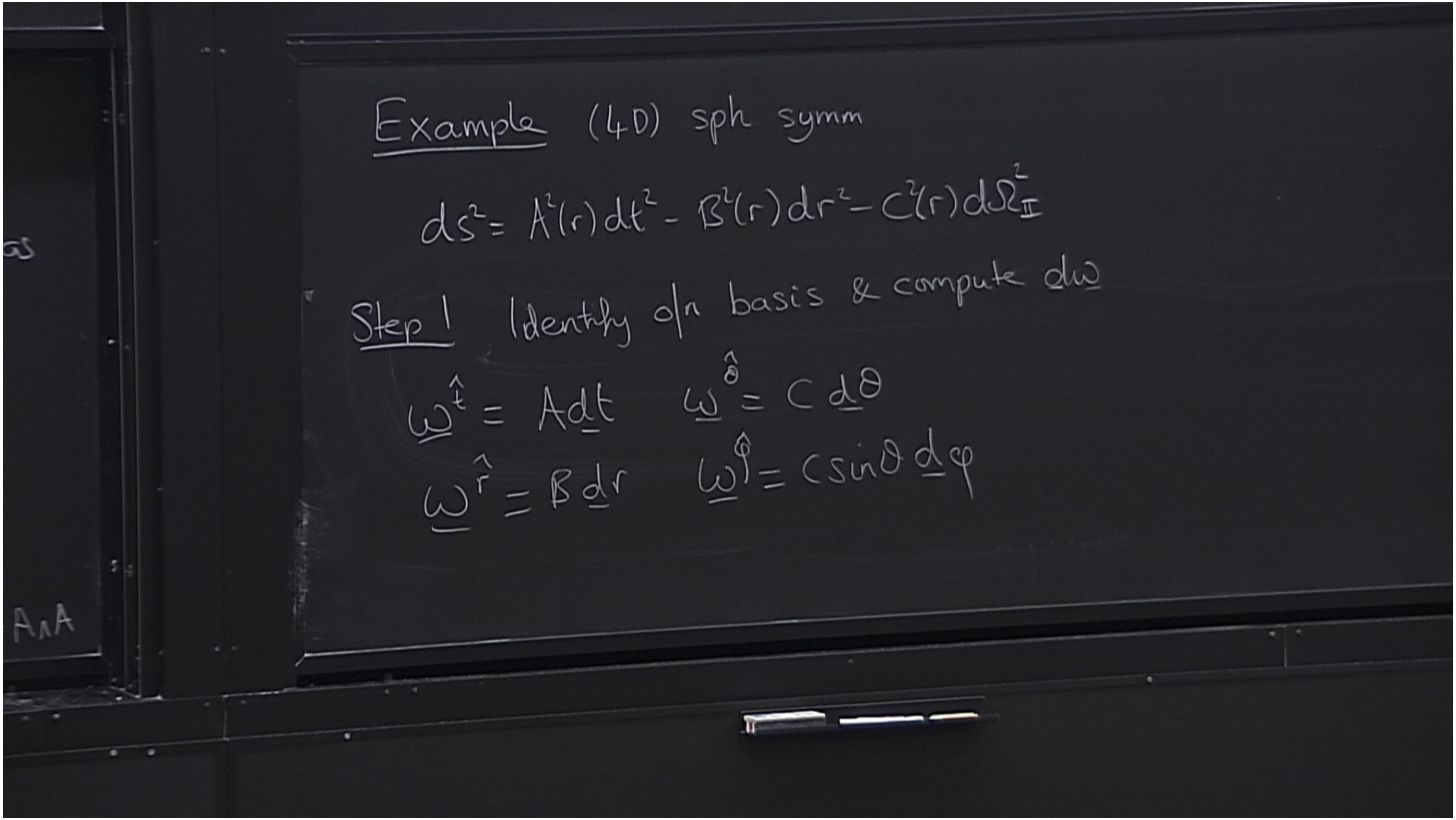
Then Cartan's 2nd eqn is

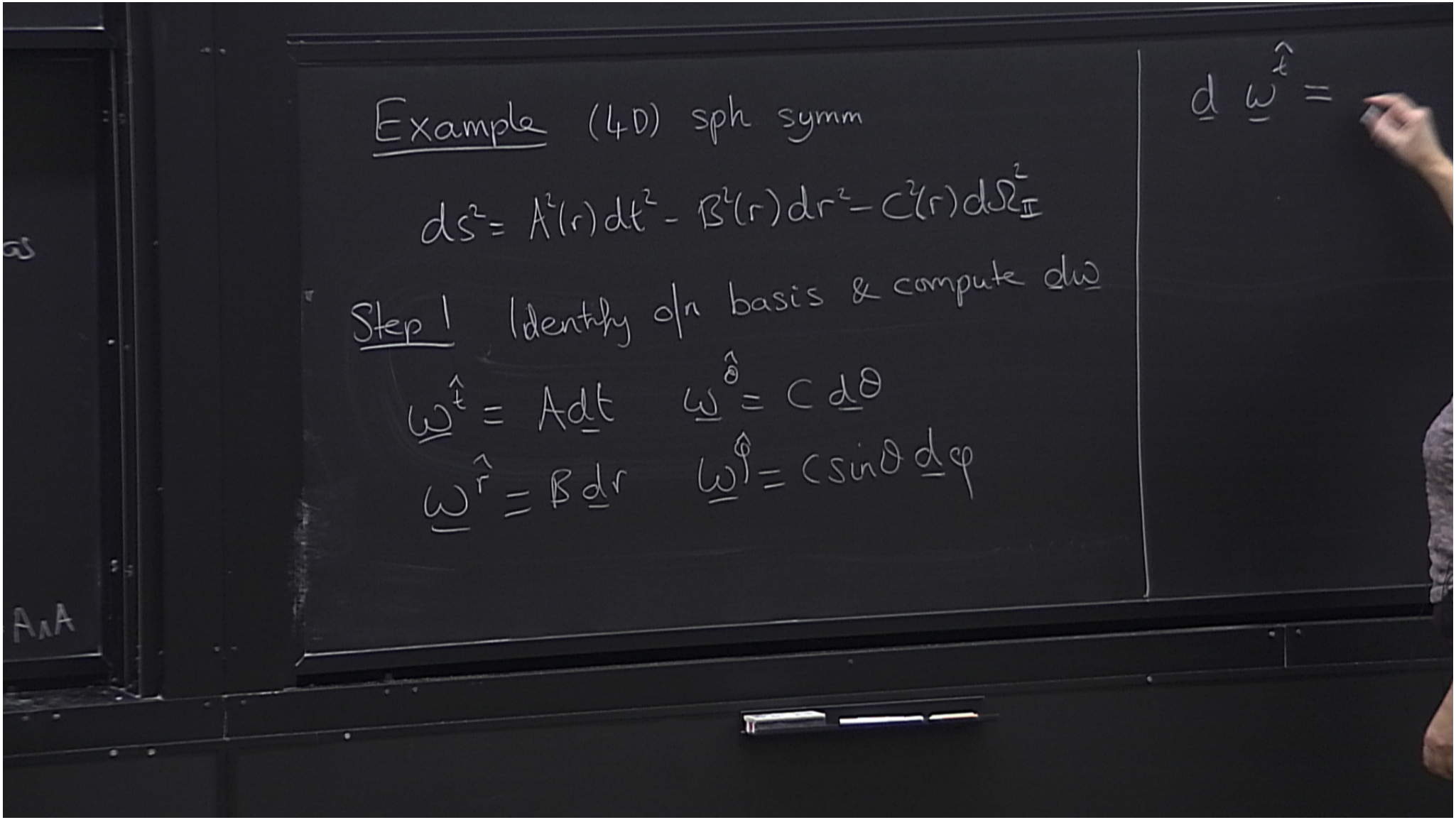
$$\underline{R}^a{}_b = \underline{d}\,\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$$

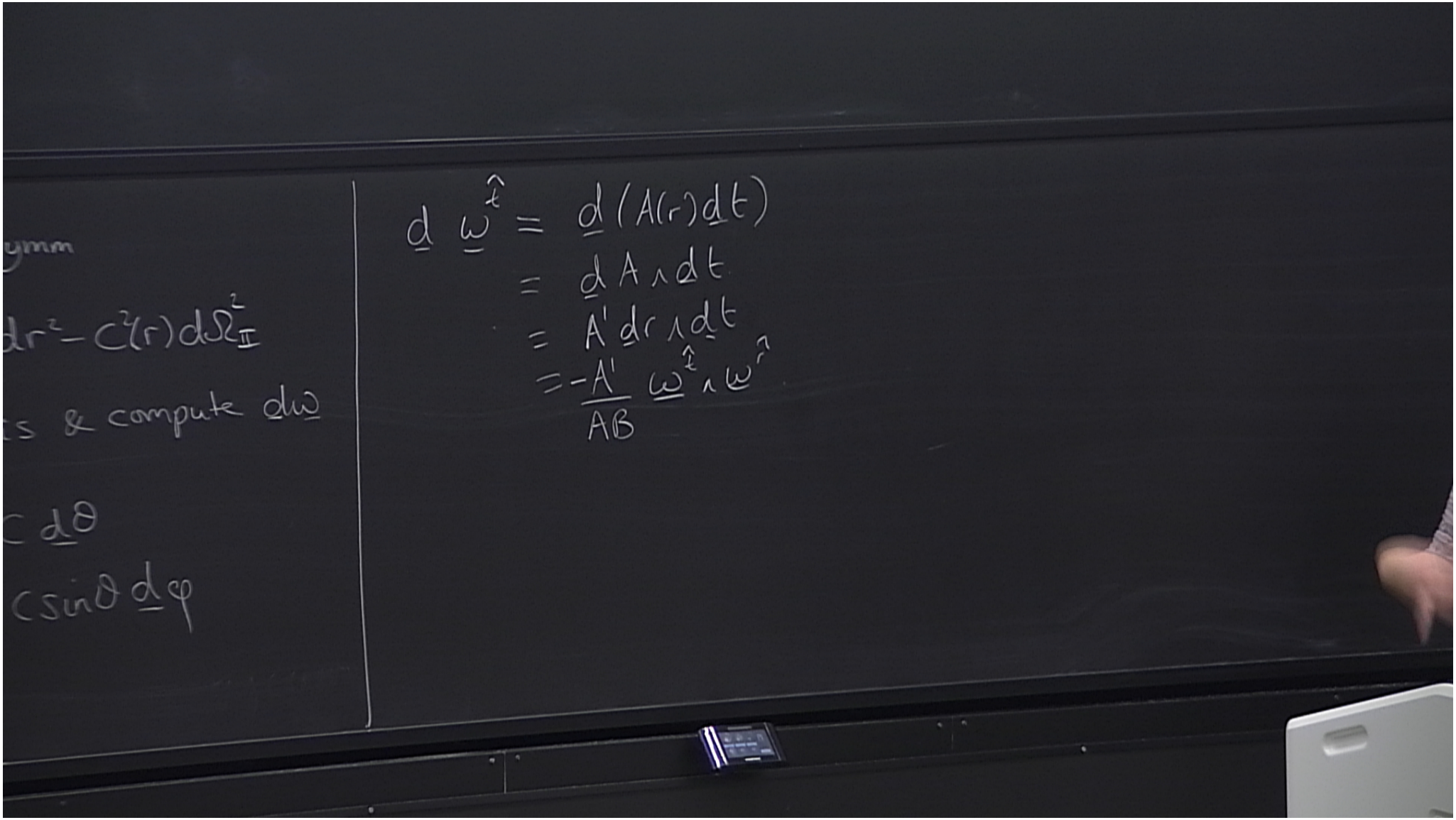
c.f $F_{\mu\nu} = [D_\mu, D_\nu]\Phi \hookrightarrow dA + A \wedge A$

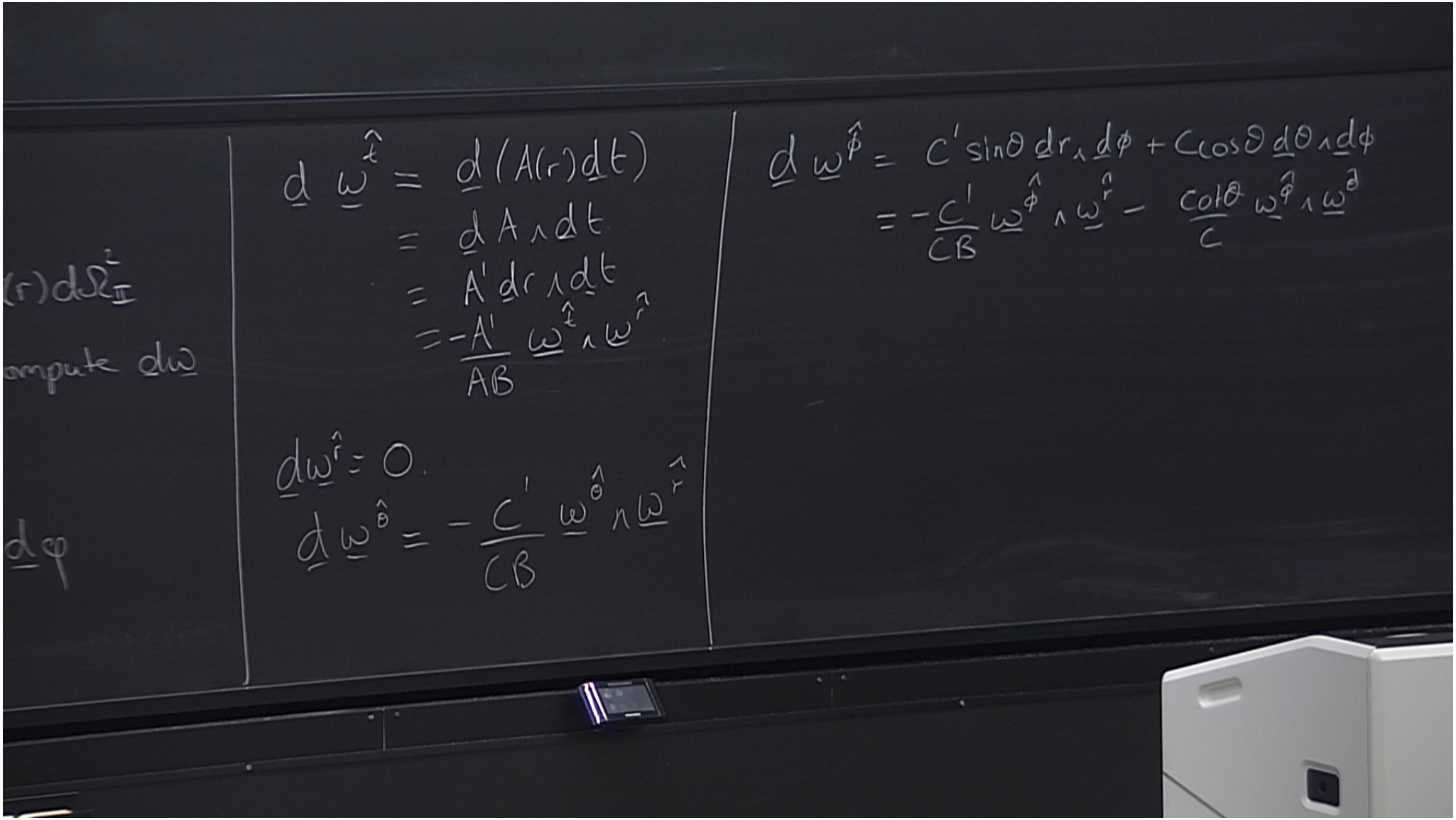

Example (4D) sph symm
$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - C^2(r)d\Omega_{II}^2$
tep 1


Example (4D) sph symm
$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - C^2(r)d\Omega_{II}^2$
Step 1 Identify o/n basis & compute $d\omega$


Example (4D) sph symm
$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - C^2(r)d\Omega_{II}^2$
Step 1 Identify o/n basis & compute $d\omega$
$\omega^{\hat{t}} = A\,dt$ $\omega^{\hat{\theta}} = C\,d\theta$
$\omega^{\hat{r}} = B\,dr$ $\omega^{\hat{\varphi}} = C\sin\theta\,d\varphi$


Example (4D) sph symm
$ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - C^2(r)d\Omega_{II}^2$
Step 1 Identify o/n basis & compute $d\underline{\omega}$
$\underline{\omega}^{\hat{t}} = A\underline{d}t$ $\underline{\omega}^{\hat{\theta}} = C\underline{d}\theta$
$\underline{\omega}^{\hat{r}} = B\underline{d}r$ $\underline{\omega}^{\hat{\varphi}} = C\sin\theta\,\underline{d}\varphi$
$\underline{d}\,\underline{\omega}^{\hat{t}} =$


ymm
$dr^2 - C^2(r)\,d\Omega_{II}^2$
s & compute $d\underline{\omega}$
$C\,\underline{d}\theta$
$C\sin\theta\,\underline{d}\varphi$
$\underline{d}\,\hat{\underline{\omega}}^t = \underline{d}(A(r)\underline{d}t)$
$= \underline{d}A \wedge \underline{d}t$
$= A'\underline{d}r \wedge \underline{d}t$
$= \frac{-A'}{AB}\,\hat{\underline{\omega}}^t \wedge \hat{\underline{\omega}}^r$


$(r)d\Omega_{II}^2$
ompute $d\underline{\omega}$
$d\underline{\varphi}$
$d\,\underline{\omega}^{\hat{t}} = d(A(r)\underline{d}t)$
$= \underline{d}A \wedge \underline{d}t$
$= A'\underline{d}r \wedge \underline{d}t$
$= \frac{-A'}{AB}\underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$
$d\underline{\omega}^{\hat{r}} = 0$
$d\,\underline{\omega}^{\hat{\theta}} = -\frac{C'}{CB}\underline{\omega}^{\hat{\theta}} \wedge \underline{\omega}^{\hat{r}}$
$d\,\underline{\omega}^{\hat{\phi}} = C'\sin\theta\,\underline{d}r \wedge \underline{d}\phi + C\cos\theta\,\underline{d}\theta \wedge \underline{d}\phi$
$= -\frac{C'}{CB}\underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{r}} - \frac{C\cot\theta}{C}\underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{\theta}}$

$= \underline{d}(A(r)\underline{d}t)$

$= \underline{d}A \wedge \underline{d}t$

$= A' \underline{d}r \wedge \underline{d}t$

$= -\frac{A'}{AB}\, \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}}$

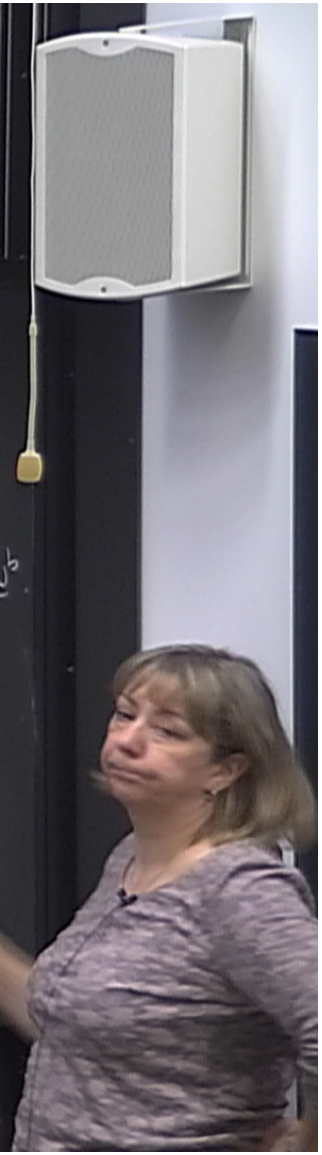
$= -\frac{C'}{CB}\, \underline{\omega}^{\hat{\theta}} \wedge \underline{\omega}^{\hat{r}}$

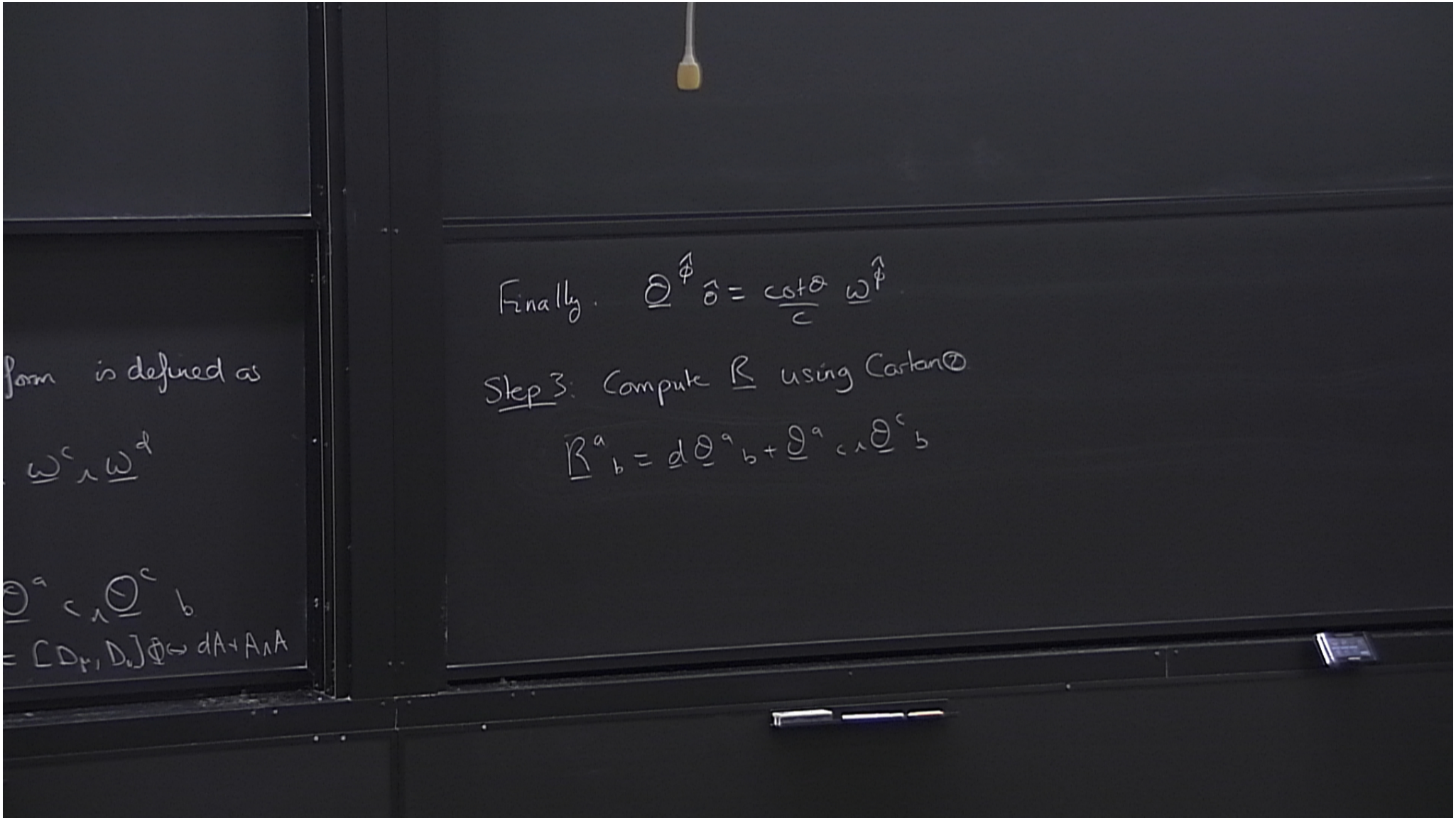
$\underline{d}\,\underline{\omega}^{\hat{\phi}} = C' \sin\theta\, \underline{d}r \wedge \underline{d}\phi + C\cos\theta\, \underline{d}\theta \wedge \underline{d}\phi$

$= -\frac{C'}{CB}\, \underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{r}} - \frac{\cot\theta}{C}\, \underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{\theta}}$

Step 2 Use Cartan ① to identify $\underline{\Theta}^{a}{}_{b}$: $\underline{d}\underline{\omega}^{a} = -\underline{\Theta}^{a}{}_{b} \wedge \underline{\omega}^{b}$.

$\underline{d}\,\underline{\omega}^{\hat{t}} = -\frac{A'}{AB}\, \underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{r}} = -\underline{\Theta}^{\hat{t}}{}_{a} \wedge \underline{\omega}^{a} = -\underline{\Theta}^{\hat{t}}{}_{\hat{r}} \wedge \underline{\omega}^{\hat{r}}$




form is defined as
$\underline{\omega}^c \wedge \underline{\omega}^d$
$\underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$
$= [D_\mu, D_\nu]\Phi \hookrightarrow dA + A \wedge A$
Finally. $\underline{\Theta}^{\hat{\phi}}{}_{\hat{\theta}} = \frac{\cot\theta}{c}\,\underline{\omega}^{\hat{\phi}}$
Step 3. Compute $\underline{R}$ using Cartan
$\underline{R}^a{}_b = d\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$

form is defined as

$\underline{\omega}^c \wedge \underline{\omega}^d$

$\underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$

$= [D_\mu, D_\nu]\Phi \hookrightarrow dA + A \wedge A$

Finally. $\underline{\Theta}^{\hat{\phi}}{}_{\hat{\theta}} = \frac{\cot\theta}{c}\,\underline{\omega}^{\hat{\phi}}$

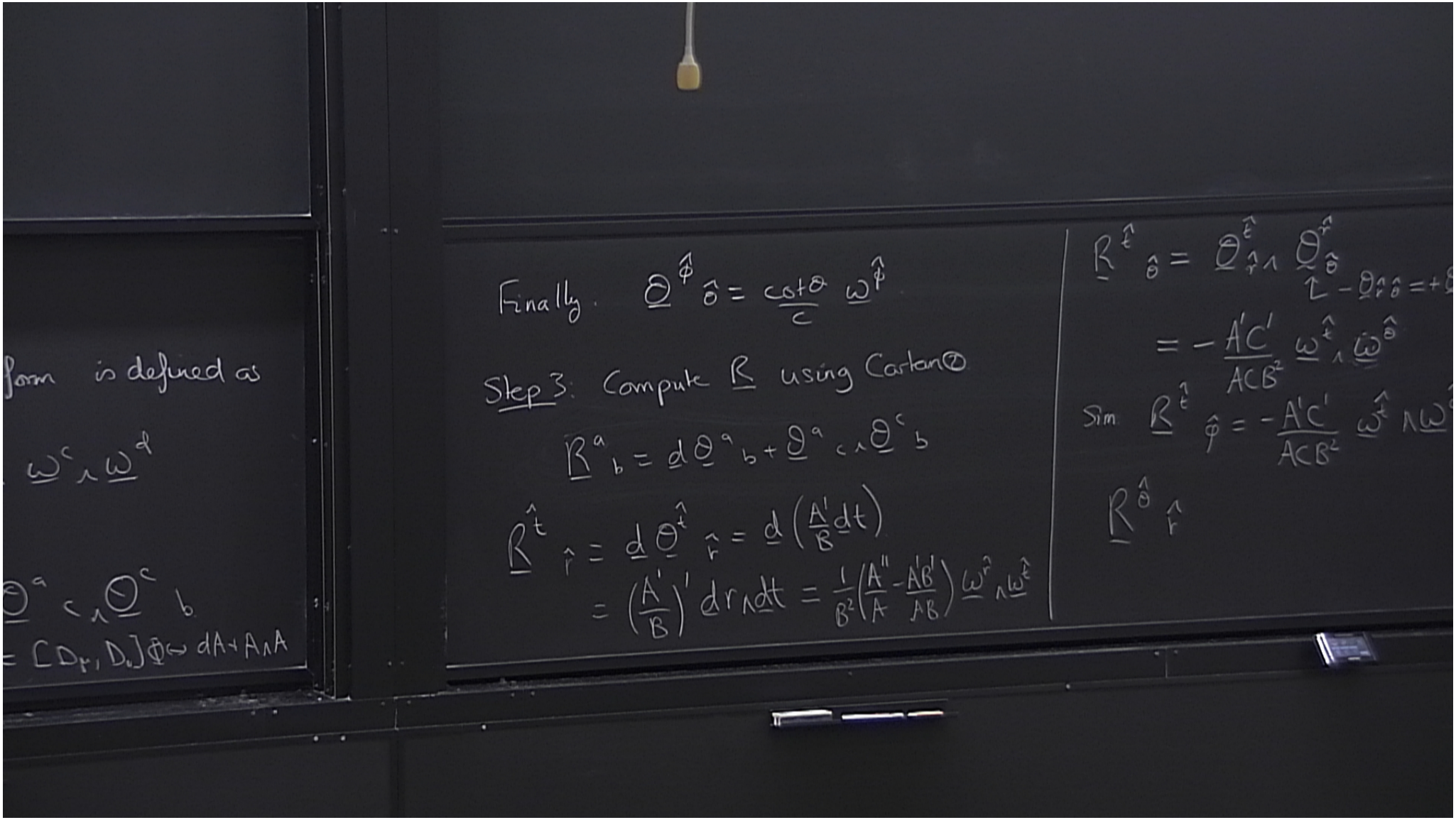
Step 3: Compute $\underline{R}$ using Cartan ②

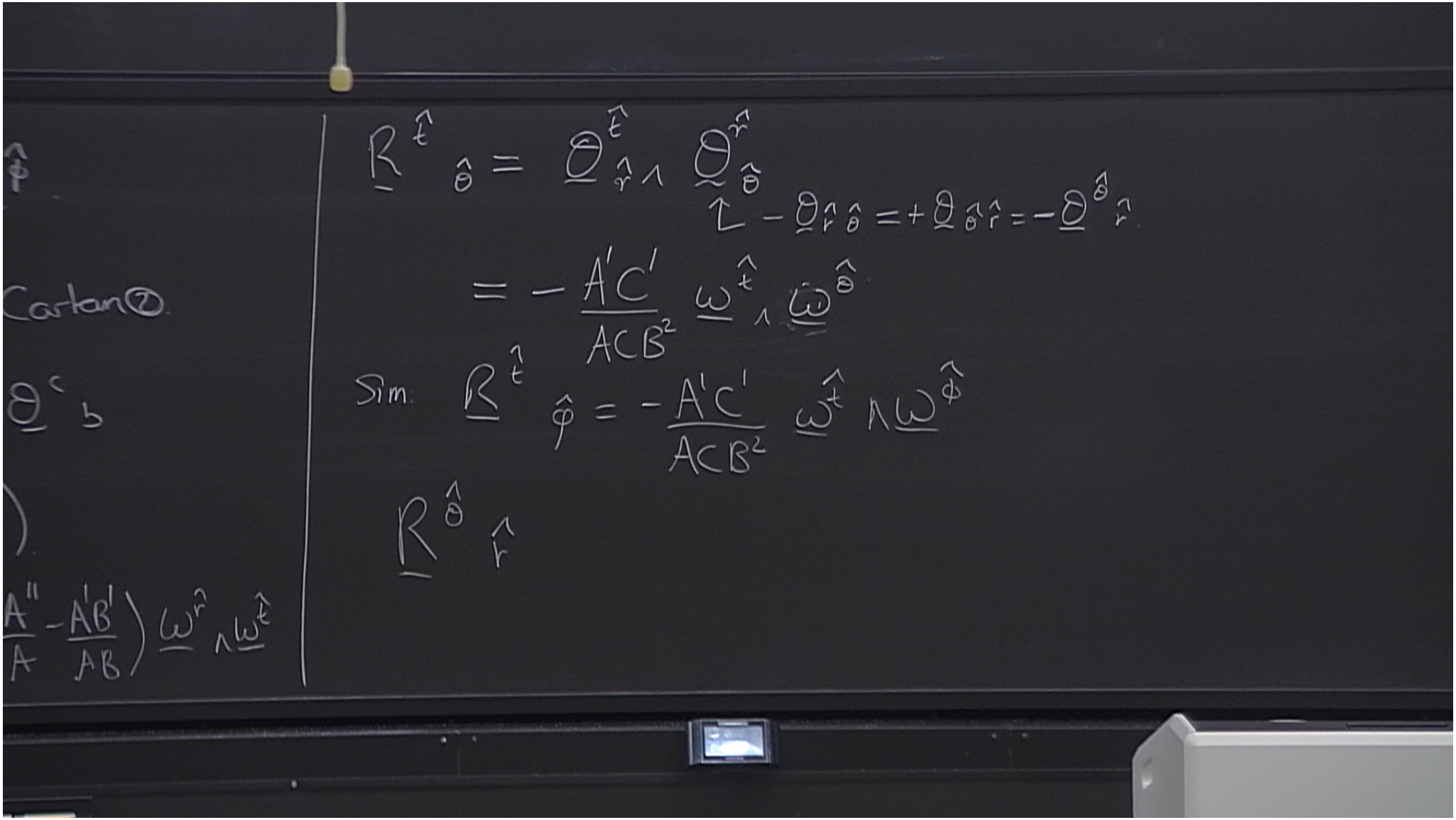
$$\underline{R}^a{}_b = d\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$$

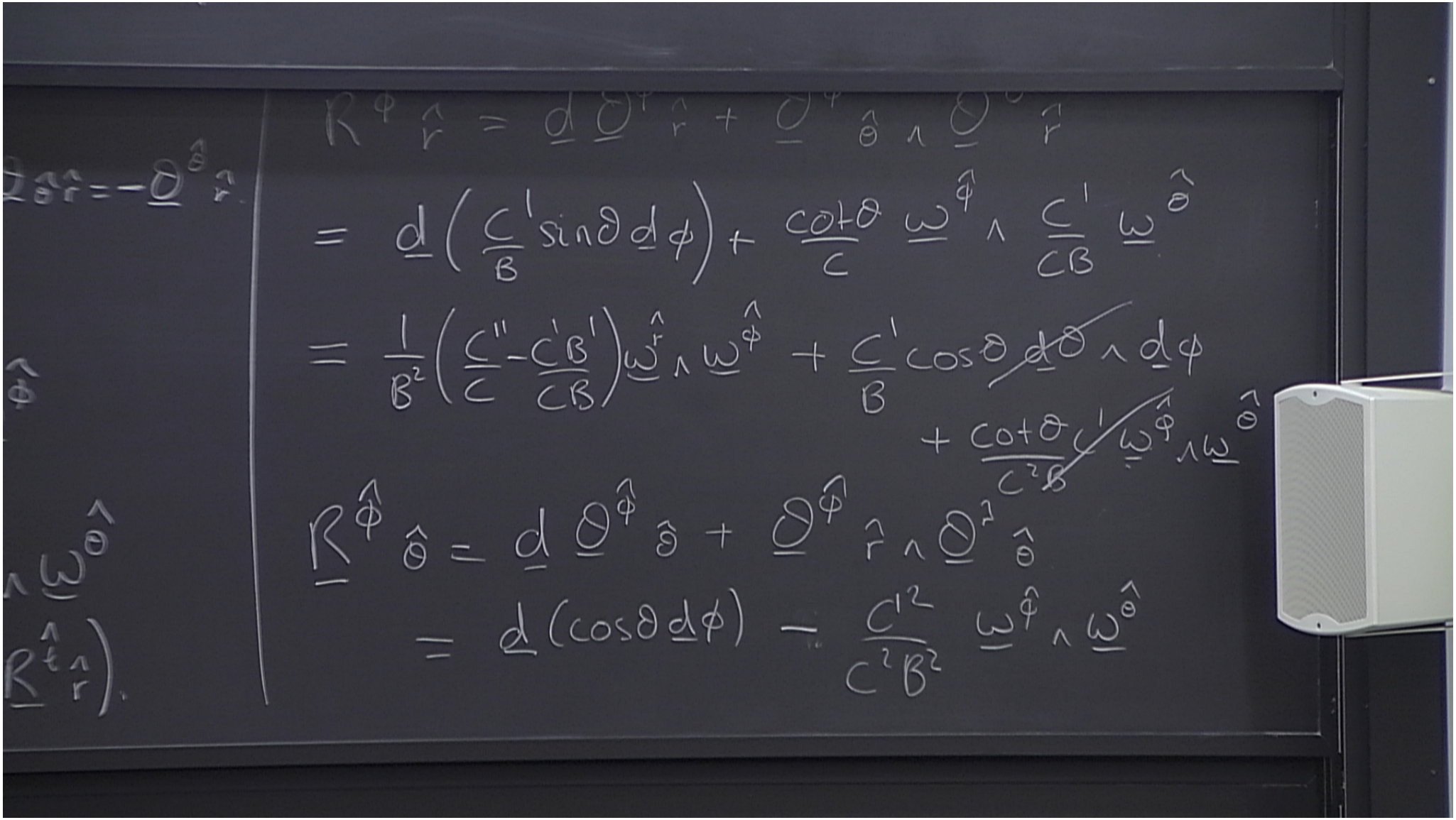
$$\underline{R}^{\hat{t}}{}_{\hat{r}} = d\underline{\Theta}^{\hat{t}}{}_{\hat{r}} = d\left(\frac{A'}{B}dt\right)$$

$$= \left(\frac{A'}{B}\right)' dr \wedge dt = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{t}}$$

$\underline{R}^{\hat{t}}{}_{\hat{\theta}}$


form is defined as
$\underline{\omega}^c \wedge \underline{\omega}^d$
$\underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$
$= [D_\mu, D_\nu]\Phi \hookrightarrow dA + A \wedge A$
Finally. $\underline{\Theta}^{\hat{\phi}}{}_{\hat{\theta}} = \frac{\cot\theta}{c}\,\underline{\omega}^{\hat{\phi}}$
Step 3. Compute $\underline{R}$ using Cartan
$\underline{R}^a{}_b = d\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$
$\underline{R}^{\hat{t}}{}_{\hat{r}} = d\underline{\Theta}^{\hat{t}}{}_{\hat{r}} = d\left(\frac{A'}{B}dt\right)$
$= \left(\frac{A'}{B}\right)' dr \wedge dt = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{t}}$
$\underline{R}^{\hat{t}}{}_{\hat{\theta}} = \underline{\Theta}^{\hat{t}}{}_{\hat{r}} \wedge \underline{\Theta}^{\hat{r}}{}_{\hat{\theta}}$
$= -\frac{A'C'}{ACB^2}\,\underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{\theta}}$
Sim $\underline{R}^{\hat{t}}{}_{\hat{\varphi}} = -\frac{A'C'}{ACB^2}\,\underline{\omega}^{\hat{t}} \wedge \underline{\omega}^{\hat{\varphi}}$
$\underline{R}^{\hat{\theta}}{}_{\hat{r}}$


Cartan②
$R^{\hat{t}}{}_{\hat{\theta}} = \Theta^{\hat{t}}{}_{\hat{r}} \wedge \Theta^{\hat{r}}{}_{\hat{\theta}}$
$\Theta_{\hat{r}\hat{\theta}} = +\Theta_{\hat{\theta}\hat{r}} = -\Theta^{\hat{\theta}}{}_{\hat{r}}$
$= -\frac{A'C'}{ACB^2}\,\omega^{\hat{t}} \wedge \omega^{\hat{\theta}}$
Sim. $R^{\hat{t}}{}_{\hat{\varphi}} = -\frac{A'C'}{ACB^2}\,\omega^{\hat{t}} \wedge \omega^{\hat{\varphi}}$
$R^{\hat{\theta}}{}_{\hat{r}}$
$\frac{A''}{A} - \frac{A'B'}{AB}\Big)\,\omega^{\hat{r}} \wedge \omega^{\hat{t}}$


$R^{\hat{\phi}}{}_{\hat{r}} = \underline{d}\,\underline{\omega}^{\hat{\phi}}{}_{\hat{r}} + \underline{\omega}^{\hat{\phi}}{}_{\hat{\theta}} \wedge \underline{\omega}^{\hat{\theta}}{}_{\hat{r}}$
$= \underline{d}\left(\frac{C'}{B}\sin\theta\,\underline{d}\phi\right) + \frac{\cot\theta}{C}\,\underline{\omega}^{\hat{\phi}} \wedge \frac{C'}{CB}\,\underline{\omega}^{\hat{\theta}}$
$= \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\phi}} + \frac{C'}{B}\cos\theta\,\underline{d}\theta \wedge \underline{d}\phi$
$+ \frac{\cot\theta\, C'}{C^2 B}\,\underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{\theta}}$
$\underline{R}^{\hat{\phi}}{}_{\hat{\theta}} = \underline{d}\,\underline{\omega}^{\hat{\phi}}{}_{\hat{\theta}} + \underline{\omega}^{\hat{\phi}}{}_{\hat{r}} \wedge \underline{\omega}^{\hat{r}}{}_{\hat{\theta}}$
$= \underline{d}(\cos\theta\,\underline{d}\phi) - \frac{C'^2}{C^2 B^2}\,\underline{\omega}^{\hat{\phi}} \wedge \underline{\omega}^{\hat{\theta}}$

$$= \left(\frac{A}{B}\right)' dr \wedge dt = \frac{1}{B^2}\left(\frac{A'}{A} - \frac{AB'}{AB}\right) \underline{\omega}^r \wedge \underline{\omega}^t$$

$$\underline{\Omega}^{\hat{\phi}}{}_{\hat{\theta}} = \left(\frac{1}{c^2} - \frac{c^{12}}{c^2 B^2}\right) \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{\theta}}$$

Step 4 Extract Riemann

$$\underline{\underline{R}}^a{}_b = \frac{1}{2} R^a{}_{bcd}\, \omega^c \wedge \omega^d$$

Read off: $R^{\hat{t}}{}_{\hat{r}\hat{r}\hat{t}} = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)$

$R^{\hat{t}}{}_{\hat{\theta}\hat{t}\hat{\theta}} =$

$R^{\hat{\theta}}{}_{\hat{r}\hat{r}\hat{\theta}} =$

$R^{\hat{\phi}}{}_{\hat{\theta}\hat{\phi}\hat{\theta}} =$

$\wedge A \wedge A$

$$= \frac{1}{B^2}\left(\frac{A'}{A} - \frac{AB'}{AB}\right) \underline{\omega}^{\hat{r}} \wedge \underline{\omega}^{\hat{t}}$$

$$(\text{of } \underline{R}^{\hat{t}}{}_{\hat{r}}).$$

$$\underline{\omega}^{\hat{\beta}} \wedge \underline{\omega}^{\hat{\theta}}$$

Riemann

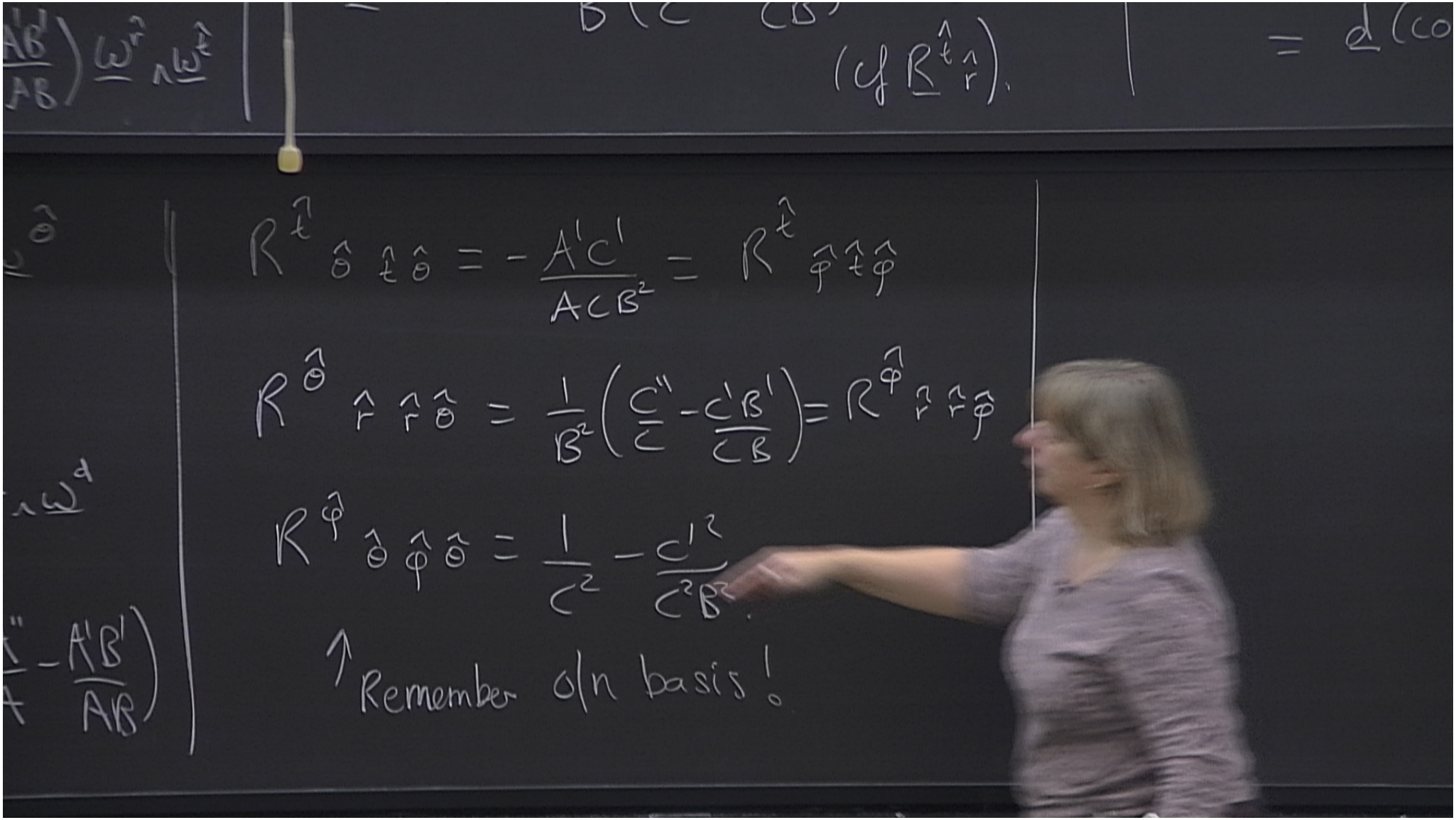
$$R^{a}{}_{bcd}\, \underline{\omega}^{c} \wedge \underline{\omega}^{d}$$

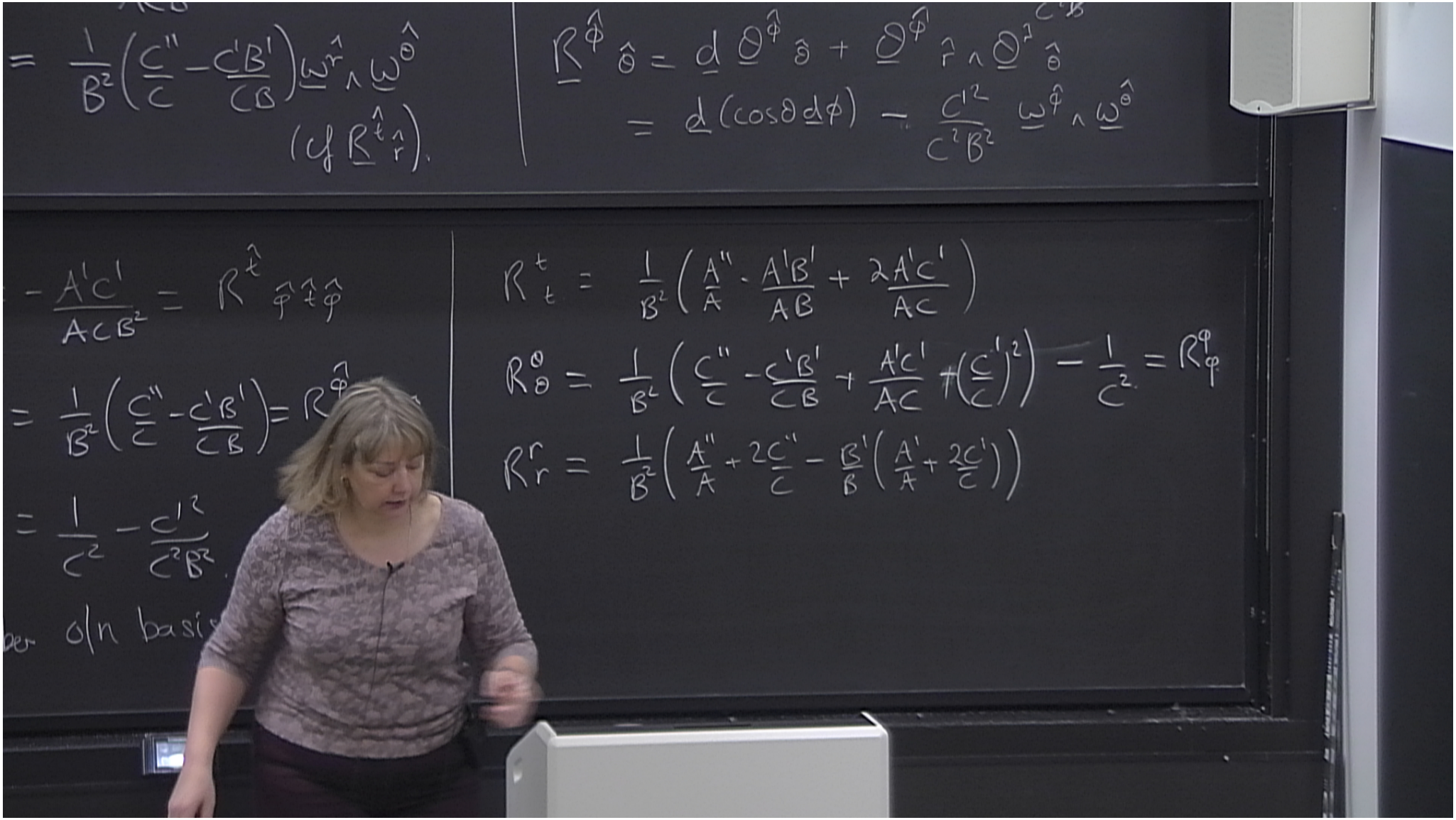
$$\hat{t} = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB}\right)$$

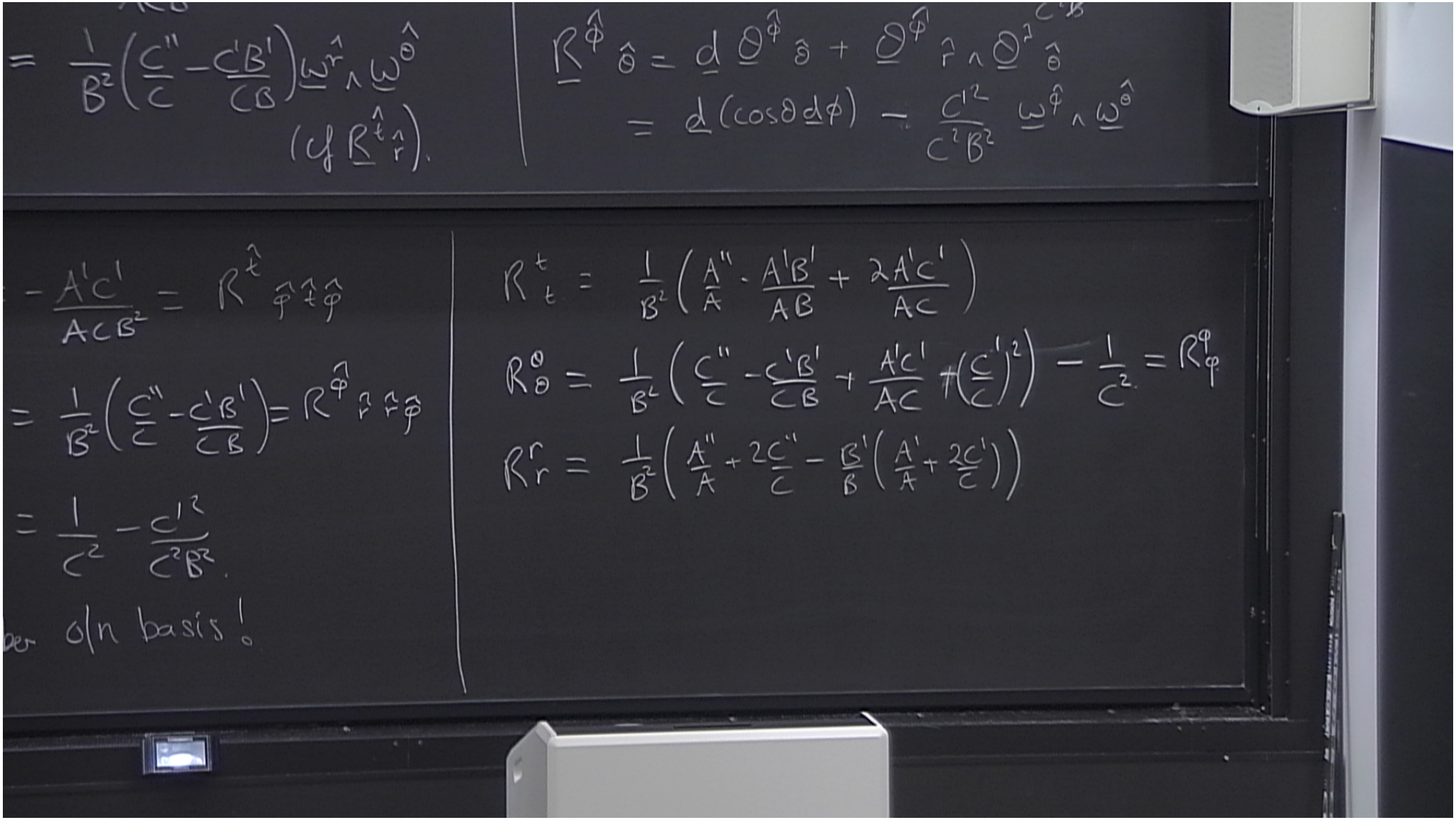
$$R^{\hat{t}}{}_{\hat{\theta}\hat{t}\hat{\theta}} = -\frac{A'C'}{ACB^2} = R^{\hat{t}}{}_{\hat{\varphi}\hat{t}\hat{\varphi}}$$

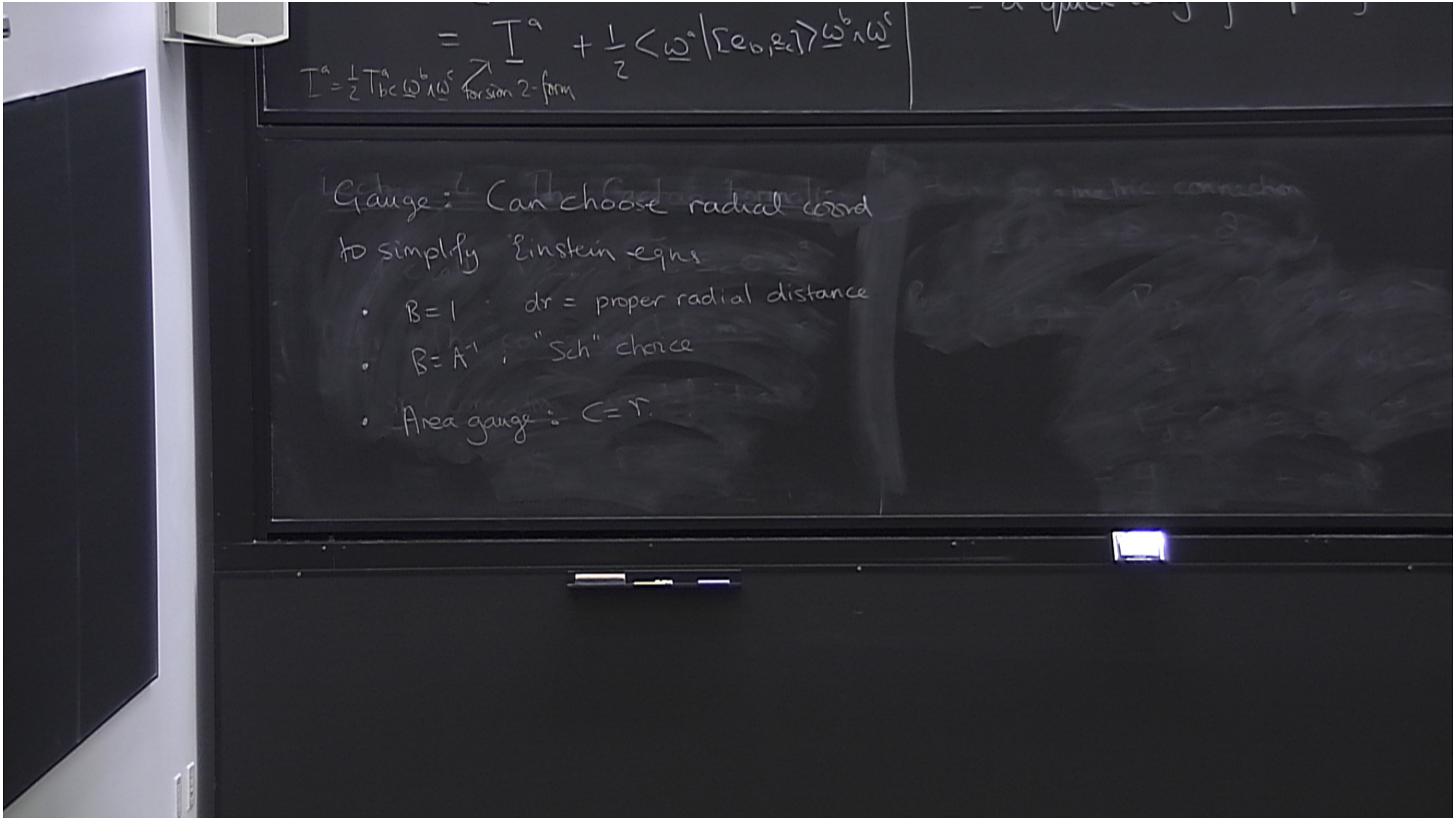
$$R^{\hat{\theta}}{}_{\hat{r}\hat{r}\hat{\theta}} = \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right) = R^{\hat{\varphi}}{}_{\hat{r}\hat{r}\hat{\varphi}}$$

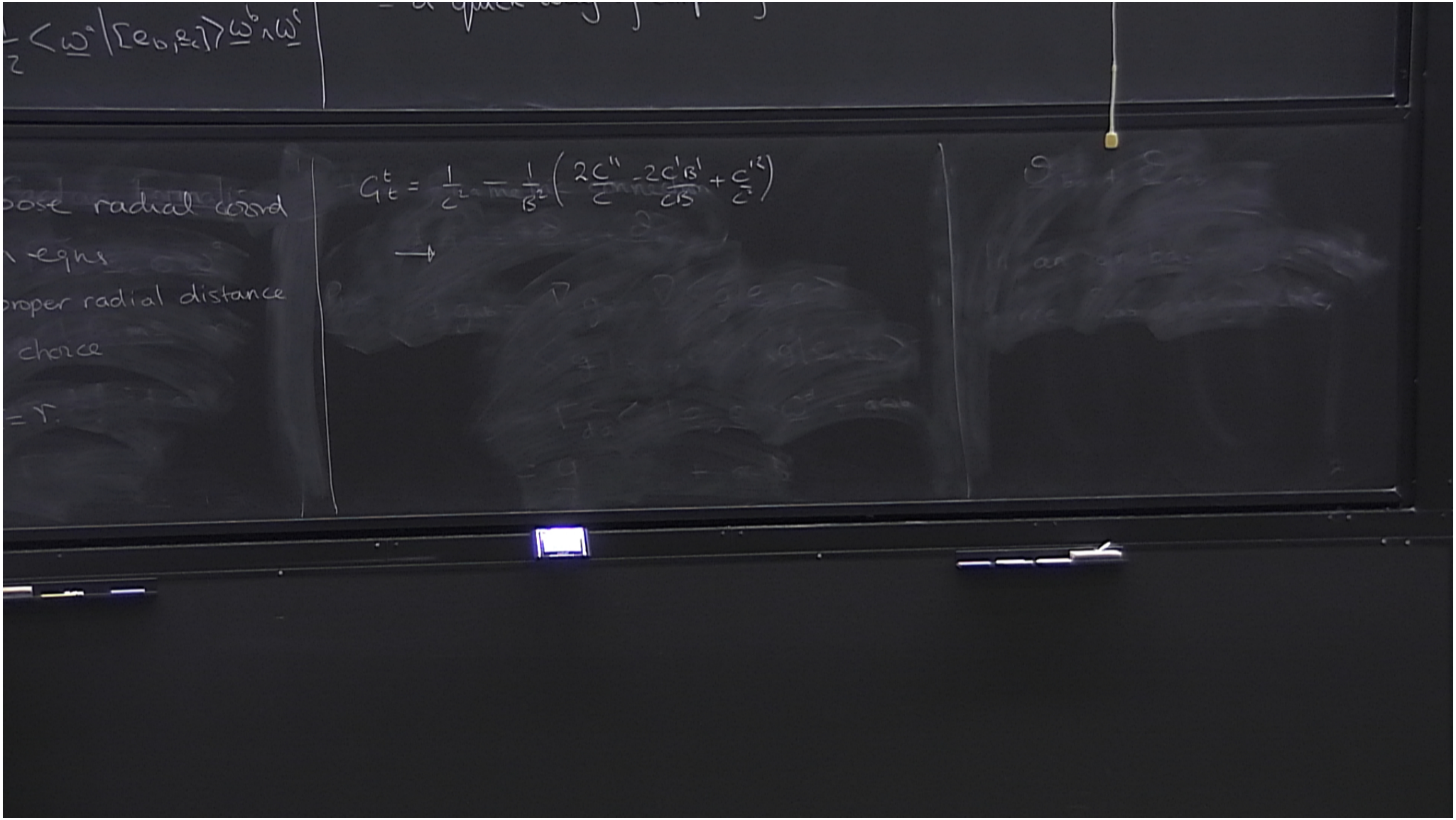
$$R^{\hat{\varphi}}{}_{\hat{\theta}\hat{\varphi}\hat{\theta}} = \frac{1}{C^2} - \frac{C'^2}{C^2B^2}.$$

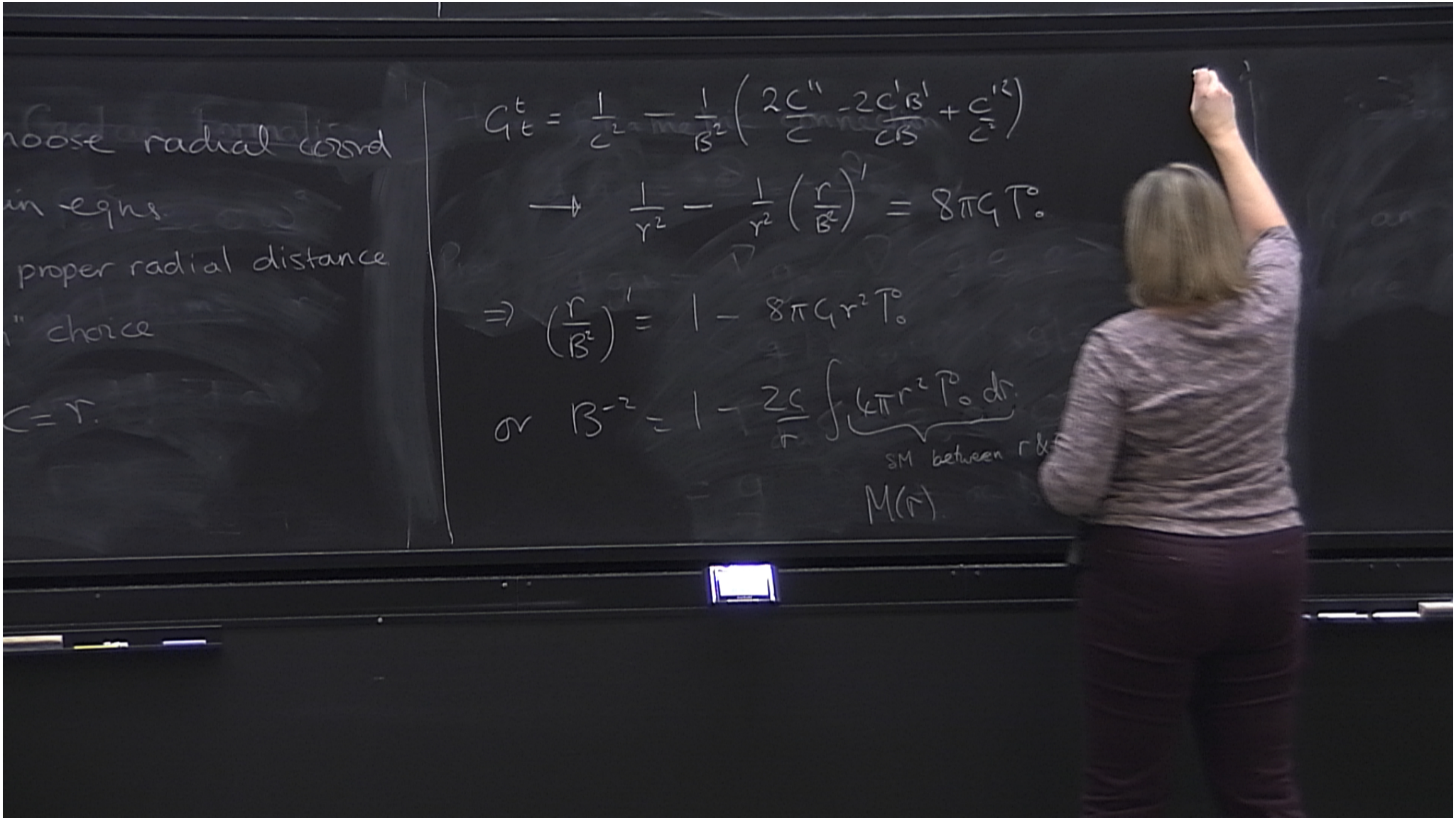

$R^{\hat{t}}{}_{\hat{\theta}\hat{t}\hat{\theta}} = -\frac{A'C'}{ACB^2} = R^{\hat{t}}{}_{\hat{\varphi}\hat{t}\hat{\varphi}}$
$R^{\hat{\theta}}{}_{\hat{r}\hat{r}\hat{\theta}} = \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right) = R^{\hat{\varphi}}{}_{\hat{r}\hat{r}\hat{\varphi}}$
$R^{\hat{\varphi}}{}_{\hat{\theta}\hat{\varphi}\hat{\theta}} = \frac{1}{C^2} - \frac{C'^2}{C^2B^2}$
↑ Remember o/n basis!

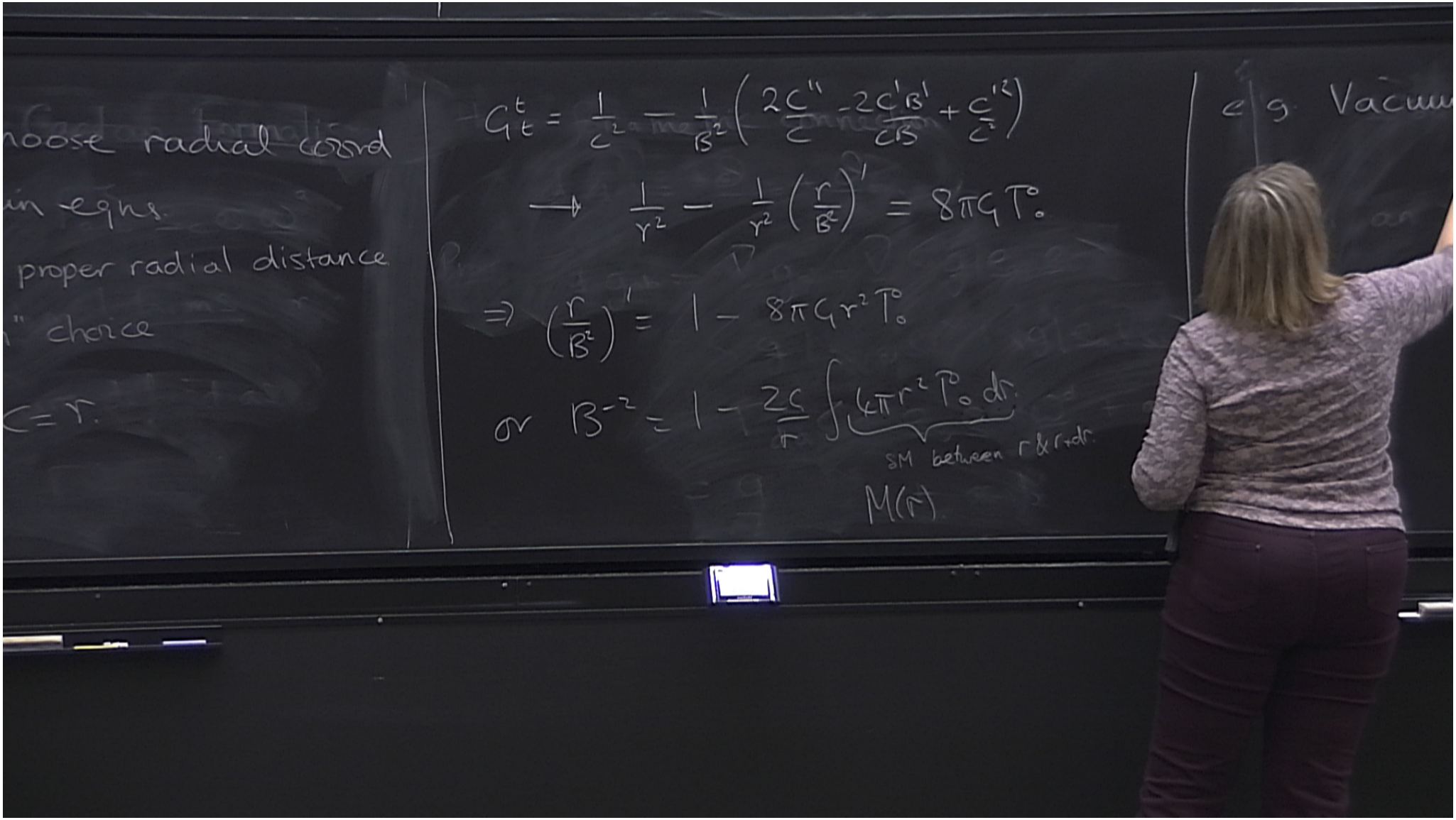

$= \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\omega^{\hat{r}} \wedge \omega^{\hat{\theta}}$
$(\text{of } R^{\hat{t}}{}_{\hat{r}})$.
$R^{\hat{\phi}}{}_{\hat{\theta}} = d\omega^{\hat{\phi}}{}_{\hat{\theta}} + \omega^{\hat{\phi}}{}_{\hat{r}} \wedge \omega^{\hat{r}}{}_{\hat{\theta}}$
$= d(\cos\theta\, d\phi) - \frac{C'^2}{C^2B^2}\omega^{\hat{\phi}} \wedge \omega^{\hat{\theta}}$
$-\frac{A'C'}{ACB^2} = R^{\hat{t}}{}_{\hat{\varphi}\hat{t}\hat{\varphi}}$
$= \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right) = R^{\hat{\varphi}}$
$= \frac{1}{C^2} - \frac{C'^2}{C^2B^2}$
$R^t{}_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + 2\frac{A'C'}{AC}\right)$
$R^\theta{}_\theta = \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB} + \frac{A'C'}{AC} + \left(\frac{C'}{C}\right)^2\right) - \frac{1}{C^2} = R^\varphi{}_\varphi$
$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + 2\frac{C''}{C} - \frac{B'}{B}\left(\frac{A'}{A} + 2\frac{C'}{C}\right)\right)$


$= \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right)\omega^{\hat{r}} \wedge \omega^{\hat{0}}$
$(= R^{\hat{t}}{}_{\hat{r}\hat{t}\hat{r}})$.
$R^{\hat{\phi}}{}_{\hat{\theta}} = d\theta^{\hat{\phi}}{}_{\hat{\theta}} + \theta^{\hat{\phi}}{}_{\hat{r}} \wedge \theta^{\hat{r}}{}_{\hat{\theta}}$
$= d(\cos\theta\, d\phi) - \frac{C'^2}{C^2B^2}\omega^{\hat{\phi}} \wedge \omega^{\hat{\theta}}$
$-\frac{A'C'}{ACB^2} = R^{\hat{t}}{}_{\hat{\phi}\hat{t}\hat{\phi}}$
$= \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB}\right) = R^{\hat{\theta}}{}_{\hat{r}\hat{\theta}\hat{r}}$
$= \frac{1}{C^2} - \frac{C'^2}{C^2B^2}$.
der o/n basis!
$R^t{}_t = \frac{1}{B^2}\left(\frac{A''}{A} - \frac{A'B'}{AB} + \frac{2A'C'}{AC}\right)$
$R^\theta{}_\theta = \frac{1}{B^2}\left(\frac{C''}{C} - \frac{C'B'}{CB} + \frac{A'C'}{AC} + \left(\frac{C'}{C}\right)^2\right) - \frac{1}{C^2} = R^\varphi{}_\varphi$
$R^r{}_r = \frac{1}{B^2}\left(\frac{A''}{A} + \frac{2C''}{C} - \frac{B'}{B}\left(\frac{A'}{A} + \frac{2C'}{C}\right)\right)$


$= \underline{T}^a + \frac{1}{2} \langle \underline{\omega}^a | [e_b, e_c] \rangle \underline{\omega}^b \wedge \underline{\omega}^c$
$\underline{T}^a = \frac{1}{2} T^a_{bc} \underline{\omega}^b \wedge \underline{\omega}^c$ torsion 2-form
Gauge: Can choose radial coord
to simplify Einstein eqns
• $B = 1$ dr = proper radial distance
• $B = A^{-1}$, "Sch" choice
• Area gauge: $C = r$.


$\frac{1}{2}\langle\omega^a|[e_b,e_c]\rangle\underline{\omega}^b\wedge\underline{\omega}^c$
$G^t_t = \frac{1}{c^2} - \frac{1}{B^2}\left(\frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2}\right)$
$\longrightarrow$
pose radial coord
eqns
proper radial distance
choice
$= r.$


hoose radial coord
in eqns
proper radial distance
" choice
C = r.
$G^t_t = \frac{1}{c^2} - \frac{1}{B^2}\left(\frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2}\right)$
$\longrightarrow \frac{1}{r^2} - \frac{1}{r^2}\left(\frac{r}{B^2}\right)' = 8\pi G T^0_0$
$\Rightarrow \left(\frac{r}{B^2}\right)' = 1 - 8\pi G r^2 T^0_0$
or $B^{-2} = 1 - \frac{2G}{r}\int \underbrace{4\pi r^2 T^0_0 dr}$
SM between r &
M(r)


hoose radial coord
in eqns
proper radial distance
" choice
C = r.
$G^t_t = \frac{1}{c^2} - \frac{1}{B^2}\left(\frac{2C''}{C} - \frac{2C'B'}{CB} + \frac{C'^2}{C^2}\right)$
$\rightarrow \frac{1}{r^2} - \frac{1}{r^2}\left(\frac{r}{B^2}\right)' = 8\pi G T^0_0$
$\Rightarrow \left(\frac{r}{B^2}\right)' = 1 - 8\pi G r^2 T^0_0$
or $B^{-2} = 1 - \frac{2C}{r}\int 4\pi r^2 T^0_0 dr$
SM between r & r+dr
M(r)
e.g. Vacuu

M(Γ)

cpts

$$R^a{}_{bcd} = \partial_c \Gamma^a_{bd} - \partial_d \Gamma^a_{bc}$$
$$+ \Gamma^a_{ce}\Gamma^e_{ab} - \Gamma^a_{de}\Gamma^e_{cb}$$
$$- C^e_{cd}\Gamma^a_{eb}$$

ysically, curvature describes
tidal forces.

Cartan's 2nd eqn

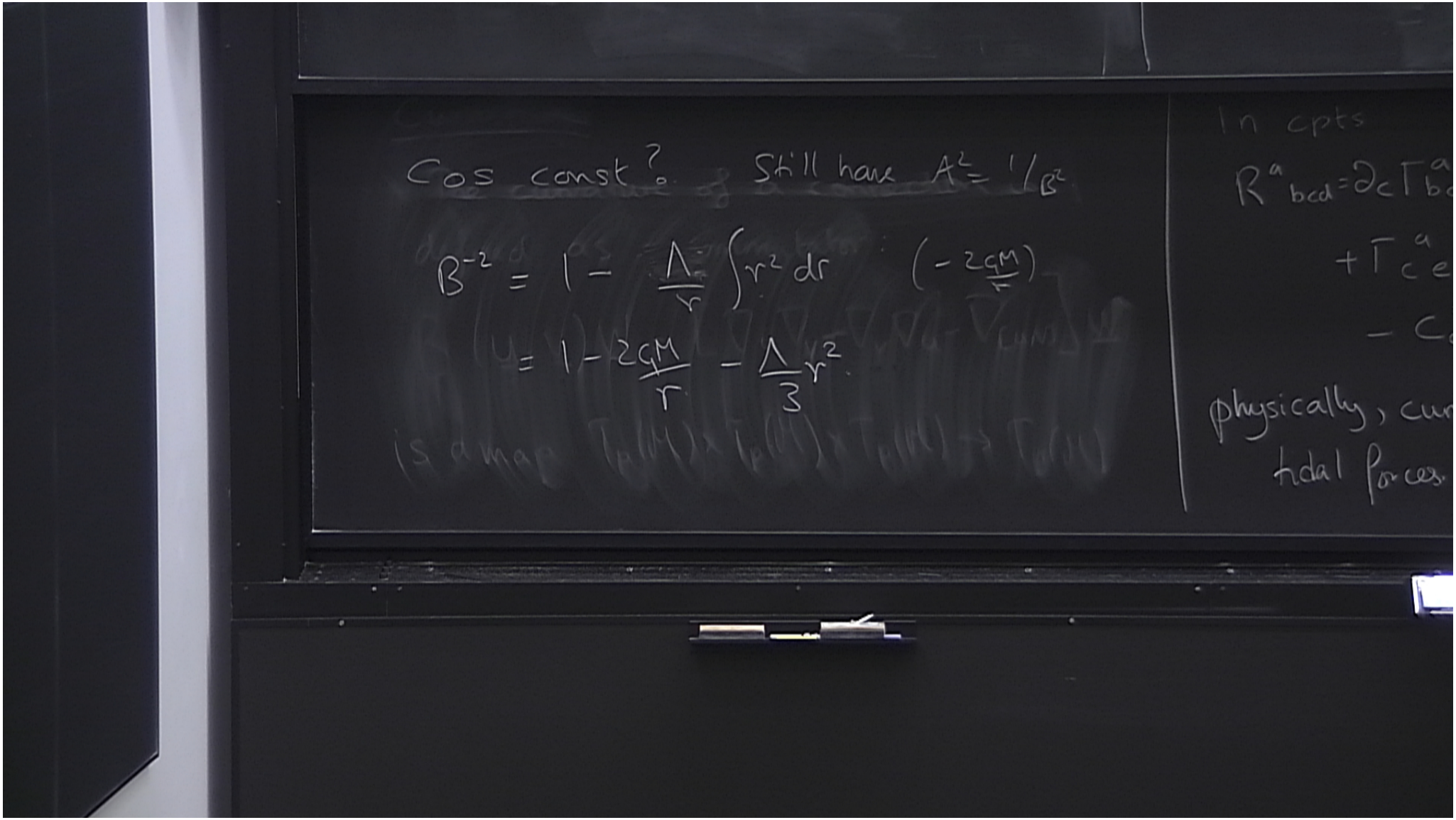
Defn The curvature 2-form is defined as

$$\underline{R}^a{}_b = \frac{1}{2} R^a{}_{bcd}\, \underline{\omega}^c \wedge \underline{\omega}^d$$

Then Cartan's 2nd eqn is

$$\underline{R}^a{}_b = \underline{d}\,\underline{\Theta}^a{}_b + \underline{\Theta}^a{}_c \wedge \underline{\Theta}^c{}_b$$

c.f $F_{\mu\nu} = [D_\mu, D_\nu]\,\phi \sim dA + A \wedge A$


Cos const? Still have $A^2 = 1/B^2$
$B^{-2} = 1 - \frac{\Lambda}{r}\int r^2 dr \quad (-\frac{2GM}{r})$
$= 1 - \frac{2GM}{r} - \frac{\Lambda}{3}r^2$
In cpts
$R^a{}_{bcd} = \partial_c \Gamma^a_{bd}$
$+ \Gamma^a_{ce}$
$- C$
physically, cur
tidal forces.